Hamiltonian Limit of Lattice QED in 2+1 Dimensions

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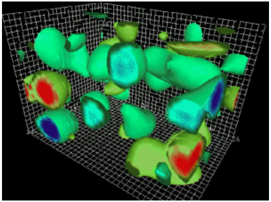
Motivation

Lagrangian

Wilson 1974 [1]

discrete time

classical computers



Deep Hybrid [2]

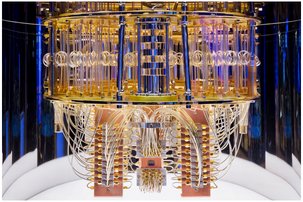
Hamiltonian

Kogut, Susskind 1975 [3]

continuous time

Exact Diagonalization

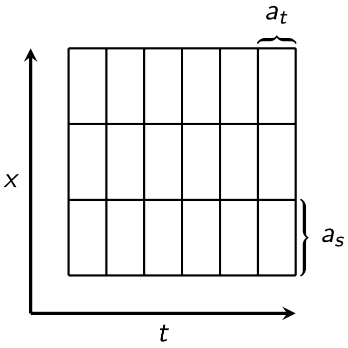
Quantum Computers



Engagdet[4]

Match to test universality

What is anisotropy?



adapted from [5], [6]

$$S = \sum_{r,i} \frac{\beta}{\xi_{\text{input}}} \operatorname{Re}\operatorname{Tr}\left(1 - P_{0i}(r)\right) + \sum_{r,i>j>0} \beta \xi_{\text{input}} \operatorname{Re}\operatorname{Tr}\left(1 - P_{ij}(r)\right)$$

$$\xi_{\text{ren}} = \frac{a_t}{a_s}$$

$$\xi_{\text{input}} \text{ set as } \frac{L}{T}$$

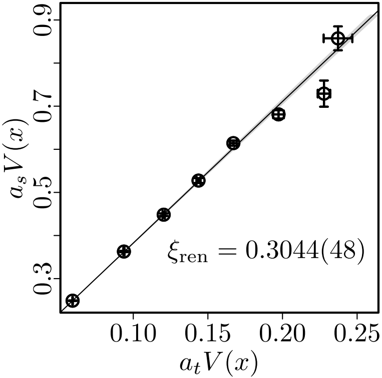
How to determine anisotropy?

$$\lim_{x\to\infty} \frac{W_{ss}(x, y+1)}{W_{ss}(x, y)} = \exp(-a_s V_s(x[a_s]))$$

$$\lim_{t\to\infty} \frac{W_{st}(x, t+1)}{W_{st}(x, t)} = \exp(-a_t V_s(x[a_s]))$$

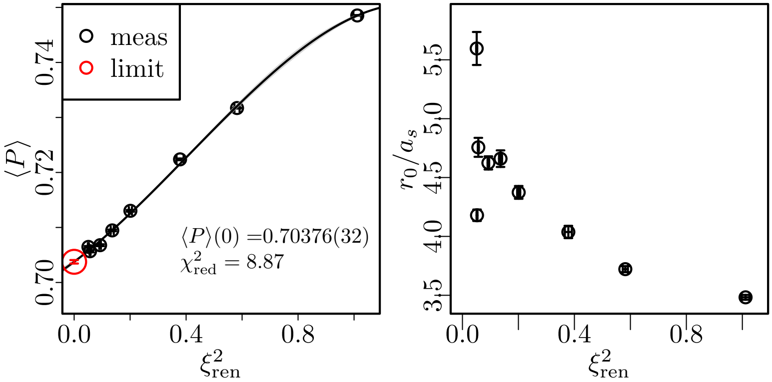
$$a_s V(x[a_s]) = \frac{1}{\xi} a_t V(x[a_s]) + se'$$

adapted from [5]



