Four-Body Nuclear Systems

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Outline

Collaborators

- L. Girlanda, L.E. Marcucci, and S. Rosati - *Pisa University (Italy)*
- A. Kievsky and R. Alvarez-Rodriguez - *INFN, Sezione di Pisa (Italy)*

Four nucleon systems - Aims

- Tests of NN & 3N interaction models
- Tests of many-body techniques
- Study of reactions of astrophysical interest
- Study of reactions for energy production

NN interaction

Phenomenological models...

- Argonne V18, CD-Bonn, Nijmengen ($\chi^2 \approx 1$)
- Fit of 3N data using non-locality in P-waves (ISuj [Doleschall, 2008])

... new developments...

- Effective field theory
  - N3LO-Jülich [Epelbaum and Coll, 1998-2006]
  - N3LO-Idaho [Entem & Machleidt, 2003]
- Low-q interaction [Bogner and Coll., 2001-2007]

and small effects

- CSB (strong + em effects)
- Parity-violating (weak interaction between nucleons)

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3N interaction

Old models...

- Tucson-Melbourne [Coon *et al*, 1979, Friar *et al*, 1999]
- Brazil [Robilotta & Coelho, 1986]
- Urbana [Pudliner *et al*, 1995]

and new developments

- Effective field theory
  - N2LO-Jülich [Epelbaum *et al*, 2002]
  - N2LO-Navratil [Navratil, 2007]
- Illinois [Pieper *et al*, 2001]
- Under progress: N3LO, Δ, CSB, ...

Theoretical methods

Kamada *et al*, PRC **64**, 044001 (2001)

Faddeev-Yakubovsky equations

Solved in configuration or momentum space

- Bochum-Cracow-Jülich, Lisboa, Grenoble, LANL, . . .

GFMC

- ANL-LANL

Variational methods

1. Gaussian basis [Kamimura, Varga]
2. HH & EIHH [Pisa, Trento-Jerulasem]
3. HO (NCSM) [Navratil and Coll.]

The HH method

HH functions

- hyperradius $\rho^2 = \frac{2}{A}\sum_{i<j} r_{ij}^2$
- hyperangles $\Omega = \{\frac{\boldsymbol{\xi}_1}{\rho}, \ldots, \frac{\boldsymbol{\xi}_{A-1}}{\rho}\}$ ($\boldsymbol{\xi}_i$ Jacobi vectors)
- $T = T_\rho + T_\Omega/\rho^2$
- The HH functions $\mathcal{Y}_{[K]}(\Omega)$ are the eigenstates of T_Ω

$$|\Psi\rangle = \sum_{n,[K]} a_{n,[K]}|n,[K]\rangle\,, \qquad \langle \boldsymbol{r}_1,\ldots,\boldsymbol{r}_A|n,[K]\rangle = L_n^{(3A-4)}(\gamma\rho)e^{-\gamma\rho/2}\mathcal{Y}_{[K]}(\Omega)$$

$$[K] \to K, L, S, T, \mu\,, \qquad T_\Omega\mathcal{Y}_{[K]}(\Omega) = (\hbar^2/M)K(K+7)\mathcal{Y}_{[K]}(\Omega)$$

Advantages - bound state

Simplified calculation of the matrix elements of

- local/non-local NN & 3N potentials
- given in coordinate/momentum space

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Degeneracy

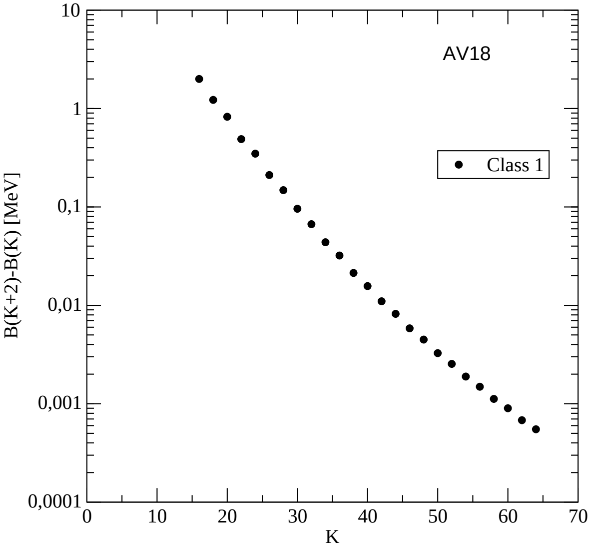
Number of independent states $\mathcal{Y}_{[K]}(\Omega)$ of given K, L, S, T, J and π

