Challenges and achievements in the ab-initio three- and four-body scattering calculations: the Coulomb force

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In collaboration with A. C. Fonseca and P. U. Sauer

Outline

- Few-body scattering with Coulomb interaction: screening and renormalization

- Applications: 3N, 4N, ...

Reactions with charged particles

- Experimental data:
  higher quality and quantity for
  ppn, ppnn, pppn systems

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$$\swarrow \qquad \searrow$$

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- analytical solution of two-body Coulomb problem

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- in the $R \to \infty$ limit physical results are recovered

Screening and renormalization

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Renormalization of the on-shell screened Coulomb transition matrix $T_R = w_R + w_R G_0 T_R$ and wave function

$$T_R z_R^{-1} \xrightarrow[R\to\infty]{} T_C \quad \text{as distribution}$$

$$(1 + G_0 T_R)|\mathbf{p}\rangle z_R^{-\frac{1}{2}} \xrightarrow[R\to\infty]{} |\psi_C^{(+)}(\mathbf{p})\rangle$$

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$$z_R \xrightarrow[R\to\infty]{} \exp(-2i(\sigma_L - \eta_{LR})) \xrightarrow[R\to\infty]{} \exp(-2i\alpha M/p\,[\ln(2pR) - C/n])$$

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Real experiment:

incoming wave packet $|\varphi_{\mathrm{in}}(\mathbf{p})\rangle$

outgoing wave packet

$$\varphi_{\mathrm{out}}(\mathbf{p}') = \int d^3\mathbf{p}\, \langle \mathbf{p}'|S|\mathbf{p}\rangle \varphi_{\mathrm{in}}(\mathbf{p})$$

$$\sim \int d^2\hat{\mathbf{p}}\, \langle \mathbf{p}'|T|\mathbf{p}\rangle \varphi_{\mathrm{in}}(\mathbf{p})$$

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$T_R z_R^{-1}\Big|_{R\to\infty}$ replace by T_C

Two-particle scattering

pp transition matrix

$$T^{(R)} = v + w_R + (v + w_R) G_0 T^{(R)}$$

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Renormalized amplitude:

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short-range part: fast convergence with R

Three-particle scattering: short-range forces

- Faddeev / Alt, Grassberger, and Sandhas equations

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_{\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$T_{\sigma} = v_{\sigma} + v_{\sigma} G_0 T_{\sigma}$$

$$G_0 = (E + i0 - H_0)^{-1}$$

Three-particle scattering

long-range part

$$T^{\text{c.m.}}_{\alpha R} = W^{\text{c.m.}}_{\alpha R} + W^{\text{c.m.}}_{\alpha R} G^{(R)}_{\alpha} T^{\text{c.m.}}_{\alpha R}$$

Three-particle scattering

Split into long-range part

$$T^{\text{c.m.}}_{\alpha R} = W^{\text{c.m.}}_{\alpha R} + W^{\text{c.m.}}_{\alpha R} G^{(R)}_{\alpha} T^{\text{c.m.}}_{\alpha R}$$

and Coulomb-distorted short-range part

$$U^{(R)}_{\beta\alpha} = \delta_{\beta\alpha} T^{\text{c.m.}}_{\alpha R} + [1 + T^{\text{c.m.}}_{\beta R} G^{(R)}_{\beta}] \tilde{U}^{(R)}_{\beta\alpha} [1 + G^{(R)}_{\alpha} T^{\text{c.m.}}_{\alpha R}]$$

$$U^{(R)}_{0\alpha} = [1 + T_{\rho R} G_0] \tilde{U}^{(R)}_{0\alpha} [1 + G^{(R)}_{\alpha} T^{\text{c.m.}}_{\alpha R}] \quad [\rho \text{ is neutral}]$$

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Renormalized amplitudes:

$$U_{\beta\alpha} = \delta_{\beta\alpha} T^{\text{c.m.}}_{\alpha C} + \lim_{R\to\infty} Z_{R\,f}^{-\frac{1}{2}} [U^{(R)}_{\beta\alpha} - \delta_{\beta\alpha} T^{\text{c.m.}}_{\alpha R}] Z_{R\,i}^{-\frac{1}{2}}$$

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Practical realization

- Calculation of short-range part using
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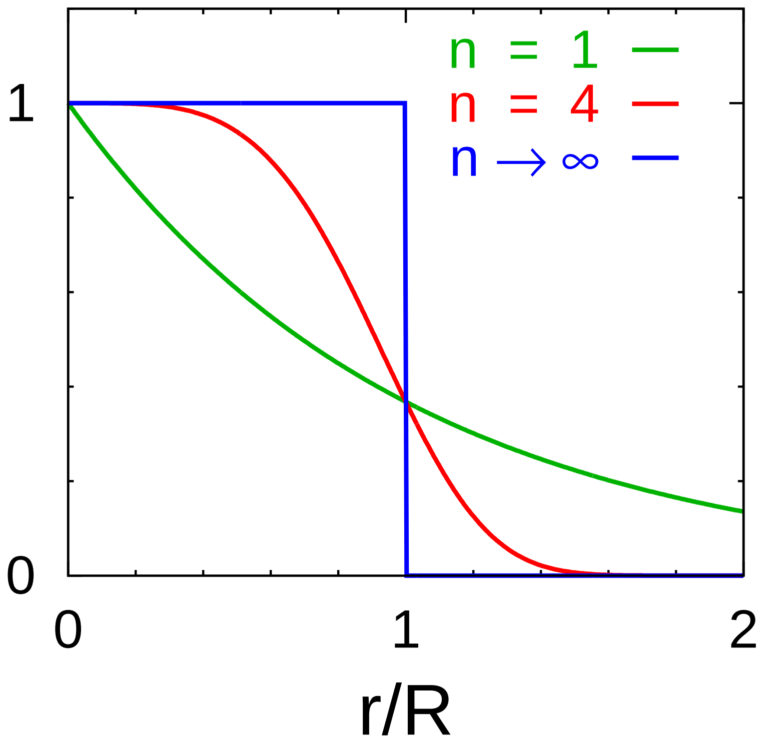
- Success of the method depends strongly on the choice of screening function

Screened Coulomb potential

$$\frac{w_R(r)}{w(r)} = e^{-\left(\frac{r}{R}\right)^{n}}$$



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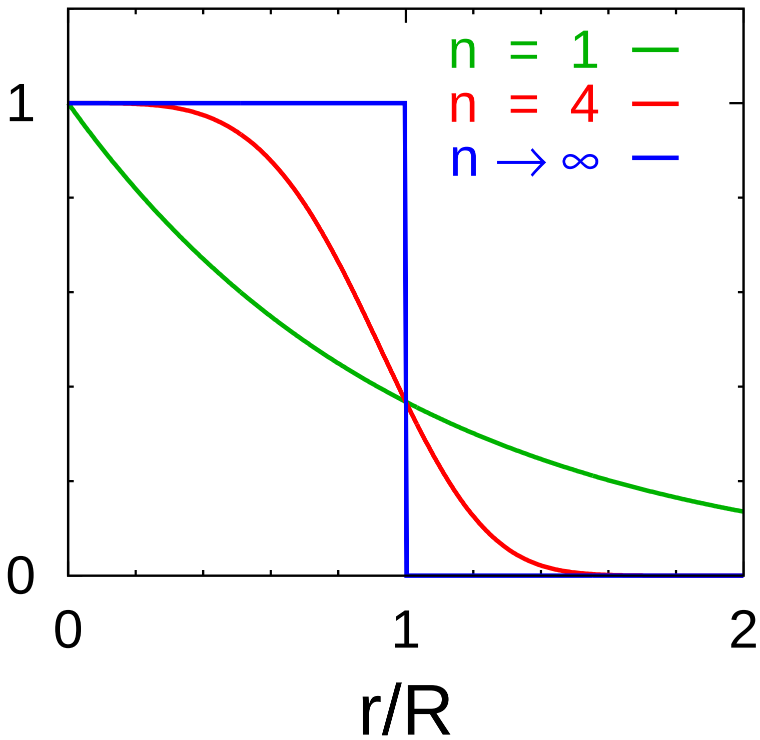

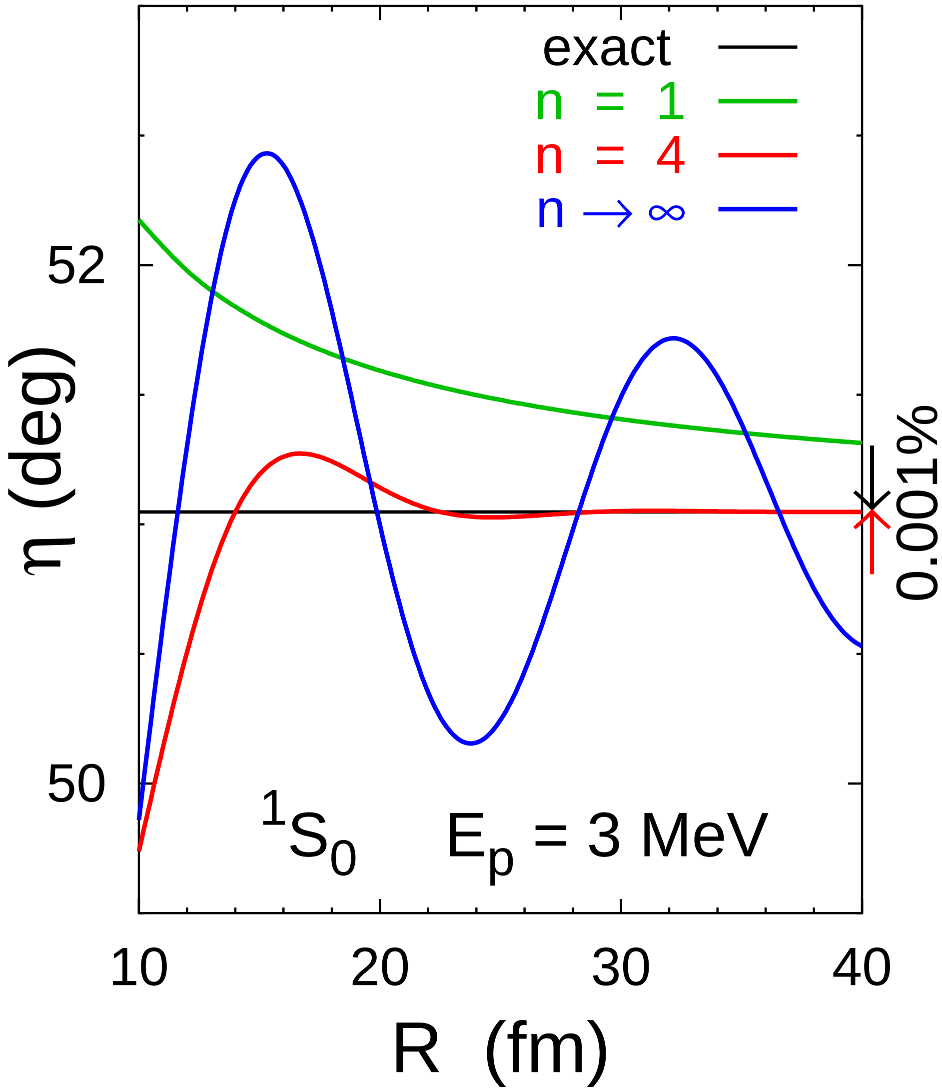

optimal choice: $3 \leq n \leq 8$

Limits of practical applicability

$p \longrightarrow 0$:

$\kappa = \alpha M/p$, $\sigma_L = \arg\Gamma(1 + L + i\kappa)$, and z_R diverge,

