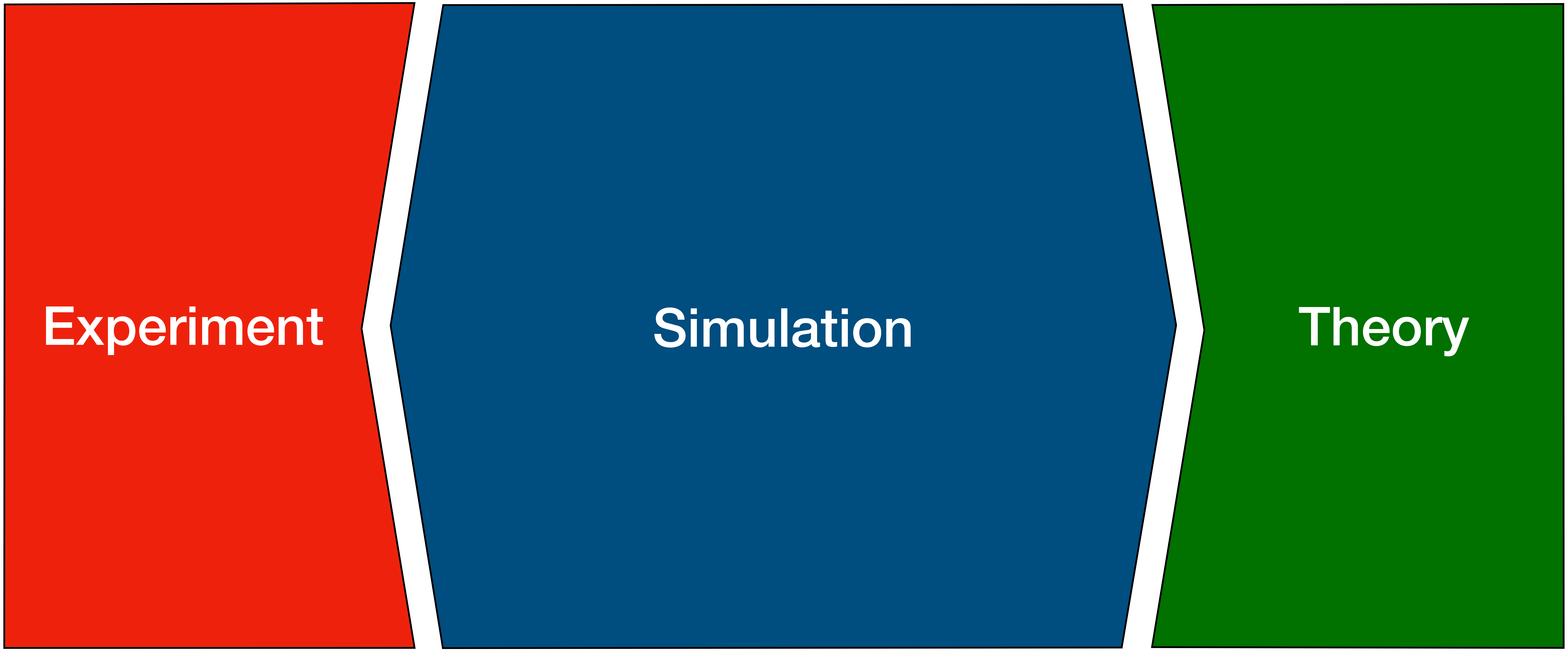
QCD and Quantum Computing

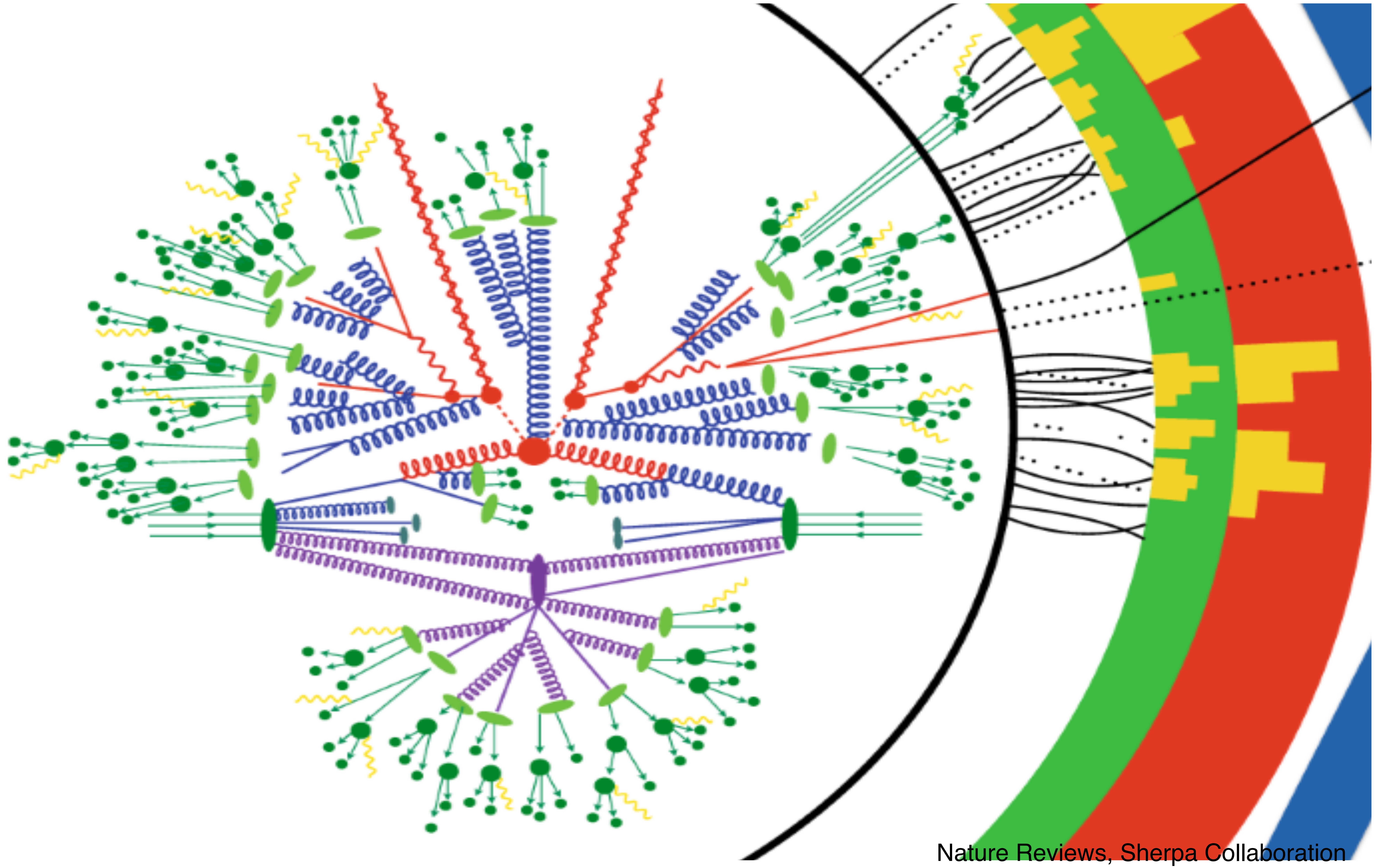
First-principles simulation of non-perturbative physics

