Gauge-scalar lattice spectroscopy

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LQCDPheno summer school • Aussois, 04.07.2023

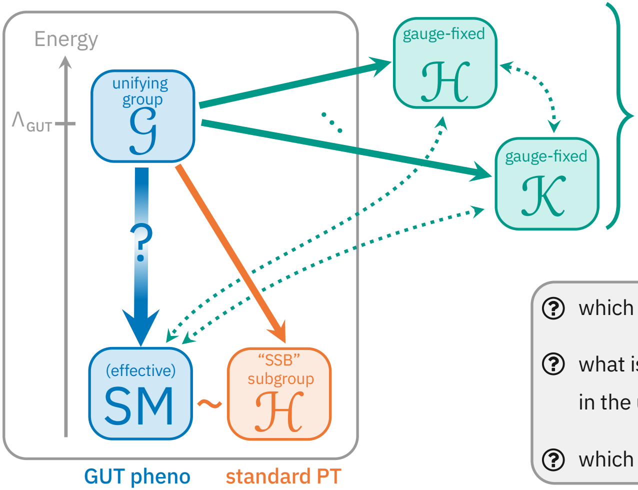
Grand Unified Theories
importance of systematic control

Gauge invariance
BRST breaks down for nonabelian theories
elementary fields are unphysical
the Fröhlich–Morchio–Strocchi mechanism

Lattice spectroscopy
toy SU(3) model to test FMS mechanism
discrepancies with naive perturbation theory

Main message:
naive perturbation theory can’t be trusted for predicting GUT spectra

Gauge-invariant approach to grand unified theories



our approach

- subgroup depends on gauge choice
- different spectra possible
- nontrivial mapping to SM spectrum
- no assumption that BEH $\Longrightarrow$ low-energy

? which assumptions carry over from SM pheno?

? what is the correspondence between bound states in the unbroken theory and SM observables?

? which GUT groups are plausible?

Elementary fields form an unphysical state space

nonabelian gauge group + local gauge-fixing condition:

no unique solutions beyond PT

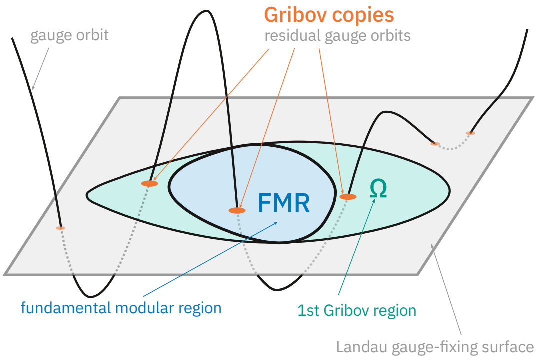
BRST insufficient to fix gauge

ξ-invariance $\nRightarrow$ gauge invariance

perturbative state space is gauge-dependent

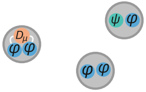
elementary fields (and e.g. Higgs vev)

are not reliable order parameters



Fröhlich–Morchio–Strocchi approach: composite states

elementary:	ψ	$W_\mu^{(a)}$	φ
composite:	$\varphi^\dagger\psi$	$i\varphi^\dagger D_\mu\varphi$	$\varphi^\dagger\varphi$
	fermion	vector boson	“Higgs”



fix to choice of gauge with $\langle\varphi\rangle \neq 0$

Higgs mechanism: $\mathcal{G}$ —breaks to→ $\mathcal{H}$

physical observable: $\mathcal{G}$-singlet $\Longleftrightarrow$ sum of $\mathcal{H}$-singlets

exact, nonperturbative relation

boundstate-boundstate correspondence after gauge-fixing is nontrivial in general:

important for BSM model building!