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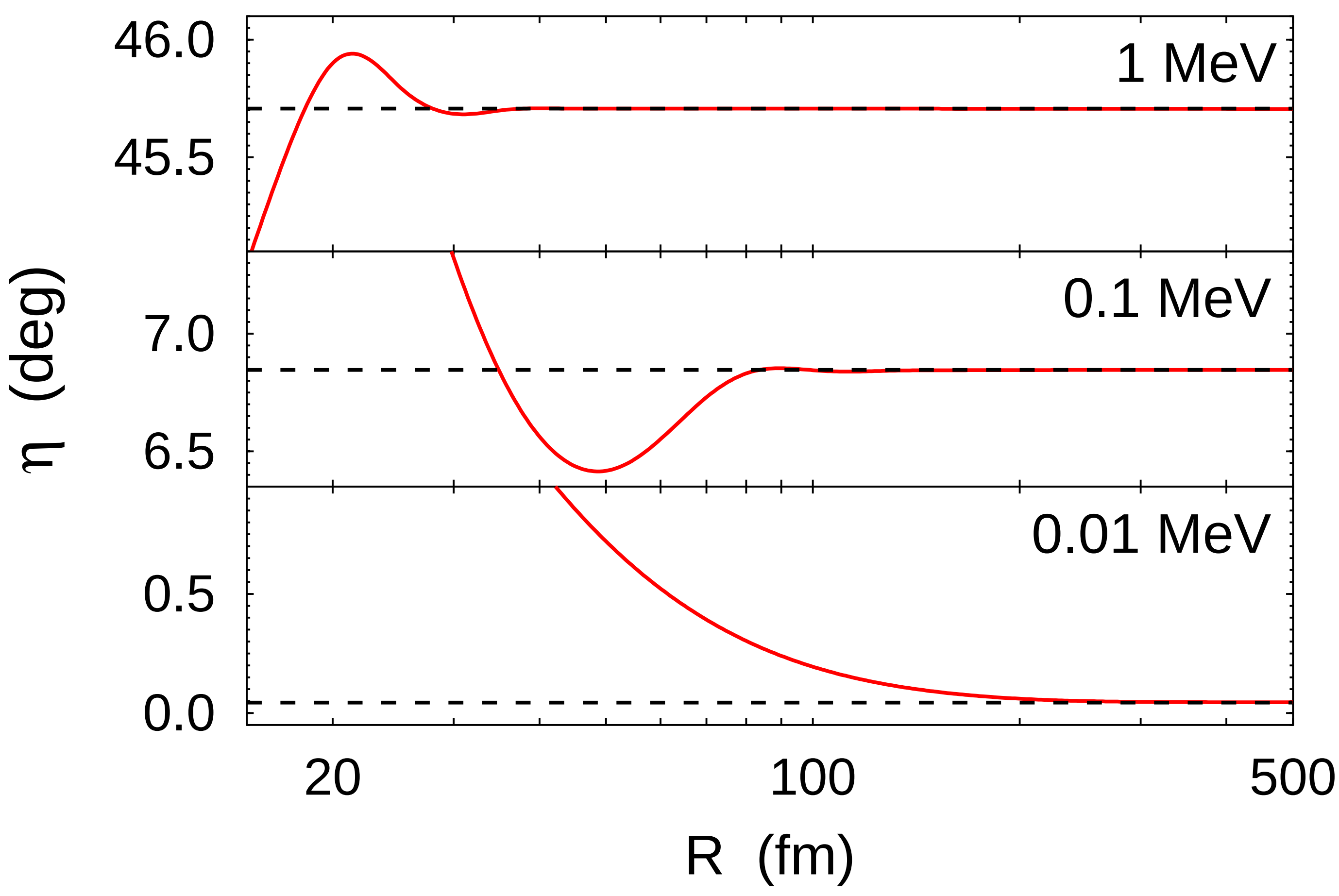
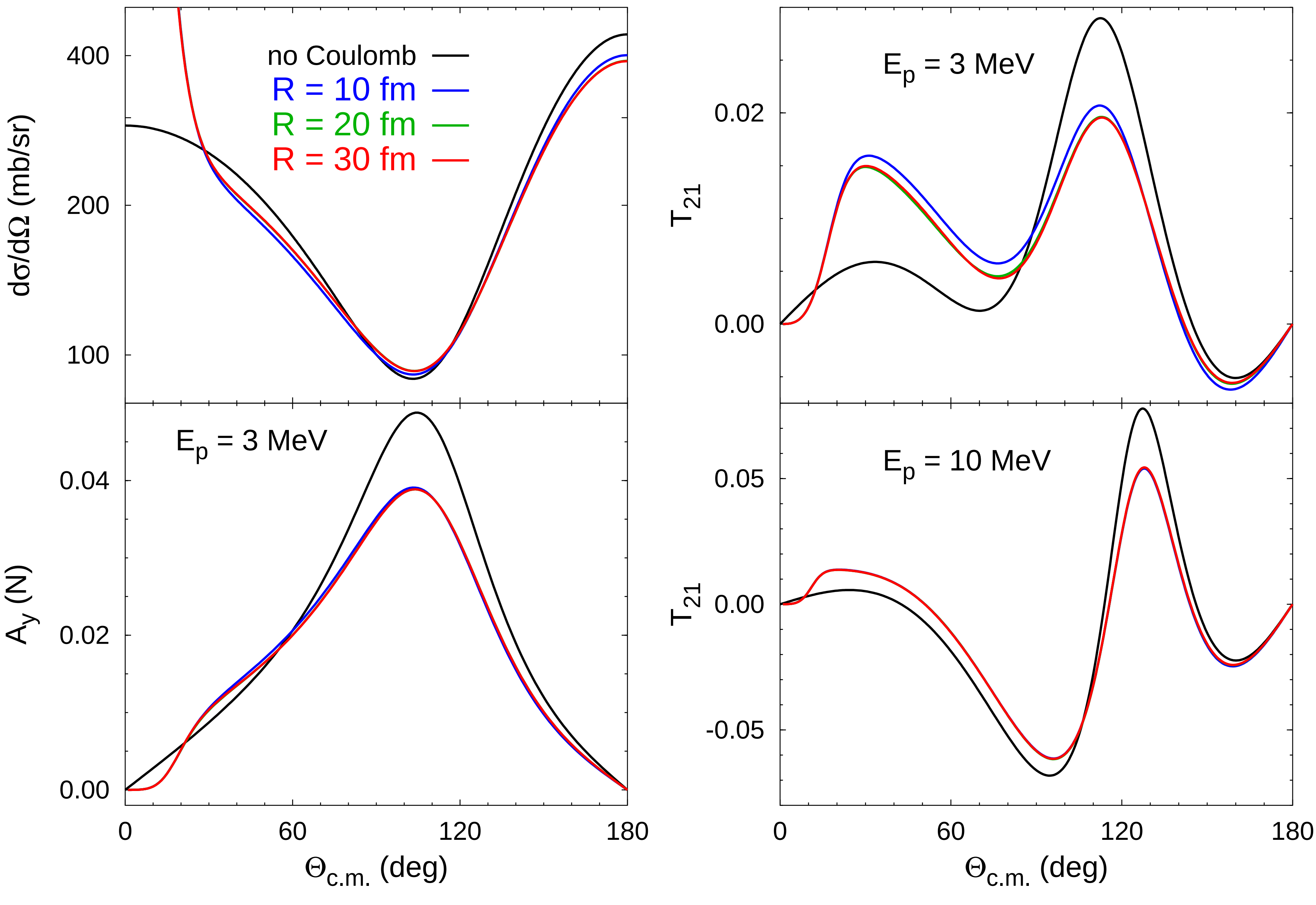
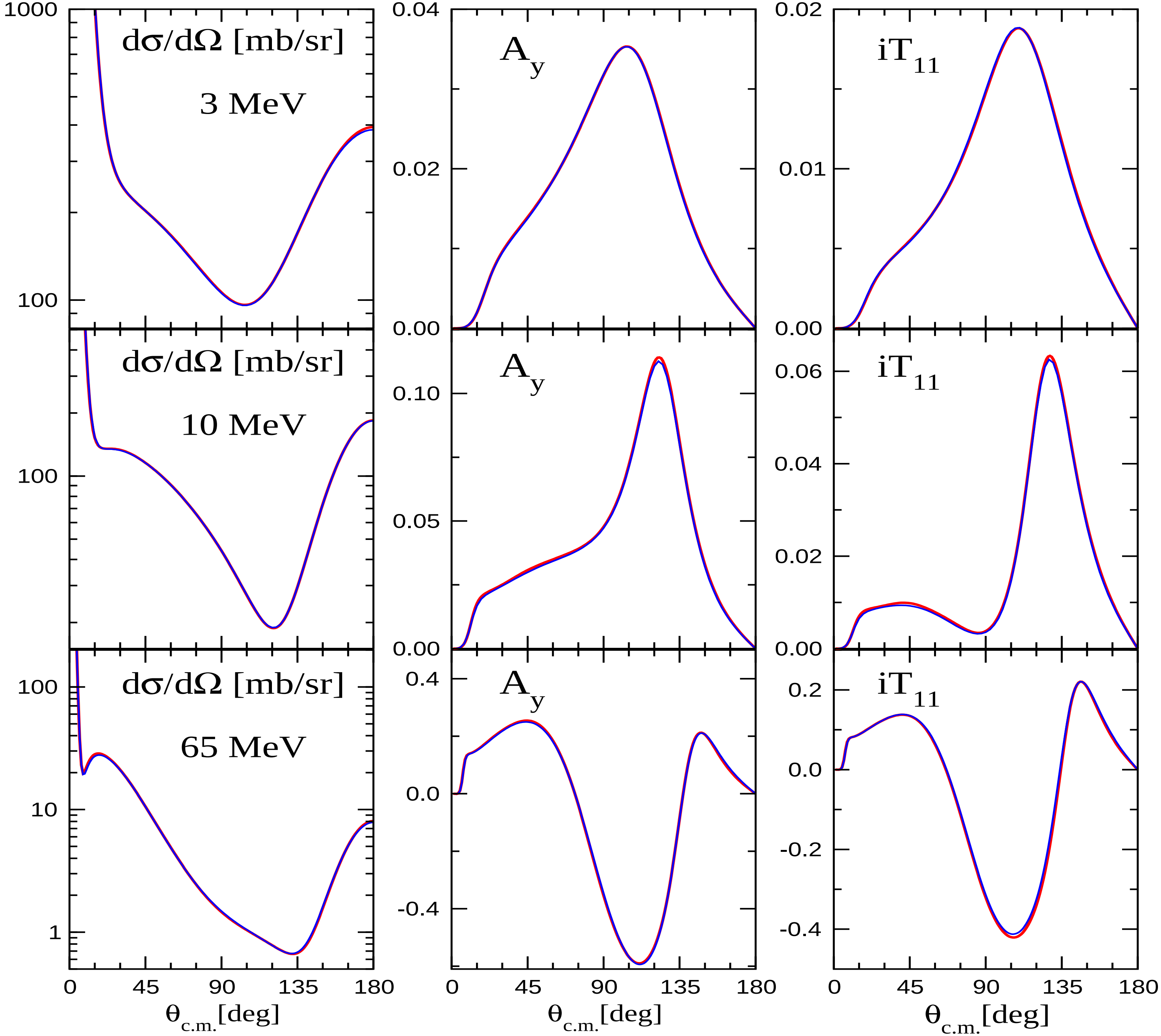
$\Rightarrow$ slow convergence with R at low relative energies



