The primordial universe as a quantum lab

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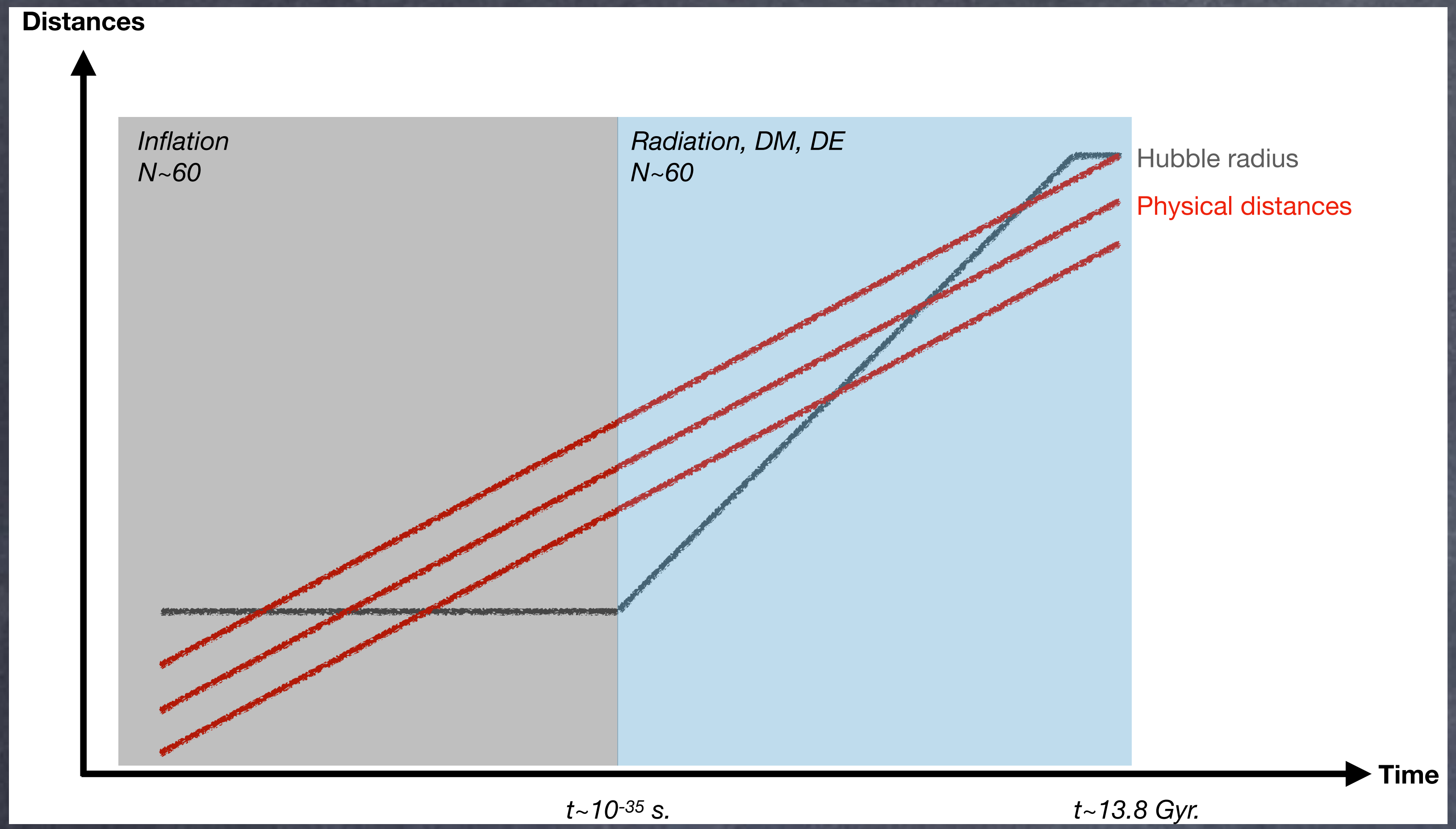
Grain, Vennin, 2020, JCAP, 02,022
Colas, Grain, Vennin, 2022, EPJC, 82, 6
Colas, Grain, Vennin, 2022, EPJC, 82, 1085
Colas, Grain, Vennin, arXiv:2212.09486

Grain, Vennin, 2017, JCAP, 05,045
Grain, Vennin, 2021, EPJC, 81, 132
Artigas, Grain, Vennin, 2022, JCAP, 02, 001

Inflation & the origin of cosmic structures

Accelerated expansion: physical scales grows exponentially faster than Hubble radius



- Observable universe contained in one Hubble patch
  —> Homogeneity of CMB

- Spatial curvature is 'diluted'
  —> exponentially suppressed

- High-energetic phenomenon

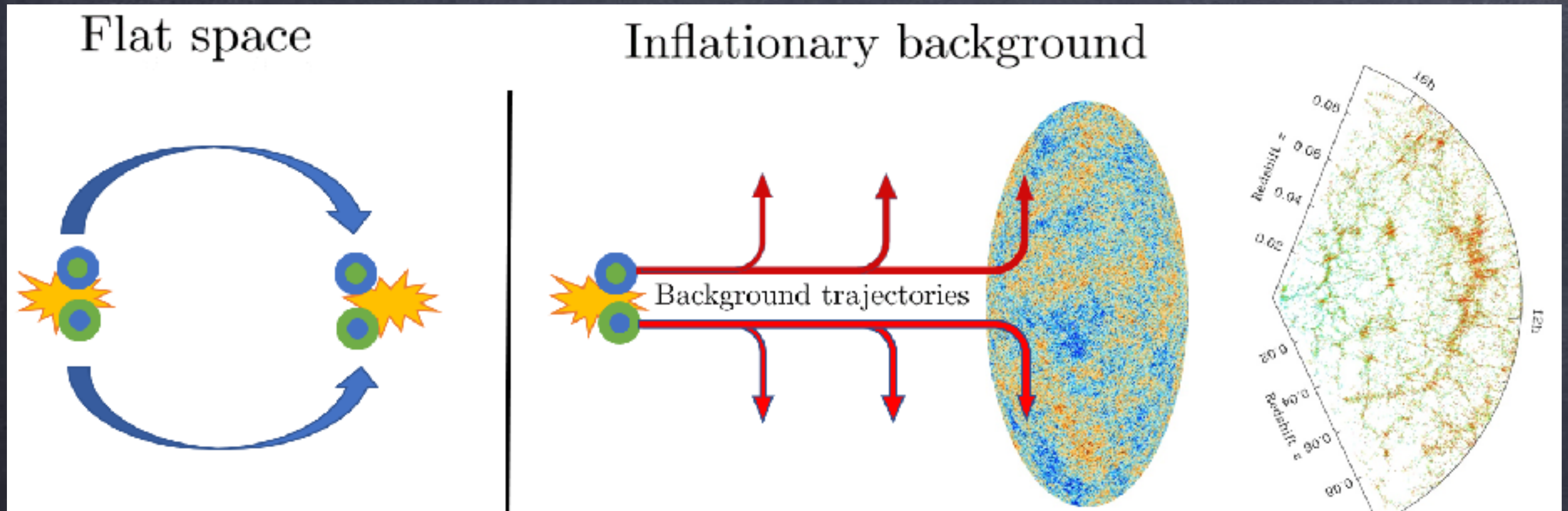
$$N_{inf} = H_{inf} \times \Delta t \quad \Longrightarrow H_{inf} \simeq 10^{16}\,\mathrm{km\,.\,s^{-1}\,.\,Mpc^{-1}} = 10^{10}\,\mathrm{GeV}$$

$$H_0 \simeq 68\,\mathrm{km\,.\,s^{-1}\,.\,Mpc^{-1}}$$

Inhomogeneities extracted out of the quantum fluctuations of density & gravitational fields

- Linearize Einstein's equation: density waves and gravitational waves
- QFT in an expanding Universe: initial vacuum for fluctuations

➡ Instability at horizon radius crossing $\omega_k^2(t) \sim k^2/a^2(t) - H^2$

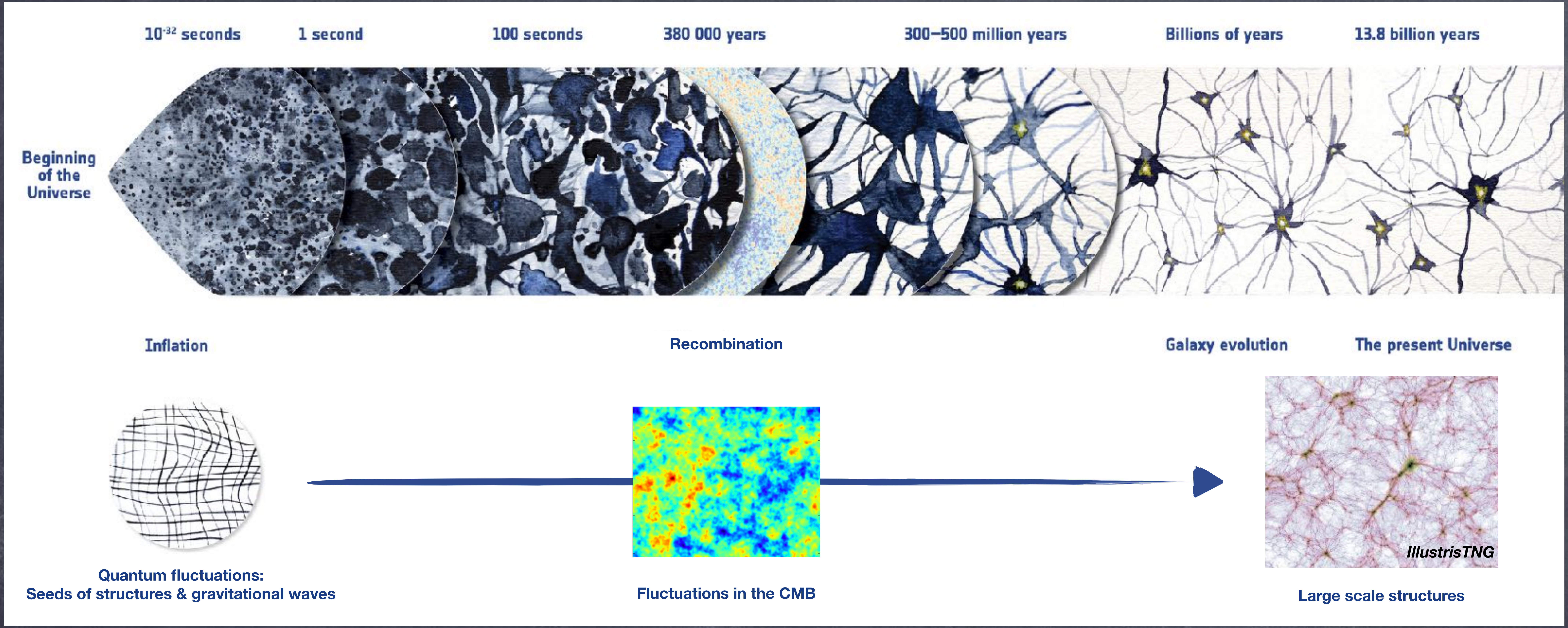


Credit T. Colas

Cosmological observables

- *N*-pt correlation functions
- Abundance of structures
- Full PDF of cosmic fluctuations

