Flavor physics for BSM searches

Jérôme Charles

Centre de Physique Théorique (Marseille)

LQCDpheno School in Aussois (2023)

cnrs
CPT Centre de Physique Théorique
Faculté des Sciences Aix*Marseille Université

Preamble

Among the ~ 20 Nobel Prizes in/around particle physics since WWII, seven are related to flavor transitions:

1957 Lee & Yang (theory of parity violation in weak currents)

1980 Cronin & Fitch (discovery of CP violation)

1988 Lederman (discovery of ν_μ and parity violation)

2002 Koshiba (discovery of neutrino oscillations)

2008 Kobayashi & Maskawa (mechanism of CP violation)

2013 Englert & Higgs (EW symmetry breaking)

2015 Kajita & McDonald (neutrino oscillations)

The Standard Model (SM) has $18 + 1 = 19$ parameters.

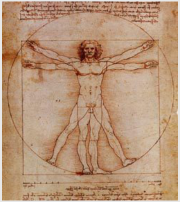
Three of them are completely flavor-blind: the gauge couplings g_S, g, g'.

The fourth, $\theta_{\rm QCD}$, violates CP but is flavor-blind (usually set to 0, experimental bound is about 10^{-11}); it is the coupling of the operator $\frac{1}{16\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu}$. [see M.-P. Lombardo's lecture]

The remaining 15 parameters are all related to electroweak and flavor symmetry breaking, through the scalar sector of the SM: the Higgs expectation value v and mass m_H, the 9 fermions masses m_f, and the four quark mixing parameters A, λ, $\bar{\rho}$, $\bar{\eta}$.

the electroweak and flavor symmetry breaking sector is the most arbitrary and the least well understood part of the Standard Model

$\Rightarrow$



?

Outline

The history of flavor physics

Antiquity

1896 discovery of the radioactivity of the uranium (Becquerel)

1898 thorium, polonium, radium (Curie^2)

1899 distinction between α and β decay (Rutherford)

1930 “invention” of the neutrino (Pauli)

Middle Age

1951-1954 CPT conservation theorem (Schwinger, Lüders & Pauli)

1956-1957 postulate and discovery of parity violation (Lee & Yang, Wu et al., Garwin & Lederman)

1964 discovery of charge $\times$ parity violation (Cronin & Fitch)

1973 mechanism(s) of CP violation in the “Standard” model (Kobayashi & Maskawa)

Modern Era

- 1998 discovery of time-reversal violation (CPLEAR)
- 1998 discovery of neutrino oscillations (Super-Kamiokande)
- 1999 direct CP violation in the kaon system (KTeV, Na48)
- 2001 mixing-induced CP violation in the B system (BaBar, Belle)
- 2004 direct CP violation in the B system (BaBar, Belle)

Postmodern Era

- 2008 Nobel Prize to Kobayashi and Maskawa for their successful mechanism of CP violation in the Standard Model
- 2014 first discovery of very rare FCNC decay $B_s \to \mu\mu$ (LHCb, CMS)
- since $\sim$ 10 years a few hints against the SM are showing up (and down)

The Standard Model

It is defined by:

- The gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$.
- The fermion content: three generations of quarks and leptons
  $\Psi_Q = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$, $\Psi_\ell = \begin{pmatrix} \nu_{\ell,L} \\ \ell_L \end{pmatrix}$, U_R, D_R, ℓ_R $(U = u, c, t,\ \ell = e, \mu, \tau)$
  where quarks (leptons) live in the fundamental (singlet) representation of $SU(3)_c$.
- The scalar sector $\Phi = \left(\phi^+, \phi^0\right)$
- The (by hand) choice of the vacuum $\langle\Phi\rangle = (0, v/\sqrt{2})$
  indeed this is the simplest way to give mass to the gauge bosons $W^\pm$, Z^0 in a gauge invariant way.

As soon as one requests that the SM is perturbatively renormalisable, all the kinetic and interaction terms follow from the above choices, ending with a Lagrangian depending on 19 free parameters.

No other model with less parameters and consistent with the data has been shown to exist so far. . .

The Yukawa sector

Because of weak chirality, naive ($L \times R$) mass terms for the fermions are forbidden by gauge symmetry; quadratic fermion terms comes from the Yukawa interactions with the Higgs doublet.

$$\mathcal{L}_Y = \bar{\Psi}_Q \Lambda_D D_R \Phi + \bar{\Psi}_Q \Lambda_U U_R \tilde{\Phi} + \bar{\Psi}_\ell \Lambda_\ell \ell_R \Phi + \mathrm{h.c.}$$

where the Λ's are 3×3 complex matrices in the family space.
Mass terms: replace the Higgs field by its expectation value;
diagonalization: $\Lambda = V \Delta W^\dagger$ where V, W are unitary and Δ is diagonal.
Consequences

- The mass eigenstates are $\hat{D}_L = V_D^\dagger D_L$, $\hat{D}_R = W_D^\dagger D_R$ and similarly for $U_{L,R}$, $\ell_{L,R}$.
- Tthe couplings of the weak current are given by the matrix $V_{\mathrm{CKM}} \equiv V_U^\dagger V_D$.
- The neutral current conserves the flavor (no FCNC at tree level).
- The massless neutrinos remain massless eigenstates in any basis.

The Higgs boson

The Yukawa interactions, hence the Higgs, determine the flavor structure of the SM; what do we know about it ?

In the physical unitary gauge, only one degree of freedom survives: there is a single neutral Higgs boson in the SM, with unpredicted mass (depending on its self-coupling).

Until 2012 it was the only unobserved particle in the Standard Model.

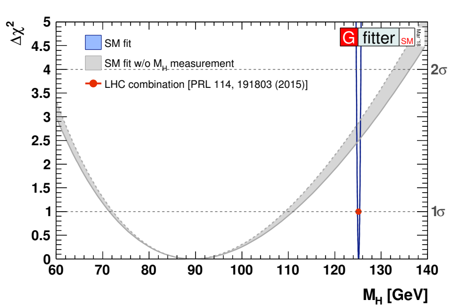
The constraint that the electroweak vacuum remains stable gives a lower bound on m_H. One finds $115\ \mathrm{GeV} \lesssim m_H$.