$L = 16,\ T = 48,\ \beta = 1.7$

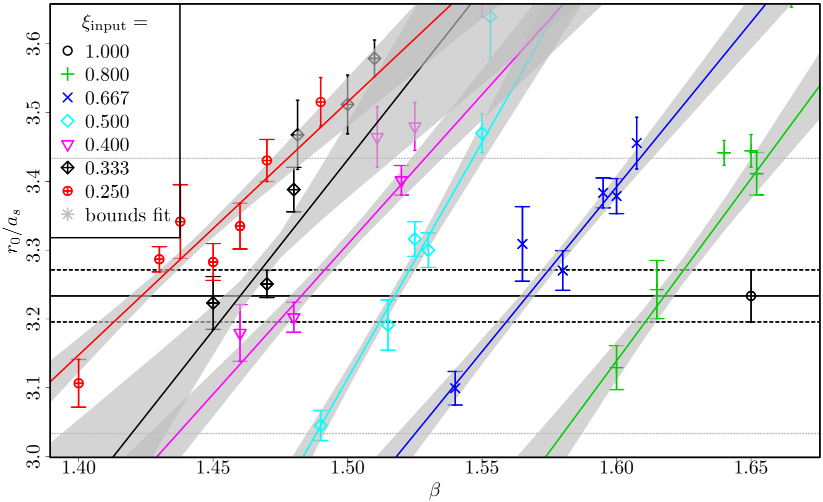
Scale setting



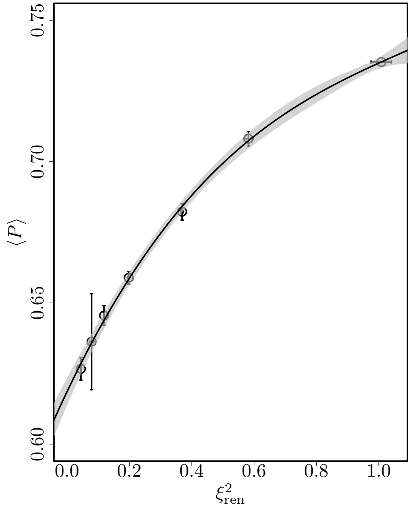
$L = 16,\ \beta = 1.7$

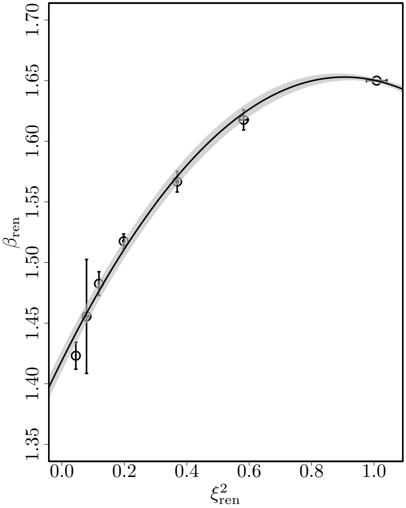
$\langle P\rangle(\xi^2_{\text{ren}}) = a_0 + a_1\xi^2_{\text{ren}} + a_2\xi^4_{\text{ren}} + a_3\xi^6_{\text{ren}}$

Scale setting



Continuum limit





Conclusion

Done

Determined anisotropy

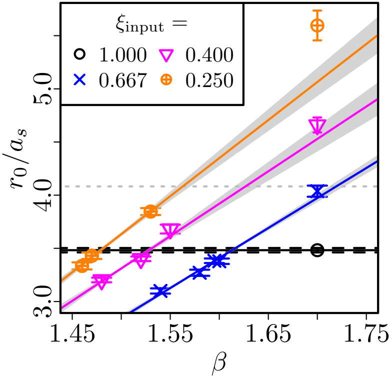
nonperturbative renormalization of ξ, β

2 continuum limits in a_t

Published proceeding[7]

To Do

Finite Volume



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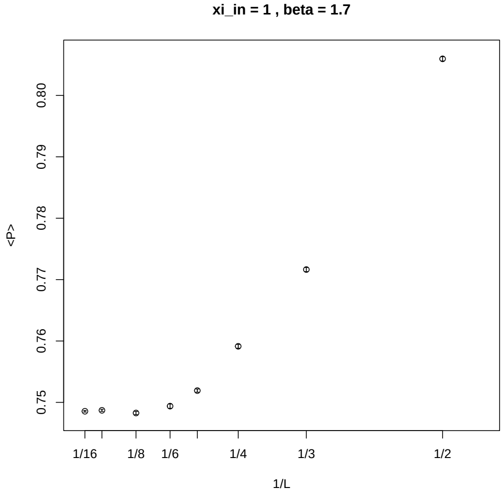
Backup slides - finite volume effects

