DIS 2024, Grenoble

April 11th, 2024

Strong CP problem and axions

Christopher Smith

- Outline

  I. Strong CP puzzle

  II. Axions

  III. Baryonic/leptonic axions

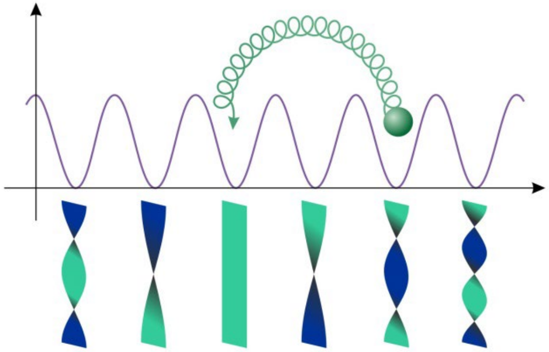
  IV. Conclusion

I. Strong CP puzzle

Origin of the strong CP puzzle - in a few words

$$\boxed{\mathcal{L}_{\not{C}\not{P}} = (\underbrace{\theta_C}_{} - \arg\det \mathrm{Y}_u - \arg\det \mathrm{Y}_d)\frac{\alpha_S}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

Non-trivial QCD topology:



Also explains η' mass ←

$G_{\mu\nu}\tilde{G}^{\mu\nu}$ is a total derivative,

Fields cancel at infinity?

No! Vacuum = pure gauge,

SU(3) pure gauge can be twisted.

Origin of the strong CP puzzle - in a few words

$$\mathcal{L}_{\not{C}\not{P}} = (\underbrace{\theta_C} - \underbrace{\arg\det Y_u - \arg\det Y_d})\frac{\alpha_S}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

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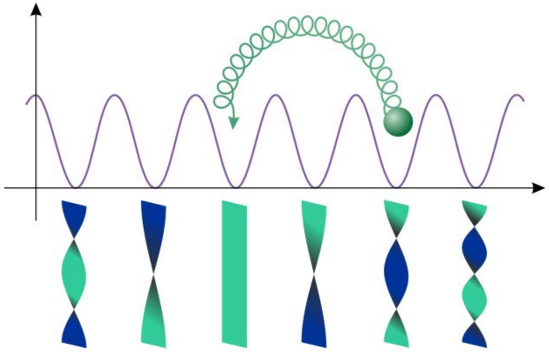

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Quark-Higgs Yukawa couplings:

We know they are complex.

↓

$\delta_{CKM} \neq 0$

from K and B physics

Origin of the strong CP puzzle - in a few words

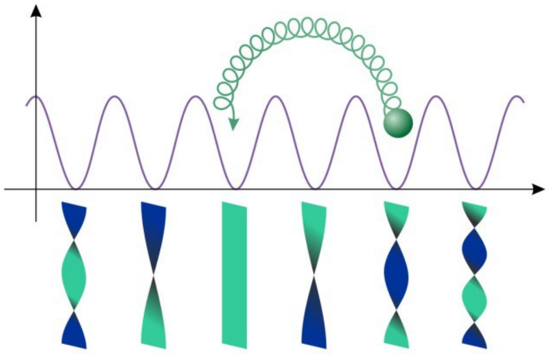
Neutron EDM implies: $\theta \equiv \theta_C - \arg\det \mathrm{Y}_u - \arg\det \mathrm{Y}_d < 10^{-10}$

↑

$$\boxed{\mathcal{L}_{\not{C}\not{P}} = (\overbrace{\underbrace{\theta_C}\,\underbrace{- \arg\det \mathrm{Y}_u - \arg\det \mathrm{Y}_d}})\frac{\alpha_S}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

↙ ↘

Non-trivial QCD topology:



Also explains η' mass ←

Quark-Higgs Yukawa couplings:

We know they are complex.

↓

$\delta_{CKM} \neq 0$

from K and B physics

Solution 1: The massless quark

Three CPV terms are present in the SM gauge Lagrangian:

$$\mathcal{L}_{\cancel{CP}} = \theta_C \frac{\alpha_S}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu} + \theta_L \frac{g^2}{16\pi^2} W_{\mu\nu}\tilde{W}^{\mu\nu} + \theta_Y \frac{g'^2}{16\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu}$$

Individual fermion rephasing $\psi \to \exp(i\alpha_\psi)\psi$ are anomalous:

$$\theta_C \to \theta_C - \sum_f (2\alpha^f_{q_L} + \alpha^f_{u_R} + \alpha^f_{d_R})$$

$$\theta_L \to \theta_L - \sum_f (3\alpha^f_{q_L} + \alpha^f_{\ell_L})$$

$$\theta_Y \to \theta_Y - \sum_f (\tfrac{1}{3}\alpha^f_{q_L} + \tfrac{8}{3}\alpha^f_{u_R} + \tfrac{2}{3}\alpha^f_{d_R} + \alpha^f_{\ell_L} + 2\alpha^f_{e_R})$$

($\mathcal{B} - \mathcal{L}$ and weak hypercharge are not anomalous)

Solution 1: The massless quark

Three CPV terms are present in the SM gauge Lagrangian:

$$\mathcal{L}_{\not{C}\not{P}}=\theta_C\frac{\alpha_S}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}+\theta_L\frac{g^2}{16\pi^2}W_{\mu\nu}\tilde{W}^{\mu\nu}+\theta_Y\frac{g'^2}{16\pi^2}B_{\mu\nu}\tilde{B}^{\mu\nu}$$

Cannot be removed,
Strong CP puzzle.

Removed thanks to $U(1)_{\mathcal{B}+\mathcal{L}}$

Removed
by partial
integration.

Individual fermion rephasing $\psi\rightarrow\exp(i\alpha_\psi)\psi$ are anomalous:

$$\theta_C\rightarrow\theta_C-\sum_f(2\alpha^f_{q_L}+\alpha^f_{u_R}+\alpha^f_{d_R})$$

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All but one phase fixed by requiring real fermion masses.

Solution 1: The massless quark

Three CPV terms are present in the SM gauge Lagrangian:

$$\mathcal{L}_{\not{C}\not{P}} = \quad \alpha_s \qquad g^2 \qquad g'^2$$

Cannot be re
Strong CP p

noved
partial
gration.

Individual fer

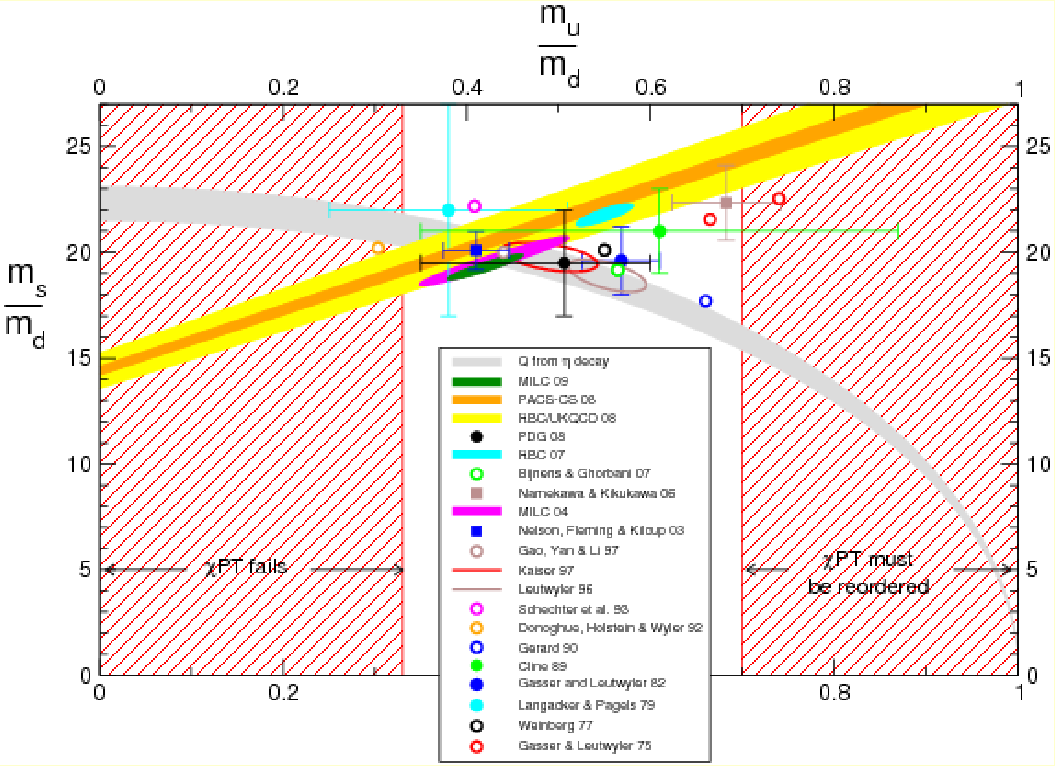
θ_C —

θ_L →

θ_Y —

All but .