Experimental constraints on m_H

The experimental measurement (ATLAS, CMS) is $m_H = 125.25 \pm 0.17$ GeV ! Hence it is perfectly compatible with the naive expectations from the SM.

Even when unseen, the Higgs boson contribute virtually (loops) to the well measured electroweak observables (LEP, TeVatron, LHC): the latter cannot be described in the SM if one neglects the Higgs (or if it's too heavy).

Gfitter global analysis



The hierarchy problem

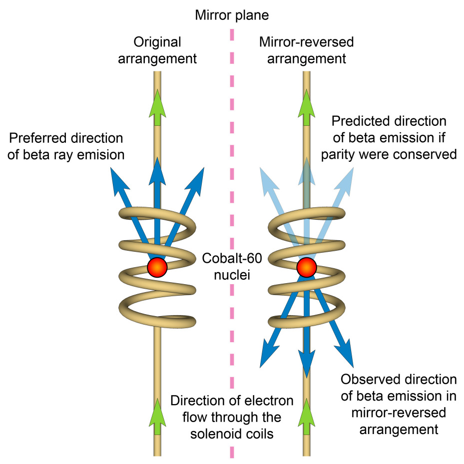
As in any non finite but renormalizable field theory, the bare Higgs mass receives divergent quantum corrections and must be renormalized.
However the scalar nature of the interaction produces quadratic instead of logarithmic divergences; regularizing these divergences with a cut-off that is interpreted as a New Physics scale, one finds

$$m_H^2 \sim (m_H^2)_{\rm bare} + \frac{\Lambda_{\rm NP}^2}{16\pi^2}$$

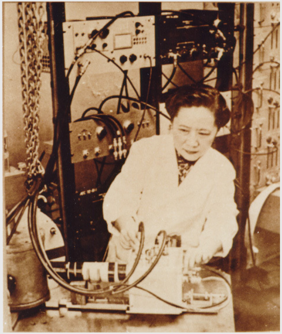
hence the Higgs mass is very sensitive to high scales: fine-tuning competition between the SM and the NP scale.
Requesting moderate fine-tuning leads to $\Lambda_{\rm NP} \sim 1$ TeV.
No evidence of such a low NP scale so far !
As we will see, flavor physics is a way to test for NP at much higher scales.

Parity violation in the Cobalt weak decay

'Left' and 'right' are not human invention: weak interactions are chiral.



Wu experiment 1956-57



immediatly confirmed by Lederman & Garwin in pion decay

CP violation

If P is not conserved, why not postulate that CP is the "correct" interpretation of the left-right symmetry ?
CP was also found to be violated in 1964 (Cronin & Fitch) in kaon decays.

$$|K^0\rangle = \bar{s}d\,, \quad |\bar{K}^0\rangle = CP|K^0\rangle = s\bar{d}$$

CP-eigenstates

$$|K_\pm\rangle = (1/\sqrt{2})(|K^0\rangle \pm |\bar{K}^0\rangle)$$

If CP were conserved, only $K_+ \to \pi\pi$ and $K_- \to \pi\pi\pi$ would be allowed

$$K_\pm \equiv K_{S,L}, \quad \tau(K_S) \ll \tau(K_L)$$

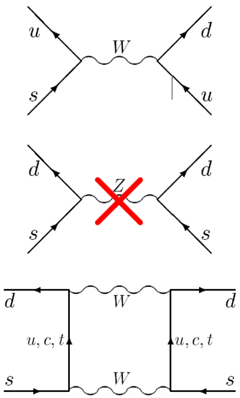
but $K_L \to \pi\pi$ was observed at the 10^{-3} level !
CP-asymmetries $\varepsilon_K \sim (K_L \to \pi\pi)/(K_S \to \pi\pi)$,
$\varepsilon' \sim (K_L \to \pi^+\pi^-) - (K_L \to \pi^0\pi^0)$
ε_K is indirect CP, while ε' comes from direct CP-violation in decay (found different from zero in 1999).

Kaon sector

Kaon rare decays and kaon mixing are the prototypes for FCNC transitions. The GIM mechanism was designed to explain the smallness of $K_L \to \mu\mu$ (Glashow, Iliopoulos & Maiani 1970).

The existence of the charm quark was predicted, and its mass estimated from the value of Δm_K (Gaillard & Lee 1974).

A genuine example of New Physics found by indirect searches in the flavor sector !



Why is *CP*-violation a fundamental phenomenon ?

Because it is one of the three ingredients for baryogenesis (10^9 times more photons than baryons in the universe - vanishingly small quantities of antimatter): Sakharov 1967

1. baryon number violating interactions
2. *C*- and *CP*-violation
3. deviation from thermal equilibrium

Actually the SM interactions to be described later contain these ingredients, but in way too small quantities.

Warning: it is actually not proven that cosmological *CP*-violation has something to do with elementary particle physics.

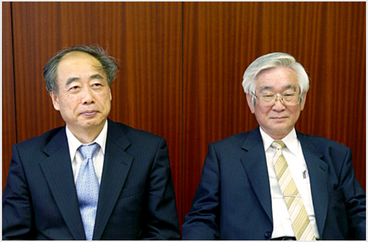
The Cabibbo-Kobayashi Maskawa matrix

Recall the diagonalization of the quadratic (mass) terms in the Yukawa Lagrangian:

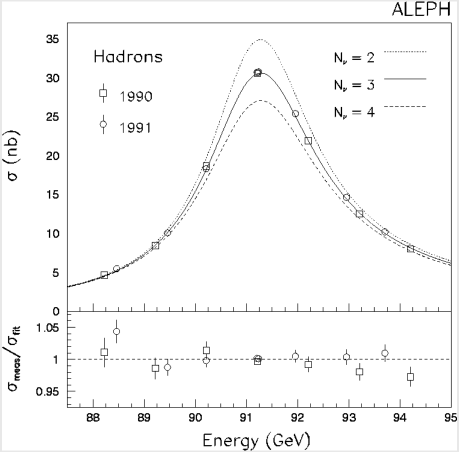
$$V_{\mathrm{CKM}} \equiv V_U^\dagger V_D = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

This matrix is unitary and since some of the phases can be reabsorbed into the quark fields it only has $n(n-1)/2$ mixing angles and $(n-1)(n-2)/2$ complex phases:

These phases can generate *CP*-violation ! (Kobayashi-Maskawa)



From parameter counting $n = 3$ is the minimal number of families that are needed to generate CP-violation through the KM mechanism.
It also happens that $n = 3$ is the number of massless neutrinos found at LEP, and more generally the number of observed fermion generation: is it a coincidence ?