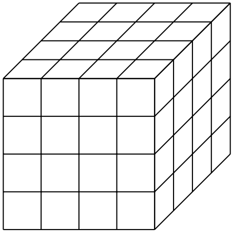


Backup slides

Master thesis talk, October 2022

Motivation

- Lattice: non-perturbatively
- Lagrangian formalism[1]: discrete time
- Hamiltonian formalism[2]: continuous time
- Test universality
- U(1) in (2+1) dimensions

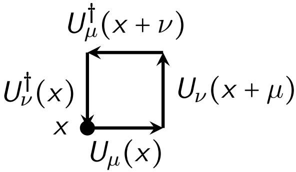


Discretizing the Action

taken from [3]

$$S[A] = \frac{1}{2g^2} \int \mathrm{d}^3x \, \mathrm{Tr}\left[F_{\mu\nu}(x) F_{\mu\nu}(x)\right]$$

$$U_\mu(n) = \exp\big(iaA_\mu(n)\big)$$



$$P_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\mu) U_\mu^\dagger(x+\nu) U_\nu^\dagger(x)$$

$$S = \beta \sum_r \sum_{\mu>\nu} \mathrm{Re}\,\mathrm{Tr}\big(1 - P_{\mu\nu}(r)\big)$$

Metropolis-Hastings

taken from [3]

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[U] O(U) \mathrm{e}^{-S[U]}$$

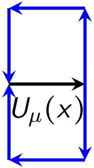
$$Z = \int \mathcal{D}[U] \mathrm{e}^{-S[U]}$$

importance sampling:

$$\langle O \rangle = \frac{1}{N} \sum_{n=1}^{N} O[U_n]$$

transition probability[6]:

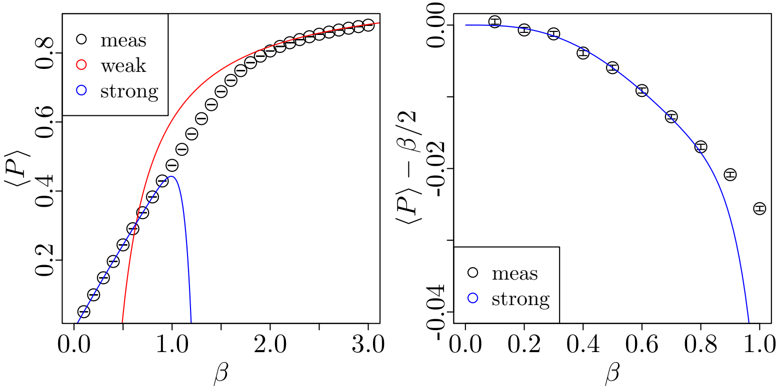
$$p = \min\left(1, \exp(-\Delta S)\right)$$



$$\Delta S/\beta$$
$$= \mathrm{Re}\,\mathrm{Tr}\Big(\left(U_\mu(x) - U'_\mu(x)\right) \cdot \Gamma_\mu(x)\Big)$$
$$U'_\mu(x) = U_\mu(x) \cdot R(\delta)$$

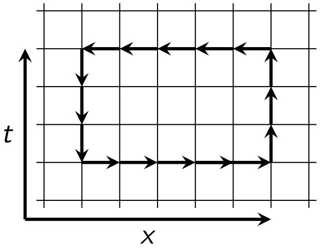
Plaquette: Code check

Configurations with SU(2)-package, analysis with hadron-package



Predictions: [7], [8], $L = 16$

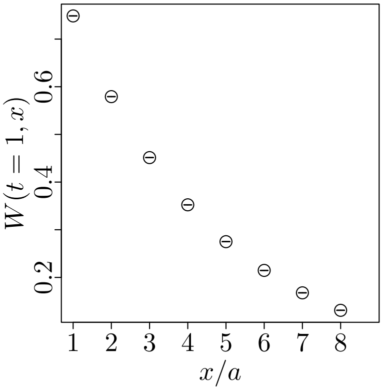
Wilson Loops and Static Potential



$$W(t,r) = \sum_{k=0}^{\infty} C(r) \exp\Big(-atE_k(r)\Big)$$

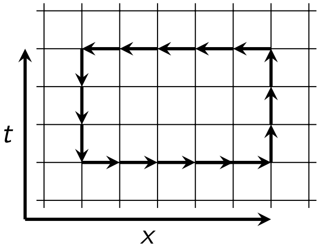
$$\lim_{t\to\infty} \frac{W(t+1,r)}{W(t,r)} = \exp\big(-aV(r[a])\big)$$

