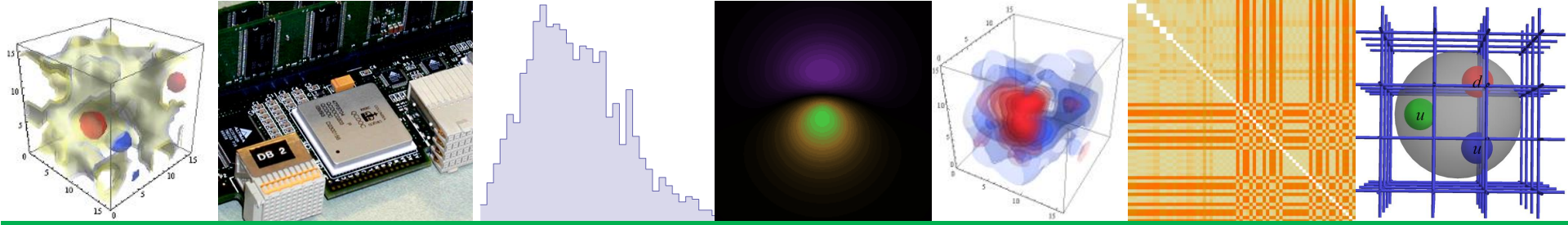


Recent Progress on Parton Distribution Functions on Lattice QCD

HUEY-WEN LIN

CTEQ

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

Outline

§ Lattice QCD in a Nutshell

§ Selected *x*-Dependent Parton Distributions

§ Impact of Lattice-QCD PDFs on Global Fits

Biased selected results toward MSULat students and postdocs

What is Lattice QCD?

§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories

§ Physical observables are calculated from the path integral

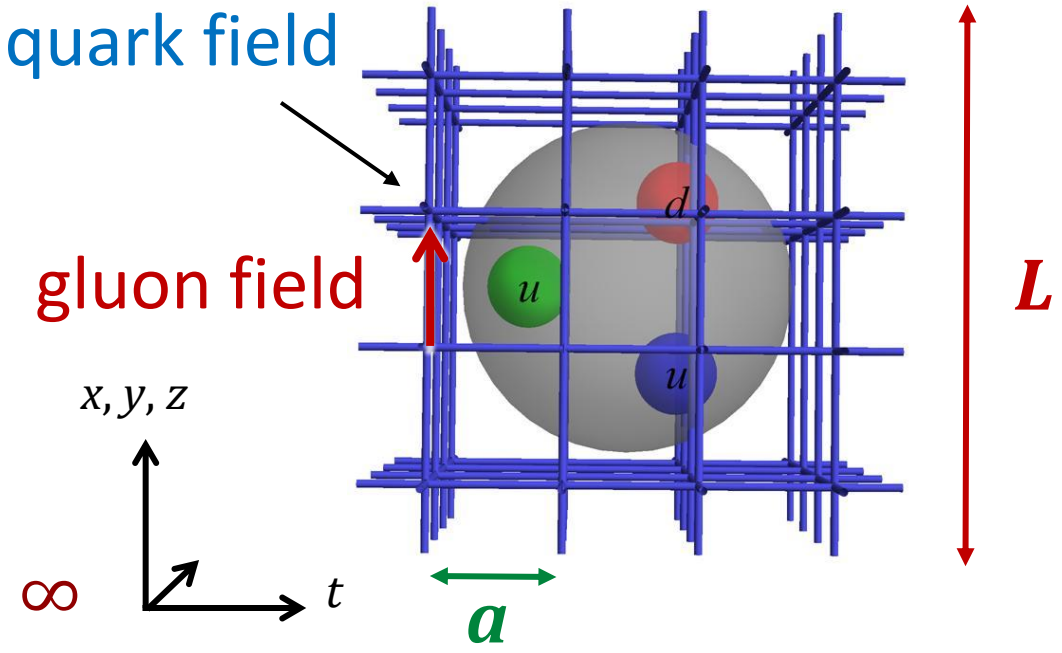
$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}O(\bar{\psi},\psi,A)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume

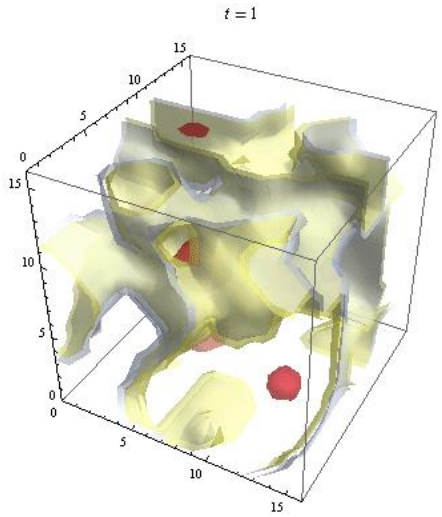
§ Recover physical limit

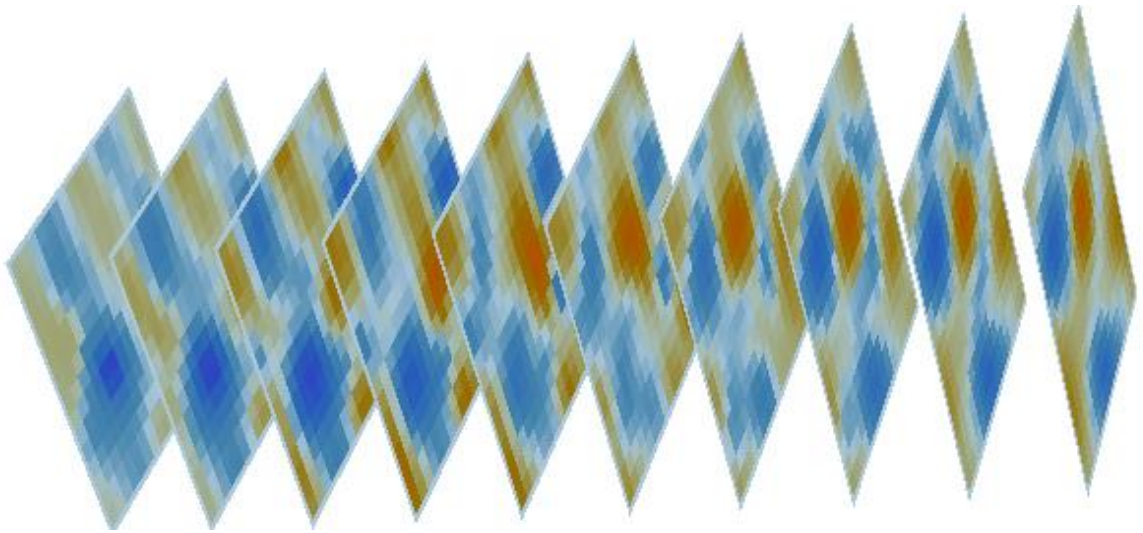
$m_\pi \to m_\pi^{\text{phys}}$, $a \to 0$, $L \to \infty$



Anatomy of a Lattice Calculation

1. Start with QCD Vacuum (gauge configurations)





Anatomy of a Lattice Calculation

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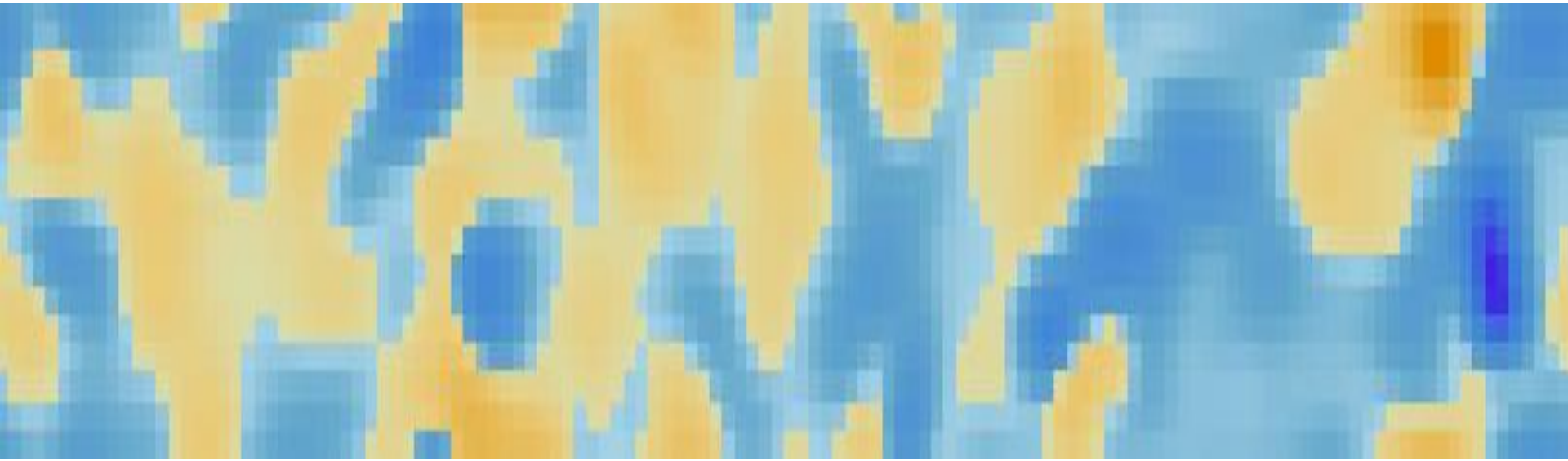
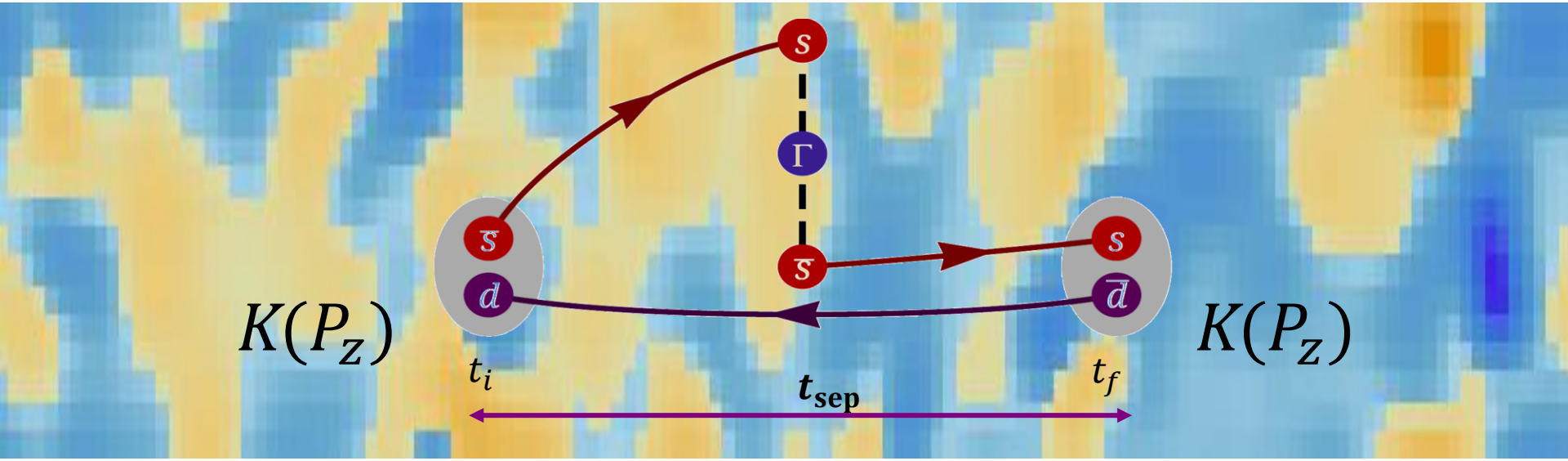


Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

Anatomy of a Lattice Calculation

2. Correlators (hadronic observables)

- Invert Dirac operator matrix (rank 10^{12})
- Combine using color, spin and momentum into hadrons

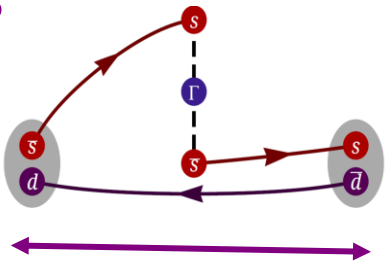


Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

Anatomy of a Lattice Calculation

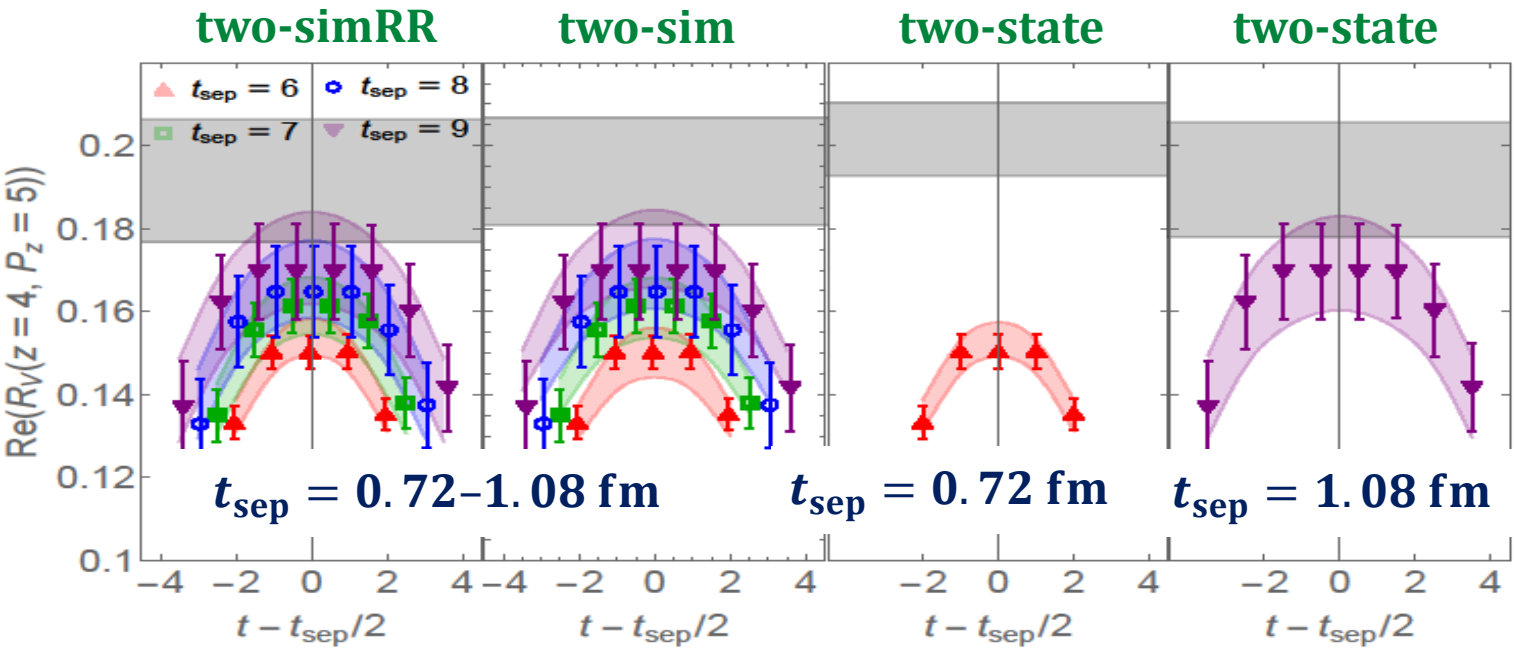
3. Extract reliable ground-state matrix elements

- Excited-state removal
  - For example, kaon matrix element at $M_\pi \approx 220$ MeV, $a \approx 0.12$ fm



t_{sep}

HL et al. (MSULat), 2003.14128



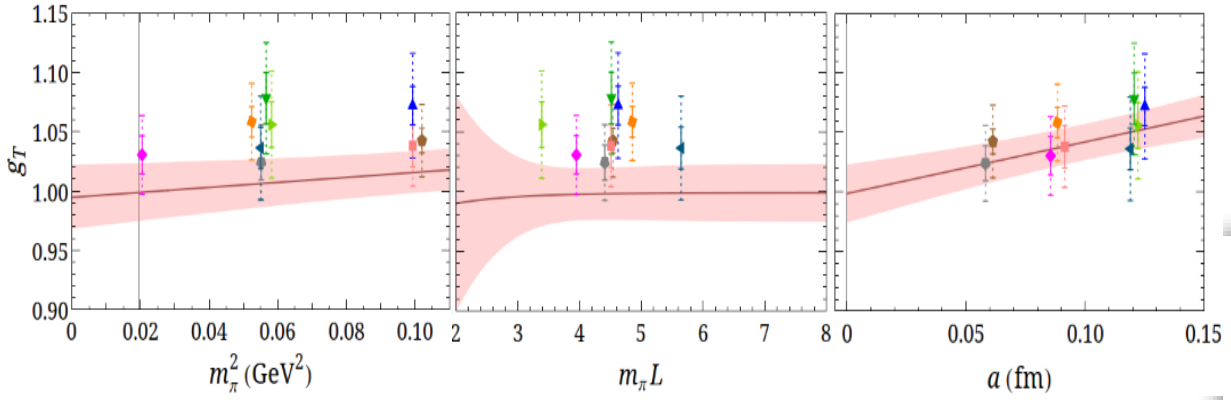
stability in extracting matrix elements

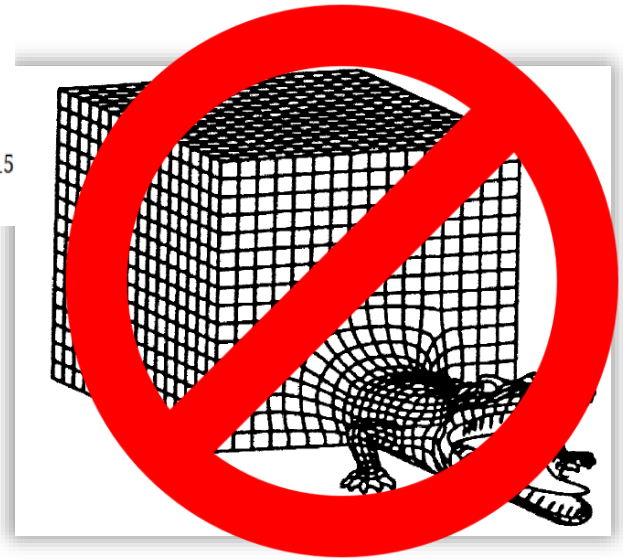
Anatomy of a Lattice Calculation

4. Systematic uncertainty (nonzero a, finite L, etc.)

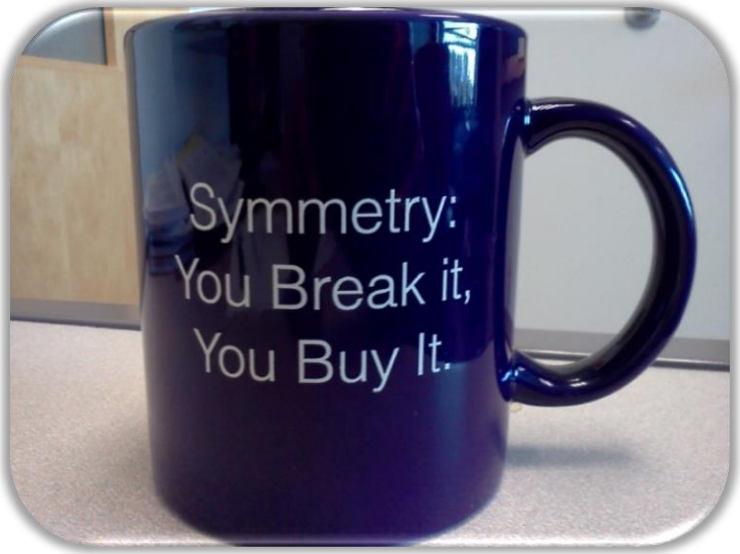
- Nonperturbative renormalization, etc
- Extrapolation to the continuum limit

  $(m_\pi \to m_\pi^{\text{phys}},\ L \to \infty,\ a \to 0)$





Lattice Structure Limitation



§ Lattice calculations rely on operator product expansion, only provide moments

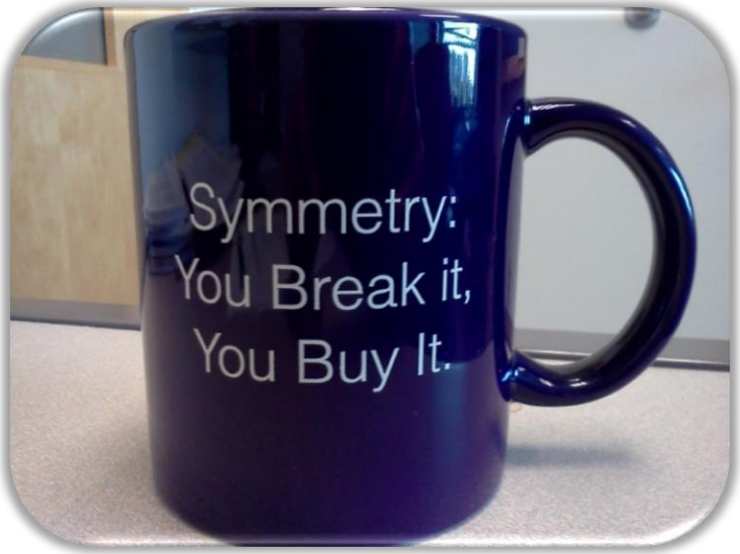
$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

§ Limited to the lowest few moments

- For higher moments, all ops mix with lower-dimension ops
- Novel proposals to overcome this problem

W. Detmold and C. Lin, Phys. Rev. D73 (2006) 014501
Z. Davoudi and M. J. Savage, Phys. Rev. D86 (2012) 054505
A. Shindler, arXiv:2311.18704 (also this Thur, WG5 @DIS2024)

Lattice Structure Limitation



§ Lattice calculations rely on operator product expansion, only provide moments

$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

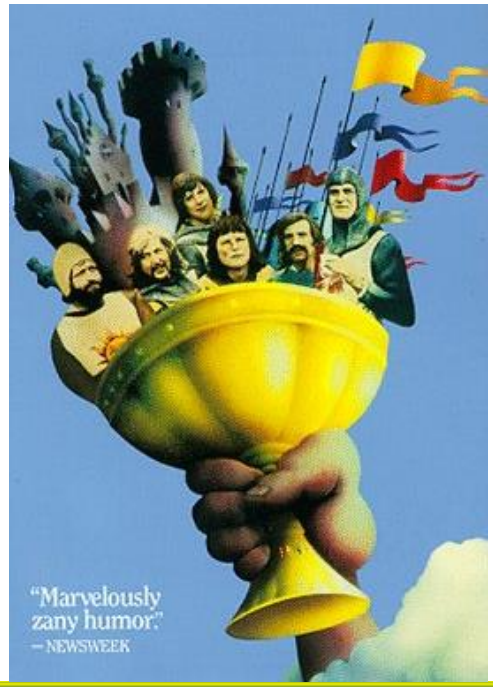
§ Longstanding obstacle!

- Holy grail of structure calculations

§ Applies to many structure quantities:

- Parton Distribution Functions (PDFs)
- Generalized parton distributions (GPD)
- Transverse-momentum distributions (TMD)

……



A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

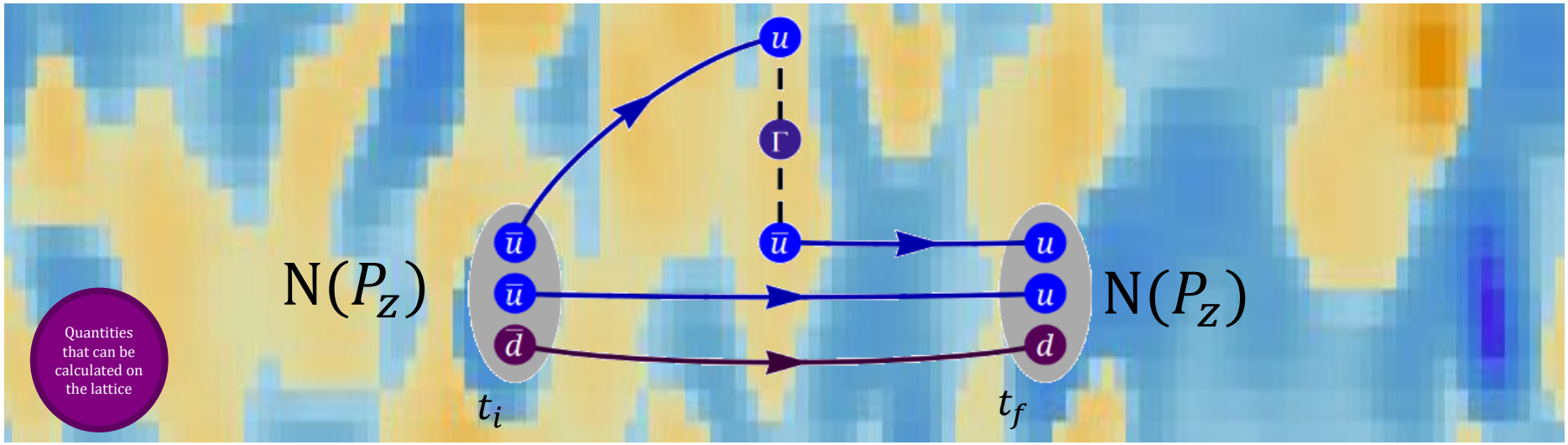
- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)
- …

Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

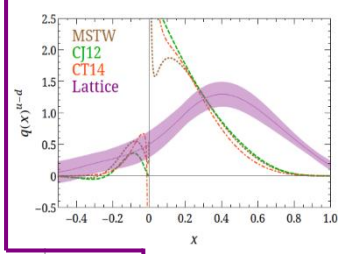
Quasi-PDF & Pseudo-PDF method

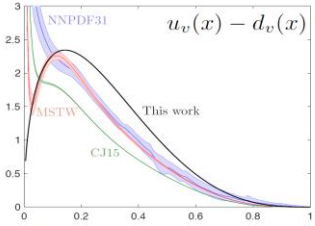


Lattice Parton Calculations

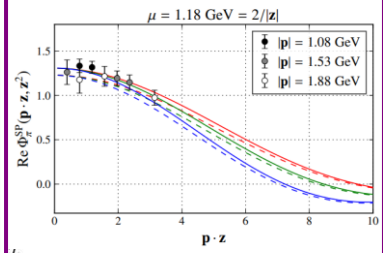
§ Rapid developments!

First unpol. PDF lattice calculation



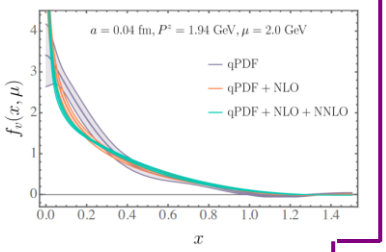


First lattice pseudo-PDFs

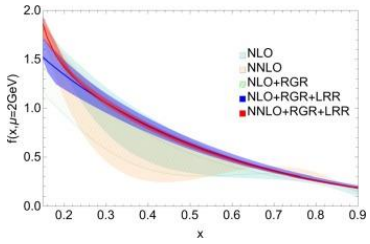


Euclidean correlation functions

1st NNLO PDF

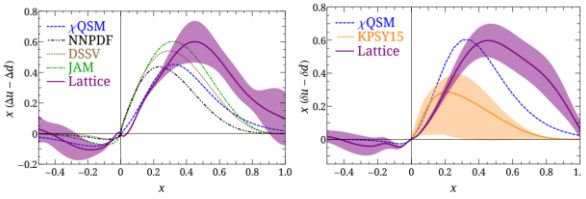


1stPDF w/ LRR+RGR

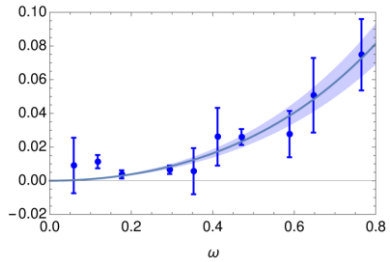


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024

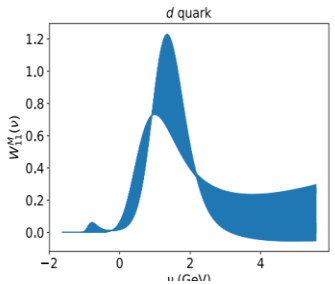
Pol. PDFs and mass corrections



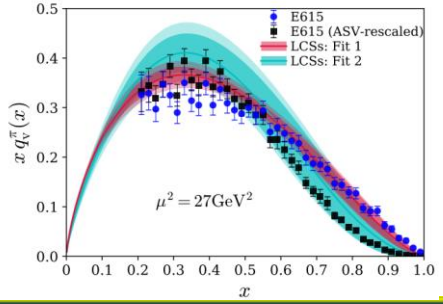
Compton amplitude



Hadronic tensor



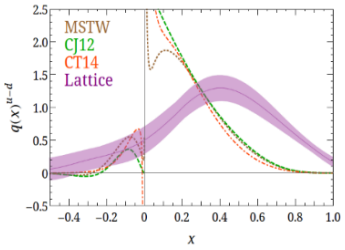
LCS



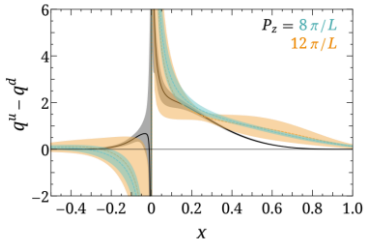
Lattice Parton Calculations

§ Physics quantity milestones

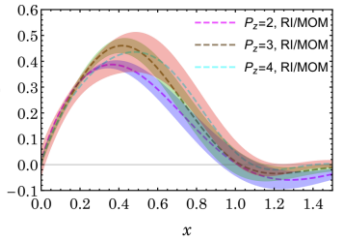
First unpol. lattice PDF



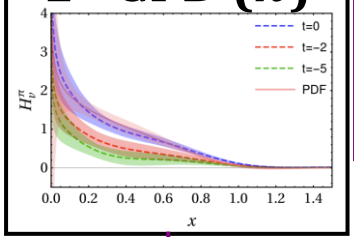
First PDFs at $M_\pi^{\rm phys}$



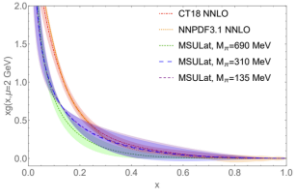
Pion v-PDF



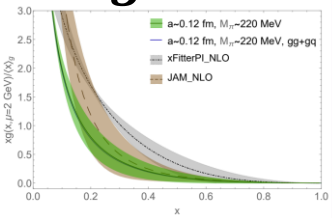
1st GPD (π)



N g-PDF



π g-PDF



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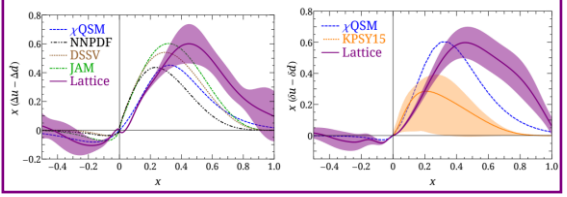
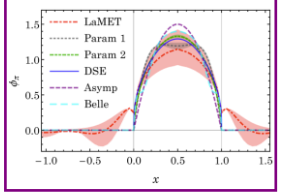
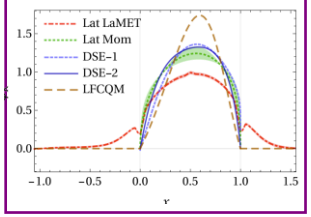


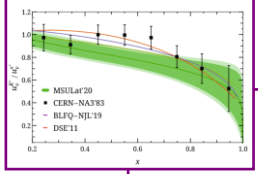
Pion DA



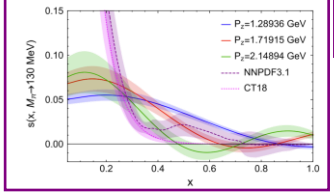
Kaon DA



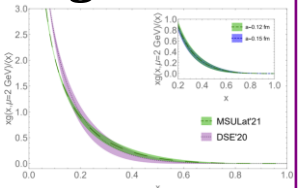
K PDF



s,c PDF



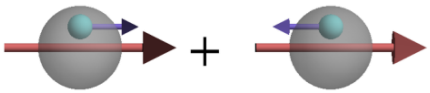
Kaon g-PDF



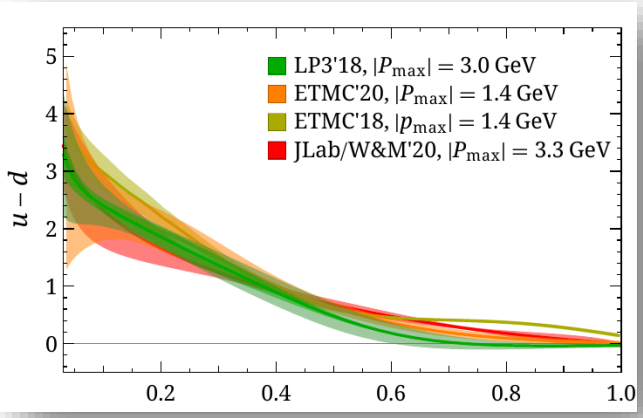
Lattice Example Results

§ Summary of physical pion mass PDFs results

unpolarized

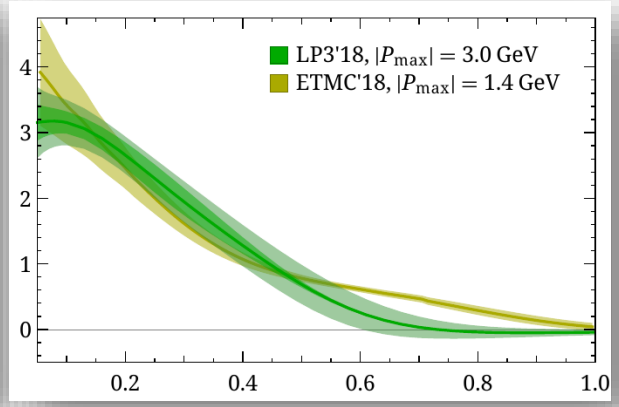


$u(x) - d(x)$



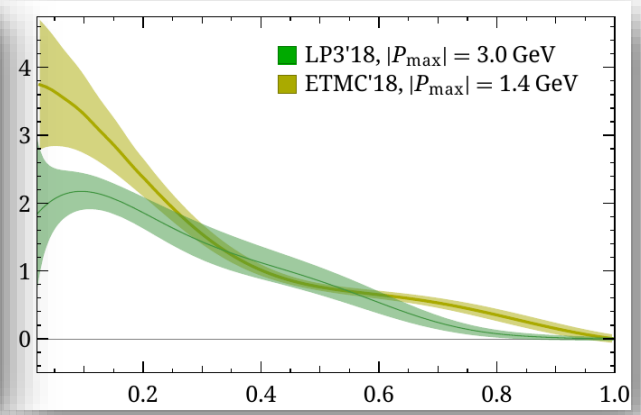
longitudinally polarized

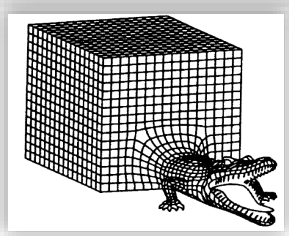
$\Delta u(x) - \Delta d(x)$



transversely polarized

$\delta u(x) - \delta d(x)$





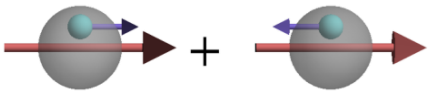
Finite volume,
Discretization,
…

2006.08636 (PDFLattice2019)