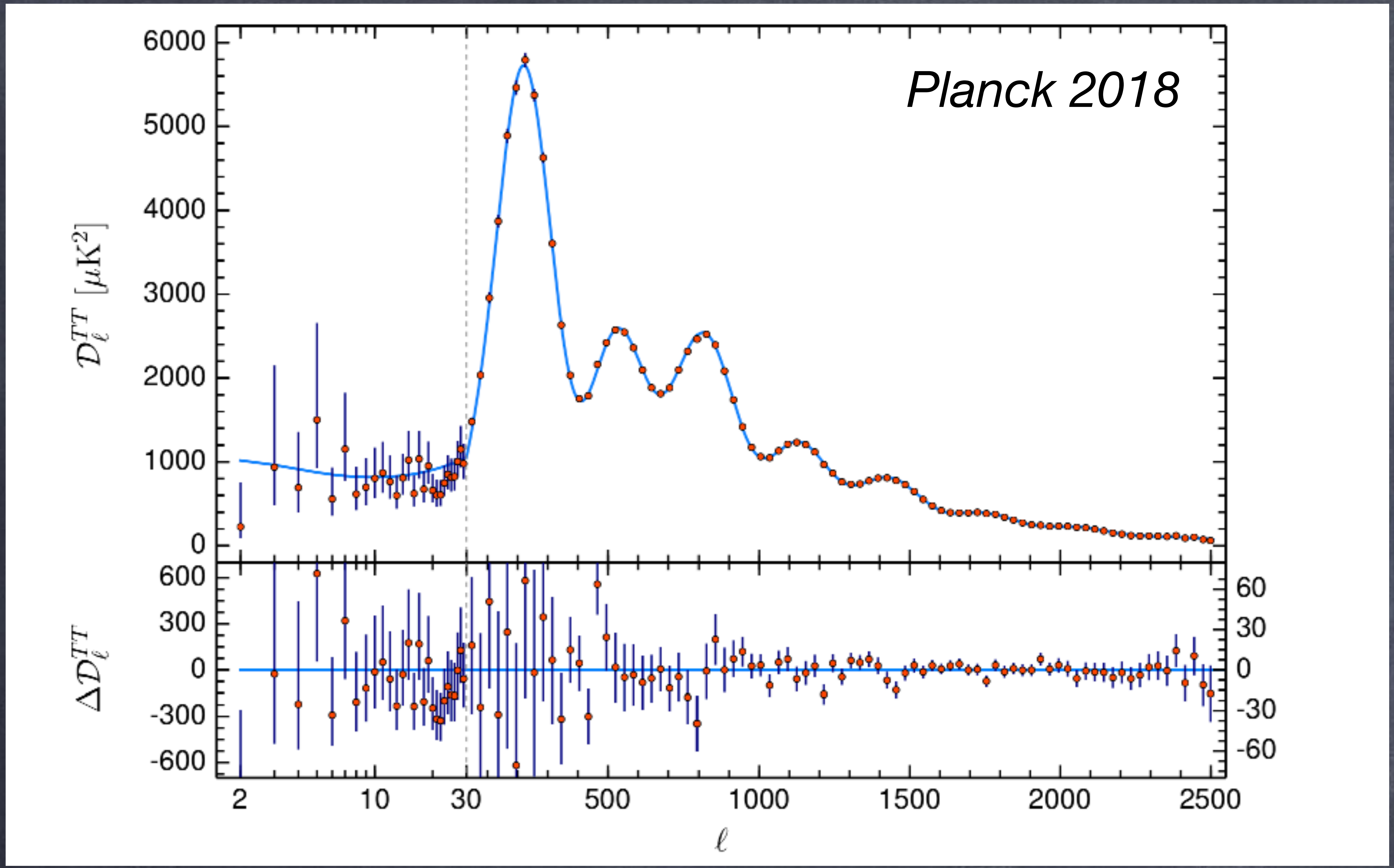
Why studying inflation



- *Sets up initial conditions for structure formation: detailed statistics*
- *High-energy physics beyond SM: number of fields, nature, couplings*
- *Laboratory for QM: QFT in curved spaces, quantum gravity (linear at least), 'classicalization'*

The origin of cosmic structures: what we know

Planck: Aghanim et al. 20
Planck: Akrami et al. 20



Density inhomogeneities:

- ✓ Superhorizon
- ✓ Adiabatic
- ✓ Almost scale-invariant
- ✓ Gaussian

$$P_\zeta(k) \sim (2 \times 10^{-9}) \left(\frac{k}{k_\star}\right)^{-0.035}$$

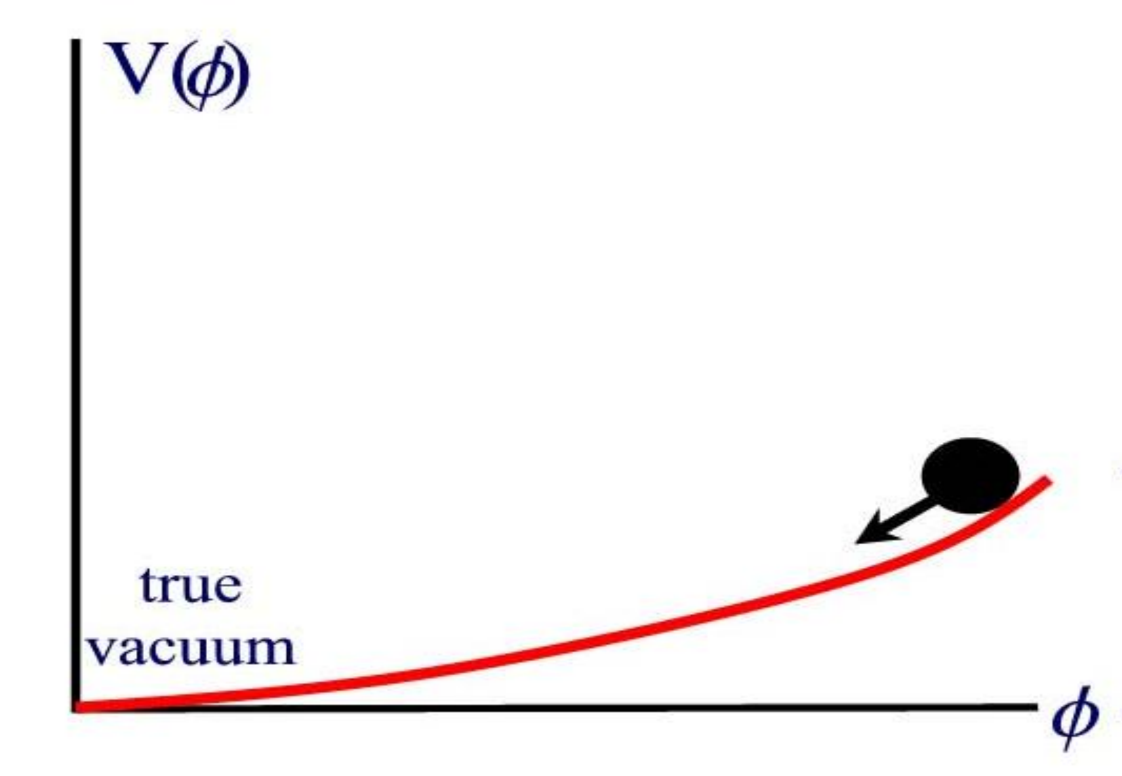
$$f_{NL}^{local} = -0.9 \pm 5.1$$
$$f_{NL}^{equil} = -26 \pm 47$$
$$f_{NL}^{ortho} = -38 \pm 68$$

Primordial gravitational waves:

- ✓ Not yet detected
- ✓ Upper bound

$$P_{GW}(k) \leq 10^{-10}$$

—> Single-field model in slow-roll



➡ *Complemented with LSS (SDSS, BOSS, DES)*

Eisenstein et al 05
Delubac et al. 15
Colas et al. 19
Doux et al. 22
D'Amico et al. 22
Cabass et al. 22
Riquelme et al. 22

Single-field inflation

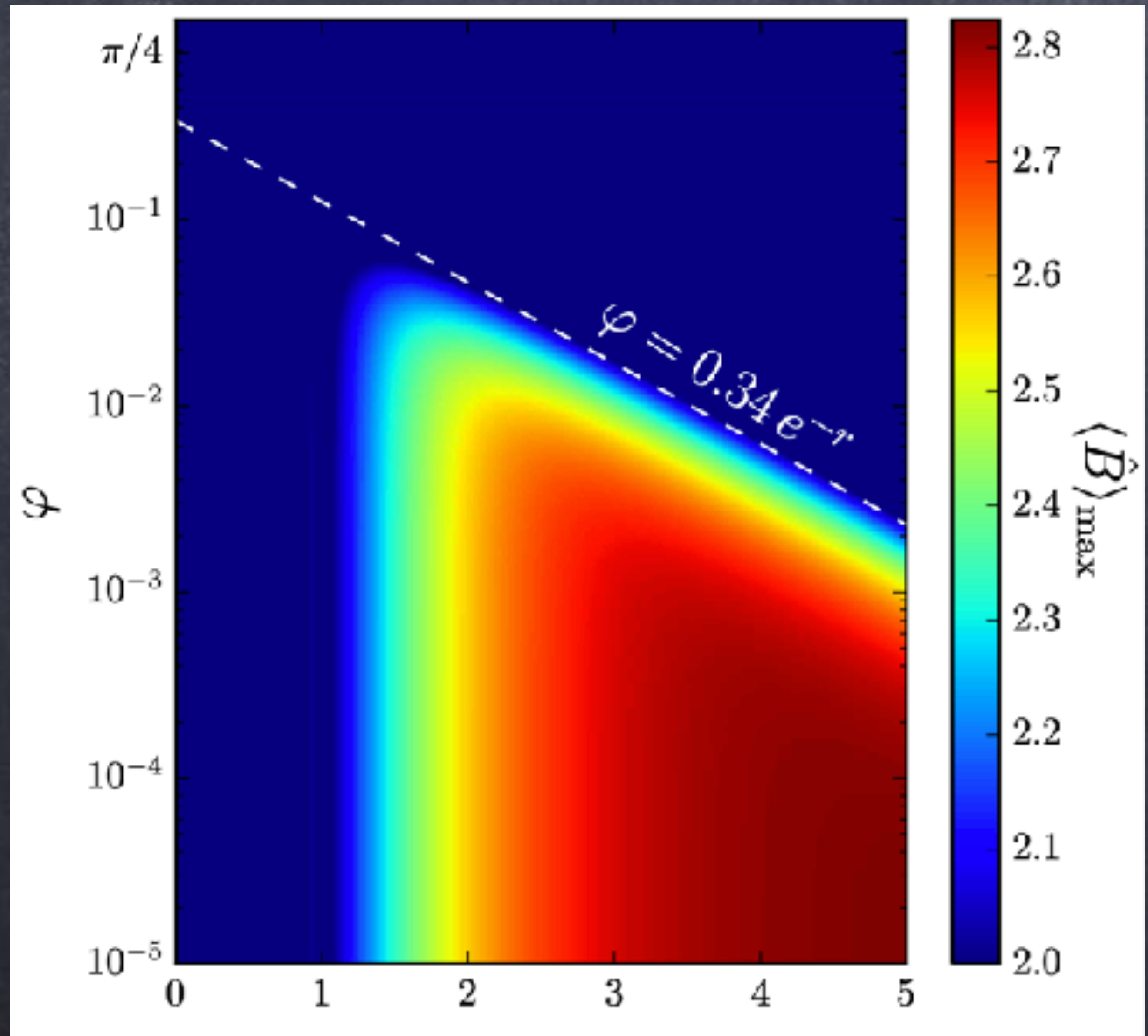
Grishchuk, Sidorov 90
Albrecht, Ferreira, Joyce, Prokopec 94
Polarski, Starobinsky 96
Lesgourgues, Polarski, Starobinsky 97
Martin, Vennin 16a, 16b, 17
Grain, Vennin 20 …