[3], [4]



$L = 16,\ \beta = 1.7$

Wilson Loops and Static Potential



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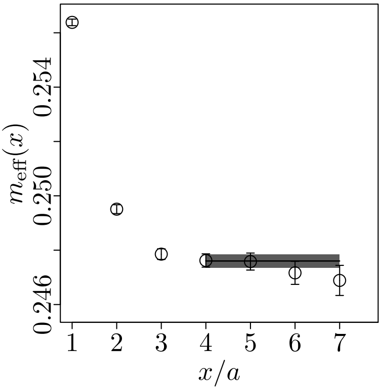
[3], [4]



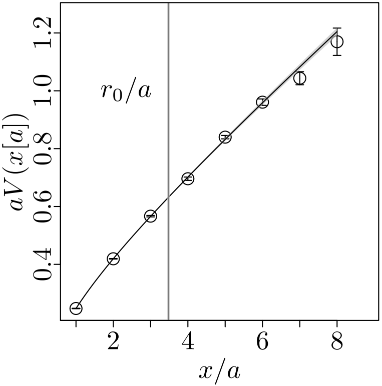
$L = 16,\ \beta = 1.7$

Setting the scale

$$V(r) = d + \sigma \cdot r + b \cdot \ln(r)$$

$$-r^2 \frac{\mathrm{d}}{\mathrm{d}r} V(r)\big|_{r=r_0} = c = -1.65$$

[9], [5]



$L = 16, \ \beta = 1.7$

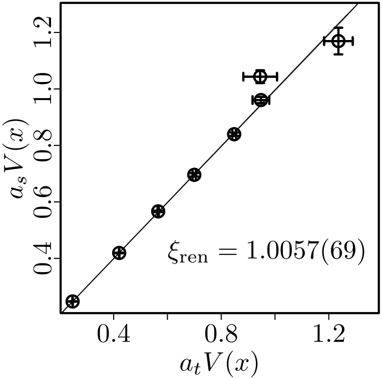
Normal potentials

$$\lim_{x\to\infty} \frac{W_{ss}(x, y+1)}{W_{ss}(x, y)} = \exp\left(-a_s V_s(x[a_s])\right)$$

$$\lim_{t\to\infty} \frac{W_{st}(x, t+1)}{W_{st}(x, t)} = \exp\left(-a_t V_s(x[a_s])\right)$$

$$a_s V(x[a_s]) = \frac{1}{\xi} a_t V(x[a_s]) + se'$$

adapted from [4]



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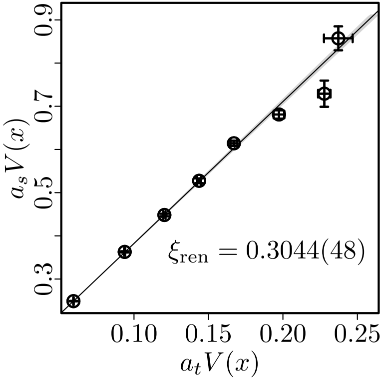


adapted from [4]

$L = 16,\ T = 48,\ \beta = 1.7$

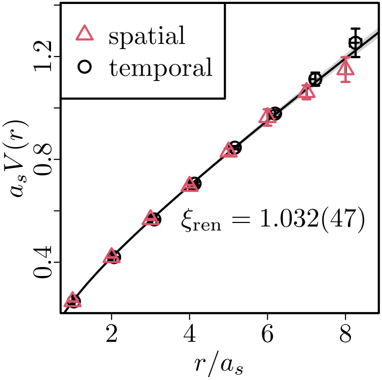
Sideways potentials

$$\lim_{x\to\infty} \frac{W_{ss}(x+1,y)}{W_{ss}(x,y)} = \exp\big(-a_s V_s(y[a_s])\big)$$

$$\lim_{x\to\infty} \frac{W_{st}(x+1,t)}{W_{st}(x,t)} = \exp\big(-a_s V_s(t[a_t])\big)$$

$$V(y[a_s]) \overset{!}{=} V\left(\frac{1}{\xi_{\text{ren}}} t[a_t]\right)$$

adapted from [10]