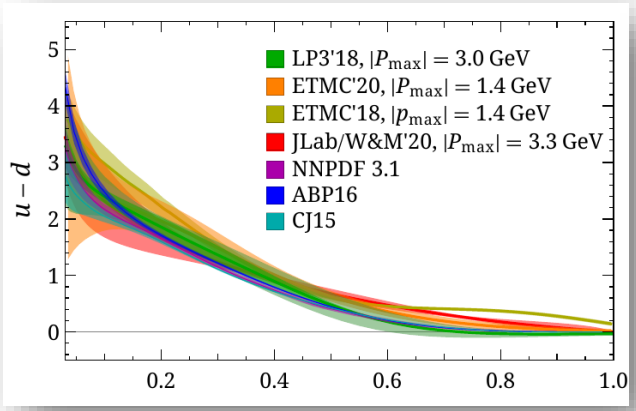
Lattice Example Results

§ Summary of physical pion mass PDFs results

unpolarized

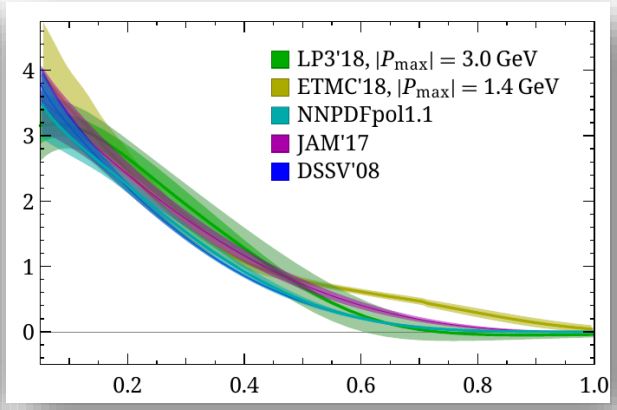


$u(x) - d(x)$



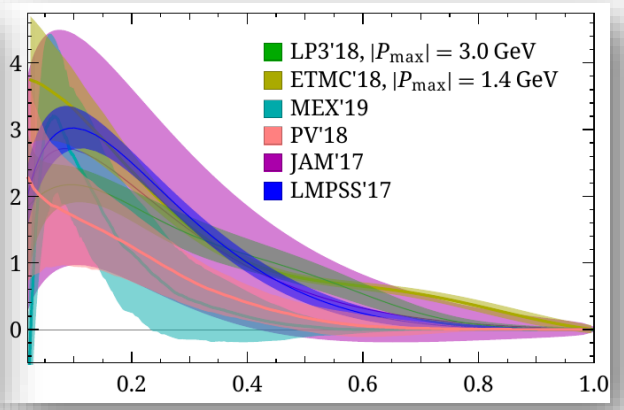
longitudinally polarized

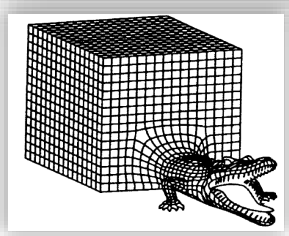
$\Delta u(x) - \Delta d(x)$



transversely polarized

$\delta u(x) - \delta d(x)$





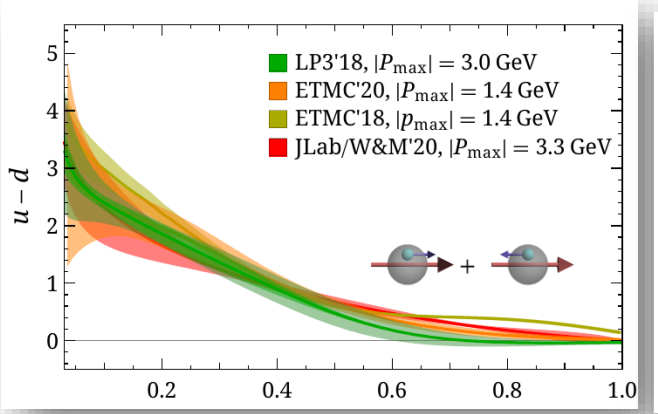
Finite volume,
Discretization,
…

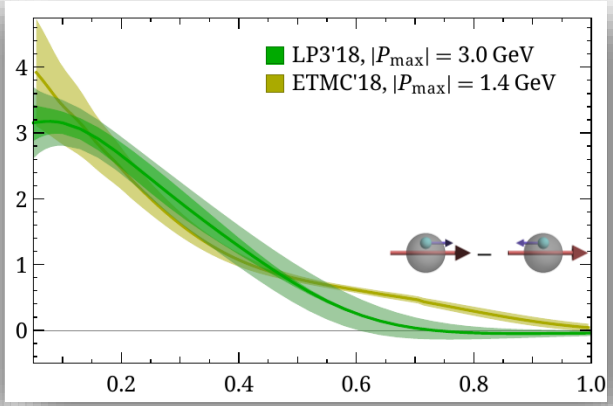
2006.08636 (PDFLattice2019)

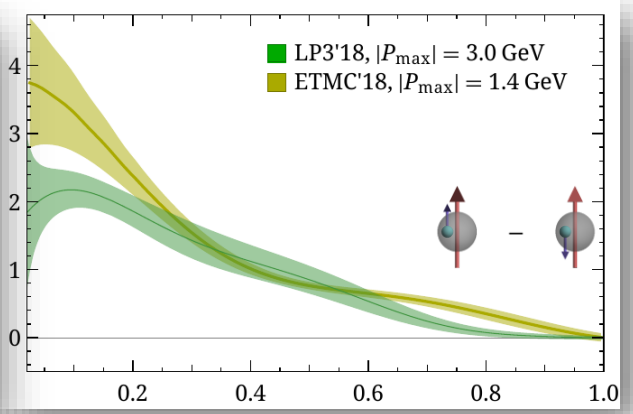
Lattice Example Results

§ Summary of physical pion mass results:

2006.08636 (PDFLattice2019)



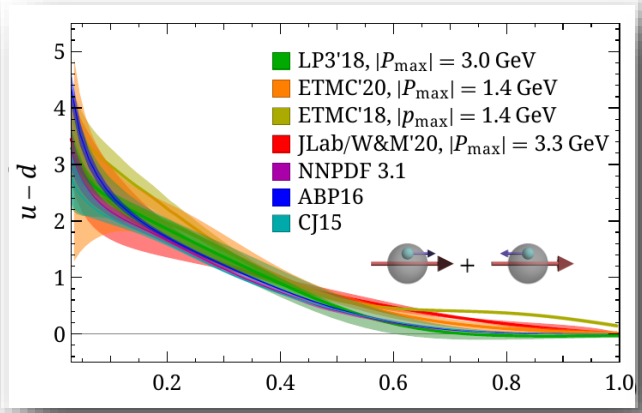


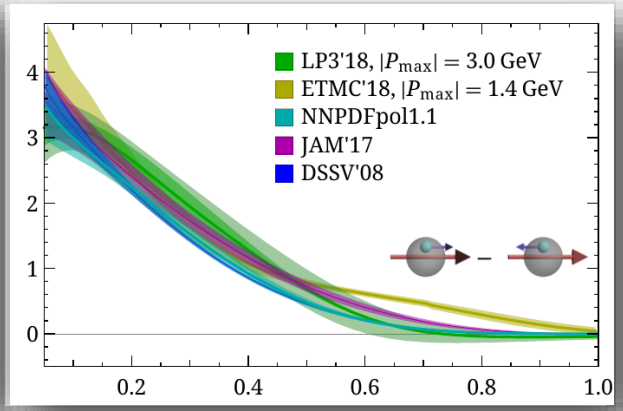


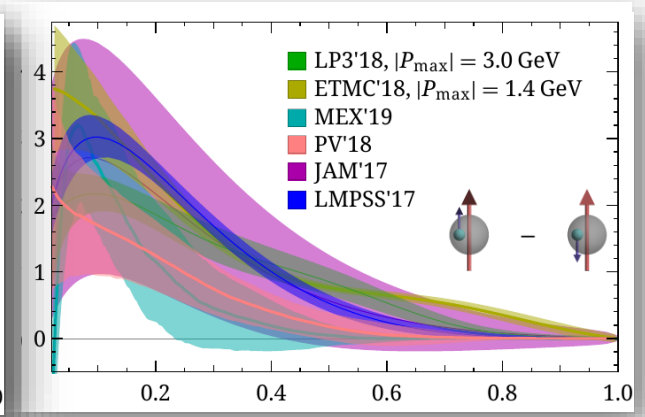
Lattice Example Results

§ Summary of physical pion mass results:

2006.08636 (PDFLattice2019)







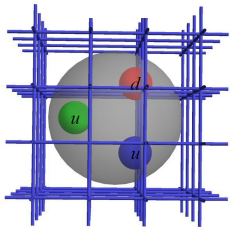
§ Complementary Lattice inputs for best PDFs

Theory Input

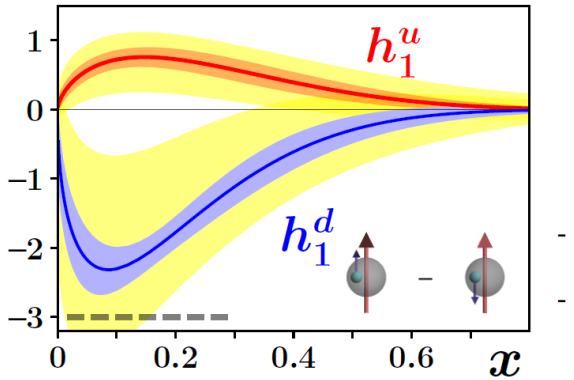
Exp't Input

Global Analysis of PDFs

+



Works have been developed in this direction



Phys. Rev. Lett. 120, 152502 (2018)

Isovector PDFs Update

§ Nucleon isovector PDF calculated directly at physical pion mass

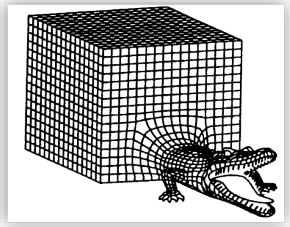
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

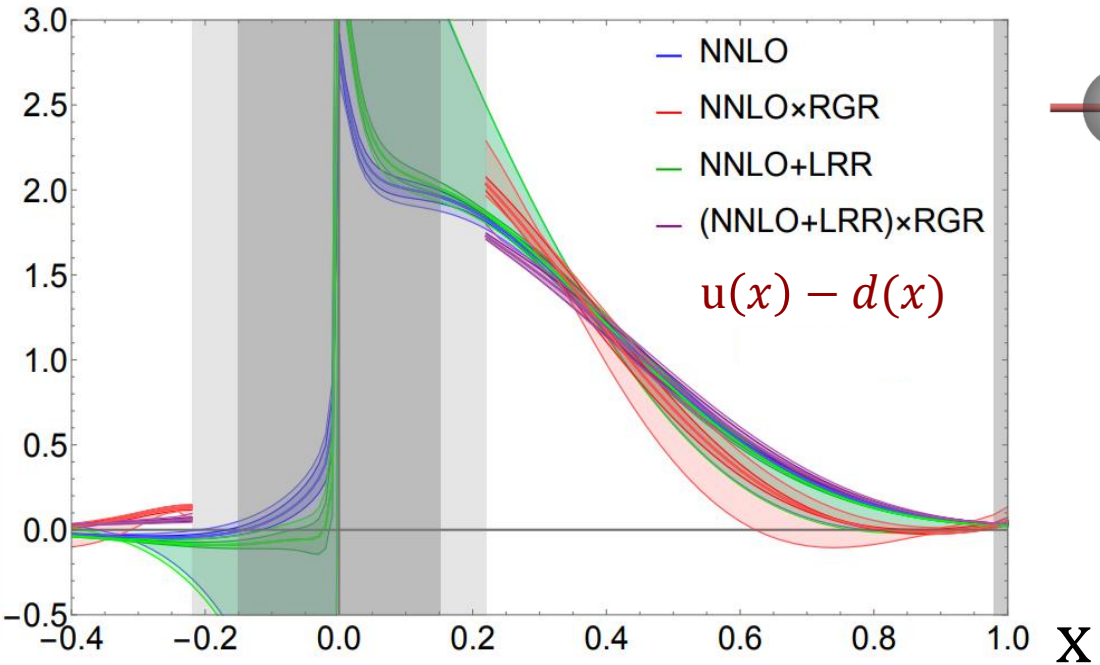
R. Zhang, et. al.
PLB 844, 138081 (2023)

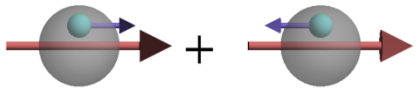
- N_f=2+1+1 clover/HISQ, a~0.09 fm

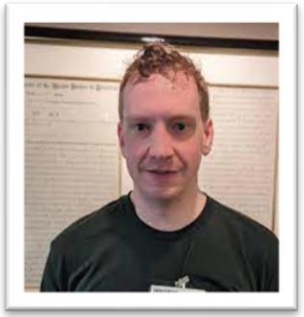
J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…









P: Jack Holligan

Isovector PDFs Update

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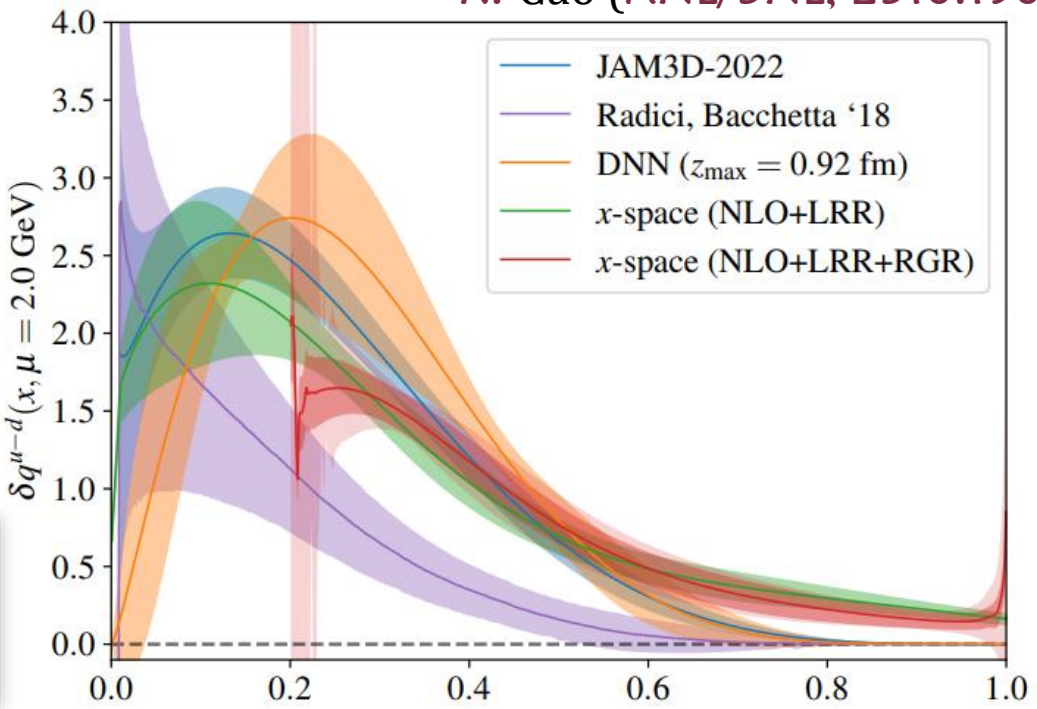
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

R. Zhang, et. al.
PLB 844, 138081 (2023)

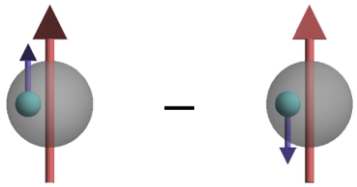
- N_f=2+1 clover/HISQ, a~0.076 fm

X. Gao (ANL/BNL, 2310.19047 [hep-lat]

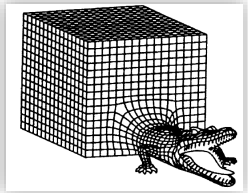
Wanted PDFs, GPDs, etc…



Transversity

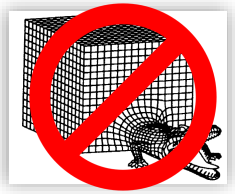


$\delta u(x) - \delta d(x)$

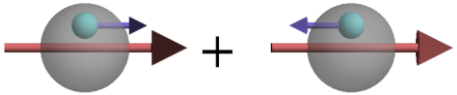


Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit

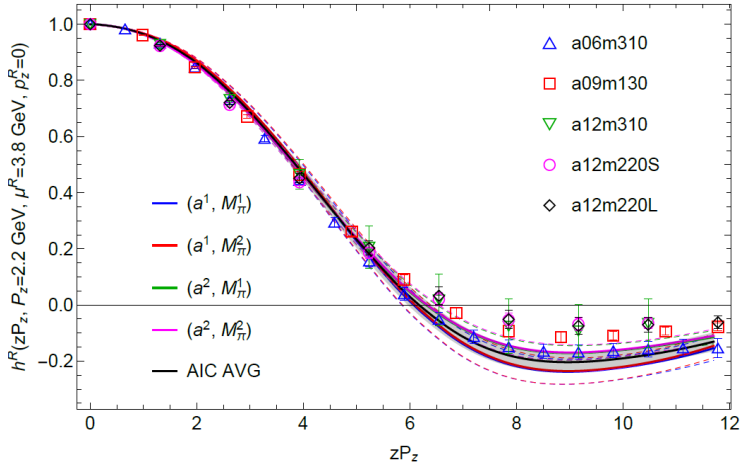


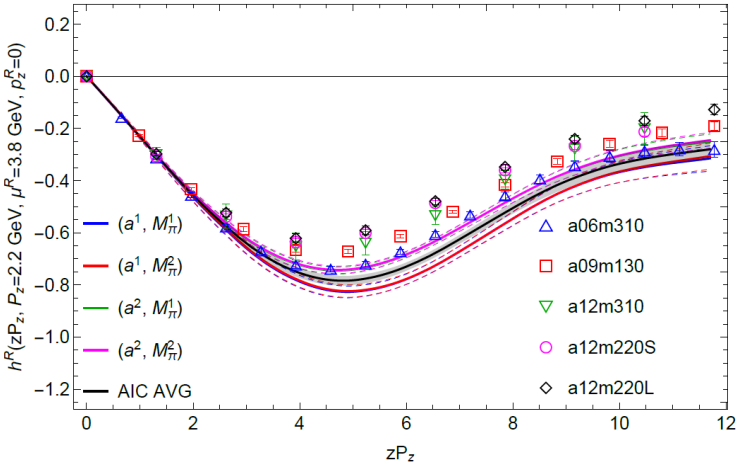
- Lattice details: clover/2+1+1 HISQ (MSULat)
  - a ≈ {0.06,0.09,0.12} fm,
  - $M_\pi \in \{\mathbf{135},220,310\}$-MeV pion,
  - $M_\pi L \in \{3.3, 5.5\}$.
  - $P_z \approx 2$ GeV 2011.14971, HL et al (MSULat)



- Naïve extrapolation to physical-continuum limit

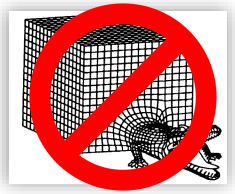
Quantities that can be calculated on the lattice





Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit



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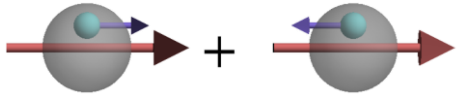
  - $P_z \approx 2$ GeV

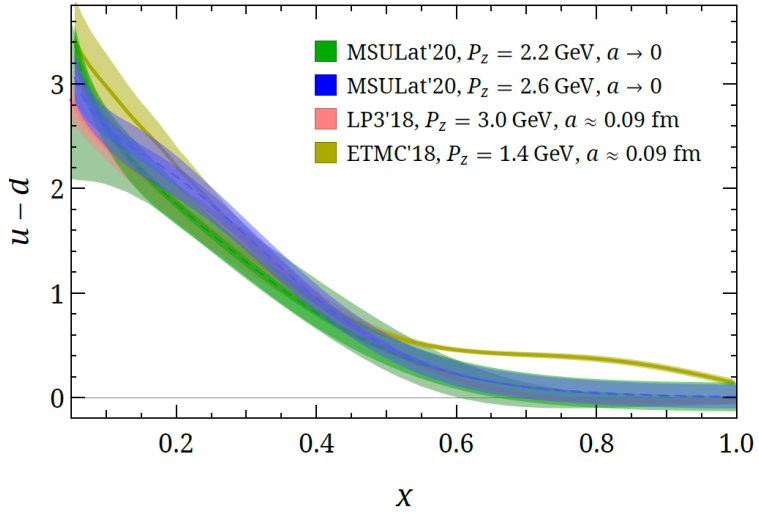
2011.14971, HL et al (MSULat)

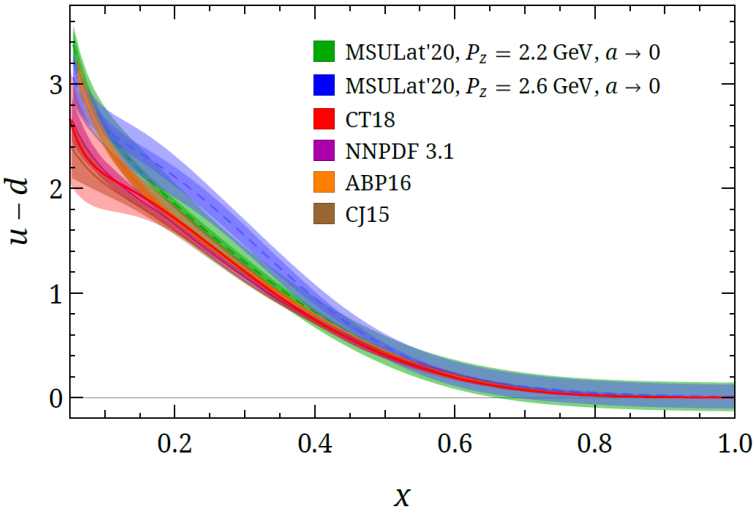


$u(x) - d(x)$

- Naïve extrapolation to physical-continuum limit

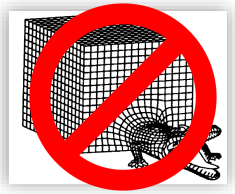
Wanted PDFs, GPDs, etc…





Continuum PDF

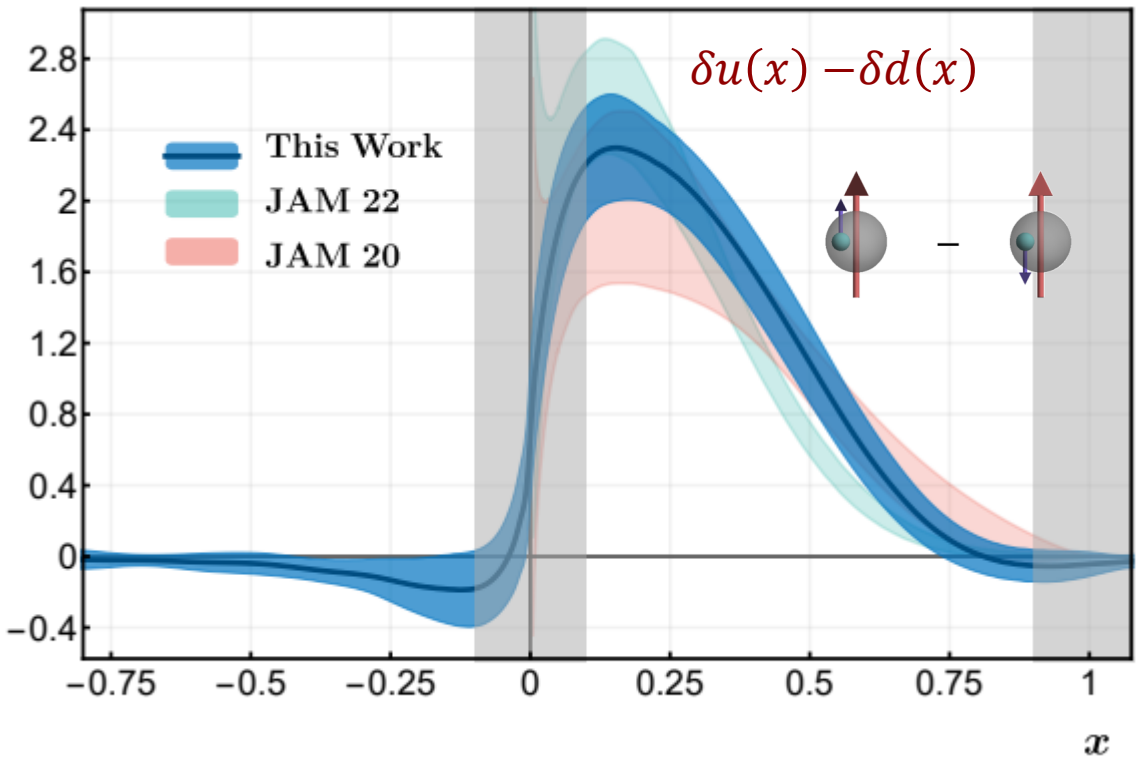
§ Nucleon PDFs using quasi-PDFs in the continuum limit



- Lattice details: clover/2+1 clover (LPC)
  - a ≈ {0.49,0.64,0.85,0.98} fm,
  - M_π ∈[222,354]-MeV pion,
  - $M_\pi L$ ∈[3.9, 8.1].
  - P_z ≈ ∈[1.8, 2.8].

F. Yao et al (LPC), 2208.08008

Wanted
PDFs, GPDs,
etc…

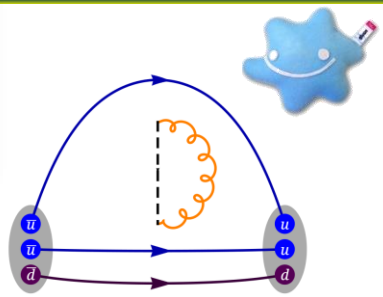


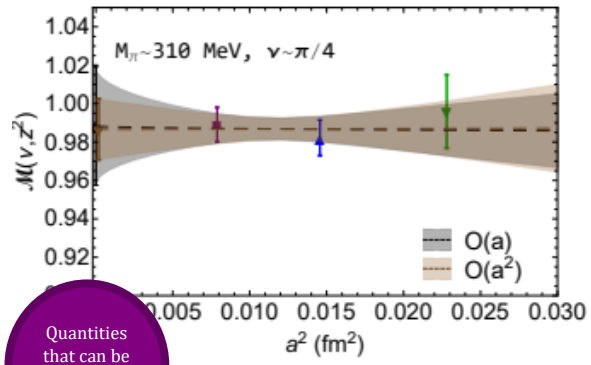
Gluon PDF in Nucleon

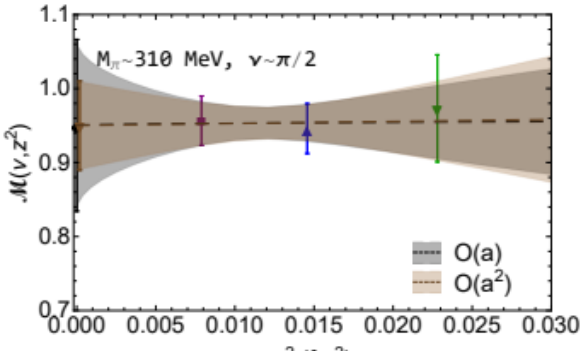
§ Continuum Gluon PDF w/ pseudo-PDF

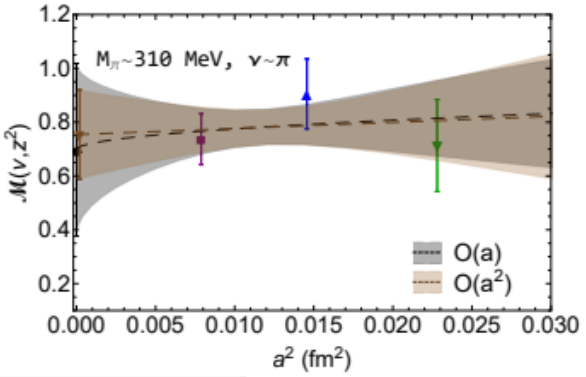
- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics

Z. Fan et al (MSULat), 2210.09985

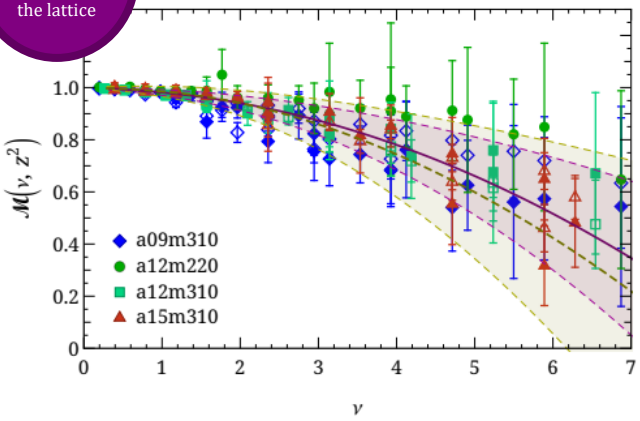


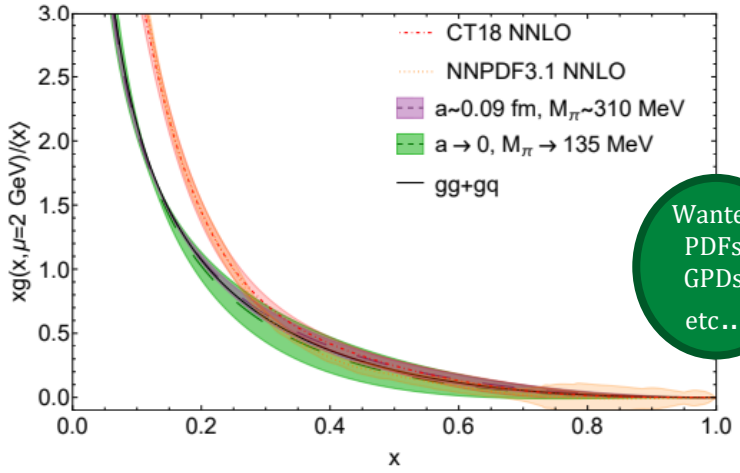






Quantities that can be calculated on the lattice





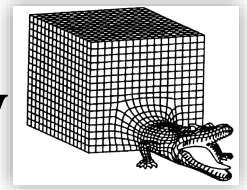
Wanted PDFs, GPDs, etc…

G: Bill Good

Pion Tomography

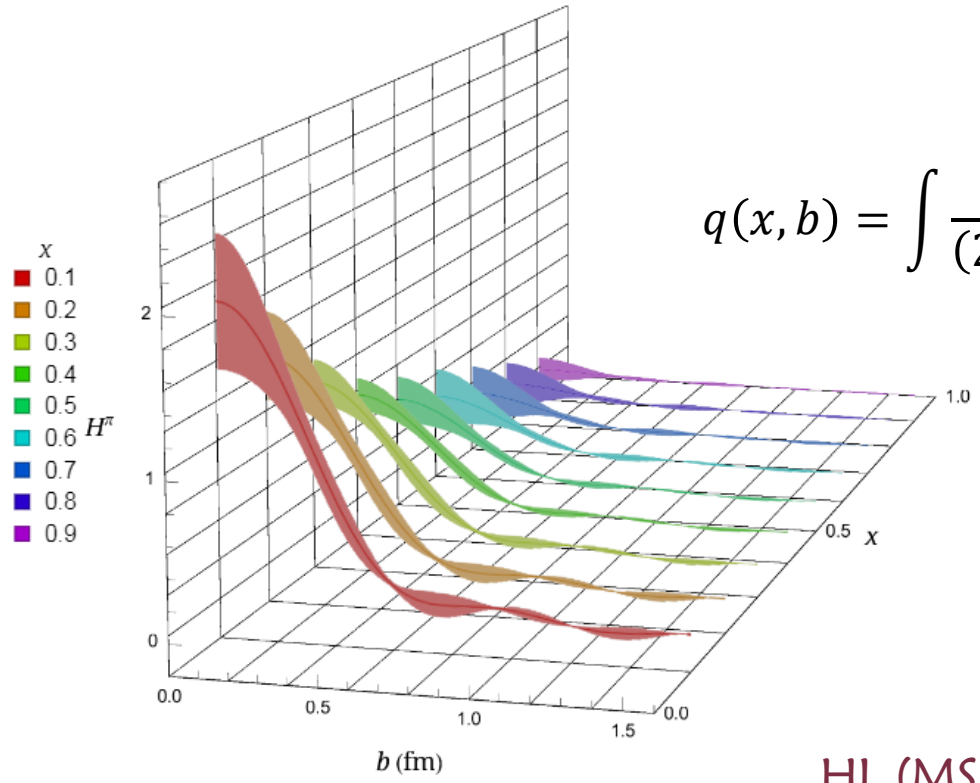
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV

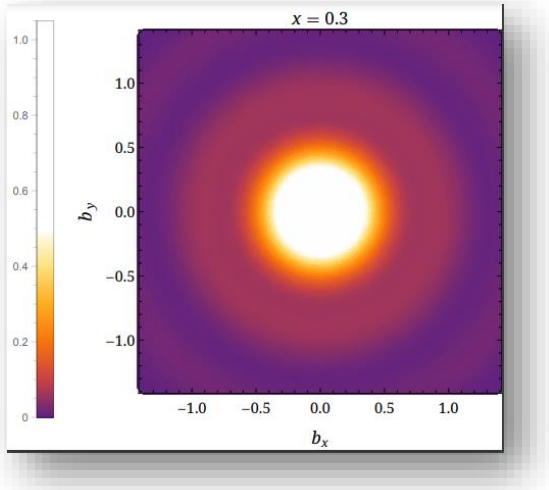


finite-volume, discretization,

- $\xi = 0$ valence-quark Pion GPD results



$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$

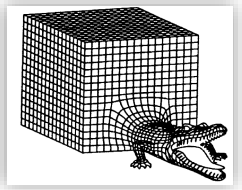


HL (MSULat), Phys. Lett. B 846 (2023) 138181

Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results



finite-volume, discretization,

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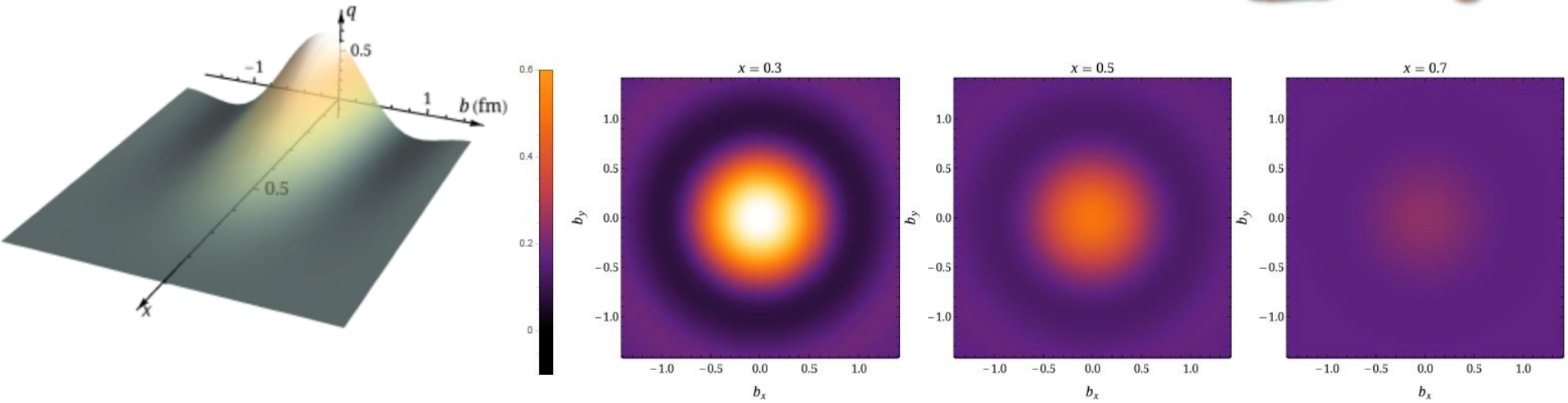
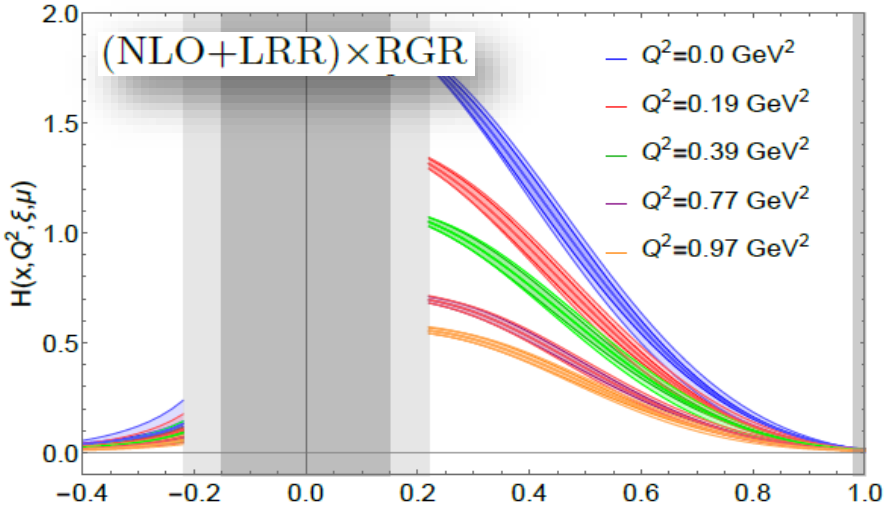
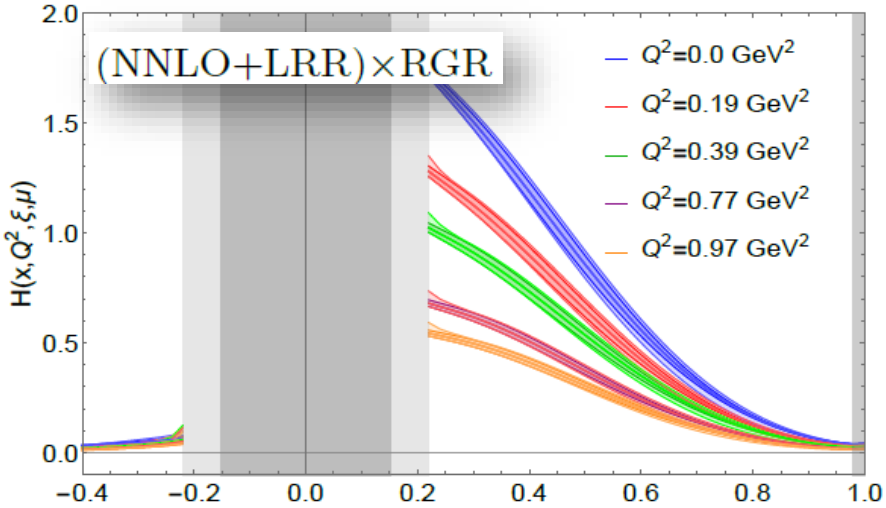