Convergence with *R*: *pd* elastic scattering



Comparison with configuration-space results

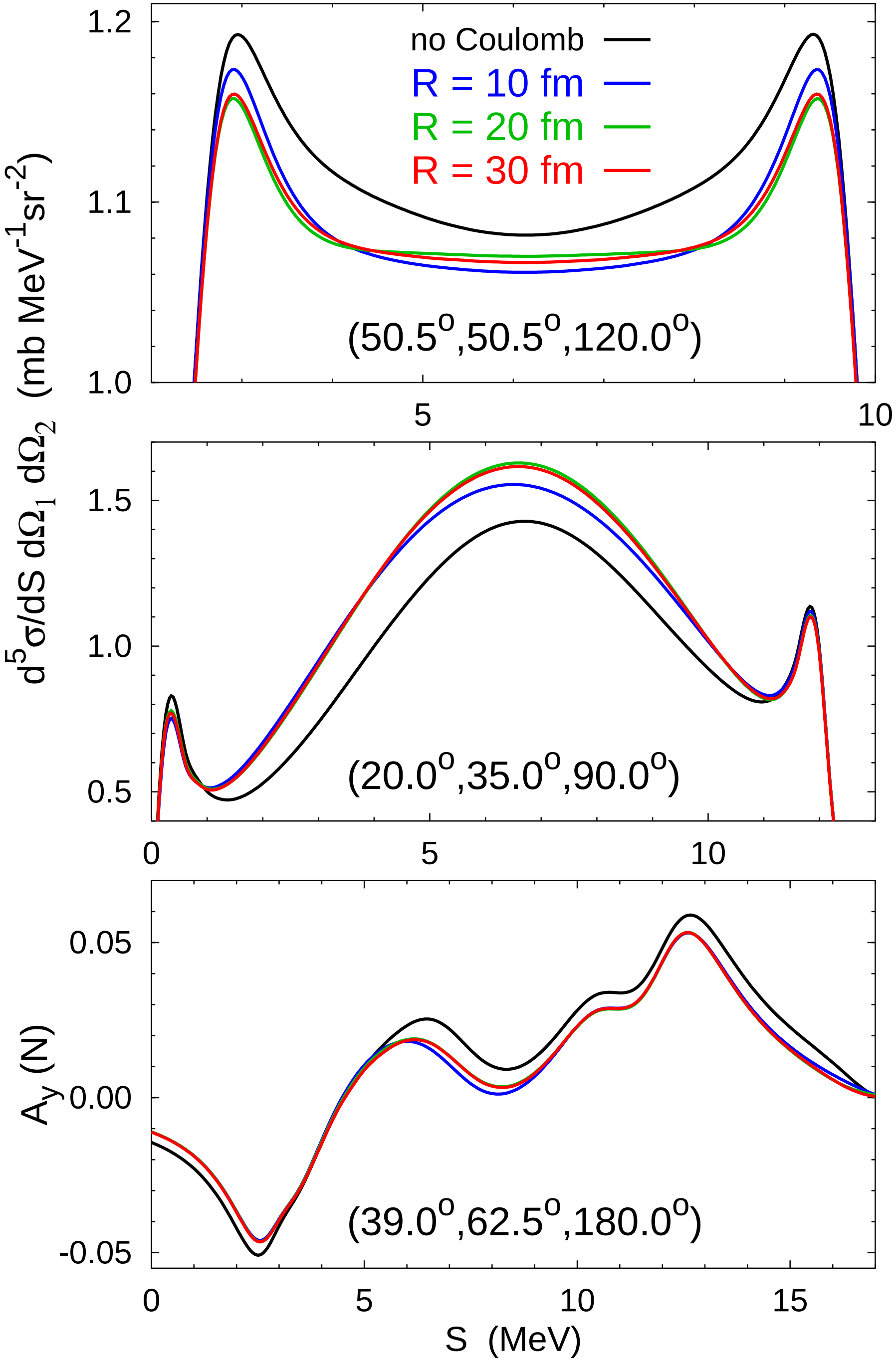


pd elastic scattering:

Kohn variational principle (Pisa)

screening and renormalization

Convergence with R: pd breakup at $E_p = 13$ MeV



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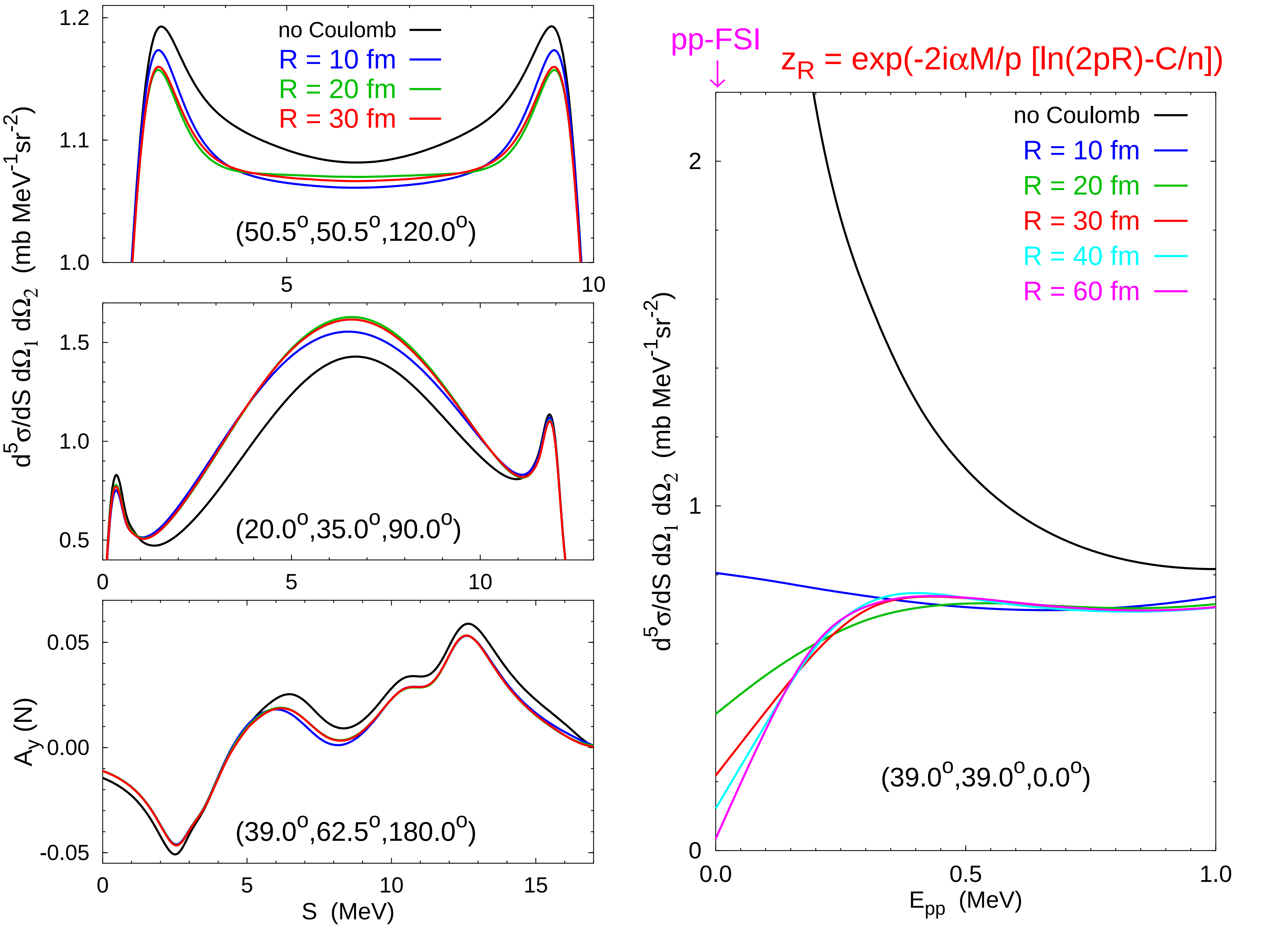
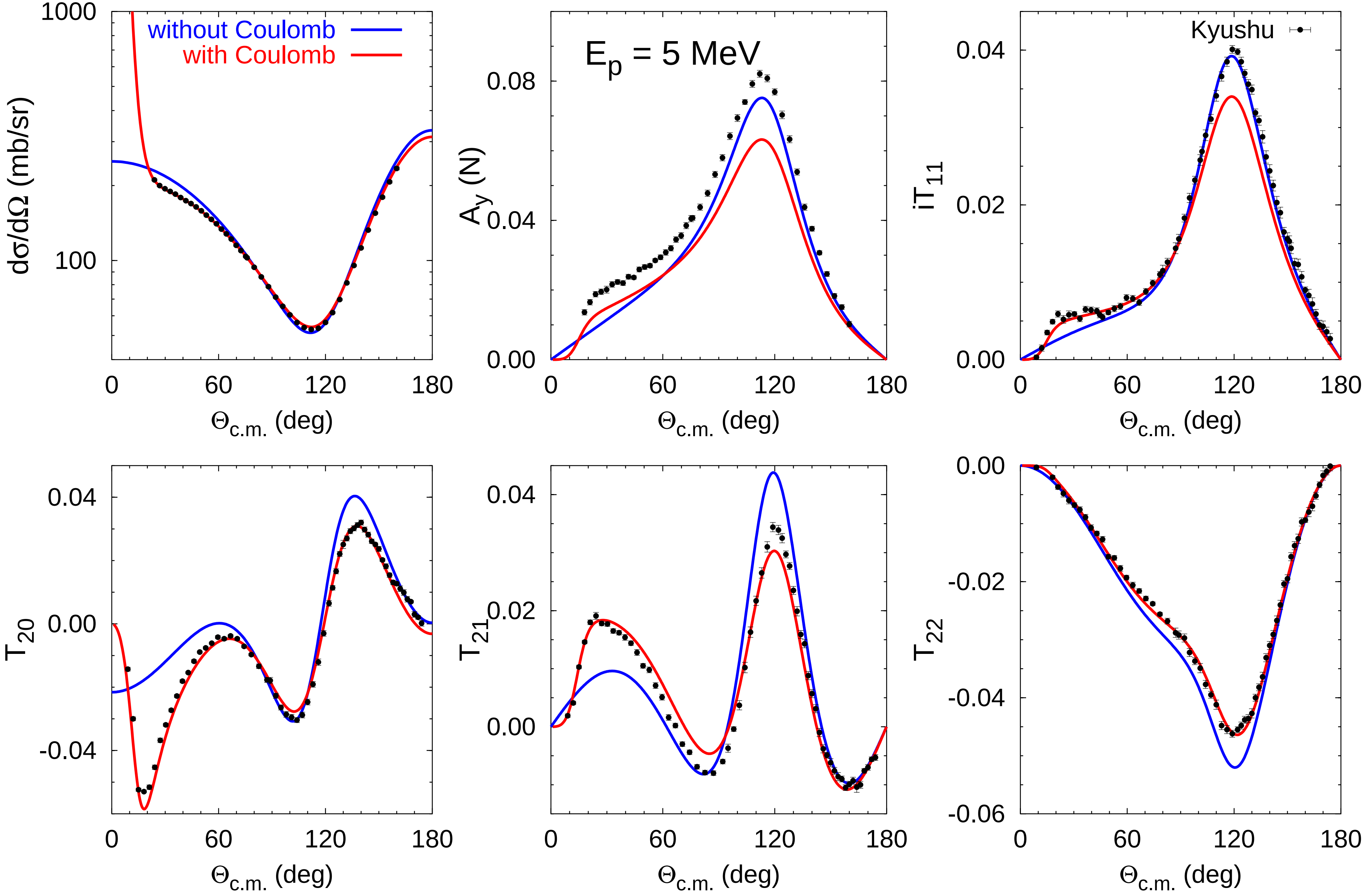
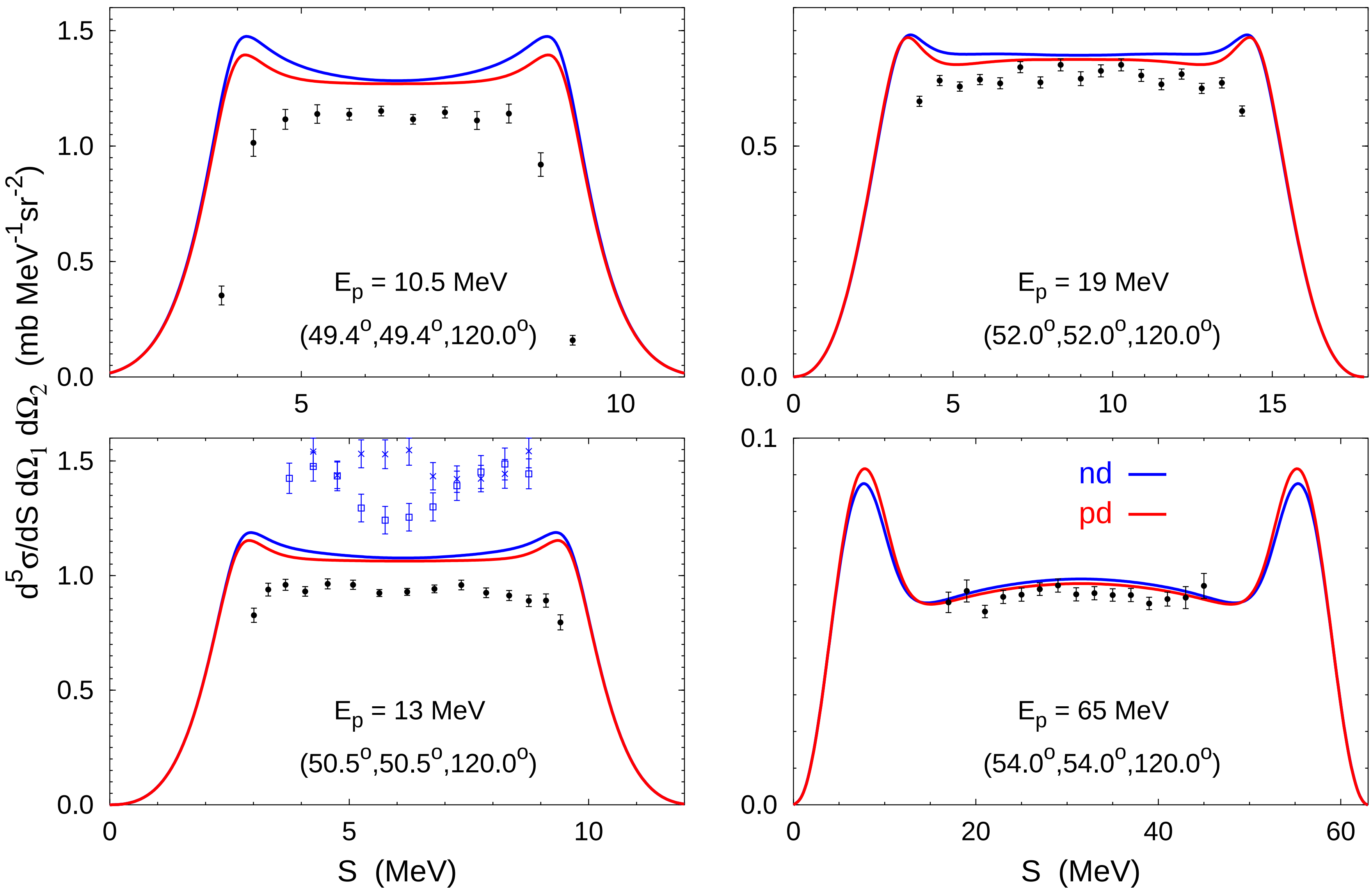