Quantum state: 2-mode squeezed state

$$|\Psi(t_f)\rangle = \prod_{\vec{k}} \frac{1}{\cosh r_k} \sum_n (-1)^n e^{2in\varphi_k} \tanh^n r_k \left| n_{\vec{k}}, n_{-\vec{k}} \right\rangle$$

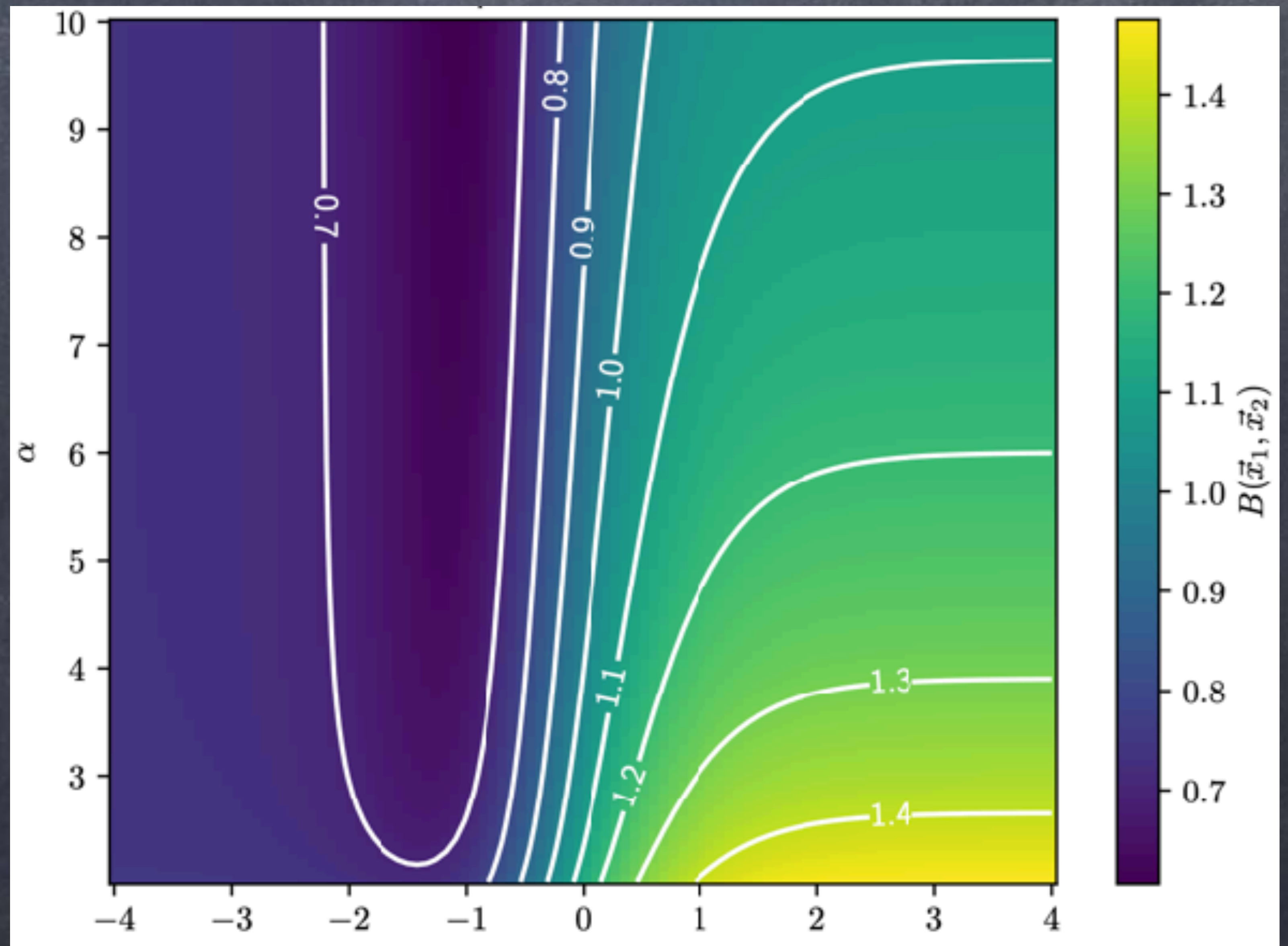
- ***Gaussian state***
- ***Quasiclassical*** (in a sense)
  - Wigner function follows classical equation
  - *2*-point corr. : equivalently described by a classical PDF
  - *N*-points corr. : equivalently described by a classical PDF if $r_k >> 1$

- ***Highly quantum***, i.e. Bell's inequalities are violated



from ***Martin, Vennin 16***

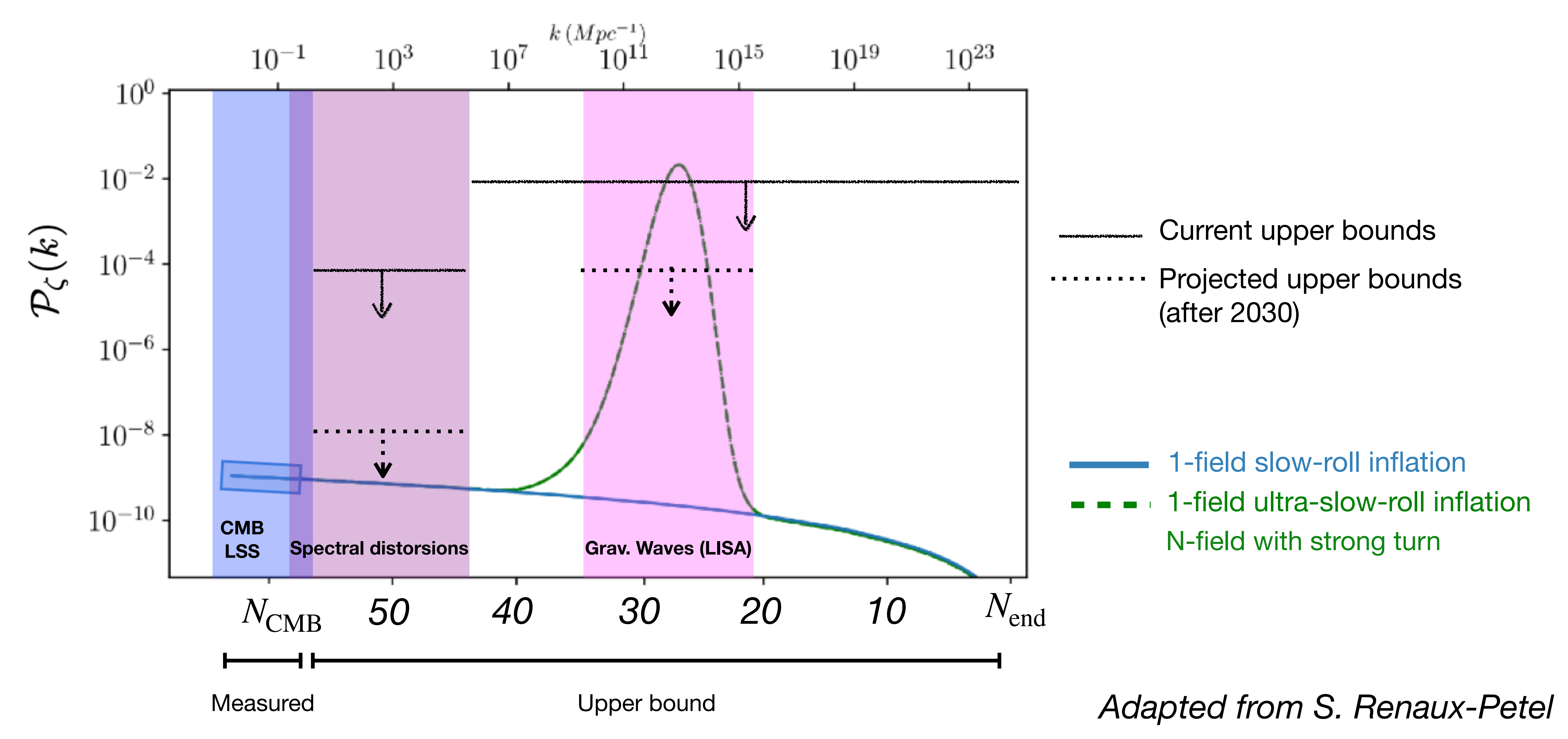
Fourier space ⟷ Real space



from ***Espinosa-Portales, Vennin 22***

The origin of cosmic structures: what we don't know

- ✓ *Type:* Gravitational waves not detected yet
- ✓ *Scales:* **~4** observed over **~26** predicted
- ✓ *Amplitude:* large-fluctuation regime not explored
- ✓ *Character:* quantumness not tested yet



Adapted from S. Renaux-Petel

Large-scale structures:

Prim. NG
Parity-odd N-pt

CMB:

B-mode
Spectral distorsions

Gravitational waves:

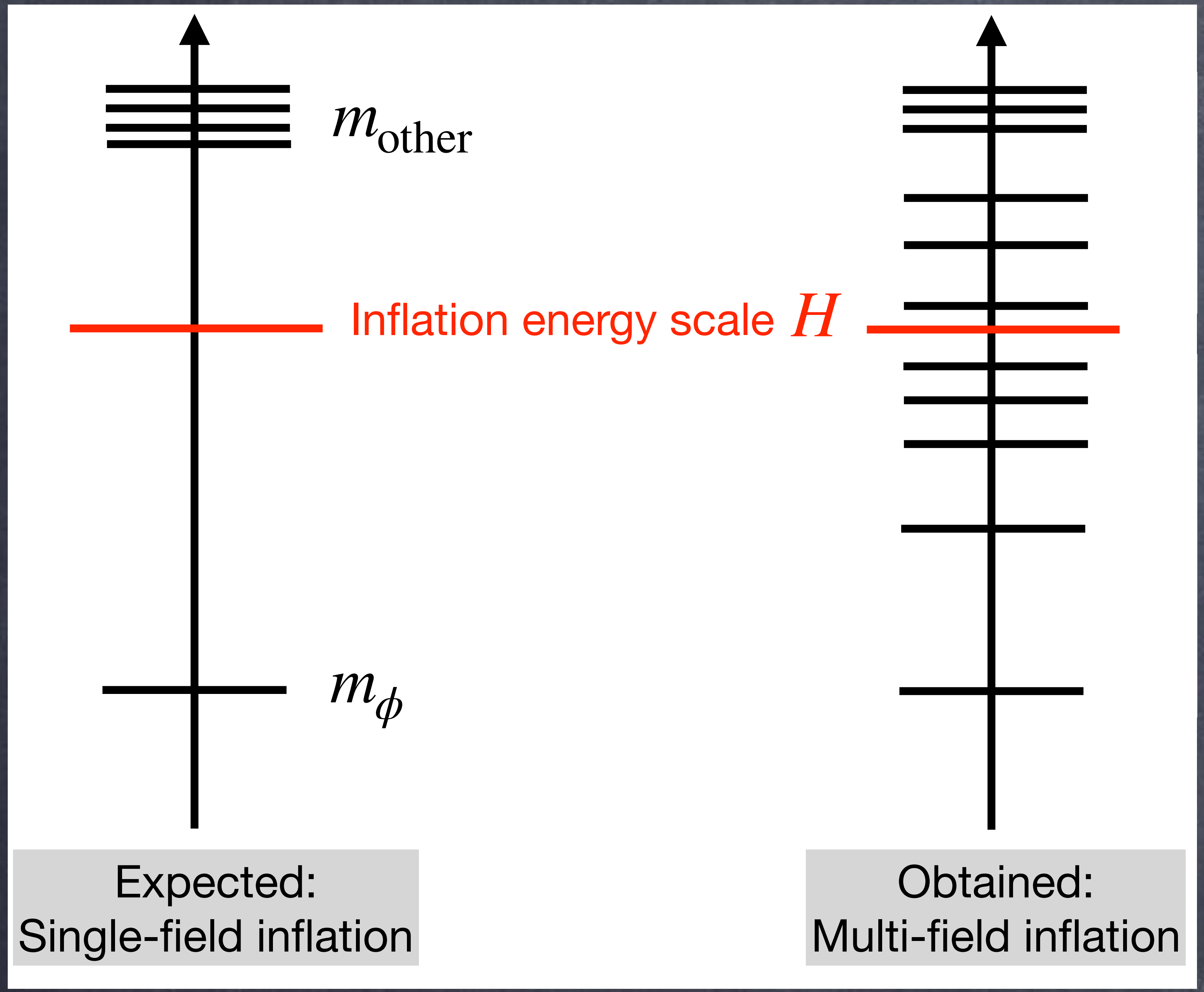
Stochastic background
Search for PBHs

Multiple degrees of freedom in inflation

Planck: 1-field inflation

High-energetic building: *N* fields, large couplings

Baumann, McAllister 14



Curved-field space:

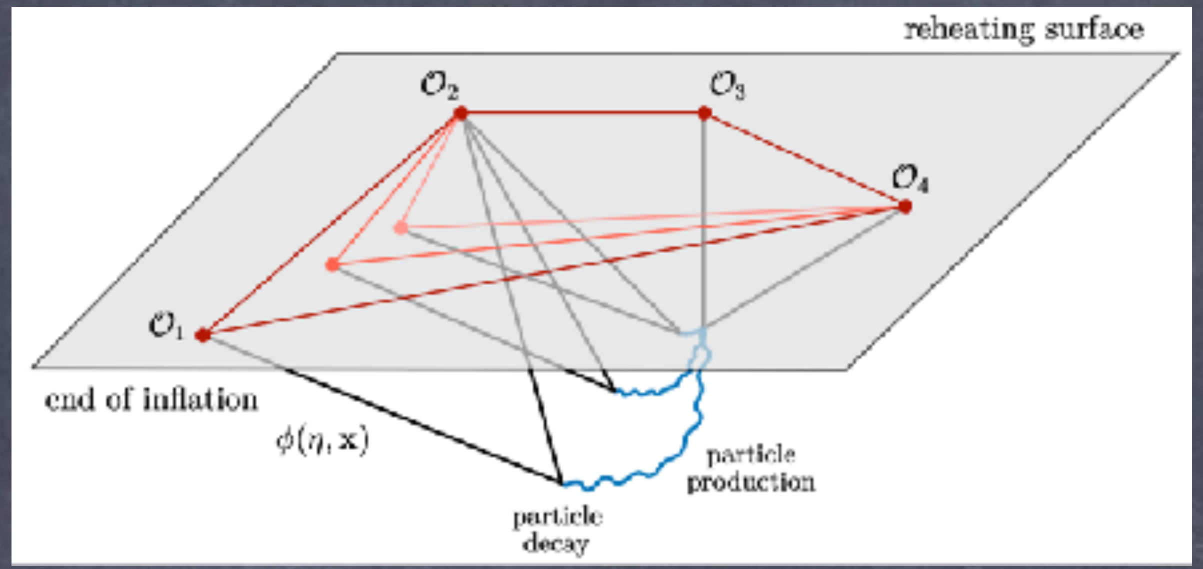
$$S_{mat} = \int \mathrm{d}^4x\sqrt{-g}\left[-G_{IJ}(\phi^K)g^{\mu\nu}\partial_\mu\phi^I\,\partial_\nu\phi^J - V(\phi^I)\right]$$

Covers a large class of HE constructions

Use to explore the rich phenomenology of multi-field inflation

Renaux-Petel, Turzynski 16
Fumagalli et al. 19
Garcia-Saenz, Pinol, Renaux-Petel 20
Fumagali, Renaux-Petel et al. 20, 21a, 21b, 22
Pinol 20
Pinol, Aoki, et al. 21 …

Cosmological collider & bootstraps:



$$\lim_{k_L\to 0}\left\langle \zeta_{k_L}\zeta_{k_S}\zeta_{k_S}\right\rangle \propto \left(\frac{k_L}{k_S}\right)^{3/2}\cos\left[\frac{M}{H}\ln\left(\frac{k_L}{k_S}\right)+\delta\right]P_s(\cos\theta)$$

Chen, Wang 09
Baumann, Green 11
Arkani-Hamed, Maldacena 15
Lee, Baumann, Pimentel 16
Pimentel 18
Goodhew, Jazayeri, Pajer 20
Jazayeri, Renaux-Petel 21 …

Single-field effective theory:

Heavy *d.o.f.*'s integrated out

Effective speed of sound

Achucarro et al. 10, 12
Cespedes at al. 12
Assassi et al. 13
Tong et al. 17 …

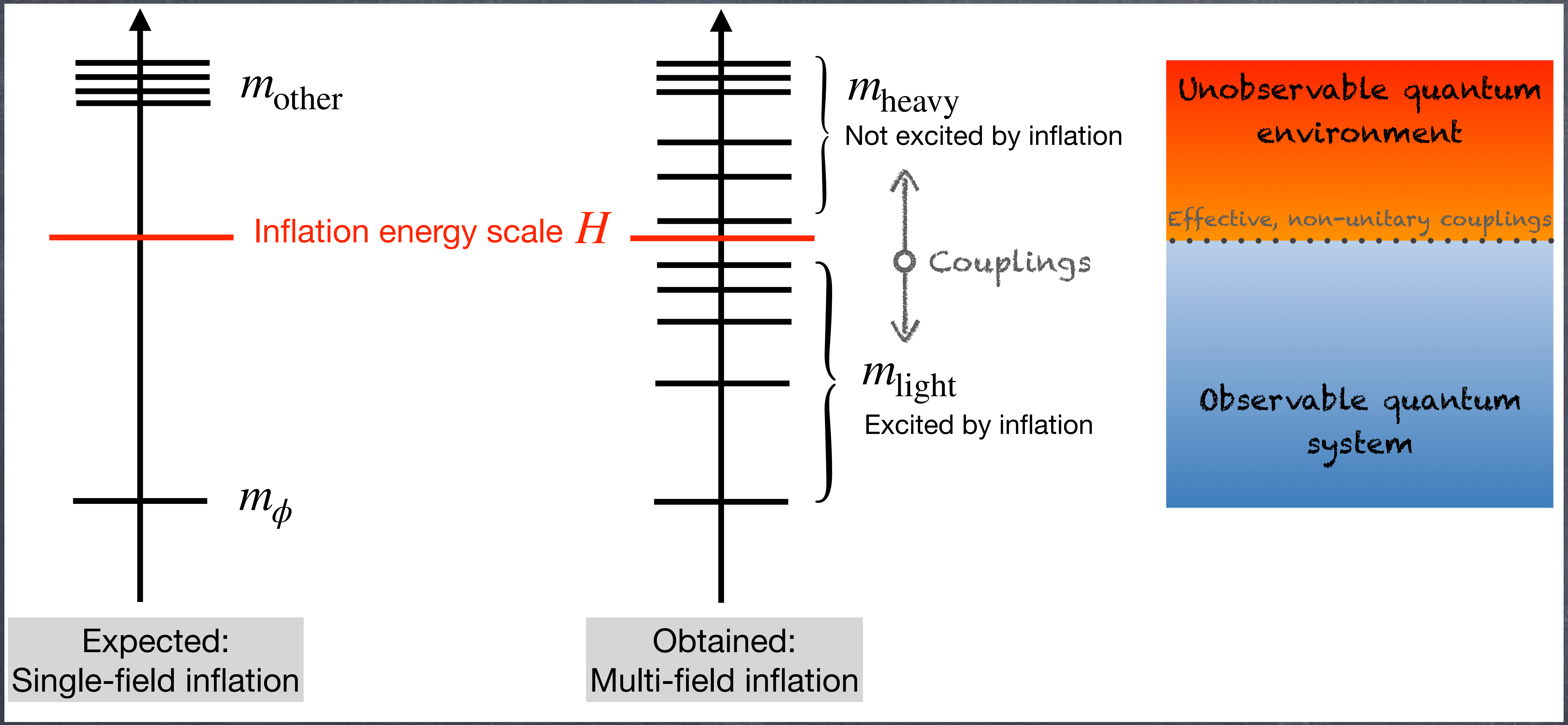
Non-linearities: many mode-couplings

General relativity (CPT at next orders): $S_3 = \int \mathrm{d}^4x f(\eta)\left(\partial\zeta\right)^2\zeta$

N-field: $S \sim \int \mathrm{d}^4x\left[-\frac{1}{2}\left(\partial\phi\right)^2 - \frac{1}{2}\left(\partial\sigma\right)^2 - \frac{1}{2}m^2\sigma^2 - \mu\sigma^3 + \rho\dot{\phi}\sigma\right]$

Multiple degrees of freedom in inflation

Many approches: explicit models, cosmological collider & bootstraps, 1-field effective theory
➡***Open quantum system*** techniques