Strong evidence no quarks are massless:



Leutwyler '09

Solution 2: The GUT paradigm

$$\left.\begin{array}{l}\text{Imagine } \theta_C = \theta_L = \theta_Y = 0 \text{ at the GUT scale} \\ \text{In some GUTs, } Y_{u,d} \text{ are hermitian, so } \arg\det Y_{u,d} = 0\end{array}\right\} \theta_{eff}(M_{GUT}) \equiv 0$$

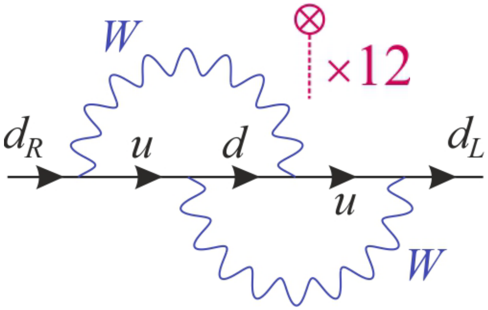
Solution 2: The GUT paradigm

Imagine $\theta_C = \theta_L = \theta_Y = 0$ at the GUT scale

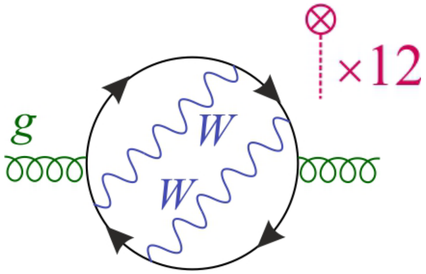
In some GUTs, $Y_{u,d}$ are hermitian, so $\arg\det Y_{u,d} = 0$

$$\left.\right\} \theta_{eff}(M_{GUT}) \equiv 0$$

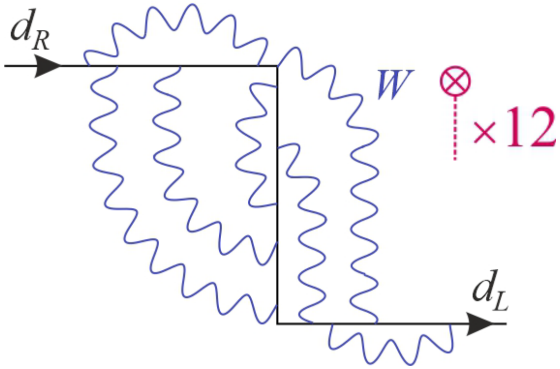
CKM-induced strong CP phases running down:



Shifts $\arg\det Y_{u,d}$



Shifts $G_{\mu\nu}\tilde{G}^{\mu\nu}$



UV divergent $\Rightarrow$

θ_{eff} = physical parameter

$$\Delta\theta_{eff} \approx \mathcal{O}(10^{-16})$$

$$\Delta\theta_{eff}(M_W) \approx \mathcal{O}(10^{-18})$$

Wilczek, '78
Ellis, Gaillard, '79
Khriplovich, Vainshtein, '93

Solution 2: The GUT paradigm

Imagine $\theta_C = \theta_L = \theta_Y = 0$ at the GUT scale

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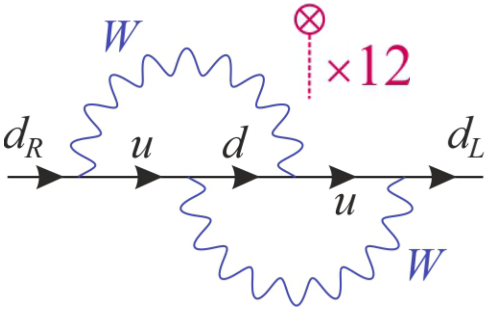

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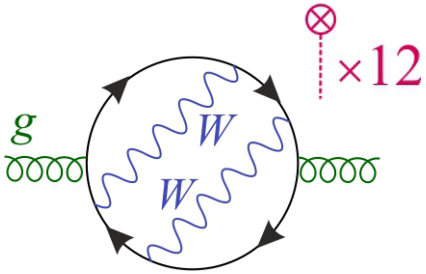

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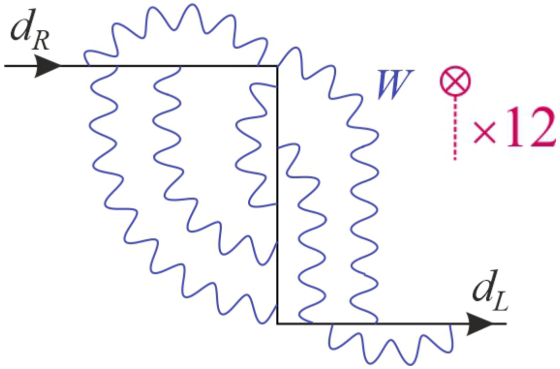

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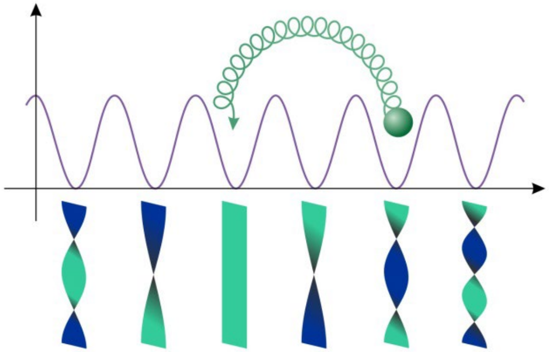
Hierarchy & fermion masses in GUT can bring new CPV sources,

How to understand the « vacuum matching » at the GUT scale?

Solution 3: Topological solution?



- Effects of θ vanish at finite perturbative order,

$$G_{\mu\nu}\tilde{G}^{\mu\nu} = dC$$

- They come from a boundary term at infinity:

$$\int_V G_{\mu\nu}\tilde{G}^{\mu\nu} = \int_V dC = \int_{\partial V} C$$

- True vacuum is an infinite sum:

$$|\theta\rangle = \sum_k e^{ik\theta}|k\rangle$$

Unobservability of the θ term is regularly put forward, e.g. recently:

Order of the limits: Taking ∂V to infinity before summing over k?

Ai, Cruz, Garbrecht, Tamarit, 2021

Do chromo-magnetic monopoles confine when $\theta \neq 0$?

Nakamura, Schierholz, 2021

Majority believes the problem exists, but this must be kept in mind…

II. Axions

The axion solution, schematically

$$\mathcal{L}_{axion} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{\alpha_S}{8\pi} \theta G_{\mu\nu} \tilde{G}^{\mu\nu} + \bar{\psi}_{L,R} i \not{D} \psi_{L,R} + y \bar{\psi}_L \psi_R \phi + V(\phi^\dagger \phi)$$

Step 1: Add ϕ to create a global U(1) PQ symmetry $\phi \to \exp(i\theta)\phi$.

Ensure it is anomalous by coupling ϕ to colored fermions:

$$\Rightarrow \partial_\mu J^\mu \sim G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Step 2: Break U(1) PQ spontaneously.

Goldstone boson is coupled to the current, $\langle 0 | J^\mu | a(p) \rangle = ivp^\mu$.

The shift symmetry permits to rotate θ away:

$$\mathcal{L}_{axion} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{\alpha_S}{8\pi} \left(\theta + \frac{a}{v} \right) G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{1}{v} \partial_\mu a J^\mu + \ldots$$

Peccei, Quinn ’77
Weinberg ’78
Wilczek ’78

The axion solution: Invisible axions

Step 3: One extra complex scalar field, but two strategies:

$$\phi = \frac{1}{\sqrt{2}}(v+\rho)\exp(i\eta / f_a) \ , \qquad f_a \equiv v \gg v_{EW}$$

KSVZ: $\phi \bar{\Psi}_L \Psi_R$

Kim ’79, Shifman,Vainshtein,Zakharov ’80

DFSZ: $\phi H_u^\dagger H_d$

Dine,Fischler,Srednicki ’81, Zhitnitsky ’80

The axion is $a = \eta$

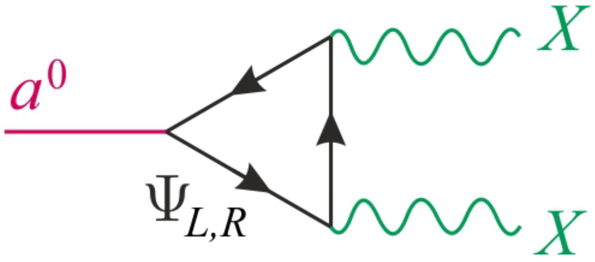
The axion is $a = \eta + \mathcal{O}\left(\frac{v_{EW}}{f_a} A^0\right)$

Couplings to gauge bosons:

Couplings to SM fermions:

$$\frac{a}{16\pi^2 f_a} \sum_{X=G,W,B} g_X^2 \mathcal{N}_X X^a_{\mu\nu} \tilde{X}^{a,\mu\nu} \quad \Longleftrightarrow \quad \frac{a}{f_a} \sum_{\psi=u,d,e} \chi_\psi \bar{\psi} \gamma_5 \psi$$

$\Uparrow$



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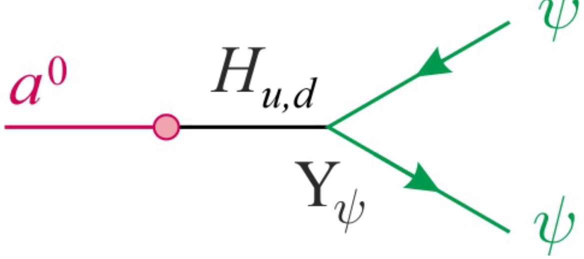
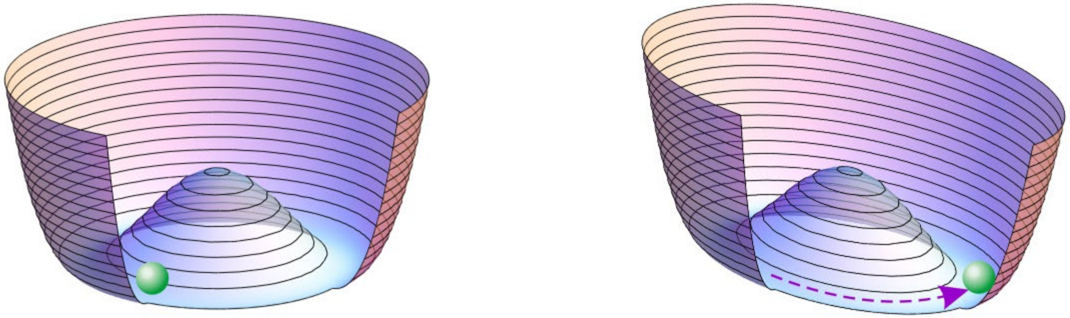


The axion solution: QCD axion

$$\mathcal{L}_{axion} \supset -\frac{\alpha_S}{8\pi}\left(\theta_S + \frac{1}{f_a}a\right)G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4}g_{a\gamma\gamma}^{SD}aF_{\mu\nu}\tilde{F}^{\mu\nu} + \ldots$$

Step 4: QCD non-perturbative effects are turned on.



Miminum at $\langle a\rangle = -f_a\theta_S$

Strong CP relaxes to zero automatically:

$$\frac{\alpha_S}{8\pi}\left(\theta_S + a/f_a\right)G_{\mu\nu}\tilde{G}^{\mu\nu} \to V_{eff}\left(\theta_S + a/f_a, \pi, \eta, \ldots\right)$$

Potential quality problem: The hadronic ‘titling’ is not that strong,

→ PQ symmetry must be quite exact !

The axion solution: QCD axion

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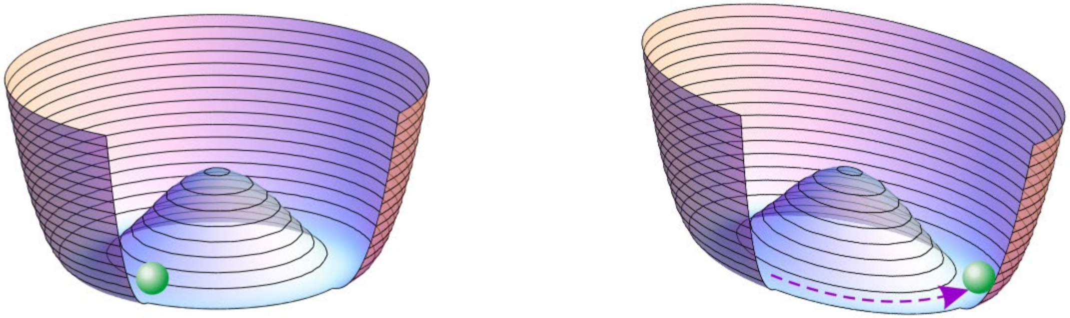

Miminum at

$$\langle a \rangle = -f_a \theta_S$$

Strong CP relaxes to zero automatically.

The axion is massive,

$$\frac{f_a^2 m_a^2}{f_\pi^2 m_\pi^2} = \frac{m_u m_d}{m_u + m_d} \quad \rightarrow m_a \approx 6\mu eV \times \frac{10^{12}\, GeV}{f_a}$$

The axion solution: QCD axion

$$\mathcal{L}_{axion} \supset -\frac{\alpha_S}{8\pi}\left(\theta_S + \frac{1}{f_a}a\right)G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4}g_{a\gamma\gamma}^{SD}aF_{\mu\nu}\tilde{F}^{\mu\nu} + \ldots$$

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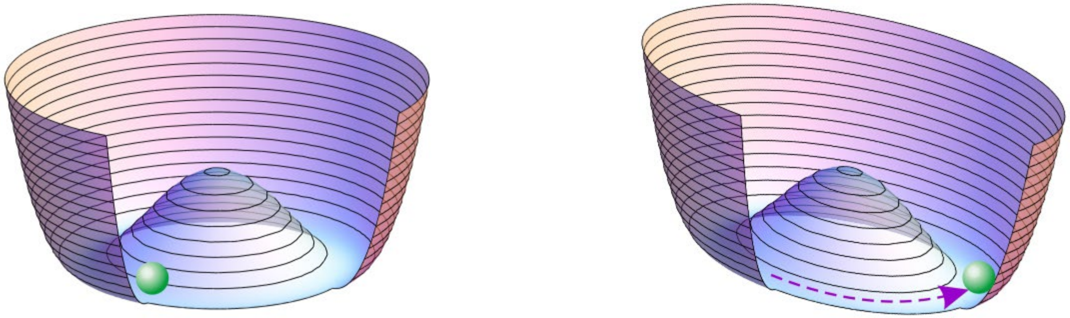
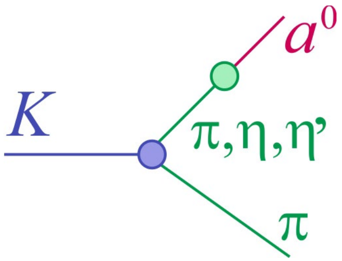


Miminum at $\langle a\rangle = -f_a\theta_S$

Strong CP relaxes to zero automatically.

The axion is massive, $m_a \approx 6\mu eV \times (10^{12}GeV / f_a)$.

The axion mixes with $\pi^0,\eta,\eta' \Rightarrow f_a > 30\times v_{EW}$ (KEK '81)



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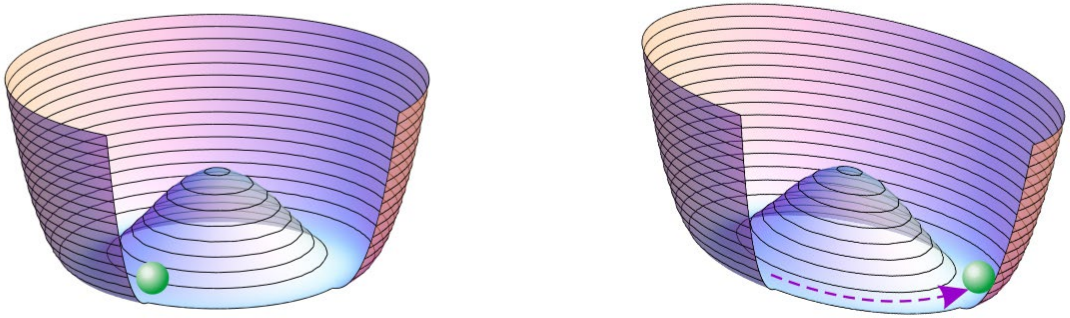


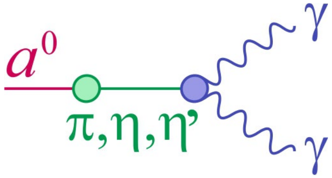
Miminum at

$\langle a \rangle = -f_a\theta_S$

Strong CP relaxes to zero automatically.

The axion is massive, $m_a \approx 6\mu eV \times (10^{12}GeV / f_a)$.