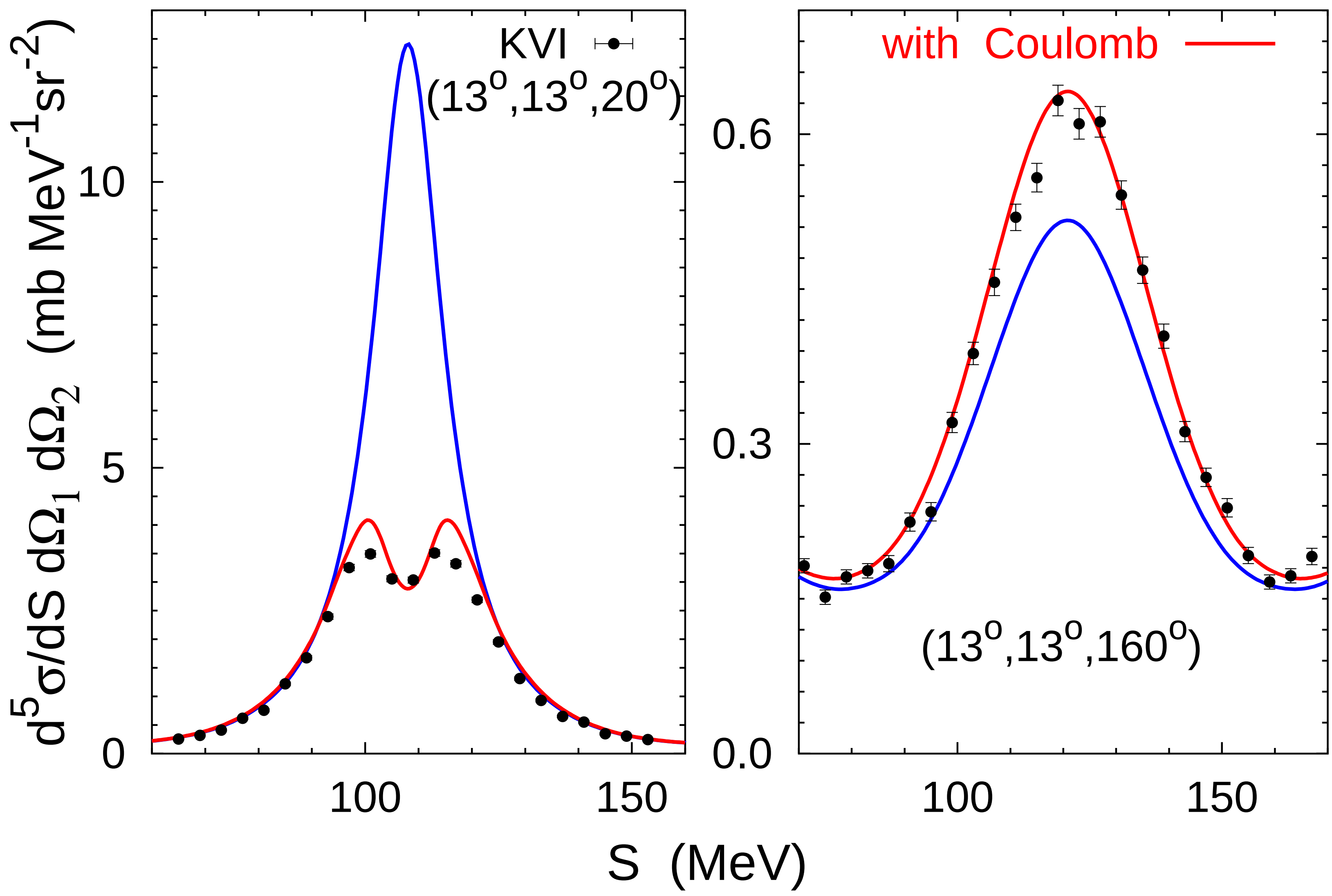
pd elastic scattering at low energies



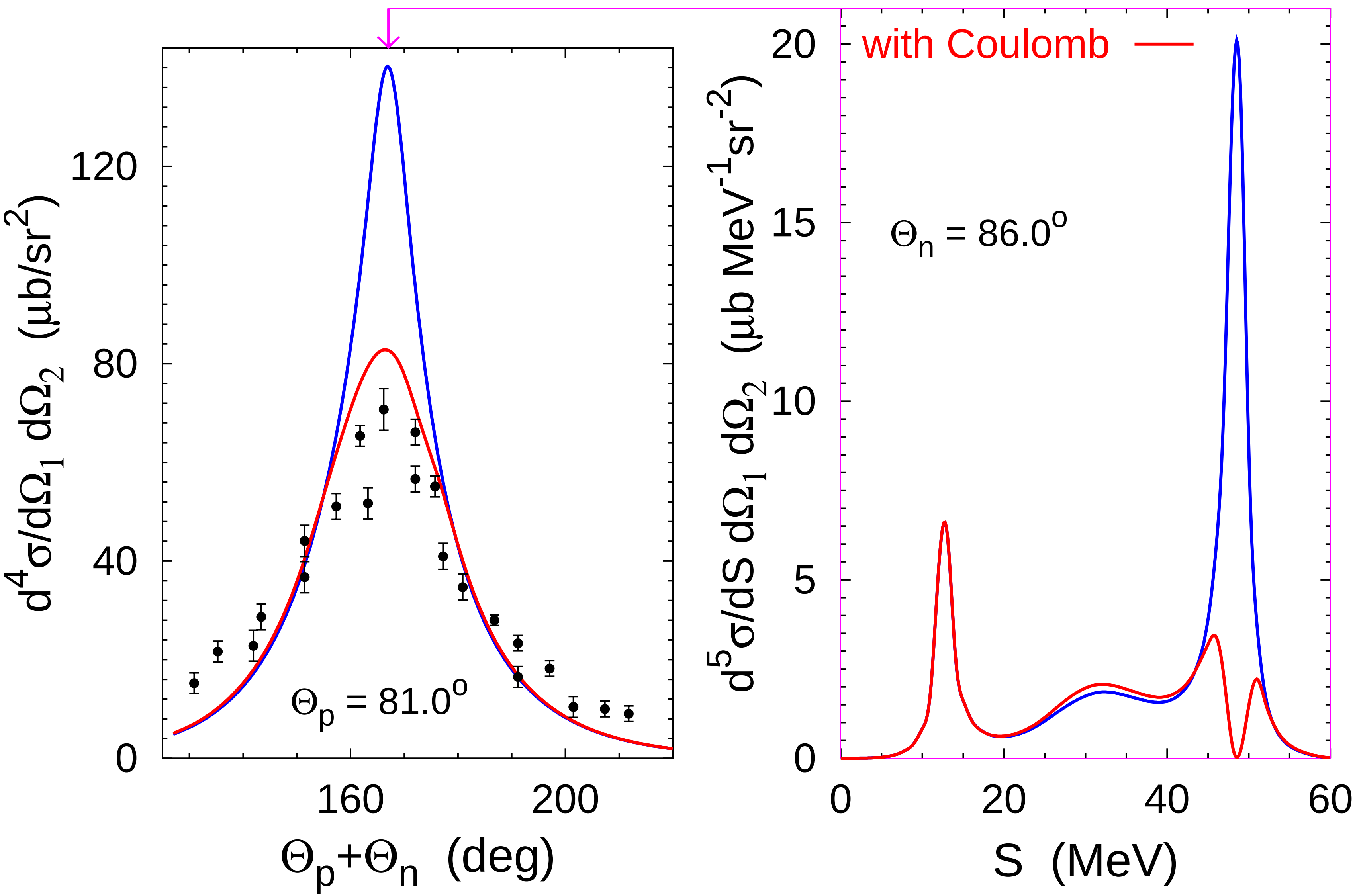
Nd breakup: space-star anomaly



dp breakup at $E_d = 130$ MeV



$^3He(\gamma, pn)p$ at $E_\gamma = 55$ MeV



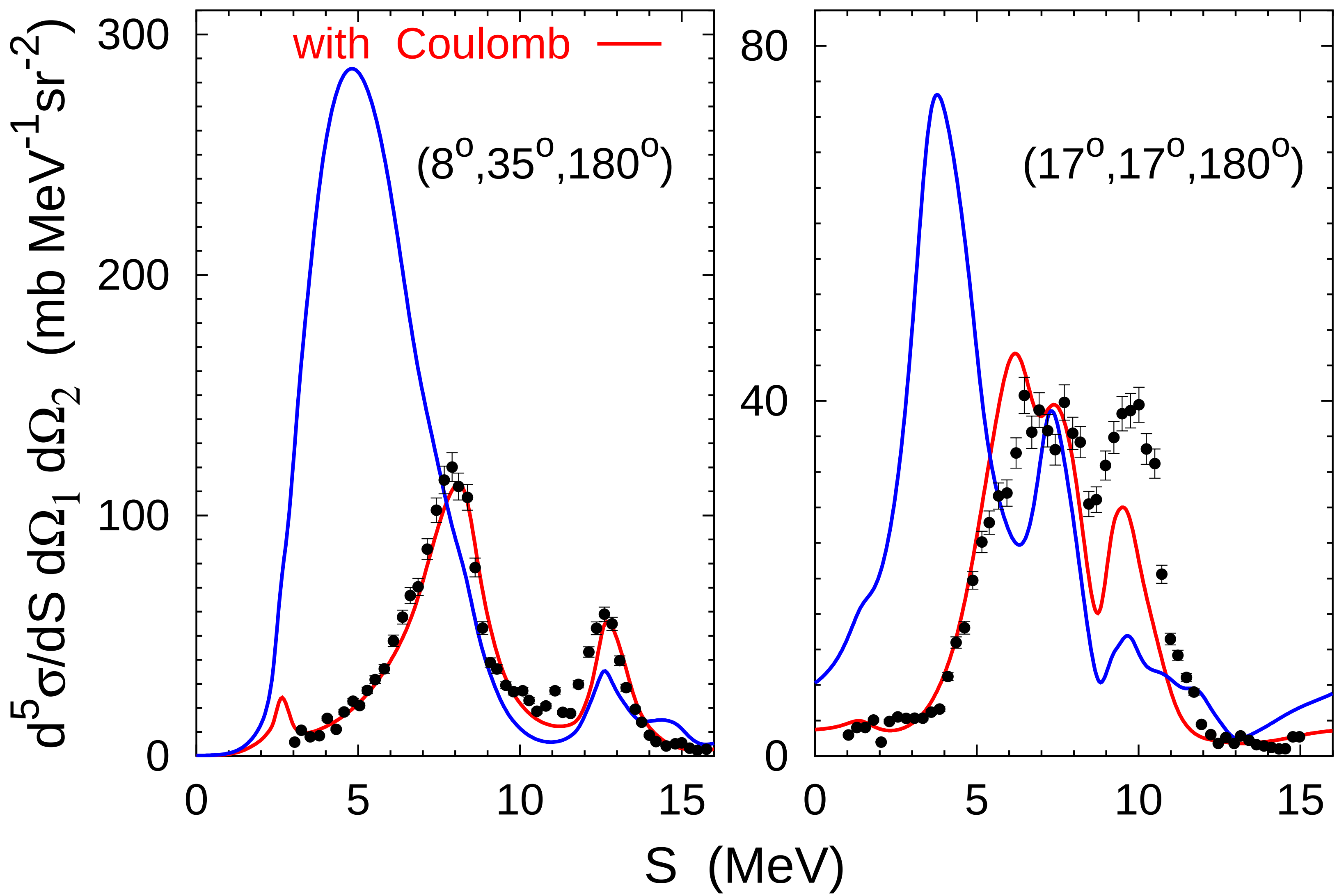
Application to 3-body nuclear reactions

$$\left.\begin{array}{l} p+(nA) \\ d+A \end{array}\right\} \longrightarrow \left\{\begin{array}{l} n+(pA) \\ p+(nA) \\ d+A \\ p+n+A \end{array}\right.$$