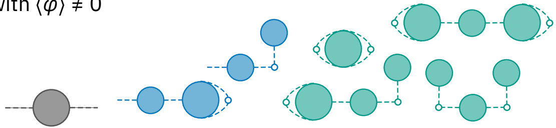
Compare: composite “Higgs” and 1^- vector singlet

[here: SU(N) Yang–Mills with single fundamental scalar]

expand in choice of gauge with $\langle\varphi\rangle \neq 0$

e.g. $\varphi(x) = v\hat{n} + \eta(x)$

$h(x) = 2\,\mathrm{Re}[\hat{n}^\dagger\eta(x)]$



$$\underbrace{\langle(\varphi^\dagger\varphi)(x)(\varphi^\dagger\varphi)(y)\rangle_c}_{\text{bound-state mass}} = v^2\langle h(x)h(y)\rangle_c + \underbrace{2v\langle h(x)(\eta^\dagger\eta)(y)\rangle_c + \langle(\eta^\dagger\eta)(x)(\eta^\dagger\eta)(y)\rangle_c}_{\text{extra terms neglected in standard picture}}$$

coincides with standard PT

$$\underbrace{\langle(\varphi^\dagger D_\mu\varphi)^\dagger(x)(\varphi^\dagger D_\nu\varphi)(y)\rangle_c}_{\text{singlet} \implies \text{only one pole}} = v^2 c^{ab}\langle W_\mu^{(a)}(x)W_\nu^{(b)}(y)\rangle_c + \underbrace{O(\eta/v) + \ldots}_{\text{don't affect pole structure}}$$

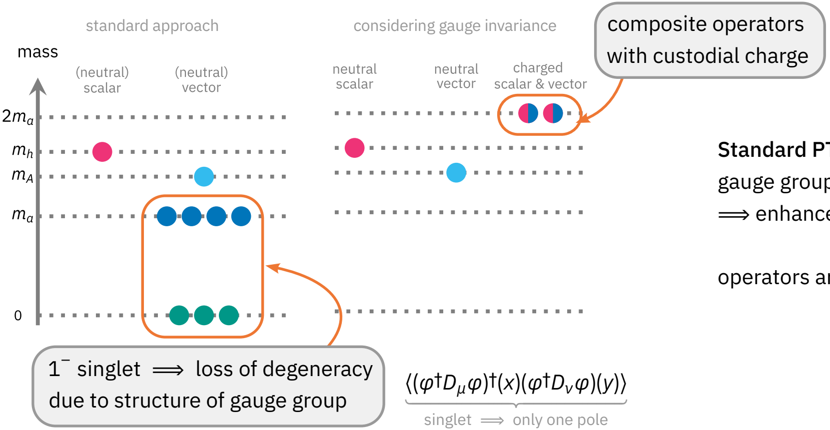
conflicts with standard PT for SU($N > 2$)

poles coincide to all orders in perturbation theory!

Maas and Sondenheimer, arXiv: 2009.06671v2 | Fröhlich, Morchio and Strocchi, Phys. Lett. B (1980) & Nucl. Phys. B (1981)

Gauge invariance qualitatively changes the spectrum

[here: SU(N) Yang–Mills with single fundamental scalar]



Standard PT still works well when:

gauge group is fundamental SU(2)

$\Longrightarrow$ enhanced charge symmetry

or

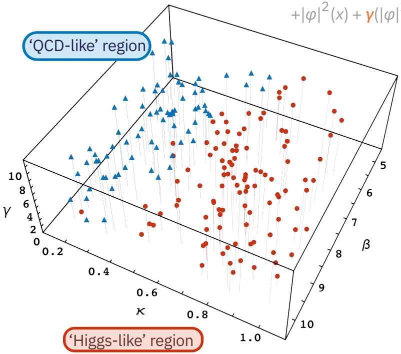
operators are explicitly gauge invariant

