Status of the muon $g-2$

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Aix*Marseille université
anr agence nationale de la recherche
CPT Centre de Physique Théorique

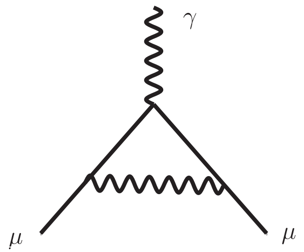
- Magnetic moment of charged leptons :

$$\vec{\mu} = g_\ell \left(\frac{Qe}{2m_\ell} \right) \vec{S}$$

- At the classical level (Dirac equation) : $g_\ell = 2$

- In the Standard Model, quantum corrections slightly shift this value

$$a_\ell = \frac{g_\ell - 2}{2} = \frac{\alpha}{2\pi} + O(\alpha^2)$$



- What is special with the muon ?

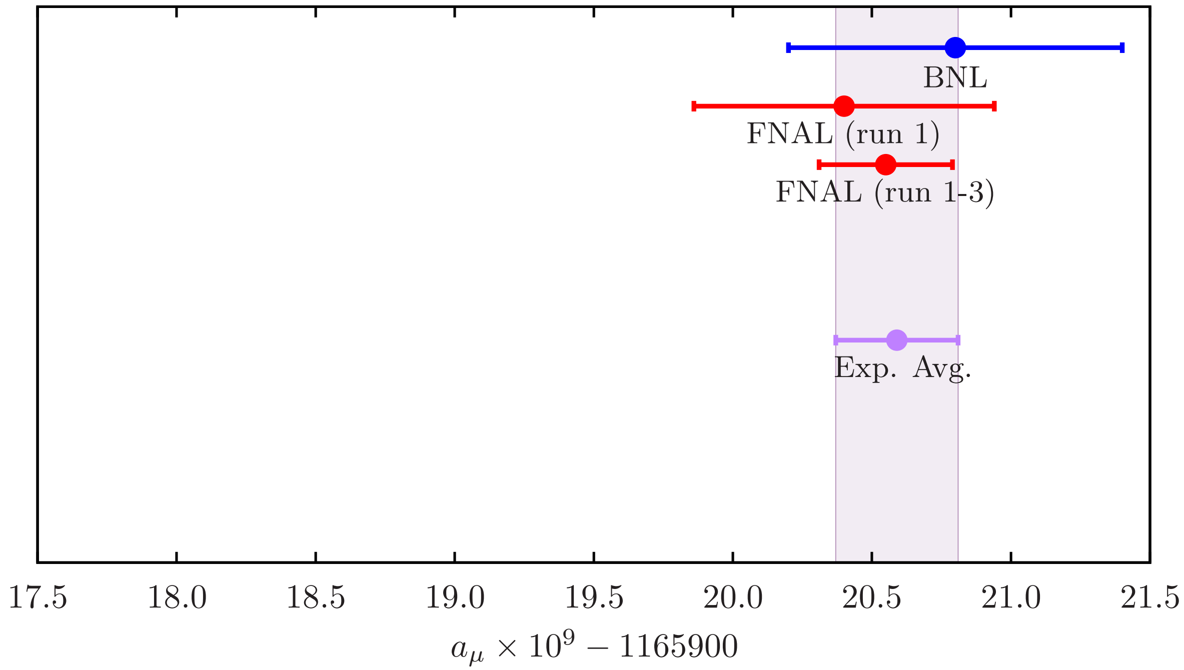
  → a_μ can be measured very precisely (0.2 ppm) ...

  → ... and can be computed with (similar) precision in the SM

  → muons are 200 heavier than electrons (and $\tau_\mu = 2.2\ \mu s \gg \tau_\tau$)

$$\delta a_\ell^{\mathrm{NP}} = \mathcal{C}\, \frac{m_\ell^2}{\Lambda_{\mathrm{NP}}^2}$$

Status after runs 1-3 at Fermilab (August 2023)



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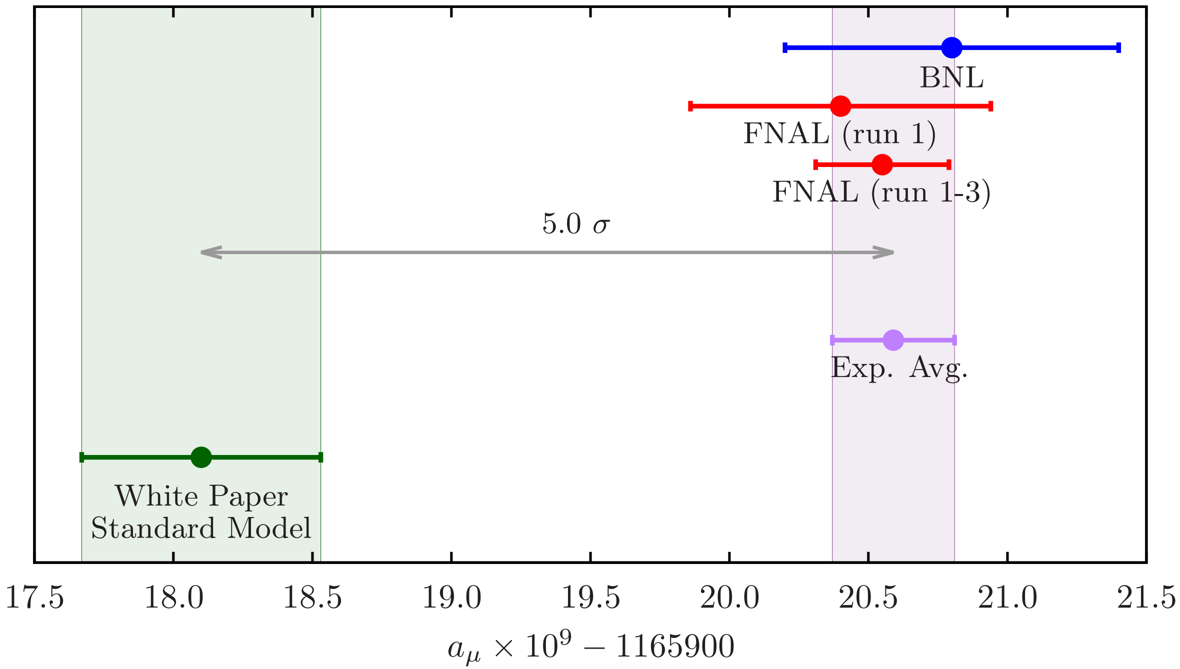