Christian Bauer
Theory Group Leader
PI Quantum Computing
Physics Division LBNL

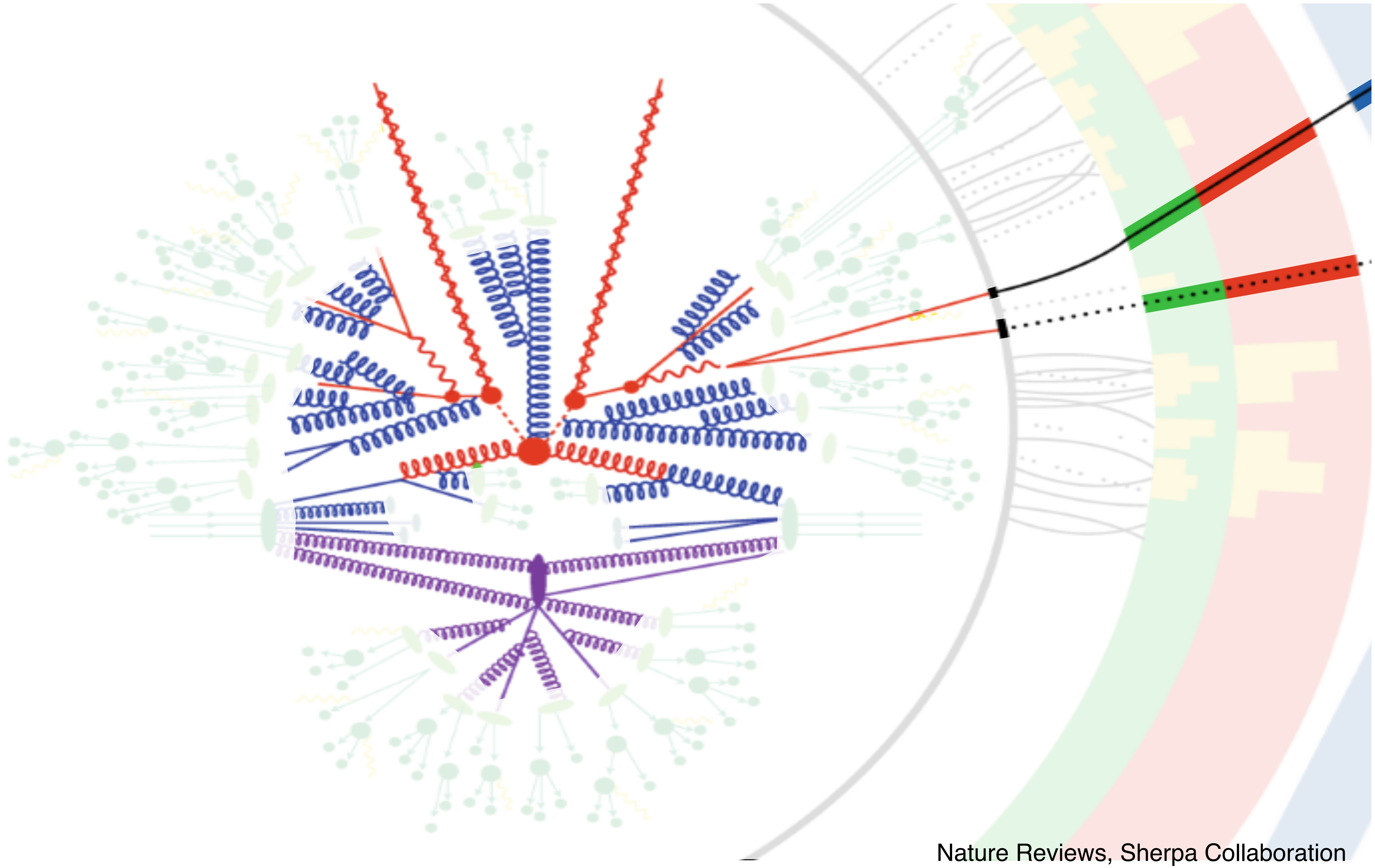
Experiment

Theory


Experiment
Simulation
Theory



Nature Reviews, Sherpa Collaboration



Nature Reviews, Sherpa Collaboration

Experiment

Simulation

Theory


Experiment
Theory


Experiment
Theory

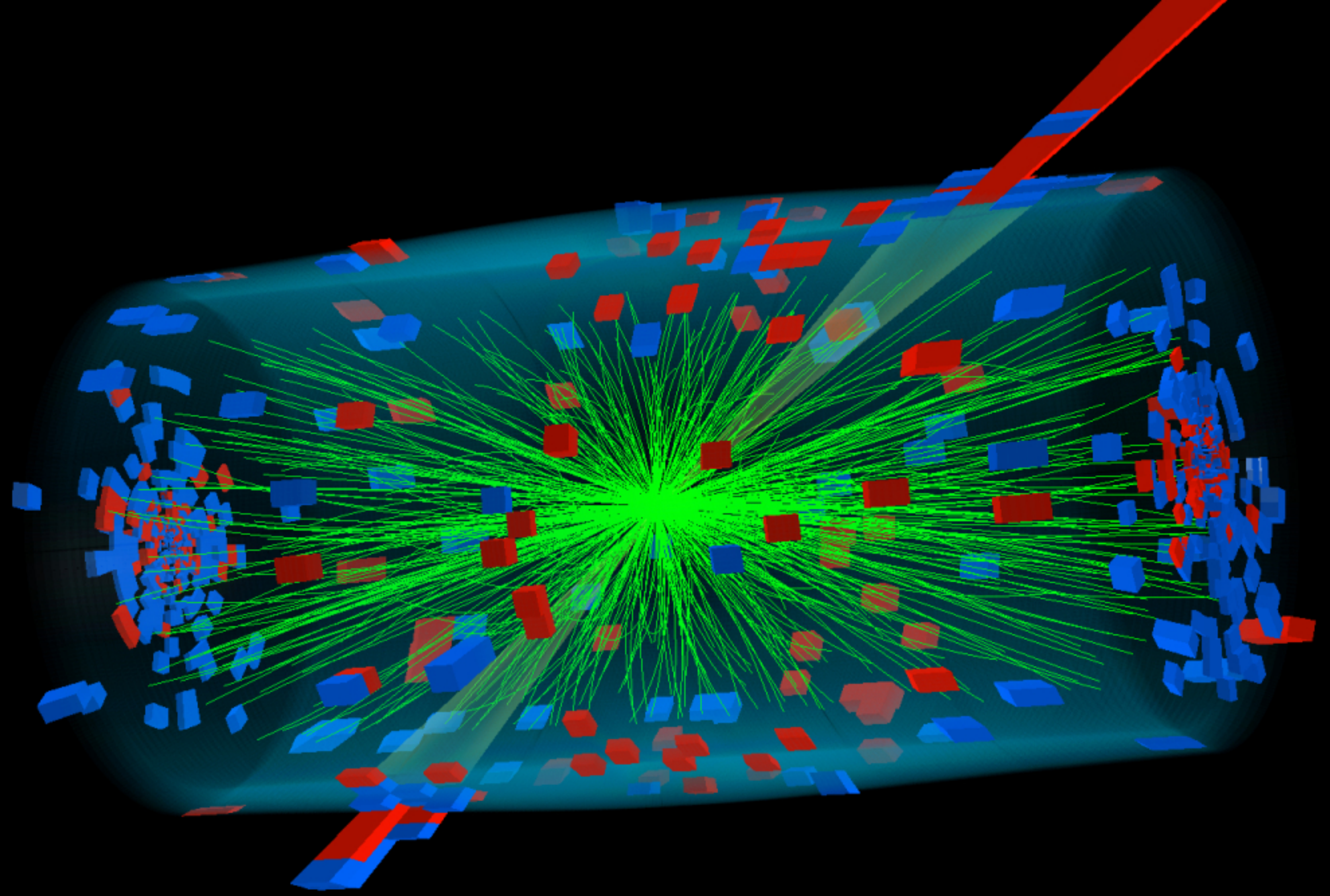
Experiment

Theory

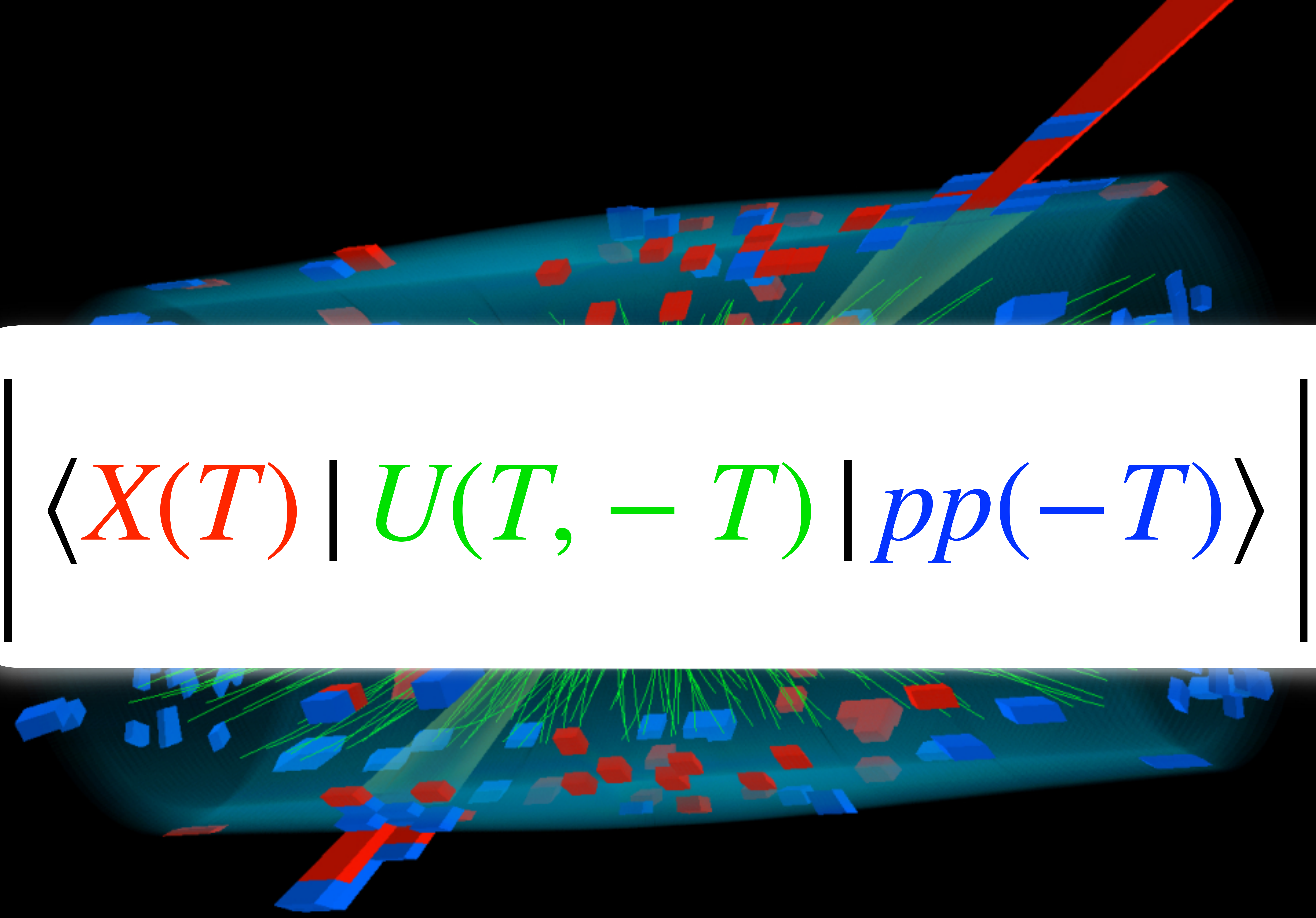
Experiment

Simulation

Theory

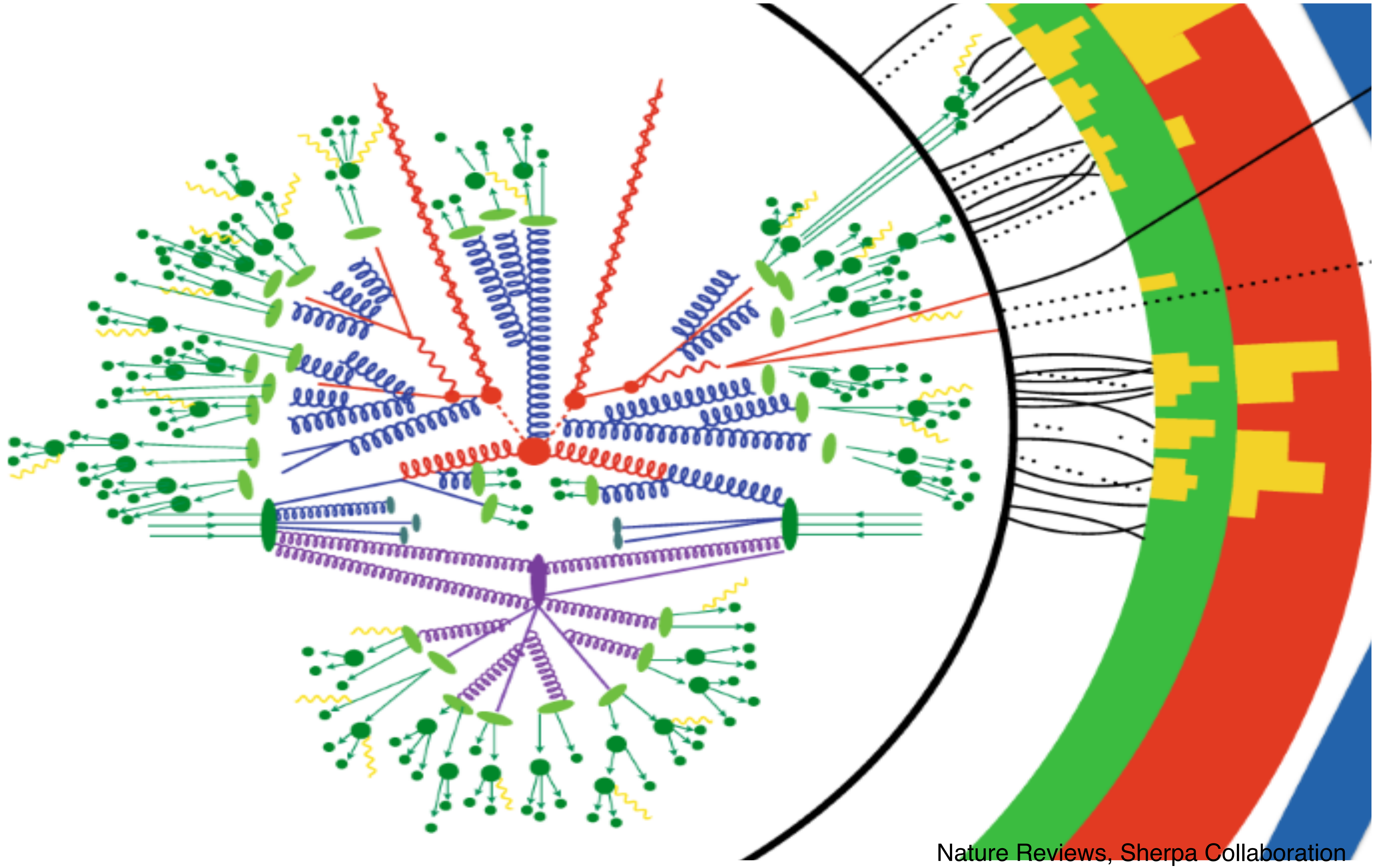


CMS Run/Event 262548/458269



$$\left| \langle X(T) | U(T, -T) | pp(-T) \rangle \right|^2$$

CMS Run/Event 262548/458269



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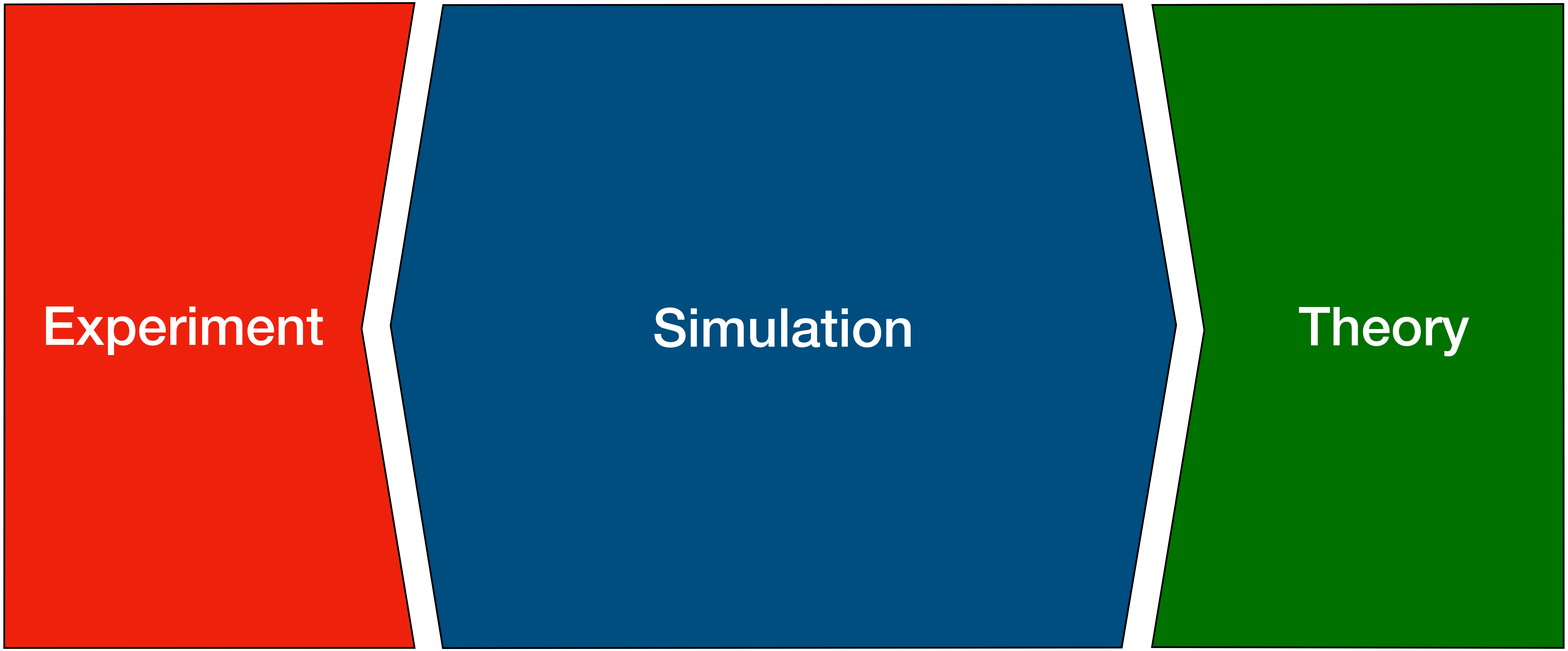
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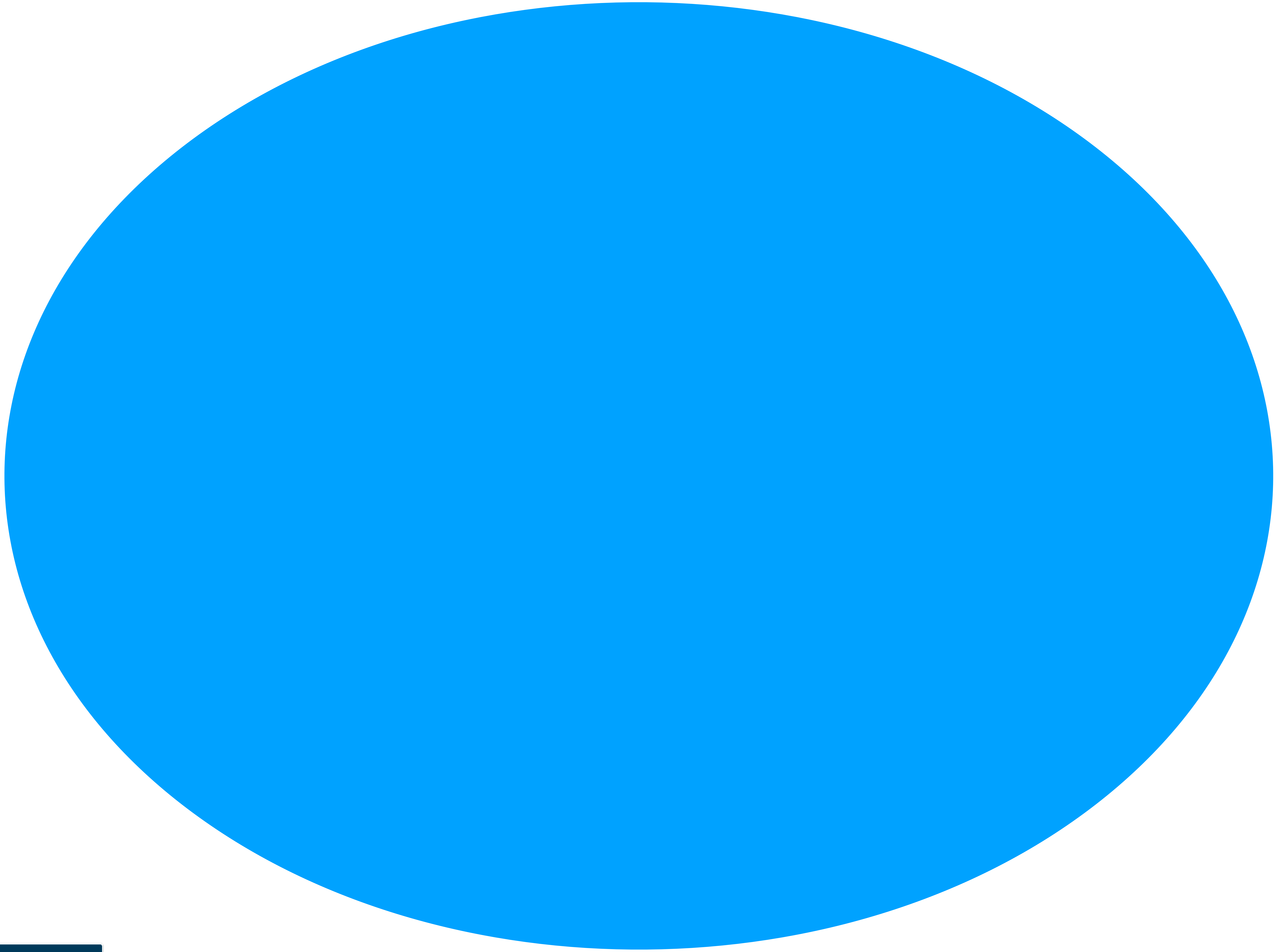
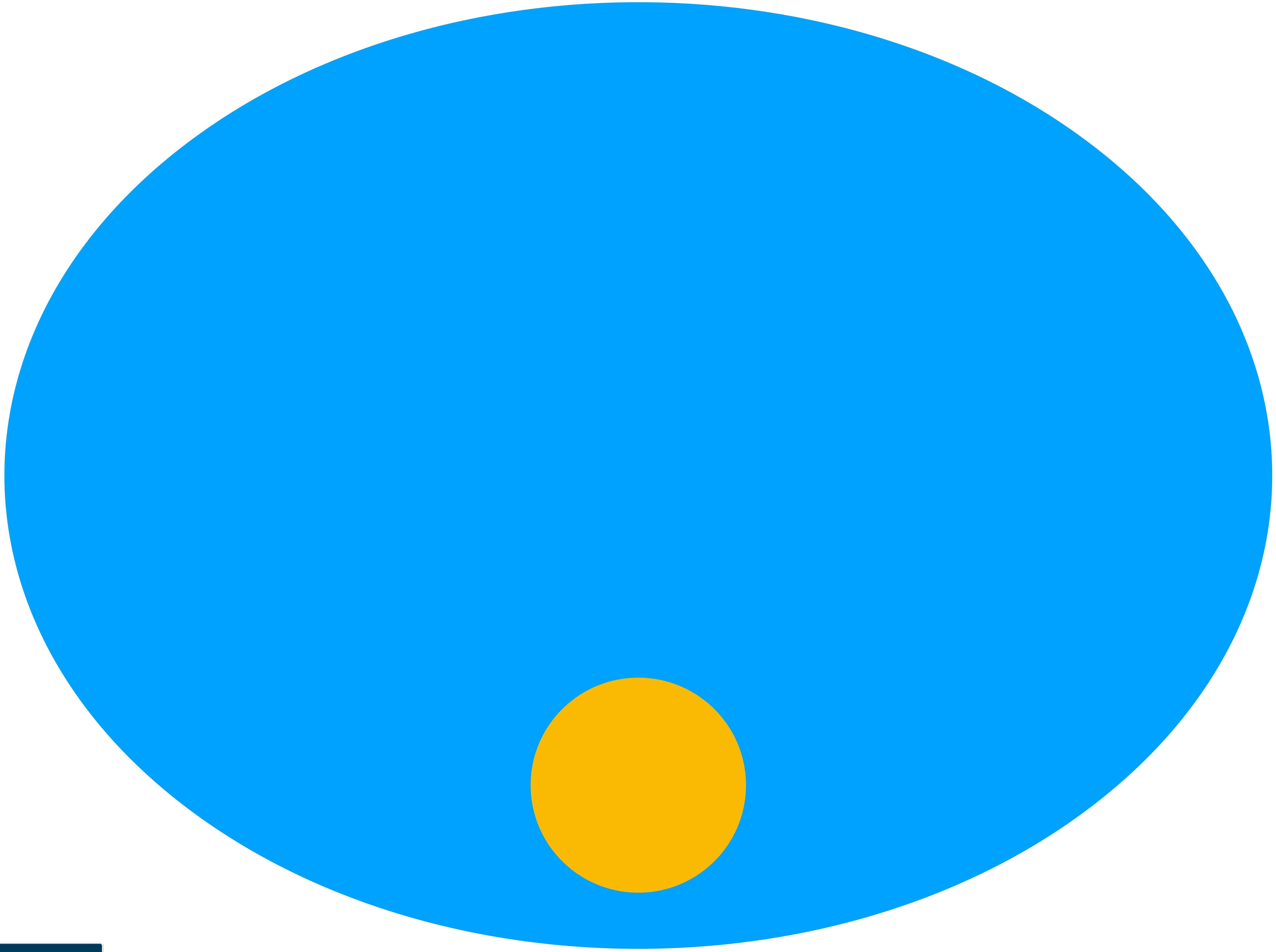
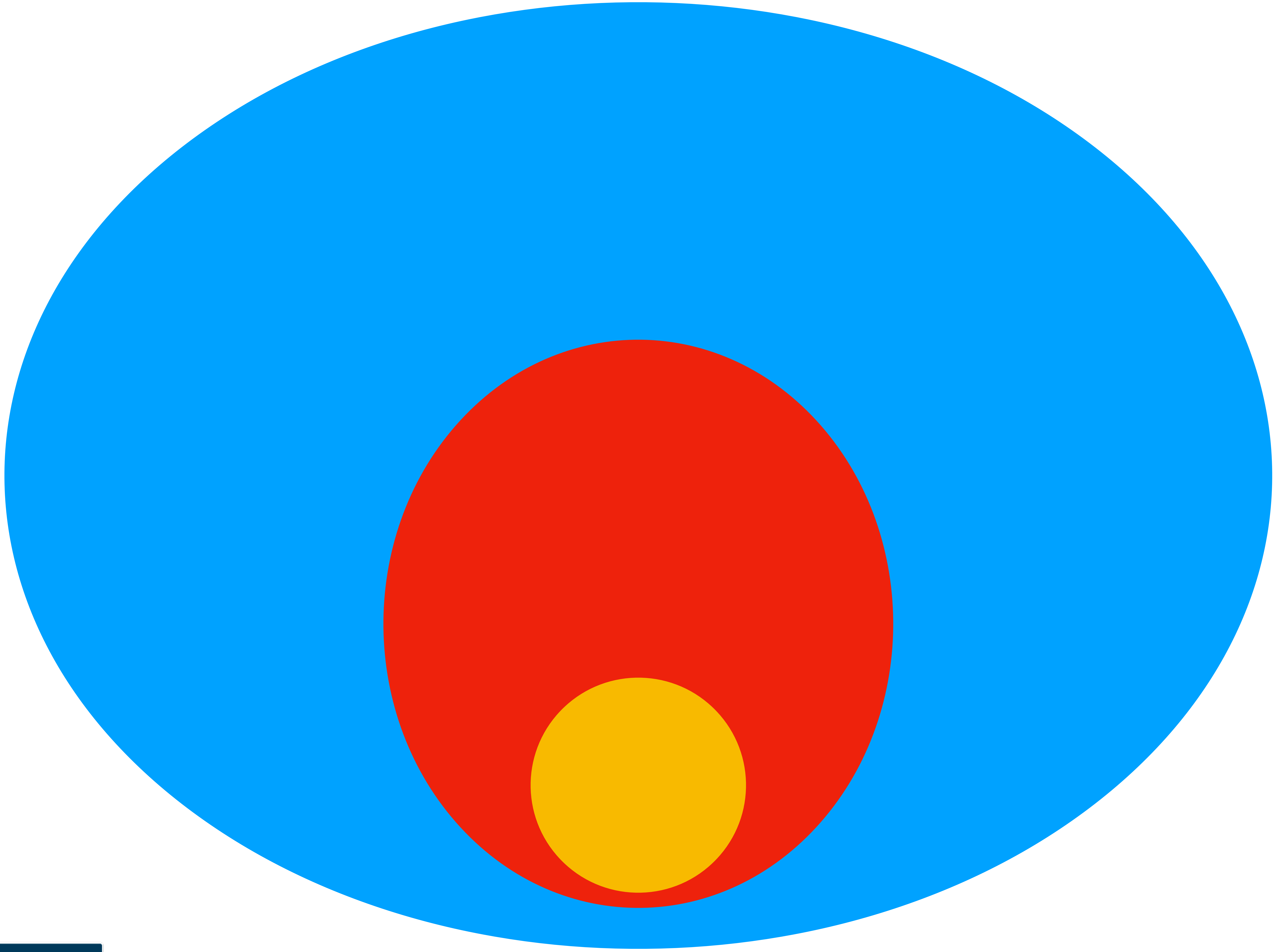