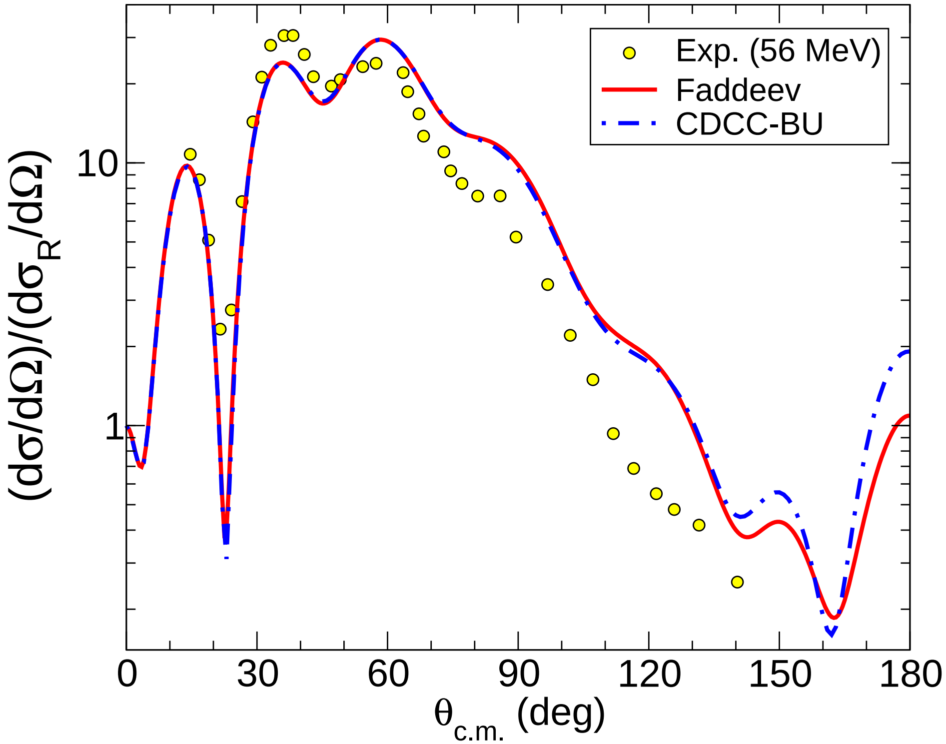
with $A = {}^{4}\mathrm{He},\ {}^{10}\mathrm{Be},\ {}^{12}\mathrm{C},\ {}^{58}\mathrm{Ni},\ \ldots$

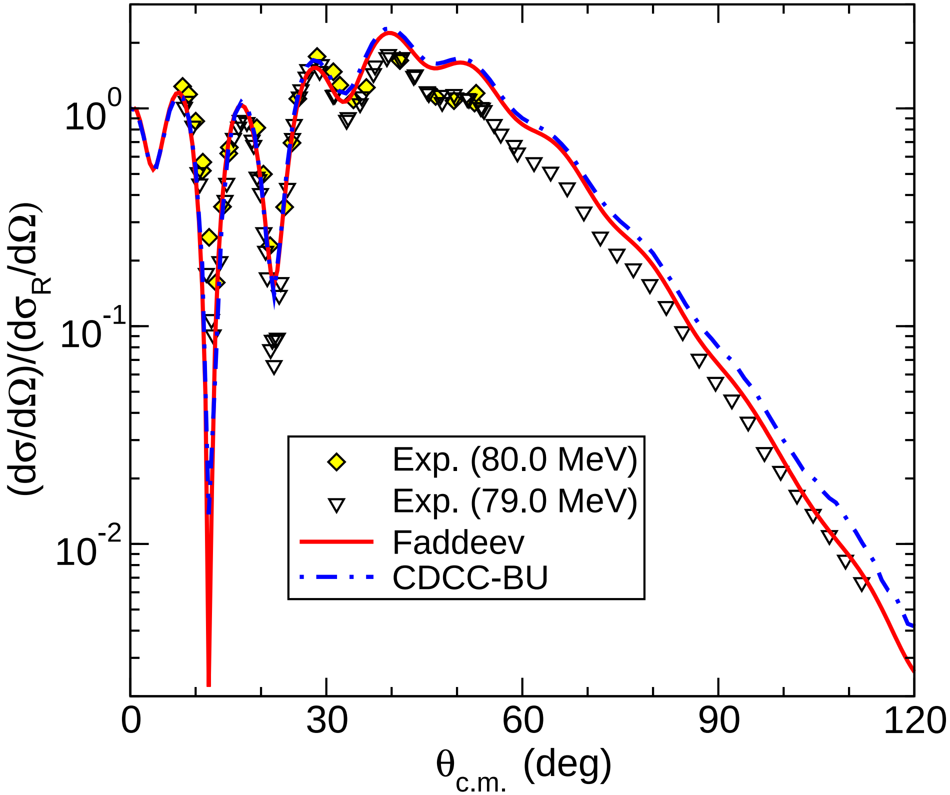
- Validity test of approximate nuclear reaction methods: CDCC, DWBA, . . .

$d(\alpha,\alpha p)n$ at $E_\alpha = 15$ MeV



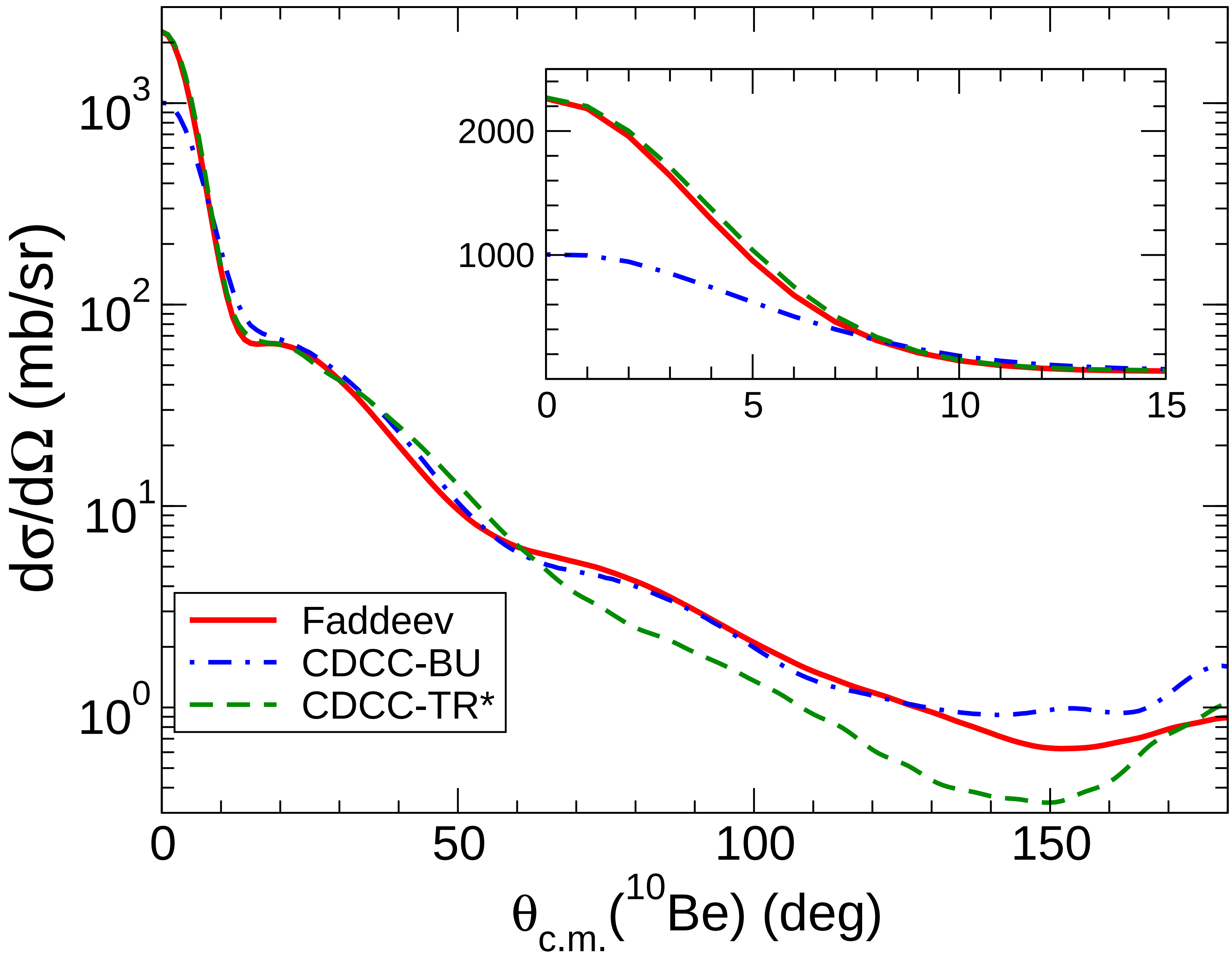
Testing CDCC: d-^{12}C and d-^{58}Ni elastic scattering





CDCC: A. M. Moro and F. M. Nunes

Testing CDCC: $p(^{11}\mathrm{Be}, ^{10}\mathrm{Be})np$ at $E/A = 38$ MeV



4N system

- "theoretical laboratory" to test models of nuclear interaction

$$n + {}^3\mathrm{H} \longrightarrow n + {}^3\mathrm{H}$$

$$p + {}^3\mathrm{He} \longrightarrow p + {}^3\mathrm{He}$$

$$\left.\begin{array}{l} p + {}^3\mathrm{H} \\ n + {}^3\mathrm{He} \\ d + d \end{array}\right\} \longrightarrow \left\{\begin{array}{l} p + {}^3\mathrm{H} \\ n + {}^3\mathrm{He} \\ d + d \end{array}\right.$$

4N scattering: symmetrized AGS equations

two-cluster 1+3 and 2+2 transition operators

$$\mathcal{U}_{11} = -(G_0 T G_0)^{-1} P_{34} - P_{34} U_1 G_0 T G_0 \mathcal{U}_{11} + U_2 G_0 T G_0 \mathcal{U}_{21}$$

$$\mathcal{U}_{21} = (G_0 T G_0)^{-1}(1 - P_{34}) + (1 - P_{34}) U_1 G_0 T G_0 \mathcal{U}_{11}$$

$$\mathcal{U}_{12} = (G_0 T G_0)^{-1} - P_{34} U_1 G_0 T G_0 \mathcal{U}_{12} + U_2 G_0 T G_0 \mathcal{U}_{22}$$

$$\mathcal{U}_{22} = (1 - P_{34}) U_1 G_0 T G_0 \mathcal{U}_{12}$$

$$U_j = P_j G_0^{-1} + P_j T G_0 U_j$$

$$P_1 = P = P_{12} P_{23} + P_{13} P_{23}$$

$$P_2 = \tilde{P} = P_{13} P_{24}$$

$$T = v + v G_0 T$$

scattering amplitude $$\mathcal{T}_{fi} = S_{fi} \langle \mathbf{p}_f \phi_f | \mathcal{U}_{fi} | \mathbf{p}_i \phi_i \rangle$$

$$|\phi_j\rangle = G_0 T P_j |\phi_j\rangle$$

Screening and renormalization in 4N scattering

$$v \longrightarrow v + w_R$$

$$T,\ U_j,\ \mathcal{U}_{fi},\ \mathcal{T}_{fi} \longrightarrow T^{(R)},\ U_j^{(R)},\ \mathcal{U}_{fi}^{(R)},\ \mathcal{T}_{fi}^{(R)}$$