Parametrization of the CKM matrix

With the mixing angles $\cos, \sin(\theta_{ij}) \equiv c_{ij}, s_{ij}$ the CKM matrix is the product of three 2x2 rotation matrices with one phase

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{23} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

However it will experimentally be found that $s_{12} \sim \lambda \sim 0.2$, $s_{23} \sim \lambda^2 \sim 0.04$, $s_{13} \sim \lambda^3 \sim 0.008$.

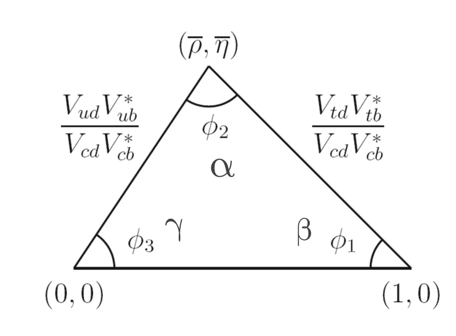
This hierarchy can be made explicit by defining the exact version of the Wolfenstein parametrization

$$\lambda^2 \equiv \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2} \qquad A^2\lambda^4 \equiv \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}$$

$$\bar\rho + i\bar\eta \equiv -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

3×3 unitarity implies six triangle relations in the complex plane; because of the λ suppression, four of these triangles are quasi-flat, and the remaining two are almost degenerate. One defines "the" Unitarity Triangle by

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

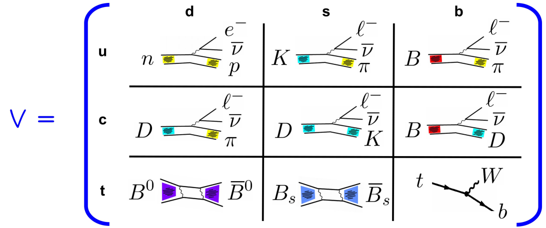


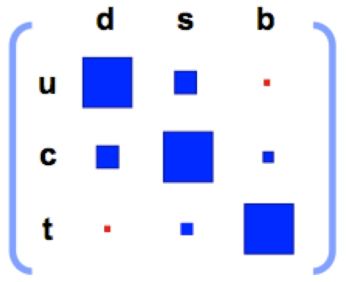
NB: $\beta, \alpha, \gamma = \phi_1, \phi_2, \phi_3$ in the Japanese notation

The prediction of CP violation from CP conserving observables only is a peculiar feature of the SM, related to the three generation KM mechanism.

Extracting CKM couplings

$$V_{\rm CKM} \equiv V_U^\dagger V_D = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





Extracting CKM couplings: QCD

In contrast to leptons, quarks are confined into non perturbative bound states (hadrons).
One does not measure directly the weak couplings of quarks, but rather matrix elements of quark operators taken between hadron states, that need to be calculated by means of theoretical methods, such as Lattice QCD.

Before that, there are a few examples in the B meson system where one can get rid of strong interaction effects, by taking advantage of the fact that QCD conserves CP (at the 10^{-11} level).
The most beautiful example is the time-dependent CP-asymmetry in $B \to J/\psi K_S$.

$B^0 - \bar{B}^0$ mixing

B^0 and $\bar{B}^0$ have the same quantum numbers from the point of view of the weak interaction, so they mix.

Mass eigenstates

$$|B_{H,L}\rangle = p|B^0\rangle \mp q|\bar{B}^0\rangle$$

Time evolution

$$i\frac{d}{dt}\begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix} = [M - (i/2)\Gamma]\begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix}$$

In practice, both theoretically and experimentally $\Gamma_{12} \ll M_{12}$, so that the solution of the diagonalization reduces to

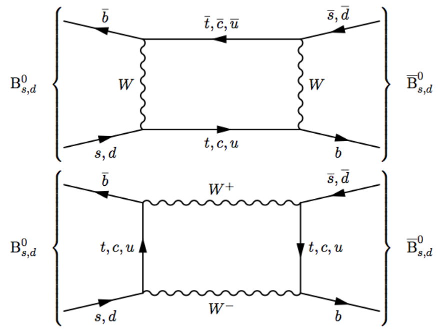
$$q/p = -\sqrt{M_{12}^*/M_{12}}$$

M_{12} is dominated by box (loop) diagrams where the top is virtual, hence

$$M_{12} \sim V_{td}^* V_{tb} \times (QCD)$$

Thus independently of the QCD matrix element, one has

$$q/p = \frac{V_{td} V_{tb}^*}{V_{td}^* V_{tb}} \simeq e^{-2i\beta}$$



Mixing-induced time-dependent *CP*-asymmetry

One defines

$$a_{CP}(t) = \frac{\Gamma(\bar{B}^0(t) \to f_{CP}) - \Gamma(B^0(t) \to f_{CP})}{\Gamma(\bar{B}^0(t) \to f_{CP}) + \Gamma(B^0(t) \to f_{CP})} = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos \Delta m t + \frac{2 \mathrm{I}m \lambda_f}{1 + |\lambda_f|^2} \sin \Delta m t$$

where

$$\lambda_f = \eta_f \frac{q}{p} \frac{\bar{A}_f}{A_f}$$

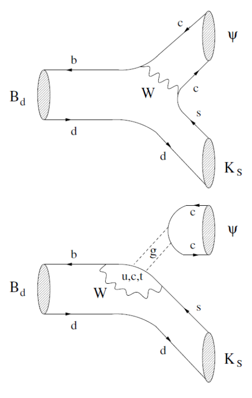
In the above expression, the coefficient of $\cos \Delta m t$ is the direct *CP*-asymmetry, while the $\sin \Delta m t$ is the mixing-induced one

The academic case is when the decay amplitude is dominated by a single CKM coupling, such that $A \sim V_{\mathrm{CKM}} \times \mathrm{QCD}$; then

$$a_{CP}(t) = \mathrm{Im} \left(\frac{V^*_{\mathrm{CKM}}}{V_{\mathrm{CKM}}} \right) \sin \Delta m t$$

