QED Corrections to Meson Masses

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QED Mass Corrections

- On the lattice, we can calculate hadron masses like m_π by fitting

$$\langle \mathcal{O}_\pi(T, \vec{0}) \mathcal{O}_\pi(-T, \vec{0}) \rangle \propto e^{-m_\pi (2T)},$$

where $\mathcal{O}_\pi$ is an interpolating operator for the pion.

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- Then

$$\langle \mathcal{O}_\pi(T,\vec{0})\mathcal{O}_\pi(-T,\vec{0})\rangle_{\mathrm{QCD+QED}}$$

$$= \frac{1}{Z}\int e^{\int d^4x\left[\mathcal{L}_{\mathrm{QCD}}+i\bar{\psi}e\gamma_\mu A_\mu\psi+\frac{1}{4}F_{\mu\nu}F_{\mu\nu}\right]}\mathcal{O}_\pi(T,\vec{0})\mathcal{O}_\pi(-T,\vec{0})$$

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$$\begin{aligned} &\langle \mathcal{O}_\pi(T,\vec{0})\mathcal{O}_\pi(-T,\vec{0})\rangle_{\text{QCD+QED}} \\ &= \frac{1}{Z}\int e^{\int d^4x\left[\mathcal{L}_{\text{QCD}}+i\bar{\psi}e\gamma_\mu A_\mu \psi+\frac{1}{4}F_{\mu\nu}F_{\mu\nu}\right]}\mathcal{O}_\pi(T,\vec{0})\mathcal{O}_\pi(-T,\vec{0}) \\ &\approx \frac{1}{Z}\int e^{\int d^4x\left[\mathcal{L}_{\text{QCD}}+\frac{1}{4}F_{\mu\nu}F_{\mu\nu}\right]}\mathcal{O}_\pi(T,\vec{0})\mathcal{O}_\pi(-T,\vec{0}) \\ &\times\left(1+\int d^4xJ_\mu A_\mu+\frac{1}{2}\int d^4xd^4yJ_\mu(x)A_\mu(x)J_\nu(y)A_\nu(y)+\ldots\right) \end{aligned}$$

QED Mass Corrections

- Now we can factor out the QED part and solve it directly

$$\langle \mathcal{O}_\pi(T,\vec{0})\mathcal{O}_\pi(-T,\vec{0})\rangle_{\text{QCD+QED}}$$

$$\approx \frac{1}{Z_{\text{QCD}}}\int e^{\int d^4x\mathcal{L}_{\text{QCD}}}\mathcal{O}_\pi(T,\vec{0})\mathcal{O}_\pi(-T,\vec{0})$$

$$\times\left(1+\frac{1}{2}\int d^4xd^4yJ_\mu(x)J_\nu(y)S_{\mu\nu}(x-y)+\ldots\right)$$

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$$=\langle \mathcal{O}_\pi(T,\vec{0})\mathcal{O}_\pi(-T,\vec{0})\rangle_{\text{QCD}}$$

$$+\frac{1}{2}\int d^4xd^4y\langle J_\mu(x)J_\nu(y)\rangle_{\text{QCD}}S_{\mu\nu}(x-y)+...$$

QED Mass Corrections

- Based on the change in $\langle \mathcal{O}_\pi(T, \vec{0}) \mathcal{O}_\pi(-T, \vec{0}) \rangle$,

$$\Delta m_\pi = \frac{1}{2} \int d^4 x \mathcal{H}_{\mu\nu}(x) S_{\mu\nu}(x),$$

where $\mathcal{H}_{\mu\nu}(x) = L^3 \frac{\langle \mathcal{O}(t+T) J_\mu(x) J_\nu(0) \mathcal{O}_\pi(-T) \rangle}{\langle \mathcal{O}_\pi(t+T) \mathcal{O}_\pi(-T) \rangle}$ on the lattice.

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- Unfortunately, if we simply calculate this integral on the lattice, we get significant finite volume effects.

QED Mass Corrections

- At large distances, $\mathcal{H}_{\mu\nu}(x) = L^3 \frac{\langle \mathcal{O}(t+T) J_\mu(x) J_\nu(0) \mathcal{O}_\pi(-T)\rangle}{\langle \mathcal{O}_\pi(t+T)\mathcal{O}_\pi(-T)\rangle}$ scales like

$$\frac{e^{-m_\pi T} e^{-m_\pi \sqrt{t^2+|\vec{x}|^2}} e^{-m_\pi T}}{e^{-m_\pi(2T+t)}} = e^{-m_\pi(\sqrt{t^2+|\vec{x}|^2}-t)}.$$

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- On the other hand, the photon propagator $S_{\mu\nu}(t, \vec{x})$ is not exponentially suppressed at large t because the photon is massless.
- Therefore, our finite volume errors in $\Delta m_\pi = \frac{1}{2} \int d^4x \mathcal{H}_{\mu\nu}(x) S_{\mu\nu}(x)$ will only be power-law suppressed.

QED Mass Corrections

- To get exponentially suppressed finite volume effects, we can reconstruct the large-distance contributions to Δm_{π}.*

* Xu Feng, Luchang Jin (2019)

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- For large t, we get

$$\mathcal{H}_{\mu\nu}(t,\vec{x}) = \int d^3\vec{x}\mathcal{H}_{\mu\nu}(t_s,\vec{x})$$

$$\times \int \frac{d^3\vec{p}}{(2\pi)^3} e^{-i\vec{p}\cdot(\vec{x}'-\vec{x})} e^{-(E_{n,\vec{p}}-m_\pi)(t-t_s)},$$

where t_s is a reference time large enough that $\mathcal{H}_{\mu\nu}(t_s,\vec{x})$ is dominated by the lowest-energy state, but small enough to be computed on the lattice.

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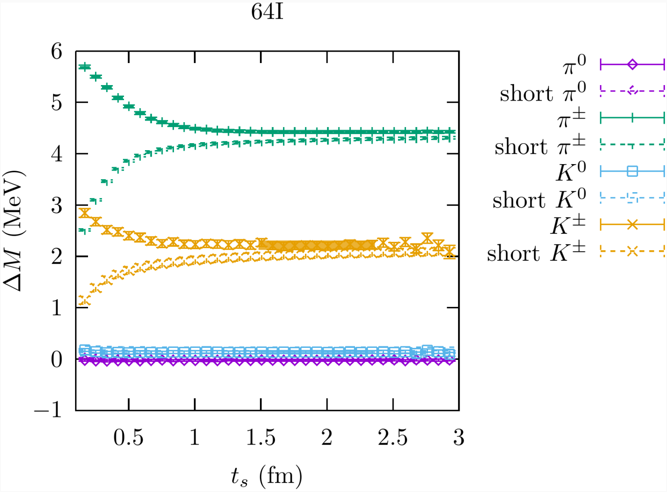

Figure 1: Δm versus t_s for various mesons on a $64^3 \times 128$ lattice using the Iwasaki gauge action