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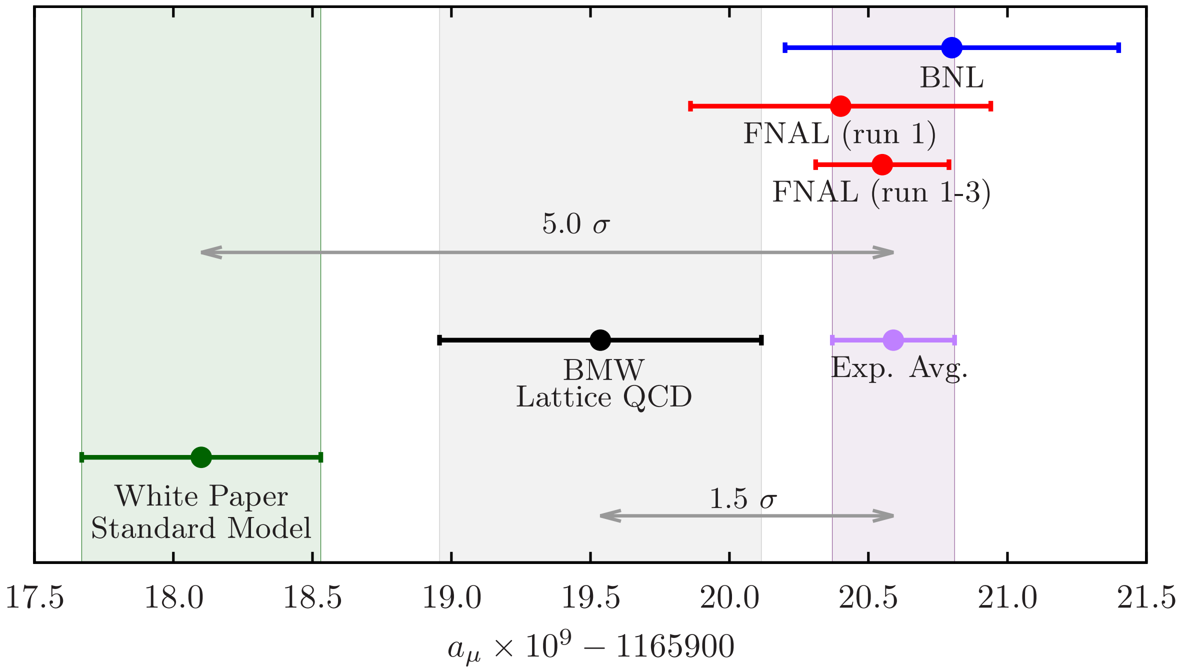

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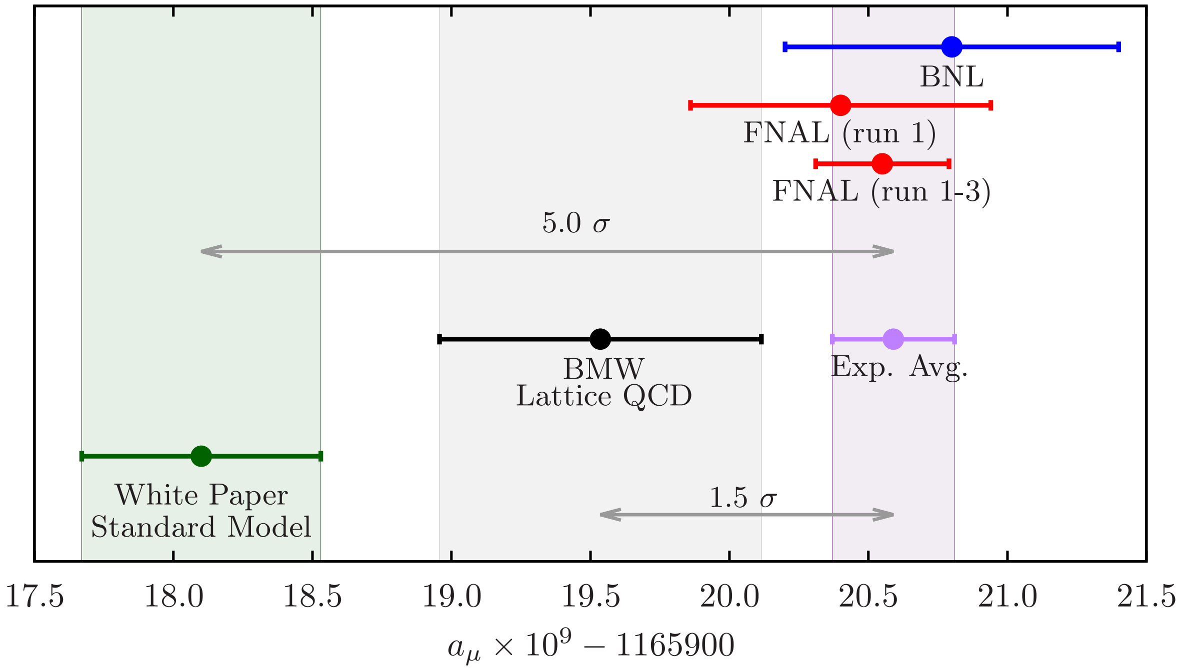

This talk : focus on the Standard Model prediction

"*The anomalous magnetic moment of the muon in the Standard Model*" [Phys.Rept. 887 (2020) 1-166]

Contribution	$a_\mu \times 10^{11}$
- **QED** (10^{th} order)	$116\,584\,718.931 \pm 0.104$
- **Electroweak**	153.6 ± 1.0
- **Strong interaction**	
HVP (LO)	$6\,931 \pm 40$
HVP (NLO + NNLO)	-85.9 ± 0.7
HLbL	92 ± 18
Standard Model	$116\,591\,810 \pm 43$
Experiment	$116\,592\,059 \pm 22$

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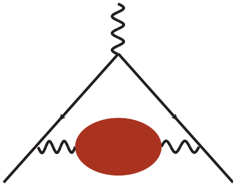
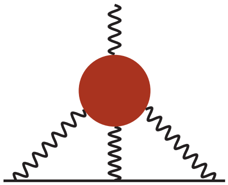


Hadronic Vacuum Polarisation

(HVP, α^2)



Hadronic Light-by-Light scattering

(HLbL, α^3)

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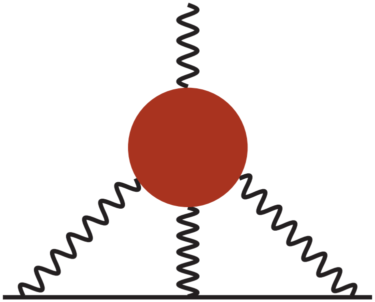
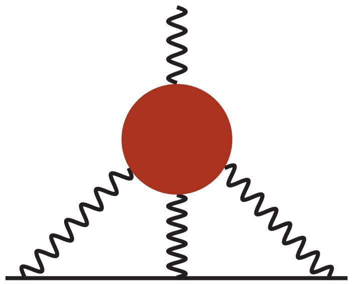
- Error budget **dominated by hadronic contributions : LO-HVP and HLbL**
  - → HVP / HLbL : dominated by low-energy physics (ρ meson / pion-pole contribution)
  - → first-principle calculations to have controlled uncertainties
    - Dispersive framework (data-driven)
    - Lattice QCD