Testing FMS on the lattice: a toy SU(3) + YM model

$$\mathcal{L} = \frac{1}{2}\,\mathrm{tr}(W_{\mu\nu}W^{\mu\nu}) + |D\varphi|^2 - \lambda(|\varphi|^2 - f^2)^2$$

$$\beta\,\mathrm{Re\,tr}\sum_{\mu<\nu}[\,\mathbb{1} - U_{\mu\nu}(x)] - \kappa\sum_{\pm\mu}\varphi^\dagger(x)U^R_\mu(x)\varphi(x+\hat{\mu})$$

$$+|\varphi|^2(x) + \gamma(|\varphi|^2(x) - 1)^2$$



$$\lambda \sim \frac{\gamma}{\kappa^2} \qquad g \sim \frac{1}{\sqrt{\beta}}$$

Generalisation of SM gauge-weak sector

single scalar $\varphi \in \mathrm{SU}(3)$ or $\varphi \in \mathrm{su}(3)$

Breaks to nontrivial gauge group

$\mathrm{SU}(3) \to \mathrm{SU}(2)$ or $\mathrm{SU}(2) \times \mathrm{U}(1)$, $\mathrm{U}(1) \times \mathrm{U}(1)$

Nontrivial custodial group

global U(1) or Z_2

- ? what is the stable spectrum?
- ? are the lighter states charged?
- ? do lattice results support FMS?

Lattice results: Maas and Törek, arXiv:1804.04453

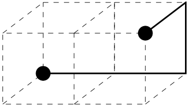
Constructing operators in different channels

$$J^{PC}_{G;\Lambda}$$

spin, parity, charge parity, lattice irrep, custodial charge

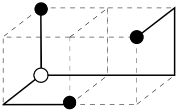
$\varphi^\dagger \cdot (D_{\mu_1} \dots D_{\mu_n} \varphi)$

U(1)-neutral, gauge-scalar



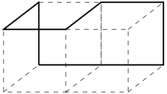
$(D_{\mu_1} \dots D_{\mu_{n_1}} \varphi) \cdot \left[(D_{\nu_1} \dots D_{\nu_{n_2}} \varphi) \times (D_{\rho_1} \dots D_{\rho_{n_3}} \varphi) \right]$

U(1)-charged, gauge-scalar



$\mathrm{tr}\left[(D_{\mu_1} \dots D_{\mu_{n_1}} F_{\nu_1 \rho_1}) \dots (D_{\sigma_1} \dots D_{\sigma_{n_R}} F_{\nu_R \rho_R}) \right]$

gaugeball



States for any (J, M) via 'ladder operators':

$\tilde{D}_\pm = \mp i (D_1 \pm i D_2)/\sqrt{2}\,, \quad \tilde{D}_0 = i D_3$

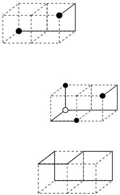
Continuum → lattice: project onto O_h irreps via Clebsch–Gordan coefficients

Project to required parity/charge parity

Smear links and scalars to enlarge basis (links: stout; scalars: APE)

Dudek et al., arXiv:1004.4930 | Wurtz and Lewis, arXiv:1307.1492

Implementation details



Setup

SU(3) + YM + single scalar

3D coupling space (β, κ, γ)

isotropic lattice: $L = 10, 12, \ldots, 32$

Gauge fixing

Landau 't Hooft or Unitary

Stochastic OR

Operator basis

(on fixed timeslice)

Heatbath + OR updates

- Cabbibo–Marinari method
- Scalar OR: rotate $\varphi(x)$ around vector $\propto \frac{\partial S}{\partial \varphi(x)}$
- Adjoint case: approx. HB/OR
  + accept/reject step

Smearing

Stout (links), APE (scalars)