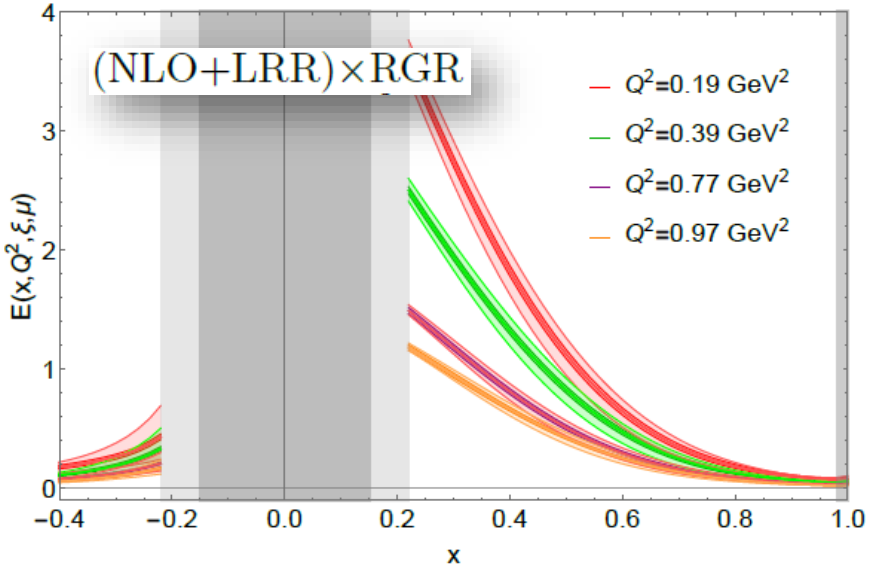
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

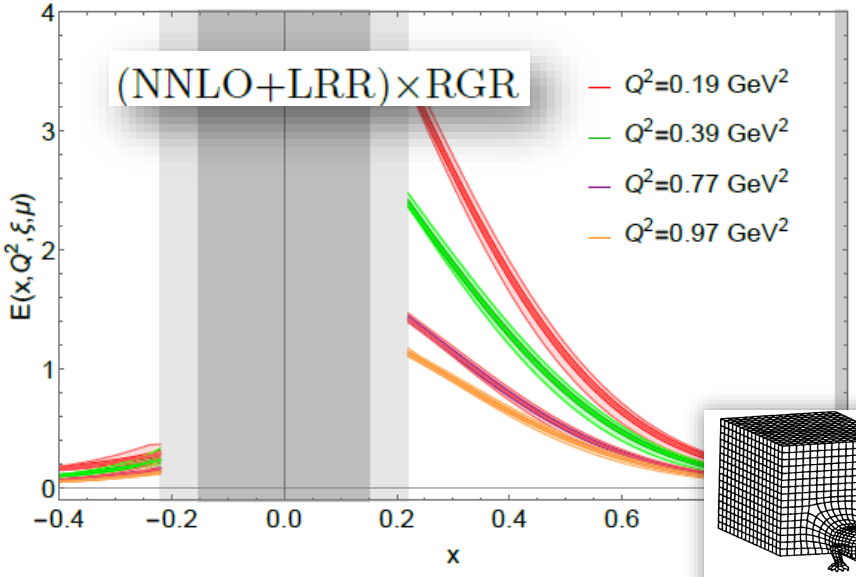
Also see work done by ANL/BNL/ETMC, 2209.05373 ,2310.13114

ξ=0 GPDs









J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Impact of Lattice-QCD PDFs on Global Fits

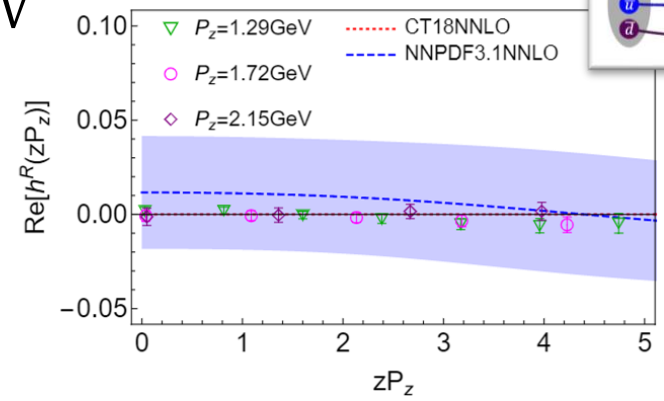
First Lattice Strange PDF

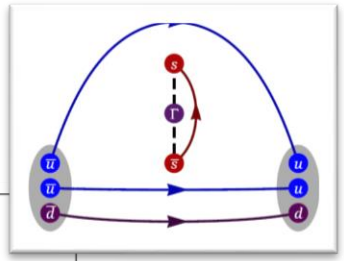
§ Results by MSULat/quasi-PDF method

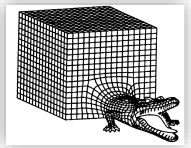
- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV

R. Zhang et al (MSULat), 2005.01124

$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x) - \bar{s}(x))\cos(xzP_z)$$







Quantities that can be calculated on the lattice

Lattice Strangeness Asymmetry Impact

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- Extrapolated to $M_\pi \approx 140$ MeV

R. Zhang et al (MSULat), 2005.01124

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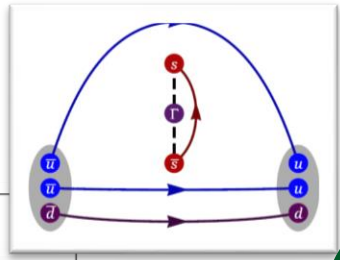
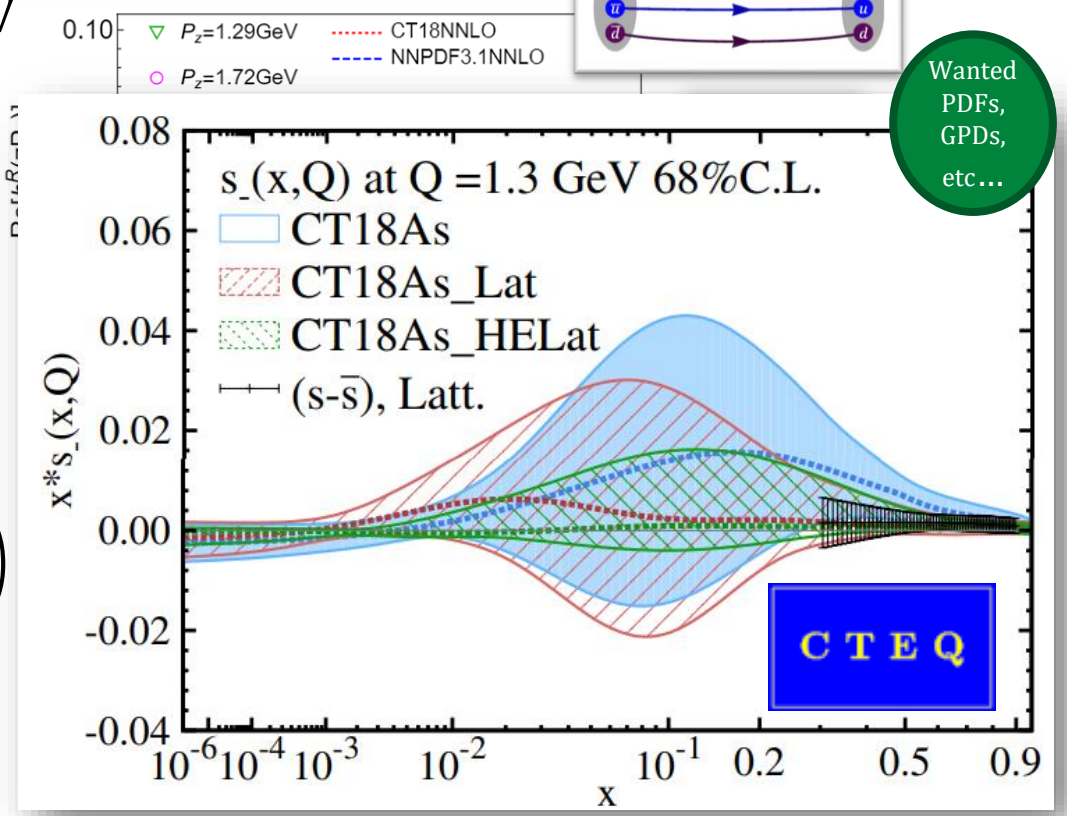




Wanted PDFs, GPDs, etc…

§ From quasi-PDF to PDF

$$\tilde{f}_q(x,P_z) = \int_{-1}^{1} \frac{dy}{|y|} f_q(y) C_{q/q}(x,y,P_z,\mu) + O\left(\frac{\Lambda^2_{\mathrm{QCD}}}{x^2P_z^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(1-x)^2P_z^2}\right)$$

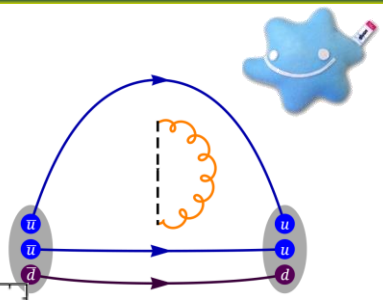
T. Hou, HL, M. Yan, C. Yuan, 2211.11064

§ The strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

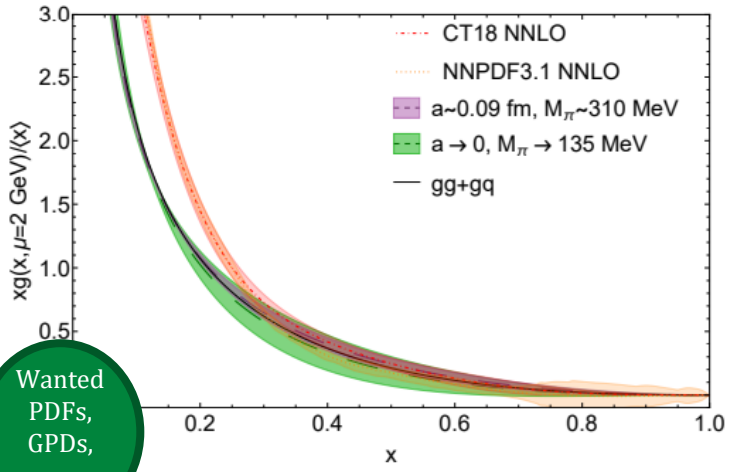
Lattice Gluon PDF Impact

§ First continuum Gluon PDF w/ pseudo-PDF

- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics



2210.09985, W. Good et al (MSULat)



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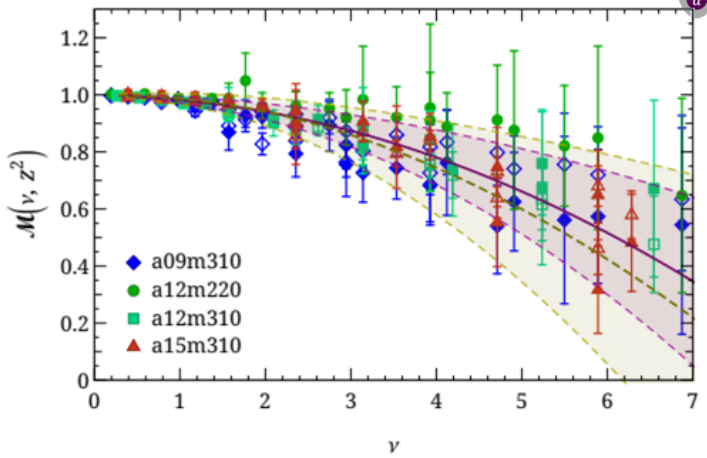
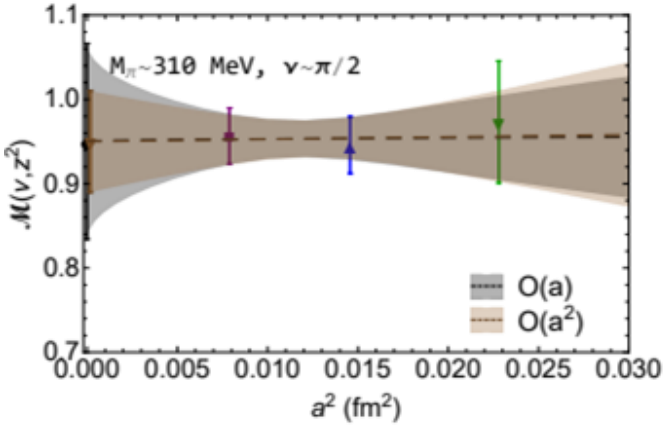


Quantities that can be calculated on the lattice



§ Can use $a \approx 0.09$ fm results as

- Best estimate of gluon PDF
- Aim for future precision

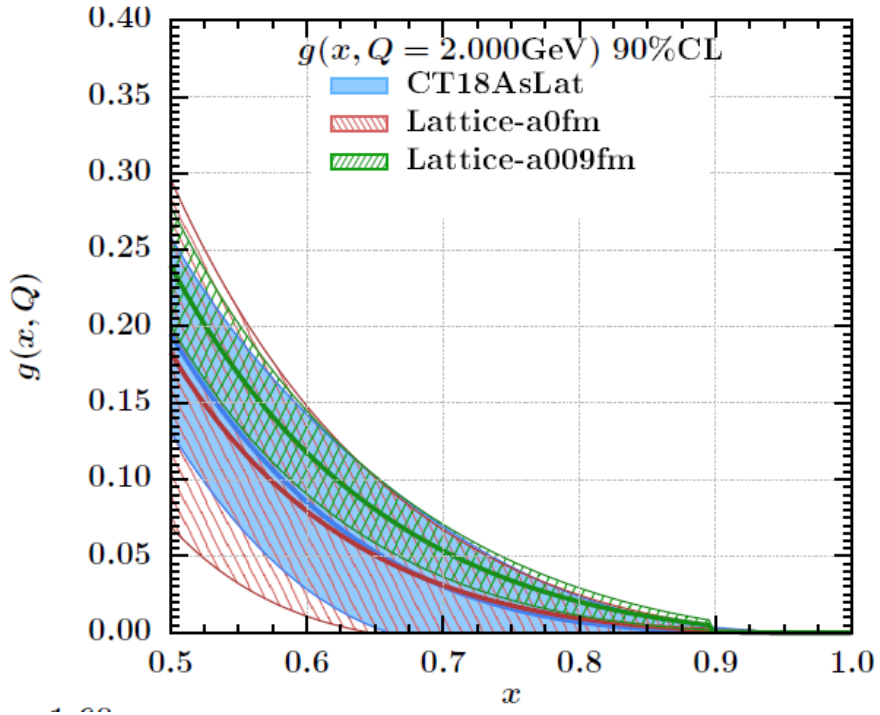
Lattice Gluon PDF Impact

§ Preliminary study with CTEQ-TEA analysis

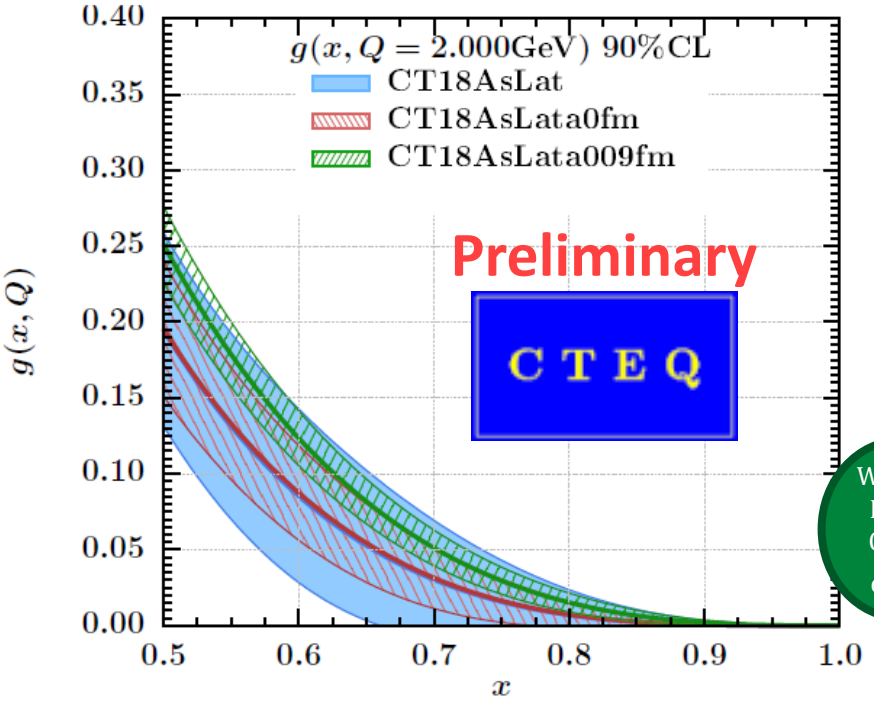
- Take lattice inputs in the region where no strong experimental data constraints, x ∈ [0.4,0.7]
- Using e-pump for re-weighting

Plots by Alim Ablat (Xinjiang U.)

Before



After

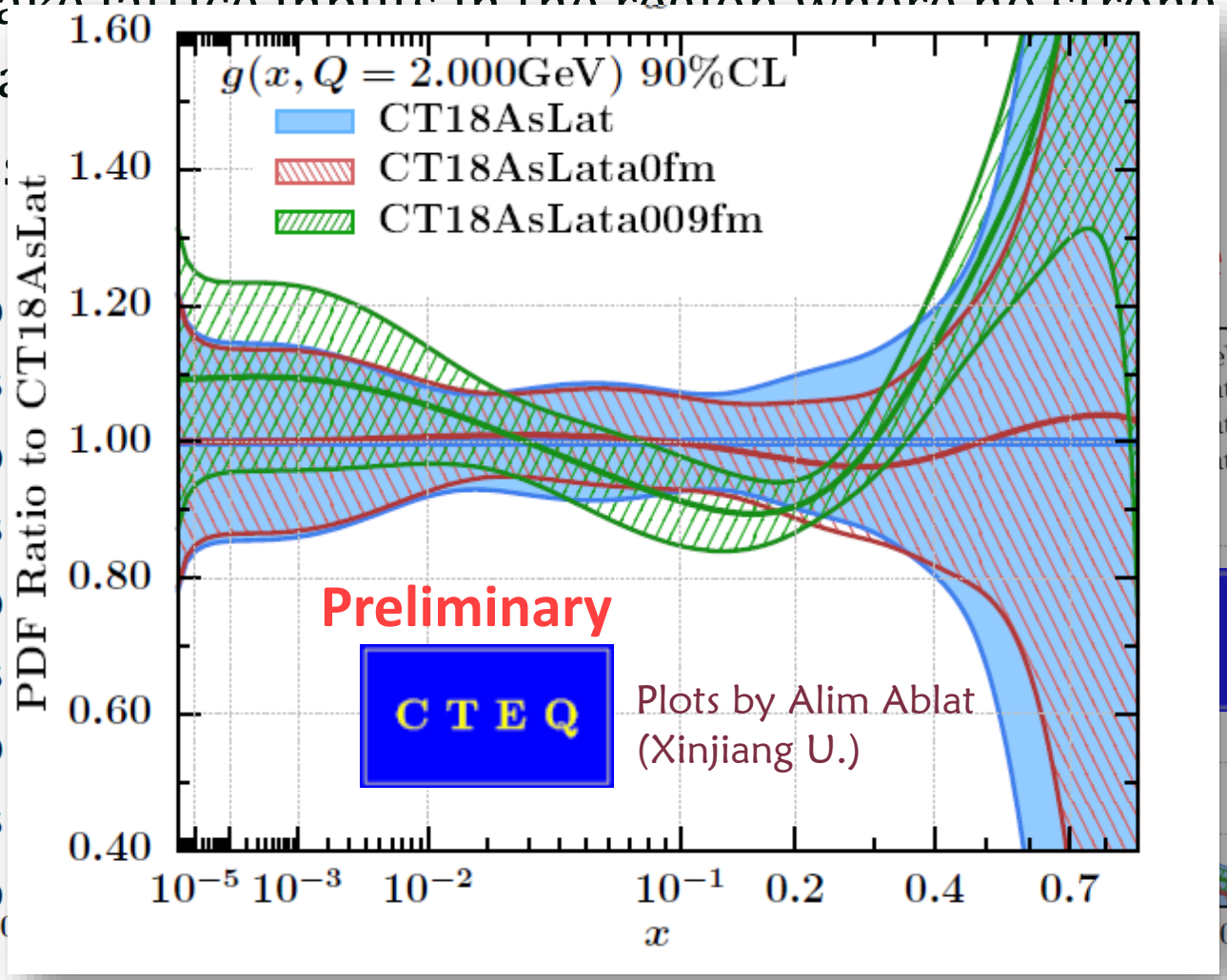


Wanted PDFs, GPDs, etc…

Lattice Gluon PDF Impact

§ Preliminary study with CTEQ-TEA analysis

- Take lattice inputs in the region where no strong experimental da
- Us



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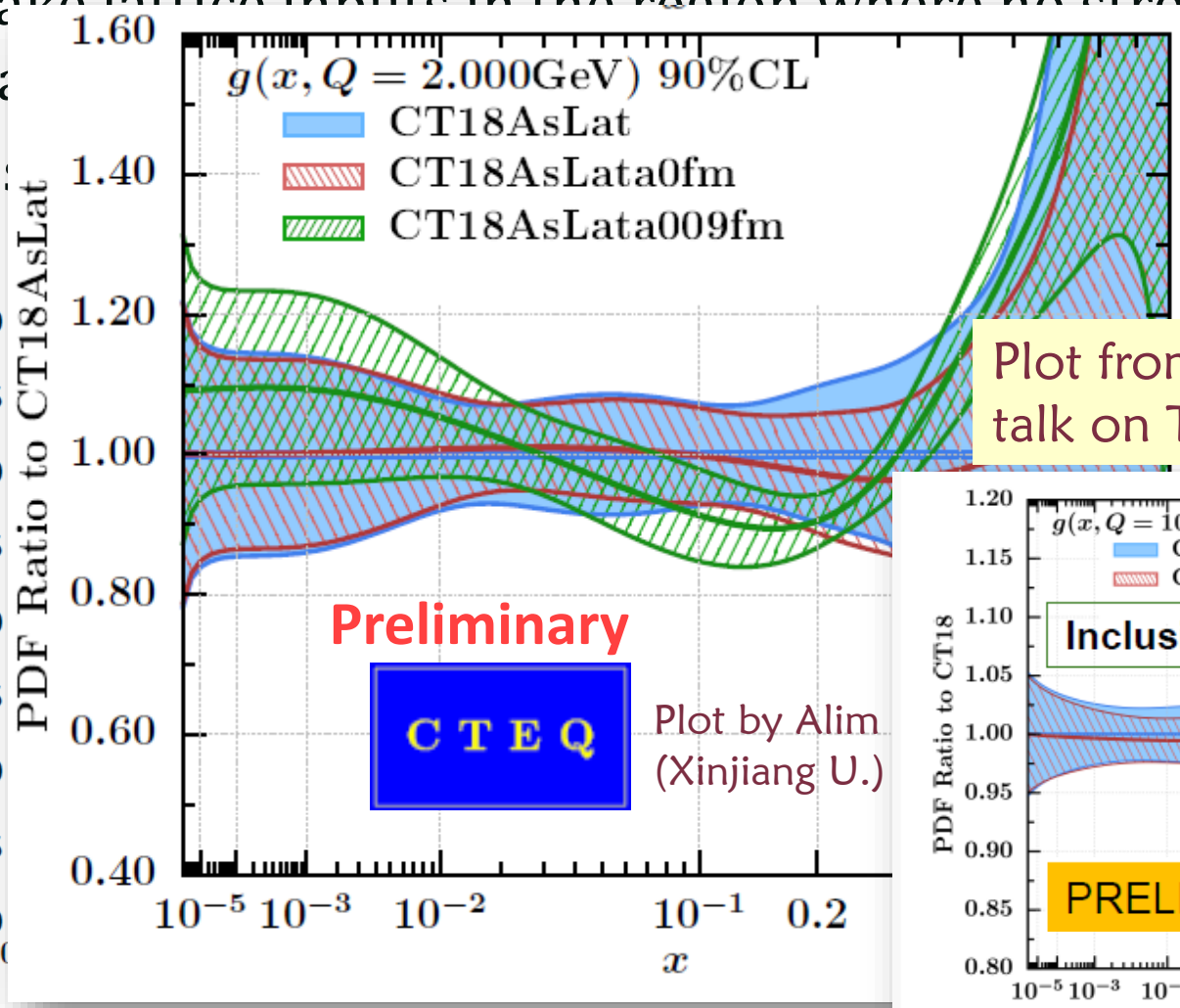
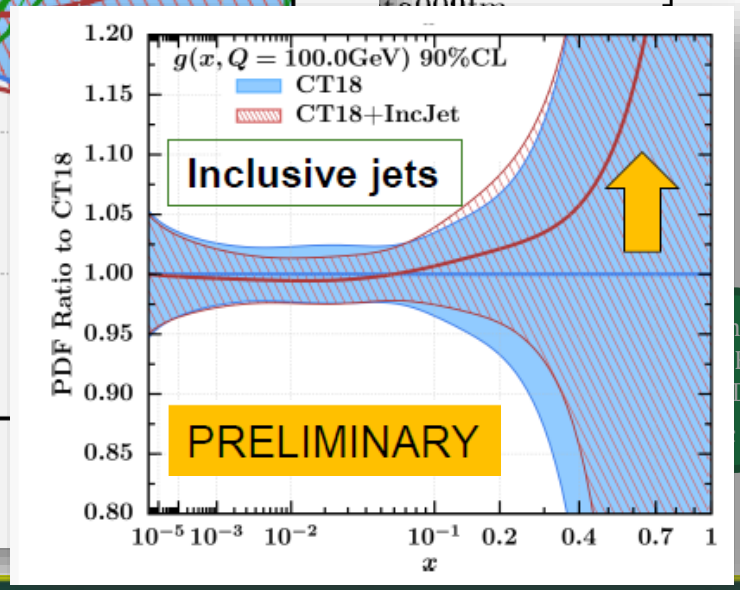


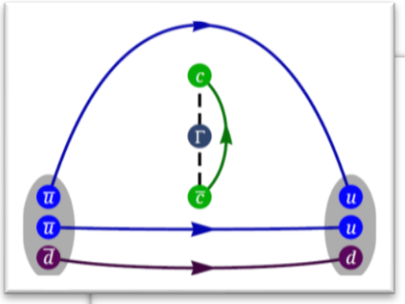
Plot from P. Nadolsky's talk on Tuesday



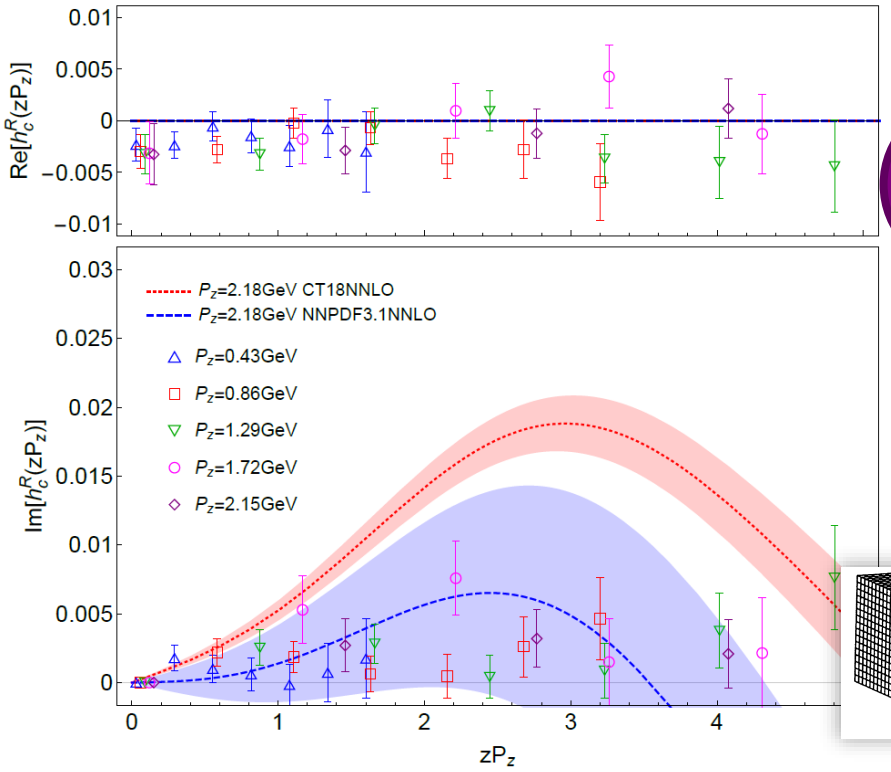
First Lattice Charm PDF

- § Large uncertainties in global PDFs
- § Results by MSULat/quasi-PDF method
  - Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

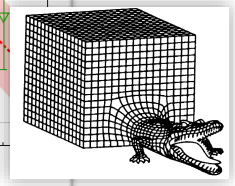
2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



Quantities that can be calculated on the lattice



First Lattice Charm PDF

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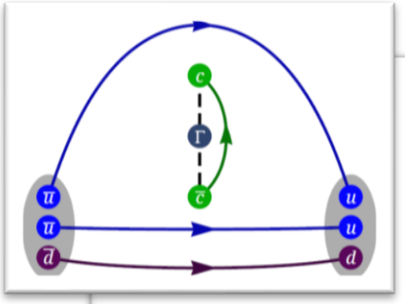

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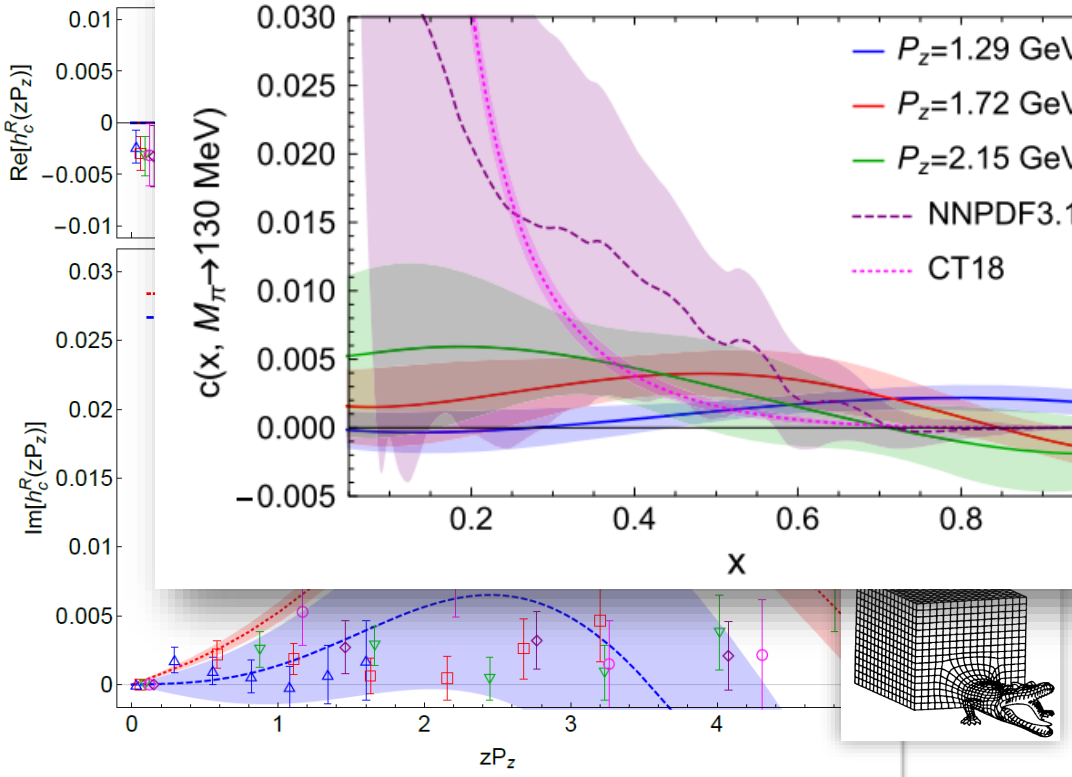


Wanted PDFs, GPDs, etc…

Lattice Progress & Challenges

§ Beyond the standard twist-2 collinear PDFs

- Generalized parton distributions (GPDs) for the pion and unpolarized/polarized nucleon
- Transverse-momentum– dependent distributions (TMDs)
  - Collins-Soper kernel, soft function and wavefunctions
- Twist-3 PDFs and GPDs

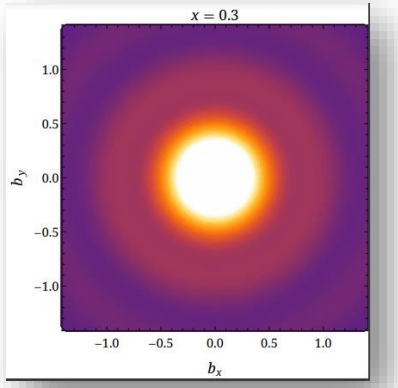
For more details and references, refer to 2202.07193

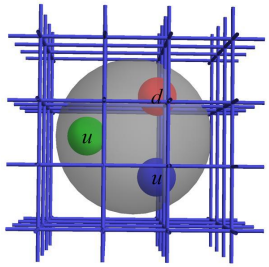
§ Challenges ahead for precision PDFs

- Large momentum is essential
  - With sufficient statistics nucleons may reach 5 GeV
- Methods for signal-to-noise improvement
  - Gluonic observables, new ideas for large momentum
- Access small-x physics; some methods have inverse problem in PDF extraction, more computational resources, etc.