All computational Problems



All computational Problems

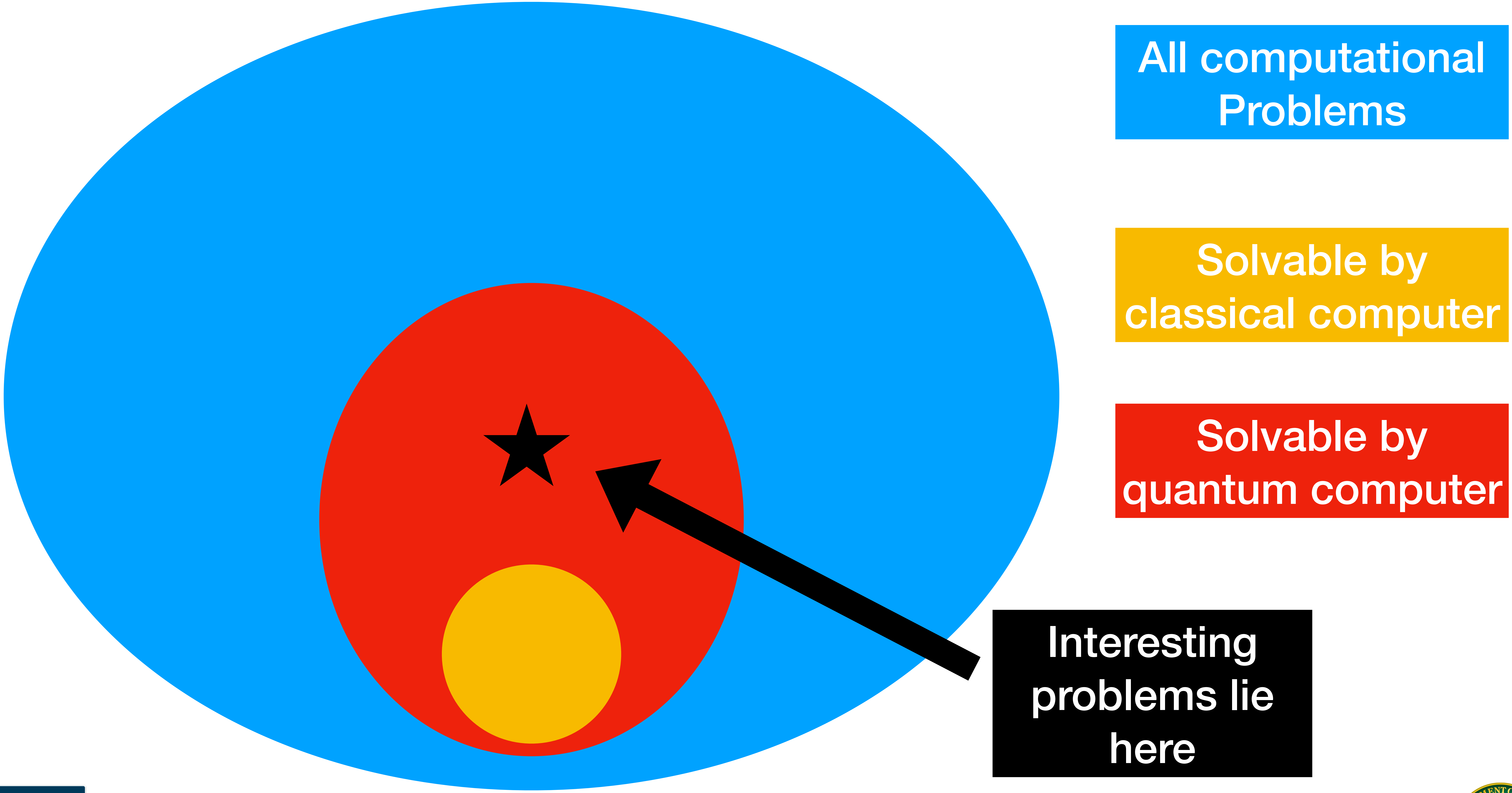
Solvable by classical computer

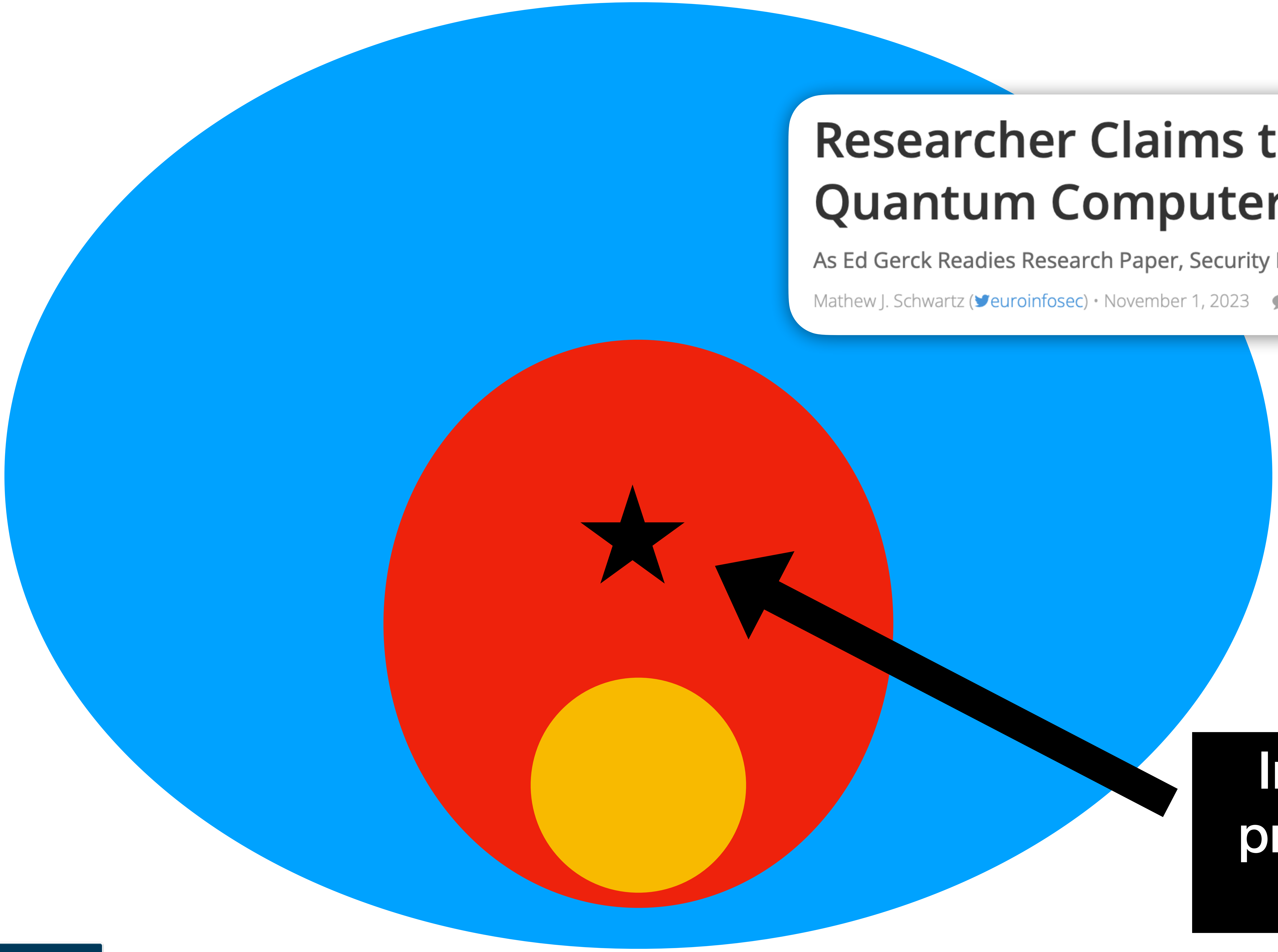


All computational Problems

Solvable by classical computer

Solvable by quantum computer


All computational Problems
Solvable by classical computer
Solvable by quantum computer
Interesting problems lie here



All computational

Researcher Claims to Crack RSA-2048 With Quantum Computer

As Ed Gerck Readies Research Paper, Security Experts Say They Want to See Proof

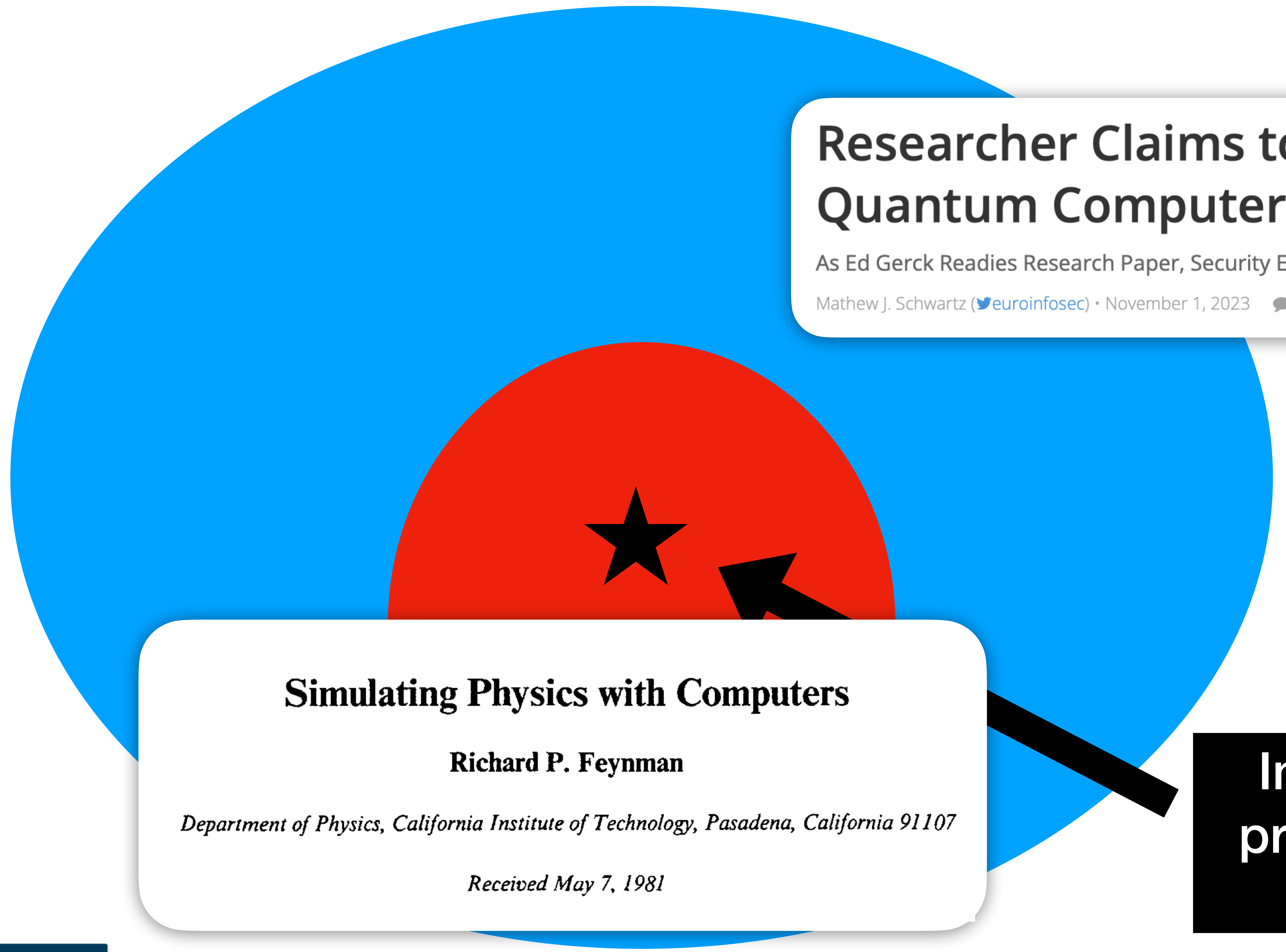
Mathew J. Schwartz (euroinfosec) • November 1, 2023

classical computer

Solvable by quantum computer

Interesting problems lie here

BERKELEY LAB



All computational

classical computer

Solvable by quantum computer

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Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

All computational

Researcher Claims to Crack RSA-2048 With

They Want to See Proof

assical computer

Solvable by quantum computer

There are many HEP problems of this kind
collider physics, neutrino physics, cosmology, early universe physics, quantum gravity etc

Recent review: CWB, Z Davoudi et al, Quantum Simulation for HEP (2204.03381)

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

Interesting problems lie here

Standard approach to nonperturbative simulations: Lattice Gauge Theory, which performs path integral using Monte-Carlo integration

Requires positive definite integrand, imaginary time

$$e^{iS[\phi_j(x_i))]} \rightarrow e^{-S[\phi_j(x_i))]}$$

Can answer many static questions, but calculating dynamics requires real time, not imaginary time

Instead of doing Monte-Carlo simulation of path integral, can try to do time evolution using Schrödinger equation

Go back to the S matrix elements mentioned before

$$\left| \langle X(T) \,|\, U(T, -T) \,|\, pp(-T) \rangle \right|^2$$

All elements in this expression in terms of fields $\phi(x)$
Both position x and field $\phi(x)$ are continuous

Discretizing position x and digitizing field value $\phi(x)$ turn continuous (QFT) problem into discrete (QM) problem

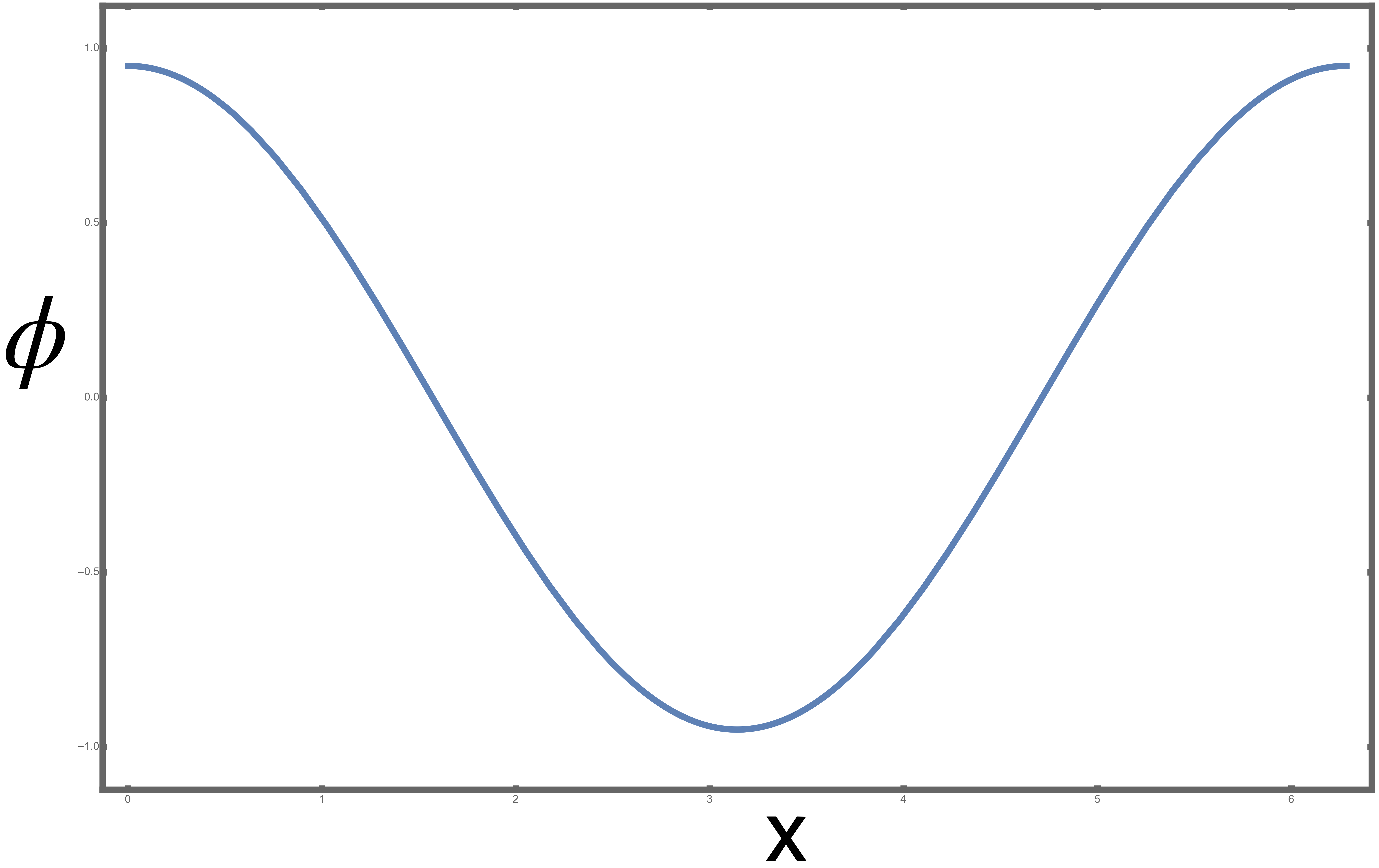
Basic idea is to map the infinite Hilbert space of QFT on a finite dimensional HS making this a QM problem

$$\left| \langle X(T) | U(T, -T) | pp(-T) \rangle \right|^2$$

3 basic steps:

1. Create an initial state vector at time (-T) of two proton wave packets
2. Evolve this state forward in time from to time T using the Hamiltonian of the full interacting field theory
3. Perform a measurement of the state

To yield finite-dimensional Hilbert space, have field configurations sample position and field values at discrete points



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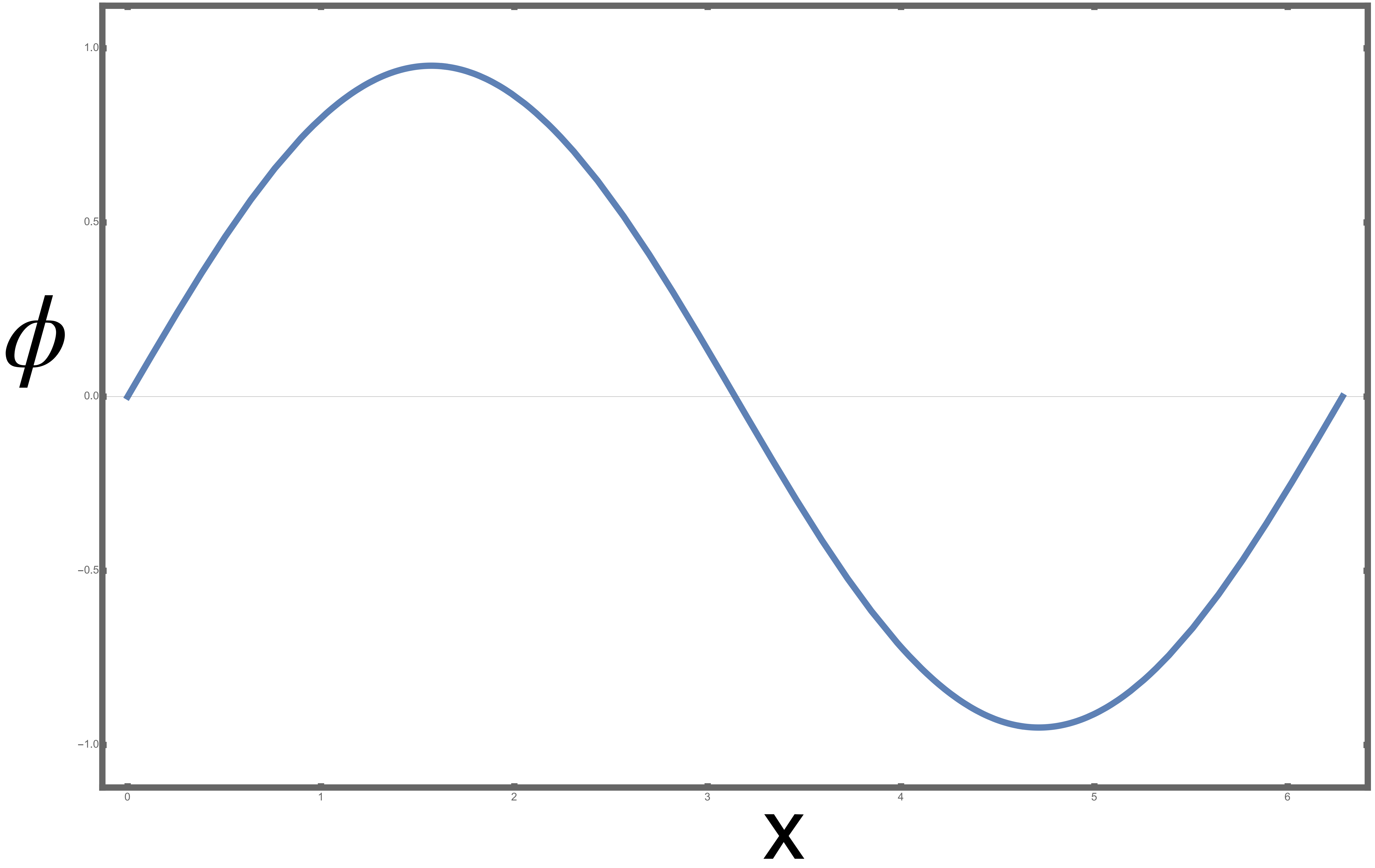

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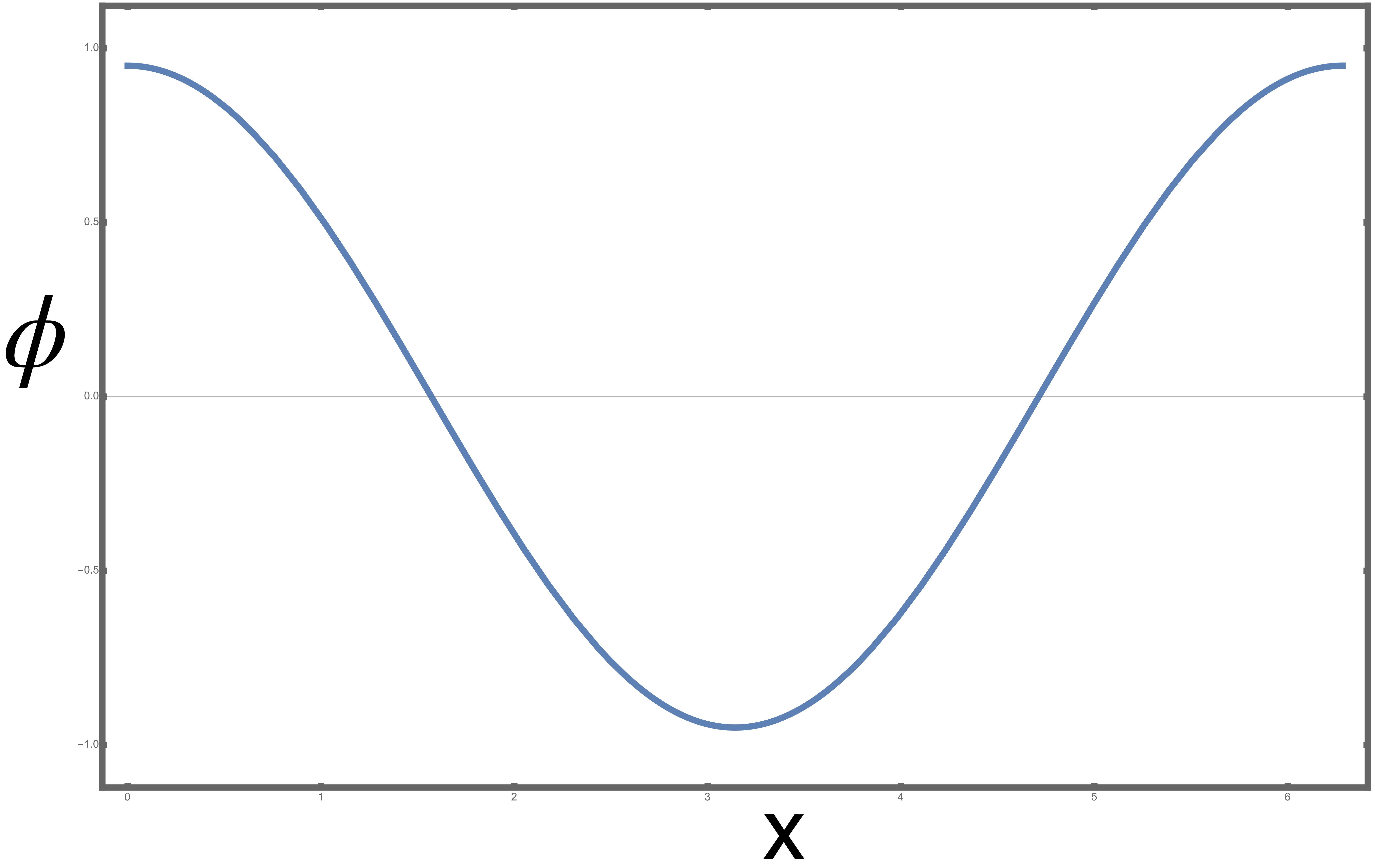

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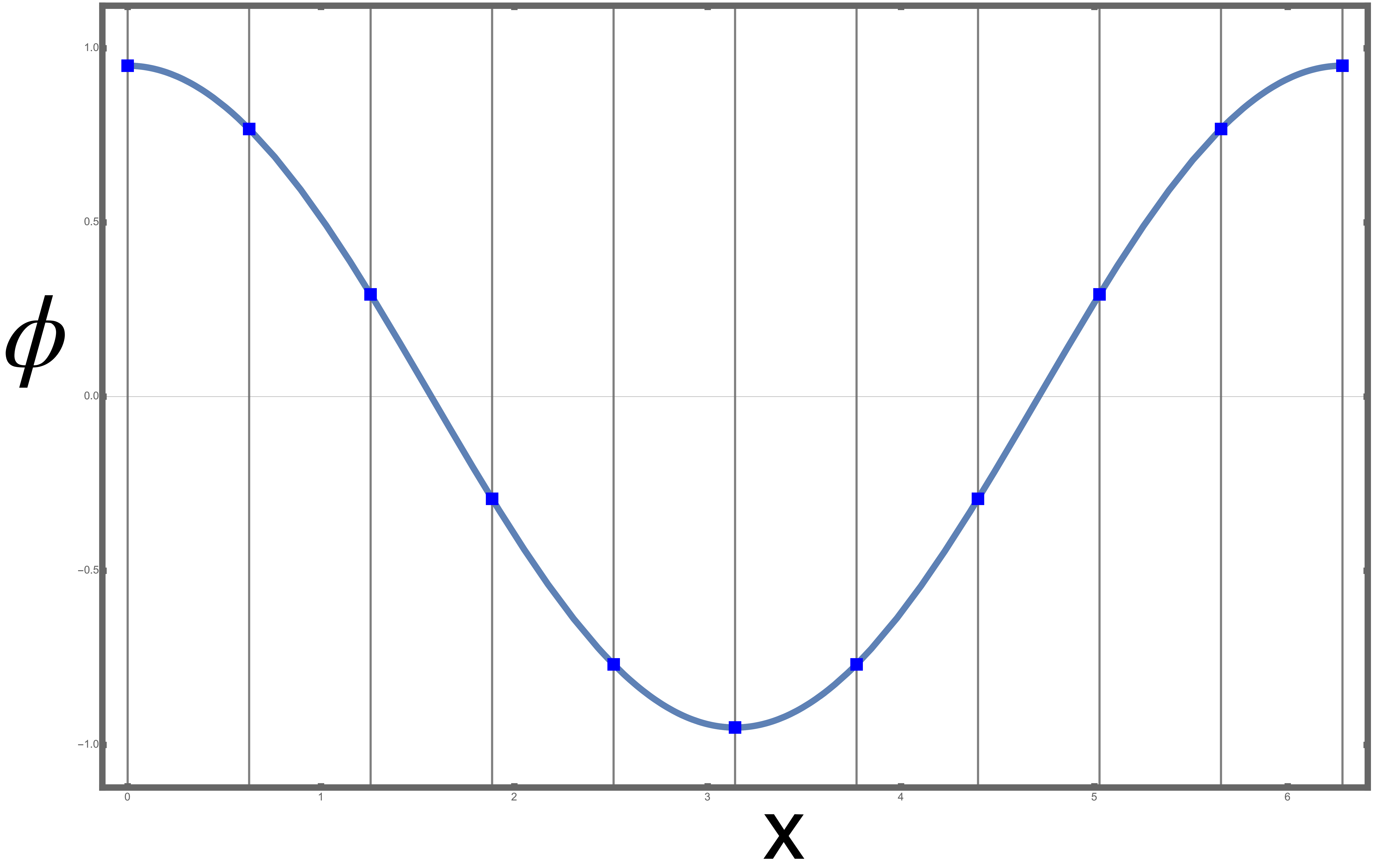

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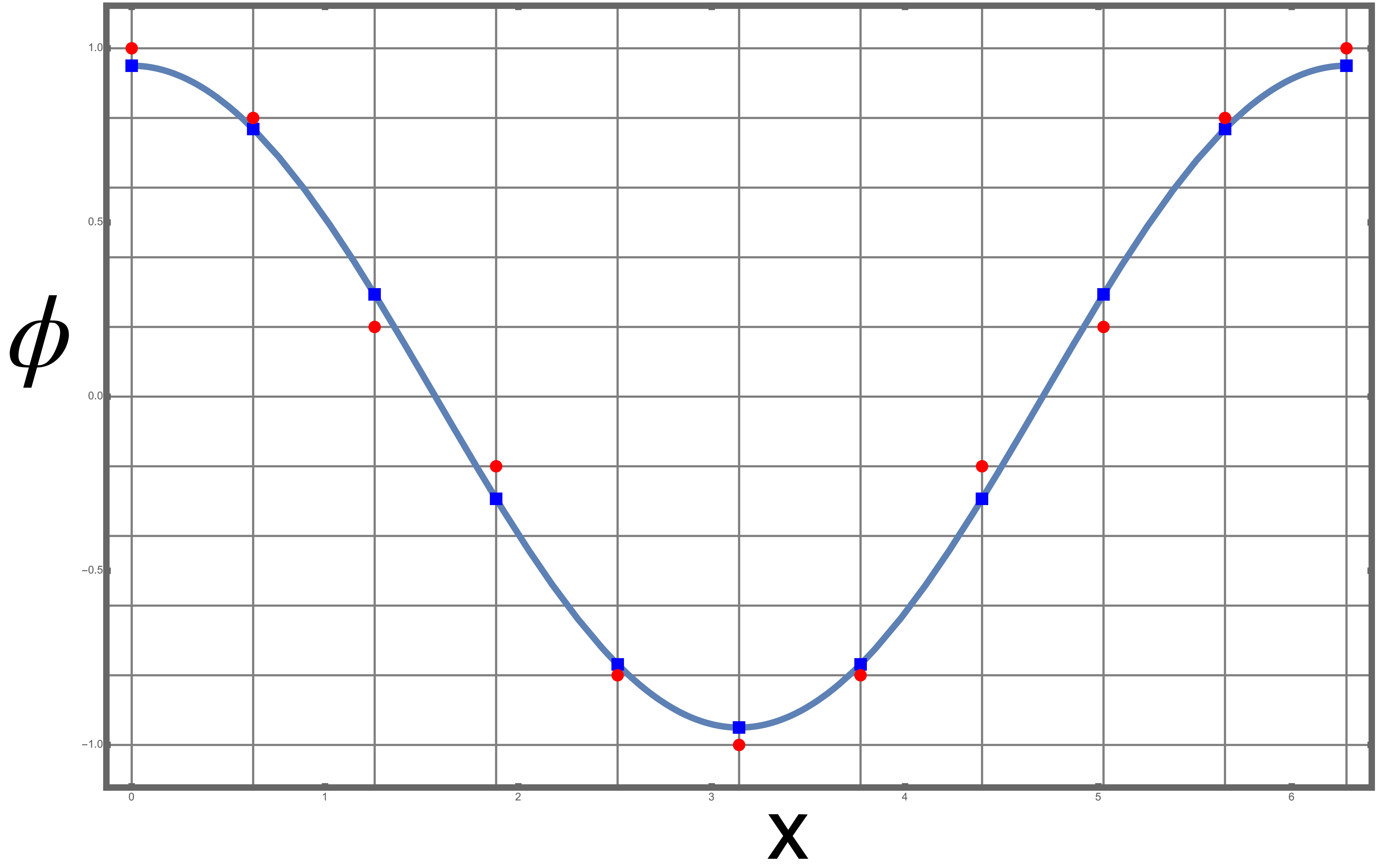