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isolate long-range interaction
and Coulomb distortion between c.m. of two clusters

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isolate long-range interaction
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Renormalization:

$$\mathcal{T}_{fi} = \lim_{R\to\infty} Z_{Rf}^{-\frac{1}{2}} \mathcal{T}_{fi}^{(R)} Z_{Ri}^{-\frac{1}{2}}$$

$$= \delta_{fi} T_{Ci}^{\mathrm{c.m.}} + \lim_{R\to\infty} Z_{Rf}^{-\frac{1}{2}} [\mathcal{T}_{fi}^{(R)} - \delta_{fi} T_{Ri}^{\mathrm{c.m.}}] Z_{Ri}^{-\frac{1}{2}}$$

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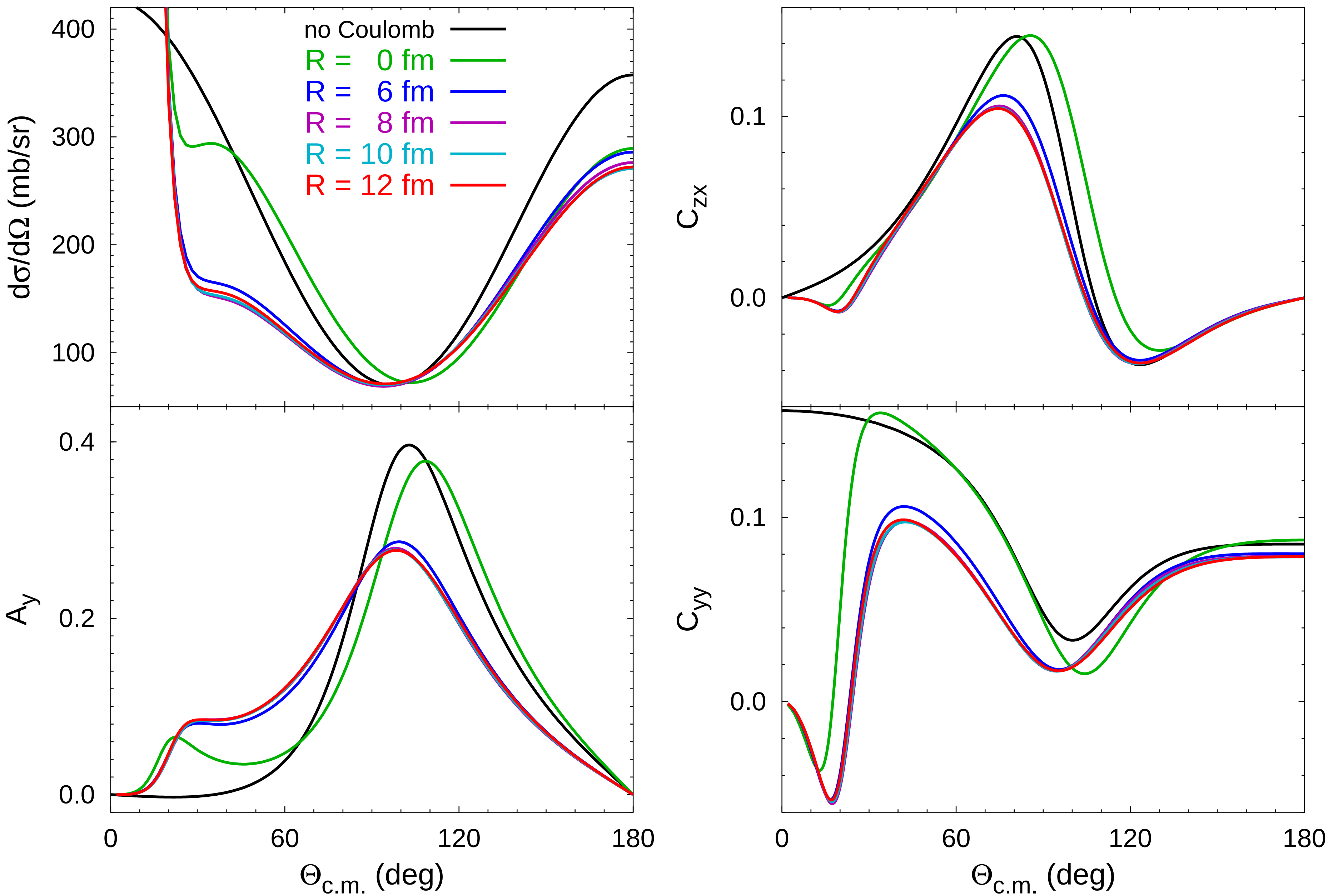
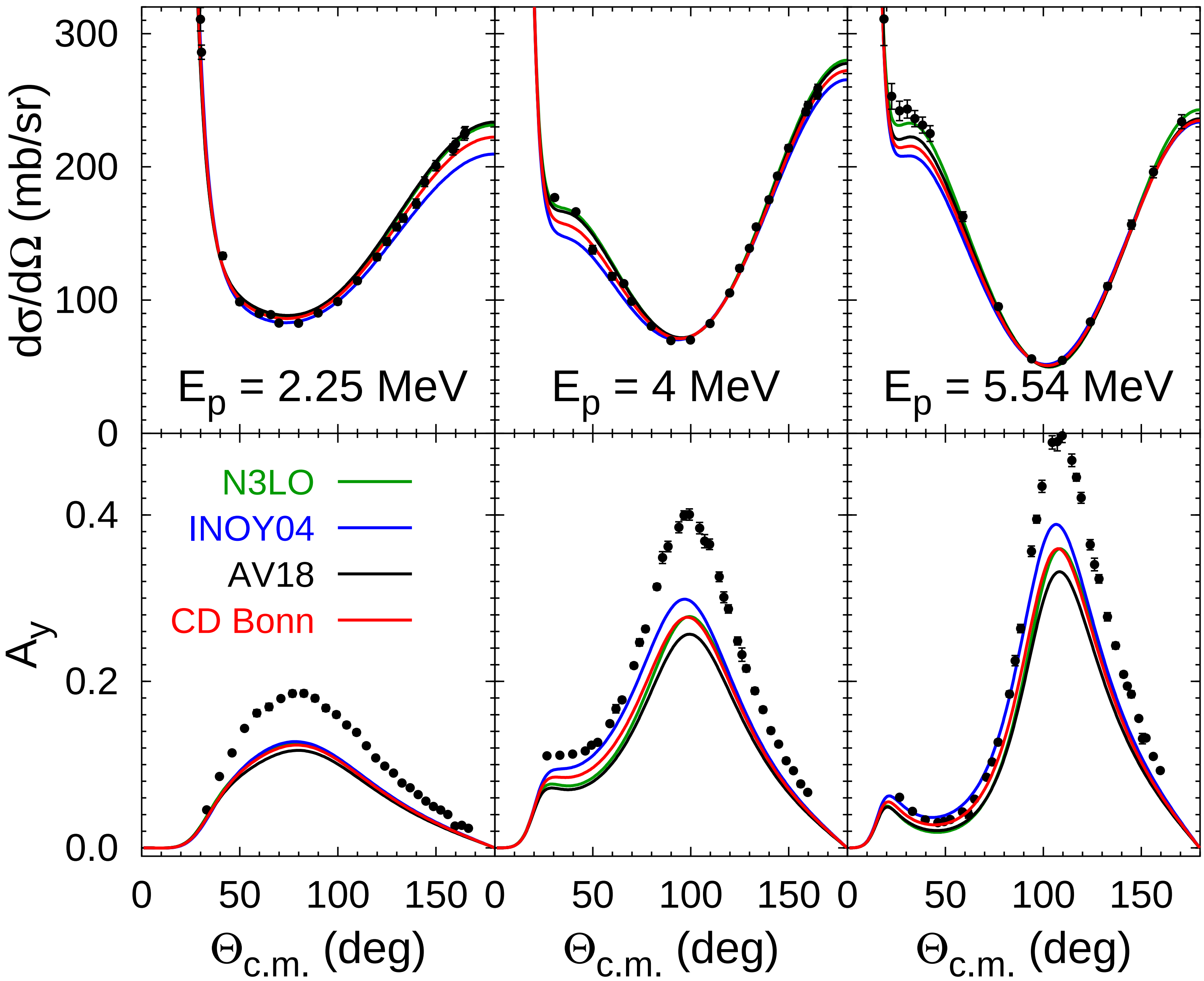
Coulomb-distorted short-range part: fast convergence with R

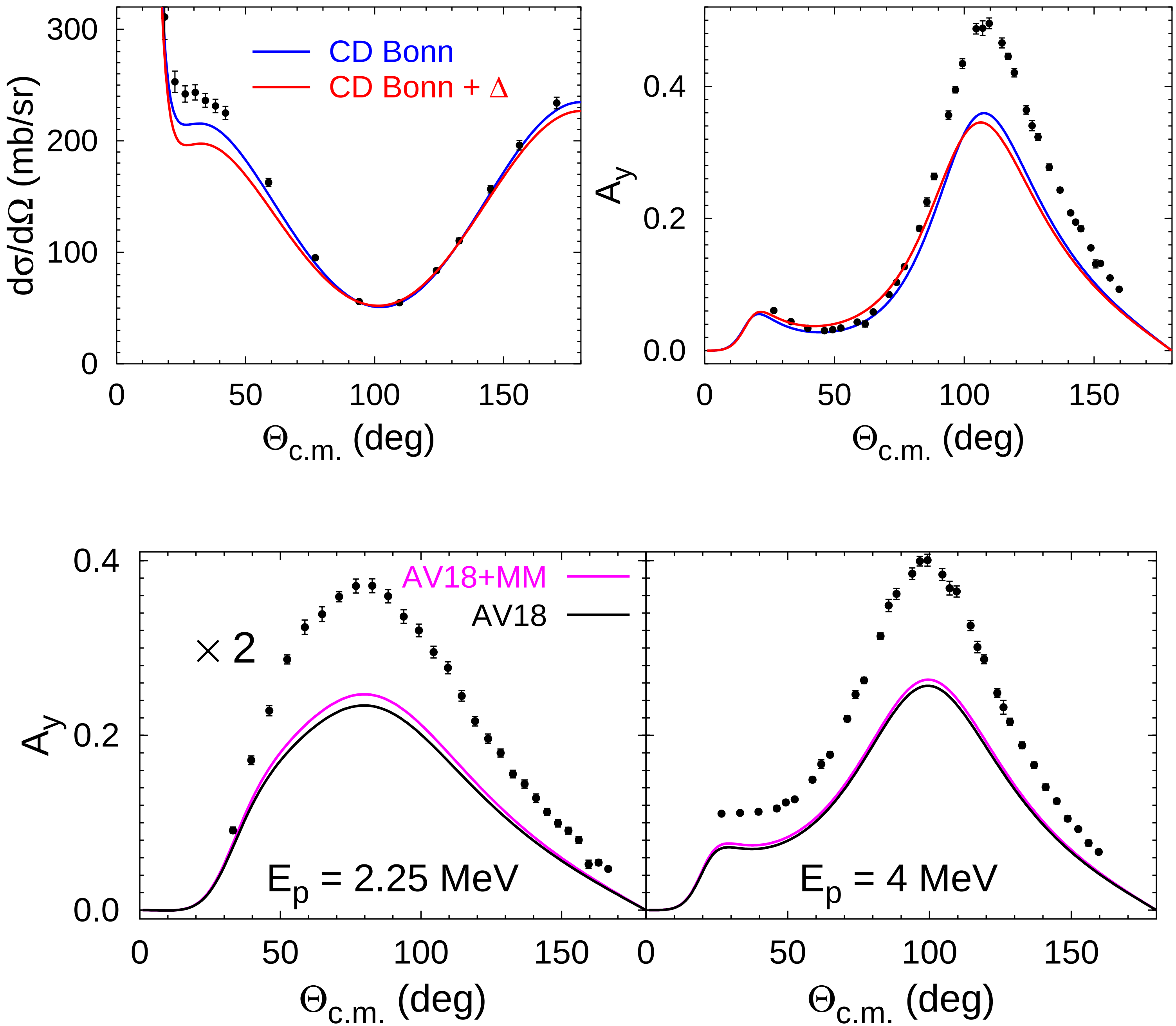
Convergence with R: p-^{3}He scattering at $E_p = 4$ MeV