Summary and Outlook

- § Exciting era using LQCD to study *x*-dependent nucleon PDFs
- § Overcoming longstanding limitations
  - Bjorken-*x* dependence of parton distributions now widely studied
  - More study of systematics planned for the near future
- § Lattice strange and gluon PDFs can have impacts
  - Treat lattice matrix elements as expt inputs in the future
- § Precision and progress are limited by resources



Theory Input → **Global Analysis of PDFs/GPDs** ← **Exp't Input** + 

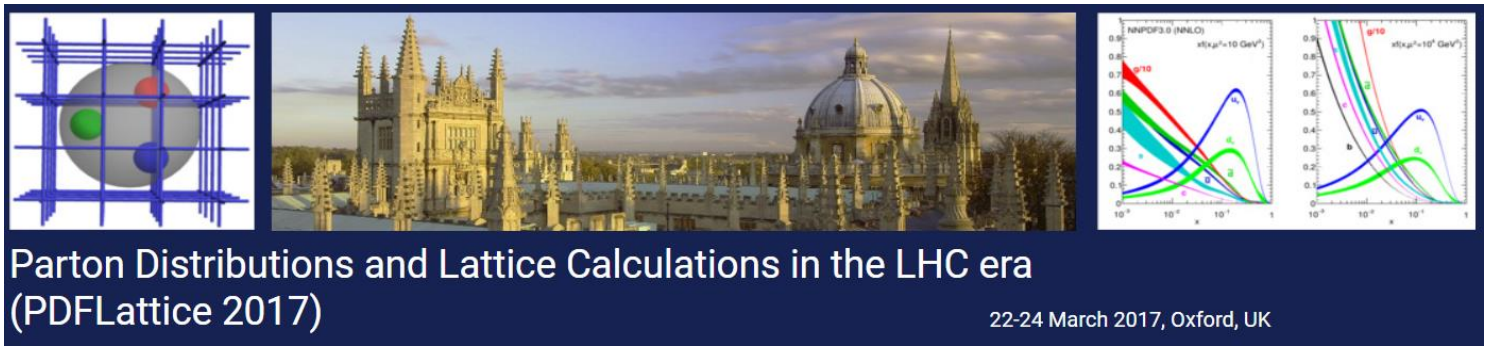
EXCLAIM

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
This work is partially sponsored by grants NSF PHY 1653405 & 1653405, DOE DE-SC0024053 & RCSA Cottrell Scholar Award

PDFLattice24 Workshop

§ November 18-20, 2024@Jefferson Lab, Newport News, VA, USA

- Joint community workshop between global-fit and lattice-QCD practitioners
- Theme: uncertainty quantification on nonperturbative correlator functions in phenomenology and lattice calculations



Students Wanted

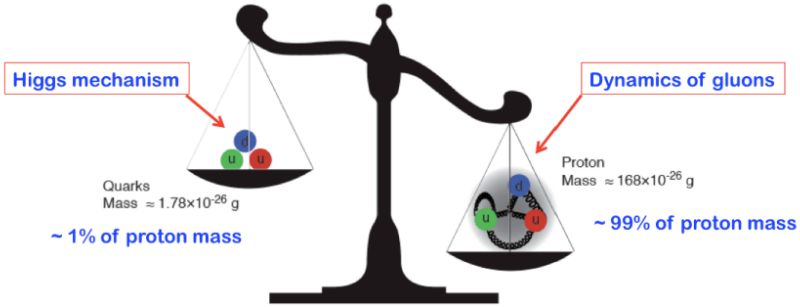
LGT4HEP website: https://lgt4hep.github.io/

Backup Slides

Backup Slides
Pion and Kaon PDFs

Motivation

- Meson structure is crucial to understand the mechanism of emergent hadron mass (EHM)
  - Help decode QCD origin of mass
- Experimentally, meson structure is harder to study
  - LQCD can provide predictions and better precision inputs
  - Quark and gluon parton distribution functions(PDFs), for example



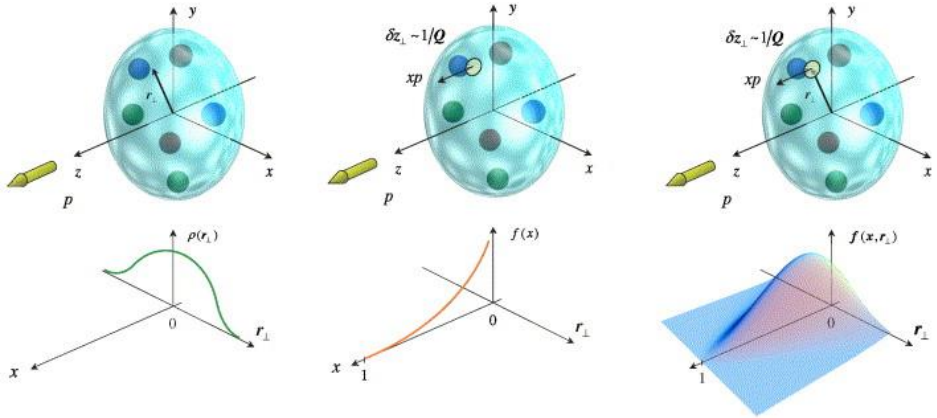


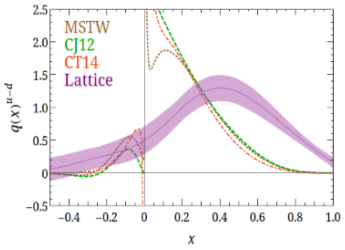
Image from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)

- Generalized parton distributions (GPDs) encode information about the spatial structure & the partonic distribution of spin and orbital angular momenta
  - US EIC, EIcC, LHeC, …

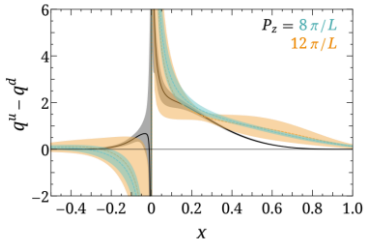
Lattice Parton Calculations

§ Physics quantity milestones

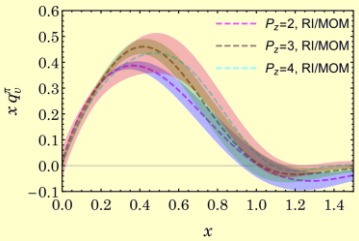
First unpol. lattice PDF



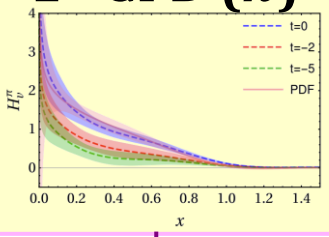
First PDFs at $M_\pi^{\rm phys}$



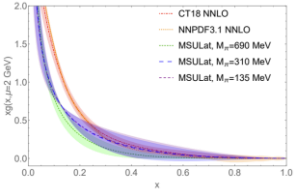
Pion v-PDF



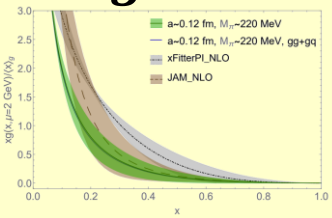
1st GPD (π)



N g-PDF

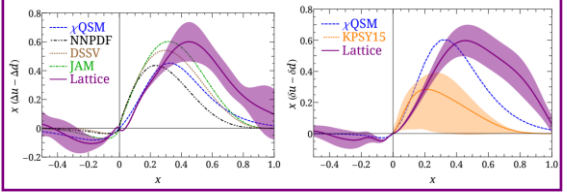


π g-PDF

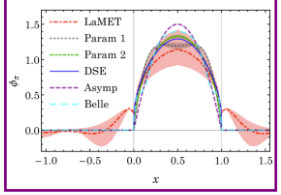


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023

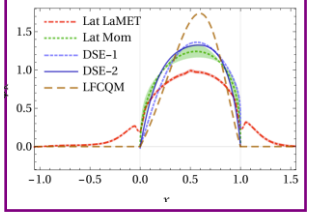
Pol. PDFs and mass corrections



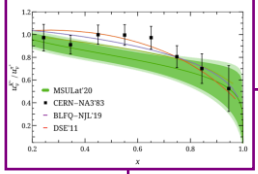
Pion DA



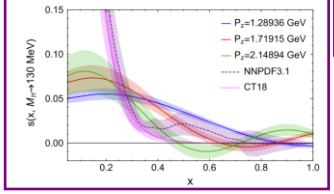
Kaon DA



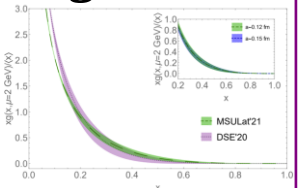
K PDF



s,c PDF



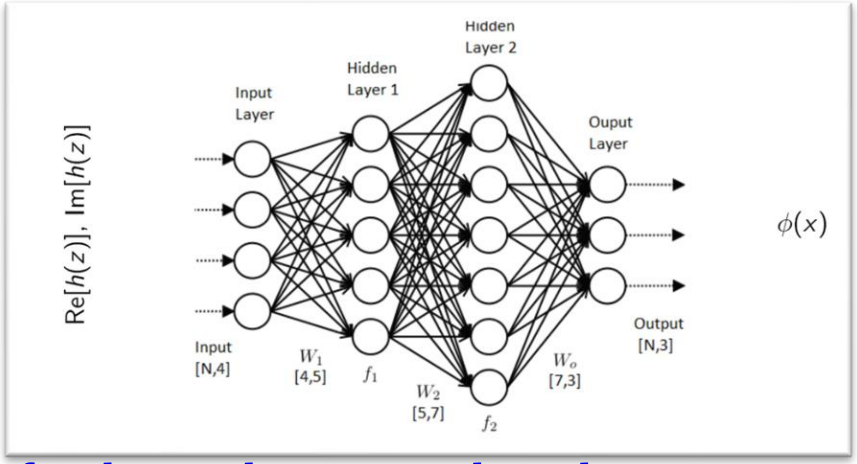
Kaon g-PDF



MSULat Pion/Kaon Structure

§ Meson distribution amplitude

- Pion Distribution Amplitude from Lattice, Phys. Rev. D 95 (2017) 9, 094514
- Kaon Distribution Amplitude from Lattice QCD and the Flavor SU(3) Symmetry, Nucl. Phys. B 939 (2019) 429-446
- Pion and kaon distribution amplitudes in the continuum limit, Phys. Rev. D 102 (2020) 9, 094519



- Precision control in lattice calculation of x-dependent pion distribution amplitude, Nucl. Phys. B 993 (2023) 116282

§ Miscellaneous

- Machine-learning prediction for quasiparton distribution function matrix elements, Phys. Rev. D 101 (2020) 3, 034516

MSULat Pion/Kaon Structure

§ Pion/kaon PDFs

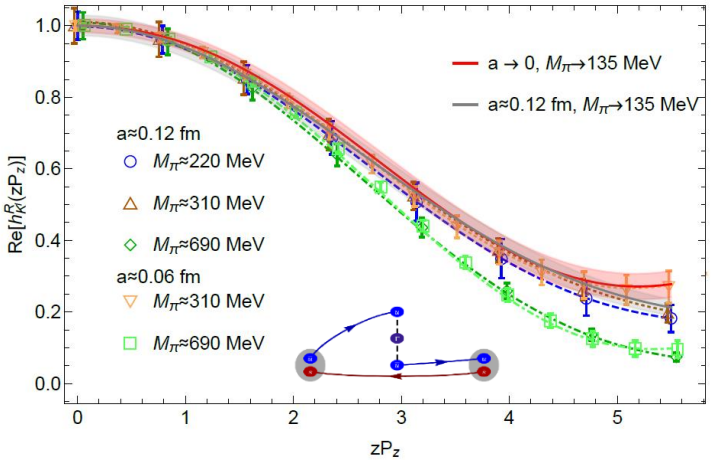
- First direct lattice-QCD calculation of the *x*-dependence of the pion parton distribution function, Phys. Rev. D 100 (2019) 3, 034505
- Valence-Quark Distribution of the Kaon and Pion from Lattice QCD, Phys. Rev. D 103 (2021) 1, 014516
- Gluon parton distribution of the pion from lattice QCD, Phys. Lett. B 823 (2021) 136778
- First glimpse into the kaon gluon parton distribution using lattice QCD, Phys. Rev. D 106 (2022) 9, 094510
- The Gluon Moment and Parton Distribution Function of the Pion from N_f=2+1+1 Lattice QCD, 2310.12034 [hep-lat]
- Pion valence quark distribution at physical pion mass of N_f=2+1+1 LQCD

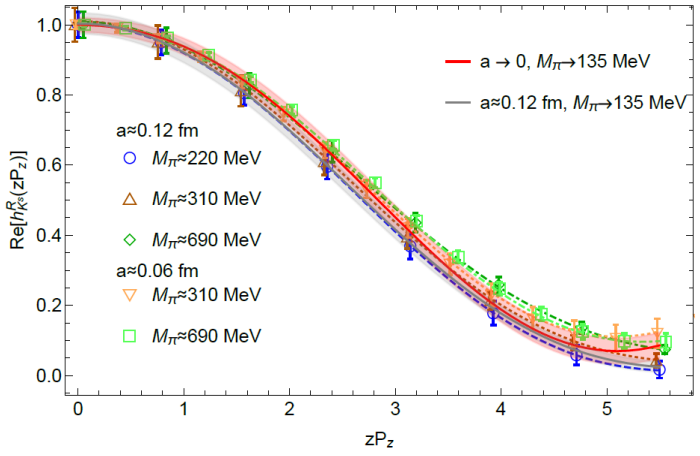
§ Pion GPD

- Pion generalized parton distribution from lattice QCD, Nucl. Phys. B 952 (2020) 114940
- Pion valence-quark generalized parton distribution at physical pion mass , Phys. Lett. B 846 (2023) 138181

Meson Valence-quark PDFs

§ Pion/kaon PDFs using quasi-PDF in the continuum limit

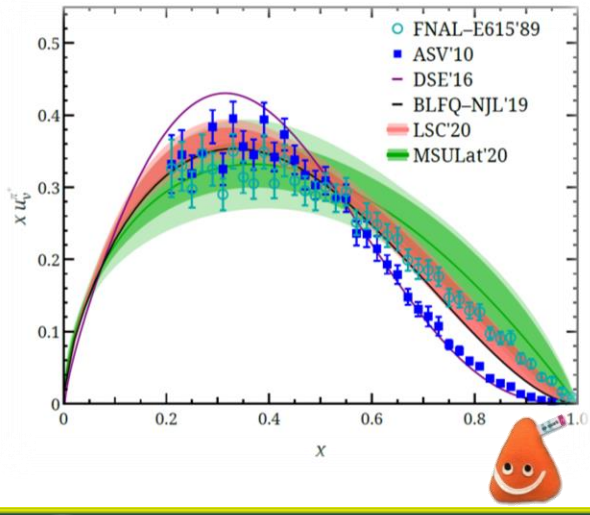


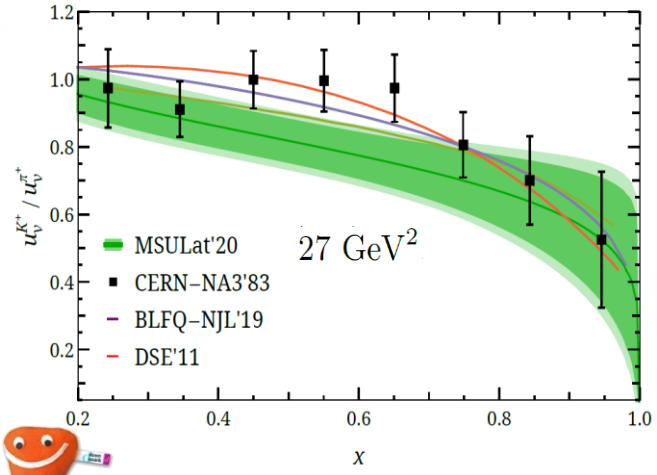


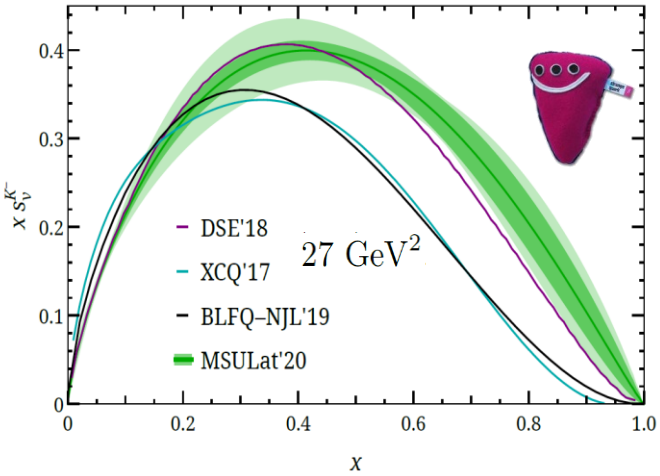
Quantities that can be calculated on the lattice

MSULat, 2003.14128

Wanted PDFs, GPDs, etc…



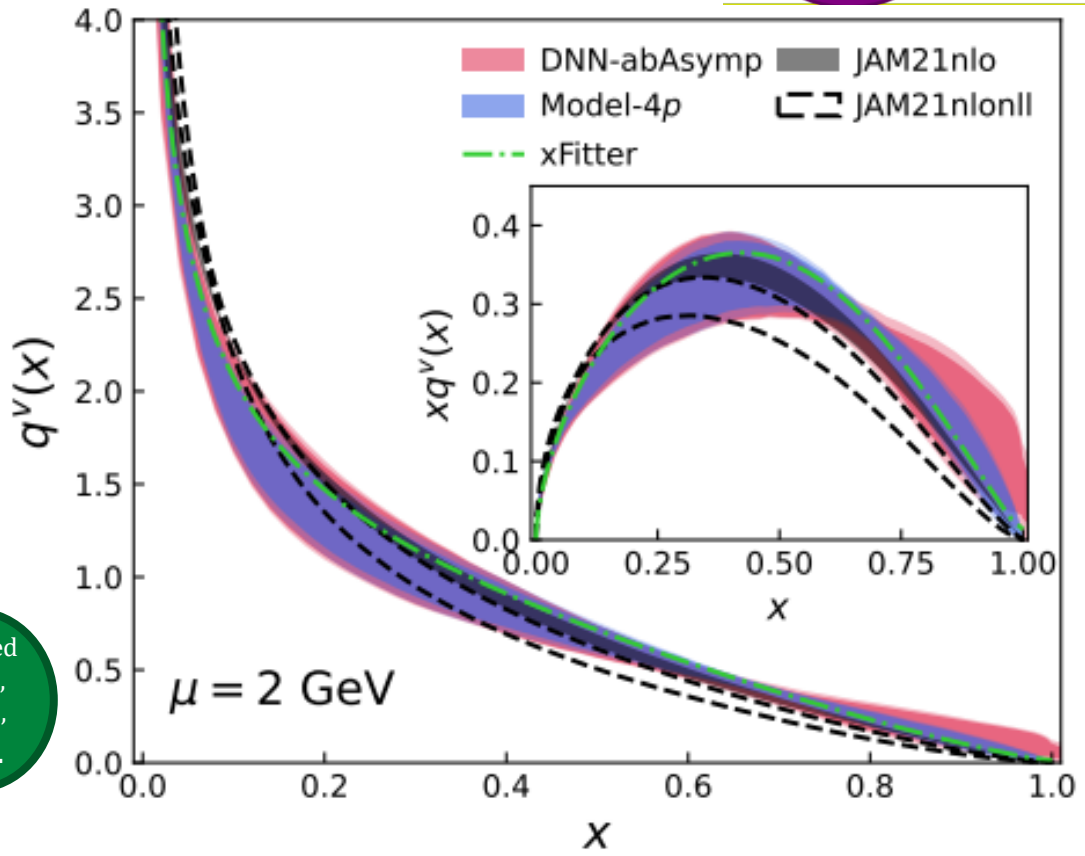




Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

- with NNLO matching



Wanted PDFs, GPDs, etc…

- N_f=2+1 clover/HISQ
  a~0.076 fm

ANL/BNL, Phys. Rev. D 106, 114510 (2022)

Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

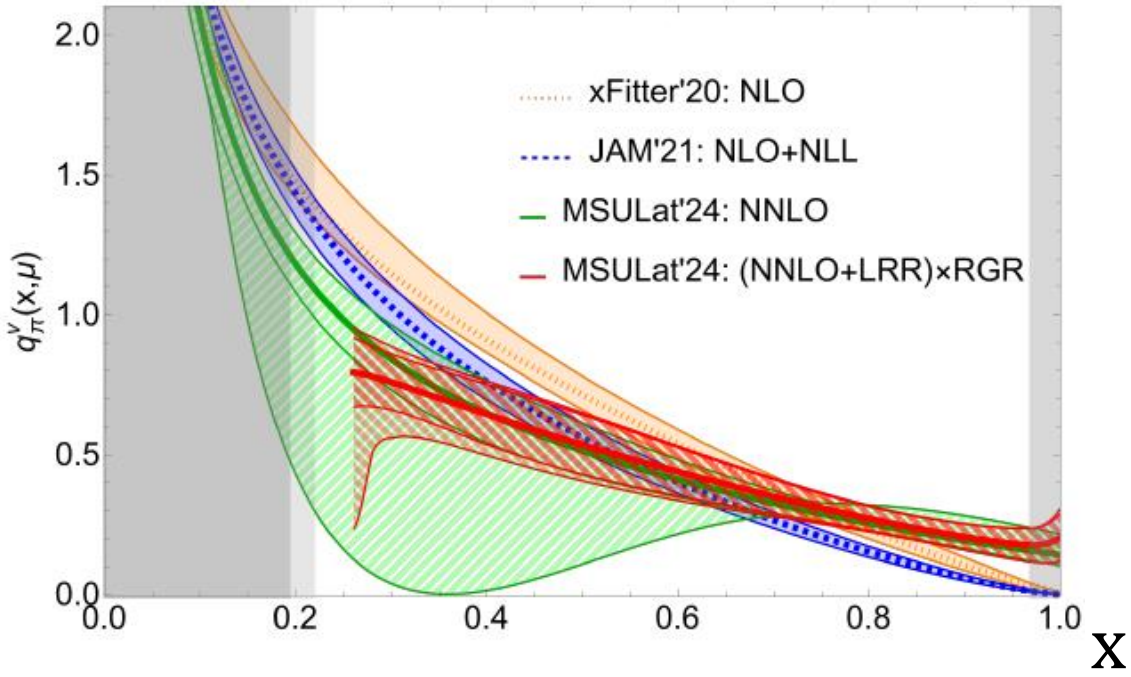
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

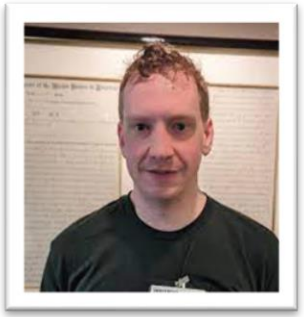
R. Zhang, et. al.
PLB 844, 138081 (2023)

- N_f=2+1+1 clover/HISQ, a~0.09 fm

J. Holligan, HL (MSULat), 10.1088/1361-6471/ad3162

Wanted PDFs, GPDs, etc…

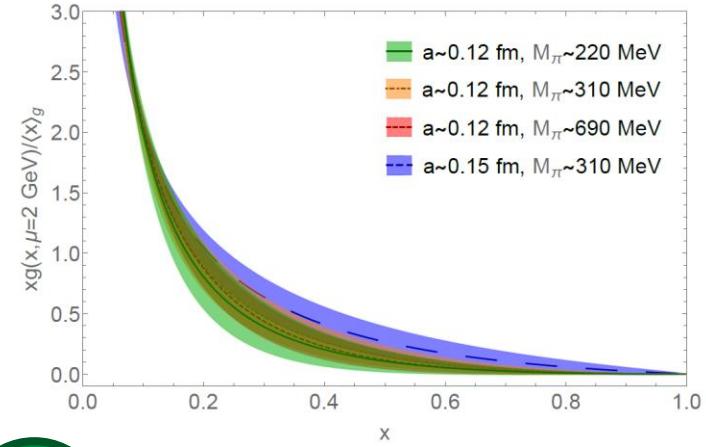


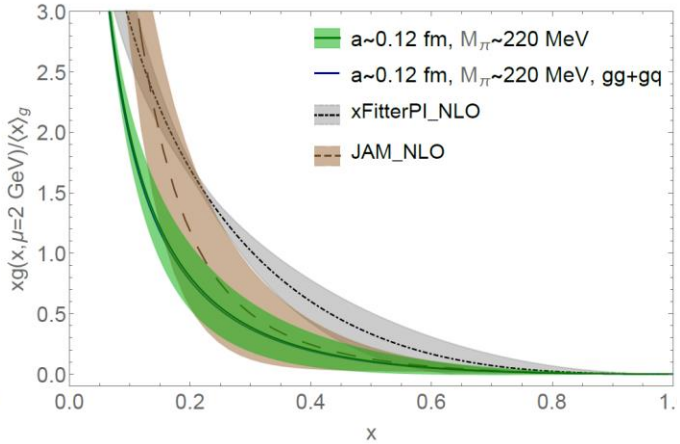


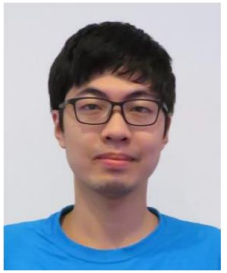
P: Jack Holligan

Meson Gluon PDFs

§ First pion and kaon gluon PDFs $g(x)/\langle x\rangle$ using pseudo-PDF



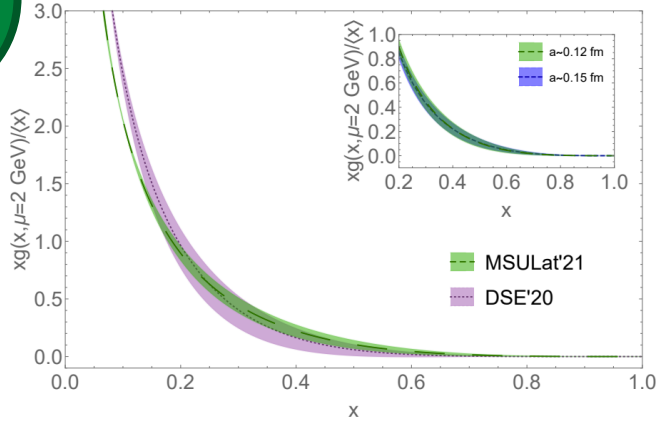




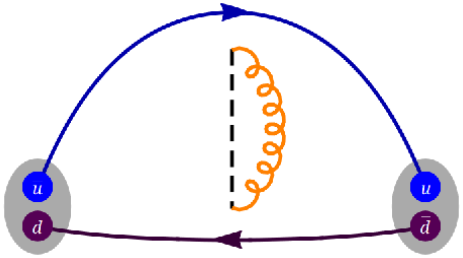
G: Zhouyou Fan

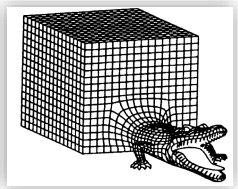
2104.06372, Fan et al. (MSULat); 2112.03124, Salas-Chavira et al. (MSULat)

Wanted PDFs, GPDs, etc…



G: Alejandro Salas-Chavira





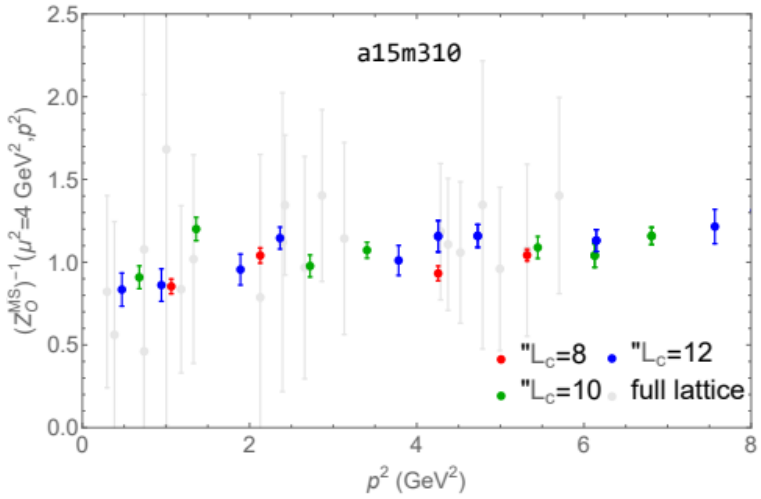
finite-volume, discretization, heavy quark mass, …

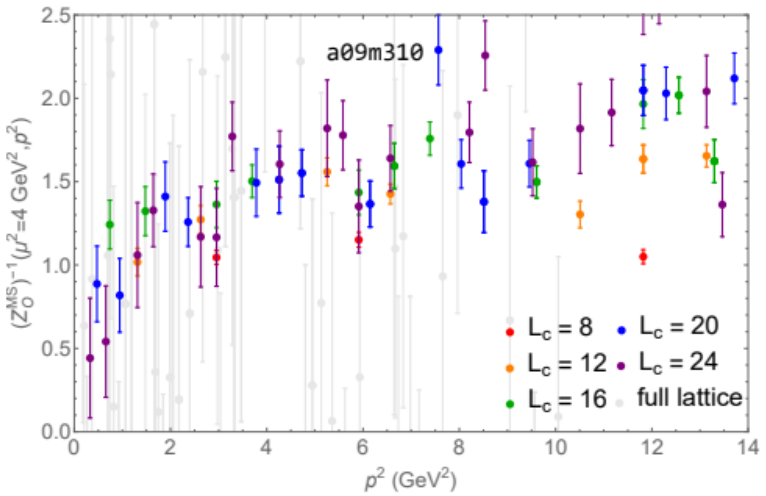
§ What does lattice QCD say about $g(x)$?

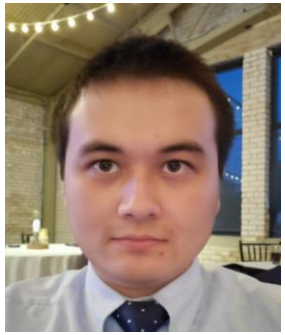
Pion Gluon PDF Update

§ Nonperturbatively renormalized $\langle x \rangle_{\{\pi,g\}}$ at the finer lattice spacing at lighter pion mass is nontrivial

- Using cluster-decomposition error reduction (CDER) to enhance the signal-to-noise ratio 1805.00531, Y. Yang et al. (χQCD)
- Lattice details: clover/HISQ, a ~{0,15, 0.12, 0.09} fm 2208.00980, Fan et al. (MSULat)

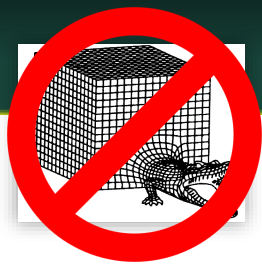




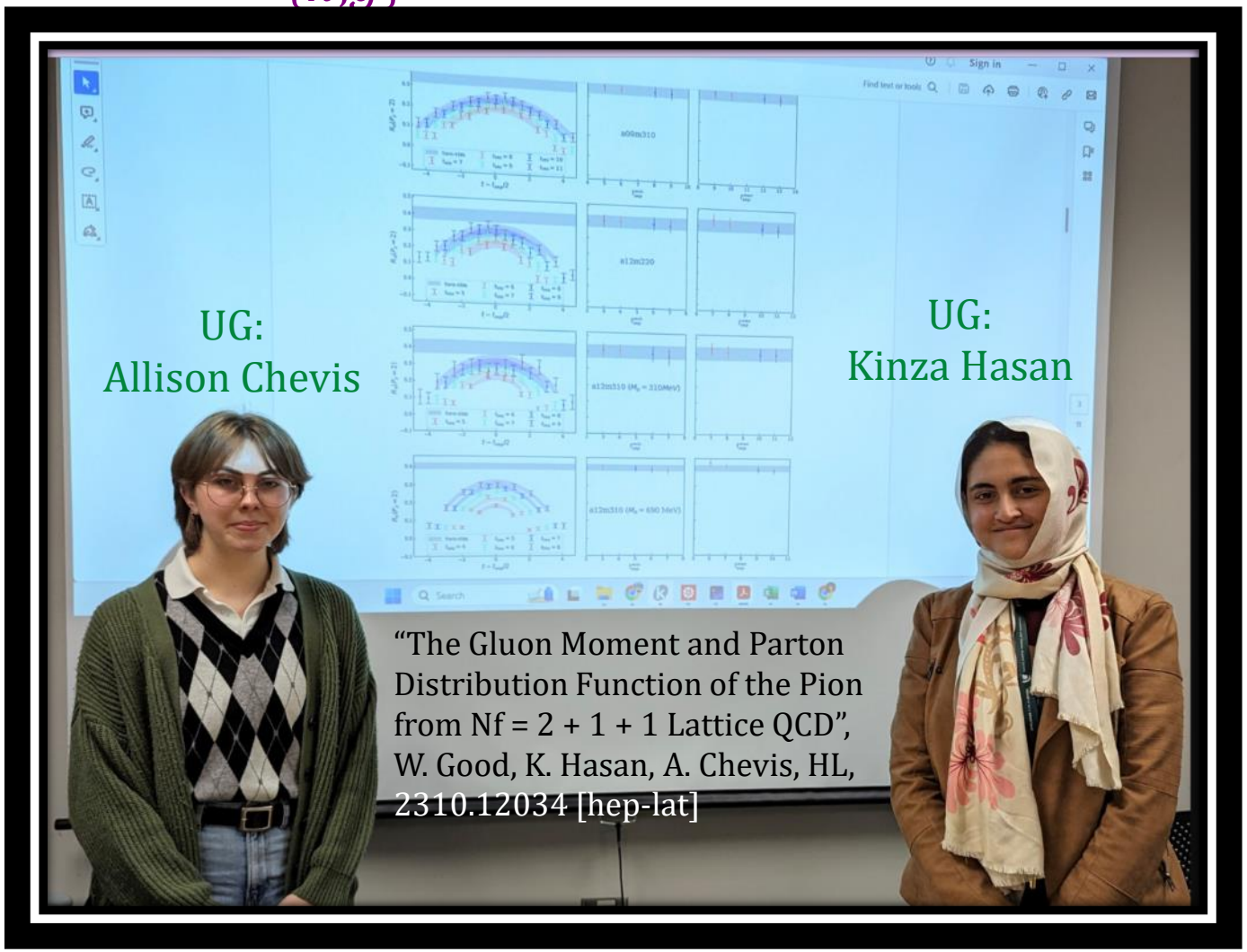


G: Matthew Zeilbeck

Pion Gluon PDF Update

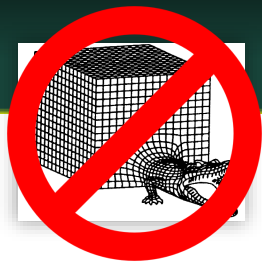


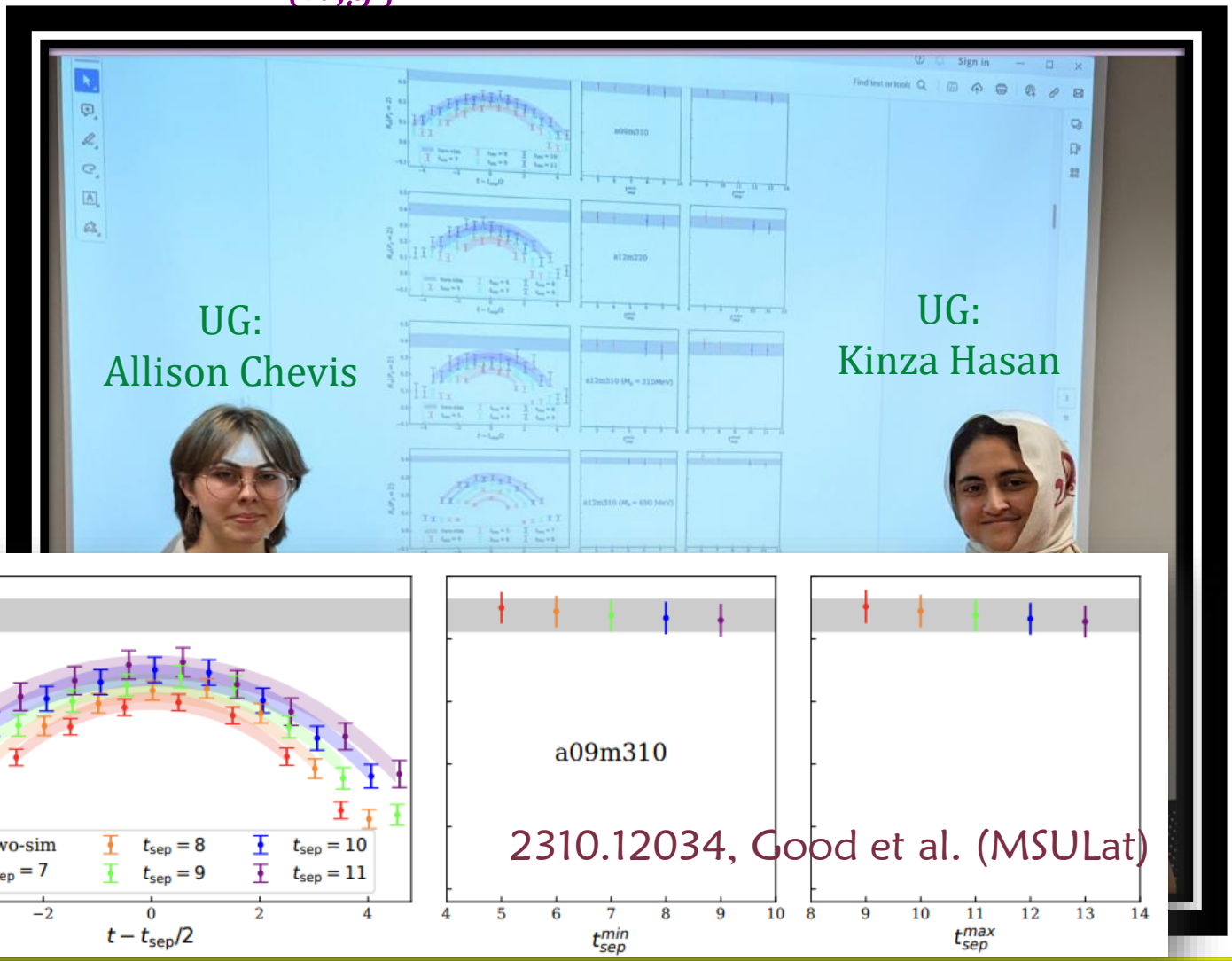
§ Study Bare $\langle x \rangle_{\{\pi,g\}}$