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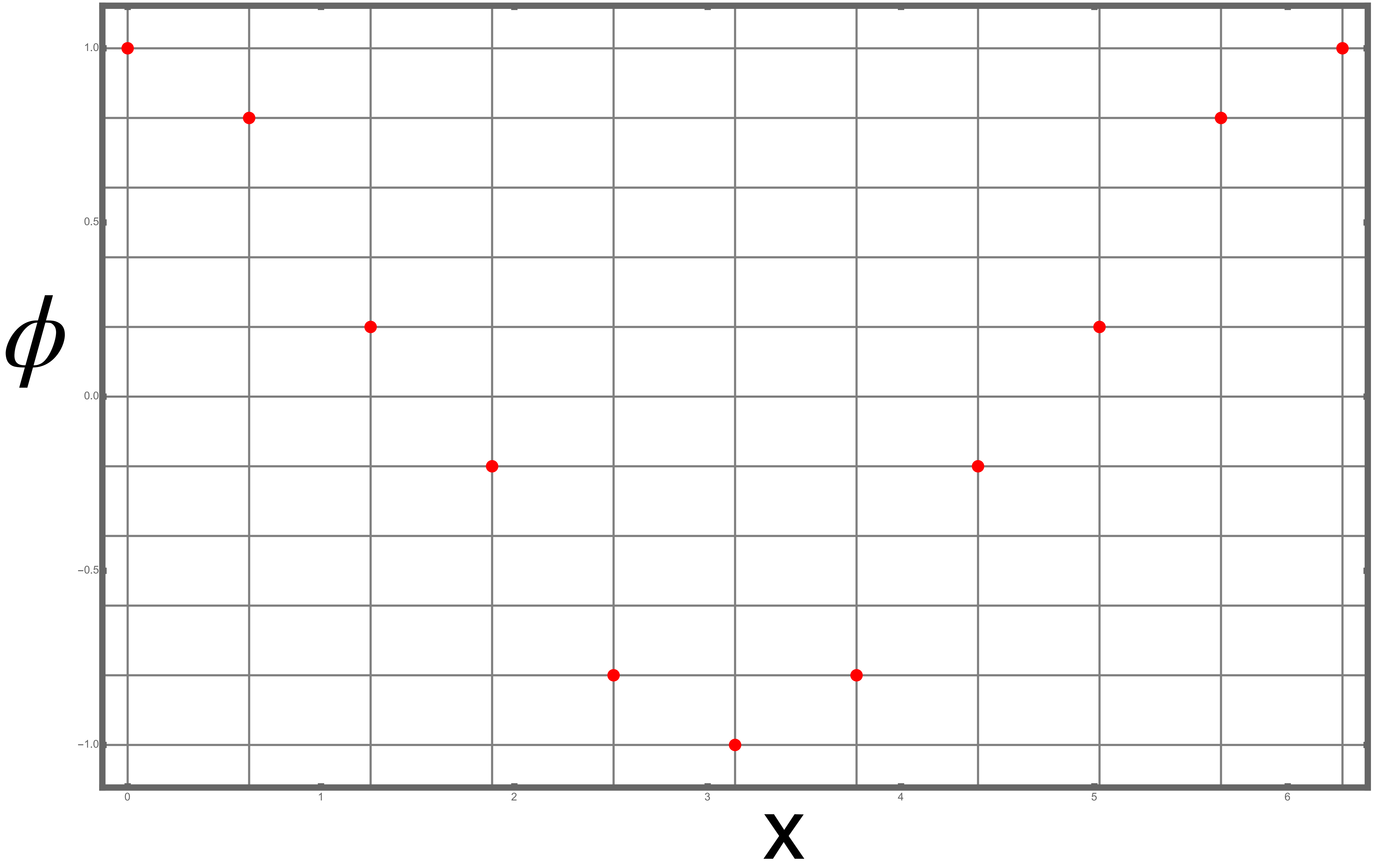

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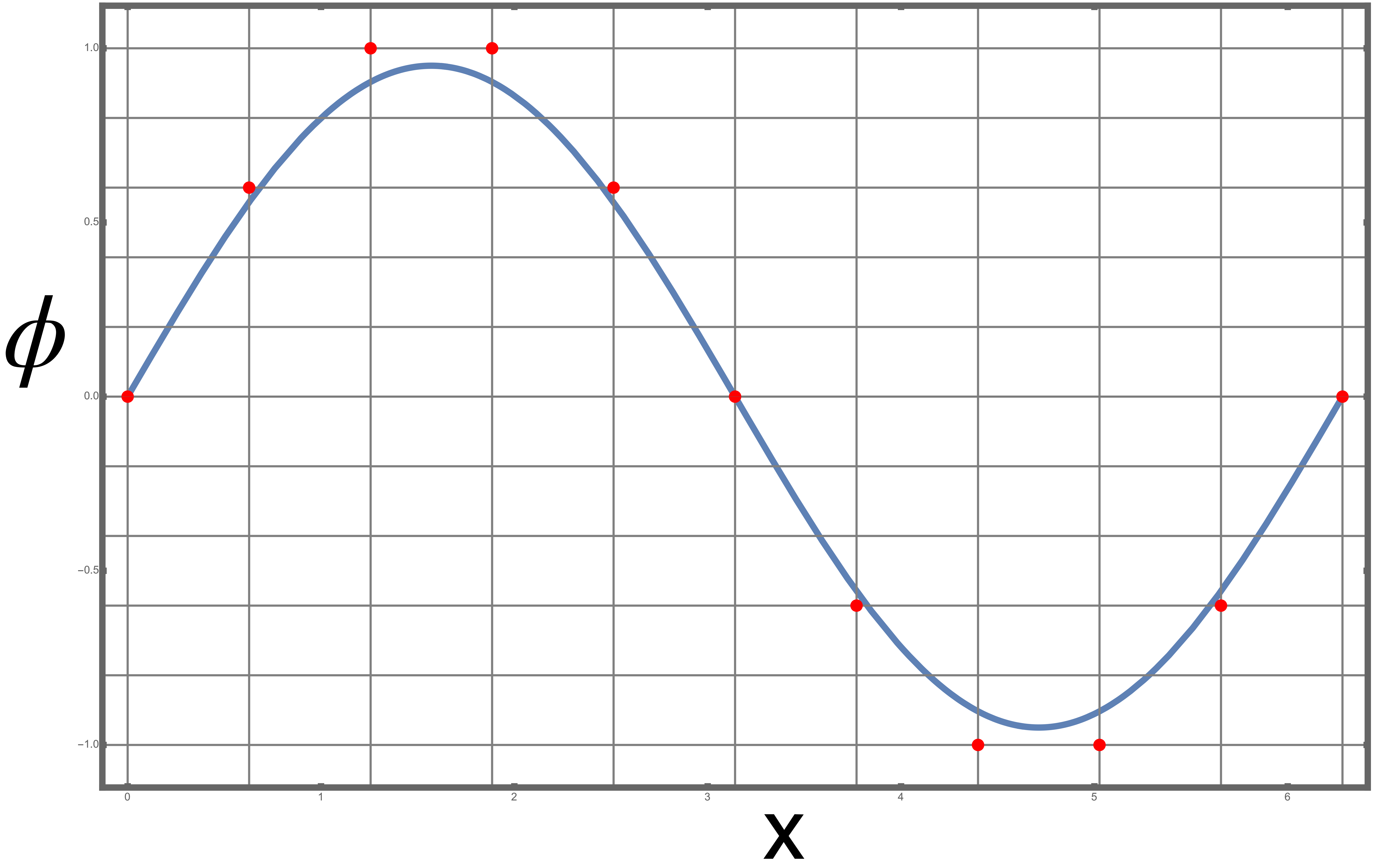

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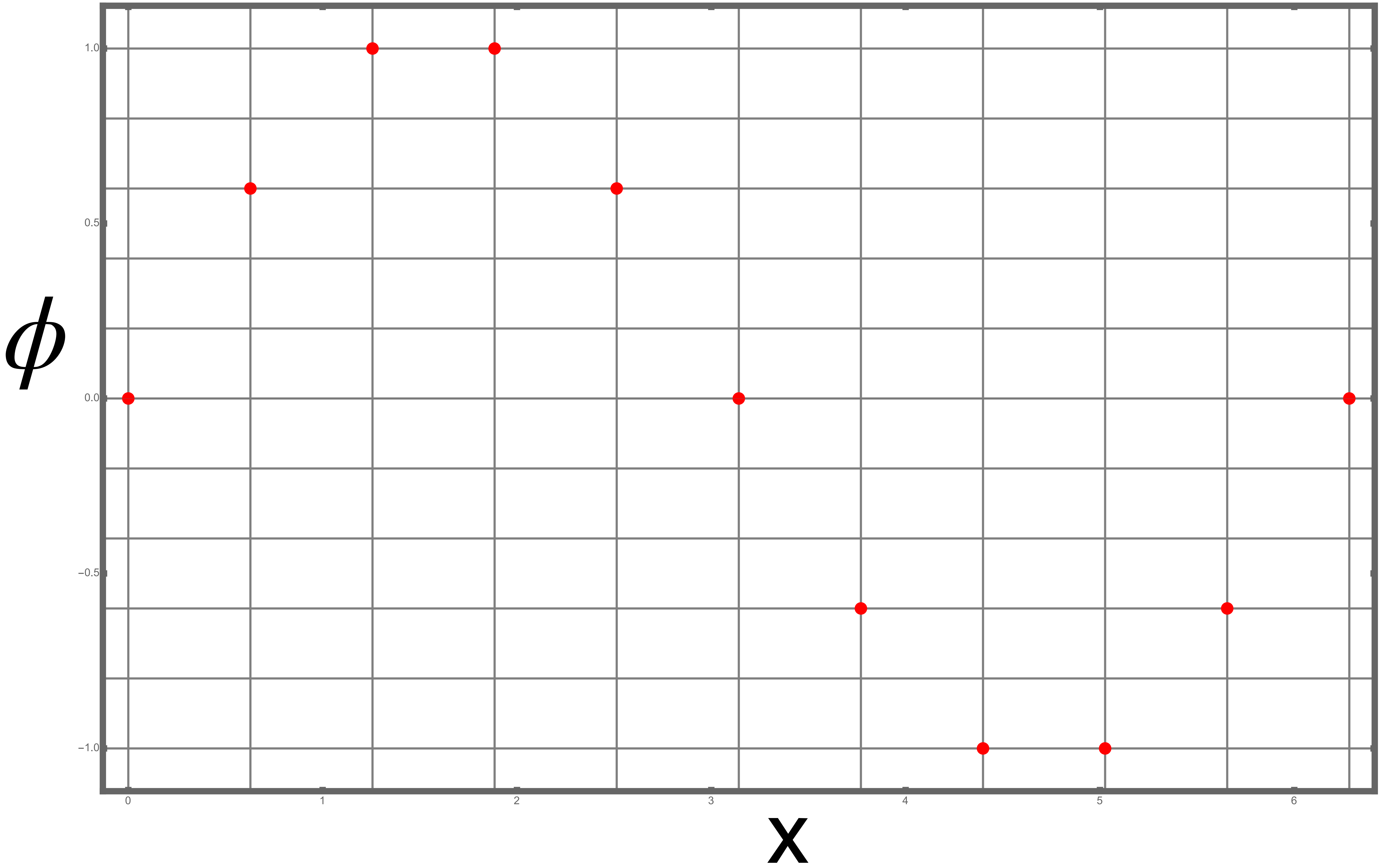

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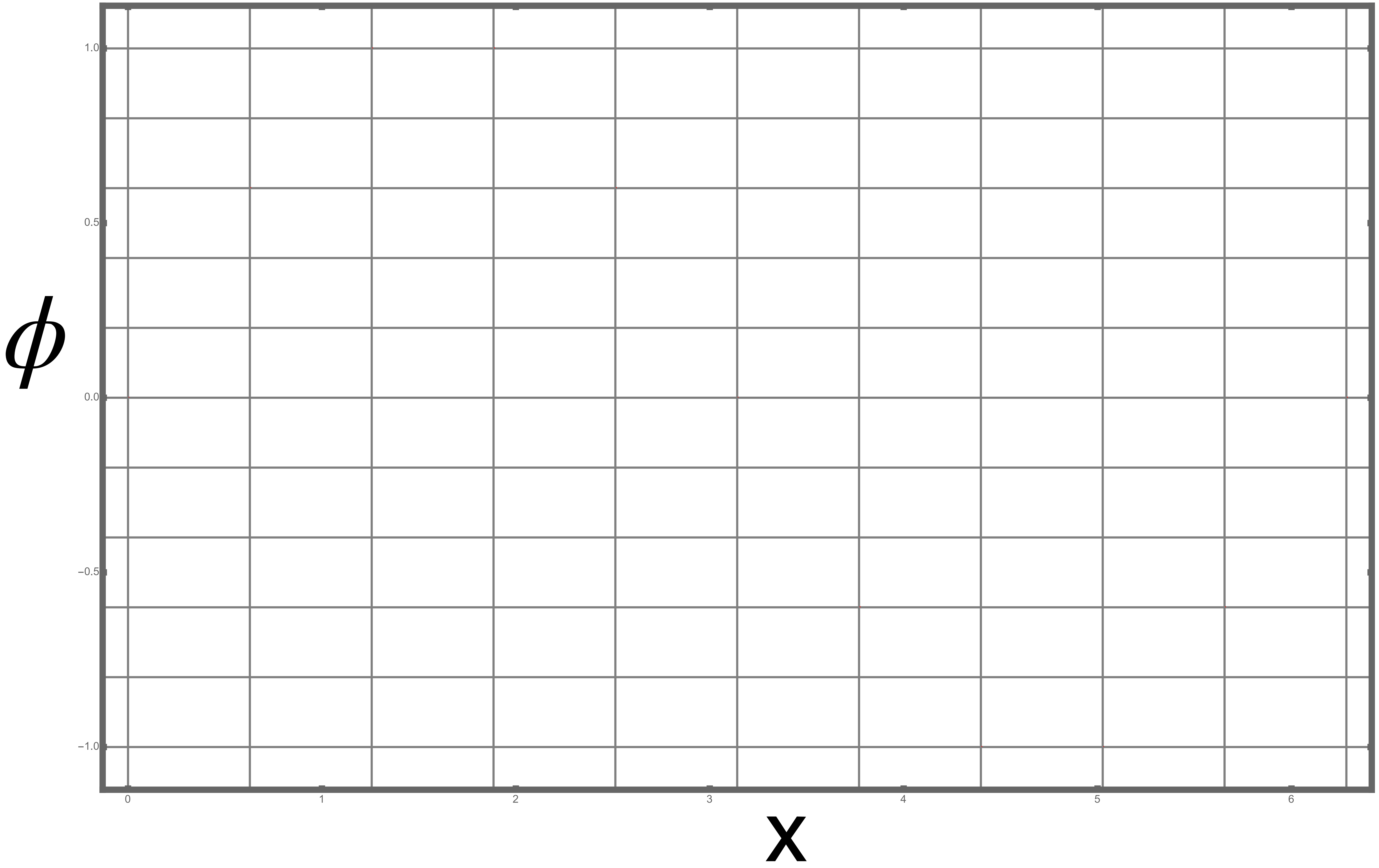

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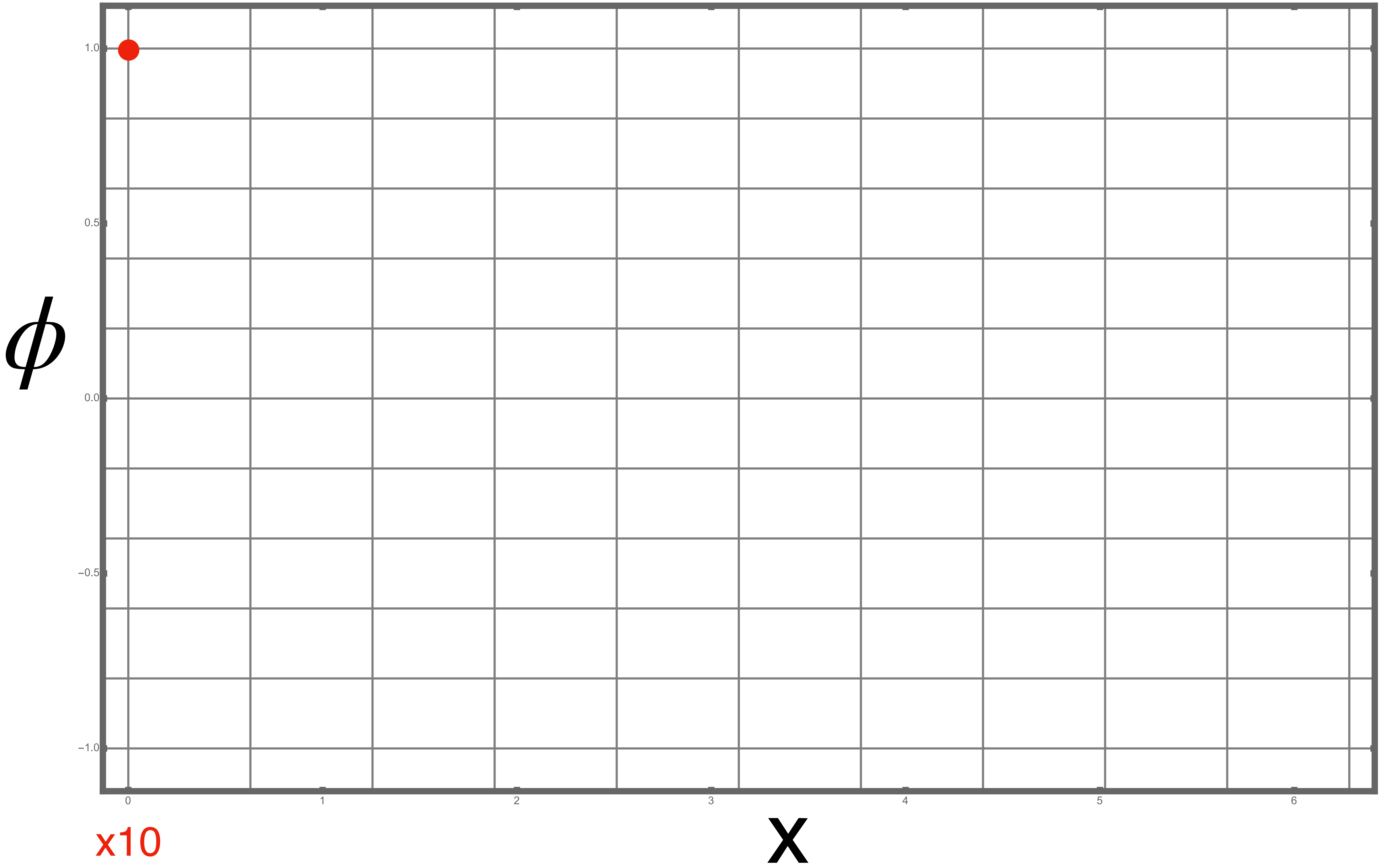