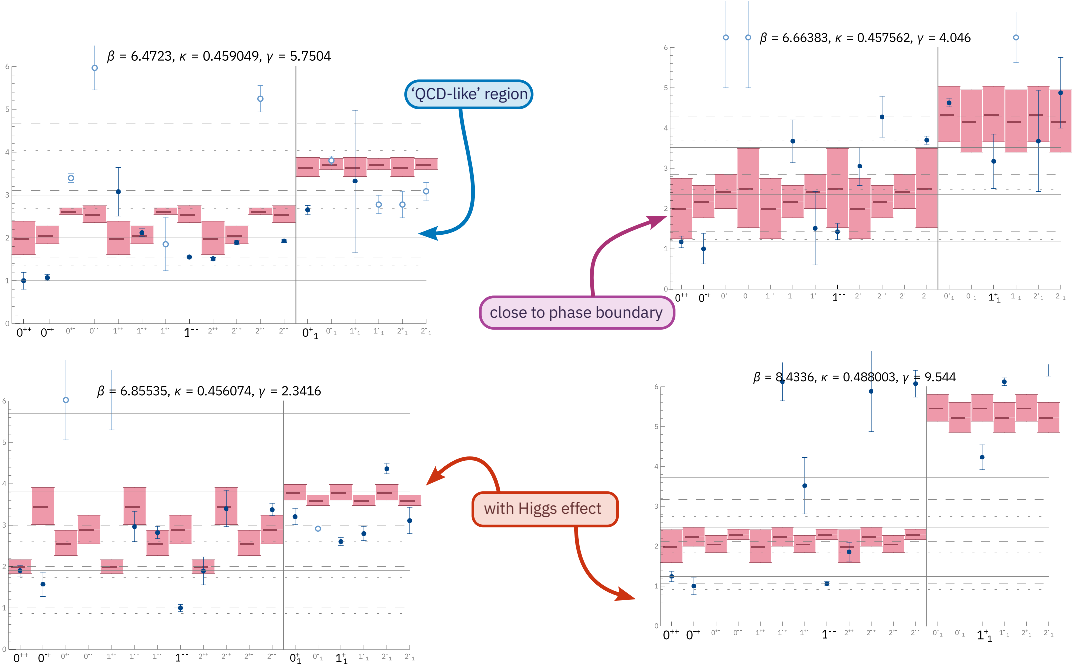
Spectroscopy

variational analysis

fitting to plateaus of $C(t)$

scattering from stable states

$V \to \infty$ extrapolation


β = 6.4723, κ = 0.459049, γ = 5.7504
'QCD-like' region
β = 6.66383, κ = 0.457562, γ = 4.046
close to phase boundary
β = 6.85535, κ = 0.456074, γ = 2.3416
with Higgs effect
β = 8.4336, κ = 0.488003, γ = 9.544
0++ 0-+ 0+- 0-- 1++ 1-+ 1+- 1-- 2++ 2-+ 2+- 2--
0+1 0-1 1+1 1-1 2+1 2-1

Extending to the adjoint-scalar case

[work in progress]

More interesting

multiple breaking patterns

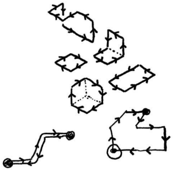
more applications to BSM physics

More difficult

massless modes

challenges in taking continuum limit

(even) noisier

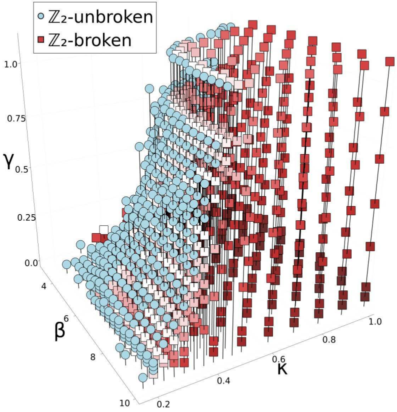


Work in progress: spectroscopy

automatising large operator basis

spectroscopy, scattering analysis

So far: phase diagram



ED, Maas and Riederer — arXiv:2211.05812 (figure)

Outlook and implications for BSM phenomenology

Systematic control matters
gauge invariance has a qualitative effect on nonperturbative spectra
qualitative differences, even at small coupling

Results
qualitative differences from pure Yang–Mills, and from SU(2)
FMS: nontrivial field theory effects can still be treated perturbatively

Applications
constraining GUTs (where lattice tests unfeasible)
meson decay, LFUV...
adjoint case: multiple symmetry breaking patterns