$B_d \to J/\psi K_S$

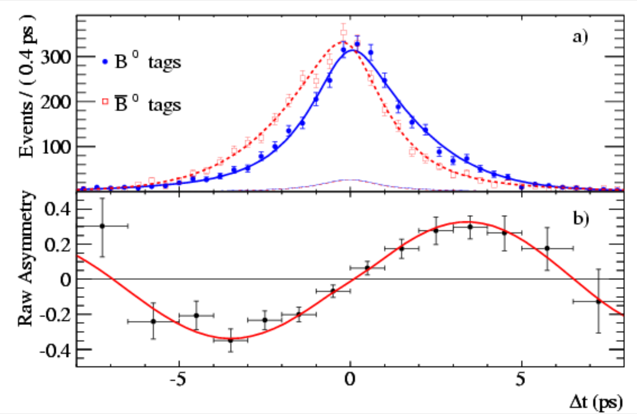
Let $B^0 \to J/\psi K_S$ interfere with $B^0 \to \bar{B}^0 \to J/\psi K_S$. The ‘tree’ diagram is, by far, dominant over the ‘penguin’ one. It is proportional to $V_{cb}V_{cs}^*$. Corrections are suppressed by both CKM ($\lambda^2 \sim 4\%$) and strong interaction effects (a few % at most).



$$
\begin{aligned}
a_{CP}(t) &= -\mathrm{Im}\left(\frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}}\right)\left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right)\left(\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\right)\sin\Delta m t \\
&= \sin 2\beta \sin\Delta m t
\end{aligned}
$$

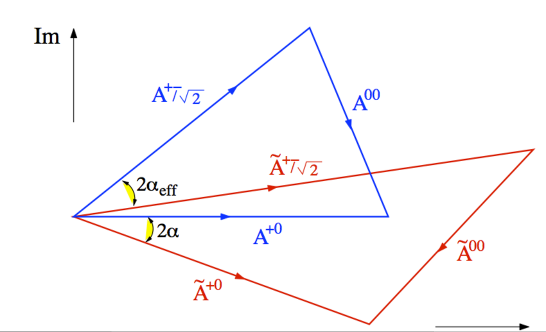
Time-dependent *CP* violation

BaBar 2009



The extraction of the angle α

It follows the same logic but in this case subdominant penguin diagrams can reach 10% or even more, and cannot be neglected.
Instead one uses the fact that the unwanted diagrams have different isospin properties than tree diagrams, and one reconstruct α as the phase between different linear combinations of decay amplitudes. Gronau London 1990
For $B \to \pi\pi$ and $B \to \rho\rho$ there are three assignments of charges and thus three amplitudes



The extraction of γ

Construct interferences between CP conjugate decay modes that differ by phase γ.

The necessary hadronic information (ratio of matrix elements) doesn't cancel but is directly taken from data (of B and/or D decays):

GLW: use $B^{\pm} \to D_{\rm CP} K^{\pm}$ to let $b \to c\bar{u}s$ interfere with $b \to u\bar{c}s$

Gronau, London, Wyler '91

ADS: use $B^{\pm} \to (D^0, \overline{D}^0) K^{\pm} \to (K^+\pi^-) K^{\pm}$ that is $(b \to c\bar{u}s) \times (c \to d\bar{u}s)$ vs. $(b \to u\bar{c}s) \times (\bar{c} \to \bar{s}u\bar{d})$

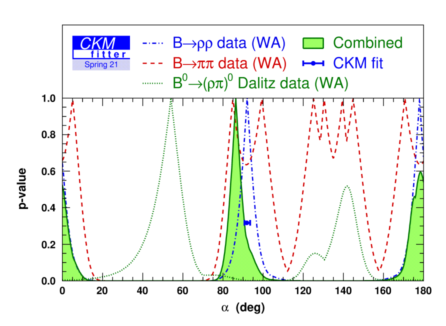
Atwood, Dunietz, Soni '96

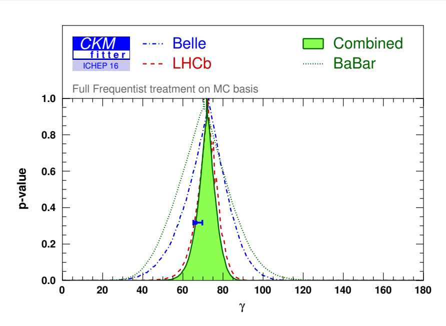
GGSZ: use instead three body decay of D, that is either described by a resonance (isobar) model, or by a binned Dalitz plot analysis

Giri, Grossman, Soffer, Zupan '03; Bondar, Poluetkov '05

Many variants (D^*, K^*, more particles in the final state...).

α and γ Grand combinations





α (direct) $= (86.4^{+4.3}_{-4.0})^\circ$

vs.

α (indirect) $= (91.9^{+1.6}_{-1.2})^\circ$

γ (direct) $= (72.1^{+5.4}_{-5.7})^\circ$

vs.

γ (indirect) $= (65.5^{+1.1}_{-2.7})^\circ$

QCD driven extraction of CKM couplings

Beyond UT angles, most of the time the flavor observables depend non trivially on both the CKM couplings (weak part) and the QCD matrix elements (strong part).

When working at low energies (wrt the weak scale) the first step is to use the Operator Product Expansion to simplify the computation.

W-mediated product of two currents

$$g^2 \int d^4x\, J_\mu(0) D_W^{\mu\nu}(x) J_\nu(x)$$

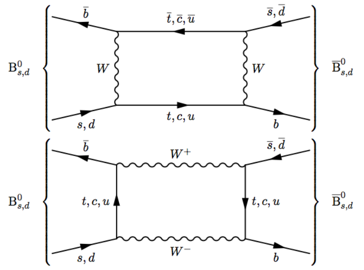
with the propagator

$$D_W^{\mu\nu}(x) = \int d^4q\, e^{iqx} \frac{ig^{\mu\nu}}{q^2 - m_W^2 + i\epsilon}$$

In the low energy limit $q^2 \ll m_W^2$ one recovers the Fermi interaction

$$\frac{g^2}{m_W^2} J^\mu(0) J_\mu(0)$$

The $\Delta B = 2$ SM operator



$$\sim (V_{td}^* V_{tb})^2 \langle \bar{B}^0 | \bar{b}\gamma^\mu (1-\gamma_5) d \bar{b}\gamma_m u (1-\gamma^5) d | B^0 \rangle \sim (V_{td}^* V_{tb})^2 (m_B)^2 f_B^2 B_B^2$$

where f_B is the B decay constant and B_B is the bag factor, both of them being complicated non perturbative quantities

When allowing for new mediators in the loop, other operators of the same dimension appear, depending on the underlying Lorentz and Dirac quantum numbers.