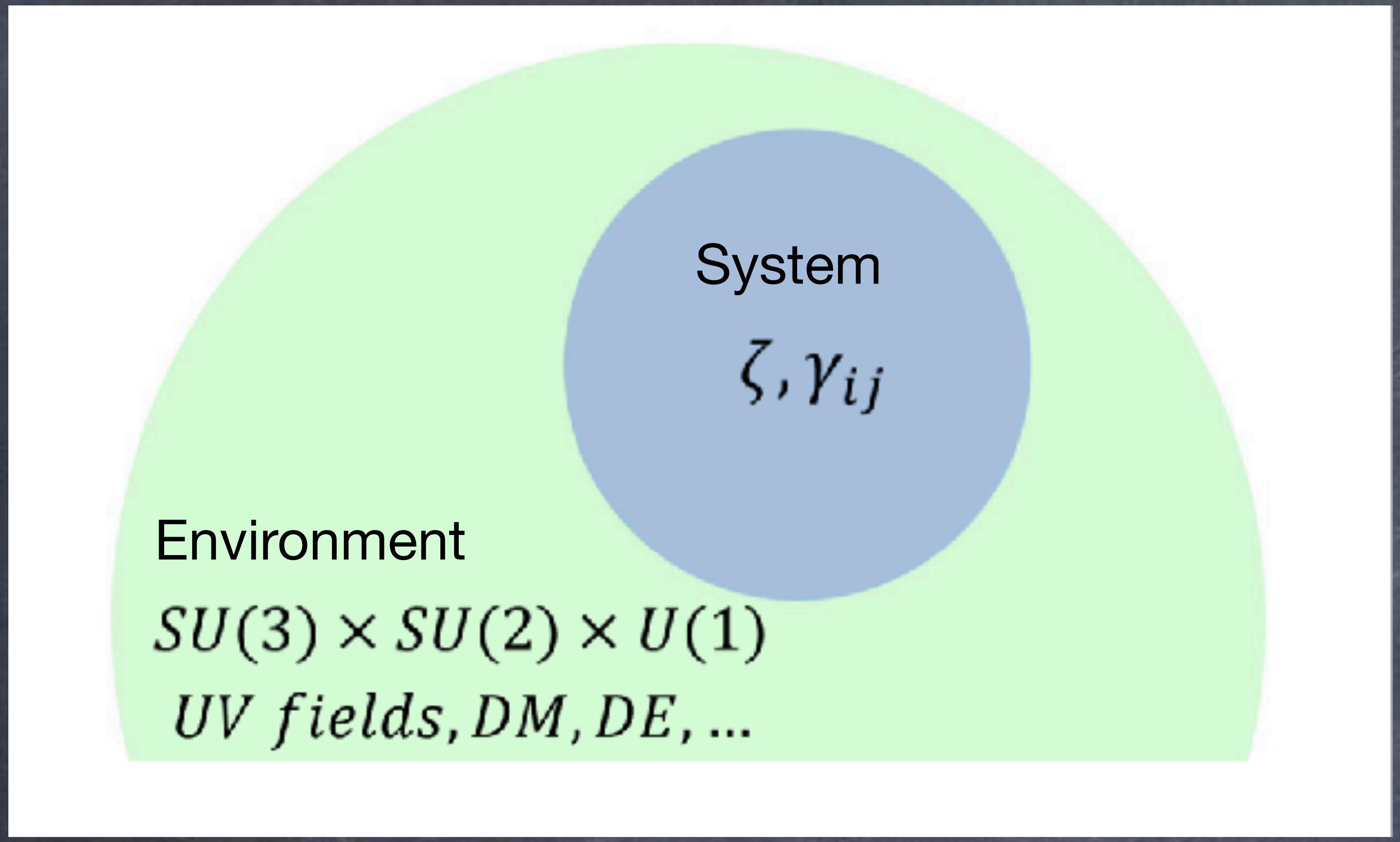


Boyanovsky 15
Martin, Vennin 18a, 18b
Colas, Grain, Vennin, 21, 22a, 22b
Martin, Micheli, Vennin 22
Burgess et al. 22
Bartolo, Daddi Hamou 22
Brahma, Berera, Calderon-Figueroa 22 …

- Predict cosmological observables
- Resum secular effects
- Keep track of quantum properties
- Explore decoherence channels

➡ ***Promising theoretical tool: cosmological observables & quantum properties***

Cosmological open quantum systems

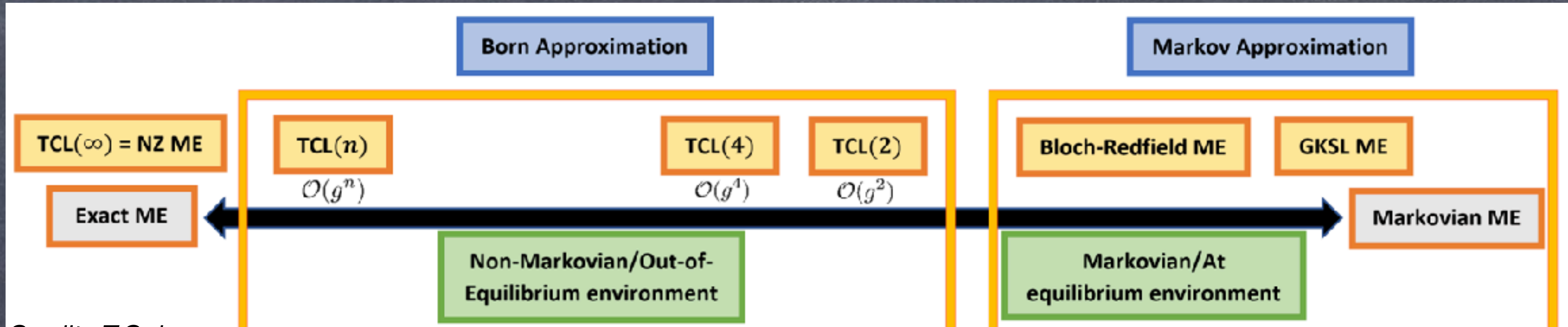


Effective master dynamical equation for the reduced density matrix $\hat{\rho}_{\text{red}}$

$$\frac{\mathrm{d}\hat{\rho}_{\text{red}}}{\mathrm{d}t} = \mathscr{L}\left[\hat{\rho}_{\text{red}}\right]$$

Whole bestiary of master equations

Breuer, Petruccione 02



Credits T.Colas

Earth-based experiments

Environment is ***thermal bath:*** **Lindblad** equation

- Equilibrium & infinitely many *d.o.f.*'s (continuous spectrum of frequencies)
- Non-unitary evolution
- Markovian evolution

Cosmological context

Dynamical background spacetime

Environment is:

- Out-of-equilibrium & small number of *d.o.f.*'s
- Driven dynamics (pairs constantly pumped out of the vacuum)
- ***Non-Markovian*** evolution

➡ ***What ME equation in cosmology?***

Benchmarking cosmological master equations

Colas, Grain, Vennin, 21

Compare ME predictions *against* exactly solvable model: 2-field with quadratic action

$$\mathscr{H} = \frac{1}{2}\int \mathrm{d}^3k \left[\vec{z}_S^{\,\dagger}\boldsymbol{H}_S(t)\vec{z}_S + \vec{z}_E^{\,\dagger}\boldsymbol{H}_E(t)\vec{z}_E + \vec{z}_S^{\,\dagger}\boldsymbol{V}(t)\vec{z}_E + \mathrm{c.c.}\right]$$

with phase-space variables $\vec{z}_i = (\phi_i, \pi_i)^{\mathrm{T}}$

Encompasses
- 2 test scalar fields
- 2-field inflation w. adiabatic vs. entropic mode
- Effective *N*-field inflation

'Free' dynamics

$$\mathscr{H}_i \propto f_i(t)\left[\hat{a}^\dagger_{i,\vec{k}}\hat{a}_{i,\vec{k}} + \hat{a}^\dagger_{i,-\vec{k}}\hat{a}_{i,-\vec{k}} + 1\right] + \left[g_i(t)\,\hat{a}^\dagger_{i,\vec{k}}\hat{a}^\dagger_{i,-\vec{k}} + \mathrm{h.c.}\right]$$

Pairs of *i*-particle

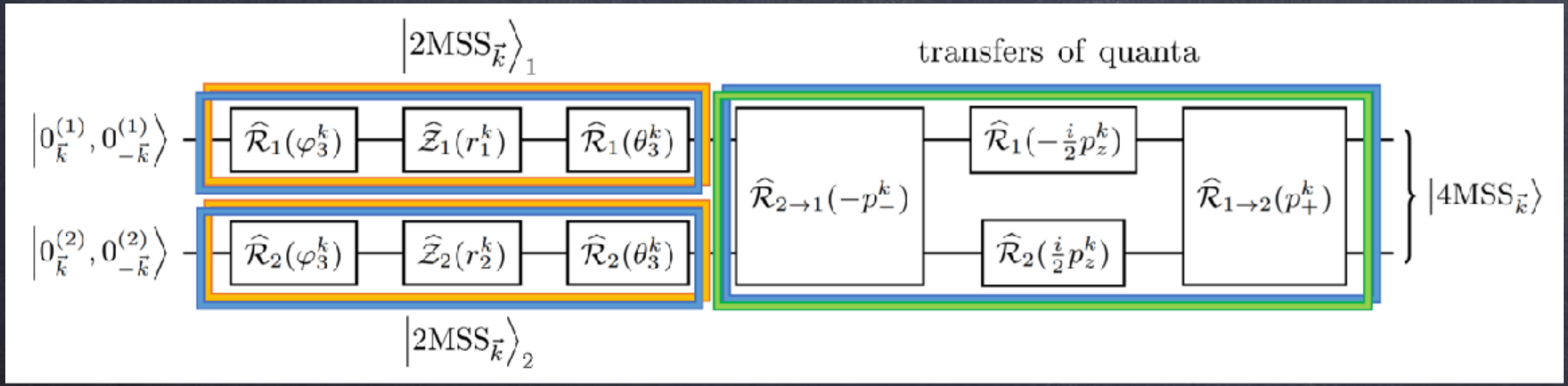
Interaction

$$\mathscr{V} \propto f(t)\left[\hat{a}^\dagger_{S,\vec{k}}\hat{a}_{E,\vec{k}} + \hat{a}^\dagger_{S,-\vec{k}}\hat{a}_{E,-\vec{k}} + 1\right] + g(t)\left[\hat{a}^\dagger_{S,\vec{k}}\hat{a}^\dagger_{E,-\vec{k}} + \hat{a}^\dagger_{E,\vec{k}}\hat{a}^\dagger_{S,-\vec{k}}\right] + \mathrm{h.c.}$$

Transfer of quanta

Entangled pairs

Evolution operator in Sp(4,R): *4-mode squeezed state*



- Most generic state in 2-field inflation
- Exact expression: sum over num. exchanged particles
- Scales as $\propto \left[\lambda(t)\right]^n$
- Gaussian state —> tracing over *E* by marginalization

Benchmarking cosmological master equations

Colas, Grain, Vennin 22a

Curved-space 'Caldeira-Leggett' model

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2}M^2\chi^2 + \lambda^2\phi\chi\right)$$

System — Environment — Interaction

Light system & heavy environment: $m \ll H \ll M$

De Sitter space: $a(\eta) = -1/(H\eta)$

➡ **Fully integrable model:** Gaussian state

Covariance matrix $\boldsymbol{\Sigma}_{\phi\phi}$

Quantum information properties $\gamma = \mathrm{Tr}\left[\hat{\rho}^2_{\mathrm{red}}\right] = \det\left(\boldsymbol{\Sigma}_{\phi\phi}\right)^{-1}/4$

ME equation at 2nd order in Born —*i.e.* Time-ConvolutionLess at $\mathcal{O}(\lambda^4)$

Unitary — Non-unitary

$$\frac{\mathrm{d}\hat{\rho}_{\mathrm{red}}}{\mathrm{d}\eta} = -i\left[\hat{H}_\phi(\eta) + \hat{H}_{\mathrm{LS}}(\eta), \hat{\rho}_{\mathrm{red}}\right] + \sum_{i,j=1}^{2}\mathcal{D}_{ij}(\eta)\left[\hat{z}_i\hat{\rho}_{\mathrm{red}}\hat{z}_j - \frac{1}{2}\left\{\hat{z}_j\hat{z}_i, \hat{\rho}_{\mathrm{red}}\right\}\right]$$

with $\hat{H}_{\mathrm{LS}}$ and $\mathcal{D}_{ij}$ at order $\mathcal{O}(\lambda^4)$