The axion mixes with $\pi^0, \eta, \eta' \Rightarrow$ $g_{a\gamma\gamma}$ correlated with m_a:

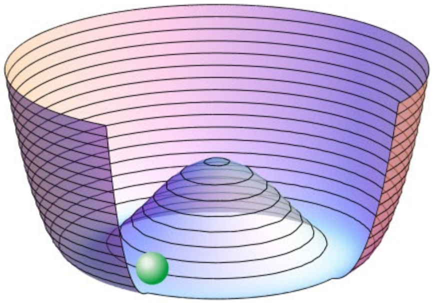


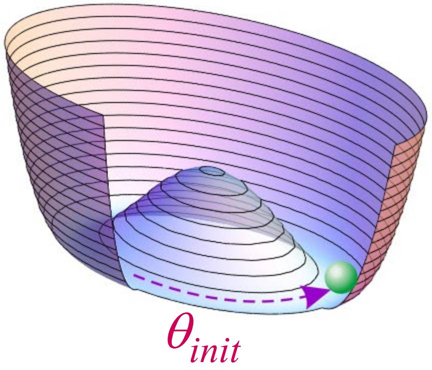
$$\Rightarrow g_{a\gamma\gamma}^{LD} = \frac{2\alpha}{6\pi f_a}\frac{4m_d + m_u}{m_u + m_d} \qquad \Rightarrow \frac{g_{a\gamma\gamma}^{LD} + g_{a\gamma\gamma}^{SD}}{m_a} \approx 10^{-10\pm1}$$

The axion solution: Dark matter axion

$$\mathcal{L}_{axion} \supset -\frac{\alpha_S}{8\pi}\left(\theta_S + \frac{1}{f_a} a\right) G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu} + \dots$$

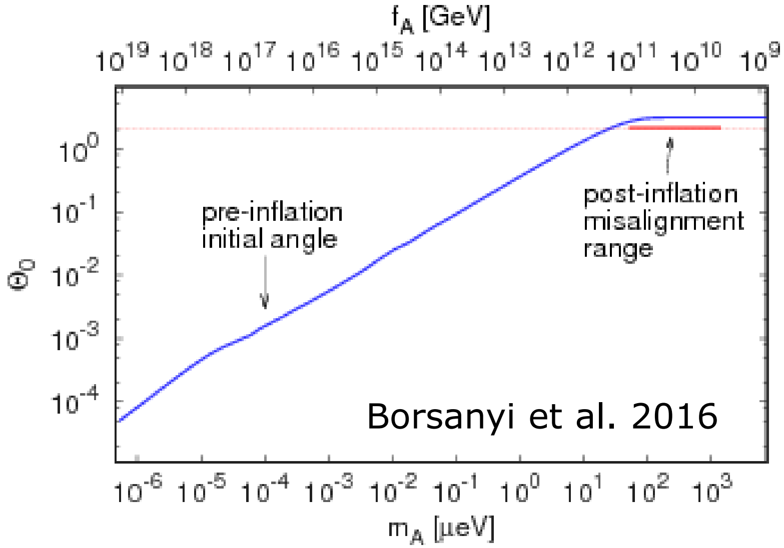
Step 5: Cosmological consequences?





Miminum at

$\langle a \rangle = -f_a \theta_S$



Borsanyi et al. 2016

The axion = cold dark matter!

$$n_{axion}(T) \sim f_a m_a(T)\theta_{init}^2 \sim \frac{1}{f_a}\chi(T)\theta_{init}^2,$$

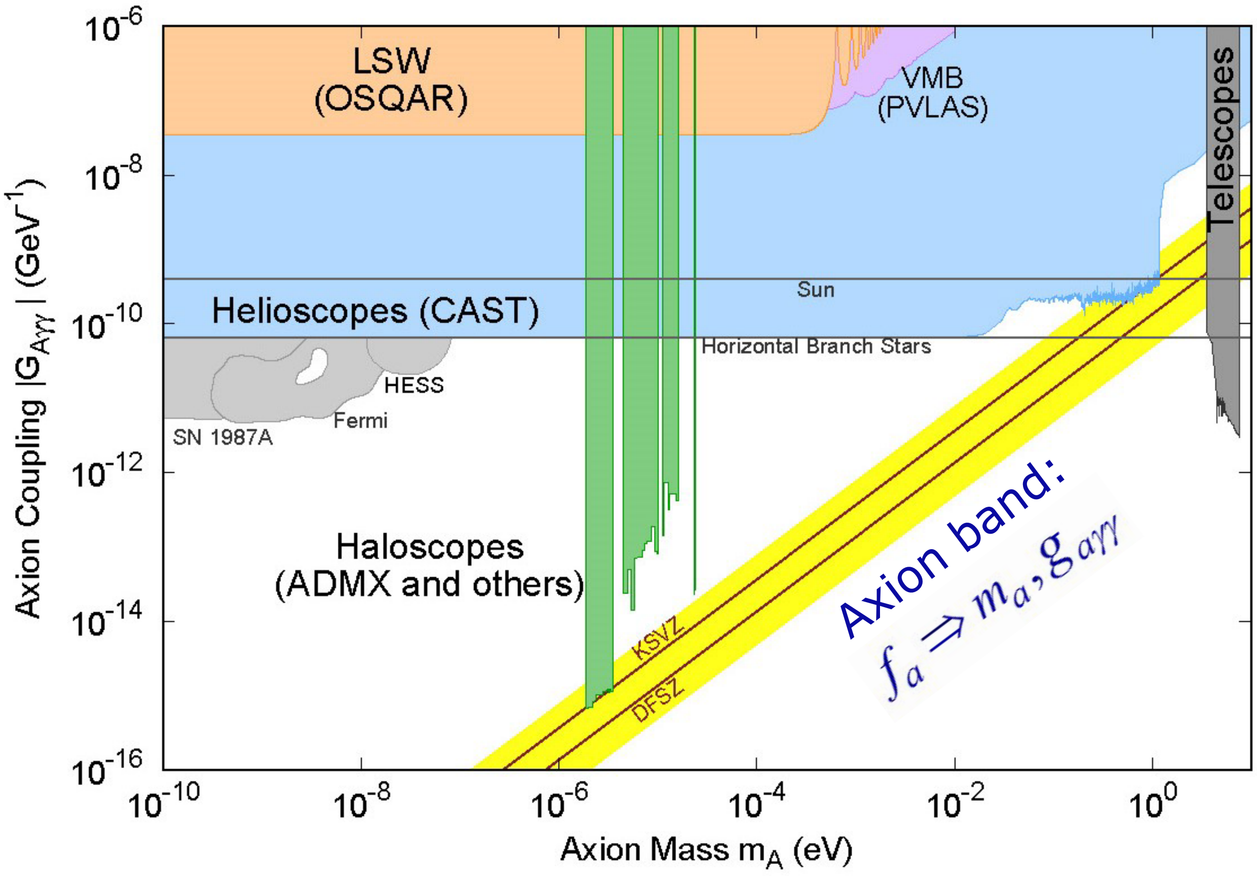
Lattice simulations give $\chi(T)$.

Initial misalignment θ_{init} unknown.

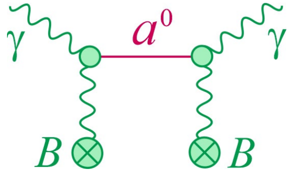
The axion solution: Experimental axions

$$\mathcal{L}_{axion} \supset -\frac{\alpha_S}{8\pi}\left(\theta_S + \frac{1}{f_a}a\right) G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4} g_{a\gamma\gamma} \boxed{aF_{\mu\nu}\tilde{F}^{\mu\nu}} + \ldots \longrightarrow \boxed{aE \cdot B}$$

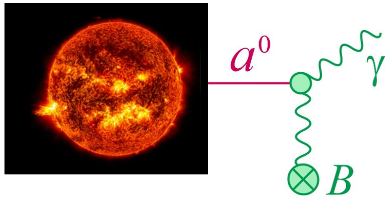
Step 6: Experimental constraints (PDG18, simplified)



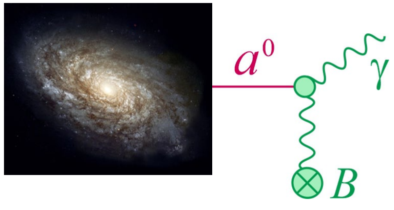
LSW, VMB:



Helioscopes:



Haloscopes:



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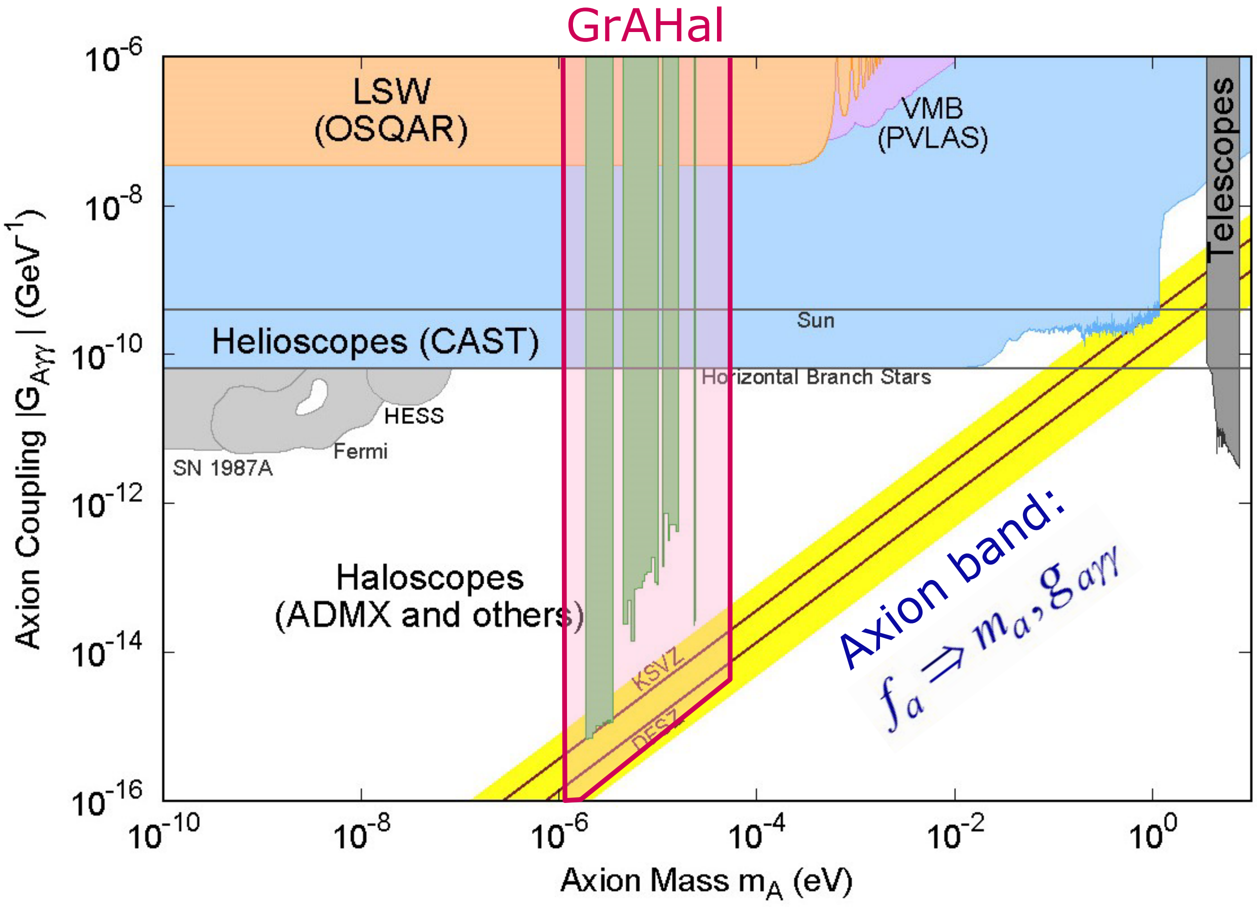

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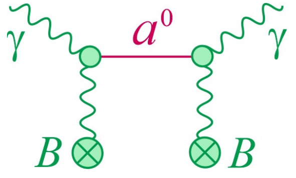

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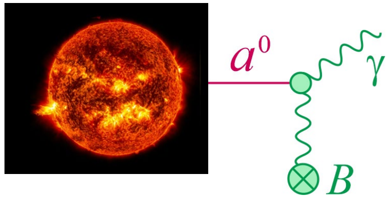


Haloscopes:

The axion solution: Axions in Grenoble

arXiv:2110.14406

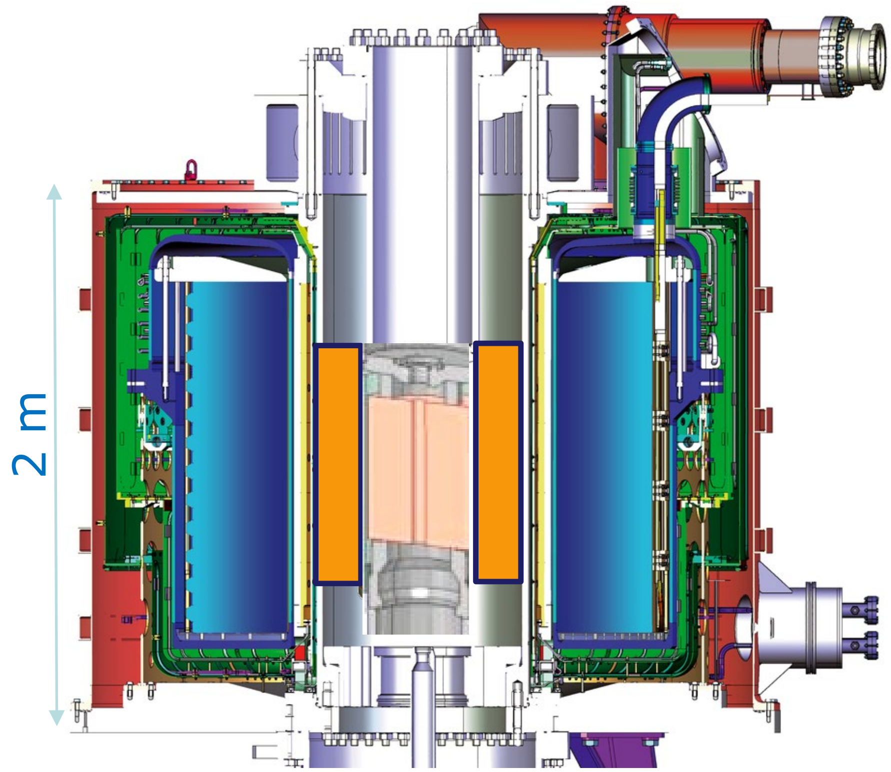
GrAHal

Grenoble Axion Haloscopes

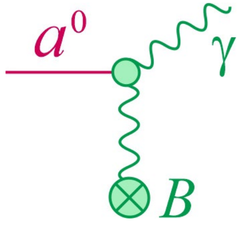
Grenet,Perrier,
Basto,Ballou,
Roch,Camus

Pugnat,Pfister,
Krämer

Barrau,Smith,
Quevillon,Martineau

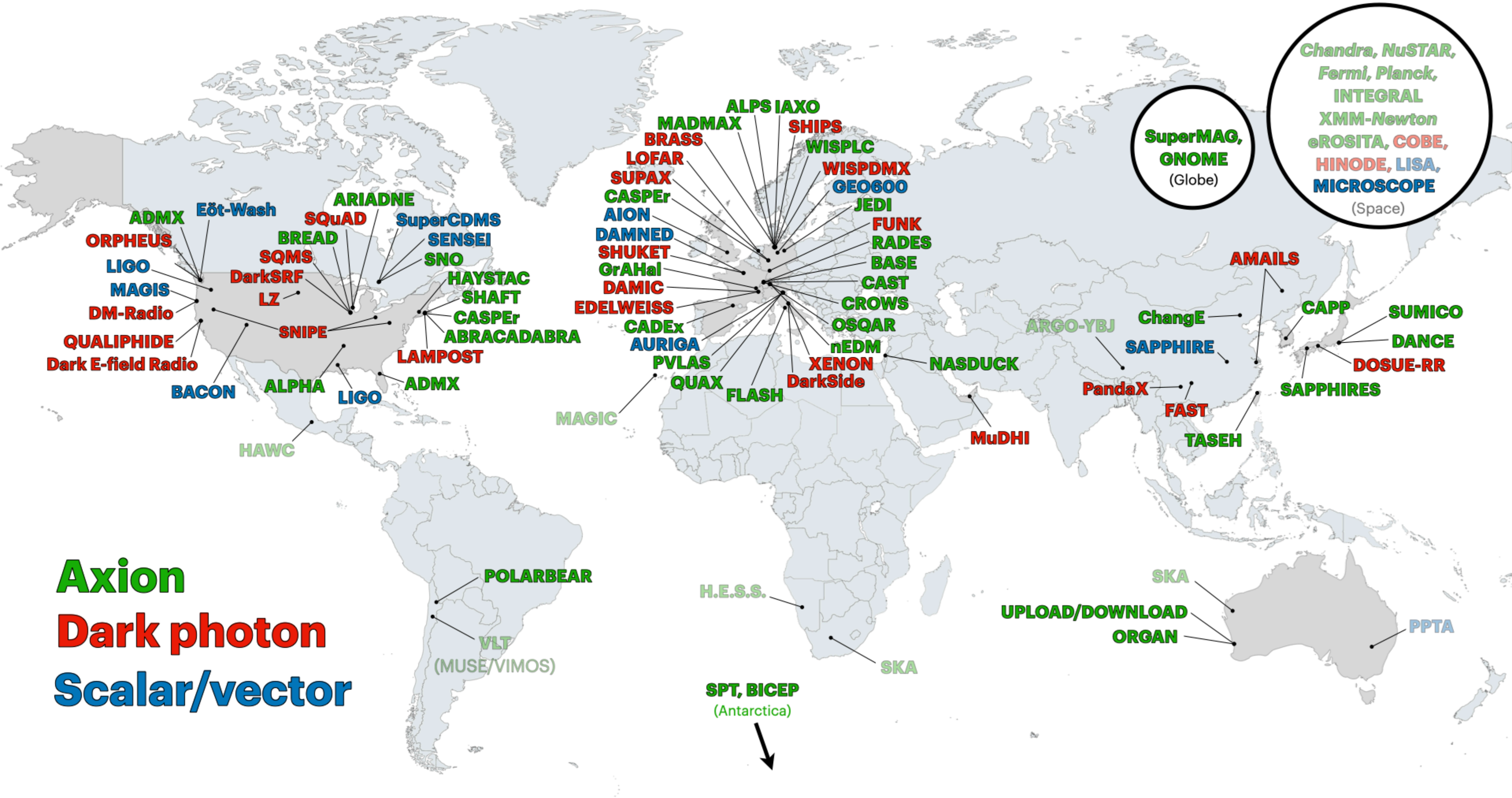


$$a(t) = \frac{\sqrt{2\rho_{DM}}}{m_a}\cos(m_a t)$$



Field	RF-cavity diameter (mm)	Frequency (GHz)	Axion mass (μeV)
43 T	8	29	118
40 T	23	10	41
27 T	110	2	8.6
17.5 T	315	0.7	3
9.5 T	675	0.34	1.4