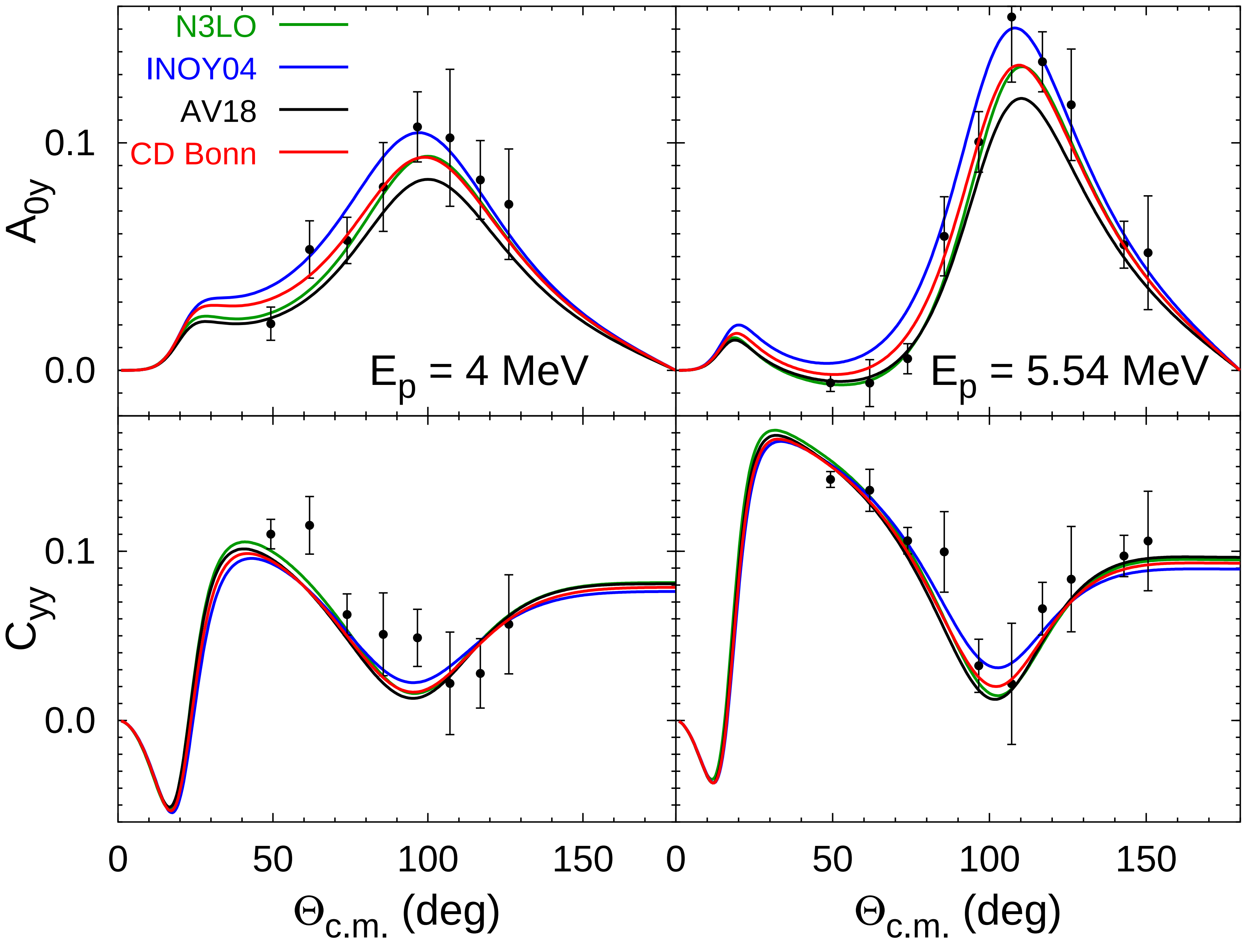
p-^{3}He scattering



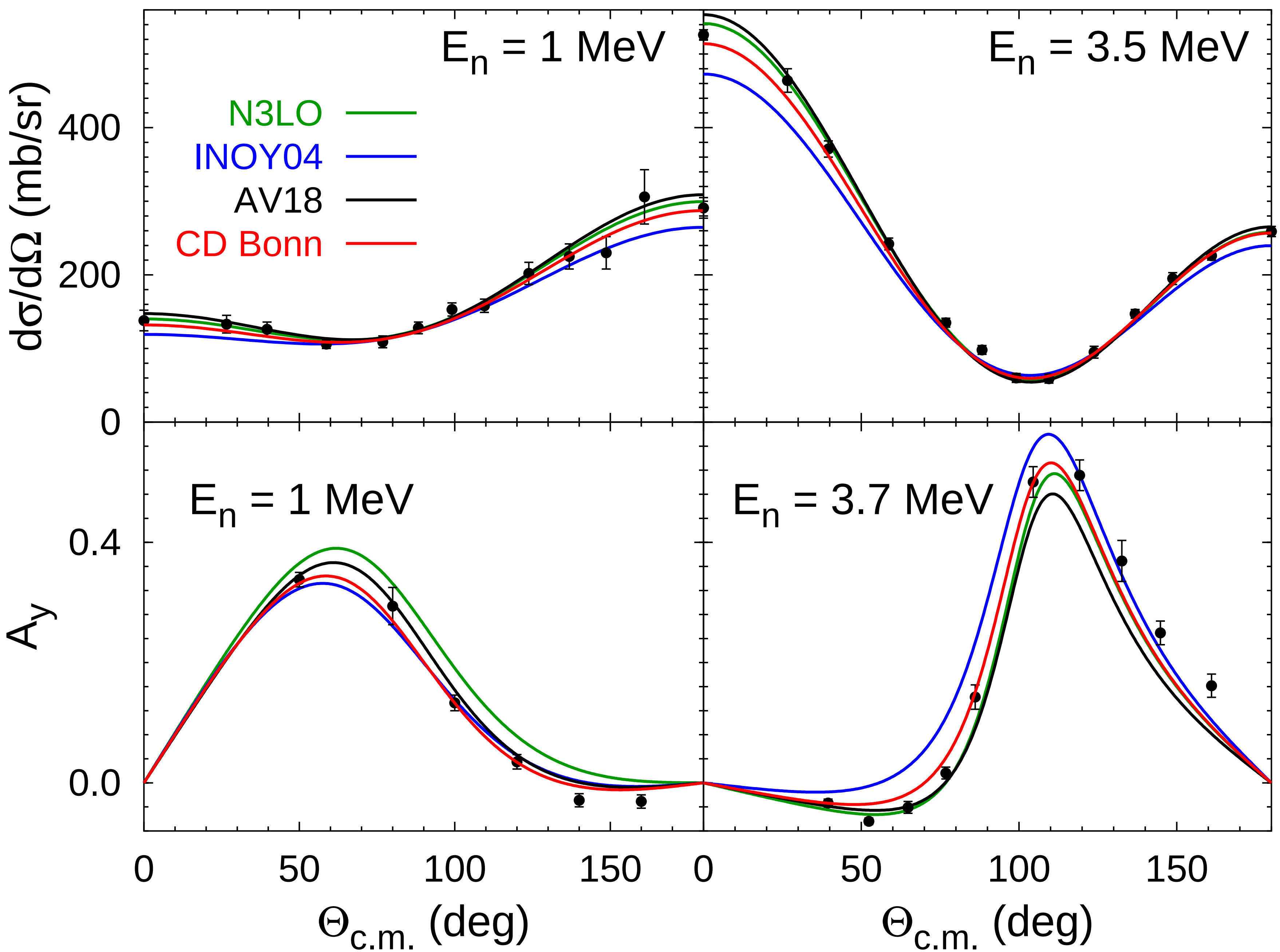
p-^{3}He A_y puzzle: 3NF, magnetic moments?