Global constraints on the CKM matrix

Goal: determine the value of the fundamental coupling constants from the measurement of experimental observables.

In order to be conservative when testing the Standard Model, one uses as experimental and theoretical inputs only the ones one thinks are well understood quantitatively.

A global statistical analysis is performed, with the best possible treatment of experimental and theoretical errors; for the latter, a model has to be defined and used.

Here the results by the CKMfitter group, based on the frequentist approach, are presented.

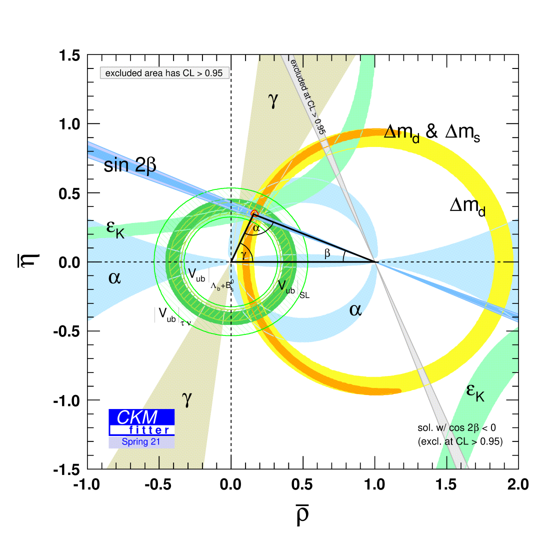
The main physics ingredients are the following

- $|V_{ud}|$, $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ from the relevant charged current, tree level weak decays; the needed strong interaction parameters are taken from Lattice QCD or other methods where necessary
- Δm_{ds} from $B_{d,s} - \bar{B}_{d,s}$ oscillation measurements and Lattice QCD
- the CP-violating angles α, β, γ from the corresponding experimental analyses; very little theoretical input is needed here
- the CP-violating asymmetry ε_K, the interpretation of which depends on the $K - \bar{K}$ mixing parameter B_K computed on the lattice

HFLAV: compilation and averages of (mostly) experimental results in the heavy flavor sector

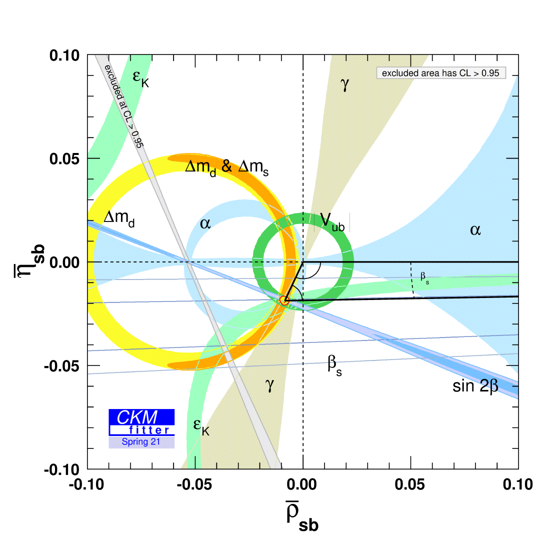
FLAG: compilation and averages of LQCD analyses in the light and heavy flavor sectors

The global CKM analysis in the B_d UT plane



all constraints together

The global CKM analysis in the B_s UT plane



all constraints together

The global CKM analysis

Wolfenstein parameters from the fit

$$A = 0.8132^{+0.0119}_{-0.0060}(1\%) \qquad \lambda = 0.22500^{+0.00024}_{-0.00022}(0.1\%)$$

$$\bar{\rho} = 0.1566^{+0.0085}_{-0.0048}(4\%) \qquad \bar{\eta} = 0.3475^{+0.0118}_{-0.0054}(2.5\%)$$

Clearly the big picture is that the CKM couplings are the dominant contribution to the physical flavor transitions, whereas the KM phase is the dominant contribution to *CP*-asymmetries.

More accurate tests can be done by comparing the indirect fit prediction for a given quantity, with its direct determination (experimental measurement or theoretical calculation).

example: $\sin 2\beta$ in 2001

indirect prediction $0.50 < \sin 2\beta < 0.86$

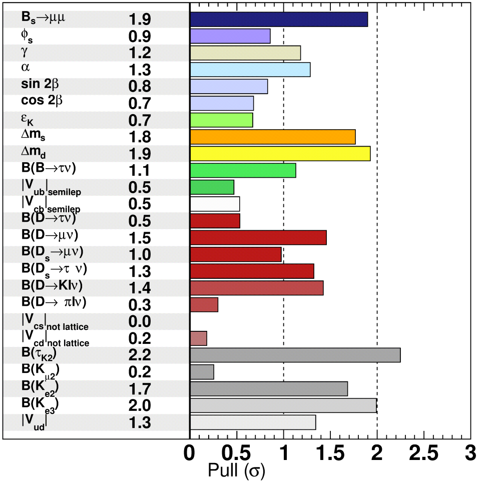
first measurements $\sin 2\beta_{\mathrm{BaBar}} = 0.59 \pm 0.14 \pm 0.05$, $\sin 2\beta_{\mathrm{Belle}} = 0.99 \pm 0.14 \pm 0.06$

now

indirect prediction $\sin 2\beta = 0.731^{+0.029}_{-0.016}$

World Average measurement $\sin 2\beta = 0.699 \pm 0.017$

Pull values for the CKM observables



Hadronic matrix elements

To leading order of the weak interaction, one has

$$\langle f|H_{eff}|i\rangle \sim V_{CKM} \times \langle f|O|i\rangle$$

where the operators O can be further decomposed with the Operator Product Expansion from the weak scale

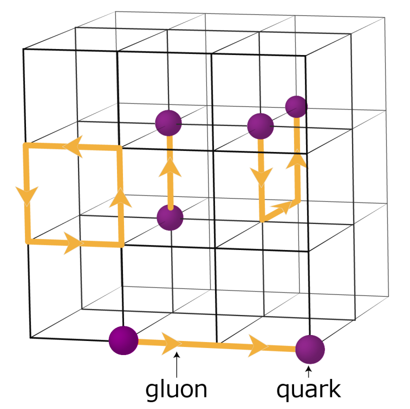
$$O \sim C_i(\mu)Q_i(\mu)$$

The $C_i(\mu)$ are renormalized Wilson coefficients that can be computed in terms of fundamental couplings in the SM and beyond, and the O_i are (renormalized) quark operators, the matrix elements of them have to be computed in QCD at low energy: they are genuinely non perturbative objects (decay constants, current form factors, non local matrix elements. . .). At a given order the list of contributing operators is finite and can be derived from EFT techniques.