Case: $J = 0$, $T = 0$, $\pi = +$

K	$L = S = 0$	$L = S = 1$	$L = S = 2$
4	4	4	3
8	14	27	18
12	41	96	63
16	90	250	158
20	176	488	321
24	282	675	445

- Problem: due to the repulsion at short interparticle distances and the strong tensor interaction, HH states with large values of K are needed
- Solution: organize the basis in *classes*
  - *class 1* ("potential basis"): take into account 2-body correlations – small subset but very slow convergence
  - *classes 2-6*: more rapid convergence

Convergence for the α-particle (1)



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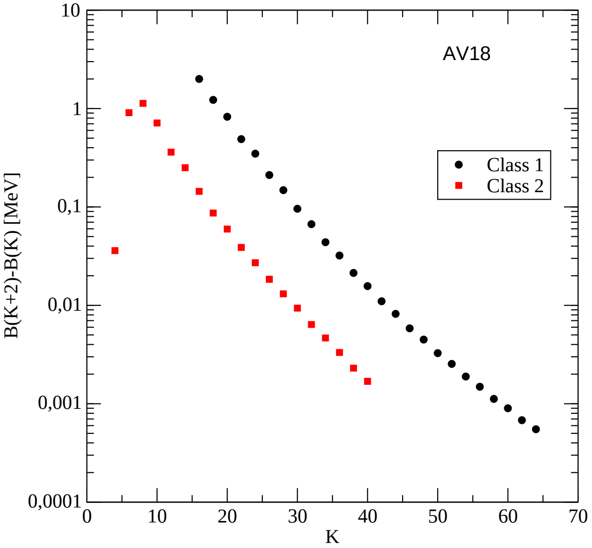

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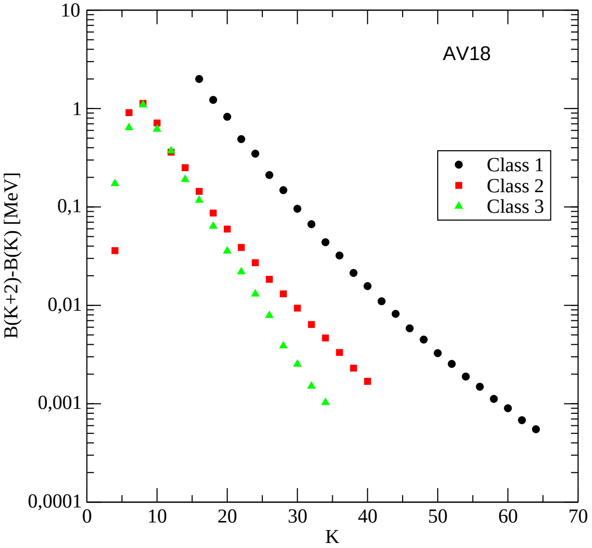

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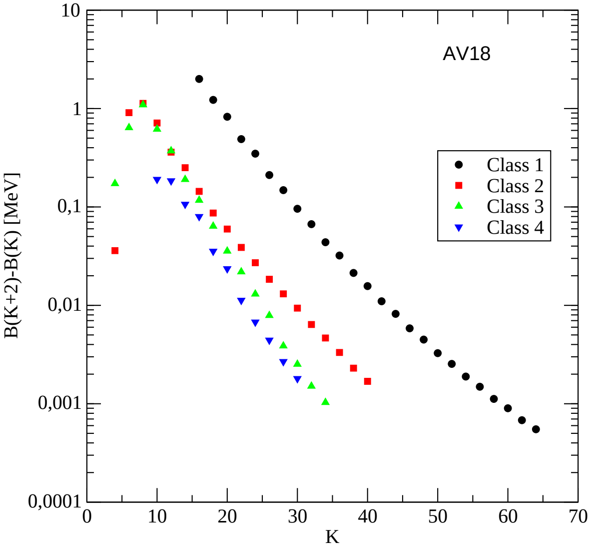
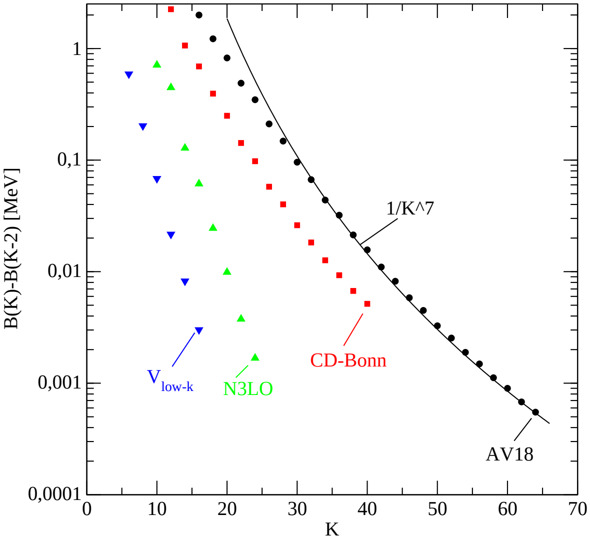