The axion solution: Axions in the World



cajohare.github.io/AxionLimits/

The axion solution: Alternatives

QCD axion models falling outside the axion band? E.g.:

Enlarging the band with many fields (larger irrep, clockwork,...)

Di Luzio, Mescia, Nardi, '16
Higaki, Jeong, Kitajima, Takahashi, '16

Ultra light axions from Z_n symmetry

Di Luzio, Gavela, Quilez, Ringwald '21

Heavy axions from a mirror QCD

Gaillard, Gavela, Houtz, Quilez, del Rey, '18

DM axions outside the misalignment mass range? E.g.:

May account only for a fraction of DM,

Misalignement with initial kinetic energy

Co, Hall, Harigaya '19

Topological defects (domain walls, axion strings)

For a review: Marsch '15

Axion-Like Particle = pseudoscalar bosons with a free mass term?

Do not solve strong CP, but quite conspicuous in string theory

For a review: Ringwald '14

Many search strategies with photons, electrons, nucleons, mesons, via cavities, NMR (EDMs), colliders, on earth or in space.

III. Baryonic/leptonic axions

- Prelude

The axion is intimately connected to CP violation.

The PQ symmetry is a flavor symmetry, like $\mathcal{B}$ and $\mathcal{L}$ numbers.

The axion is a viable dark matter candidate.

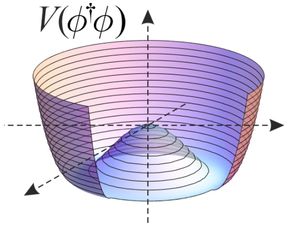
Wouldn't it be nice if it could also induce baryogenesis?

Could this explain the closeness of DM and baryonic relic densities?

Let's try to incorporate $\mathcal{B}$ and/or $\mathcal{L}$ violation *within* axion models!

Toy model for a “QED axion”

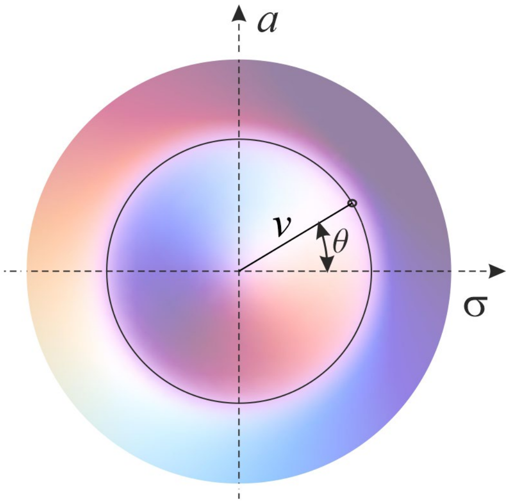
$$\mathcal{L}_{axion} = \frac{e^2}{16\pi^2}\theta F_{\mu\nu}\tilde{F}^{\mu\nu} + \bar{\psi}_{L,R}\, i\not{D}\,\psi_{L,R} - (y\bar{\psi}_L\psi_R\phi + h.c.) + \partial_\mu\phi^\dagger\partial^\mu\phi - V(\phi^\dagger\phi)$$



Three equivalent representations in the broken phase:

Usual renormalizable representation:

$$\phi = \frac{1}{\sqrt{2}}(\sigma + ia + v) \qquad \mathcal{L}_{linear} = \frac{\alpha\theta}{4\pi}F_{\mu\nu}\tilde{F}^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi - m\frac{a}{v}\bar{\psi}i\gamma_5\psi + \ldots$$



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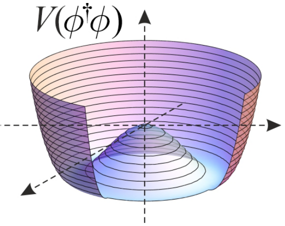

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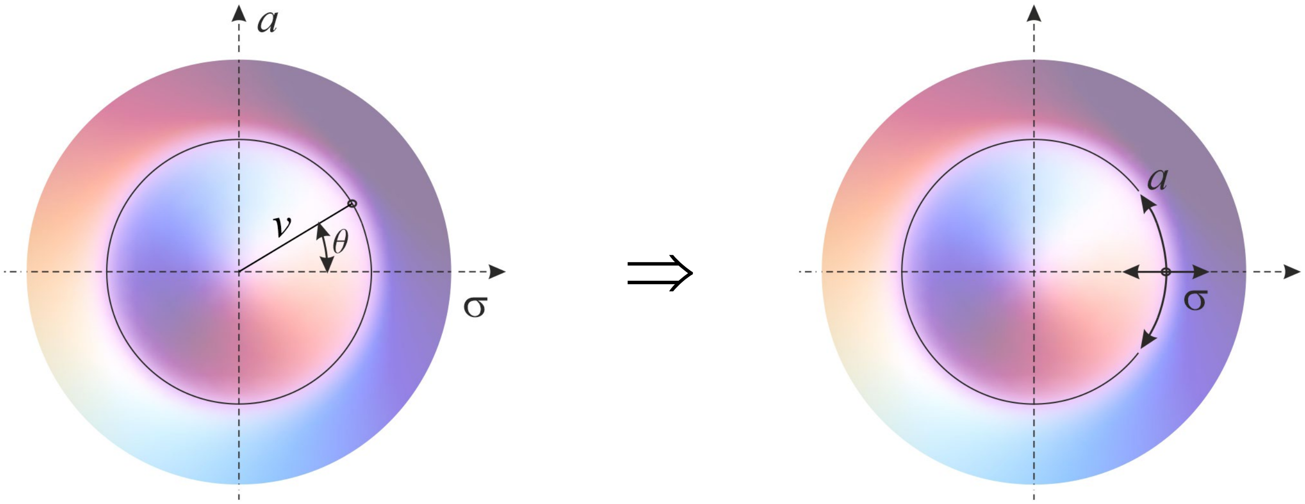

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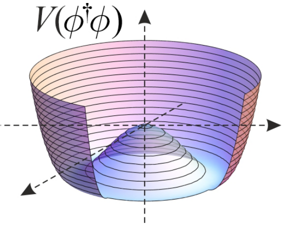

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Shift-symmetric polar representation, with σ integrated out:

$$\phi = \frac{1}{\sqrt{2}}(\sigma + v)e^{ia/v} \qquad \mathcal{L}_{polar} = \frac{\alpha\theta}{4\pi}F_{\mu\nu}\tilde{F}^{\mu\nu} + \bar{\psi}\left(i\not{D} - me^{ia\gamma^5/v}\right)\psi + \ldots$$

Derivative representation by making the fermion PQ-neutral:

$$\psi \to e^{-ia\gamma^5/2v}\psi \qquad \mathcal{L}_{der} = \frac{\alpha}{4\pi}\left(\theta - \frac{a}{v}\right)F_{\mu\nu}\tilde{F}^{\mu\nu} + \bar{\psi}\left(i\not{D} - m + \frac{\partial_\mu a\gamma^\mu\gamma^5}{2v}\right)\psi + \ldots$$

Typical Axion effective Lagrangian

Georgi,Kaplan,Randal, ‘86

Anomalous couplings to gauge bosons:

$$\mathcal{L}_{Jac} = \frac{a^0}{16\pi^2 f_a}\left(g_s^2 \mathcal{N}_C G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + g^2 \mathcal{N}_L W^i_{\mu\nu}\tilde{W}^{i,\mu\nu} + g'^2 \mathcal{N}_Y B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$$

...with $\mathcal{N}_X = \sum_{\psi} Q_\psi C_X(\psi)$.