Lattice QCD and flavor physics

A particle can be correctly simulated iff its mass 'fits' on the lattice

$$a \ll \frac{1}{m} \ll L$$



In particular, on present lattices physical pions remain costly [see C. Urbach's lecture], and physical b hadrons out of reach.

Flavor physics is intrinsically a multi scale problem: a challenge for LQCD.

Example: $B \to \pi\pi$ (for CKM angle α) is a triple nightmare for LQCD: 1) The B is too heavy; 2) The π's are very light; 3) Maiani-Testa theorem states that the infinite volume limit corresponds to a configuration where the pions are at rest, which is unphysical.

Extending the applicability of LQCD

For too light/heavy particles, one can instead perform a simulation at a different mass, and then use a power expansion to extrapolate to the physical region. Light/heavy effective theories can be used to control the extrapolation.

To evade Maiani-Testa, one can use a ‘small’ lattice box, in which multihadron states are discrete and sufficiently well separated. A result by Lellouch & Lüscher relate these states to the physical configurations at infinite volume.

Heavy quark symmetry

Let's consider a heavy meson $(Q\bar{q})$. In the limit $m_Q \gg \Lambda$ where Λ is a typical interaction scale, the heavy quark becomes static: properties of the meson do not depend on m_Q anymore. In addition spin effects are suppressed by $1/m_Q$ (as in the hydrogen atom). [Isgur & Wise 1989]

Heavy quark symmetry = spin-flavor symmetry. Consequence: B, B^*, D and D^* are essentially the same bound state (up to a calculable m_Q scaling).

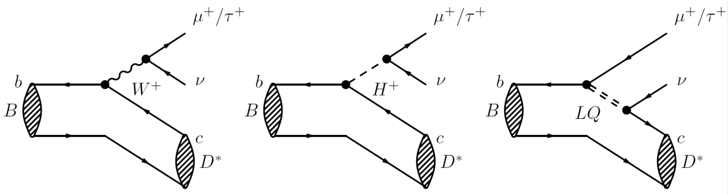
More generally, B and D bound states can be categorized in degenerate spin doublets, and mass differences between the doublets do not depend on the b or c flavor.

Heavy mass expansion

The heavy mass expansion is not only a symmetry: it is also a rigorous tool to expand many quantities in inverse powers of the heavy mass, leading to the construction of an effective theory (HQET).

The coefficients of these expansions are defined in the heavy mass limit, so they obey HQS relations. This allows to constrain them from data when they are too complicated to be computed theoretically.

Heavy-to-heavy form factors



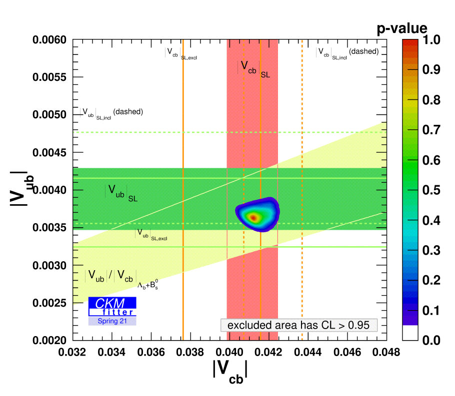
Whatever the mediator, the hadronic part is a matrix element of a two quark current

$$\langle D^{(*)}|\bar{c}\Gamma b|B\rangle$$

Heavy quark symmetry, in the $m_{b,c} \to \infty$ limit, essentially predicts that the initial and final states are the same: hence the matrix element is fully described by a single elastic form factor $\xi(w)$, $w = v_B \cdot v_D$, that is normalized at zero recoil $\xi(1) = 1$ ($w \leftrightarrow q^2$, the lepton invariant mass). Hence in principle in the heavy quark limit $|V_{cb}|$ can be extracted from the measurement of the zero recoil rate without any input from QCD calculations !

The extraction of $|V_{cb}|$

In practice one needs to control the corrections to the heavy mass limit to reach an accuracy of a few %. Form factors are calculated in LQCD, with the help of heavy quark expansion to extrapolate to the physical b mass. $|V_{cb}|$ is the most precisely known short-distance quantity in the B meson sector



Heavy-to-light form factors

Consider a $b \to u$ transition

$$\langle \pi(\rho) | \bar{u} \Gamma b | B \rangle$$

or a $b \to s$ one

$$\langle K^{(*)} | \bar{s} \Gamma b | B \rangle$$

In this case strict HQS is only useful for the heavy initial state, predictions are much looser than in the heavy-to-heavy case. However one can perform a combined $m_b \to \infty$ and $E \to \infty$ expansion, where E is the energy of the final meson in the B rest frame,

$$E = v_b \cdot p_K = (m_B/2)(1 - q^2/mB^2)$$

(E large $\Leftrightarrow q^2 \sim \Lambda^2$ or $q^2 \sim m_B \Lambda$): in this limit the 3 (for $B \to K$) $+7$ (for $B \to K^*$) form factors reduce to three independent ζ, $\zeta_\perp$, $\zeta_\|$ 'soft' form factors, that obey well-defined scaling laws in E. [JC *et al.* '99]

SCET

Effective field theory implementation: Soft-Collinear Effective Theory (SCET). [Bauer *et al.* '00,'01]

The SCET limit is best used in the moderate to large recoil region, $q^2 \sim m_B \Lambda$, where LQCD has no access.