Work in progress
better statistics → excited states
adjoint spectroscopy (more complex)
SSB for global U(1) symmetry

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gauge invariance

↓

nontrivial field-theory effects
(even at weak coupling)

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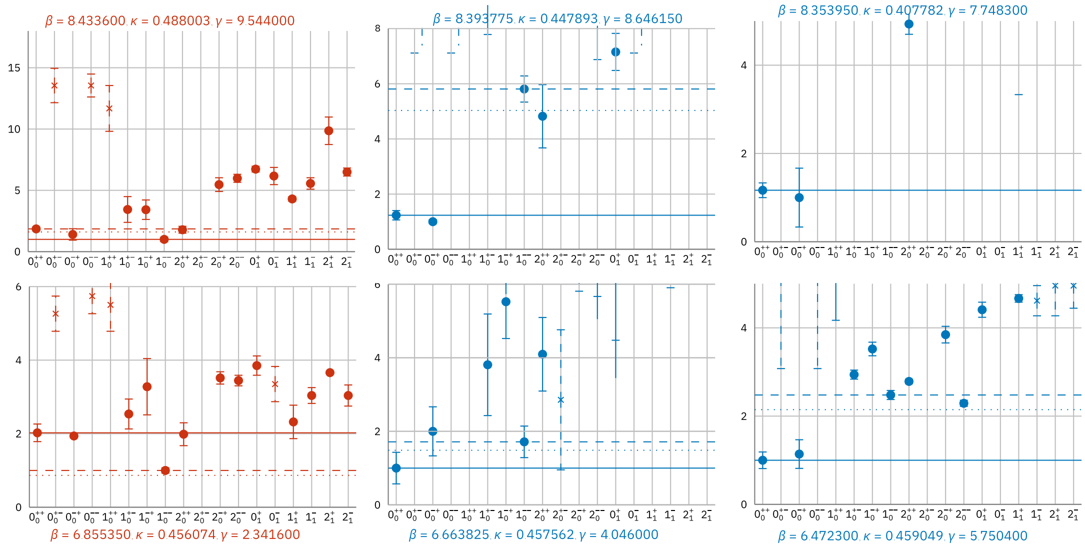
naive perturbation theory can’t be
trusted for BSM model-building

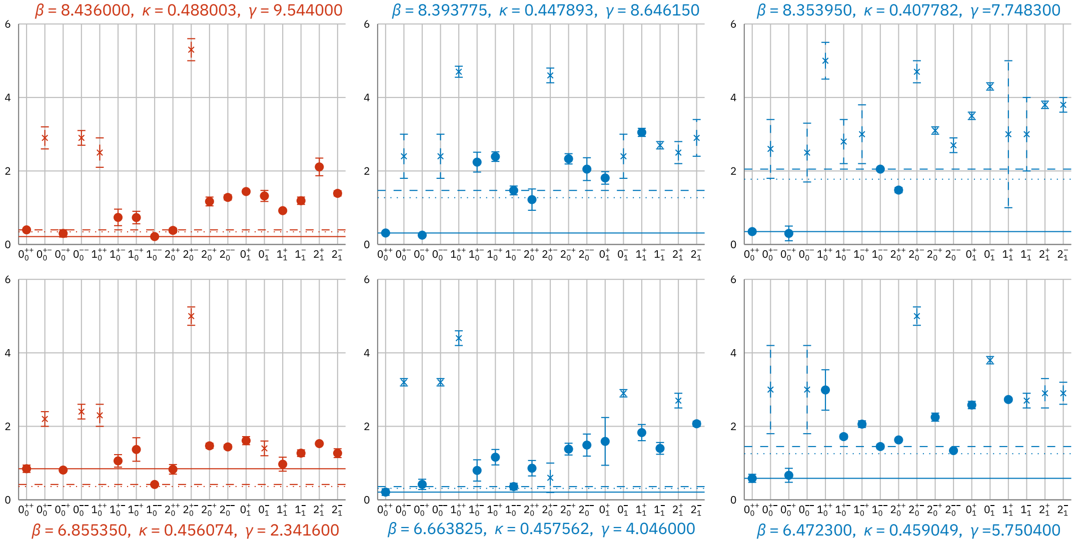
Phase transition: Higgs-like → “QCD-like”