Derivative couplings to fermions (and other PQ-charged fields):

$$\mathcal{L}_{Der} = -\frac{1}{f_a}\partial_\mu a J^\mu_{PQ} \quad ,$$

...with $J^\mu_{PQ} = \sum_{\psi=\psi_{L,R}} Q_\psi \bar{\psi}\gamma^\mu\psi + \ldots$

$\longrightarrow 0$ (partial integration + CVC)

$$= \sum_{\psi=u,d,e,\nu}\left[(Q_{\psi_R}+Q_{\psi_L})\bar{\psi}\gamma^\mu\psi + (Q_{\psi_R}-Q_{\psi_L})\bar{\psi}\gamma^\mu\gamma_5\psi\right] + \ldots$$

All this seems ok... but actually there is a serious ambiguity issue!

Problem: The fermion PQ charges are ill-defined

Scalars have well defined PQ charges, but fermions do not:

KSVZ: $\phi \bar{\Psi}_L \Psi_R$

	Ψ_L	Ψ_R	q_L	u_R	d_R	ℓ_L	e_R	ν_R
$U(1)_{PQ}$	α	$\alpha-1$	β	β	β	γ	γ	γ
$U(1)_Y$	Y	Y	$1/3$	$4/3$	$-2/3$	-1	-2	0

DFSZ: $\phi H_u^\dagger H_d$

$x \equiv v_u / v_d$

	q_L	u_R	d_R	ℓ_L	e_R	ν_R
$U(1)_{PQ}$	β	$\beta+x$	$\beta-1/x$	γ	$\gamma-1/x$	$\gamma+x$
$U(1)_Y$	$1/3$	$4/3$	$-2/3$	-1	-2	0

Free parameters reflects the conservation of Ψ, $\mathcal{B}$, $\mathcal{L}$ numbers.

Consequence: Ambiguous DFSZ Axion couplings for SM gauge bosons

Anomalous couplings to gauge bosons:

$$\begin{aligned}\mathcal{L}_{Jac} = & \frac{a}{16\pi^2 f_a} g_s^2 \mathcal{N}_C G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \\ & + \frac{a}{16\pi^2 f_a} g^2 \mathcal{N}_L W^i_{\mu\nu} \tilde{W}^{i,\mu\nu} \\ & + \frac{a}{16\pi^2 f_a} g'^2 \mathcal{N}_Y B_{\mu\nu} \tilde{B}^{\mu\nu}\end{aligned}$$

$$\mathcal{N}_C = \frac{1}{2}\left(x + \frac{1}{x}\right)$$

$$\mathcal{N}_L = -\frac{1}{2}(3\beta + \gamma)$$

$$\mathcal{N}_Y = \frac{1}{2}(3\beta + \gamma) + \frac{4}{3}\left(x + \frac{1}{x}\right)$$

Derivative couplings to SM fermions:

$$\mathcal{L}_{Der} = -\frac{1}{2f_a} \partial_\mu a^0 \sum_{u,d,e,\nu} \chi_V^f \bar{\Psi}_f \gamma^\mu \psi_f + \chi_A^f \bar{\Psi}_f \gamma^\mu \gamma_5 \psi_f$$

	u	d	e	ν
χ_V	$2\beta + x$	$2\beta + \frac{1}{x}$	$2\gamma + \frac{1}{x}$	γ
χ_A	x	$\frac{1}{x}$	$\frac{1}{x}$	$-\gamma$

Both manifestly $SU(2)_L \otimes U(1)_Y$ symmetric, both ambiguous!

Consequence: Ambiguous DFSZ Axion couplings for SM gauge bosons

Axion couplings in the polar/linear representation (= THDM!!!)

$$\begin{aligned}\mathcal{L}_{polar} =& \frac{a}{16\pi^2 f_a} g_s^2 \mathcal{N}_C G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} \\ &+ \frac{a}{16\pi^2 f_a} e^2 \mathcal{N}_{em} F_{\mu\nu} \tilde{F}^{\mu\nu} \\ &+ \frac{a}{16\pi^2 f_a} \frac{2e^2}{c_W s_W} (\mathcal{N}_0 - s_W^2 \mathcal{N}_{em}) Z_{\mu\nu} \tilde{F}^{\mu\nu} \\ &+ \frac{a}{16\pi^2 f_a} \frac{e^2}{c_W^2 s_W^2} (\mathcal{N}_1 - 2 s_W^2 \mathcal{N}_0 + s_W^4 \mathcal{N}_{em}) Z_{\mu\nu} \tilde{Z}^{\mu\nu} \\ &+ \frac{a}{16\pi^2 f_a} 2g^2 \mathcal{N}_2 W_{\mu\nu}^+ \tilde{W}^{-,\mu\nu}\end{aligned}$$

$$\mathcal{N}_C = \frac{1}{2}\left(x + \frac{1}{x}\right)$$

$$\mathcal{N}_{em} = \frac{4}{3}\left(x + \frac{1}{x}\right)$$

$$\mathcal{N}_0 = \frac{1}{2}\left(x + \frac{1}{x}\right)$$

$$\mathcal{N}_1 = \frac{1}{12}\left(3x + \frac{4}{x}\right)$$

$$\mathcal{N}_2 = \frac{1}{4}\left(x + \frac{3}{2x}\right)$$

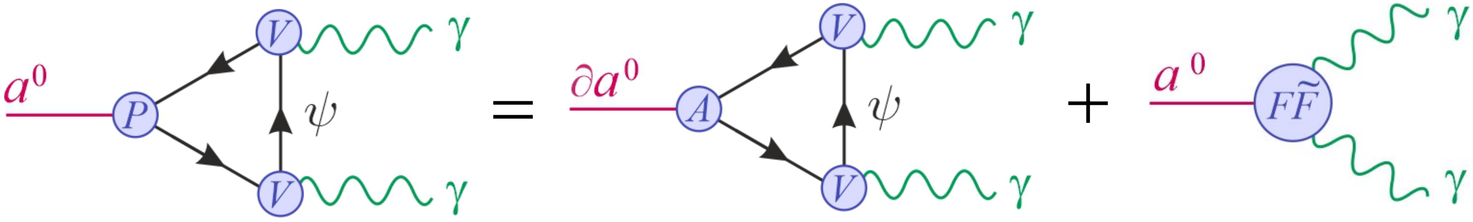
Gunion, Haber, Kao ‘91

Not ambiguous, but does not match $\mathcal{L}_{Jac}$:

$$\mathcal{N}_{em} = \mathcal{N}_L + \mathcal{N}_Y \quad \text{but} \quad \mathcal{N}_0 \neq \mathcal{N}_1 \neq \mathcal{N}_2 \neq \mathcal{N}_L = -\frac{1}{2}(3\beta + \gamma)$$

Solution: Violations of Sutherland-Veltman theorem

Axial current anomalies: $2imP = \partial_\mu A^\mu - \frac{1}{8\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$



Not anomalous

Anomalous

Anomaly

$\partial_\mu A^\mu \to 0$ when $m \to \infty$

True coupling, $\mathcal{L}_{linear}$

$\mathcal{L}_{Der}$

$\mathcal{L}_{Jac}$

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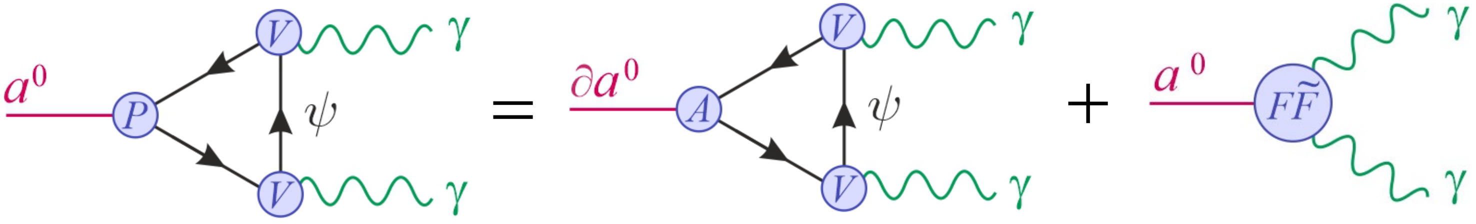

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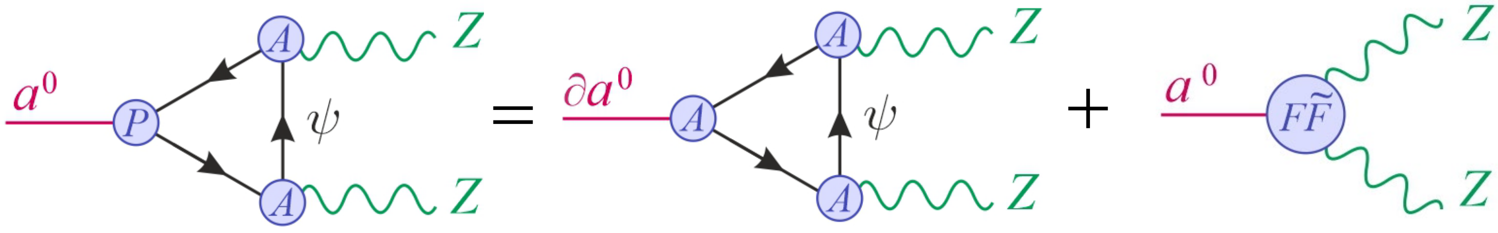