Hadronic Light-by-Light Scattering



$a_\mu^{\mathrm{HLbL}} = (92 \pm 18) \times 10^{-11}$
$a_\mu^{\mathrm{Exp.}} = (116\ 592\ 059 \pm 22) \times 10^{-11}$

$\rightarrow$ we need $< 10\%$ precision



- Very challenging to compute

  $\rightarrow$ hadronic light-by-light tensor $\Pi_{\mu\nu\lambda\sigma}(p_1, p_2, p_3) = \int_{x,y,z} \Pi_{\mu\nu\lambda\sigma}(x,y,z) e^{-i(q_1 x + q_2 y + q_3 z)}$

  $\rightarrow$ multi-scale system

- Until 2016 : mostly based on model estimates

$$a_\mu^{\mathrm{HLbL}} = 105(26) \times 10^{-11} \qquad \text{[Prades, de Rafael, Vainshtein '09]}$$

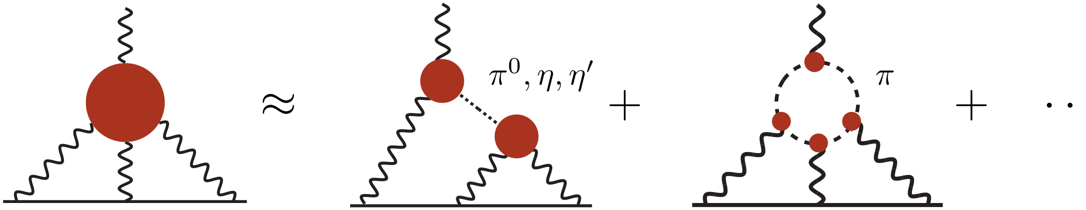
$$a_\mu^{\mathrm{HLbL}} = 116(39) \times 10^{-11} \qquad \text{[Jegerlehner, Nyffeler '09]}$$

- Precision goal : below 10% (with controlled uncertainties)

  $\rightarrow$ requires first principle approach : data-driven dispersive framework / lattice QCD

Recent calculations

$a_\mu^{\mathrm{HLbL}} =$



Dispersive framework ('21)	$a_\mu \times 10^{11}$
π^0, η, η'	93.8 ± 4
pion/kaon loops	-16.4 ± 0.2
S-wave $\pi\pi$	-8 ± 1
axial vector	6 ± 6
scalar + tensor	-1 ± 3
q-loops / short. dist. cstr	15 ± 10
charm + heavy q	3 ± 1
sum (dispersive)	92 ± 19

Lattice QCD	
Mainz '22	109.6 ± 15.9
RBC/UKQCD '23	124.7 ± 15.2

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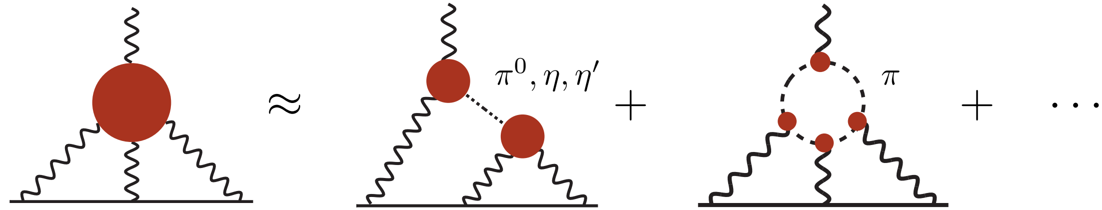

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Lattice QCD		
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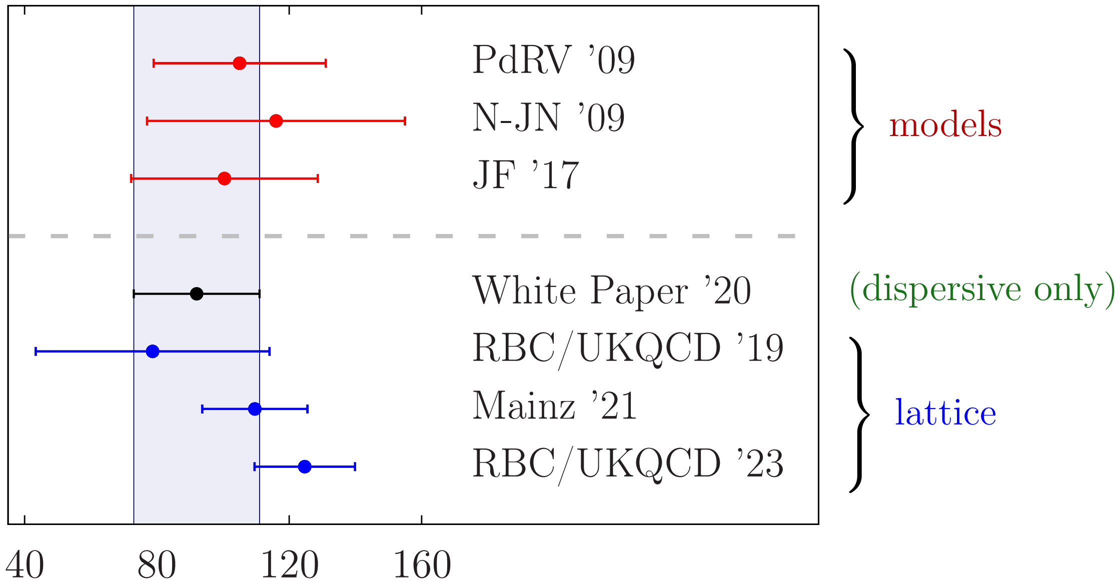
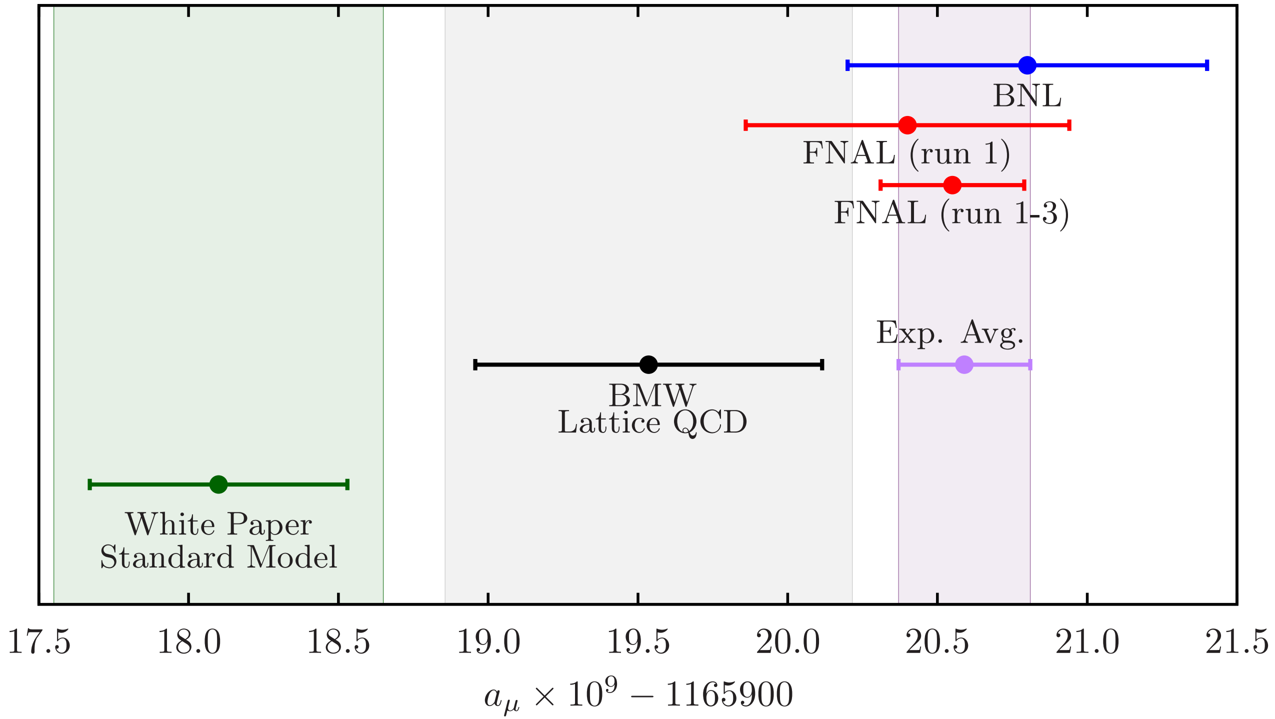
$$a_\mu^{\mathrm{HLbL}} \times 10^{11}$$