Pion Gluon PDF Update

§ Study Bare $\langle x \rangle_{\{\pi,g\}}$



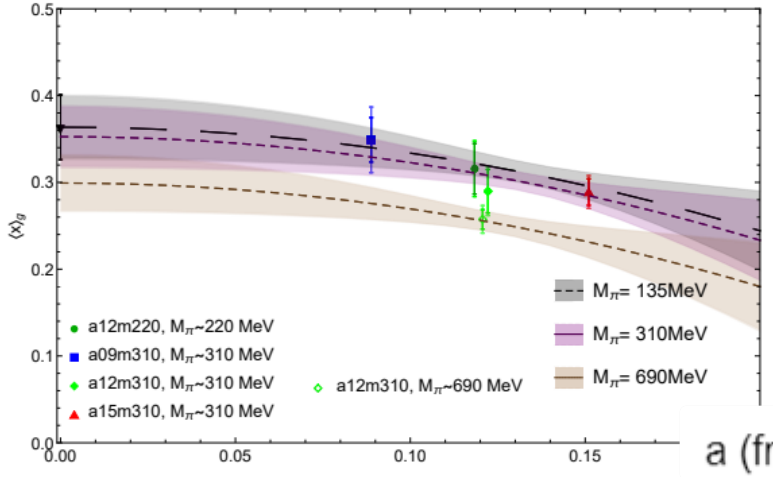


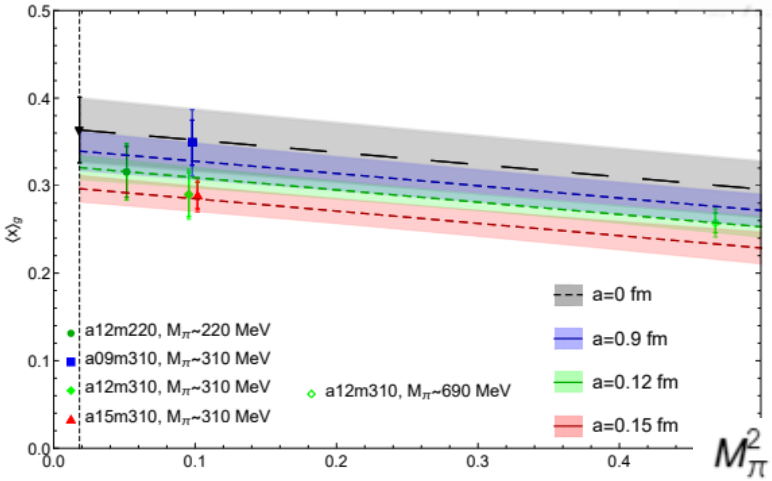
2310.12034, Good et al. (MSULat)

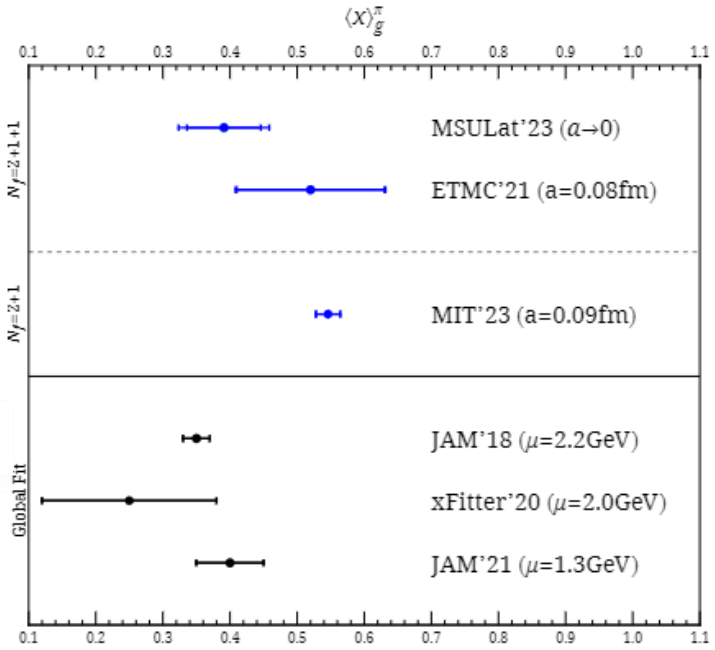
Pion Gluon PDF Update

§ Study discretization systematic in $\langle x \rangle_{\{\pi,g\}}$

- Lattice details: clover/HISO. HISO. a ~{0.15. 0.12. 0.09} fm







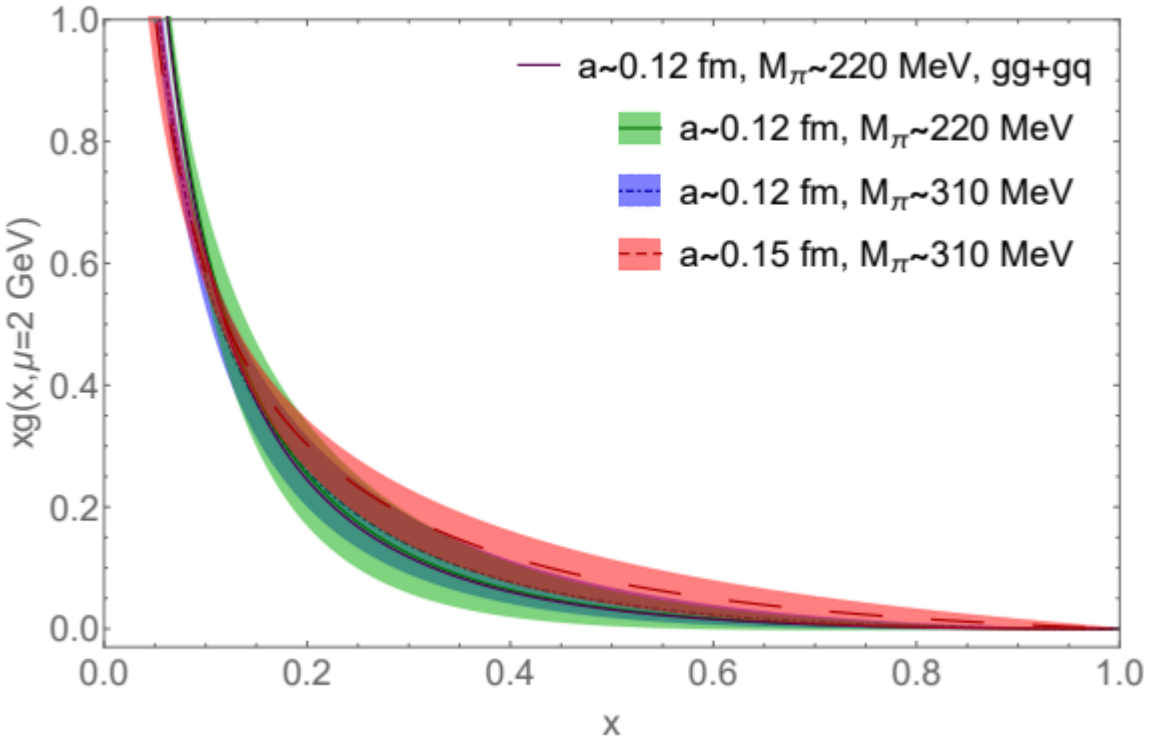
2310.12034, Good et al. (MSULat)

G: Bill Good

Pion Gluon PDF Update

§ Back to Pion gluon PDF $g(x)$

- Update previous calculated $g(x)/\langle x \rangle$ in 2021



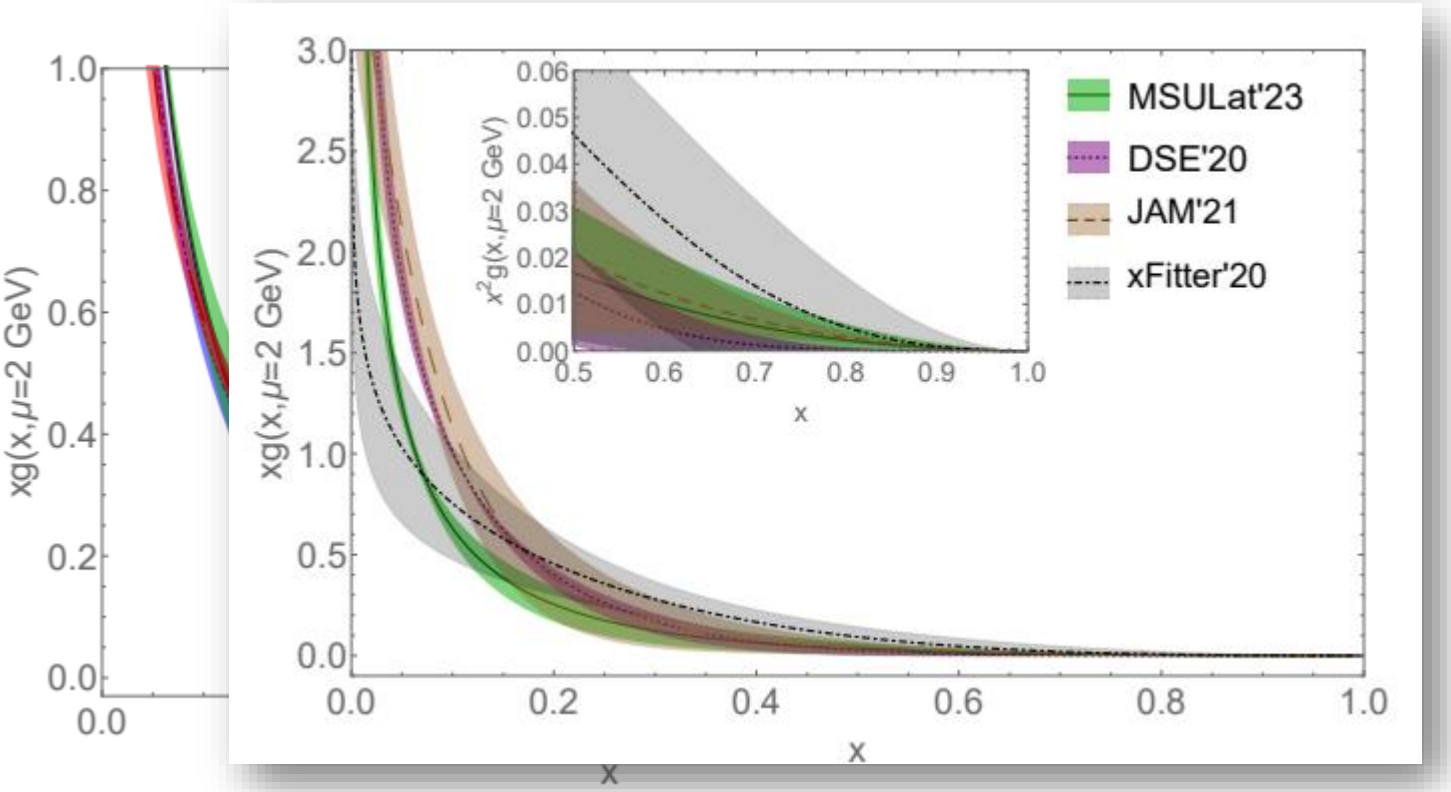
2310.12034, Good et al. (MSULat)

G: Bill Good

Pion Gluon PDF Update

§ Back to Pion gluon PDF $g(x)$

- Update previous calculated $g(x)/\langle x \rangle$ in 2021

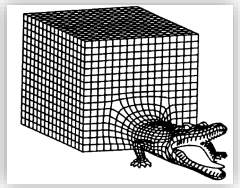


2310.12034, Good et al. (MSULat)

G: Bill Good

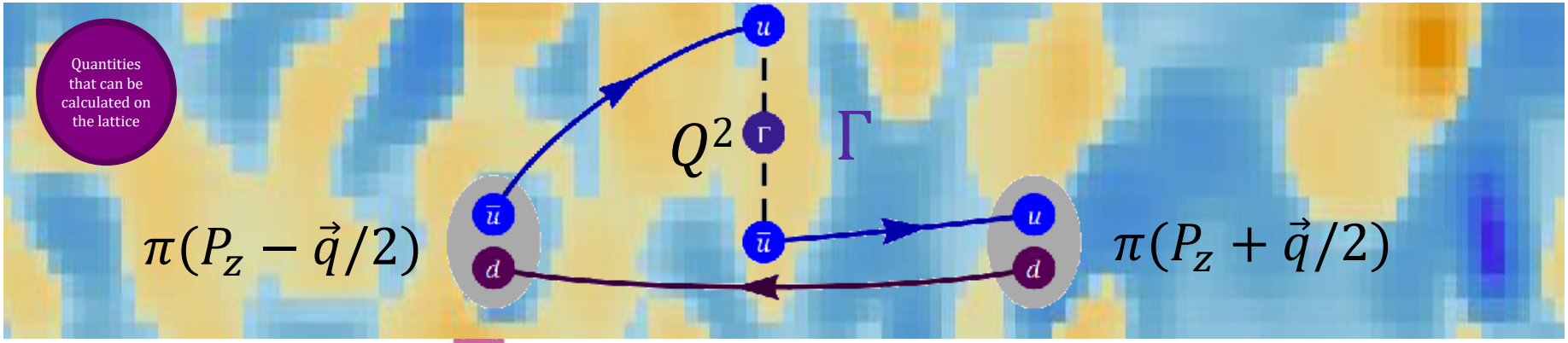
Generalized Parton Distributions

Single-ensemble result



finite-volume,
discretization,
heavy quark mass,
...

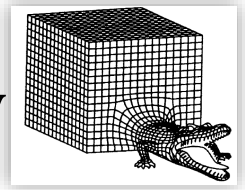
$$H_q^{\pi}(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right) \gamma^+ \Gamma \left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$



Valence-Quark Pion GPD

§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

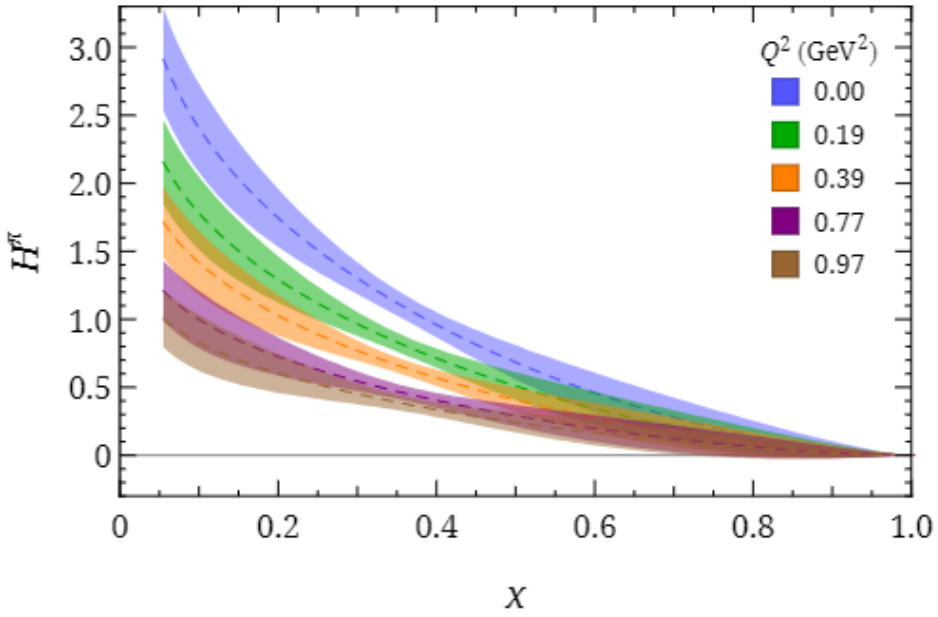
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV

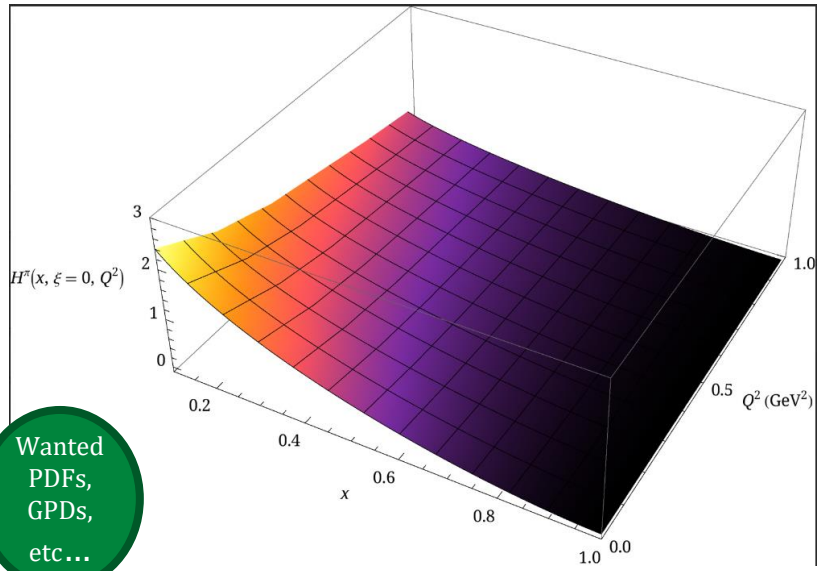


finite-volume, discretization,

- $\xi = 0$ valence-quark Pion GPD results

HL (MSULat), Phys. Lett. B 846 (2023) 138181



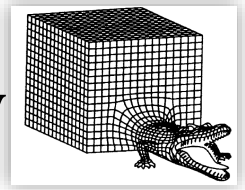


Wanted PDFs, GPDs, etc…

Valence-Quark Pion GPD

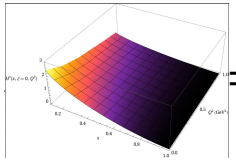
§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

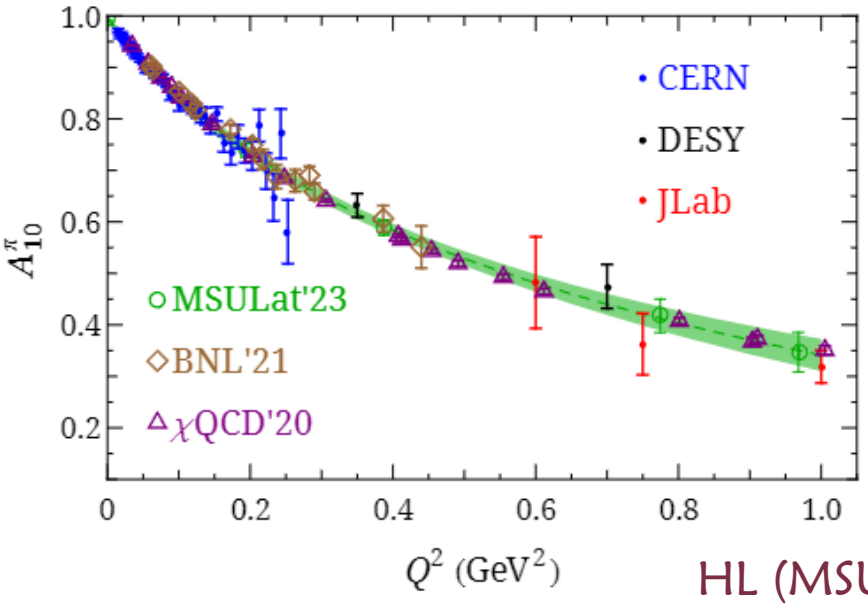
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV



finite-volume, discretization,

- $\xi = 0$ valence-quark Pion GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} \; \text{  } \; = A_{ni}^{\pi}(t)$$

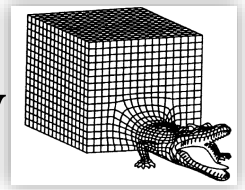


HL (MSULat), Phys. Lett. B 846 (2023) 138181

Pion Tomography

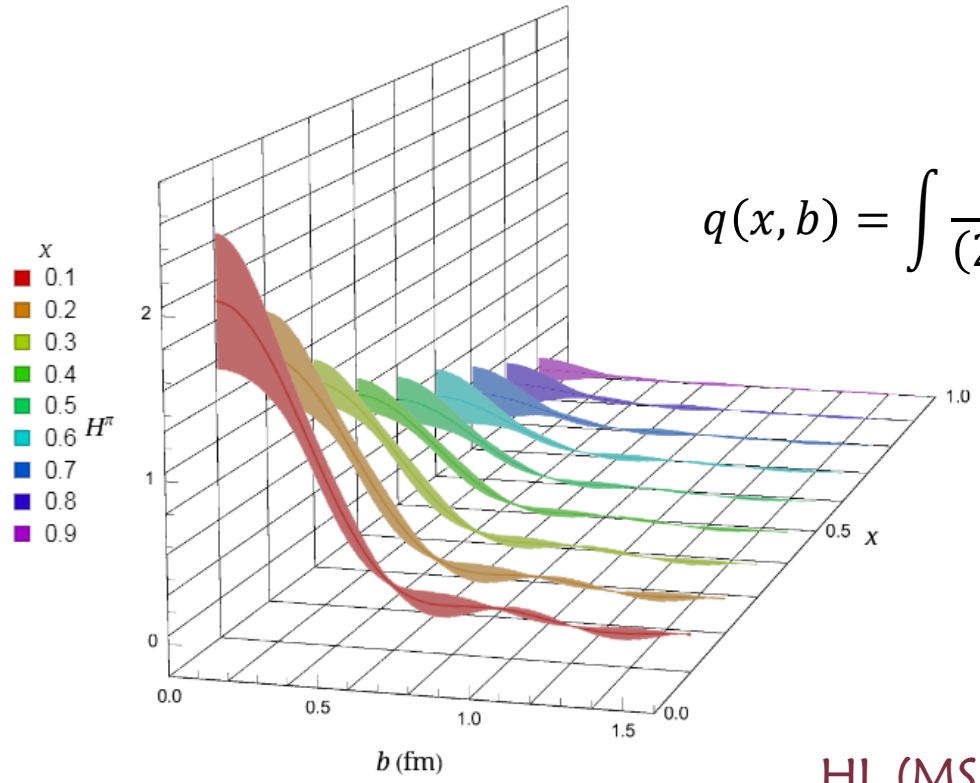
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV

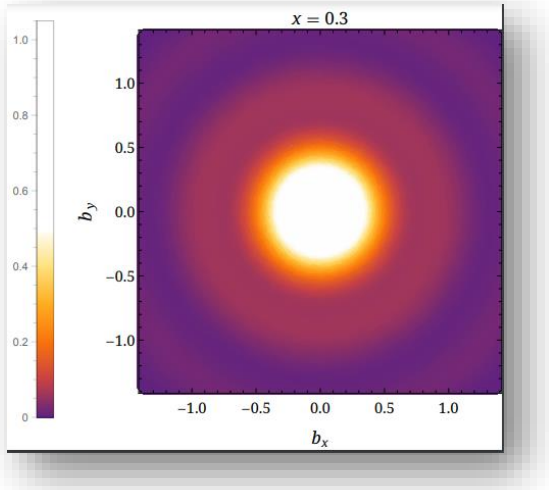


finite-volume, discretization,

- $\xi = 0$ valence-quark Pion GPD results



$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$



HL (MSULat), Phys. Lett. B 846 (2023) 138181

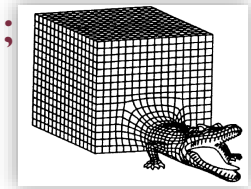
First Lattice GPDs

§ First glimpse into pion GPD using Quasi-PDF/LaMET

- Lattice details: clover/HISQ, **0.12fm**, **310-MeV** pion mass

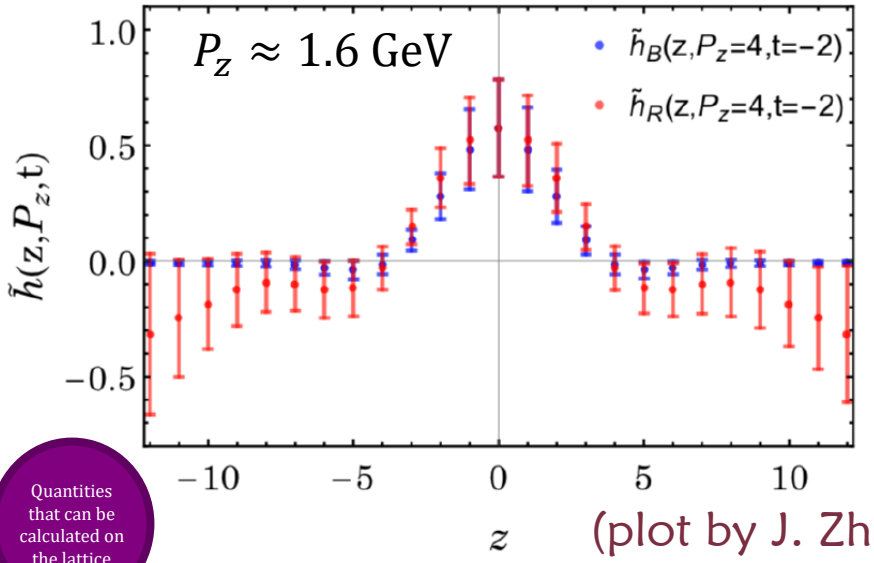
$P_z \approx 1.3, 1.6$ GeV

MILC, Phys. Rev. D, 82 (2010), 074501;
Phys. Rev. D, 87 (2013), 0545056

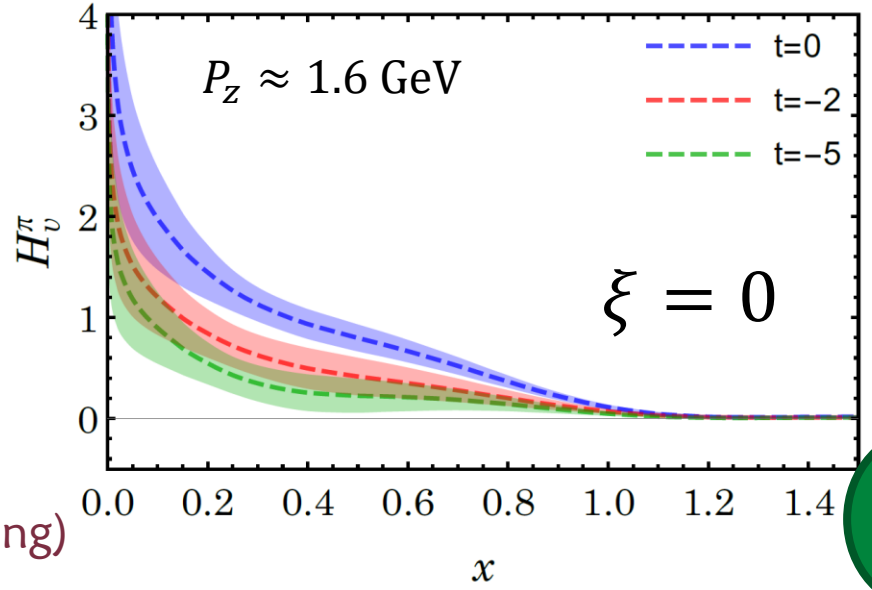


J. Chen, HL, J. Zhang, 1904.1237;

$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right) \gamma^+ \Gamma \left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$



(plot by J. Zhang)



Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

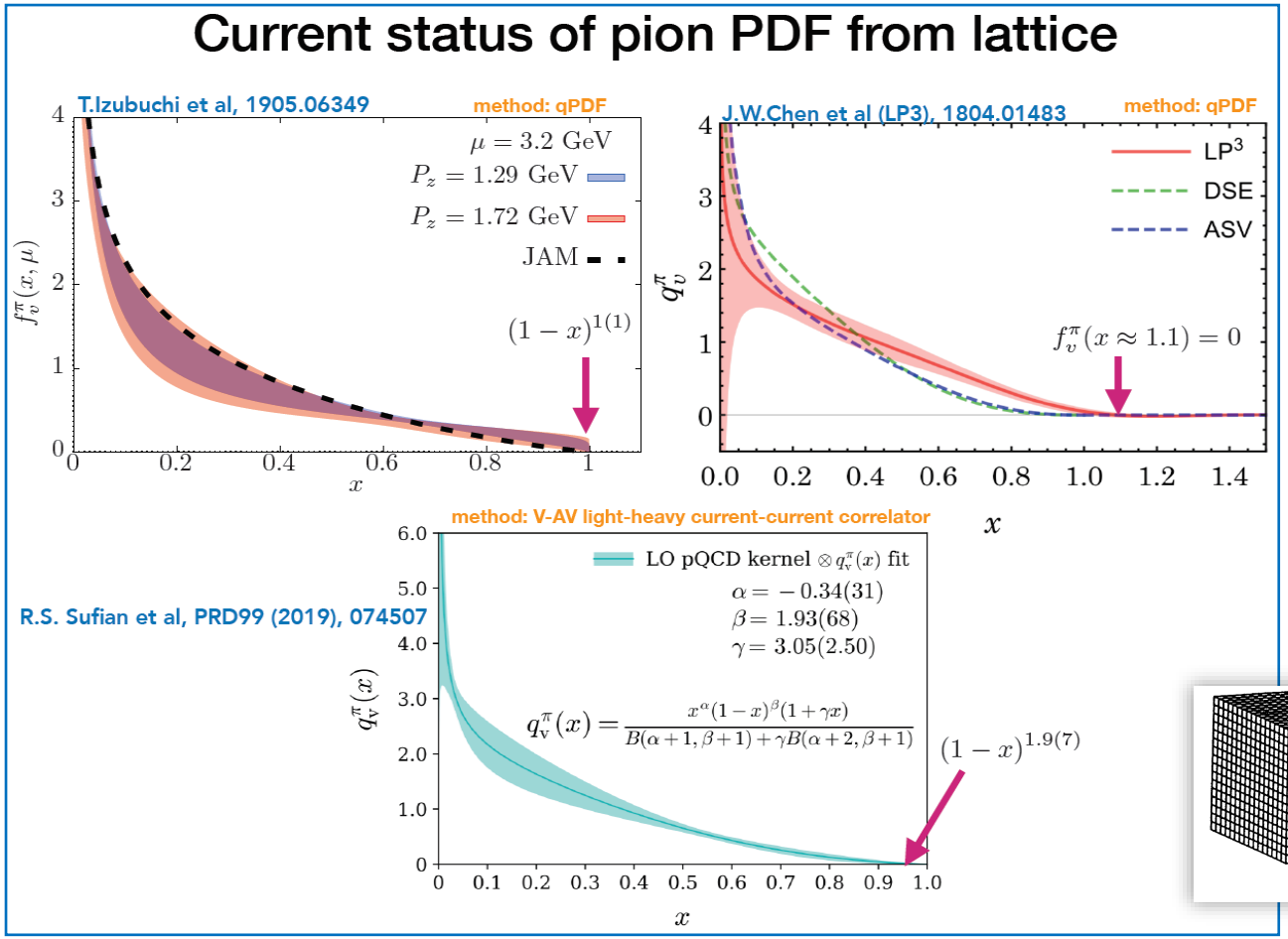
Pion Valence-Quark PDF

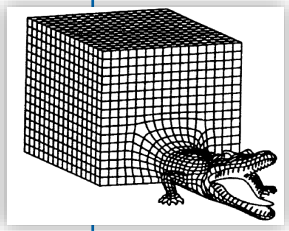
§ Status as of Summer 2019

Slide by Nikhil Karthik @ Lattice 2019

$M_\pi \approx 310$ MeV

$M_\pi \approx 426$ MeV





§ Single-ensemble calculation

- Non-physical pion mass, single lattice spacing, single volume

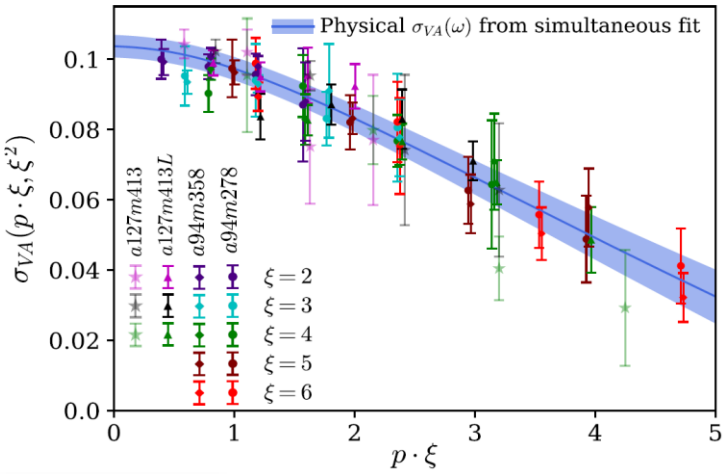
Pion Valence-Quark PDF

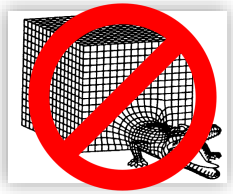
§ Results from JLab-W&M/ **LCS** method

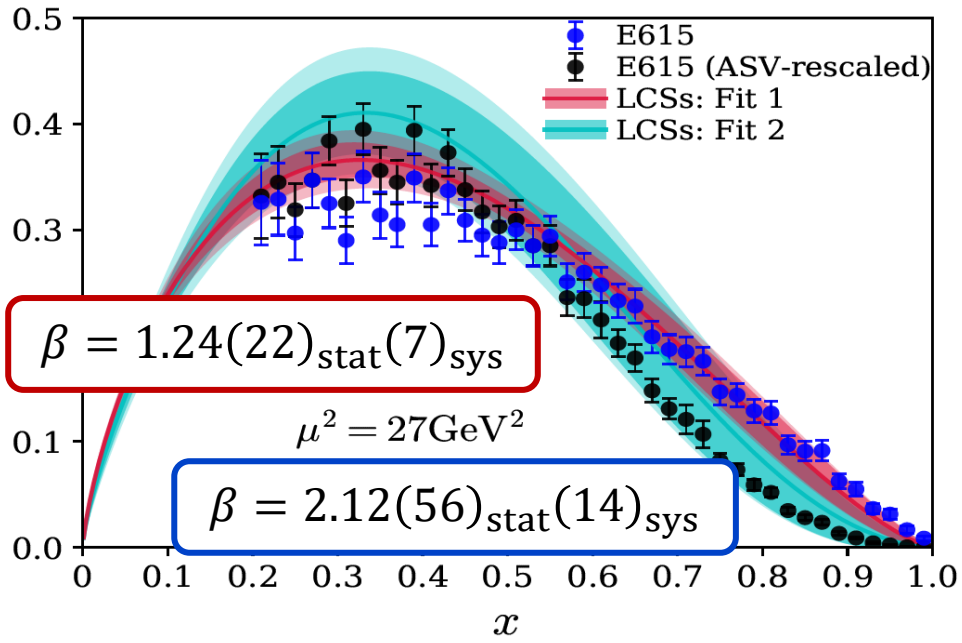
- $M_\pi = \mathbf{278}, 358, 413$ MeV with $a = 0.094, 0.127$ fm
- Extrapolated to physical limit (shown as blue band)
- Renormalized $Z_{V,A}$ in RI/MOM, matched to $\overline{\text{MS}}$, run to 27 GeV^2

R. S. Sufian, et al, 2001.04960

$$q_v^\pi(x) = \frac{x^\alpha (1-x)^\beta (1+\gamma x)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+2,\beta+1)}$$





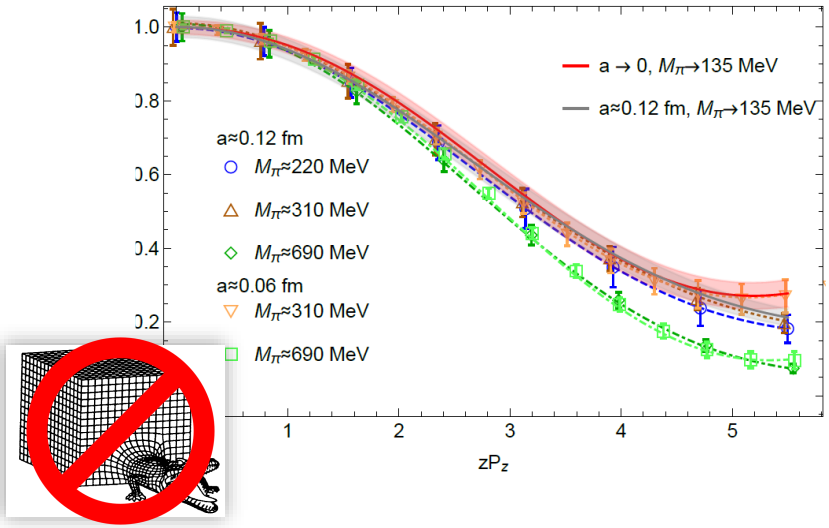


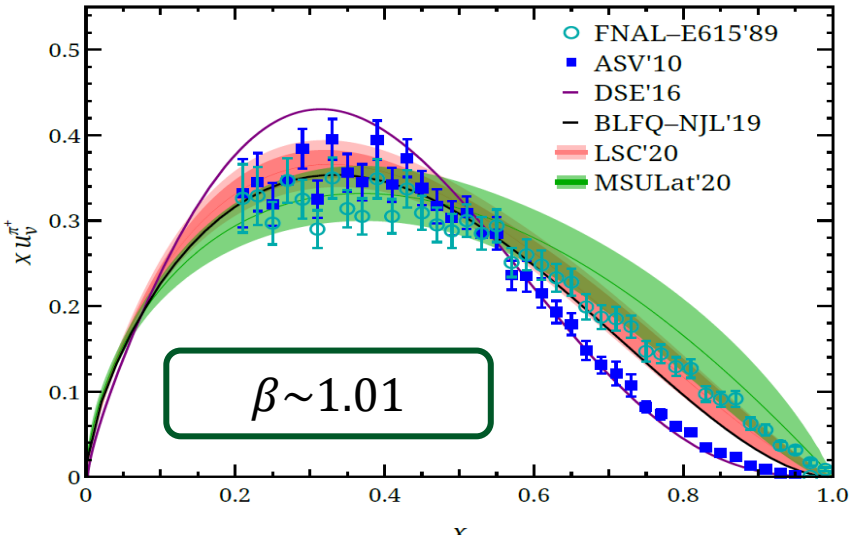
Pion Valence-Quark PDF

§ Results from MSULat/quasi-PDF method

- $M_\pi = \mathbf{220}, 310, 790$ MeV with $a = 0.06, 0.12$ fm
- Extrapolated to physical limit (shown as pink/green band)
- Renormalized in RI/MOM, matched to $\overline{\text{MS}}$, run to 27 GeV^2

H. Lin et al. (MSULat), 2003.14128





J. S. Conway et al., PRD39, 92 (1989).
M. Aicher et al, PRL105, 252003 (2010), 1009.2481.
C. Chen et al, PRD93, 074021 (2016), 1602.01502.

