α_s from Lattice QCD

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Hands on session

1 Basic formulas

- The definition of the Λ parameter

$$\frac{\Lambda}{\mu} = \left[b_0\bar{g}^2(\mu)\right]^{-\frac{b_1}{2b_0^2}} e^{-\frac{1}{2b_0\bar{g}^2(\mu)}} \exp\left\{-\int_0^{\bar{g}(\mu)} \mathrm{d}x \left[\frac{1}{\beta(x)} + \frac{1}{b_0x^3} - \frac{b_1}{b_0^2 x}\right]\right\}. \quad (1)$$

- The definition of the step scaling function

$$\sigma(u) = \bar{g}^2(\mu/2)\Big|_{\bar{g}^2(\mu)=u}. \quad (2)$$

2 "Analytic" problems

Problem 1 *The asymptotic expansion of the β-function reads*

$$\beta(x) \overset{x\to 0}{\sim} -x^3(b_0 + b_1x^2) + O(x^7). \quad (3)$$

Show that the coefficients b_0, b_1 are universal, i.e. that under a change of scheme

$$\bar{g}^2_{s'}(\mu) \overset{\mu\to\infty}{\sim} \bar{g}^2_s(\mu) + c_{ss'}\bar{g}^4_s(\mu) + \dots, \quad (4)$$

they do not change.

Problem 2 *Show that under a change of scheme*

$$\bar{g}^2_{s'}(\mu) \overset{\mu\to\infty}{\sim} \bar{g}^2_s(\mu) + c_{ss'}\bar{g}^4_s(\mu) + \dots, \quad (5)$$

the Λ parameter changes according to

$$\frac{\Lambda_{s'}}{\Lambda_s} = \exp\left(\frac{-c_{ss'}}{2b_0}\right) \quad (6)$$

3 "Numeric" problems

Problem 3 *God comes and tells us that for $N_\mathrm{f} = 3$ QCD*

$$\bar{g}^2_{\overline{\mathrm{MS}}}(\mu) = \pi \quad (7)$$

for $\mu = 2900.0$ MeV.

1. *How much is the value of $\Lambda^{(3)}_{\overline{\mathrm{MS}}}$?*

2. *What if it is an imperfect God that only knows that* $\bar{g}^2_{\overline{\rm MS}}(\mu) = 3.1400(314)$ *for* $\mu =$ 2900.0(29.0) *MeV ? What contributes more to the error?*

3. *Imagine that we know* $\bar{g}^2_{\overline{\rm MS}}(\mu) = 4, 3, 2, 1$ *with a 1% error. What are the errors in* $\Lambda^{(3)}_{\overline{\rm MS}}/\mu$

NOTE: *the known perturbative coefficients are:*

$$b_0 = 0.056993165798815; \qquad b_1 = 0.002566495563671084$$
$$b_2 = 0.00016349868861254387 \qquad b_3 = 1.9442896826846894 \times 10^{-5}; b_4 = 1.3280696495205313 \times 10^{-6}\,.$$

and God herself did not gave us any more coefficients...

Problem 4 *After a lot of effort, we have determined the step scaling function at high energies* $\sigma(u)$ *in the GF scheme for* $N_{\rm f} = 0$ *QCD in the range* $\bar{g}^2_{\rm GF} \in [1.0, 2.5]$. *This parametrization is given by*

$$\sigma(x) = \sum_{n=1}^{6} p_i x^i \tag{8}$$

(see coefficients and covariance below)

On the other hand, the relation between couplings is given by

$$\bar{g}^2_{\overline{\rm MS}}(\mu) \overset{\mu\to\infty}{\sim} \bar{g}^2_{\rm GF}(\mu) - 0.09708\,\bar{g}^4_{\rm GF}(\mu) + 0.0325890528\,\bar{g}^6_{\rm GF}(\mu) + \dots \tag{9}$$

1. *If we fix* μ_0 *by the condition* $u_0 \equiv \bar{g}^2_{\rm GF}(\mu_0) = 4\pi/5$ *What value for* $\Lambda^{(0)}_{\overline{\rm MS}}/\mu_0$ *do you obtain if you match with the* $\overline{\rm MS}$ *scheme at a scale given by* $\bar{g}^2_{\rm GF}(\mu) = \sigma^{-k}(u_0)$ *for k=1,...,6?*

NOTE: *if propagating errors seems difficult, just ignore the covariance and compute central values.*

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## Fit parameters (first 4 given by PT)
  Parameter 1: 1.0
  Parameter 2: 0.09656692756243895
  Parameter 3: 0.014995603833711159
  Parameter 4: -1.7630643103448565e-5
  Parameter 5: 0.0009946526673356919 +/- 0.00036588940929584726
  Parameter 6: -0.00018554377625936878 +/- 0.00015733625418245799
## Covariance between fit parameters
  cov(p5,p6): -5.6230216171133985e-8
## Coefficients of the beta function (nf=0)
 - b0: 0.06965831375410722
- b1: 0.00409035230460079
- b2: 0.00036276139272537237
- b3: 4.702648076511487e-5
- b4: 5.470115382072742e-6
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