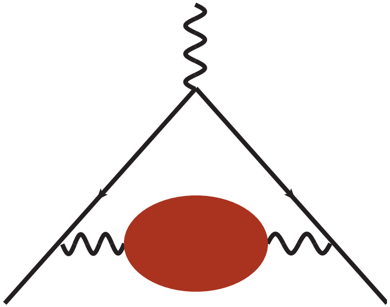


- First complete lattice QCD results are now published
  - → good agreement with the dispersive framework (precision $\sim$ 15%)
- Close to the target precision : 10%
  - → new lattice results expected soon

- Current situation, ignoring improvements on the HLbL calculations



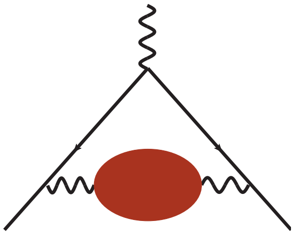
Hadronic Vacuum Polarization



$a_\mu^{\rm hvp} = (6\ 931 \pm 40) \times 10^{-11}$
$a_\mu^{\rm Exp.} = (116\ 592\ 059 \pm 22) \times 10^{-11}$

$\rightarrow$ we need few permil precision

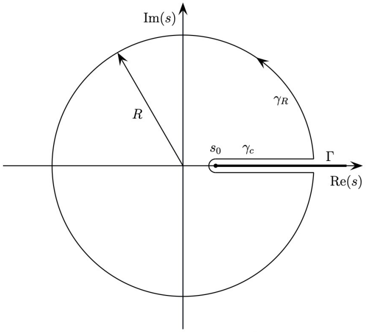
$$a_\mu^{\rm HVP} = 4\alpha^2 \int_0^\infty \mathrm{d}Q^2 \; f(Q^2) \; \left(\Pi(Q^2) - \Pi(0)\right)$$



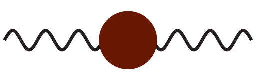
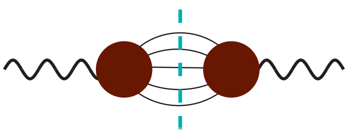
$$\Pi_{\mu\nu}(Q) = \int \mathrm{d}^4x \, e^{iQ\cdot x} \; \langle J_\mu(x) J_\nu(0) \rangle = \left(Q_\mu Q_\nu - \delta_{\mu\nu} Q^2\right) \Pi(Q^2)$$

- Use analyticity

$$\Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{s_{\rm th}}^\infty \frac{\mathrm{Im}\Pi(s')}{s'(s'-s-i\epsilon)} \mathrm{d}s'$$



- Optical theorem (unitarity)

Im  $\propto \sum_n$ 

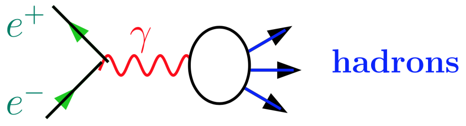
$$\mathrm{Im}\,\Pi(s) \propto \sigma(e^+e^- \to \gamma^* \to \text{hadrons})$$

- Insert the VP in the definition of a_μ to get

$$a_\mu^{\rm LO-HVP} = \frac{m_\mu^3}{12\pi^2} \int_{s_{\rm th}}^\infty \mathrm{d}s \frac{K(s)}{s} \sigma(e^+e^- \to \text{hadrons})$$

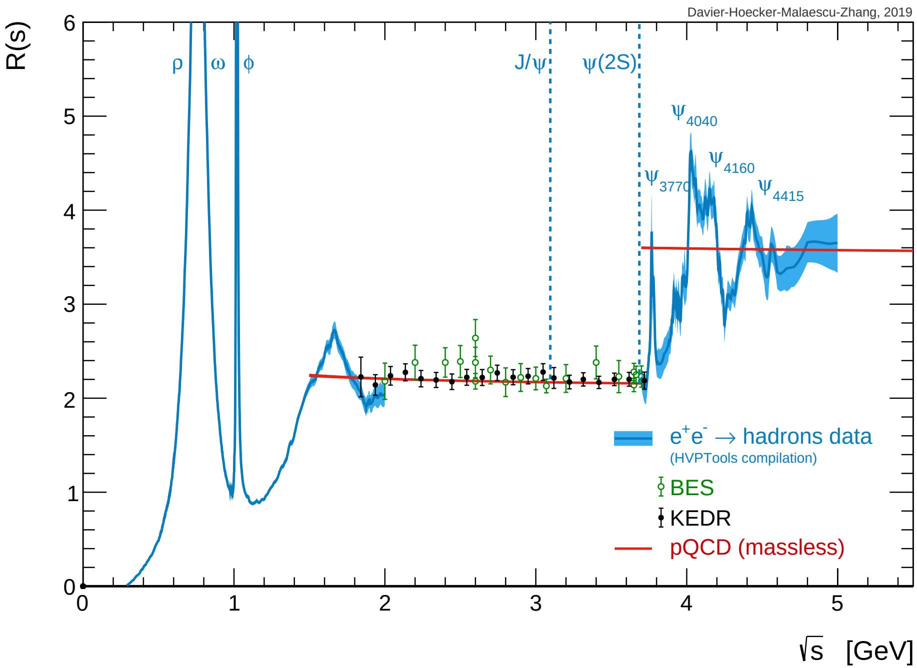
- R-ratio

$$R_{\text{had}}(s) = \frac{\sigma^0(e^+e^- \to \gamma^* \to \text{hadrons})}{(4\pi\alpha^2/3s)}$$