Convergence for the α-particle (2)



Convergence of class 1

$1/K^n$ law:
[Schneider, 1972], [Demin, 1977]

Triton and ^{4}He binding energies (1)

Potential	Method	$B(^3H)$	$B(^4He)$
AV18	HH	7.624	24.22
	FE/FY Nogga	7.621	24.23
	FE/FY Deltuva	7.621	24.24
	FE/FY Lazauskas	7.616	24.22
CDBonn	HH	7.998	26.13
	FE/FY Nogga	8.005	26.23 (26.16)
	FE/FY Deltuva	7.998	26.11
	NCSM Navratil	7.99(1)	
N3LO-Idaho	HH	7.854	25.38
	FE/FY Nogga	7.854	25.37
	FE/FY Deltuva	7.854	25.38
	NCSM Navratil	7.852(5)	25.39(1)

Triton and ^{4}He binding energies (2)

Potential	Method	$B(^3H)$	$B(^4He)$
AV18/UIX	HH	8.479	28.47
	FE/FY Nogga	8.476	28.53
	FE/FY Lazauskas	8.473	
CDBonn/TM	HH	8.474	29.00
	FE/FY Nogga	8.482	29.09
N3LO-Idaho/N2LO	HH	8.474	28.37
	NCSM Navratil	8.473(5)	28.34(2)

Scattering states

$$\Omega^{\pm}_{LSJJ_z} = \frac{1}{\sqrt{4}} \sum_{i=1}^{4} \Big[[s_i \otimes \phi_{jkl}]_S \otimes Y_L(\hat{\mathbf{y}}_i) \Big]_{JJ_z} \left(f_L(y_i) \frac{G_L(\eta, qy_i)}{qy_i} \pm \mathrm{i} \frac{F_L(\eta, qy_i)}{qy_i} \right)$$

$$|\Psi_{LSJJ_z}\rangle = \sum_{n,[K]} a_{LSJJ_z,[K]} |n,[K]\rangle + \left[\delta_{LL'} \delta_{SS'} |\Omega^{-}_{LSJJ_z}\rangle - \mathcal{S}^{J}_{LS,L'S'} |\Omega^{+}_{L'S'JJ_z}\rangle \right]$$

Method of calculation: projection of the asymptotic states in partial waves $\ell \leq l_{\mathrm{max}}$

Simplified calculation of the matrix elements of

- local/non-local NN & 3N potentials
- given in coordinate/momentum space
- Coulomb interaction

The breakup channels can be taken into account (work in progress)

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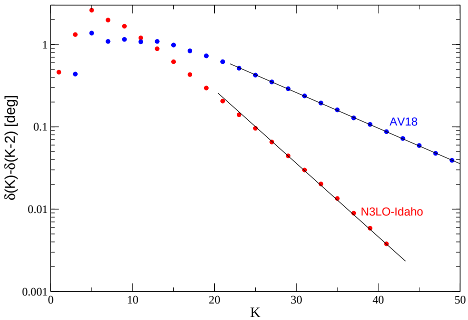
Convergence