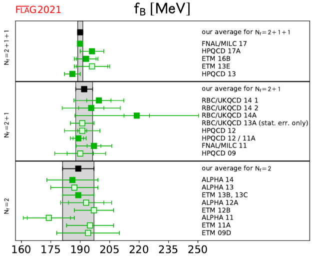
In the low recoil region $q^2 \sim m_B^2$, SCET does not apply. However in this region the form factors can be computed directly on the lattice.

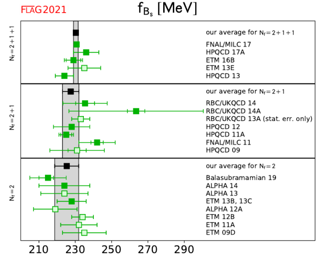
In contrast to most effective theories, SCET is an expansion with respect to kinematical variables (E or q^2), not to constants (masses). For this reason it is significantly more complicated.

Heavy and/or energetic quarks on the lattice

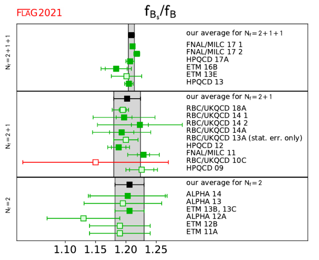
Heavy quark effective theories (HQET or NRQCD) can be directly implemented on the lattice: this evades the $a \ll m$ constraint.
However power corrections involve more and more complicated operators.
An alternative is to simulate QCD heavy quarks that are lighter than physical masses, and extrapolate to the physical point using behavior predicted by EFT.
SCET for energetic light quarks is a further challenge, as it is intrinsically Minkovskian and non local; it has strong similarities with PDFs [see L. del Debbio's lecture].

Matrix elements on the lattice





Ratios of matrix elements may be more accurate (cancellation of stat/syst uncertainties). However not all LQCD groups compute the same combinations. Full correlation matrices are needed to guarantee no loss of information.



Matrix elements on the lattice

New Physics contributions generate new operators that are absent in the SM (5 operators for $B\bar{B}$ mixing). Matrix elements of them can introduce additional difficulties in LQCD calculations, with the consequence that metrology of NP models is often less precise than of the SM.

We have introduced the theoretical tools needed to evaluate hadronic matrix elements relevant for flavor physics especially in the B meson sector.

Now we can consider not only the determination of the SM couplings, but also try to test for the presence of New Physics contributions.

Where could be New Physics ?

First answer: *a priori* anywhere; imagine there are right-handed currents, then all the flavor observables are impacted, and we presumably do not have enough theoretical and experimental information to extract both left- and right-handed couplings simultaneously; in other words, the CKM matrix is unknown, and the apparent successes so far are accidental.
Generally speaking, if New Physics is generic and impact many kind of observables, then many things have to be recalculated and a completely global analysis is needed; this is actually very challenging and is not the common practice.
However one may assume that dominant New Physics effects occurs in SM amplitudes that are small because of its specific properties; in particular since Flavor Changing Neutral Currents are suppressed by quantum loops in the SM, this is the first place to look at.

The flavor problem

Recall that the (FCNC) meson mixing operators come with a coupling of the form

$$\frac{g^2}{m_W^2}(V_i V_j^*)^2$$

Similarly, New Physics will contribute to the same operators with couplings

$$\frac{c_{ij}}{\Lambda_{NP}^2}$$

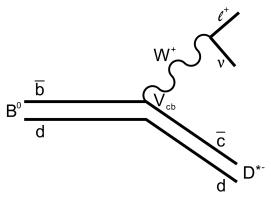
Thus to avoid that NP contributions are larger than SM ones, one needs

$$\frac{\Lambda_{NP}}{\sqrt{c_{ij}}} \gtrsim \frac{4\mathrm{TeV}}{|V_i V_j|}$$

$K\bar{K}$ mixing	10^4 TeV	$D\bar{D}$ mixing	10^4 TeV
$B_d\bar{B}_d$ mixing	10^3 TeV	$B_s\bar{B}_s$ mixing	10^2 TeV

In other words, either New Physics is very far and cannot solve the Higgs mass hierarchy problem, or the new flavor couplings are very small.

Lepton universality: $R(D)$ and $R(D^*)$

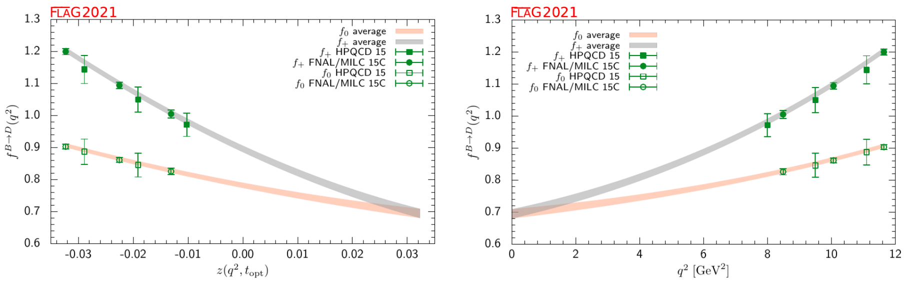


We have seen that $b \to c$ transitions are much constrained by HQS.
Semileptonic decays with a light lepton pair leads to an excellent determination of $|V_{cb}|$.
First measurement of

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)}$$

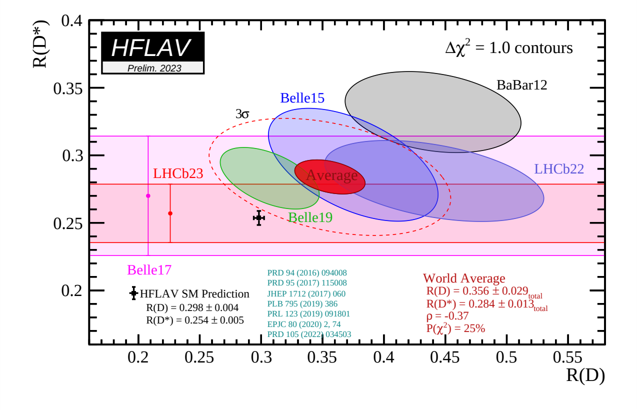
by BaBar in 2012.