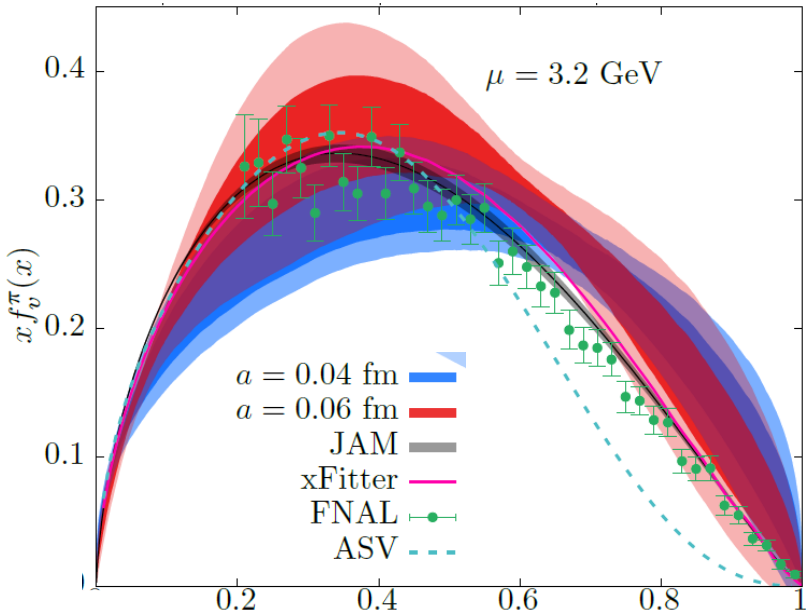
J. Lan, et al, PRL122, 172001 (2019), 1901.11430;
PRD101, 034024 (2020), 1907.01509.
R. S. Sufian, et al, 2001.04960

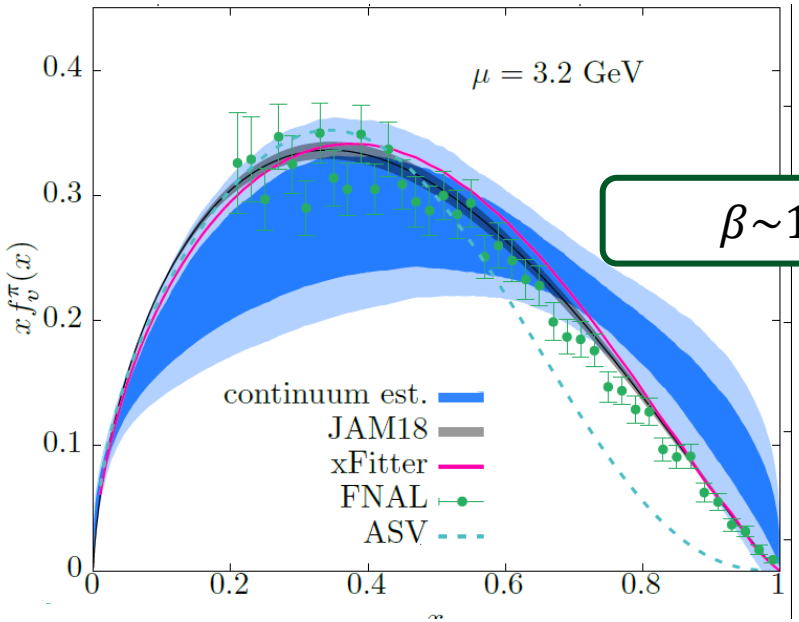
Pion Valence-Quark PDF

§ Results from BNL/quasi-PDF method

- $M_\pi = \mathbf{300}$ MeV with $a = 0.04, 0.06$ fm
- Extrapolated to continuum limit
- Renormalized in RI/MOM, matched to $\overline{\text{MS}}$ at 10 GeV^2

X. Gao et al. 2007.06590



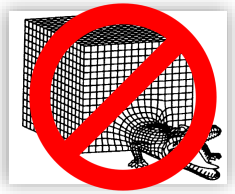


$\beta \sim 1$

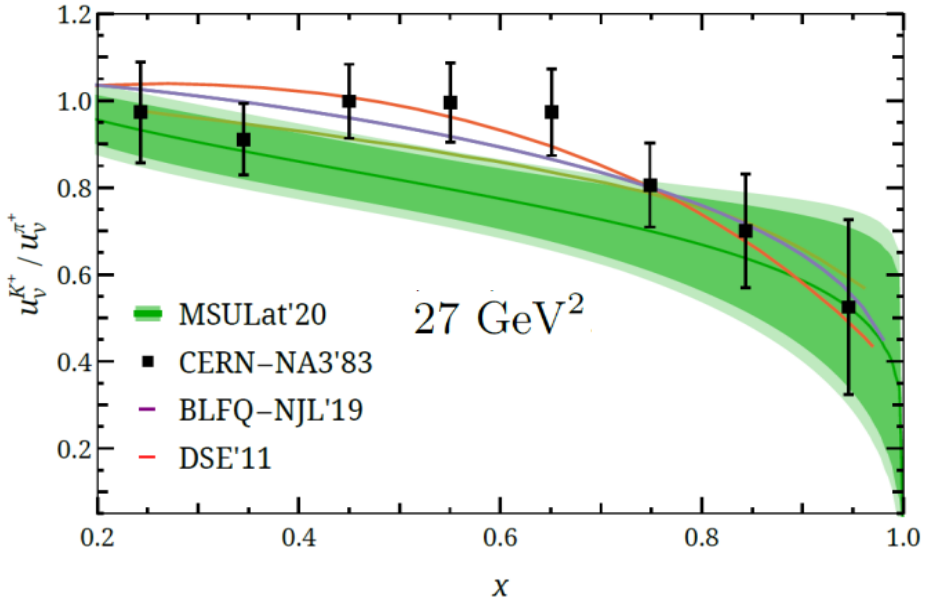
Kaon Valence-Quark PDFs

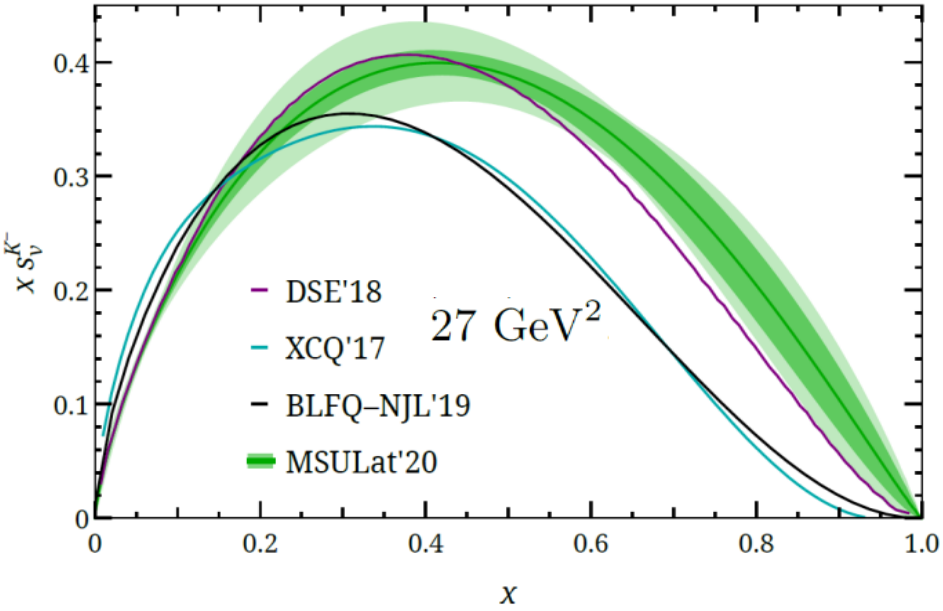
§ Pion/**kaon** PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_z \approx \{1.3, 1.7\}$ GeV



2003.14128 HL et al (MSULat)

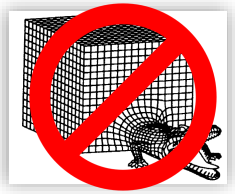




Kaon Valence-Quark PDFs

§ Pion/**kaon** PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_z \approx \{1.3, 1.7\}$ GeV



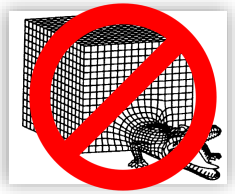
§ First LQCD calculation $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$ 2003.14128 HL et al (MSULat)

n	$\langle x^n \rangle (u_v^{K^+})$	$\langle x^n \rangle (s_v^{K^-})$
1	$0.192(8)_{\text{stat}}(6)_{\text{syst}}$	$0.261(8)_{\text{stat}}(8)_{\text{syst}}$
2	$0.080(7)_{\text{stat}}(6)_{\text{syst}}$	$0.120(7)_{\text{stat}}(9)_{\text{syst}}$
3	$0.041(6)_{\text{stat}}(4)_{\text{syst}}$	$0.069(6)_{\text{stat}}(8)_{\text{syst}}$

Kaon Valence-Quark PDFs

§ Pion/**kaon** PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_z \approx \{1.3, 1.7\}$ GeV



§ First LQCD calculation $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$

§ Later ETMC **260**-MeV results on $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$

2003.14128 HL et al (MSULat)

n	$\langle x^n \rangle (u_v^{K^+})$	$\langle x^n \rangle (s_v^{K^-})$
1	$0.192(8)_{\text{stat}}(6)_{\text{syst}}$	$0.261(8)_{\text{stat}}(8)_{\text{syst}}$
2	$0.080(7)_{\text{stat}}(6)_{\text{syst}}$	$0.120(7)_{\text{stat}}(9)_{\text{syst}}$
3	$0.041(6)_{\text{stat}}(4)_{\text{syst}}$	$0.069(6)_{\text{stat}}(8)_{\text{syst}}$

2010.0349, 2104.02247

n	$\langle x^n \rangle (u_v^{K^+})$	$\langle x^n \rangle (s_v^{K^-})$
1	$0.246(2)_{\text{stat}}(2)_{\text{syst}}$	$0.317(2)_{\text{stat}}(1)_{\text{syst}}$
2	$0.093(5)_{\text{stat}}(3)_{\text{syst}}$	$0.134(5)_{\text{stat}}(2)_{\text{syst}}$
3	$0.035(6)_{\text{stat}}(3)_{\text{syst}}$	$0.075(5)_{\text{stat}}(1)_{\text{syst}}$

First Pion Gluon PDF

§ Pion GLUON PDFs using pseudo-PDF

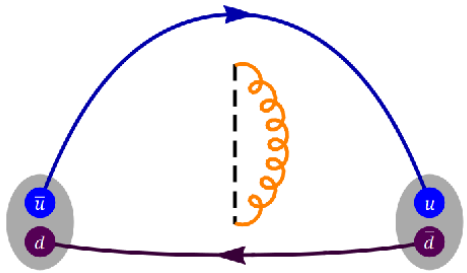
- Lattice details: clover/2+1+1 HISQ (MSULat)

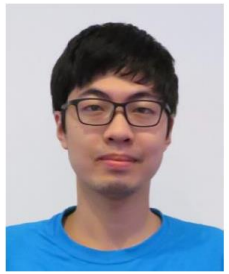
  $a \approx \{0.12, 0.15\}$ fm,

  $M_\pi \in \{220, 310, 690\}$-MeV pion

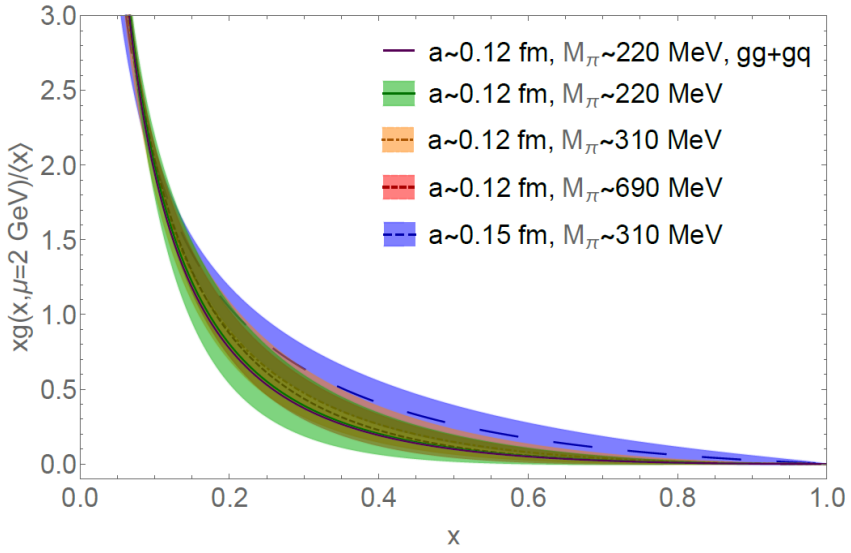
  $P_{z,\max} \approx 2.3$ GeV

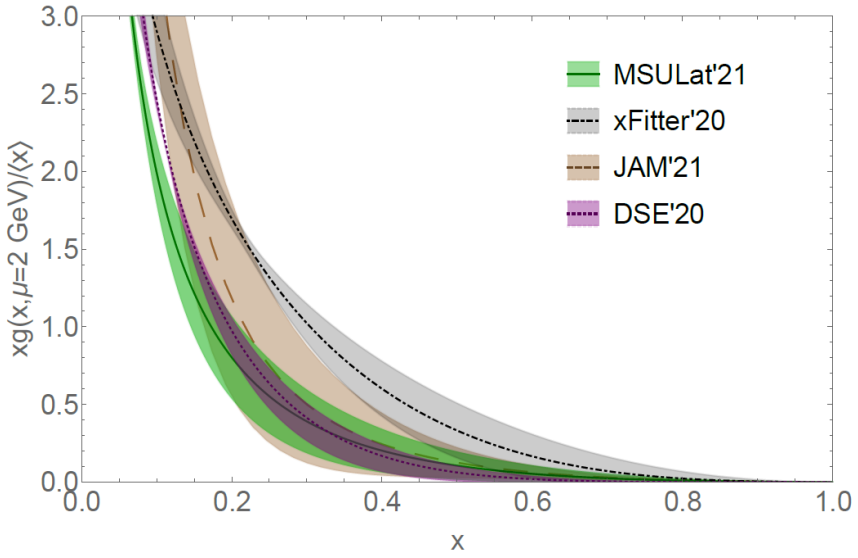
2104.06372, Fan, HL(MSULat)





Zhouyou Fan (MSU)



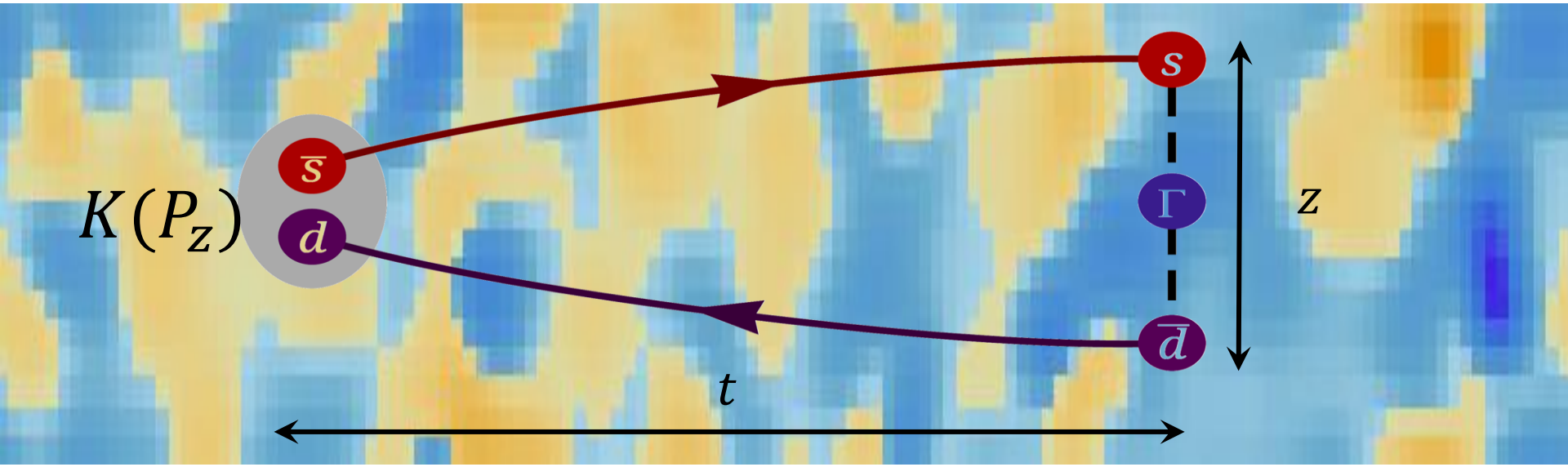


Pion and Kaon DA

§ The first continuum-limit study of *x*-dependent meson DA on the lattice

- $M_\pi \in \{\mathbf{310}, 690\ (\eta_s)\}$ MeV
- $a \in \{0.06, 0.09, 0.12\}$ fm
- $M_\pi^{\min} L = 4.5$

$$C_M^{\mathrm{DA}}(z,P,t) = \left\langle 0 \left| \int d^3y\, e^{i\,\vec{P}\cdot\vec{y}} \bar{\psi}_1(\vec{y},t)\gamma_z\gamma_5 U(\vec{y},\vec{y}+z\,\hat{z})\psi_2(\vec{y}+z\,\hat{z},t)\bar{\psi}_2(0,0)\gamma_5\psi_1(0,0) \right| 0 \right\rangle$$



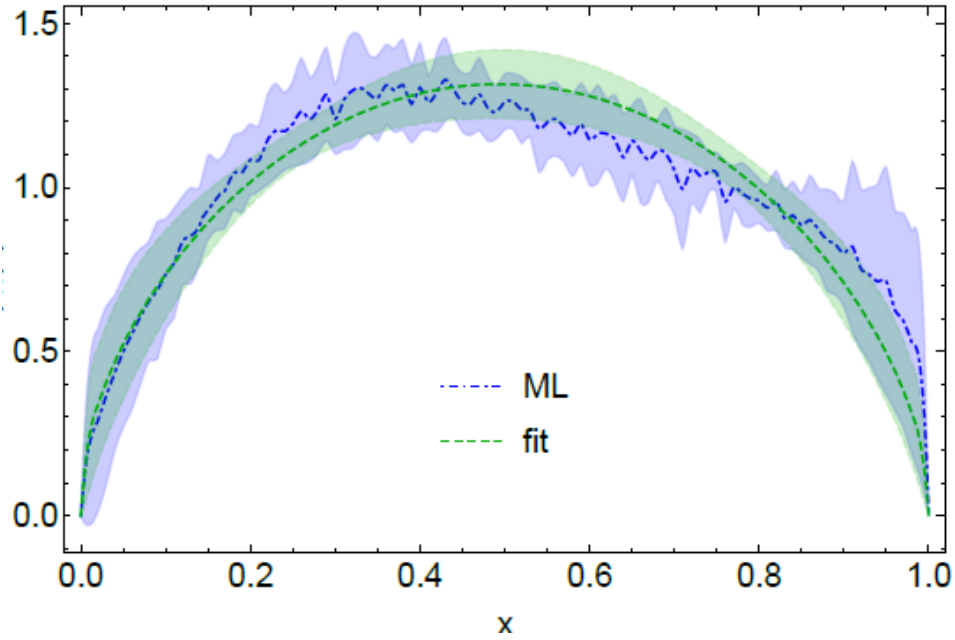
Pion and Kaon DA

§ Extract the DA distribution from the physical-continuum matrix elements

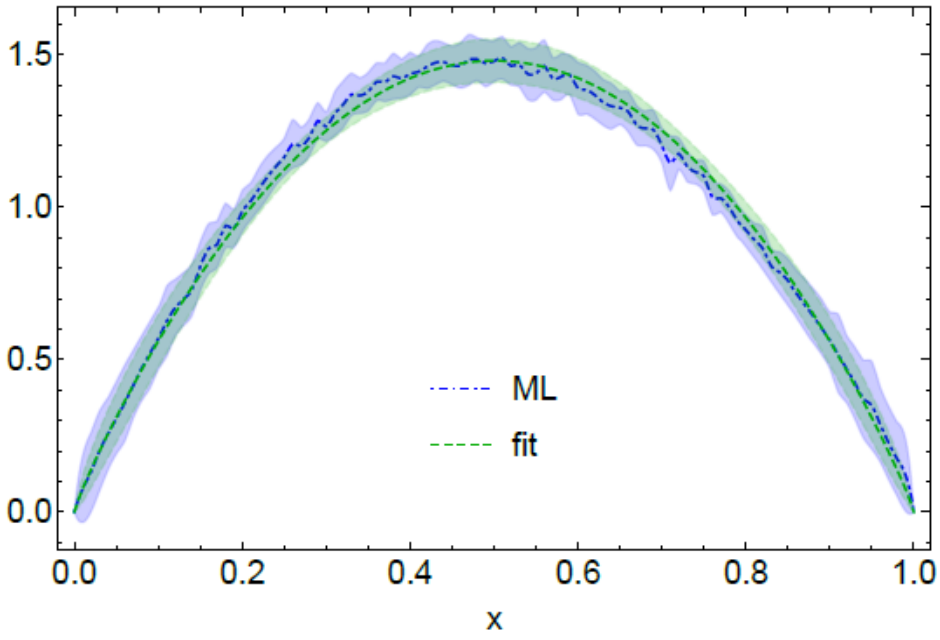
R. Zhang et al. (MSULat), 2005.13955

$$h(z,\mu^R,p_z^R,P_z)=\int_{-\infty}^{\infty}dx\int_0^1 dy\, C\left(x,y,\left(\frac{\mu^R}{p_z^R}\right)^2,\frac{P_z}{\mu^R},\frac{P_z}{p_z^R}\right)f_{m,n}(y)e^{i(1-x)zP_z}$$

Pion



Kaon



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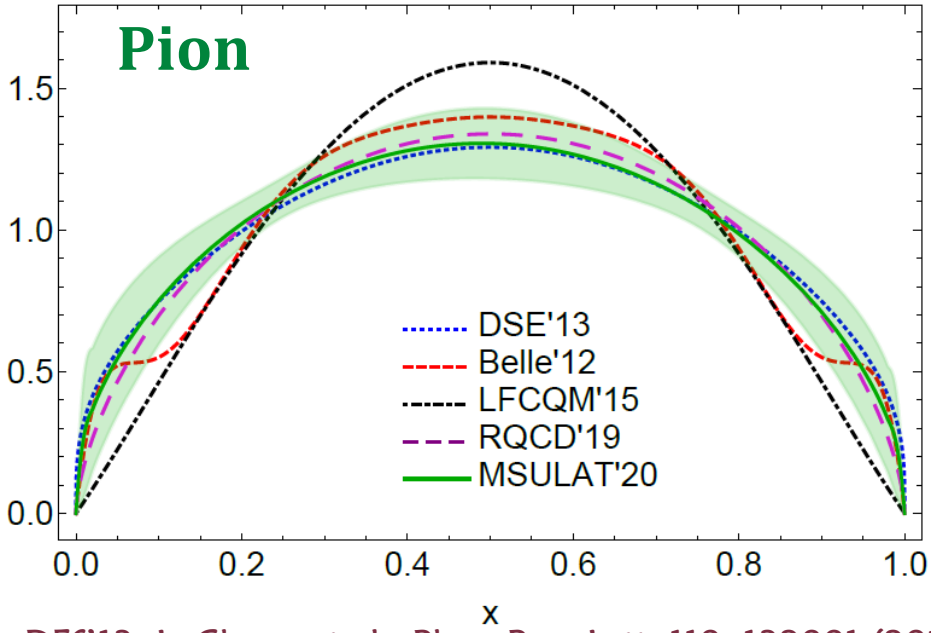
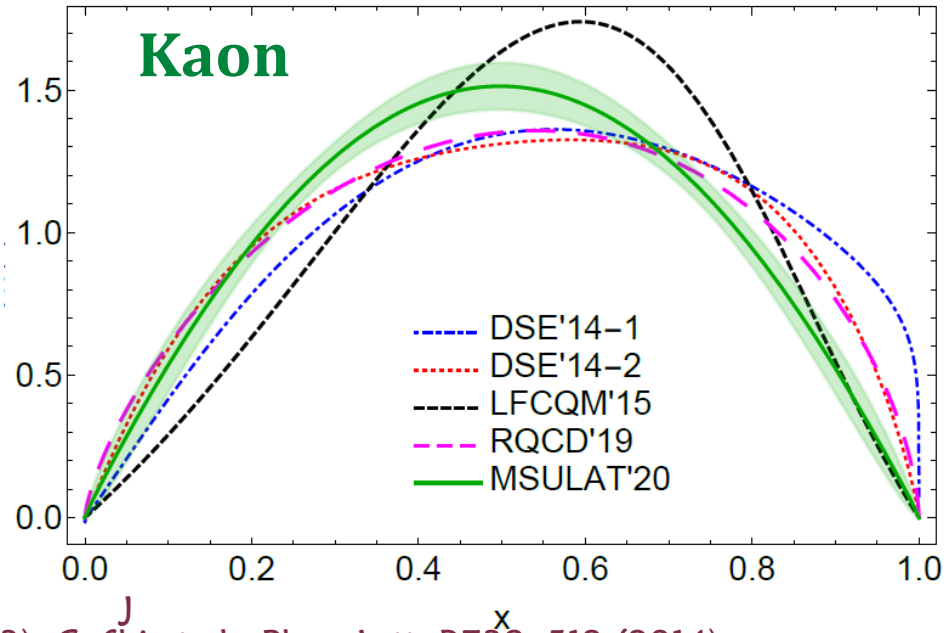


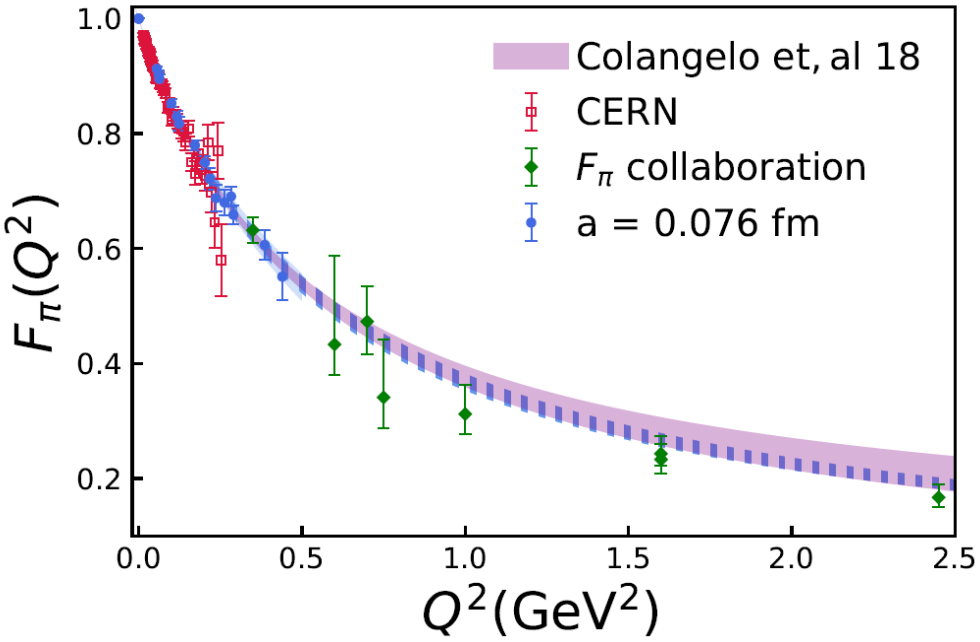


DES'13: L. Chang et al., Phys. Rev. Lett. 110, 132001 (2013); C. Shi et al., Phys. Lett. B738, 512 (2014)
Belle'12: S. Agaev et al., Phys. Rev. D86, 077504 (2012);
LFCQM'15:J. P. B. C. de Melo et al., AIP Conf. Proc. 1735, 080012 (2016);
RQCD'19: G. S. Bali et al.,J HEP 08, 065 (2019); DSE'14:

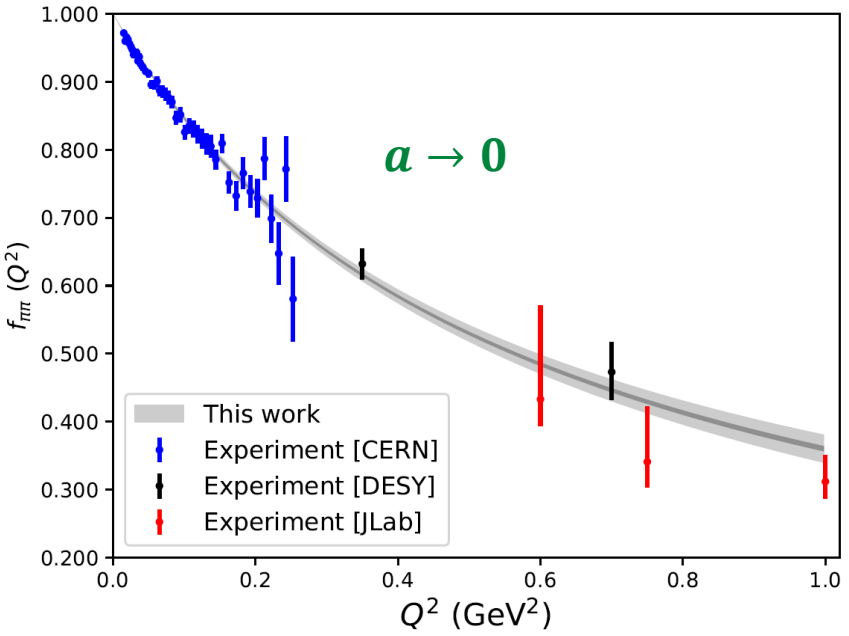
Pion Form Factors

§ Two new lattice pion form factors calcs at physical pion

- χQCD: 2+1f, overlap/DWF,
  $a \approx [0.08,0.2]$ fm, $M_\pi \in [139,340]$-MeV
- BNL: 2+1+1,clover/HISQ,
  $a \approx [0.04,0.08]$ fm, $M_\pi \in \{135, 300\}$-MeV



X. Gao et al (BNL), 2102.06047



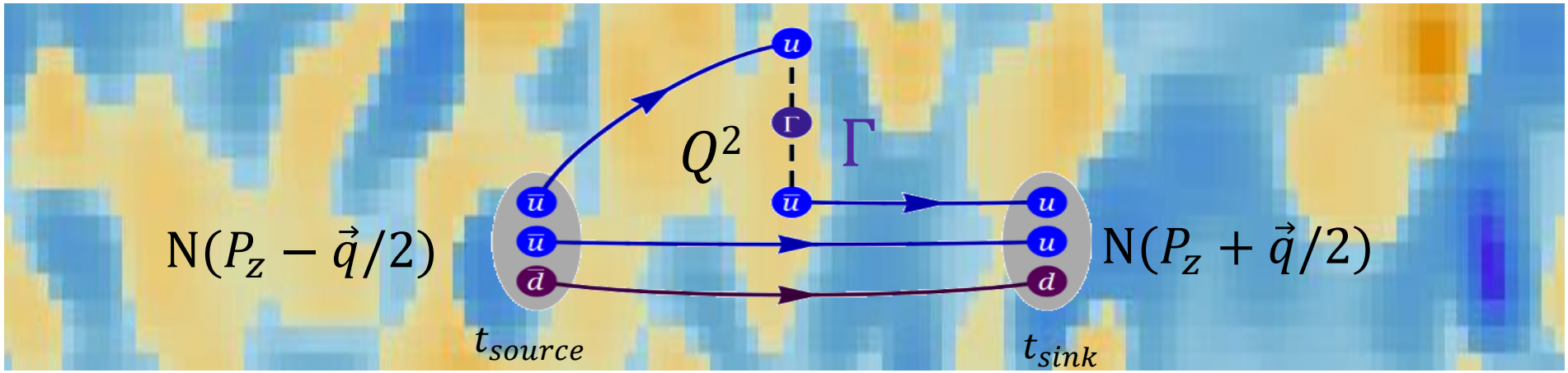
G. Wang et al (XQCD), 2006.05431

Backup Slides
Updates on Nucleon GPDs

2020: Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results



$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(H(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + E(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