Lamb-Shift
rescaling of energy levels

Diffusion & dissipation
Non-Markovian for $\mathcal{D}$ non-positive semidefinite

Benchmarking cosmological master equations

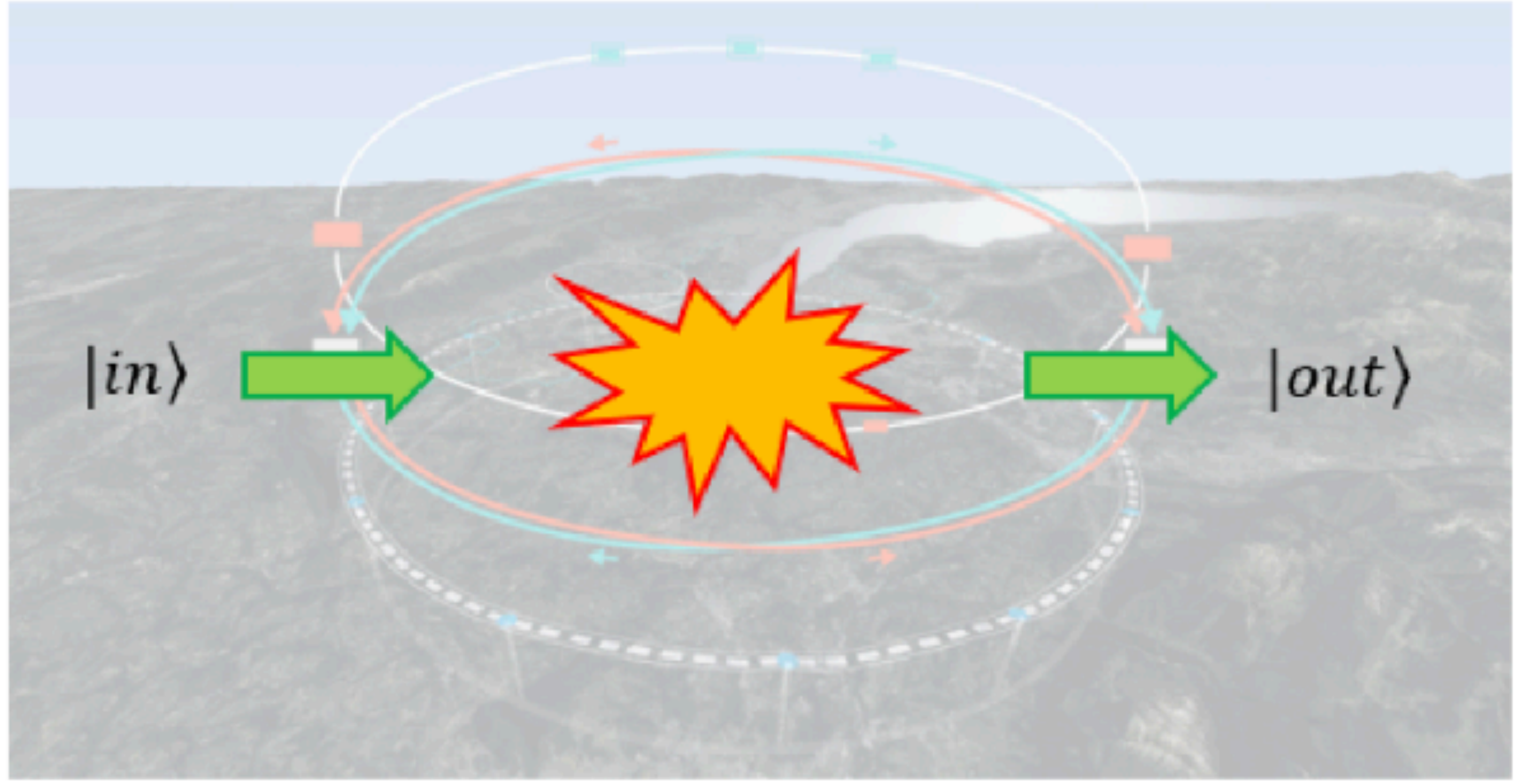
Colas, Grain, Vennin 22a

Curved-space ‘Caldeira-Leggett’ model

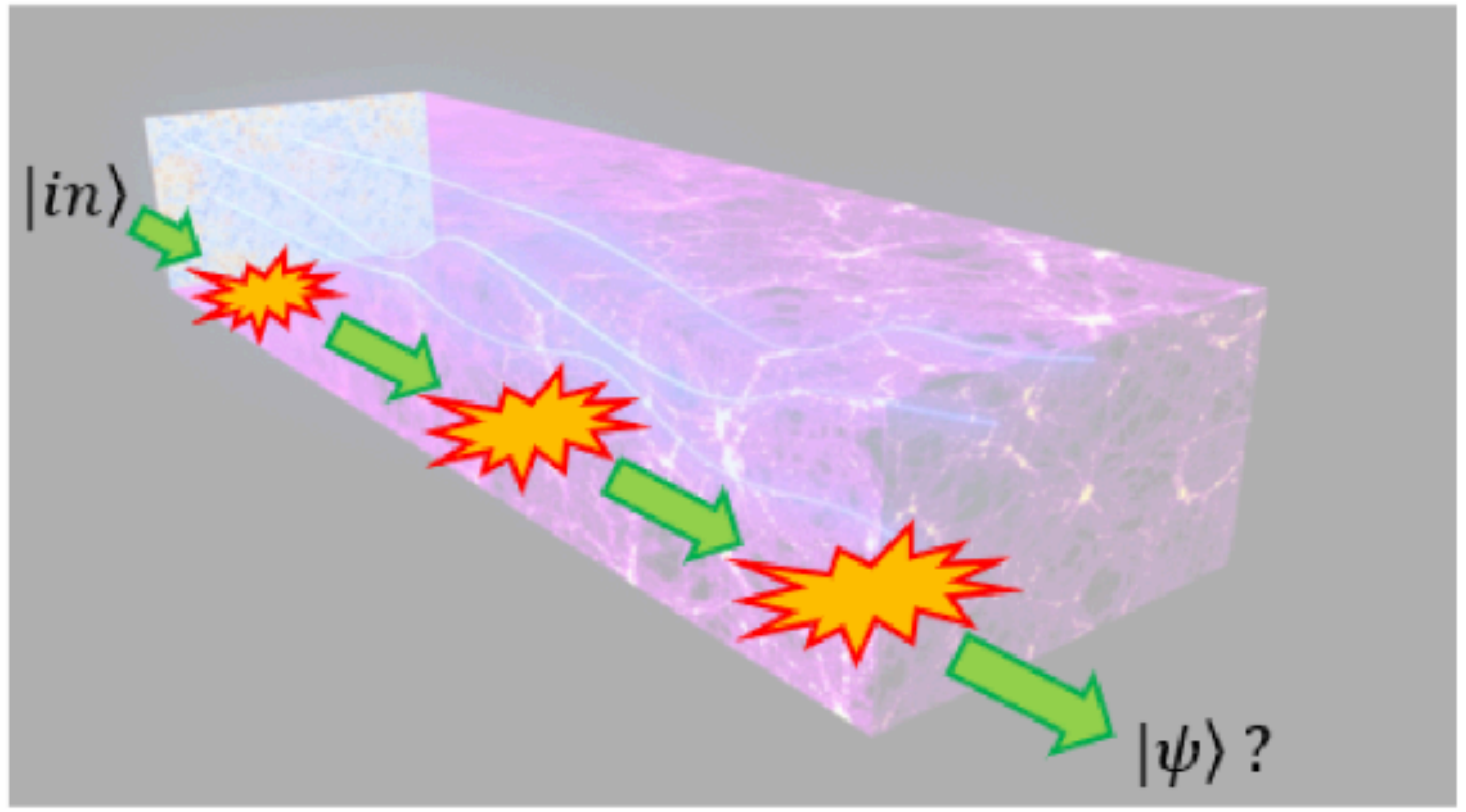
$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2}M^2\chi^2 + \lambda^2\phi\chi\right)$$

Light system & heavy environment: $m \ll H \ll M$

De Sitter space: $a(\eta) = -1/(H\eta)$



Credits : FCC, CERN



Credits : Planck, ESA

- Exact solution
- Standard perturbation theory: perturbative solution of ME at $\mathcal{O}(\lambda^4)$ exactly SPT
- Non-perturbative solution of ME: ***Partial resummation***

  TCL_n all the terms at order $(\lambda^2)^n$ and some of $(\lambda^2)^{m>n}$

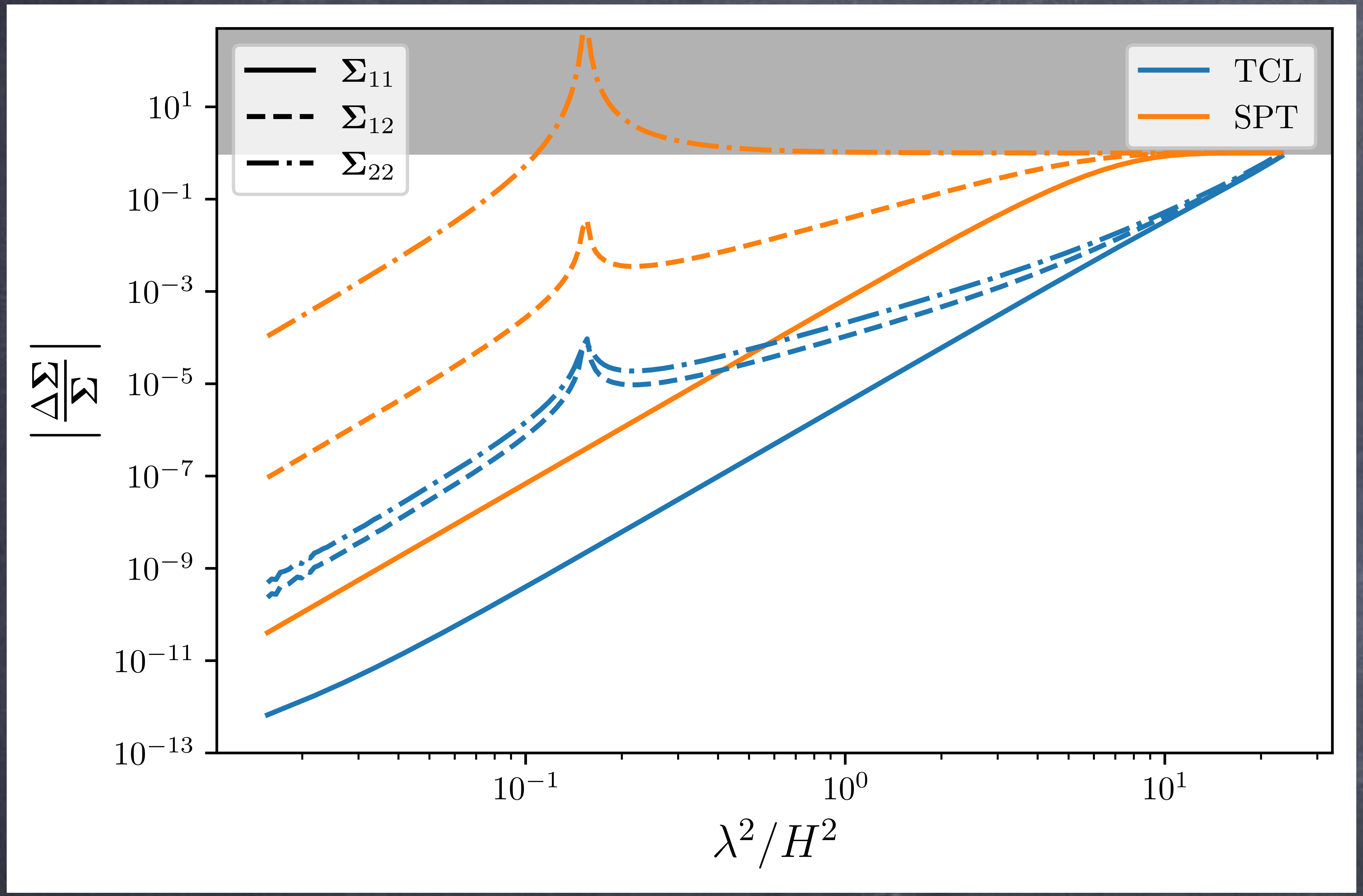
  Breaks some order-by-order cancellations