Dependence to form factors is much reduced in these ratios, which allows a test of the universality of lepton couplings to quarks (a quite accidental prediction of the Standard Model). Following SM prediction comes from HFLAV 2023, based on 2016-2022 LQCD analyses.



On the experimental side the measurement is challenging, because of missing energy in τ decay: excited D states constitute a significant background to the tauonic mode, especially at hadron colliders (LHCb).

World summary of $R(D)$ and $R(D^*)$

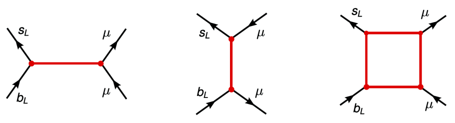


Full combination is more than 3 away from the SM prediction (precise value depends a bit on the treatment of form factors)

A very intriguing anomaly because it is large, robust, and observed in a charged current transition !

FCNC $b \to s$ transitions

2nd $\leftrightarrow$ 3rd quark generation: so far the least well known sector of weak quark decays.

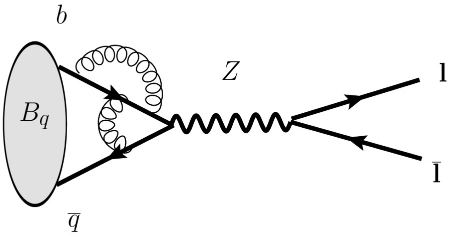


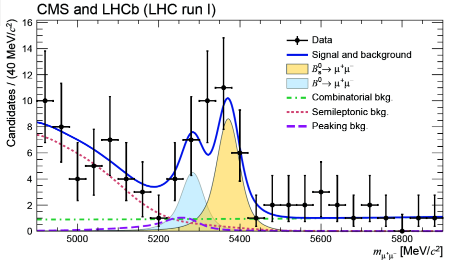
Large NP effects are still allowed, and quite naturally predicted by scenarios where non standard couplings are larger for the heaviest generations.

The very rare $B_s \to \mu^+\mu^-$ decay

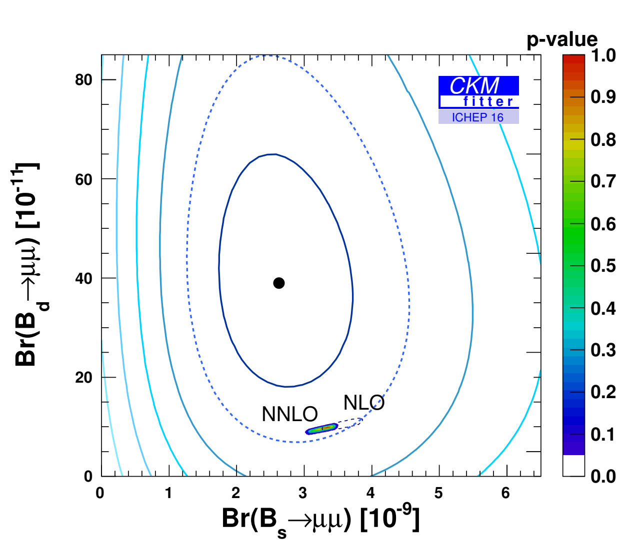
This is the rarest decay that comes with both a non trivial measurement and a non trivial theoretical prediction. It is very sensitive to new particles, *e.g.* a charged Higgs.

Hadronically, it only depends (even outside SM !) on the f_{B_s} decay constant that is well computed on the lattice. Perturbative contributions have been computed up to NLO-EW and NNLO-QCD (Buchalla *et al.*, Bobeth *et al.*, Hermann *et al.*)



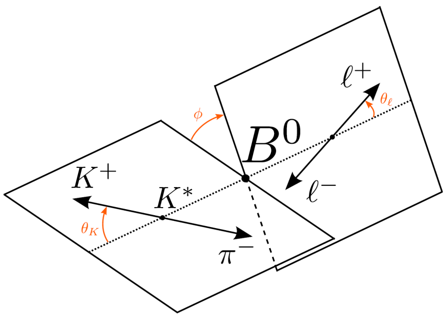


2D constraint on $B_{d,s} \to \mu^+\mu^-$ decays



$B \to K^{(*)}\ell\ell$ angular observables

Experiments can measure 4-dimensional distribution



$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_{K^*}\, d\cos\theta_\ell\, d\phi} \sim \sum_i I_i f_i(\Phi)$$

where the linear coefficients I_i are angular observables that can be expressed in terms of $B \to K^*$ matrix elements: much more information than just a BR !

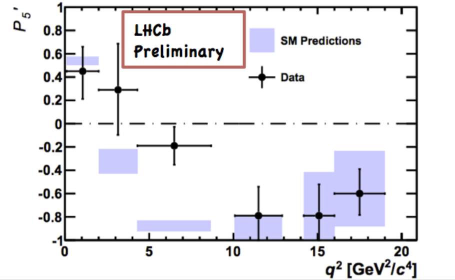
Optimized angular observables in $B \to K^{(*)}\ell\ell$

Independent amplitude combinations are related to each other thanks to SCET form factor relations.

This allows the construction of 'optimized' observable ratios, that are asymptotically independent of form factors. First one was the forward-backward asymmetry Ali *et al.* '00.

This can be made very general, by taking appropriate ratios of angular observables Krüger *et al.* '12, Descotes-Genon *et al.* '12.

Experimental results



First significant tension: 2-3σ in third bin of P_5' (LHCb '13)

This was the motivation for more sophisticated global analyses and refined measurements. Now individual anomalies of $b \to s$ transitions reach a large significance, even larger than 5σ depending on the treatment of hadronic uncertainties !

However dominant uncertainties comes from the contribution of long distance and non local charm loop contributions, which are still out of reach of LQCD possibilies.

Conclusion

Flavor physics is a particularly interesting field to constrain Beyond the Standard Model contributions

- large CP violation (in the B sector)
- sizable FCNC transitions
- 3 generation democracy
- sensitivity to weak-like New Physics
- sensitivity to very massive mediators
- interesting strong interaction properties

In the last few years a few anomalies have shown up against SM predictions. Some of them have washed out, but others are more robust and are very intriguing.

From the point of view of the strong interaction quark flavor systems are an opportunity to develop a variety of theoretical tools to systematically approach the difficult non perturbative problems.

Altogether many important questions are waiting to be addressed !