$n - {}^3\mathrm{H}$ at $E_{cm} = 3$ MeV, ${}^3P_2$ wave

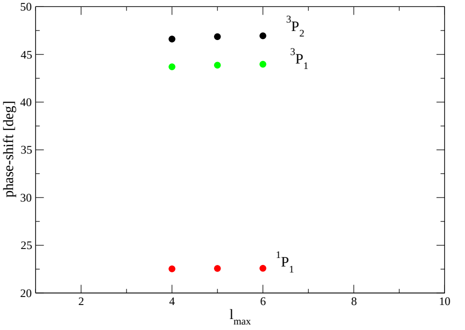


Convergence of class 1

Convergence

$n - {}^3\mathrm{H}$ at $E = 3$ MeV, ${}^3P_2$, ${}^3P_1$, and ${}^1P_1$ waves

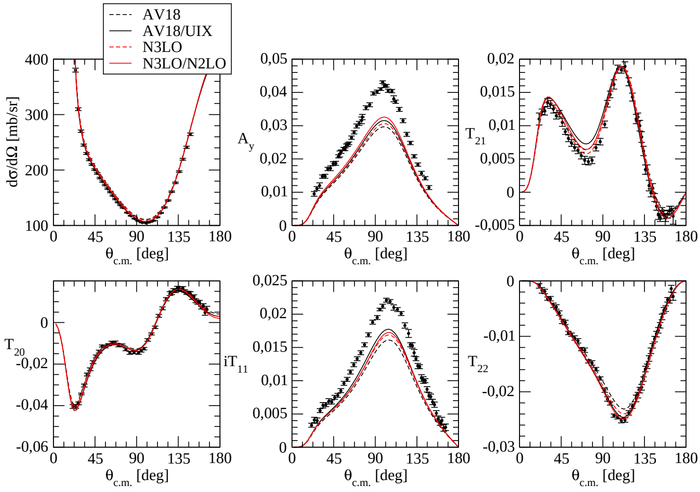
Truncation of the partial wave expansion of $\Omega^{\pm}_{LSJJ_z}$ up to l_{max}



N-d elastic scattering

First results with non-local potentials

p-d scattering at $E_{\text{c.m.}} = 1.666$ MeV



$n - {}^3\mathrm{H}$ elastic scattering

Comparison with the FY calculation by Deltuva & Fonseca, 2007

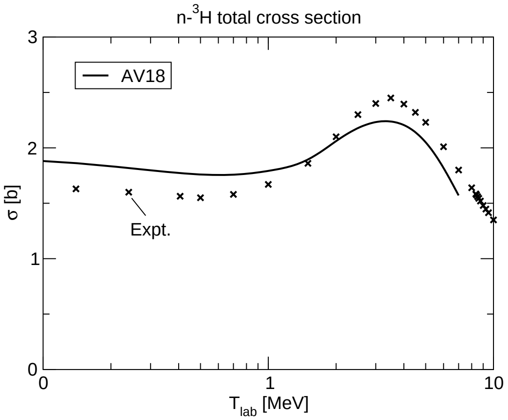
N3LO-Idaho potential, $E_{\mathrm{c.m.}} = 3$ MeV

preliminary

Phase-shift	HH	FY	Phase-shift	HH	FY
${}^1S_0$	-69.3	-69.1	${}^3P_0$	23.2	23.3
${}^3S_1$	-61.4	-61.2	${}^1P_1$	22.7	22.5
${}^3D_1$	-1.14	-1.10	${}^3P_1$	44.4	44.5
ϵ	0.77	0.80	ϵ	9.80	9.64
${}^1D_2$	-1.72	-1.90	${}^3P_2$	48.4	48.7
${}^3D_2$	-0.94	-1.01	${}^3F_2$	0.07	0.09
ϵ	2.74	2.81	ϵ	1.24	1.26

$n - {}^3\mathrm{H}$ elastic scattering

Total cross section (Exp. data: [Phillips et al., 1980])



See also [Lazauskas & Carbonell, 2004], [Deltuva & Fonseca, 2007]

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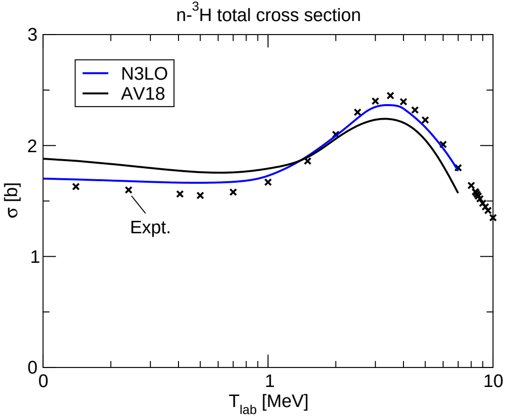