$L = 16,\ T = 16,\ \beta = 1.7$

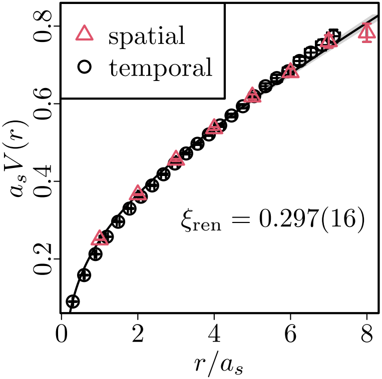
Sideways potentials

$$\lim_{x\to\infty} \frac{W_{ss}(x+1,y)}{W_{ss}(x,y)} = \exp\bigl(-a_s V_s(y[a_s])\bigr)$$

$$\lim_{x\to\infty} \frac{W_{st}(x+1,t)}{W_{st}(x,t)} = \exp\bigl(-a_s V_s(t[a_t])\bigr)$$

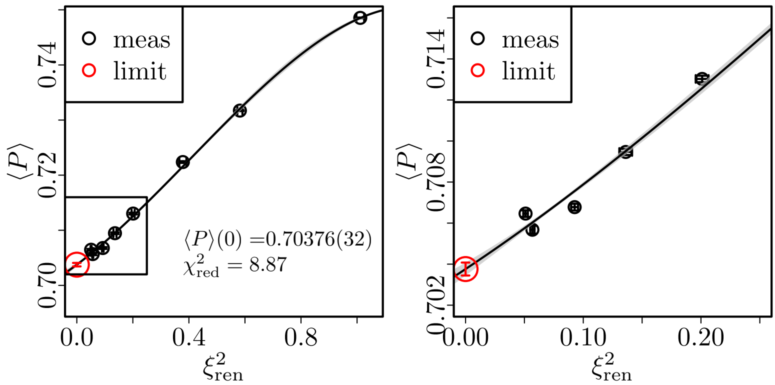
$$V(y[a_s]) \stackrel{!}{=} V\left(\frac{1}{\xi_{\text{ren}}} t[a_t]\right)$$

adapted from [10]



$L = 16,\ T = 48,\ \beta = 1.7$

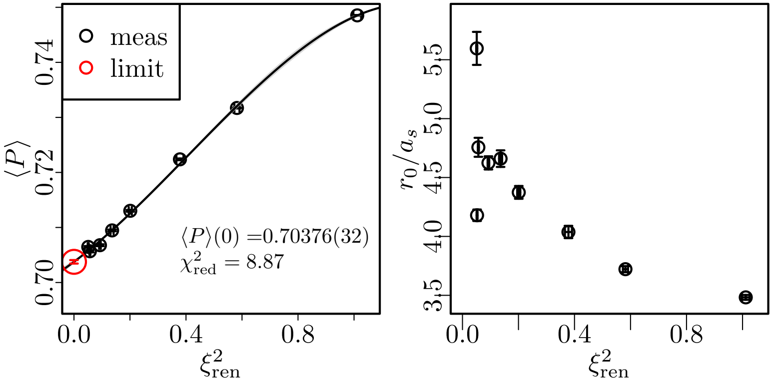
Naive continuum limit



$L = 16,\ \beta = 1.7$

$$\langle P \rangle(\xi^2_{\text{ren}}) = a_0 + a_1\xi^2_{\text{ren}} + a_2\xi^4_{\text{ren}} + a_3\xi^6_{\text{ren}}$$

Naive continuum limit

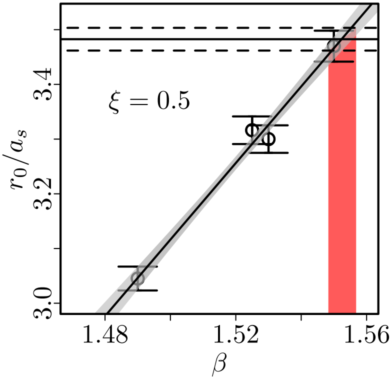


$L = 16,\ \beta = 1.7$

$\langle P\rangle(\xi^2_{\text{ren}}) = a_0 + a_1\xi^2_{\text{ren}} + a_2\xi^4_{\text{ren}} + a_3\xi^6_{\text{ren}}$