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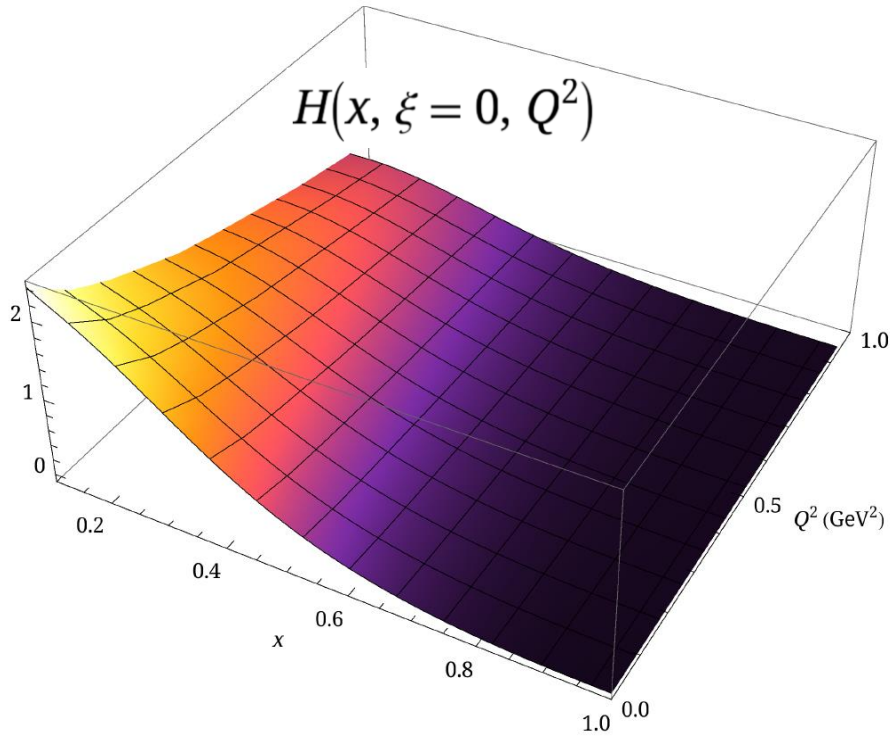

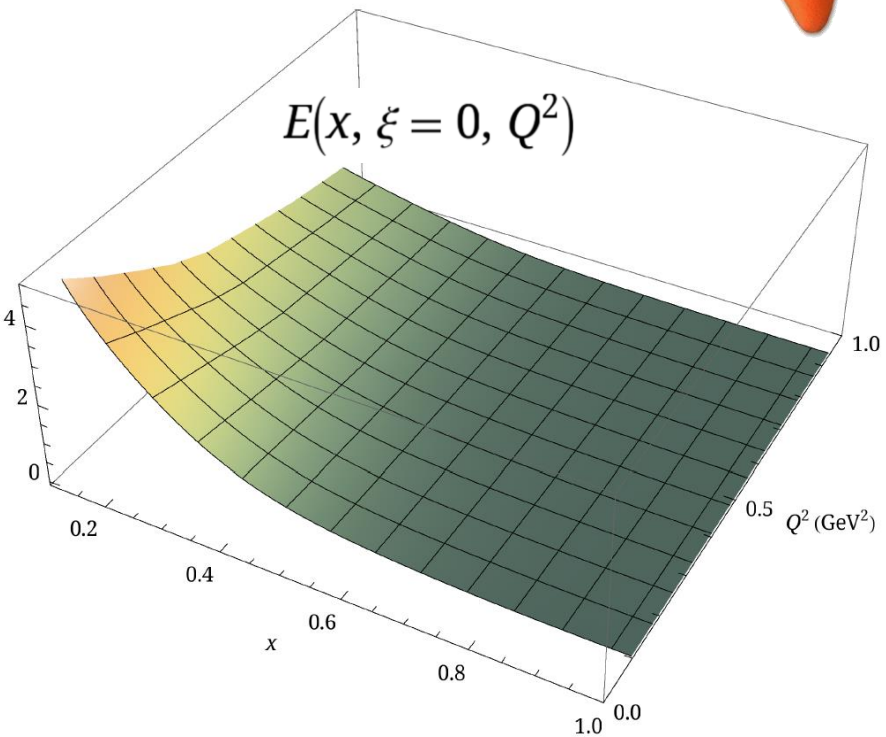

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

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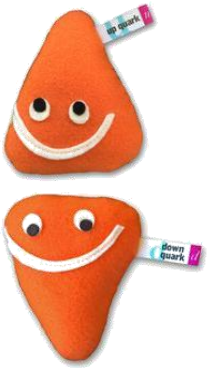
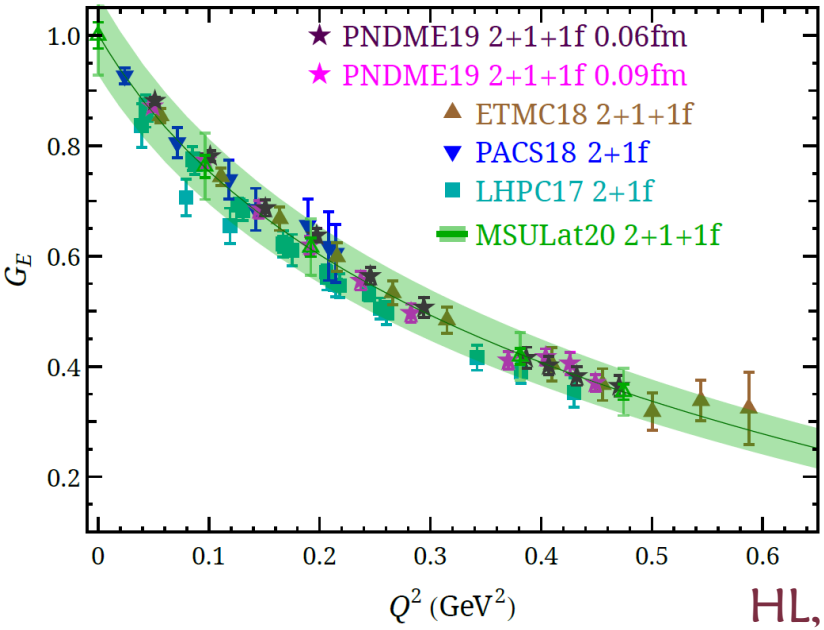


$n = 1$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

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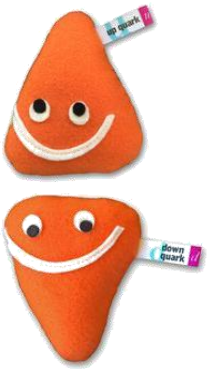
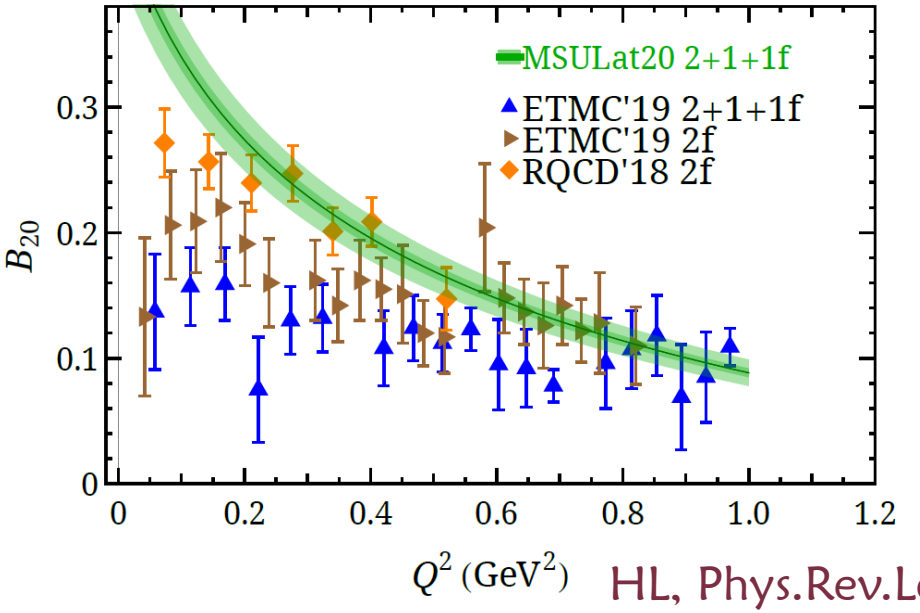


$n = 2$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon Tomography

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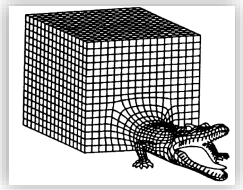
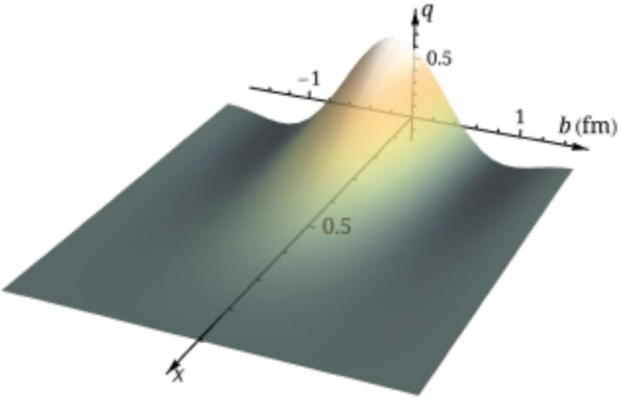
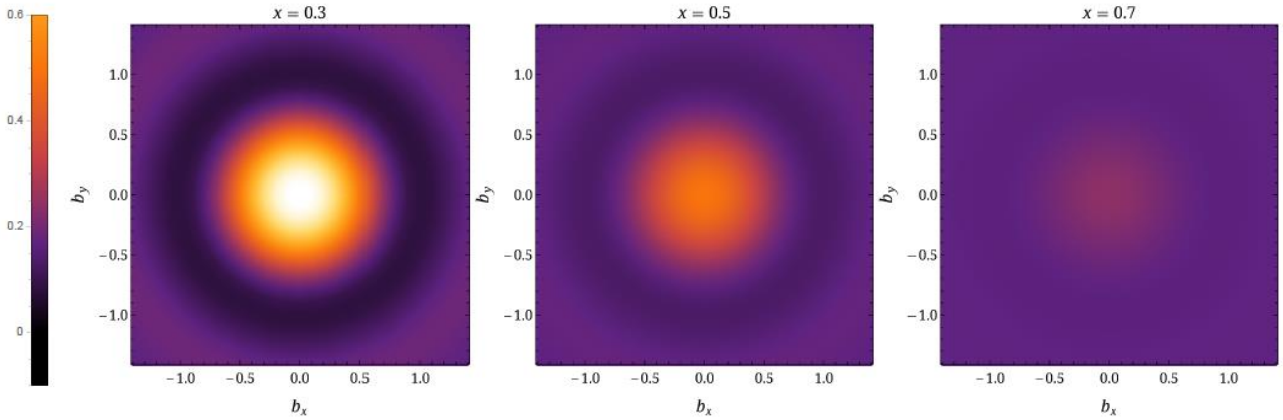


finite-volume, discretization,

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x,\xi=0,t=-\vec{q}^{\,2}) e^{i\vec{q}\cdot\vec{b}}$$





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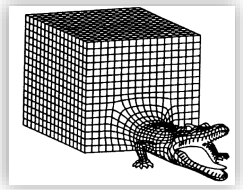

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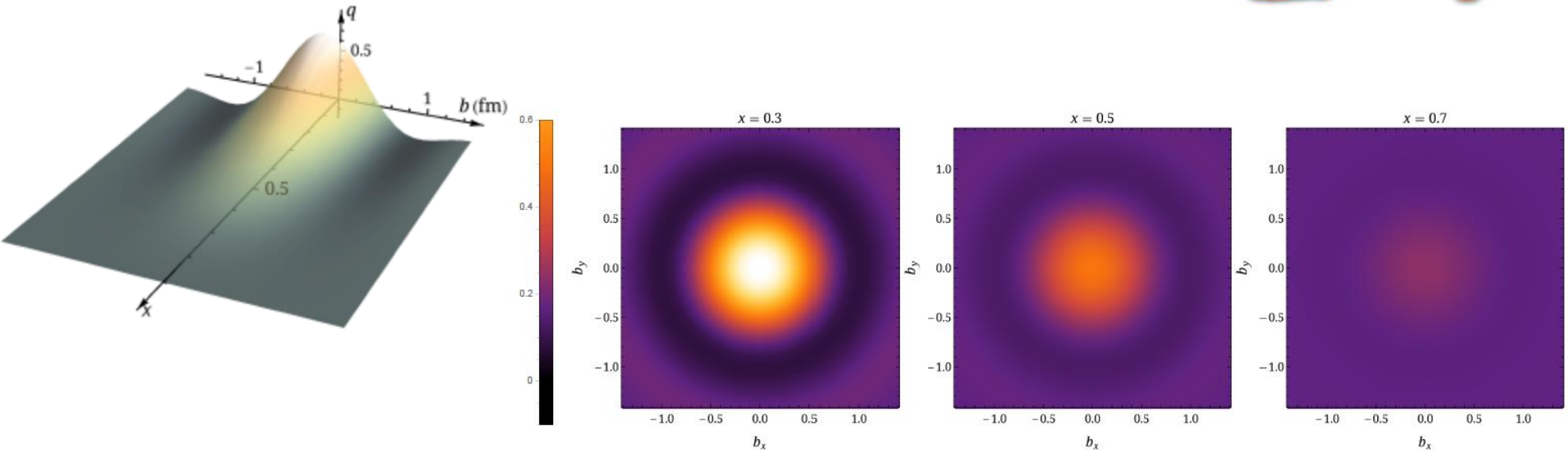

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ

  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right]$$

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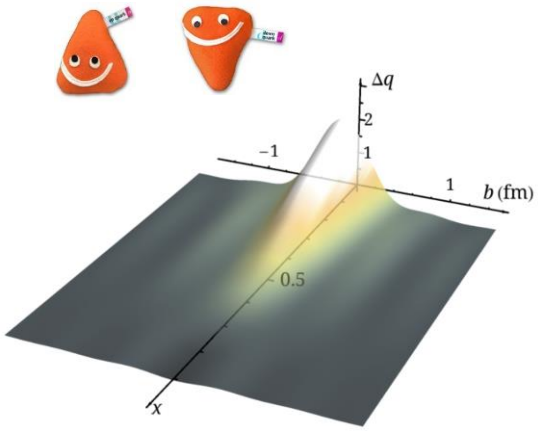
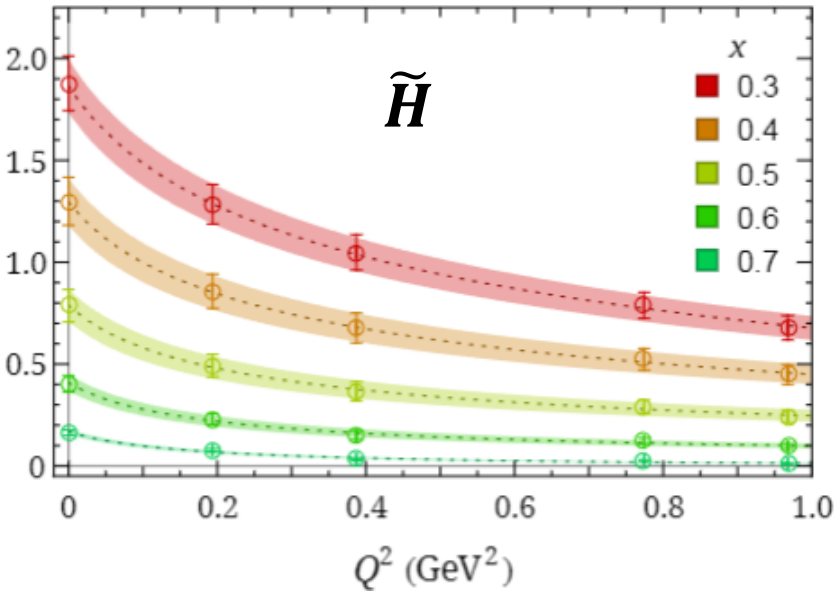
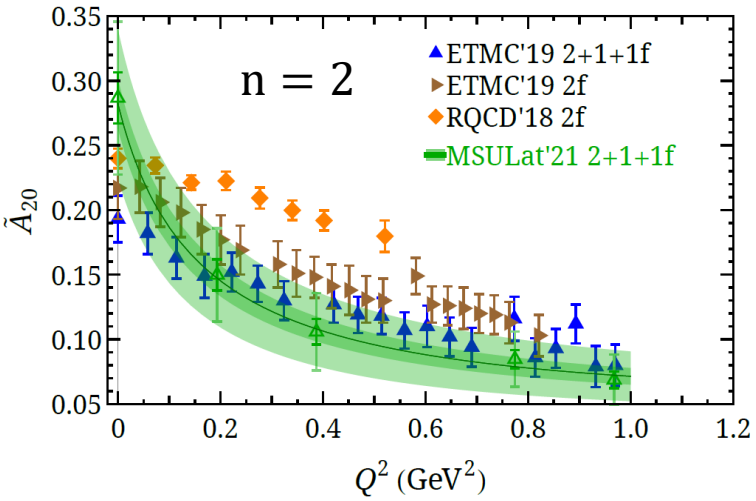
HL (MSULat), Phys.Lett.B 824 (2022) 136821





- Take the integral to form moments



Caveats

§ Systematics in our earlier quasi-PDF calculation

- Renormalization: non-perturbative RI/MOM renormalization
  - State of the art: hybrid-ratio renormalization

    X. Ji et. al. NPB 964, 115311 (2021)
- Next-leading order (NLO) matching only
  - State of the art: NNLO matching kernel available

    X. Gao, PRL 128, 142003 (2022)
- Did not teat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

    R. Zhang, et. al. PLB 844, 138081 (2023)
- For the rest of this presentation, we will focus on the uncertainties from the above (rather than typical lattice-calculation precision or systematics)

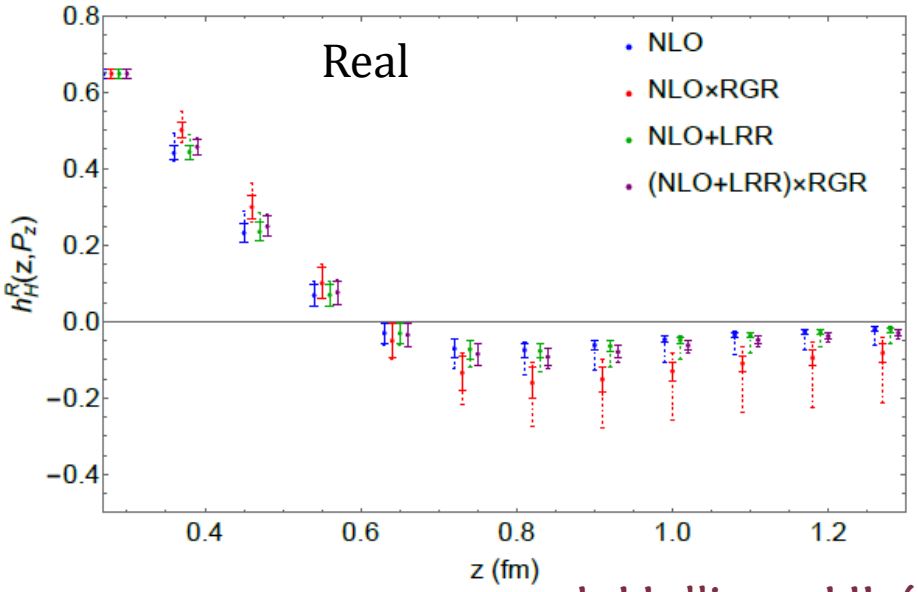
Forward-Limit Case: PDF

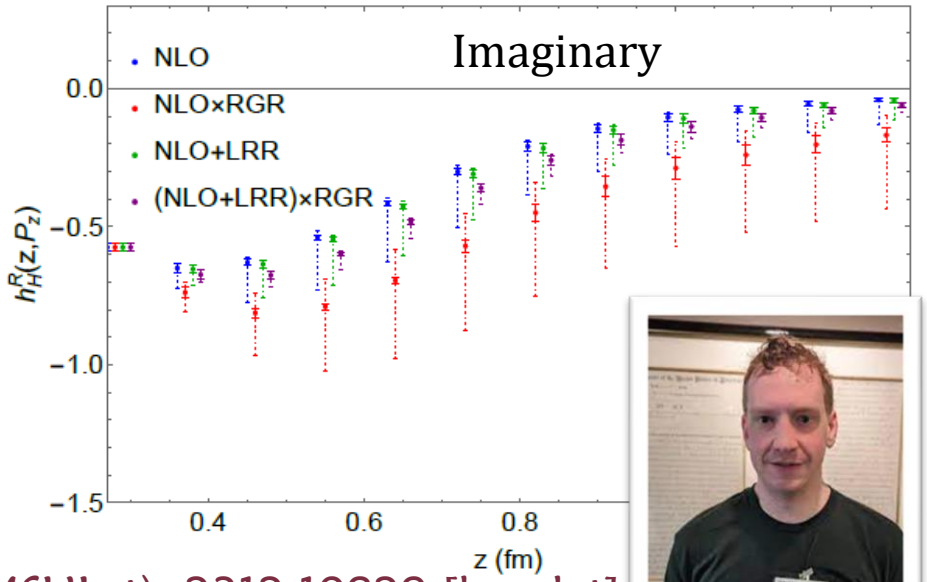
§ NLO hybrid-ratio renormalized matrix elements

$$h^R(z,P_z) = \begin{cases} N\dfrac{h^B(z,P_z)}{h^B(z,P_z=0)} & \text{for } z < z_s \\ Ne^{(\delta m+m_0)(z-z_s)}\dfrac{h^B(z,P_z)}{h^B(z,P_z=0)} & \text{for } z \geq z_s \end{cases}$$

Remove the linear divergence & renormalon ambiguity at large distances

- Vary the scale within $[0.75, 1.5]$: $\approx 15\%$ variation $\alpha_s(\mu = 2.0\ \text{GeV})$
- Systematic errors shown below:



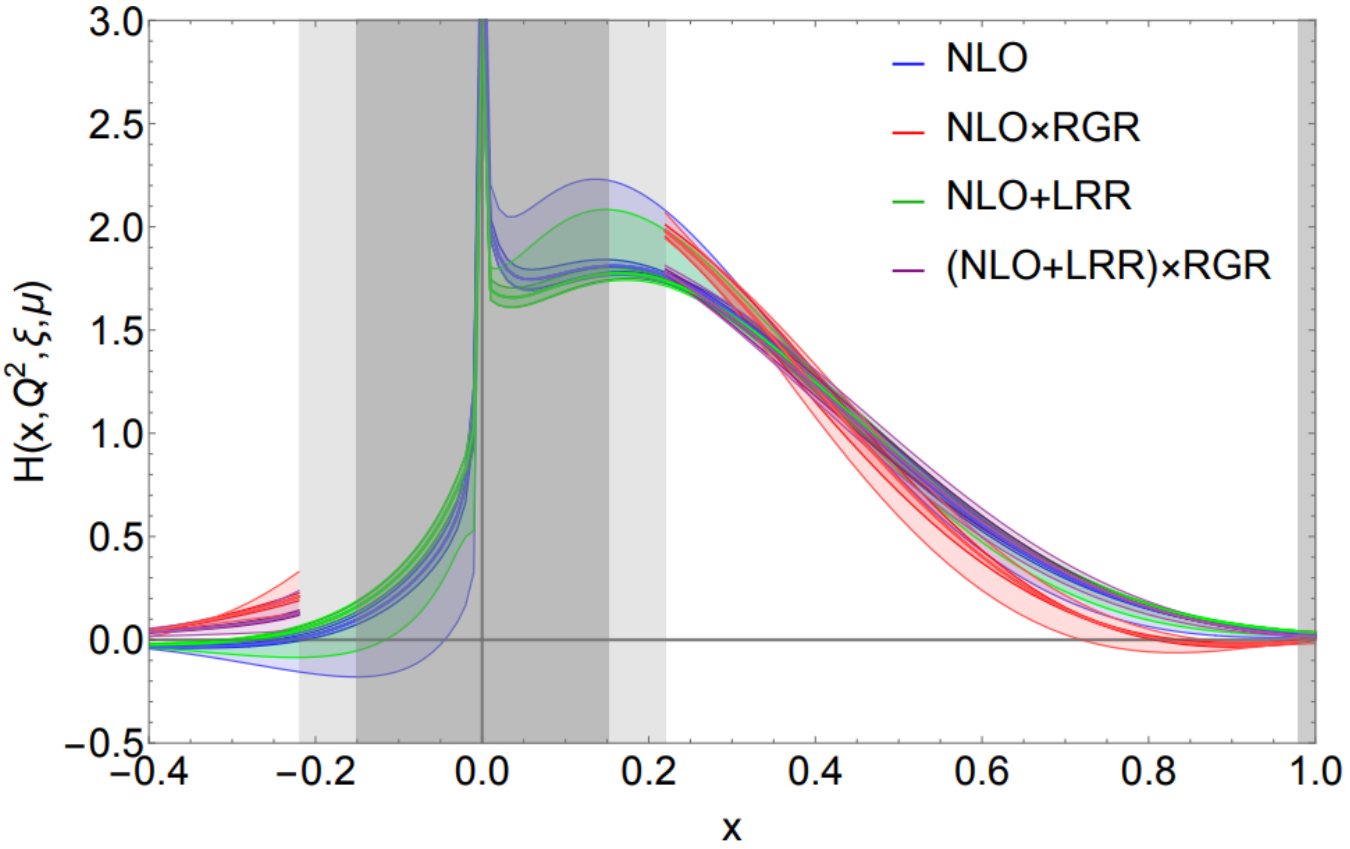


J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Forward Limit Case: PDF

§ NLO isovector nucleon $H(\xi = 0, Q^2 = 0, x)$

- RGR process: DGLAP equation breaks down for $|x| \lesssim 0.2$ with $\mu = c' \times 2xP_z$



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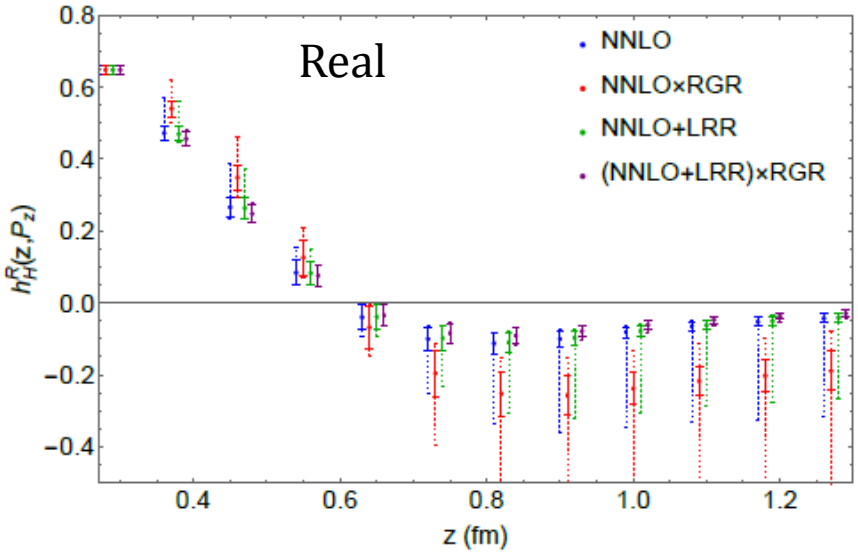

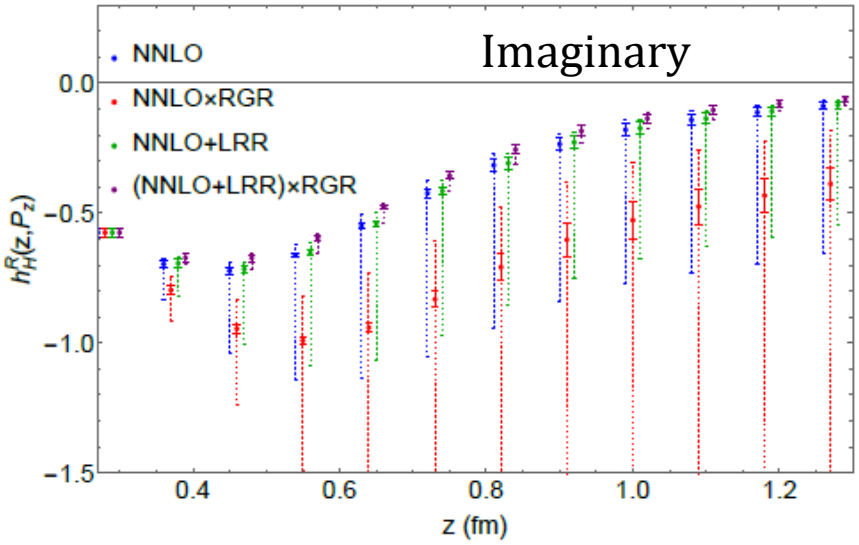

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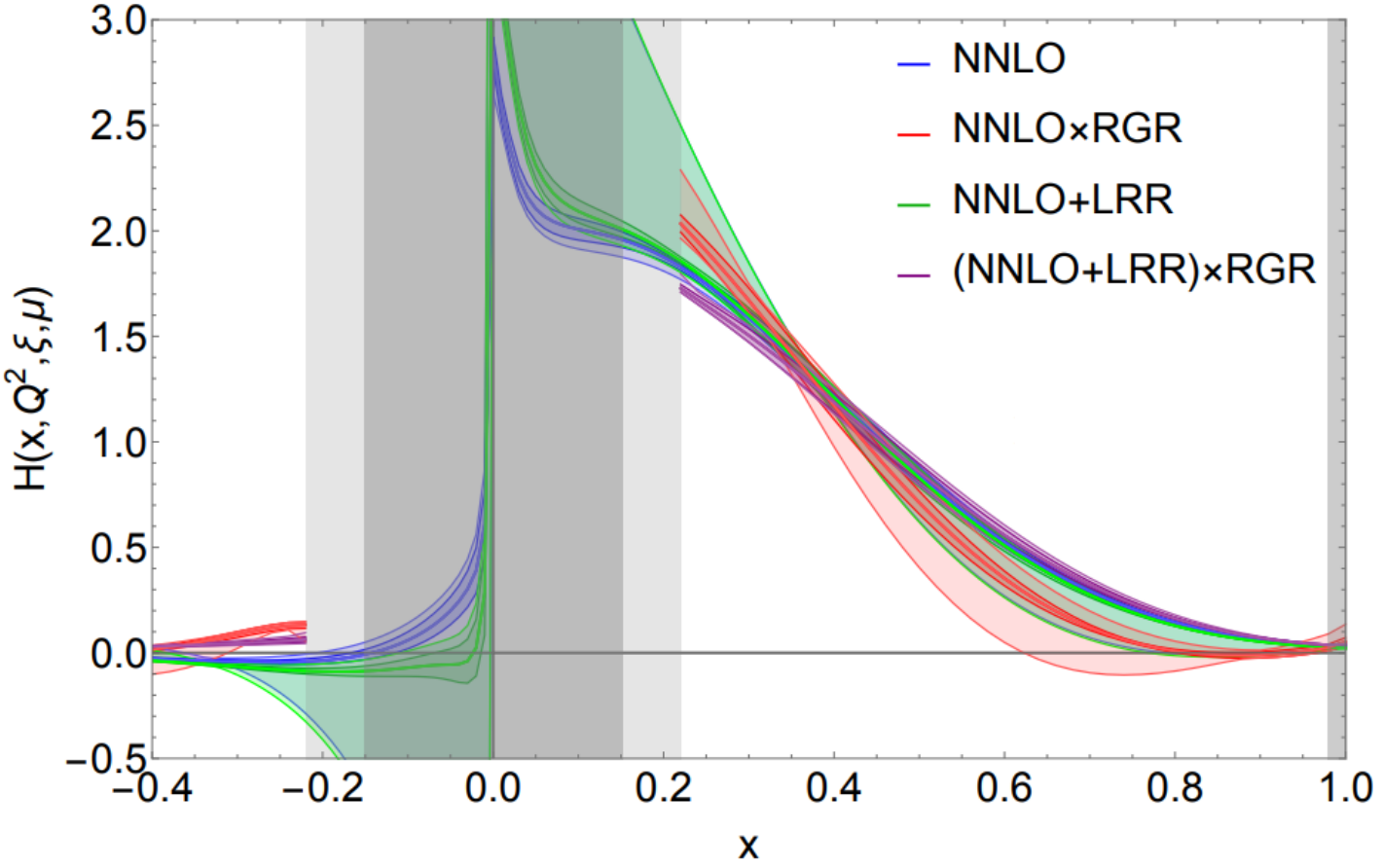
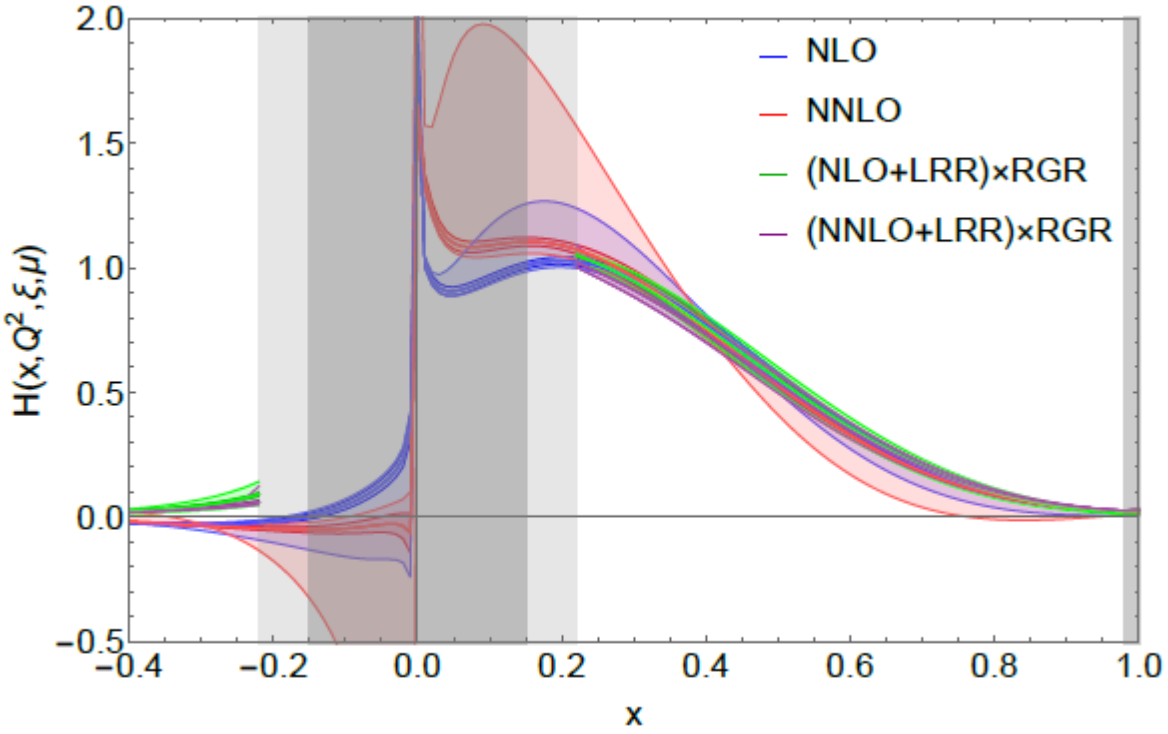