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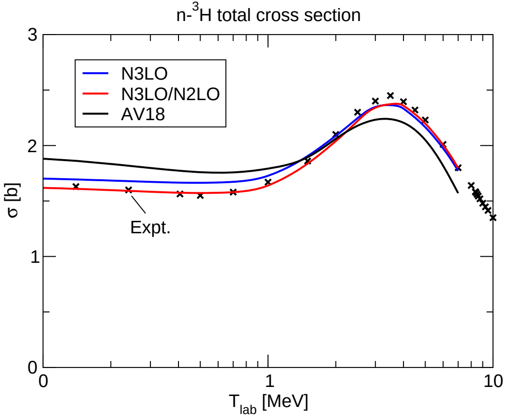

See also [Lazauskas & Carbonell, 2004], [Deltuva & Fonseca, 2007]

Contribution of *S* and *P* waves

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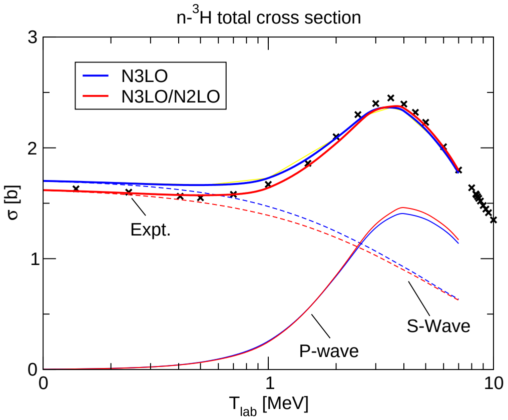
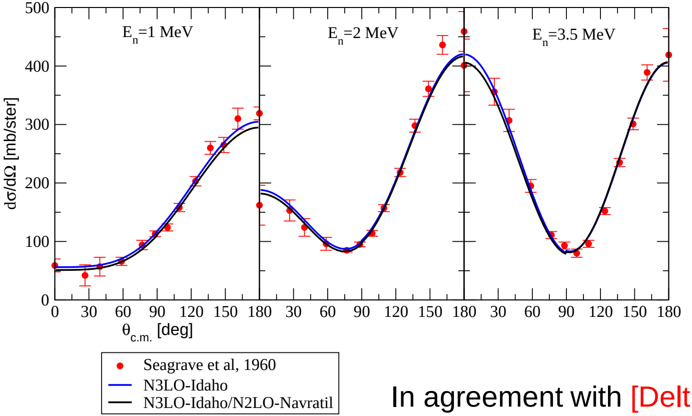


$n - {}^3\mathrm{H}$ elastic scattering

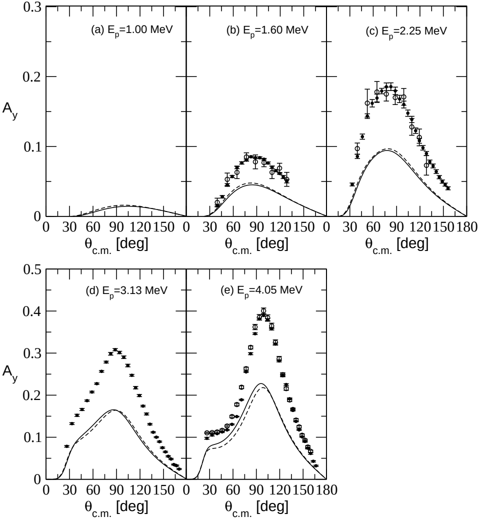
Expt: Seagrave *et al.*, 1960



In agreement with [Deltuva & Fonseca, 2007]

$p - {}^3\mathrm{He}$ elastic scattering - A_y puzzle

old HH calculation - TUNL data Fisher *et al*, 2006



Neutron star crust

- The tetraneutron is not bound [Pieper, 2003] [Lazauskas & Carbonell, 2005]
- May neutron clusters exist in the neutron star crust?