Renormalization of β

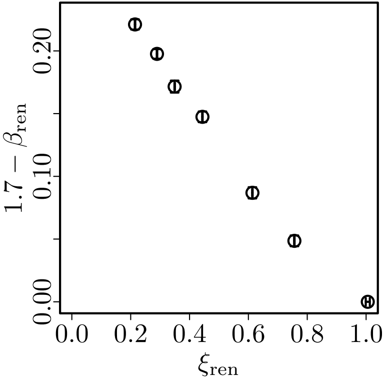
- r_0/a_s for constant lattice spacing
- Linear interpolations of $r_0/a_s(\beta)$, $\langle P\rangle(\beta)$, $\xi_{\text{ren}}(\beta)$
- $\beta_{\text{ren}} : r_0(\beta_{\text{ren}}) = r_0(\xi_{\text{input}} = 1)$



$L = 16, \ T = 32$

Renormalization of β

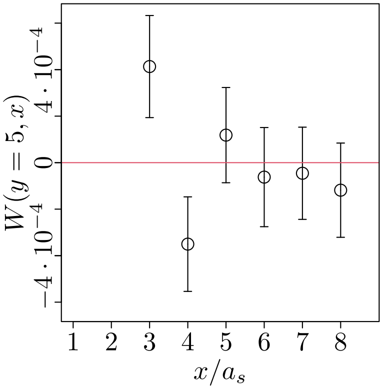
- r_0/a_s for constant lattice spacing
- Linear interpolations of $r_0/a_s(\beta)$, $\langle P\rangle(\beta)$, $\xi_{\text{ren}}(\beta)$
- $\beta_{\text{ren}} : r_0(\beta_{\text{ren}}) = r_0(\xi_{\text{input}} = 1)$
- Pairs $(\xi_{\text{ren}}, \langle P\rangle)(\beta_{\text{ren}})$



$L = 16$

Difficulties with this method

Fluctuations at small β

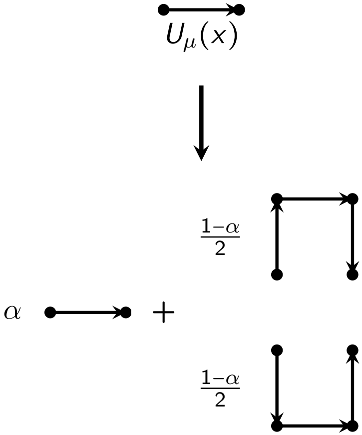


$L = 16,\ T = 16,\ \beta = 1.0$

Difficulties with this method

Fluctuations at small β

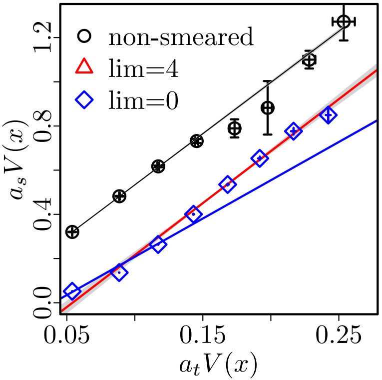
- smearing[5]



Difficulties with this method

Fluctuations at small β

- smearing[5]

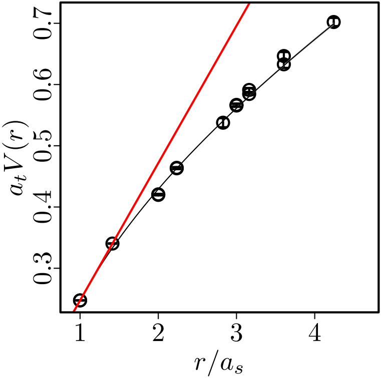


$\beta = 1.48, \ n = 3, \ \alpha = 0.7$

Difficulties with this method

Fluctuations at small β

- smearing[5]
- use non-integer distances



$L = 16$, $T = 16$, $\beta = 1.7$