Benchmarking cosmological master equations

Colas, Grain, Vennin 22a

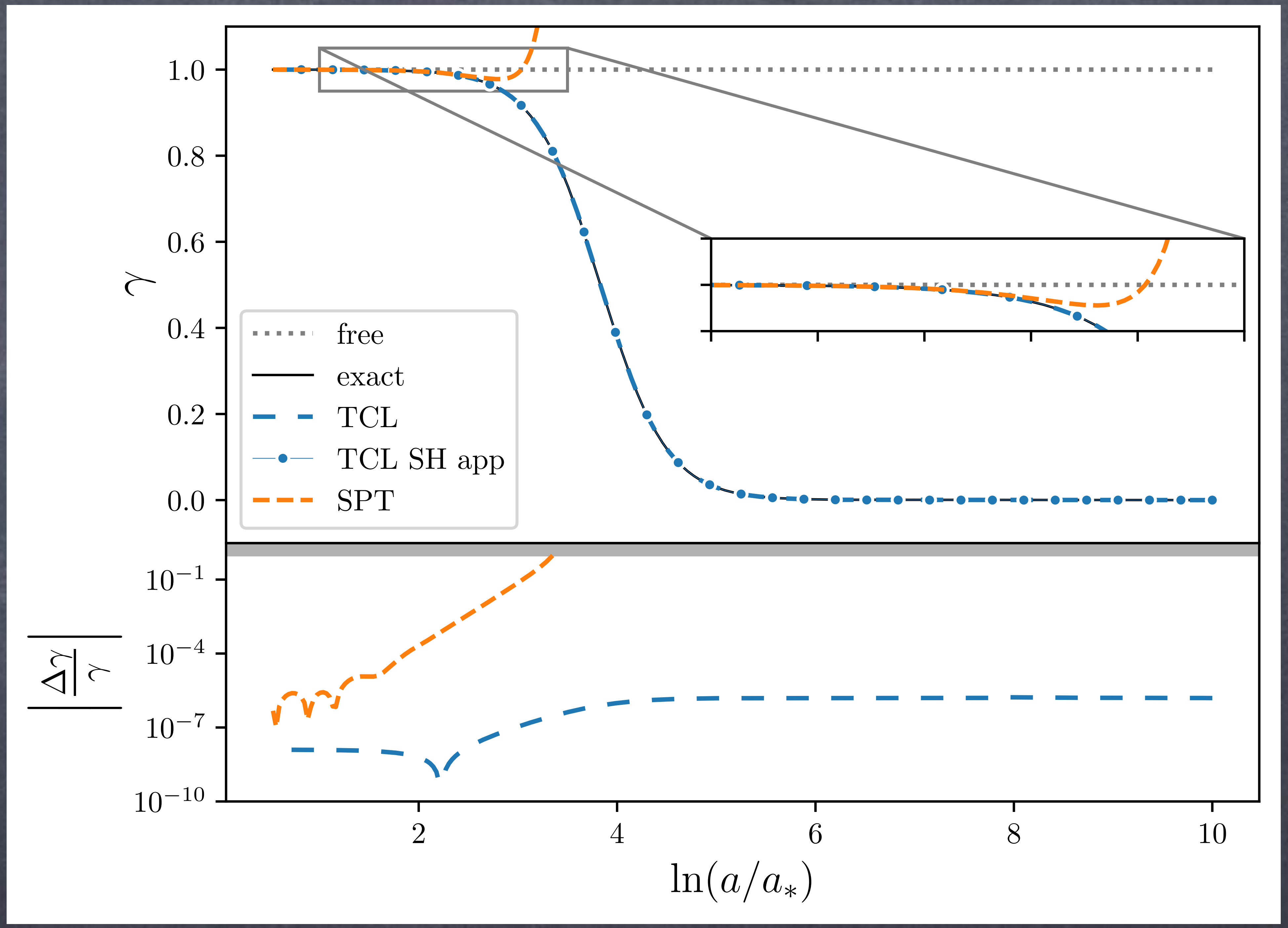
Curved-space ‘Caldeira-Leggett’ model

Power spectra



- ME systematically performs better
- Performs amazingly well (given that 1 d.o.f. in environment)
- SPT already performs well

Self-coherence



- SPT rapidly fails systematically performs better
- ME still performs very well

➡ Partial resumation necessary for purity

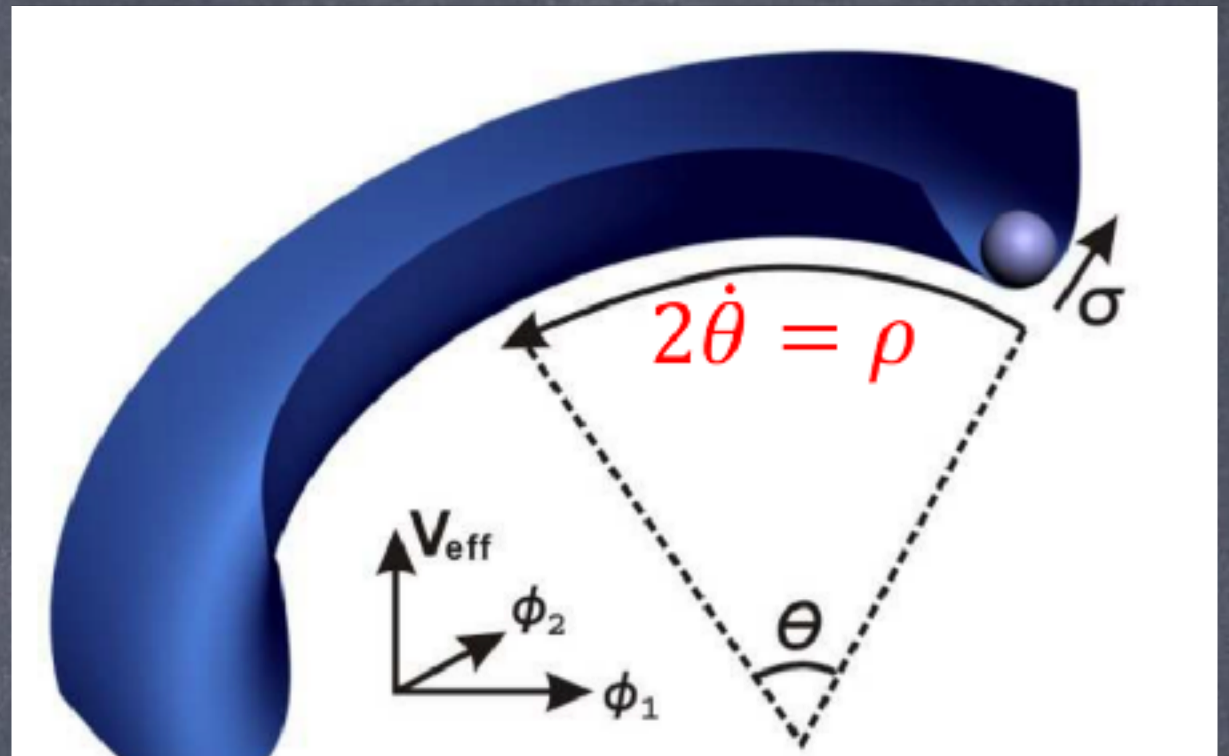
Recoherence in the early universe

Colas, Grain, Vennin 22b

Most general setup for curvature pert. coupled to entropic mode

- Massless curvature pert.: $S_\zeta = \int \mathrm{d}^4x \epsilon a^2 M_{Pl}^2 \left[\zeta'^2 - (\partial_i\zeta)^2\right]$
- Massive entropic pert.: $S_{\mathcal{F}} = \int \mathrm{d}^4x \frac{a^2}{2} \left[\mathcal{F}'^2 - (\partial_i\mathcal{F})^2 - m^2a^2\mathcal{F}^2\right]$
- Interaction via momentum: $S_{\mathrm{int}} = \int \mathrm{d}^4x \rho a^3 \sqrt{2\epsilon} M_{Pl} \zeta' \mathcal{F}$