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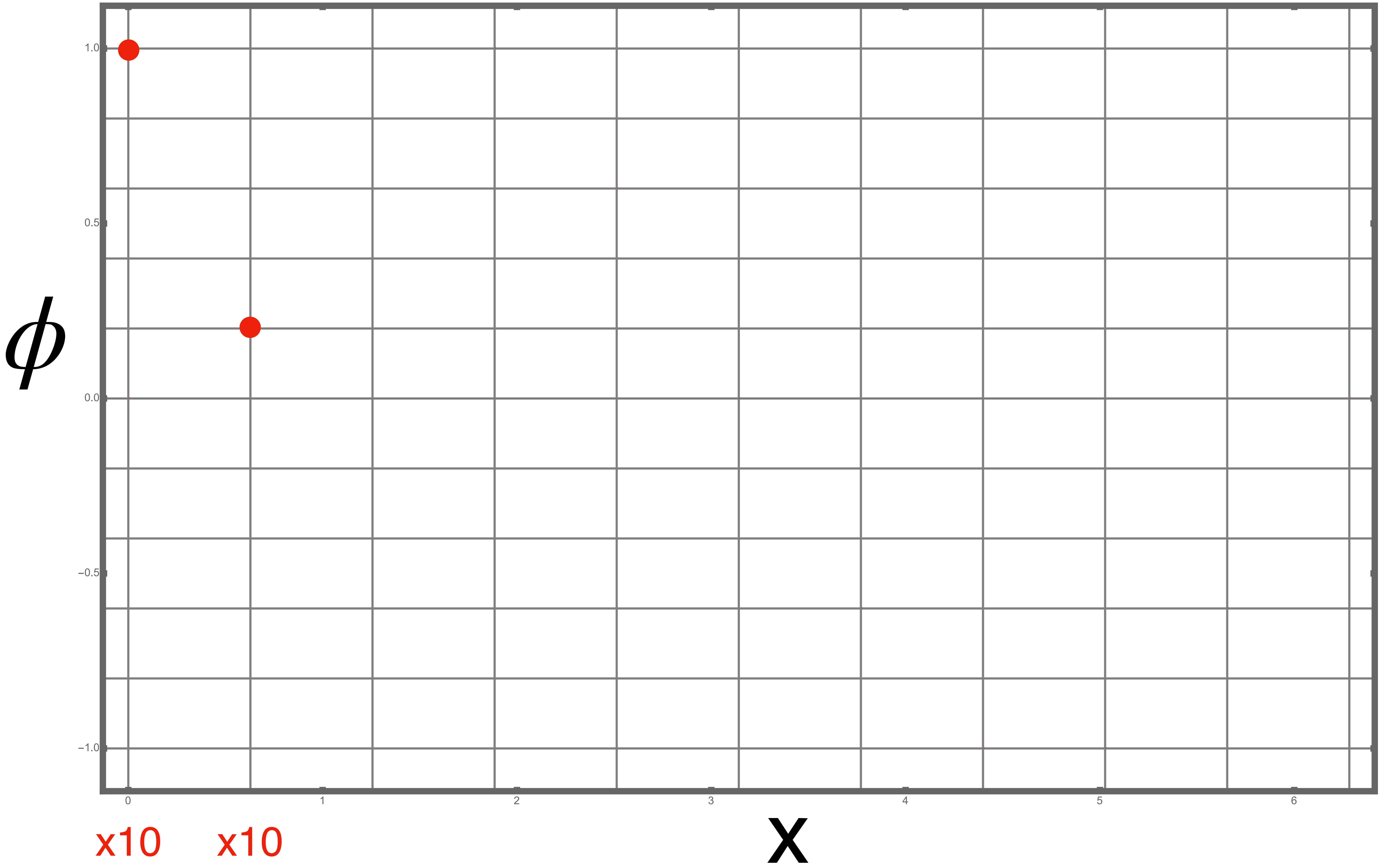

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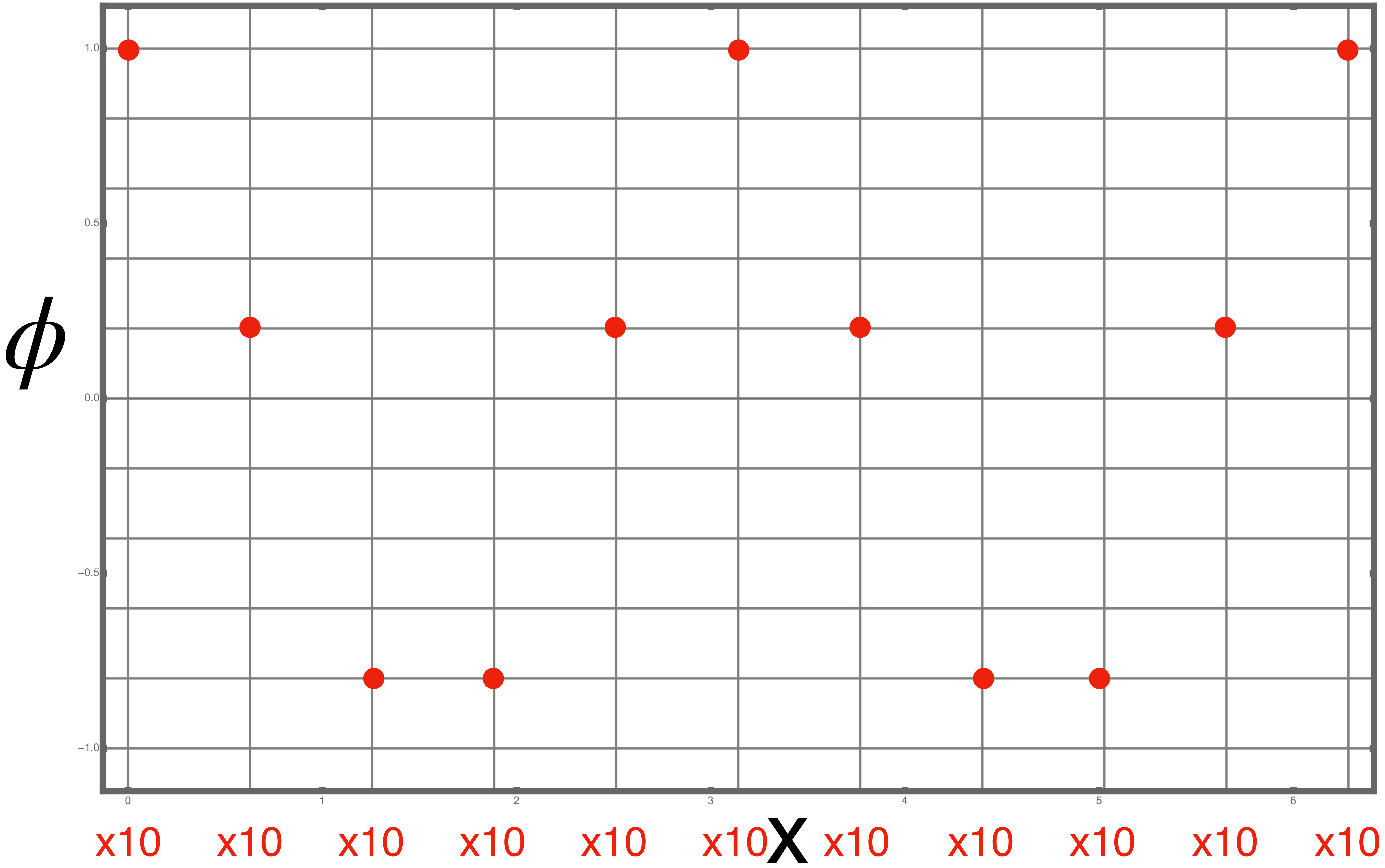

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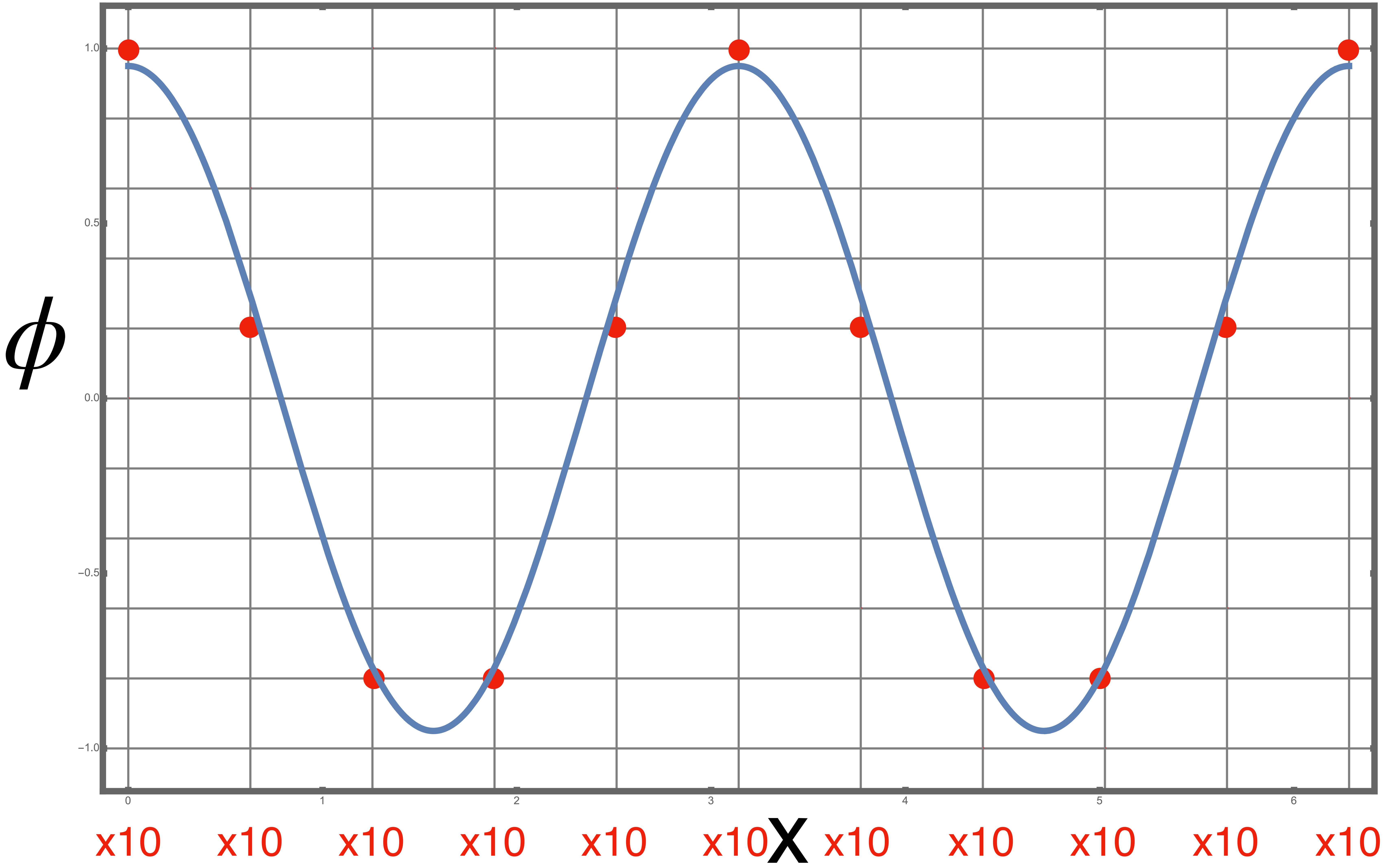

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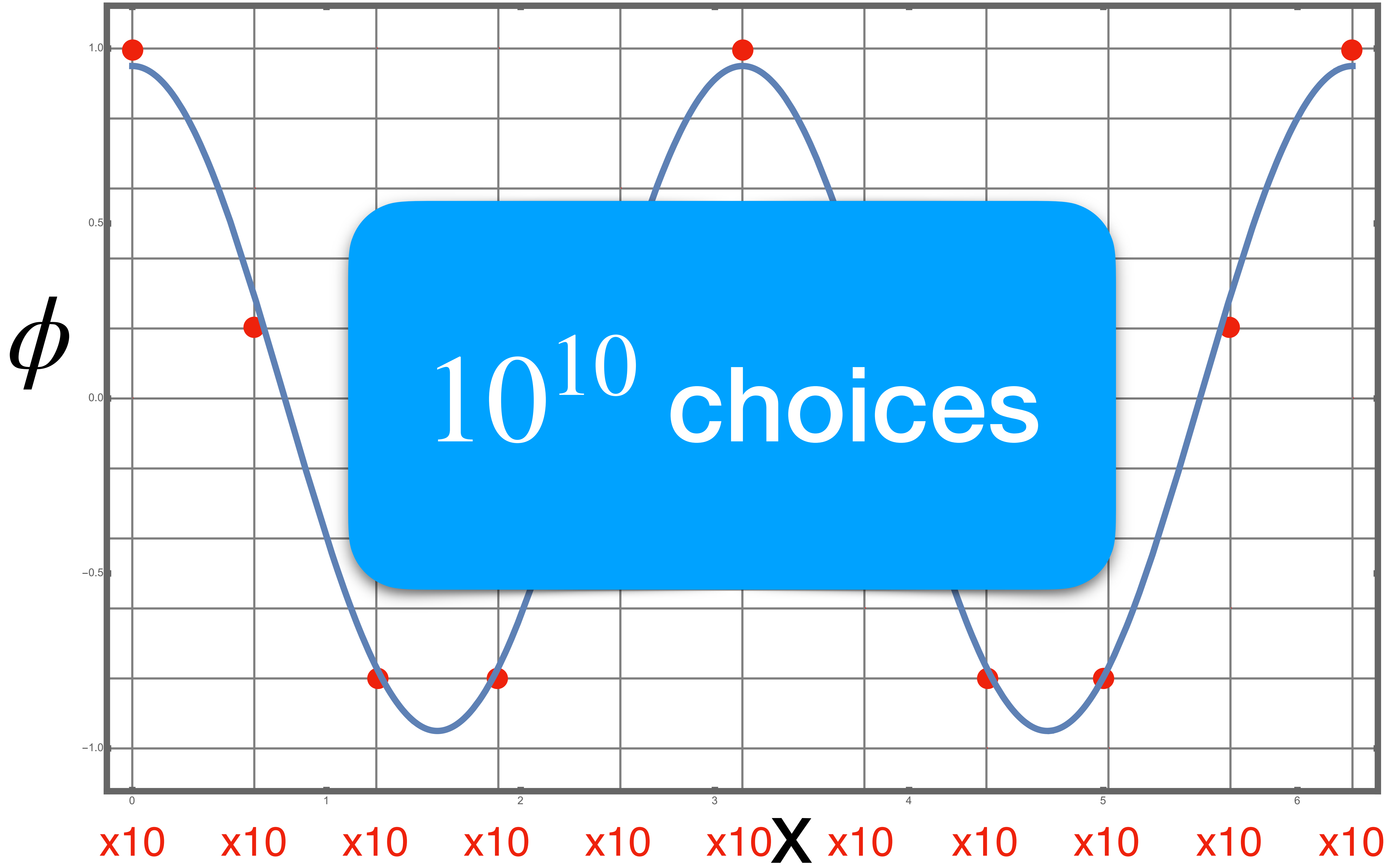


To yield finite-dimensional Hilbert space, have field configurations sample position and field values at discrete points

Size of Hilbert space:

$$n = n_j^V$$

This complexity is completely unmanageable for classical computers, which explains why this has not been pursued

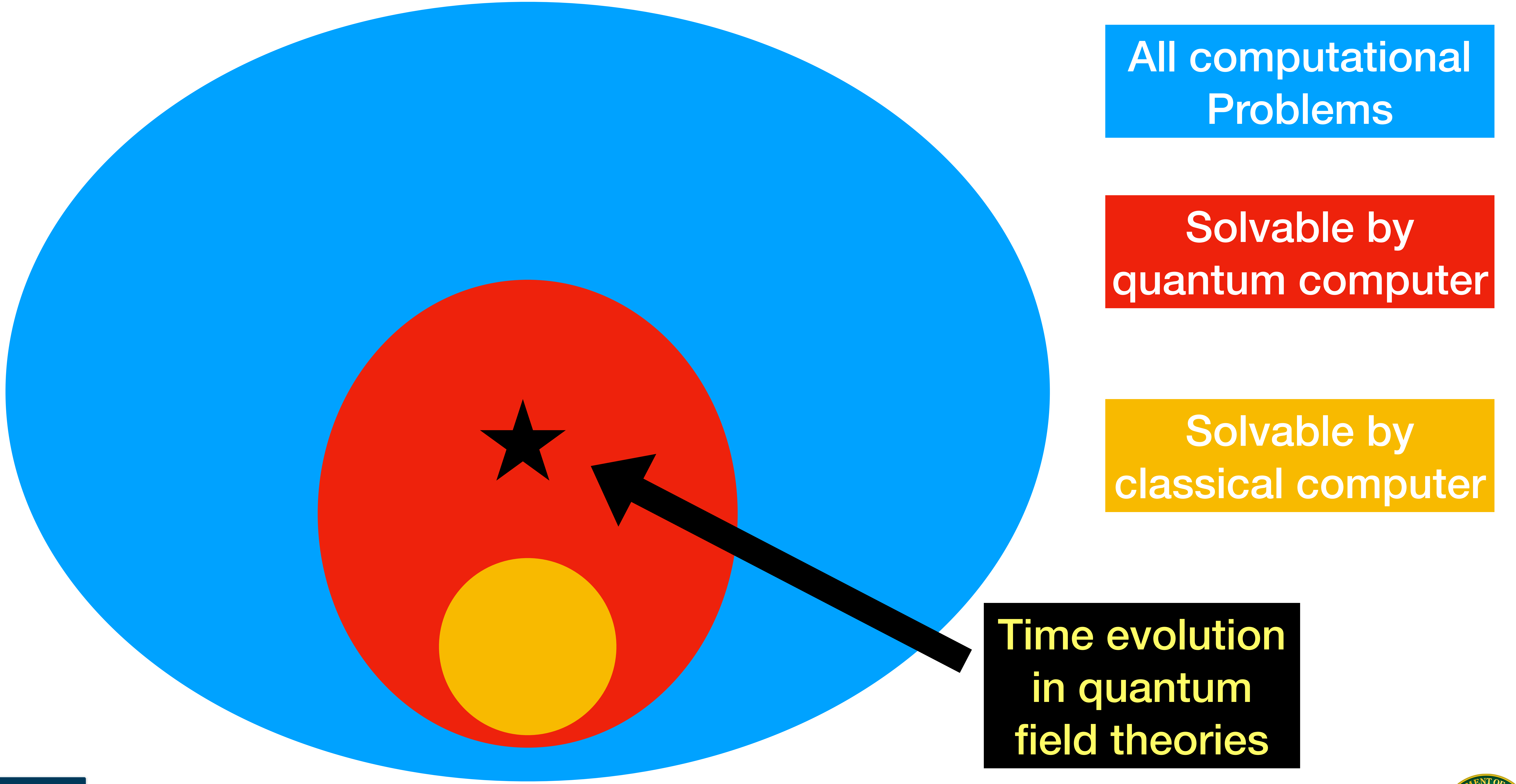
	Classical computer
nL=2	10^5
nL=3	10^18
nL=4	10^43
nL=5	10^83
nL=6	10^150

Quantum Algorithms for Quantum Field Theories

Stephen P. Jordan,[1*] Keith S. M. Lee,[2] John Preskill[3]

Quantum field theory reconciles quantum mechanics and special relativity, and plays a central role in many areas of physics. We developed a quantum algorithm to compute relativistic scattering probabilities in a massive quantum field theory with quartic self-interactions (ϕ^4 theory) in spacetime of four and fewer dimensions. Its run time is polynomial in the number of particles, their energy, and the desired precision, and applies at both weak and strong coupling. In the strong-coupling and high-precision regimes, our quantum algorithm achieves exponential speedup over the fastest known classical algorithm.