- Compilation of experimental data from many experiments

[Davier, Hoecker, Malaescu, Zhang, 2019]



- 2020 White paper average for the dispersive approach (CMD3 data not included)

$$a_\mu^{\rm hvp} = 693.1(2.8)_{\rm stat}(0.7)_{\rm DV+QCD}(2.8)_{\rm KLOE/BABAR} \times 10^{-10} \quad [0.58\%]$$

[Davier et al. '19] [Keshavarzi et al. '20]

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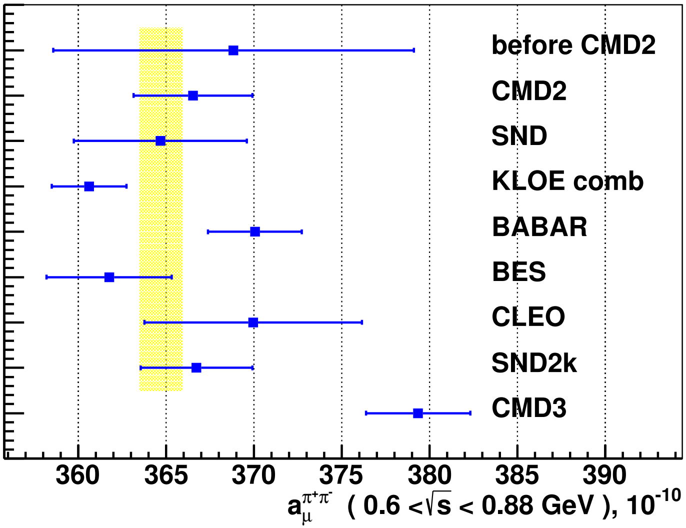
- But large tensions between different experimental data sets

→ mostly problematic for the dominant $\pi\pi$ channel, region $\sqrt{s} \in [0.6 : 0.9]$ GeV

[CMD3 '23 [2302.08834]]



Difference pheno / exp for the $g-2$:

$$a_\mu^{\mathrm{SM}} - a_\mu^{\mathrm{exp.}} = 28(8) \times 10^{-10}$$

→ $\pi^+\pi^-$: 73% of the total contribution

→ CMD3 ('23) results remove the tension

→ The 5σ tension should be taken with extreme caution

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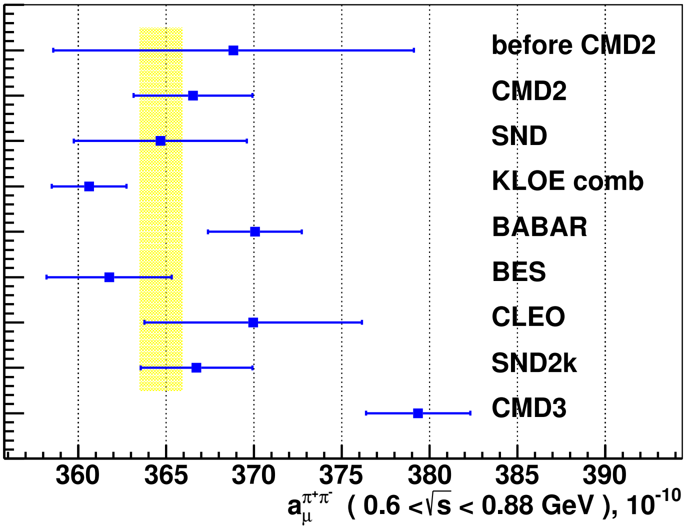

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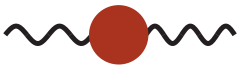
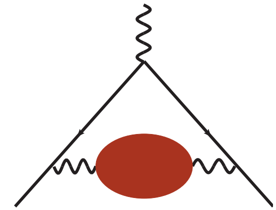
- Strong motivation for independent lattice calculations (systematically improvable)

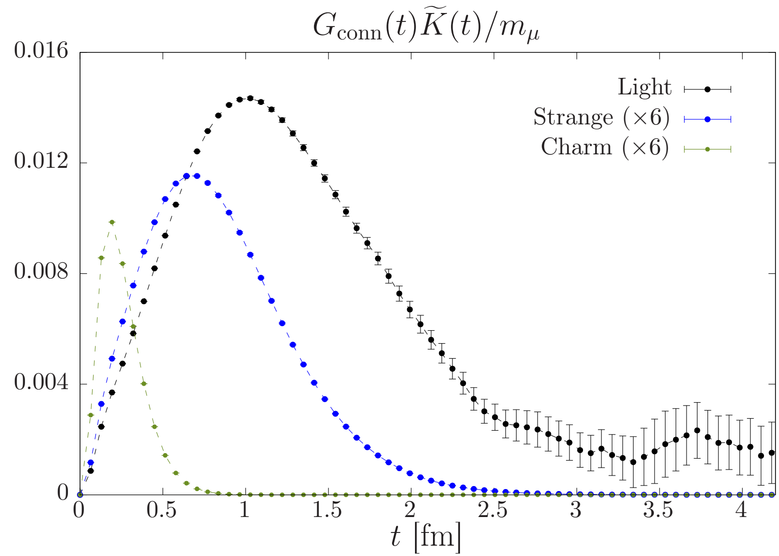
$\Pi_{\mu\nu}(Q) =$  $= \int \mathrm{d}^4x\, e^{iQ\cdot x}\, \langle J_\mu(x) J_\nu(0) \rangle$



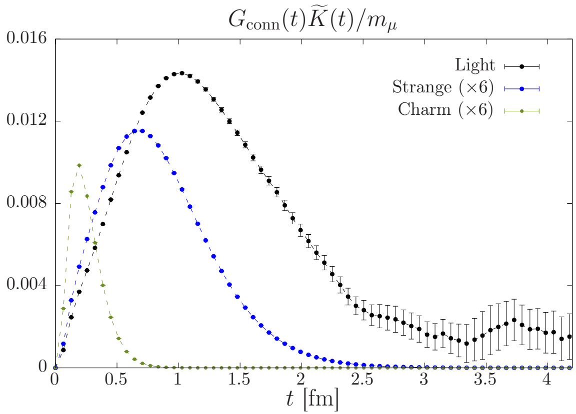
- The time-momentum representation [Blum '02] [Bernecker, Meyer '11]

$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}t\, \widetilde{K}(t)\, G(t)\,, \qquad G(t) = -\frac{1}{3}\sum_{k=1}^{3}\sum_{\vec{x}} \langle J_k(x) J_k(0) \rangle$$