Chen, Wang 09, 10
Baumann, Green 11
Assassi et al. 13
Pinol 18 …



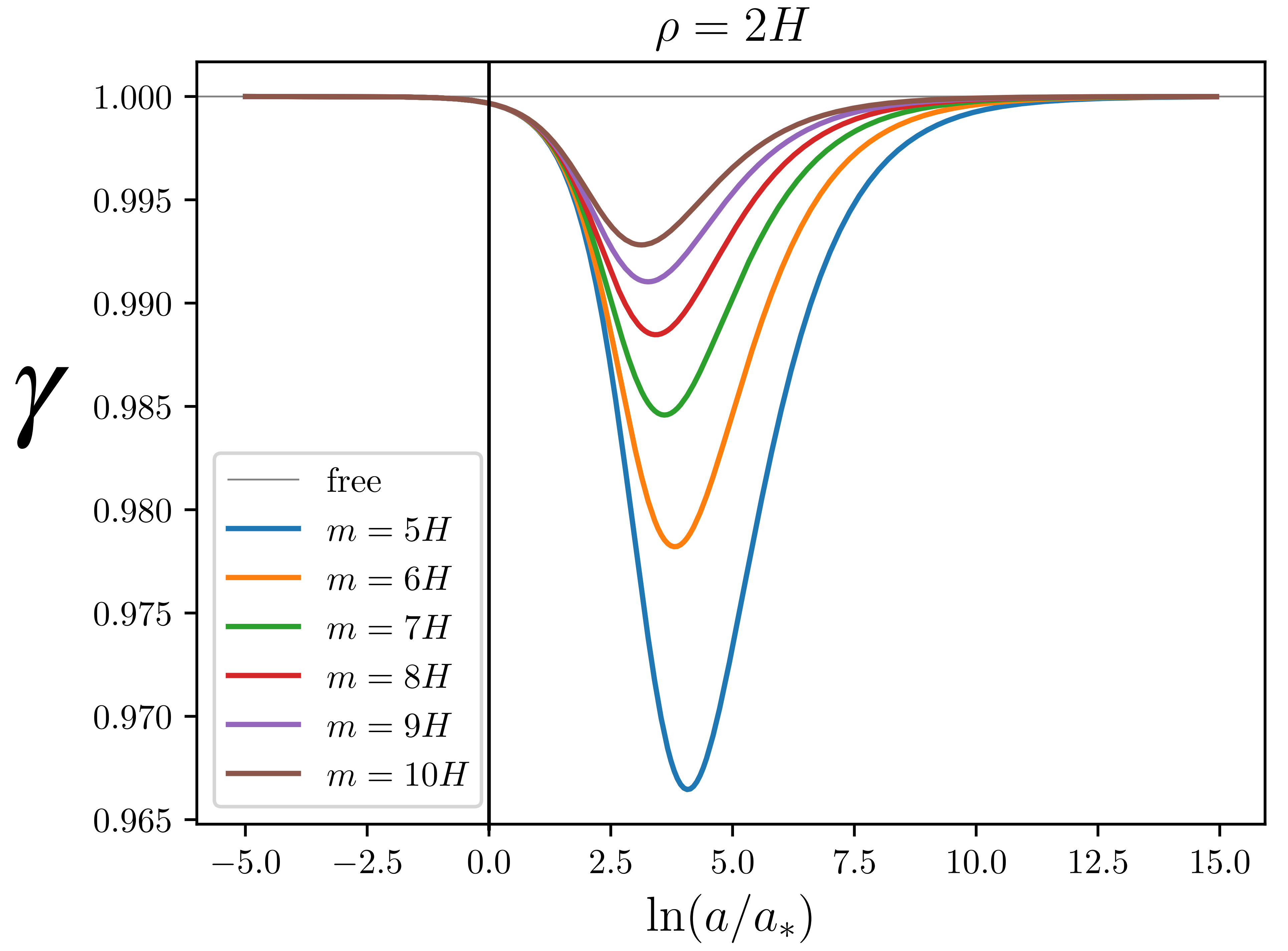
Credit Chen, Wang 10

Power spectrum: *effective speed of sound* $c_s^2 = 1 - (\rho/m)^2$

$$\mathcal{P}_\zeta = \mathcal{P}_\zeta^{(0)} \left[1 + \rho^2/(2m^2)\right]$$

➡No dramatic resummations

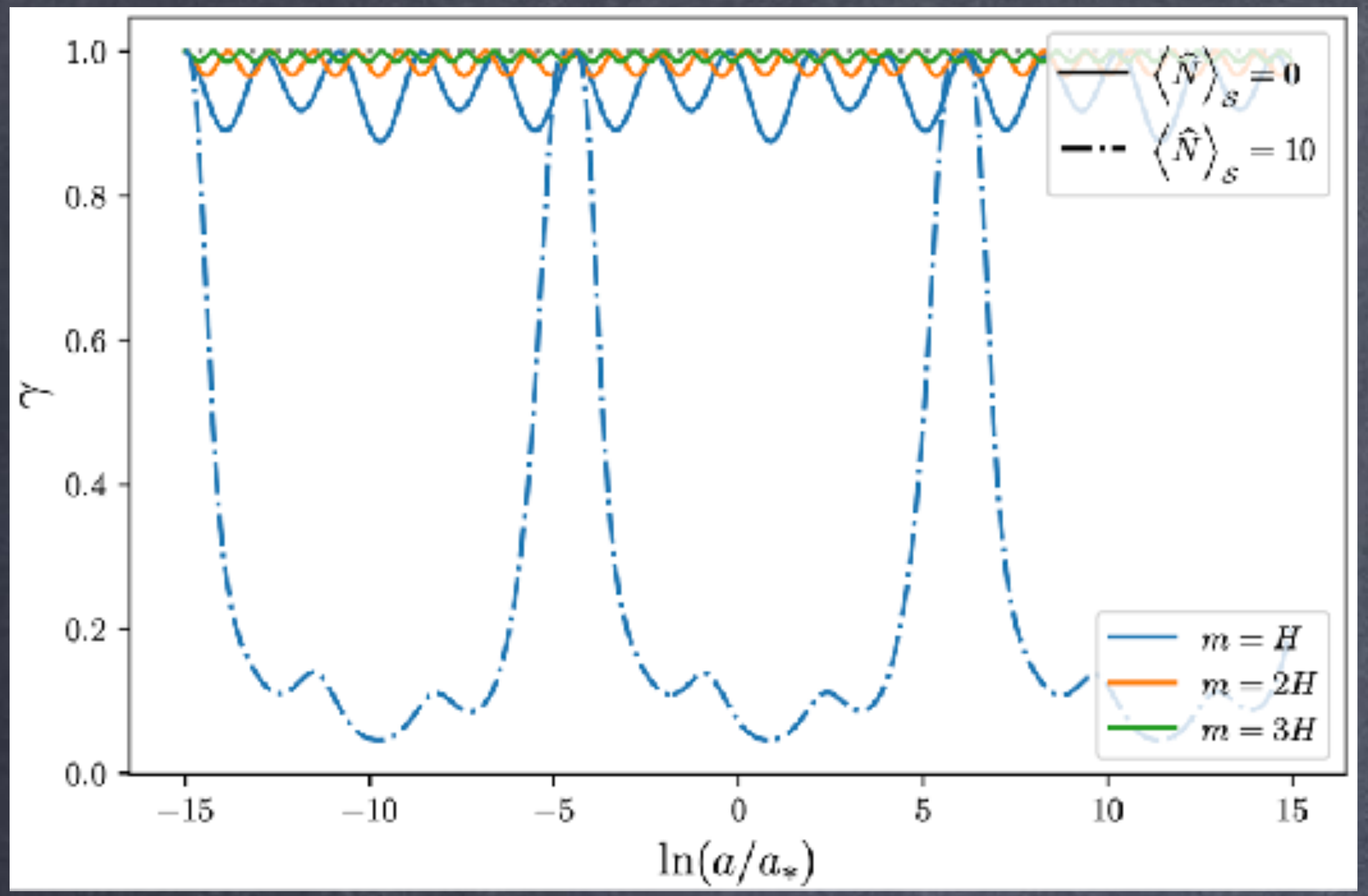
Quantum information properties: *recoherence*



Recoherence in the early universe

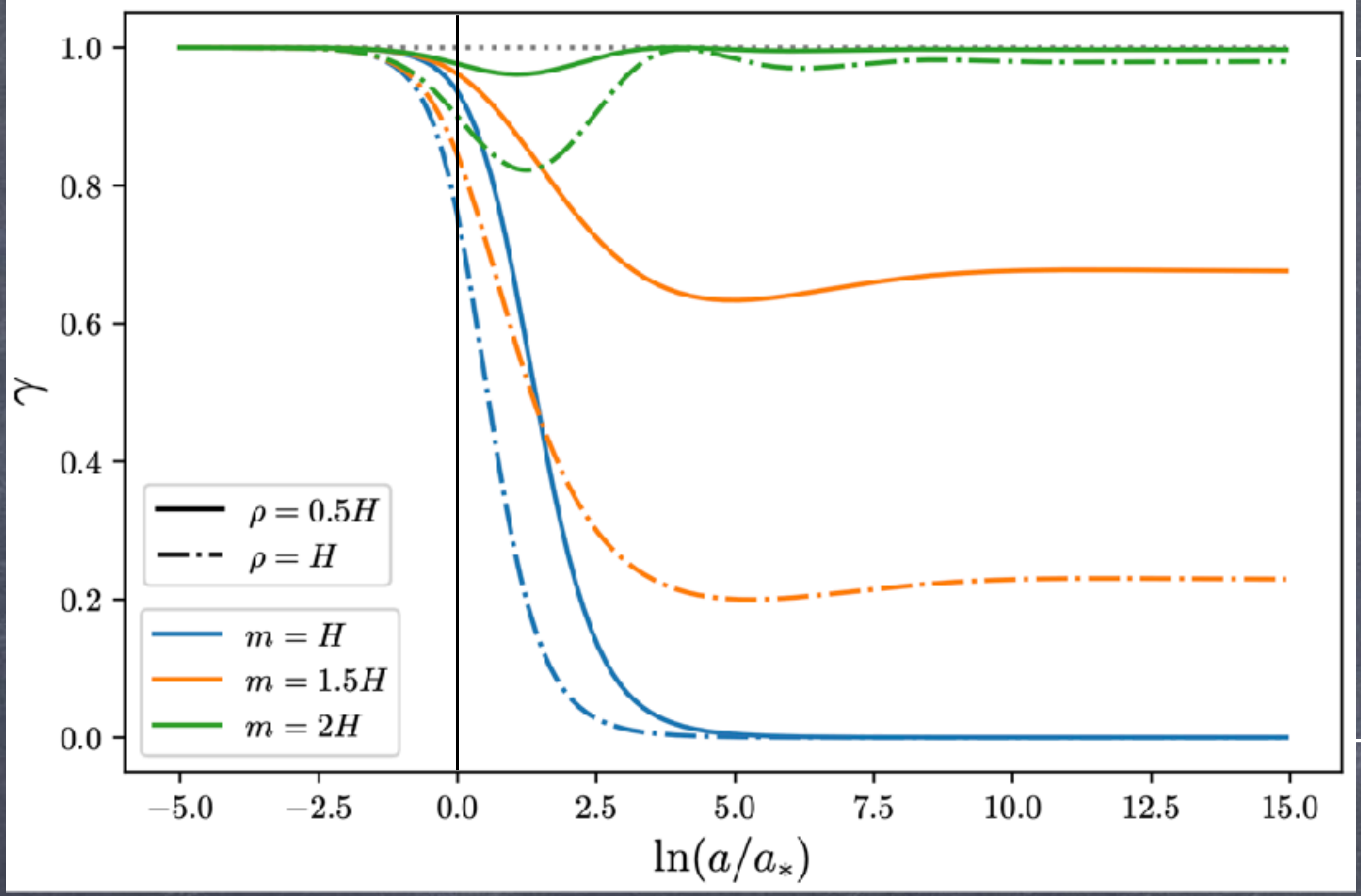
Colas, Grain, Vennin 22b

Flat space analog (2 H.O.'s)



Heavy *vs.* light environment

$$\mathscr{L}_{\rm int} = \rho a^3 \sqrt{2\varepsilon} M_{Pl} \zeta' \mathscr{F}$$



ζ' decays
$\mathscr{F}$ ***oscillates***
➡ $\mathscr{L}_{\rm int}$ is suppressed

ζ' decays
$\mathscr{F}$ ***grows***
➡ $\mathscr{L}_{\rm int}$ is not suppressed

Heavy entropic mode: interaction effectively shuts down
➡ Decoupling both for cosmological observables ***and*** quantum properties
➡ Recoherence: Non-markovian process which can't be captured with Lindblad

Conclusion

- Quantum properties remain elusive to our observations

- Single field inflation: serious obstructions to e.g. Bell's inequalities tests

Leggett-Garg inequalities ***Martin, Vennin 16***
Analytic properties of higher-order statistics ***Green, Porto 20***

- Many extra d.o.f.'s in inflation: ***Cosmological open quantum system*** *programme*

Cosmological observables with (partial) non-perturbative resumation
Tracks the quantum information properties w. decoherence channels
➡ Environment: out-of-equilibrium & small ; driven dynamics
➡ Non-Markovian evolution: recoherence process

- Further developments

Formal: link with EFT and stochastic inflation, Born *vs.* Markovian expansion
Markovianity with *N*-fields: is decoherence inevitable?
Non-linearities: higher-order statistics (TCL_3), markovianity regained