Science 336 (2012) 1130



All computational Problems
Solvable by quantum computer
Solvable by classical computer
Time evolution in quantum field theories

Quantum computers therefore put first principles calculations of scattering cross sections (and other observables) in the realm of possibility

	Classical computer	Quantum Computer
nL=2	10^5	10^1
nL=3	10^18	10^1
nL=4	10^43	10^2
nL=5	10^83	10^2
nL=6	10^150	10^2

Identify the right questions to address

Find Theory Formulation for SU(3)

Quantum Simulations Research

Find efficient Quantum algorithms

Obtain results on realistic machines (with noise)

Identify the right questions to address

Find Theory Formulation for SU(3)

Quantum Simulations Research

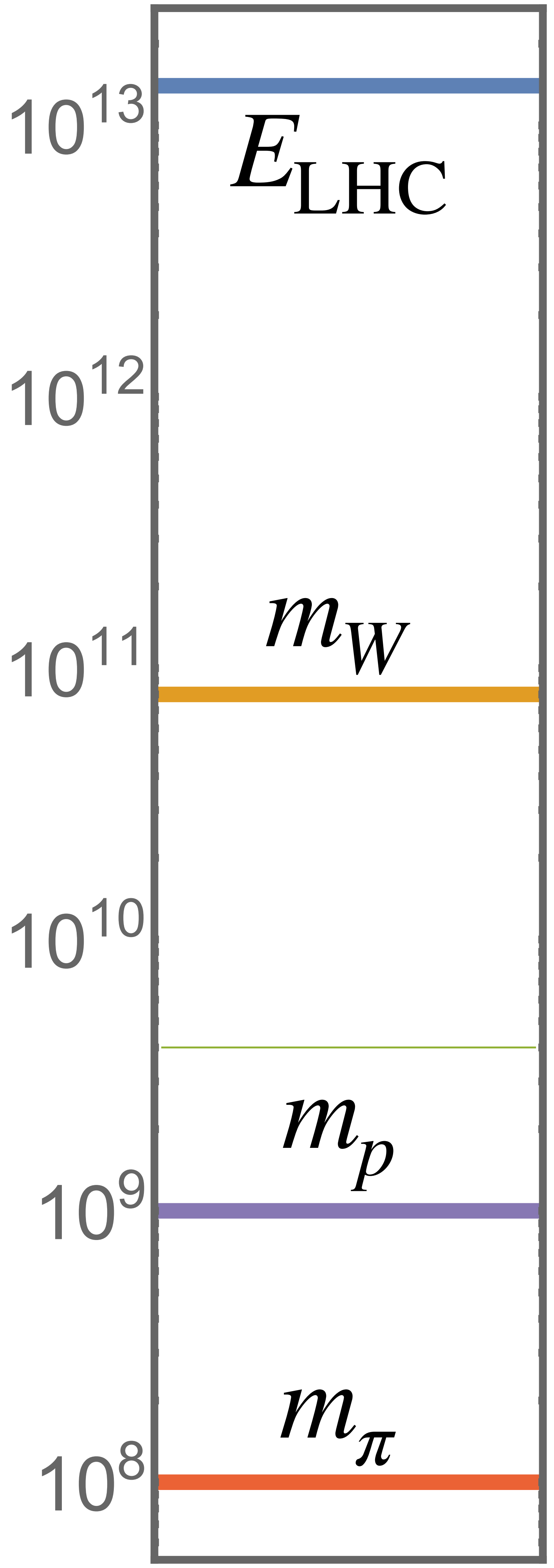
Find efficient Quantum algorithms

Obtain results on realistic machines (with noise)

Identify the right questions to address

Quantum Simulations Research

There are many energy scales that are present in LHC events, and all need to be accounted for in an adequate description



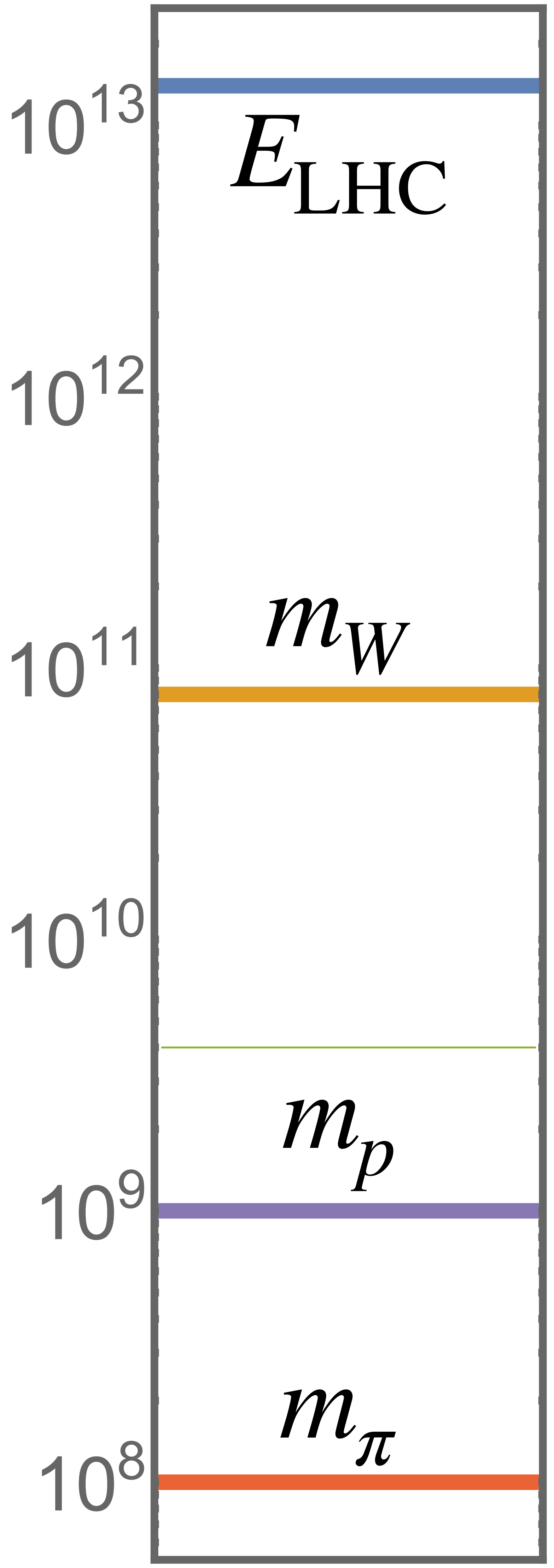
Energy of colliding protons

Scale of electroweak gauge bosons

Mass of the proton

Mass of the pion, the lightest hadron

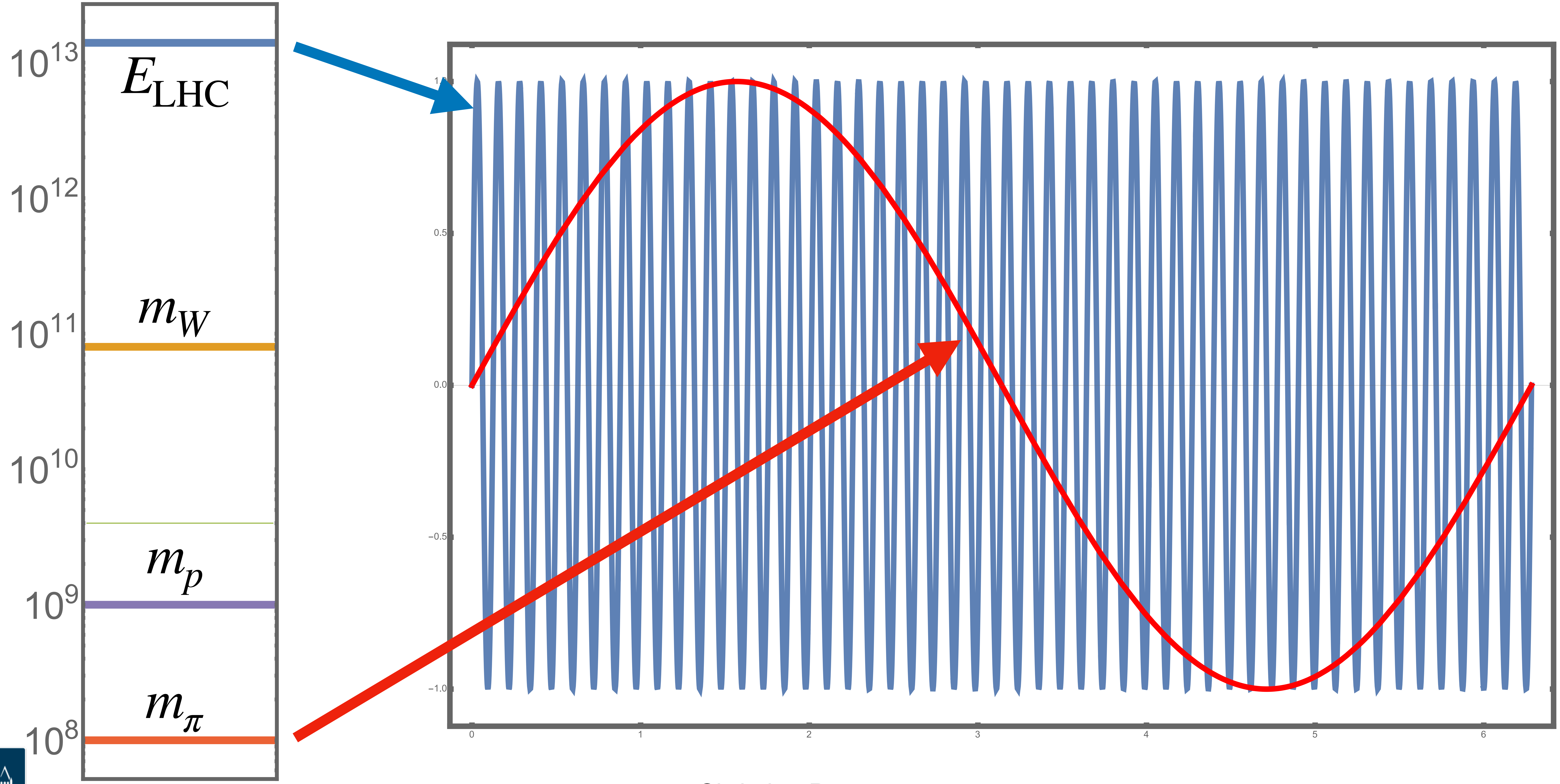
There are many energy scales that are present in LHC events, and all need to be accounted for in an adequate description



Field configurations corresonding to given energy have wavelength

$$l \sim 1/E$$

The largest and smallest energy scales set the maximum and minimum wavelength of field configurations that need to be considered



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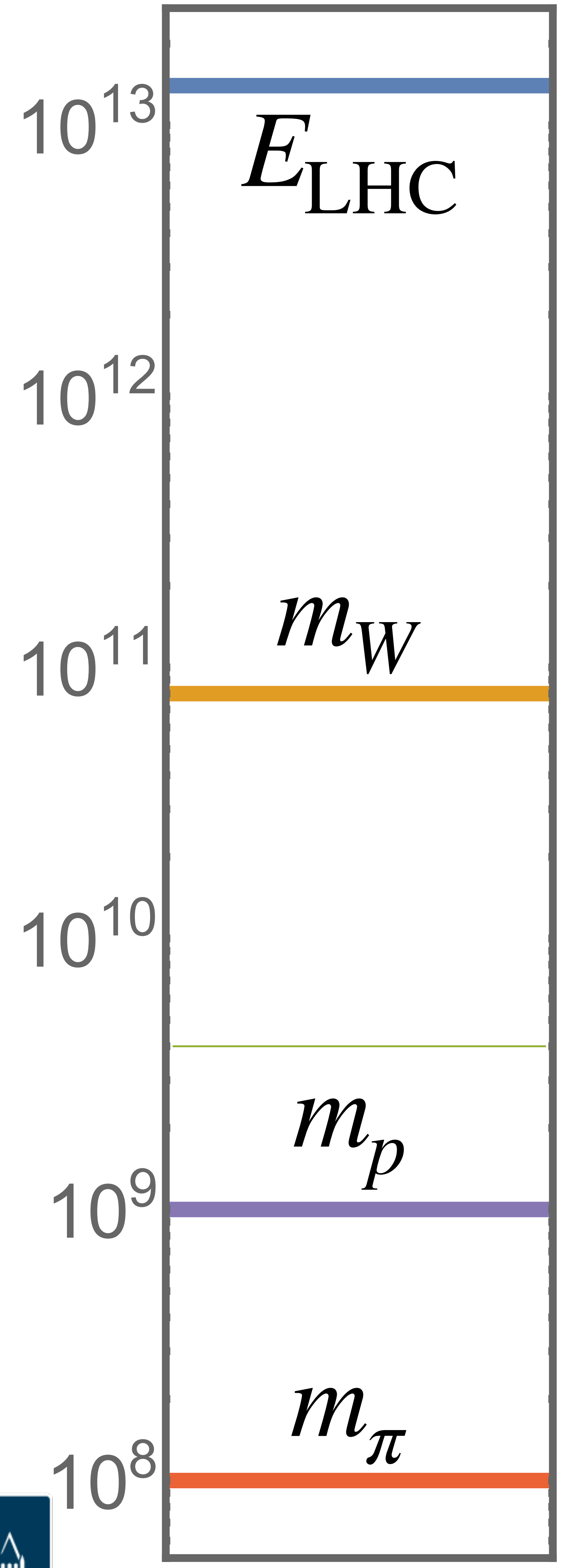