The neutron star crust

- Density $10^6 \div 10^{10}$ g cm^{-3}
  - cubic Coulomb lattice of fully ionized (neutron rich) nuclei
  - a gas of "free" (delocalized) neutrons
- It explains a variety of observations (pulsar glitches, giant flares, X-ray bursts, etc) [see, for example, Chamel, `ArXiv:0709.3798`]
- The neutrons can be "confined" by a variety of hypothetical effects
  - "neutron-lattice" interaction
  - very intense magnetic fields
  - holes in the lattice

Question: which is the magnitude of the force needed to bind 4 neutrons?

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Adiabatic method

1. Schrödinger equation

$$\left\{-\frac{\hbar^2}{M}\left(\frac{\partial^2}{\partial\rho^2}+\frac{8}{\rho}\frac{\partial}{\partial\rho}+\frac{\Lambda^2(\Omega)}{\rho^2}\right)+V(\rho,\Omega)-E\right\}\Psi=0$$

2. Adiabatic HH functions

$$\left\{-\frac{\hbar^2}{M}\frac{\Lambda^2(\Omega)}{\rho^2}+V(\rho,\Omega)\right\}\Phi_m(\rho,\Omega)=U_m(\rho)\Phi_m(\rho,\Omega)$$

3. Use the HH expansion $\Phi_m(\rho,\omega)=\sum_{[K]}^{K_{max}} a_{m,\mu}(\rho)\mathcal{Y}_\mu(\Omega)$
4. Search $\Psi=\sum_m f_m(\rho)\Phi_m(\rho,\Omega)$ and solve the coupled equations

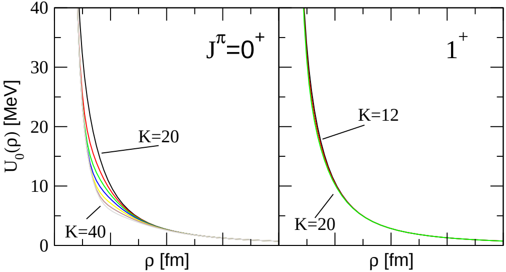
$$\left[-\frac{\hbar^2}{M}f''_m(\rho)+U_m(\rho)\right]-\frac{\hbar^2}{M}\sum_{m'}\left[2P_{m,m'}(\rho)\frac{d}{d\rho}+Q_{m,m'}(\rho)\right]f_{m'}(\rho)=Ef_m(\rho)$$

5. To give an idea, look at the lowest eigenpotential $U_0(\rho)$

Lowest adiabatic potential $U_0(\rho)$

Very preliminary

AV18 potential States $T = 2$, $T_z = -2$



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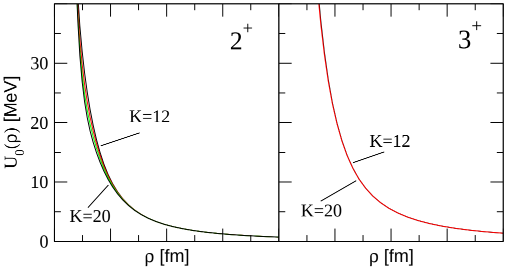

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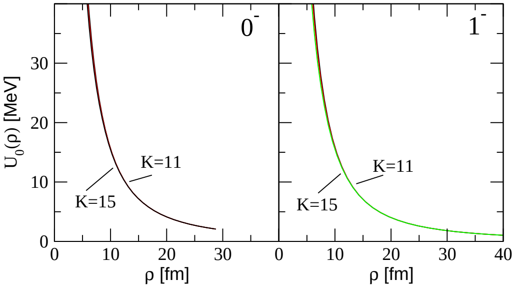

Magnitude of the extra confining potential $\sim 5 \div 10$ MeV, for interparticle distances ~ 2 fm

Future work: add 3N force / try different interaction models

Work in progress

- Four-body scattering:
  - Benchmark with Deltuva & Fonseca:
  - N3LO-Idaho & AV18
  - NN+3N vs explicit inclusion of Δ
- Study of parity-violation in few-nucleon systems
  - Neutron spin rotation in $\vec{n} - d$ scattering PRC **78**, 014002 (2008)

Future work

- breakup channels (starting with $N + d \to N + p + n$)
- Extension to $A > 4$ - See talk by Gattobigio (INL, Nice) - in collaboration with P. Barletta (UCN, London) & C. Romero-Redondo (CSIC, Madrid)

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