Not anomalous

Anomalous

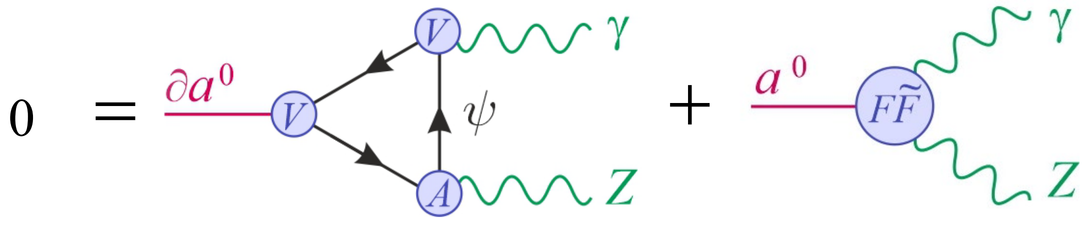
Anomaly

$\partial_\mu A^\mu \not\to 0$ when $m \to \infty$

Solution: Violations of Sutherland-Veltman theorem

Axial current anomalies: $2imP = \partial_\mu A^\mu - \frac{1}{8\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$

Vector current anomalies: $0 = \partial_\mu V^\mu - \frac{1}{8\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$



$\partial_\mu V^\mu \nrightarrow 0$ when $m \to \infty$

$\mathcal{L}_{Der}$

$\mathcal{L}_{Jac}$

$\mathcal{B}$ and $\mathcal{L}$ ambiguities cancel out between $\mathcal{L}_{Jac}$ and $\mathcal{L}_{Der}$.

The electroweak couplings $a^0 + \gamma Z, ZZ, WW$ are not given by $\mathcal{L}_{Jac}$.

Quevillon,CS ‘19, Bonnefoy,Di Luzio,Grojean,Paul,Rossia ‘20, Quevillon,CS,Vuong ‘21

Using the ambiguities to entangle PQ with $\mathcal{B}$ and $\mathcal{L}$

Fermionic PQ charges are ambiguous.

	ϕ	H_u	H_d	q_L	u_R	d_R	ℓ_L	e_R	ν_R
$U(1)_{PQ}$	1	x	$-1/x$	β	$\beta+x$	$\beta-1/x$	γ	$\gamma-1/x$	$\gamma+x$

The incorrect way to use these parameters:

Set them to some value, $\beta=\gamma=0$ say, which forbids for example:

$$\mathcal{L}^{eff}_{Majo} = \frac{1}{M}(\bar{\ell}^C_L H^T_u)(H_u \ell_L) \sim m_\nu \bar{\nu}^C_L \nu_L \quad \rightarrow PQ(\mathcal{L}^{eff}_{Majo}) = 2(\gamma + x)$$

Yet, in the linear representation, adding this operator is harmless!

The correct way to use these parameters:

Keep them free to accommodate possible $\mathcal{B}$ and/or $\mathcal{L}$ violations.

If two models differ by their values: equivalent phenomenology!

Using the ambiguities to entangle PQ with $\mathcal{B}$ and $\mathcal{L}$

- Example: KSVZ axion as majoron

Langacker et al. '86
Shin '87
Clarke,Volkas '16

$$\boxed{\phi\bar{\Psi}_L\Psi_R+\phi\overline{\nu}_R^C\nu_R}$$

$$f_a\overline{\nu}_R^C\nu_R\rightarrow m_\nu\sim\frac{v^2}{f_a}Y_\nu^TY_\nu$$

$$a\overline{\nu}_R^C\nu_R\leftarrow$$ no f_a suppression.

The PQ current eats up the lepton current:

	ϕ	H	Ψ_L	Ψ_R	q_L	u_R	d_R	ℓ_L	e_R	ν_R
$U(1)_{PQ=\mathcal{L}}$	1	0	α	$\alpha-1$	β	β	β	$-1/2$	$-1/2$	$-1/2$

- In general, SSB along two combinations of $\mathcal{B}$ and $\mathcal{L}$ only.

Beware though that EW instantons $\Rightarrow\mathcal{L}_{eff}\sim(\ell_Lq_L^3)^3\Rightarrow3\beta+\gamma=0$.

Too much $\mathcal{B}$ and/or $\mathcal{L}$ violations can make the axion massive.

Quevillon,CS '20

Arias-Aragón, CS, ‘22

Rich phenomenology with leptoquarks & diquarks

- Spontaneous proton decay

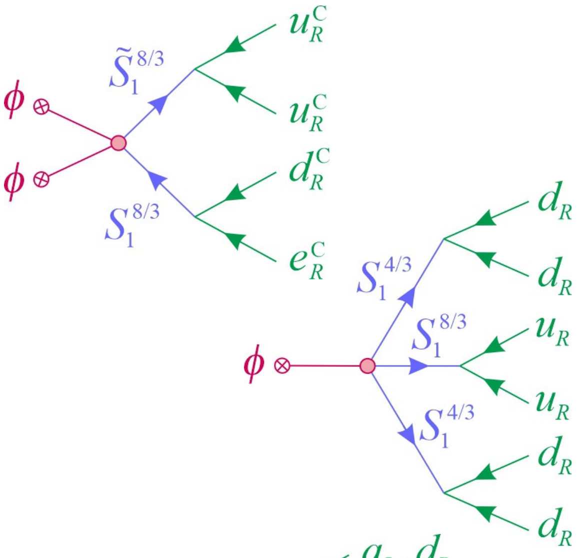
$$S_1^{8/3}\overline{d}_R e_R^C + \tilde{S}_1^{8/3}\overline{u}_R^C u_R + \phi^2 S_1^{8/3\dagger}\tilde{S}_1^{8/3}$$

(Very similar to Reig, Srivastava, ’18)

- Spontaneous neutron-antineutron oscillation

$$S_1^{4/3}\overline{d}_R^C d_R + S_1^{8/3}\overline{u}_R^C u_R + \phi S_1^{4/3} S_1^{4/3} S_1^{8/3}$$

(Kind of similar to Barbieri, Mohapatra ’81)

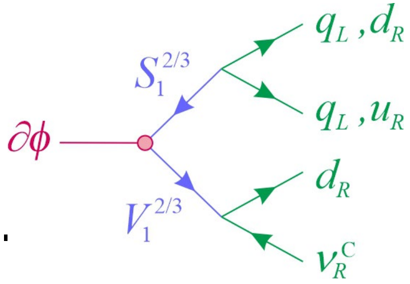


- ALP and the neutron lifetime puzzle

$$S_1^{2/3}\overline{d}_R^C u_R + V_{1,\mu}^{2/3}\overline{d}_R\gamma^\mu \nu_R + \partial^\mu\phi S_1^{2/3\dagger}V_{1,\mu}^{2/3}$$

$B(n \to \nu a) \sim 1\%$ with $p \to e\gamma\gamma > 10^{34}\, yr$ if $m_p < m_a < m_n$.

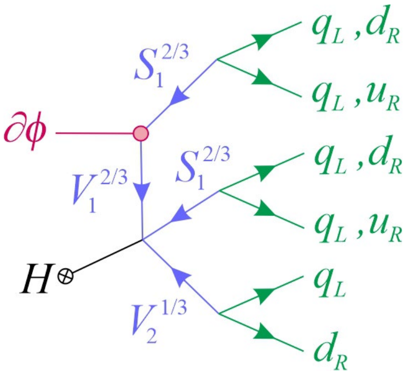
(see Fornal, Grinstein, ’18)



- Intense antimatter production?

$$S_1^{2/3}\overline{d}_R^C u_R + V_{2,\mu}^{1/3}\overline{d}_R^C\gamma^\mu q_L + \partial^\mu\phi V_{1,\mu}^{2/3\dagger}S_1^{2/3} + HV_{1,\mu}^{2/3}S_1^{2/3}V_{2,\mu}^{1/3}$$

Resonant $n \to a^0\overline{n}$ if $\delta m_{n-\overline{n}} \approx B\times 10^{-7}\,\mathrm{eV} = m_a$ (precisely!)



IV. Conclusion

The axion is currently the best solution to the strong CP puzzle,

Great deal of freedom in implementing such a mechanism,

Theoretical description is more delicate than its seems.

The axion should not be a «one-problem solution»:

It is now one of the best DM candidate,

Could play a role in the origin of neutrino masses,

Could it also induce lepto/baryogenesis?

Theoretical and experimental efforts are well under way in many fields

Axion physics transcends all energy frontiers and relates
low-energy QCD to cosmology, via atomic physics,
EW and colliders, and the physics of stars and galaxies.