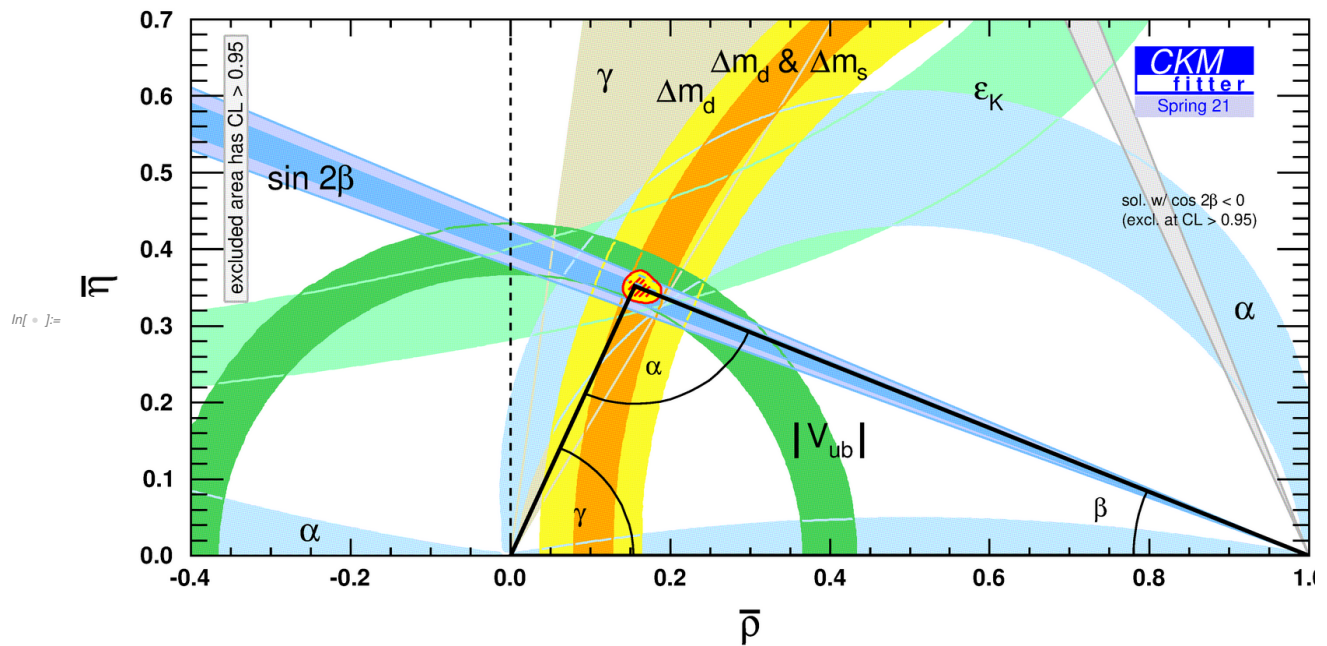


A simple CKM fit

The purpose of this exercise is to perform a simple CKM fit, exhibiting a few issues:

- mirror solutions
- compatibility of different inputs and pull values



Useful formulae

p-value from χ^2 with N dof degrees of freedom

$$p[\chi^2, \text{dof}] := \frac{\Gamma\left(\frac{\text{dof}}{2}, \frac{\chi^2}{2}\right)}{\Gamma\left(\frac{\text{dof}}{2}\right)}$$

inverse conversion in number of standard deviations

$$\sigma \text{Unit}[p] := \sqrt{2} \operatorname{erf}^{-1}(1 - p)$$

Wolfenstein parametrization of CKM matrix elements

$$V_{ud} = 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8}$$

$$V_{us} = \lambda$$

$$V_{cb} = A \lambda^2$$

$$V_{ub} = A \lambda^3 \sqrt{\rho \text{bar}^2 + \eta \text{bar}^2}$$

Wolfenstein parametrization of UT angles

$$\ln[\dots] := \alpha = \tan^{-1}(\eta\text{bar}^2 - \rho\text{bar}(1 - \rho\text{bar}), \eta\text{bar})$$

$$\sin 2\beta = -\frac{2\eta\text{bar}(\rho\text{bar} - 1)}{\eta\text{bar}^2 + (\rho\text{bar} - 1)^2}$$

Fit inputs

$$V_{ud} = 0.97373 \pm 0.00031 \text{ (PDG)}$$

$$V_{us} = 0.2243 \pm 0.0008 \text{ (PDG)}$$

$$V_{cb} = (40.8 \pm 1.4) \times 10^{-3} \text{ (PDG)}$$

$$V_{ub} = (4.05 \pm 0.64) \times 10^{-3} \text{ (FLAG B} \rightarrow \tau \nu \text{)}$$

$$\sin 2\beta = 0.699 \pm 0.017 \text{ (PDG)}$$

$$\alpha = (85.5 \pm 4.6)^\circ \text{ (PDG)}$$

Q1

- construct a least-square fit with the following inputs

$$V_{ud} \ V_{us} \ V_{cb} \ \sin 2\beta \ V_{ub}$$

- minimize the corresponding χ^2 function, compute the goodness-of-fit in terms of a p-value

answer

Q2

- construct a least-square fit with the following inputs

$$V_{ud} \ V_{us} \ V_{cb} \ \sin 2\beta \ \alpha$$

- minimize the corresponding χ^2 function, compute the goodness-of-fit in terms of a p-value

- is there a degeneracy ?

answer

Q3

- construct a χ^2 test for the compatibility between V_{ud} and V_{us} . Compare with the previous fits and discuss the interpretation

answer

Q4

- construct a least-square fit with all inputs and minimize the χ^2 . How can it be used to test the compatibility of the LQCD determination of V_{ub} with the fit from Q2 ?

answer