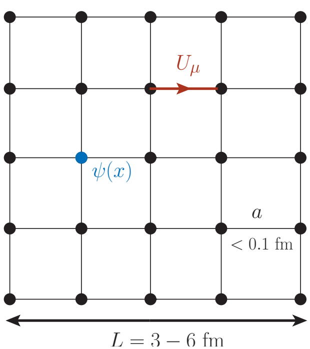
- $J_k = \frac{2}{3}\overline{u}\gamma_k u - \frac{1}{3}\overline{d}\gamma_k d - \frac{1}{3}\overline{s}\gamma_k s + \frac{2}{3}\overline{c}\gamma_k c + \cdots$



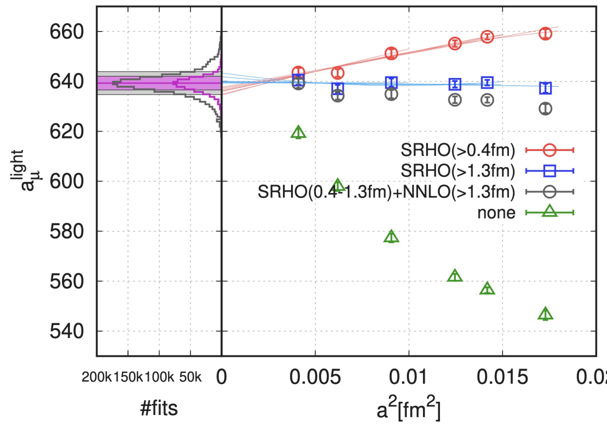
- Noise problem (light-quark contribution)



- Finite-volume effects $\mathcal{O}(3\%)$



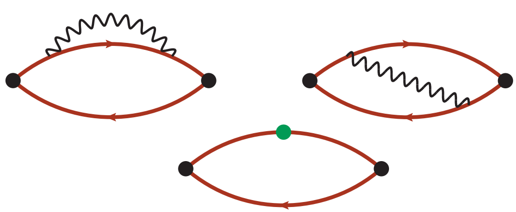
- Continuum extrapolation [BMW '20]

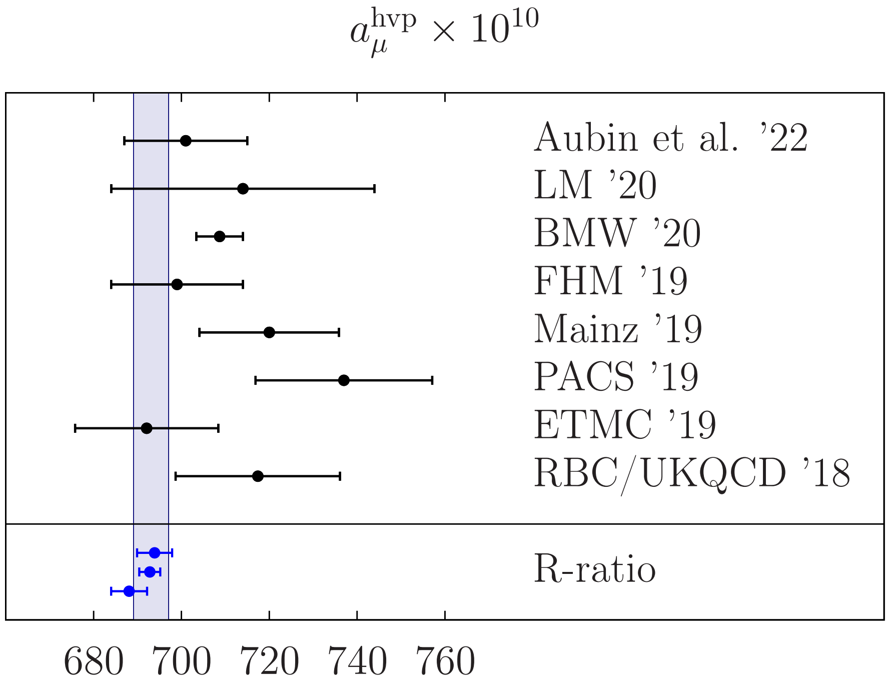


- QED / strong isospin breaking corrections

$m_u \neq m_d$: $\mathrm{O}(\frac{m_u - m_d}{\Lambda_{\rm QCD}}) \approx 1/100$

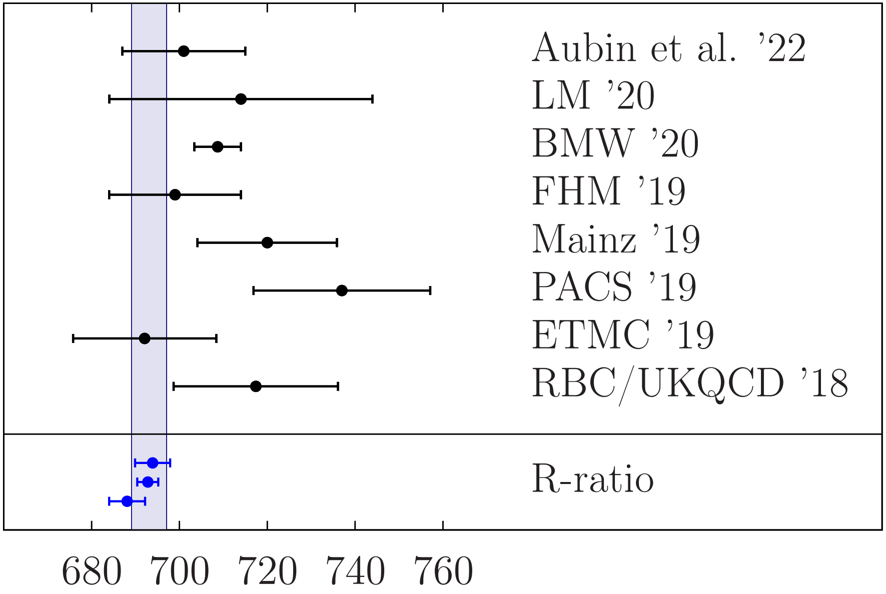
$Q_u \neq Q_d$: $\mathrm{O}(\alpha_{\rm em}) \approx 1/100$





- many lattice calculations (precision $\sim 2\%$)
- first sub-percent calculation by the BMW collaboration
- the tension with R-ratio not (yet ?) conclusive (1.8σ)

$a_\mu^{\mathrm{hvp}} \times 10^{10}$



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Idea : look at a similar but simpler observable