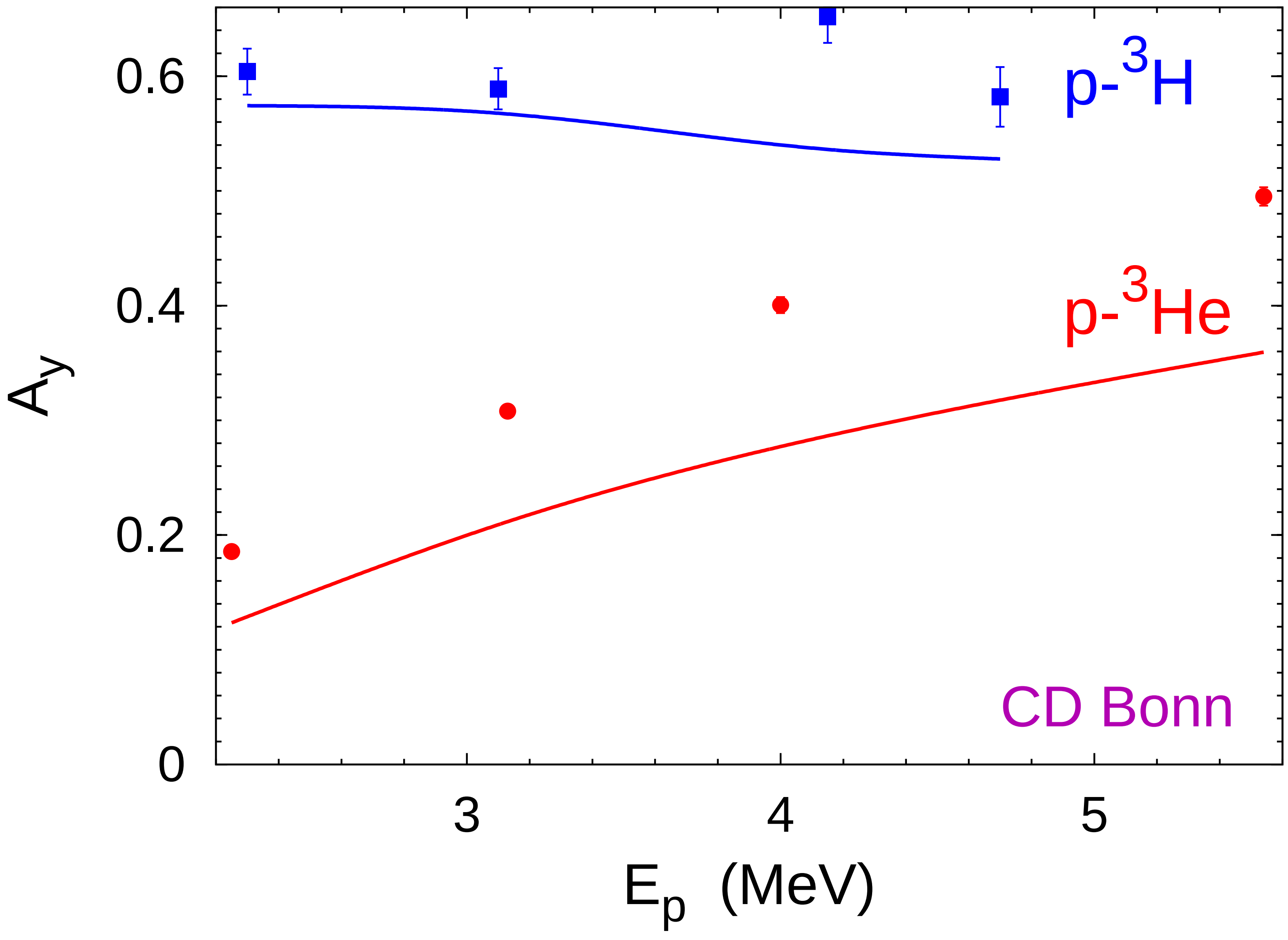
p-^{3}He scattering



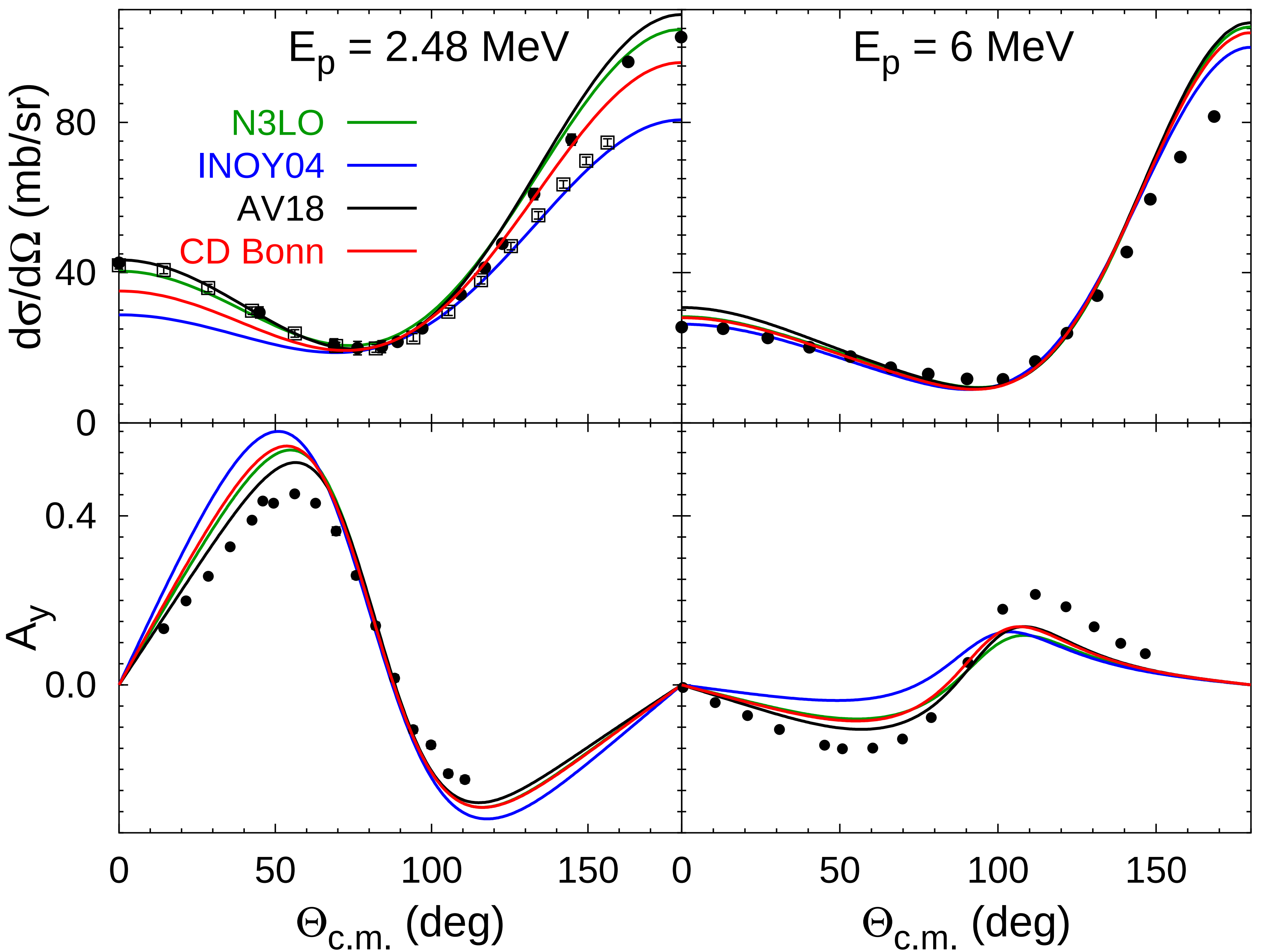
n-^{3}He elastic scattering



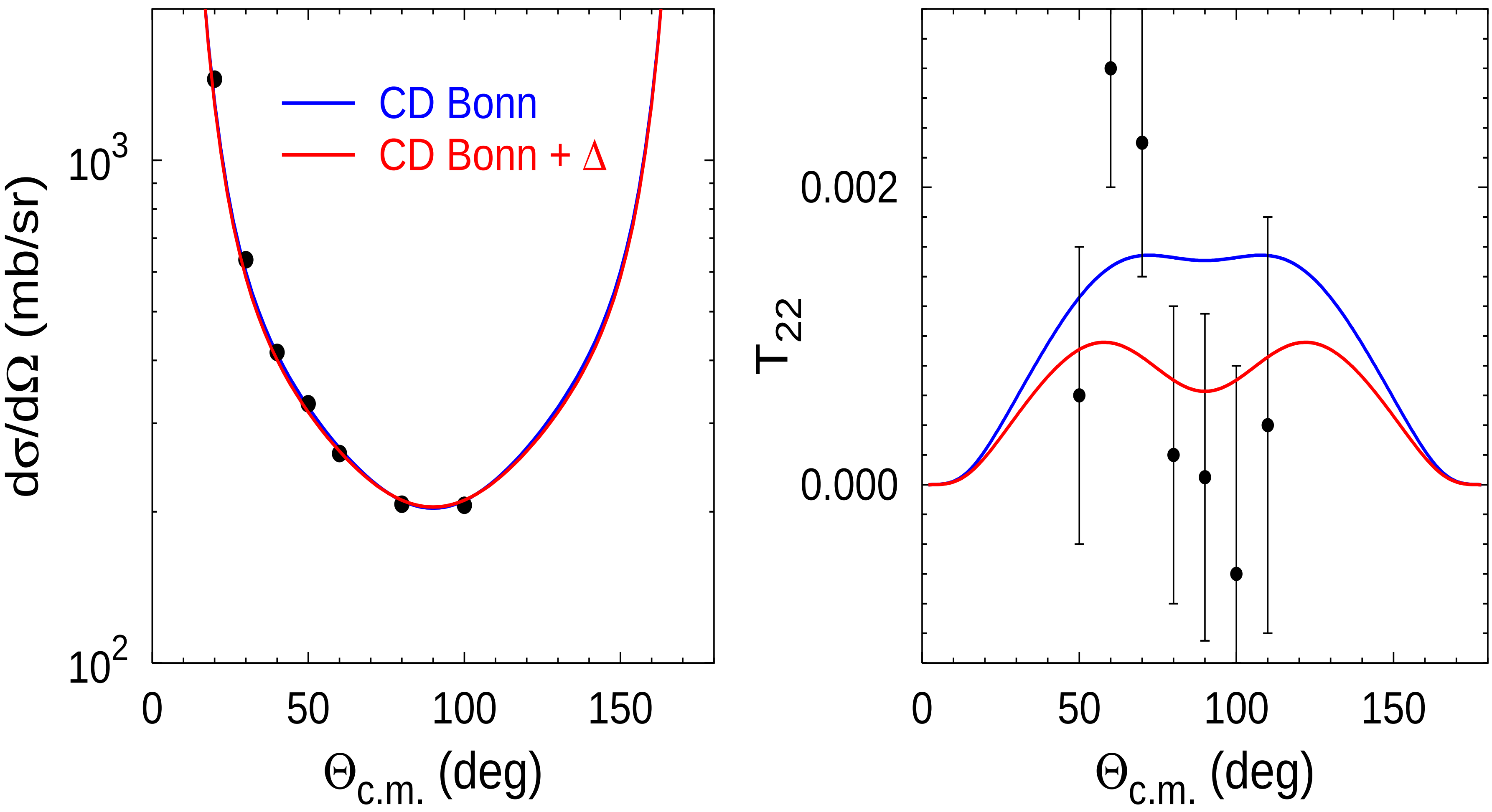
A_y maximum in p-^{3}He and p-^{3}H elastic scattering



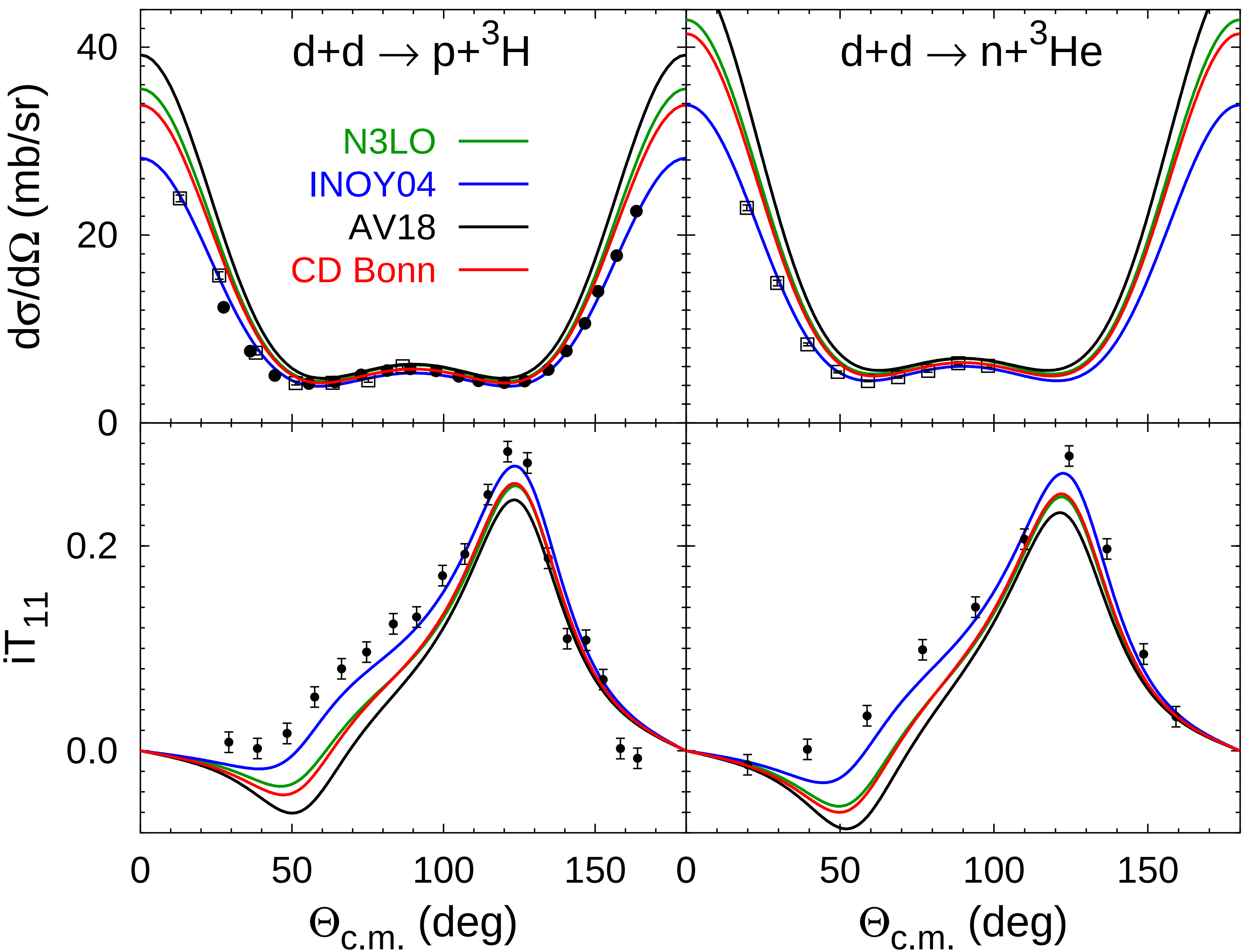
Transfer reaction $p + {}^3\text{H} \rightarrow n + {}^3\text{He}$



d-d elastic scattering at $E_d = 3$ MeV



$d + d \to N + [3N]$ **transfer at $E_d = 3$ MeV**



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- old idea of screening and renormalization
  + novel practical realization
  $\Rightarrow$ momentum-space description of few-body reactions
  including Coulomb interaction

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  + novel practical realization
  $\Rightarrow$ momentum-space description of few-body reactions
  including Coulomb interaction
- hadronic and electromagnetic 3N reactions
- 3-body nuclear reactions
- low energy 4N scattering