J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

$\xi=0$, $Q^2=0.39$ GeV^2 GPDs

§ Repeat the procedure for nonzero transfer momentum



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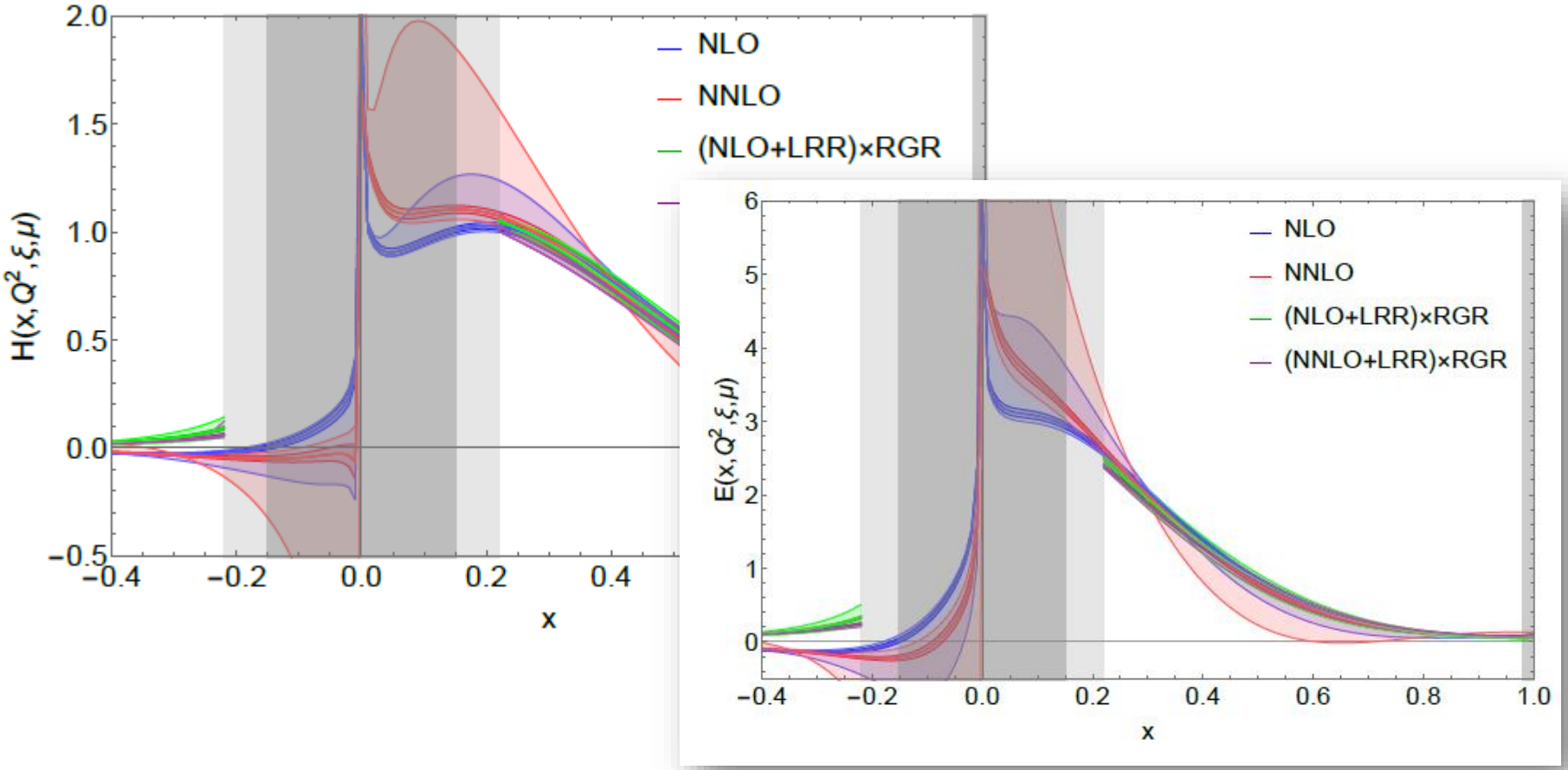

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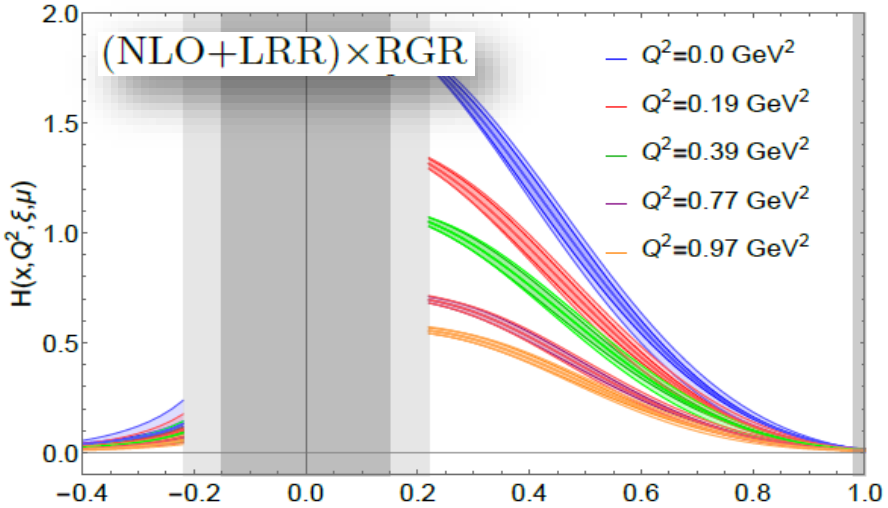
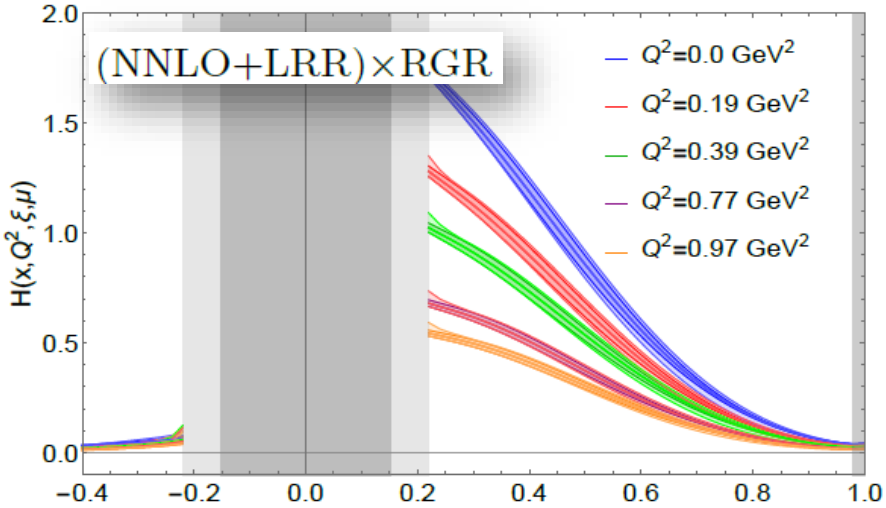
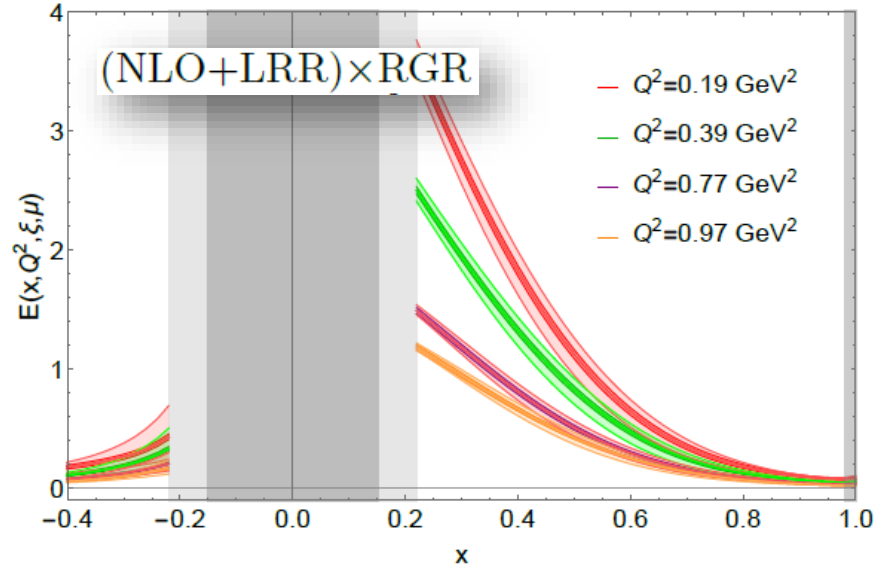
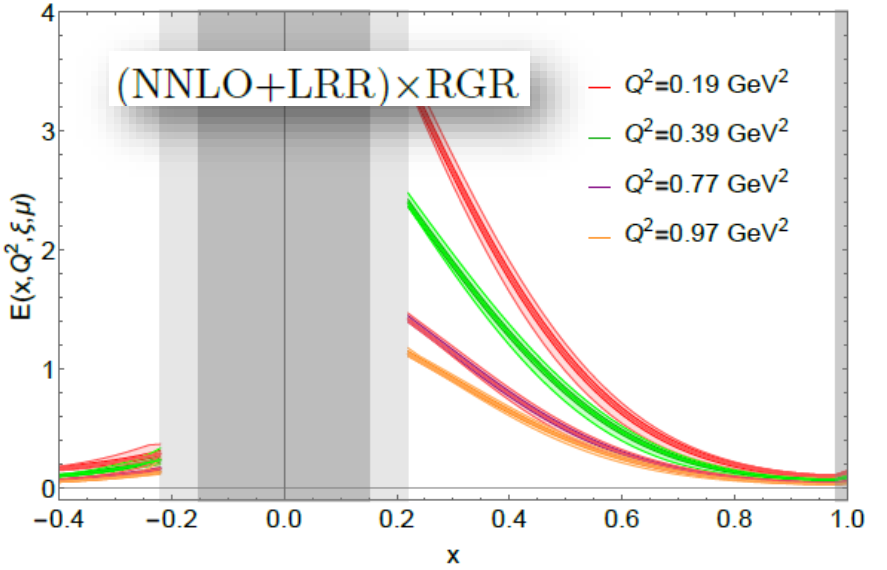
ξ=0 GPDs









J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

$\xi \neq 0$ GPDs

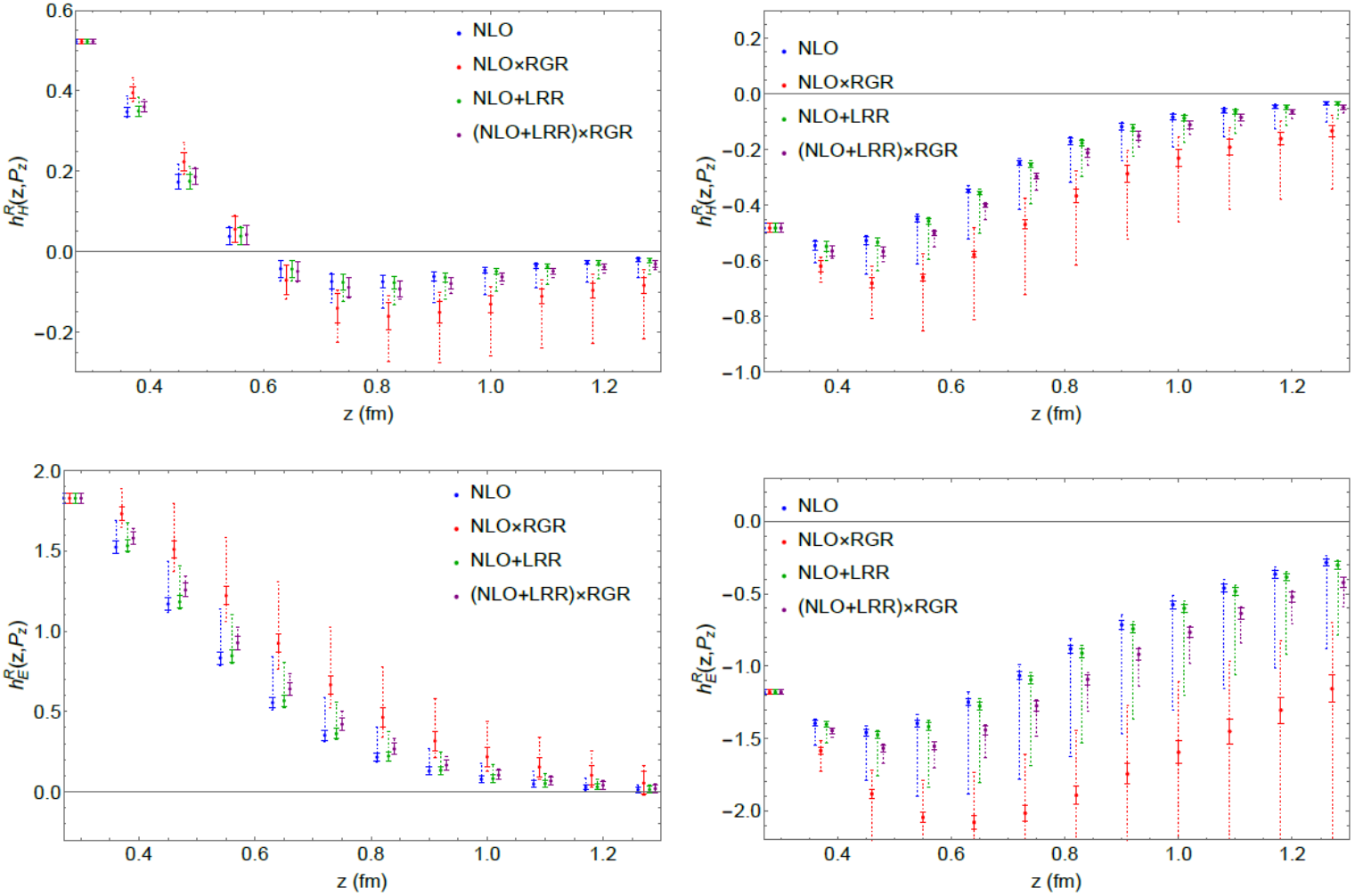
§ Only the NLO matching kernel is available

$$
\begin{aligned}
&\mathcal{K}(x, y, \mu, \xi, P_z) \\
&= \delta(x-y) \\
&+ \frac{\alpha_s C_F}{4\pi}\Bigg[\left(\frac{|\xi+x|}{2\xi(\xi+y)} + \frac{|\xi+x|}{(\xi+y)(y-x)}\right)\left(\ln\left(\frac{4y^2(\xi+x)^2P_z^2}{\mu^2}\right)-1\right) \\
&+ \left(\frac{|\xi-x|}{2\xi(\xi-y)} + \frac{|\xi-x|}{(\xi-y)(x-y)}\right)\left(\ln\left(\frac{4y^2(\xi-x)^2P_z^2}{\mu^2}\right)-1\right) \\
&+ \left(\left(\frac{\xi+x}{\xi+y}+\frac{\xi-x}{\xi-y}\right)\frac{1}{|x-y|} - \frac{|x-y|}{\xi^2-y^2}\right)\left(\ln\left(\frac{4y^2(x-y)^2P_z^2}{\mu^2}\right)-1\right)\Bigg]
\end{aligned}
$$

F. Yoa et al, JHEP 11(2023) 021

$\xi \neq 0$ GPDs

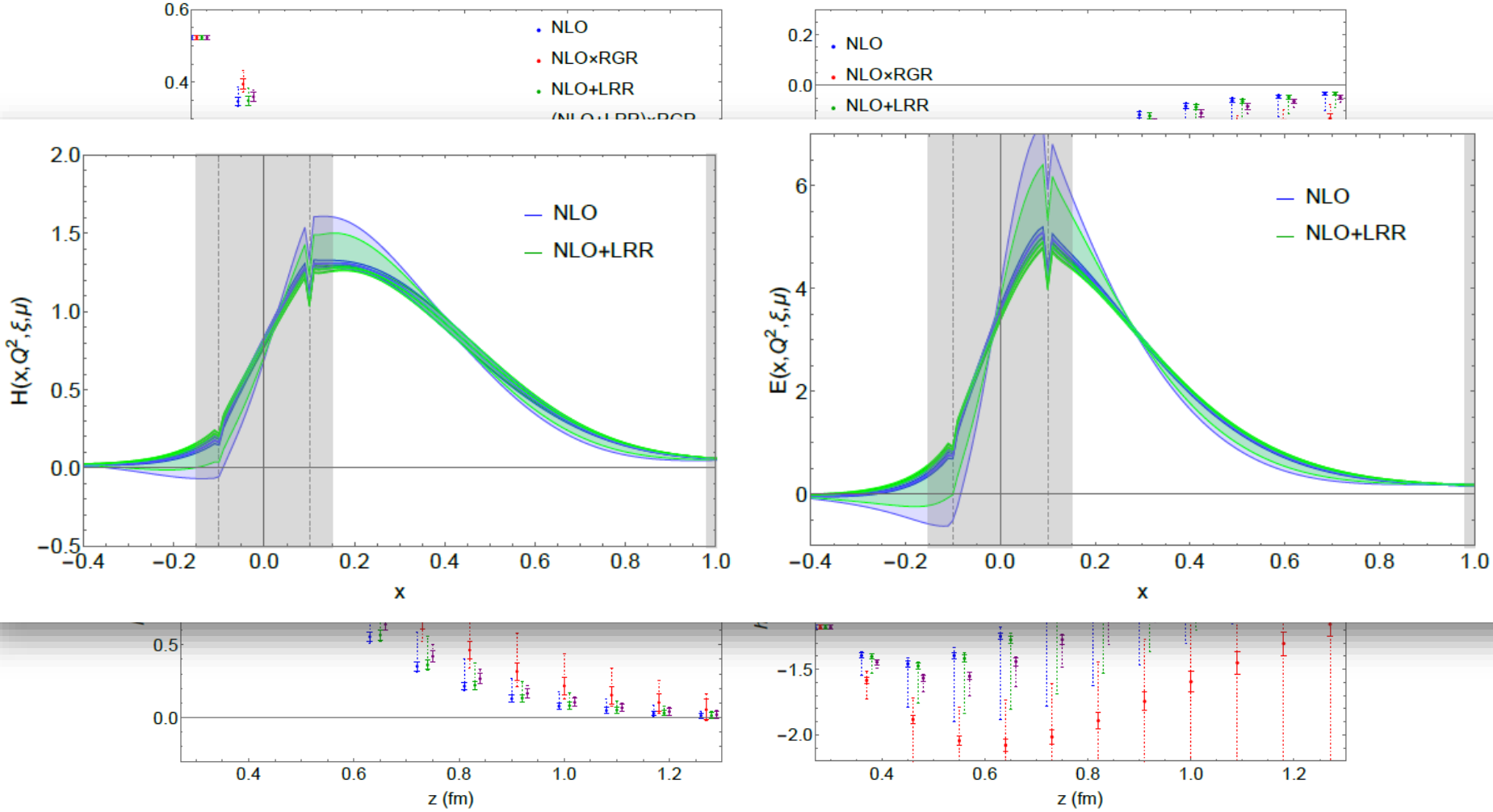
§ NLO $\xi = 0.1$, $Q^2 = 0.23\ \mathrm{GeV}^2$



J. Holligan, HL (MSULat), in preparation

$\xi \neq 0$ GPDs

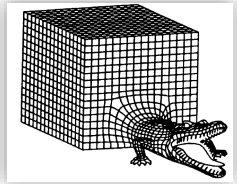
§ NLO $\xi = 0.1$, $Q^2 = 0.23\ \mathrm{GeV}^2$



J. Holligan, HL (MSULat),2312.10829 [hep-lat]

Generalized Parton Distributions

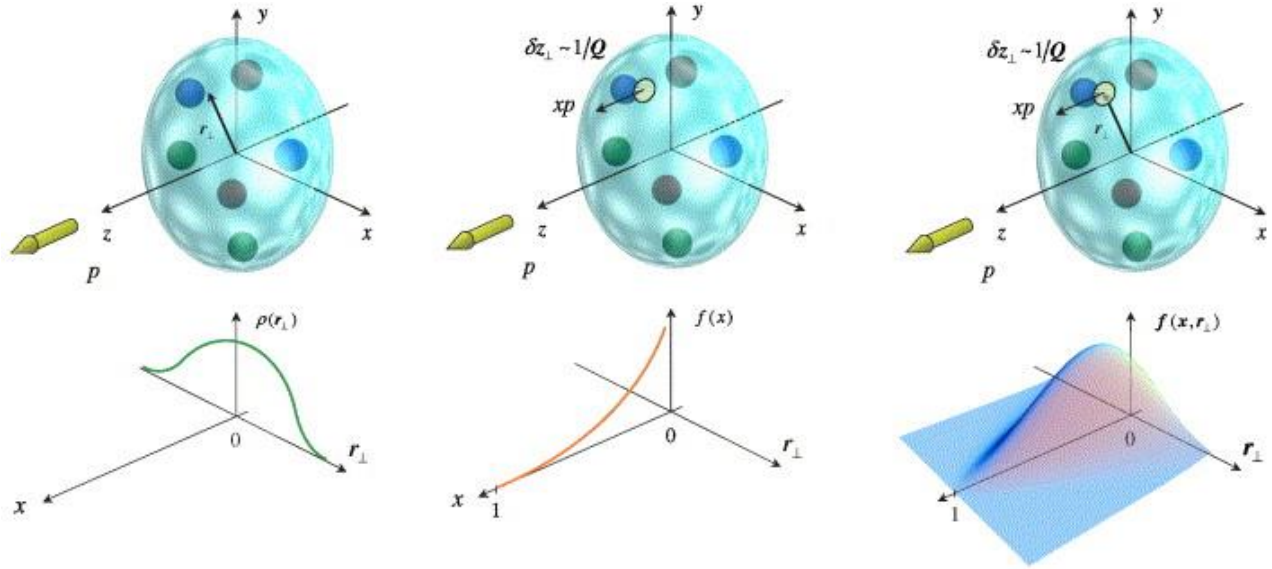
Single-ensemble result



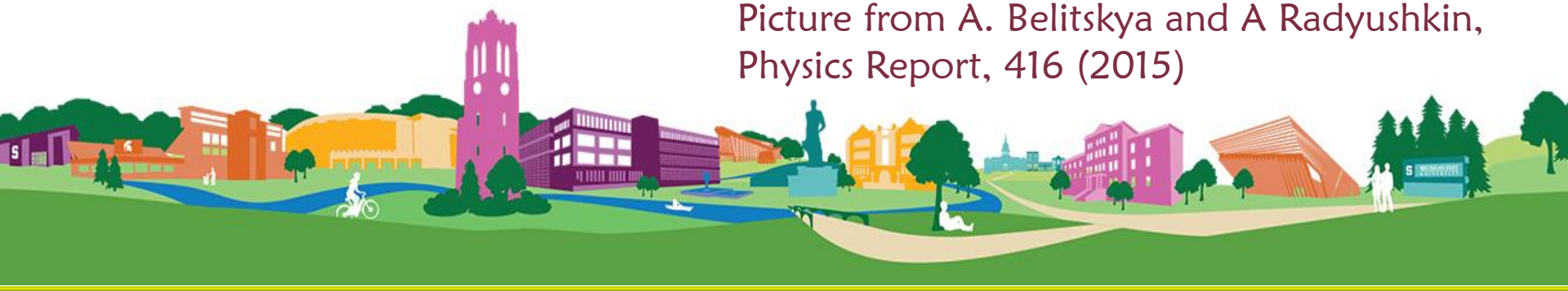
finite-volume,
discretization,
heavy quark mass,
...

Biased selected/highlighted results

Bjorken-x Dependent GPDs

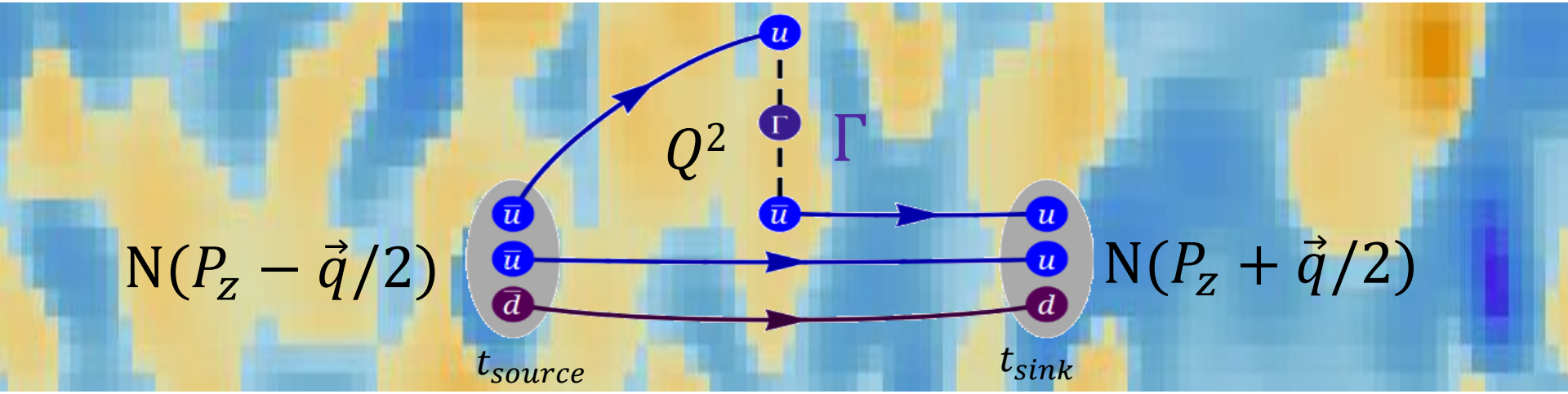


Picture from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)



Generalized Parton Distributions

§ On the lattice, one needs to calculate the following (nucleon example)

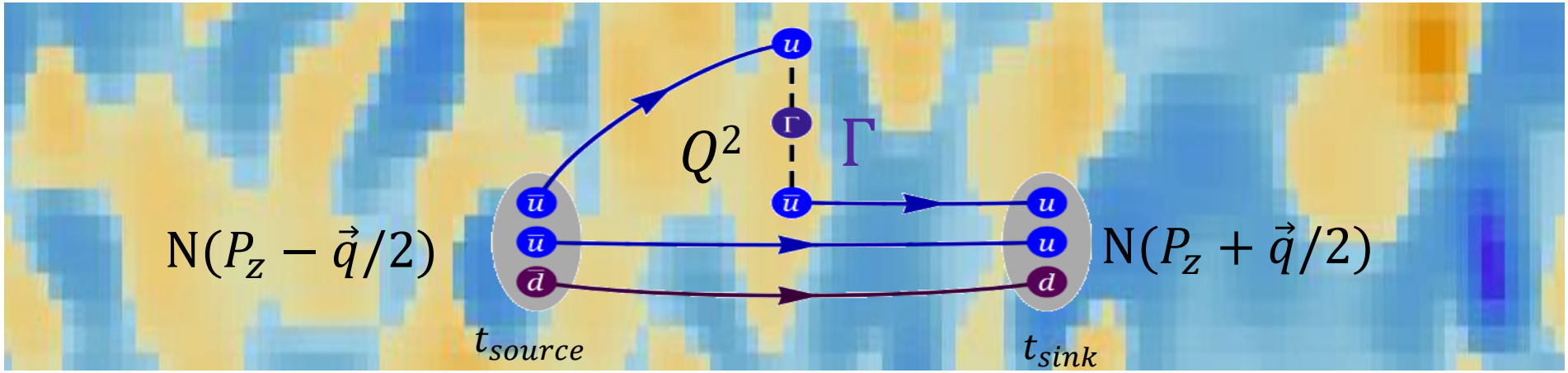


$$\tilde{F}\left(x,\tilde{\xi},t,\bar{P}_z\right) = \frac{\bar{P}_z}{\bar{P}_0}\int\frac{dz}{4\pi}e^{ixz\bar{P}z}\left\langle P'\middle|\tilde{O}_{\gamma_0}(z)\middle|P\right\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(H\left(x,\tilde{\xi},t,\bar{P}_z\right)\gamma^0 + E\left(x,\tilde{\xi},t,\bar{P}_z\right)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

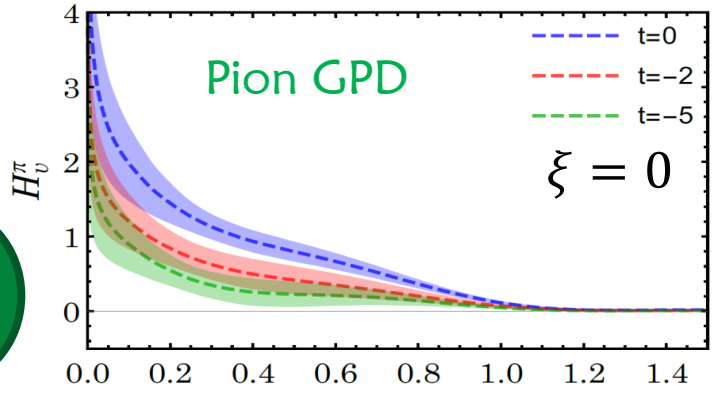
$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

Generalized Parton Distributions

§ On the lattice, one needs to calculate the following

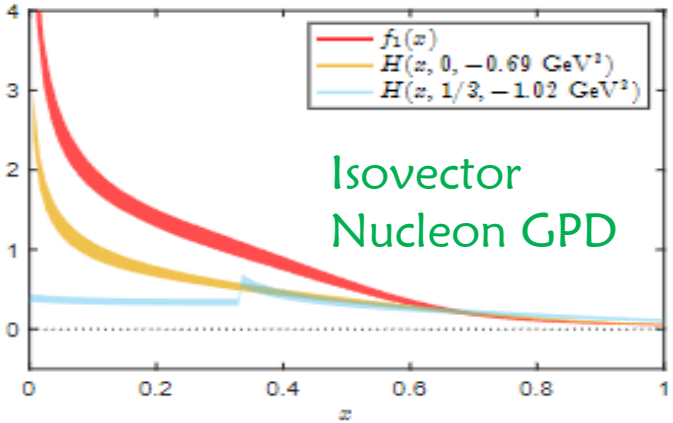


§ Heavy pion-mass results



Wanted PDFs, GPDs, etc…

J. Chen, HL, J. Zhang, 1904.12376



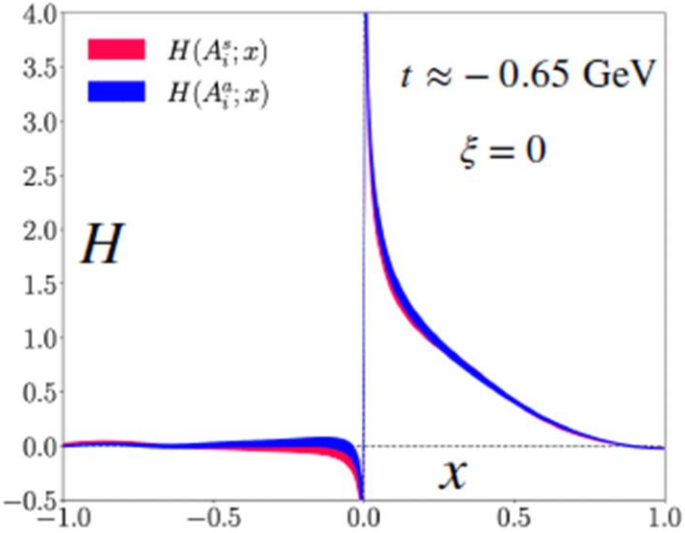
C. Alexandrou et al, 2008.10573

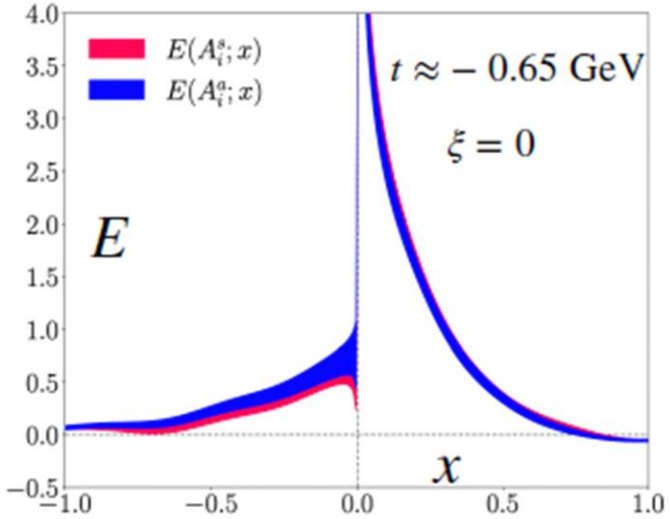
Ge

§ Or

See Th

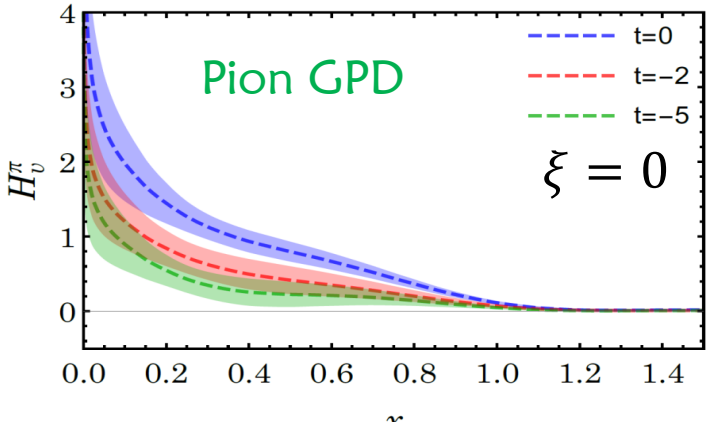
Shohini Bhattacharya *et al.*, arXiv:2209.05373 [hep-lat]



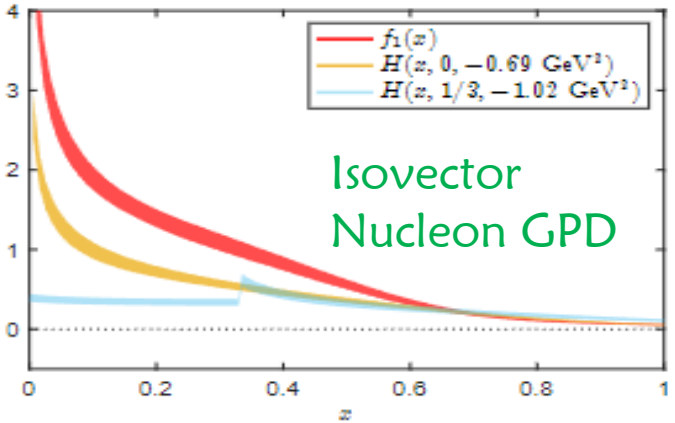


260-MeV results for asymmetric setup for GPD calculations to save computing time

§ Heavy pion-mass results



J. Chen, HL, J. Zhang, 1904.12376



C. Alexandrou et al, 2008.10573

2008.10573

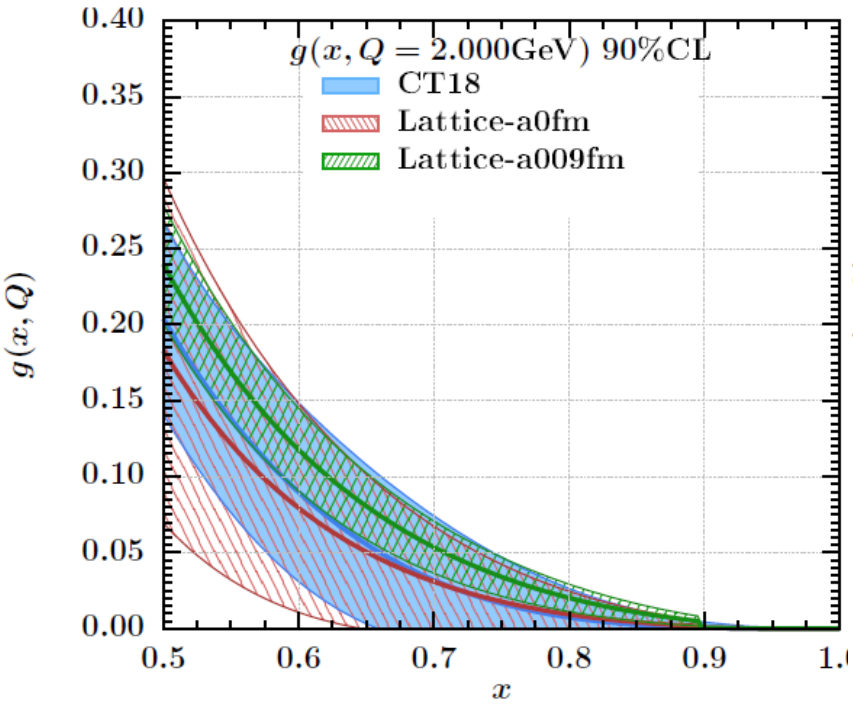
Lattice Gluon PDF Impact

§ Preliminary study with CTEQ-TEA analysis

- Take lattice inputs in the region where no strong experimental data constraints, x ∈ [0.4,0.7]
- Using e-pump for re-weighting

Plots by Alim Ablat (Xinjiang U.)

Before



After