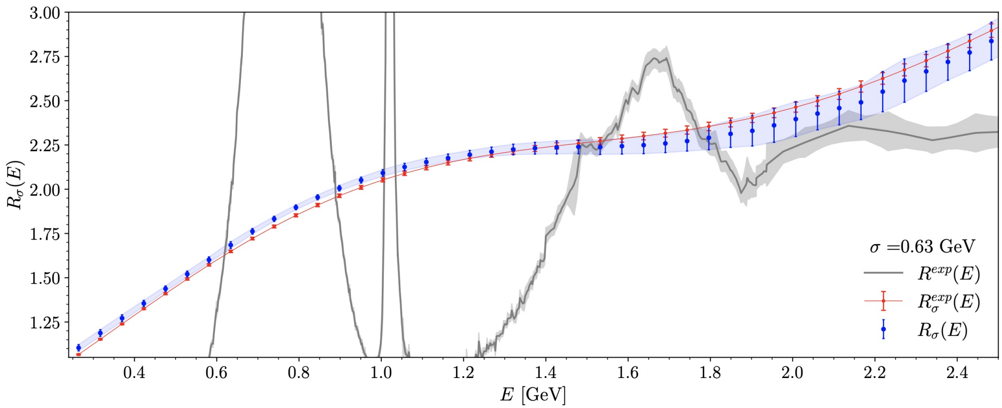
Relation between the lattice correlator and the R-ratio is given by a Laplace transform :

$$G(t) = \frac{1}{12\pi^2} \int_{E_{\rm th}}^{\infty} \mathrm{d}\omega\ \omega^2\ R(\omega) e^{-t\omega}$$

$$G(t) = \frac{1}{3} \sum_{k=1}^{3} \int \mathrm{d}^3x\, \langle J_k(\mathbf{x},t) J_k(0) \rangle\,.$$

- $G(t)$ is known on a finite value of timeslices and at finite lattice spacing
- and is affected by statistical errors

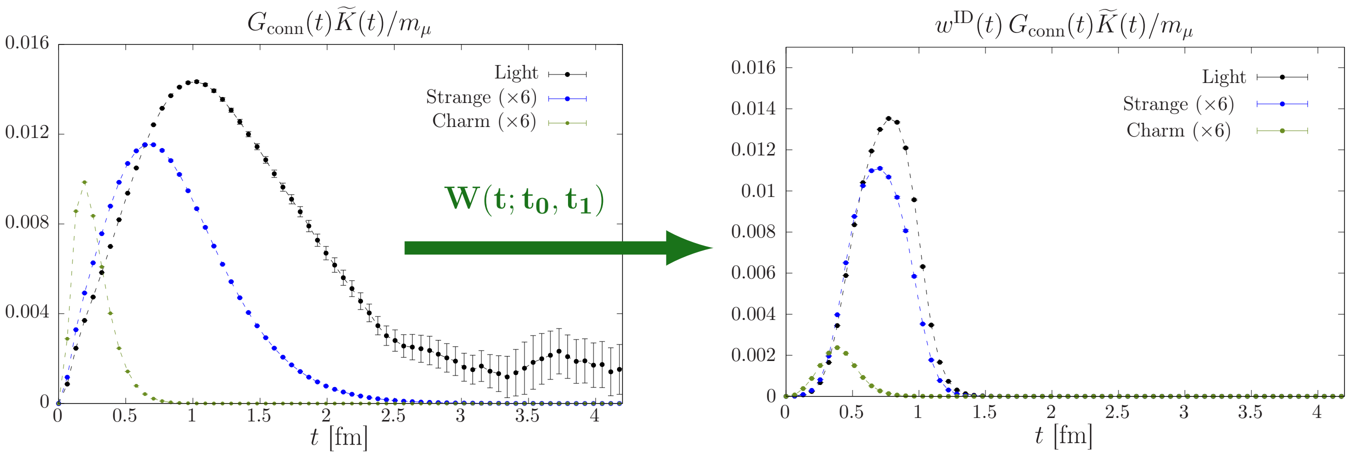
The inverse Laplace transform is an ill-defined problem



[Phys.Rev.Lett. 130 (2023)]

The intermediate window observable

$$a_\mu^{\text{win}} = \left(\frac{\alpha}{\pi}\right)^2 \sum_t G(t)\,\widetilde{K}(t)\,\mathbf{W(t;t_0,t_1)}$$

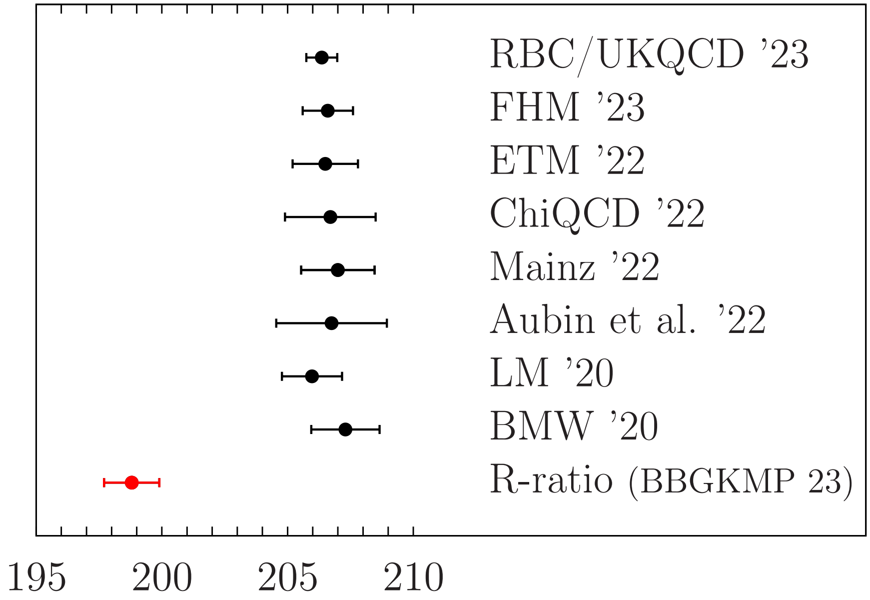


- **Intermediate window** : $\sim 30\%$ of the total contribution
- Easier to compute on the lattice (and accessible from R-ratio data !)
- Data-driven : 2π contribution in the region $600\ \text{MeV} \leq \sqrt{s} \leq 900\ \text{MeV}$ (around the ρ peak) :

  $\rightarrow$ relative contribution of 55%-60% to both $a_\mu^{\text{LO-HVP}}$ and a_μ^{win} !