[$V\to\infty$, normalised to lightest mass]



Phase transition: Higgs-like → "QCD-like"

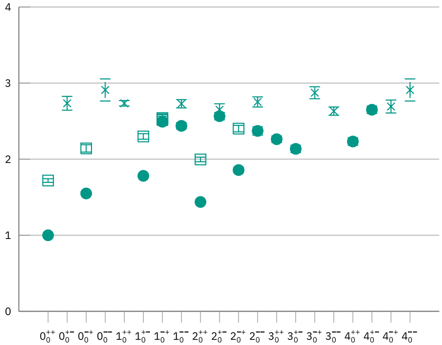
[energy scale = am_{eff} , $V\to\infty$]



Comparison to pure Yang–Mills

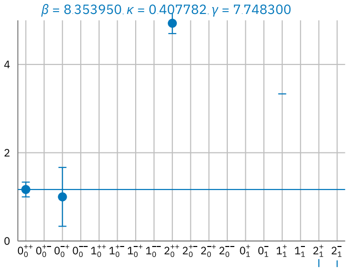
[normalised to lightest mass]

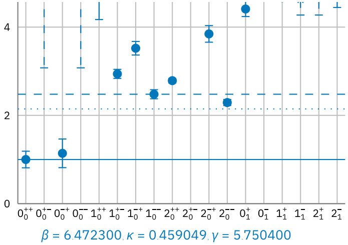
Pure-YM SU(3) case



SU(3) YM + scalar

(deep QCD-like region)

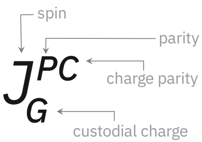


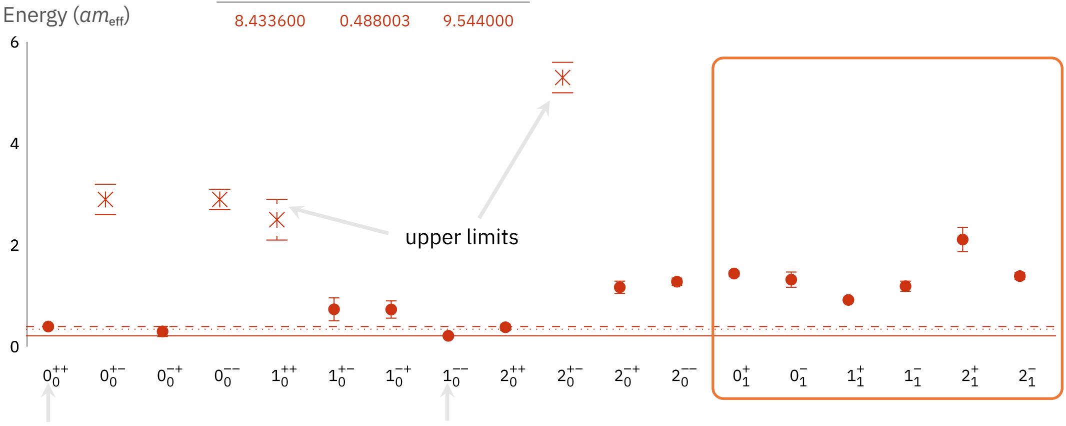


data (left) from Athenodorou and Teper, arXiv:2106.00364

SU(3) fundamental spectrum: additional U(1)-charged states

β	κ	γ
8.433600	0.488003	9.544000





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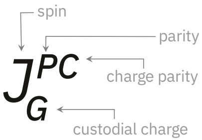

β	κ	γ
8.433600	0.488003	9.544000
6.472300	0.459049	5.75040