  $\rightarrow$ $\sqrt{s} \leq 600$ MeV slightly suppressed, $\sqrt{s} \geq 900$ MeV slightly enhanced.

$$\left(a_\mu^{\rm hvp}\right)^{{\rm win},l} \times 10^{10}$$



- significant tension between (all !) lattice calculations vs data-driven approach
  - → here shown for the light-quark connected contribution in the isospin limit
  - → 3.7σ tension for the window observable
  - → CMD3 data not included

The tension between lattice QCD simulations and the data-driven approach needs to be understood to further reduce the error

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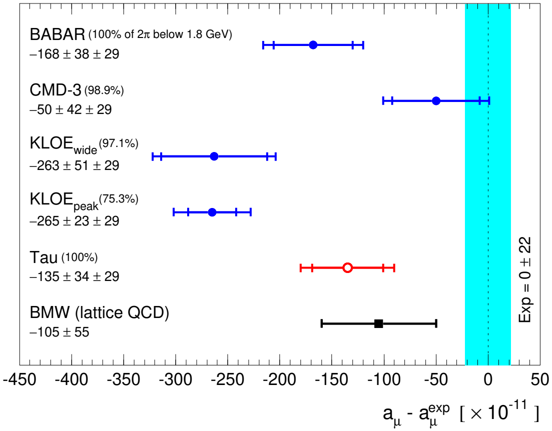
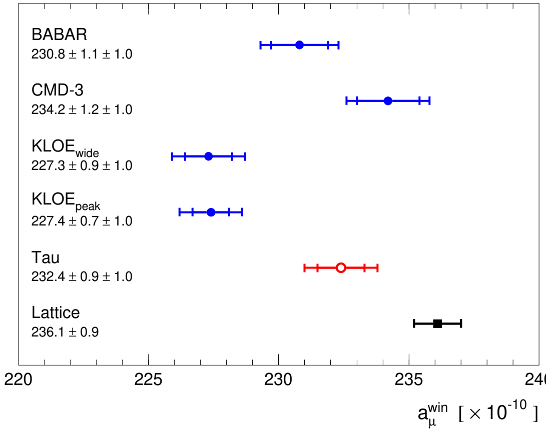
- Updated analysis using τ data [Davier et al. 2312.02053] , [Masjuan et al. 2305.20005]





→ if only BaBar/CMD3/τ : agreement with lattice (the tension with Exp. is reduced to 2.8σ)
→ but still some tension for the intermediate window observable.

- Smeared R-ratio from Lattice QCD [ETMc Phys.Rev.Lett. 130 (2023)]

- Combined analysis with different observable $a_\mu^{\rm hvp}$, $a_\mu^{\rm win}$, running α [Davier et al. 2308.04221 [hep-ph]]
→ would require an increase of $\pi\pi$ cross sections around the ρ peak by almost 5%

- Standard Model estimate dominated by hadronic uncertainties

- **HLbL contribution** : already close to the target precision of 10%.

  $\rightarrow$ good agreement between calculations (at the level of 15%)

- **HVP contribution**

  $\rightarrow$ target precision : a few-permille

  $\rightarrow$ significant progress on the lattice : now competitive in terms of error

  $\rightarrow$ ... but this increase of precision comes with a new puzzle : tension with the R-ratio

  $\rightarrow$ need to be understood to agree on the Standard Model estimate

  - Discrepancy KLOE / BaBar / CMD3 close to the ρ peak
  - Comparison Lattice / R-ratio : closer look at the region around the ρ
  - Need more lattice QCD calculations with sub-percent precision

- Final result from Fermilab expected in $\sim$ one year !

  $\rightarrow$ New experiment at J-PARC E34 in 2025

Backup slides

Hadronic contributions are dominated by low energy physics ≤ 1 GeV

- Estimator for the HVP contribution

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}t \, K(t) \, G(t) \, , \quad G(t) = -\frac{1}{3} \sum_{\vec{x},k} \langle J_k(x) J_k(0) \rangle_{\mathrm{QCD}}$$

- Estimator for the HLbL contribution

$$a_\mu^{\mathrm{HLbL}} = -\frac{me^6}{3} \int \mathrm{d}^4 y \int \mathrm{d}^4 x \int \mathrm{d}^4 z \, \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y) \, z_\rho \, \langle J_\mu(x) J_\nu(y) J_\sigma(z) J_\lambda(0) \rangle_{\mathrm{QCD}}$$

- Hadronic contributions are evaluated non-perturbatively using numerical methods

  $\longrightarrow$ lattice regularization : numerical evaluation of the path integral using MC techniques

  $\longrightarrow$ large-scale simulations ($\sim 10^3 - 10^4$ CPU cores)

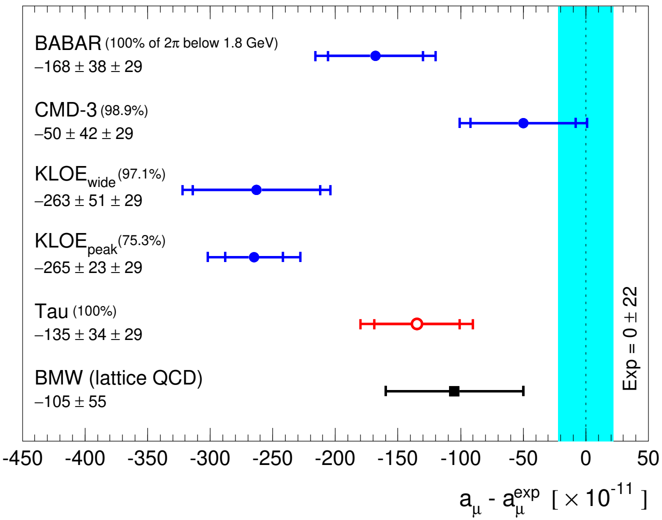
  $\longrightarrow$ statistical + systematic uncertainties

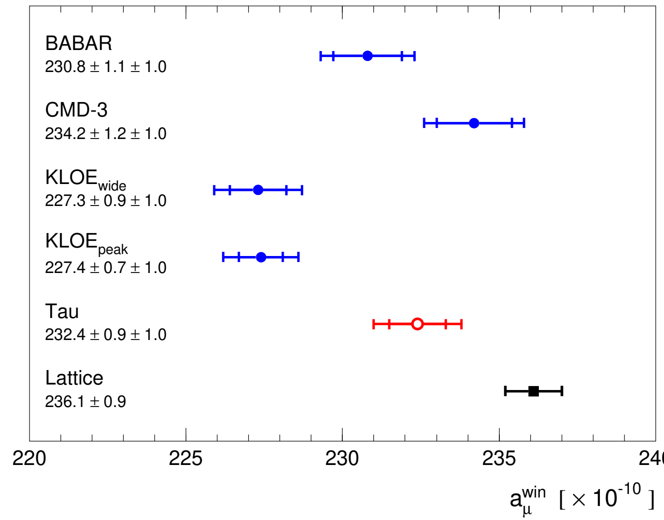
- The QED part of the diagram is computed perturbatively

  $\longrightarrow$ in the continuum and infinite volume limits

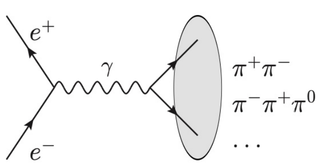
  $\longrightarrow$ weights the non-perturbative hadronic correlator

[Davier et. al arXiv :2312.02053]



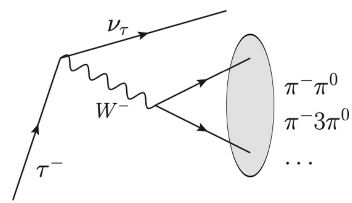


Isospin corrections are needed when using τ data (Figures from Mattia Bruno) :



EM current

Final states $I = 0, 1$ neutral



$V - A$ current

Final states $I = 1$ charged