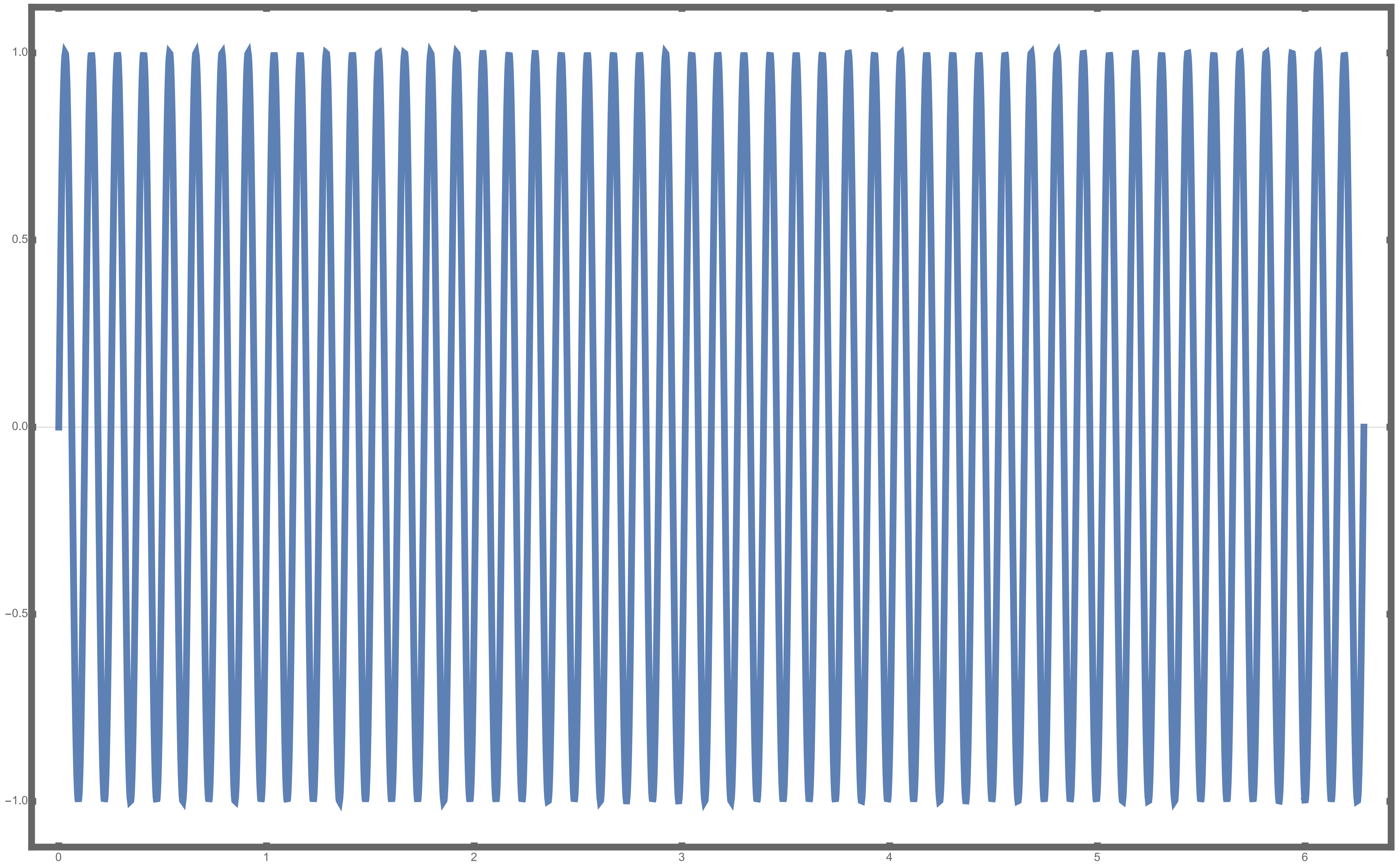

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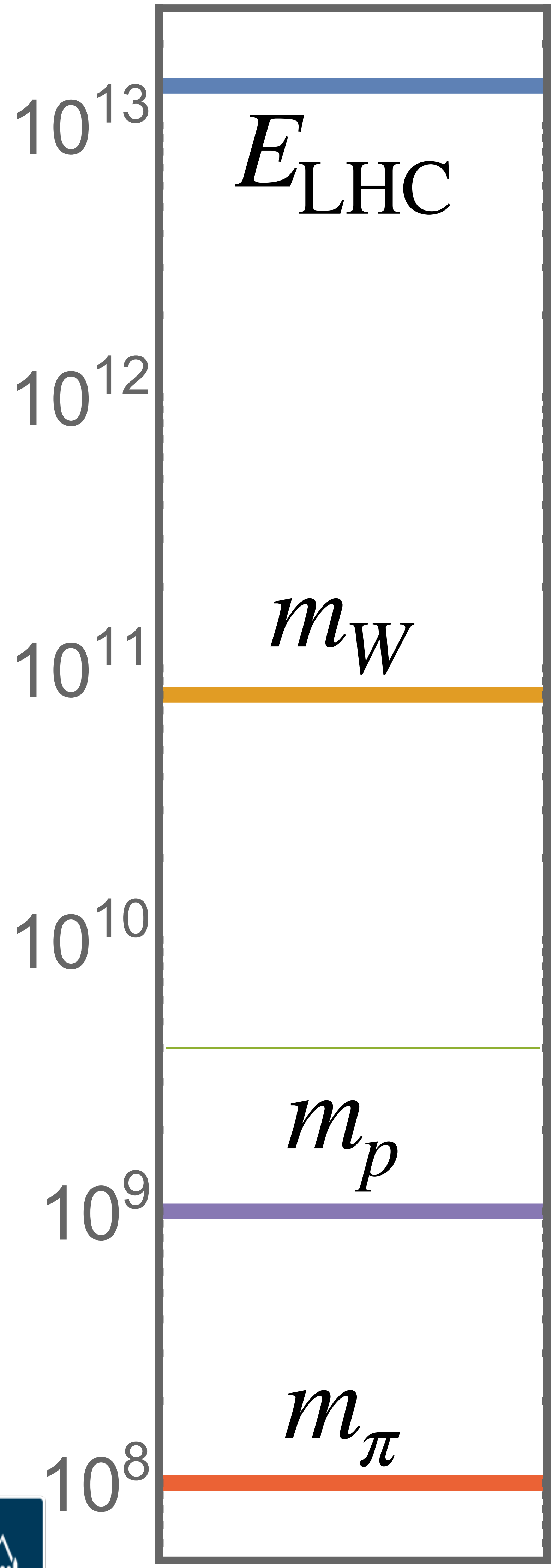

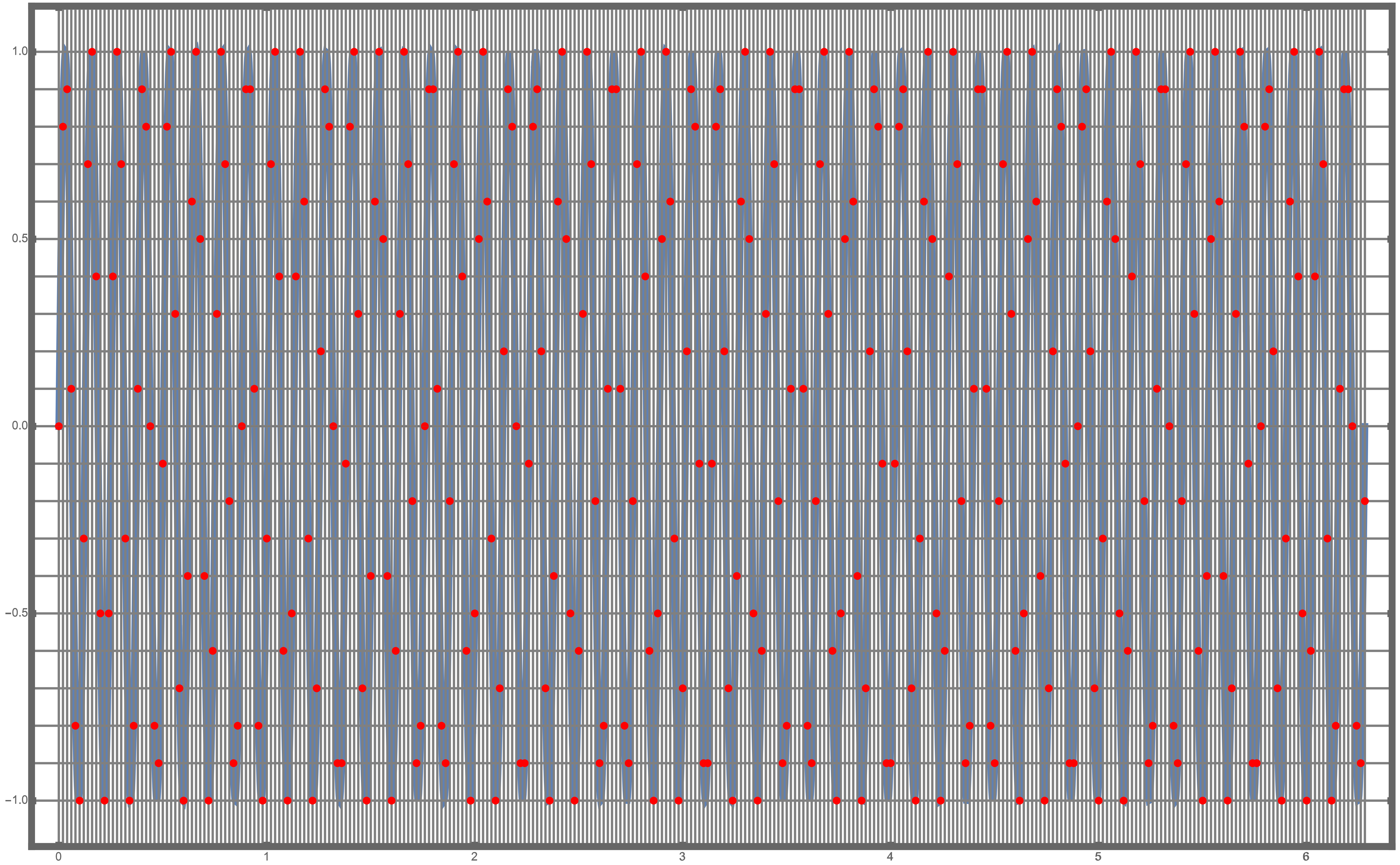

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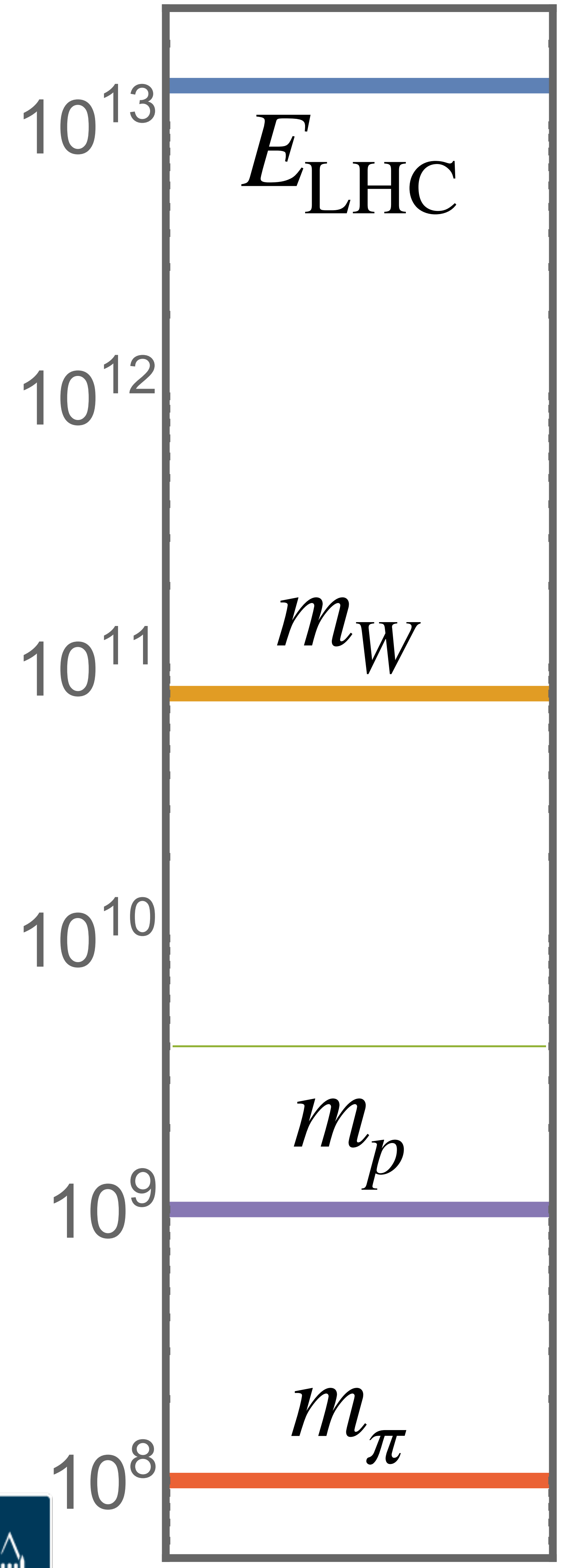
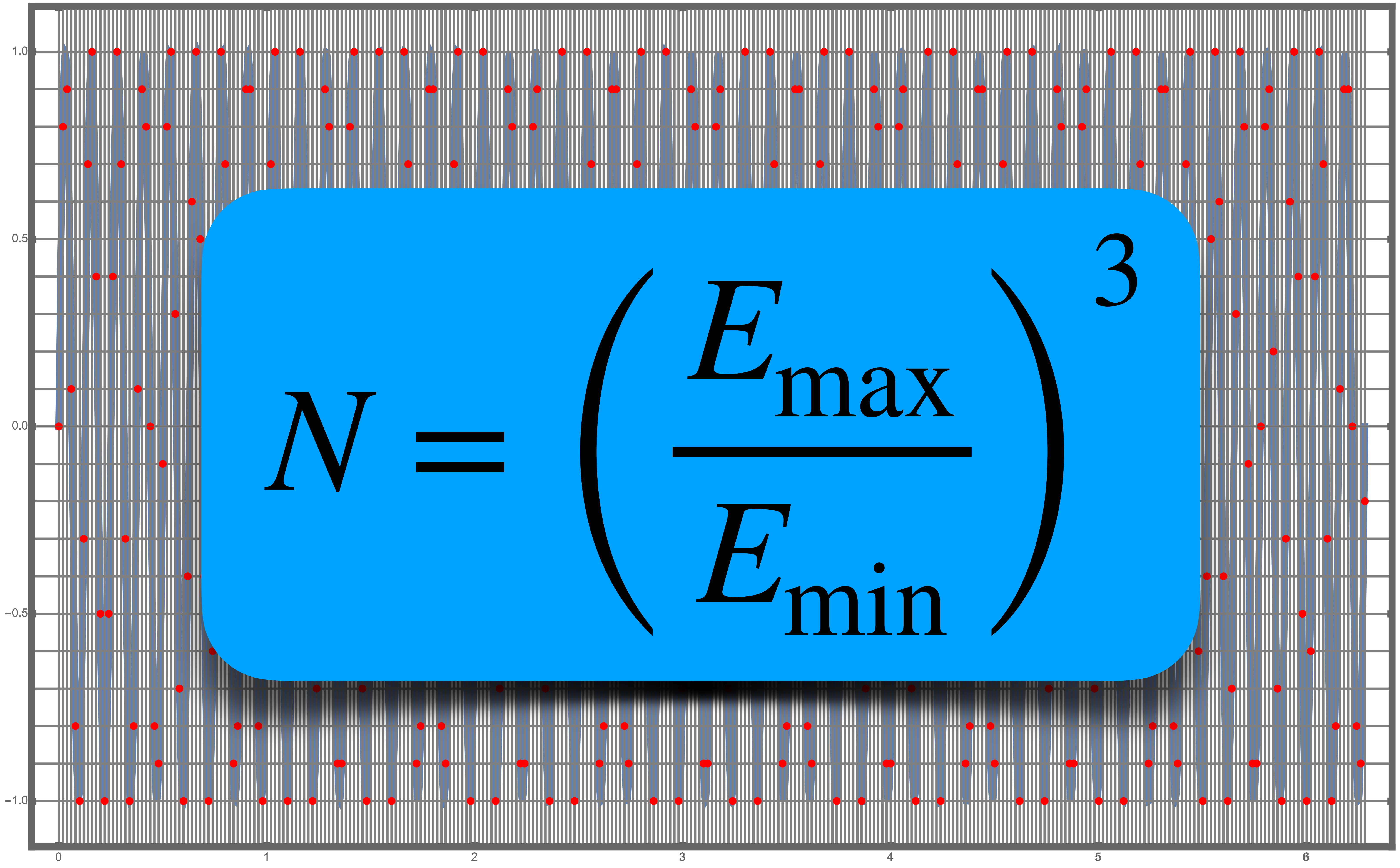
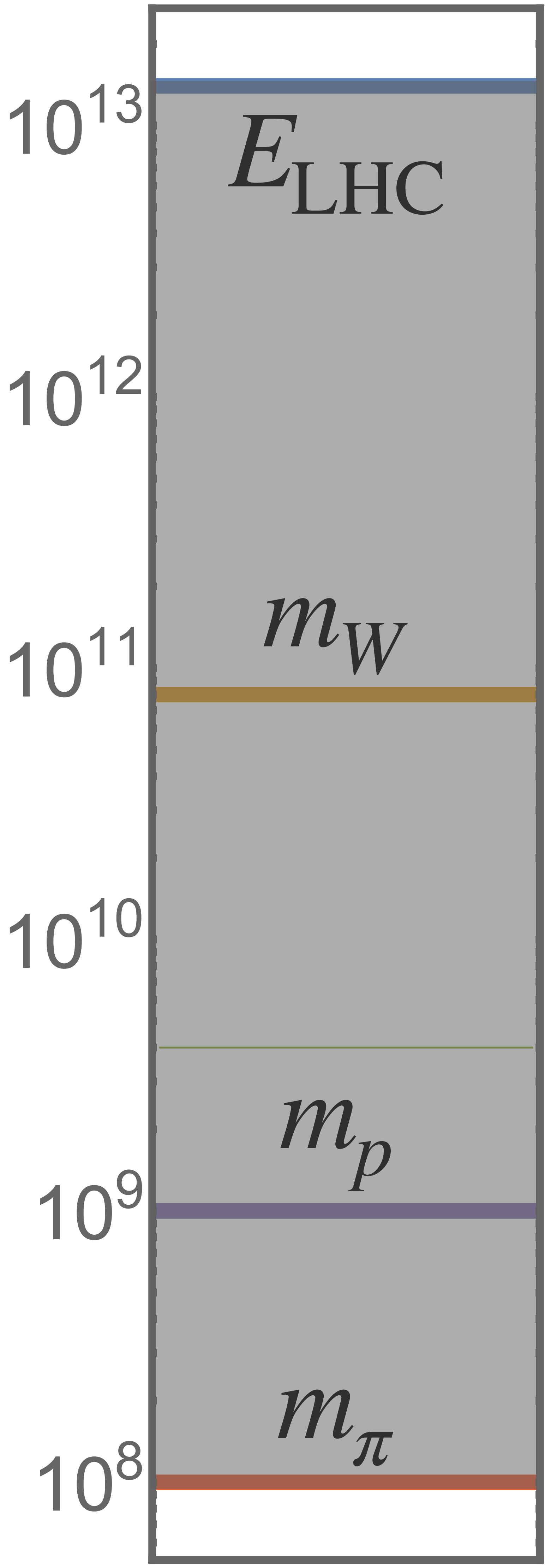
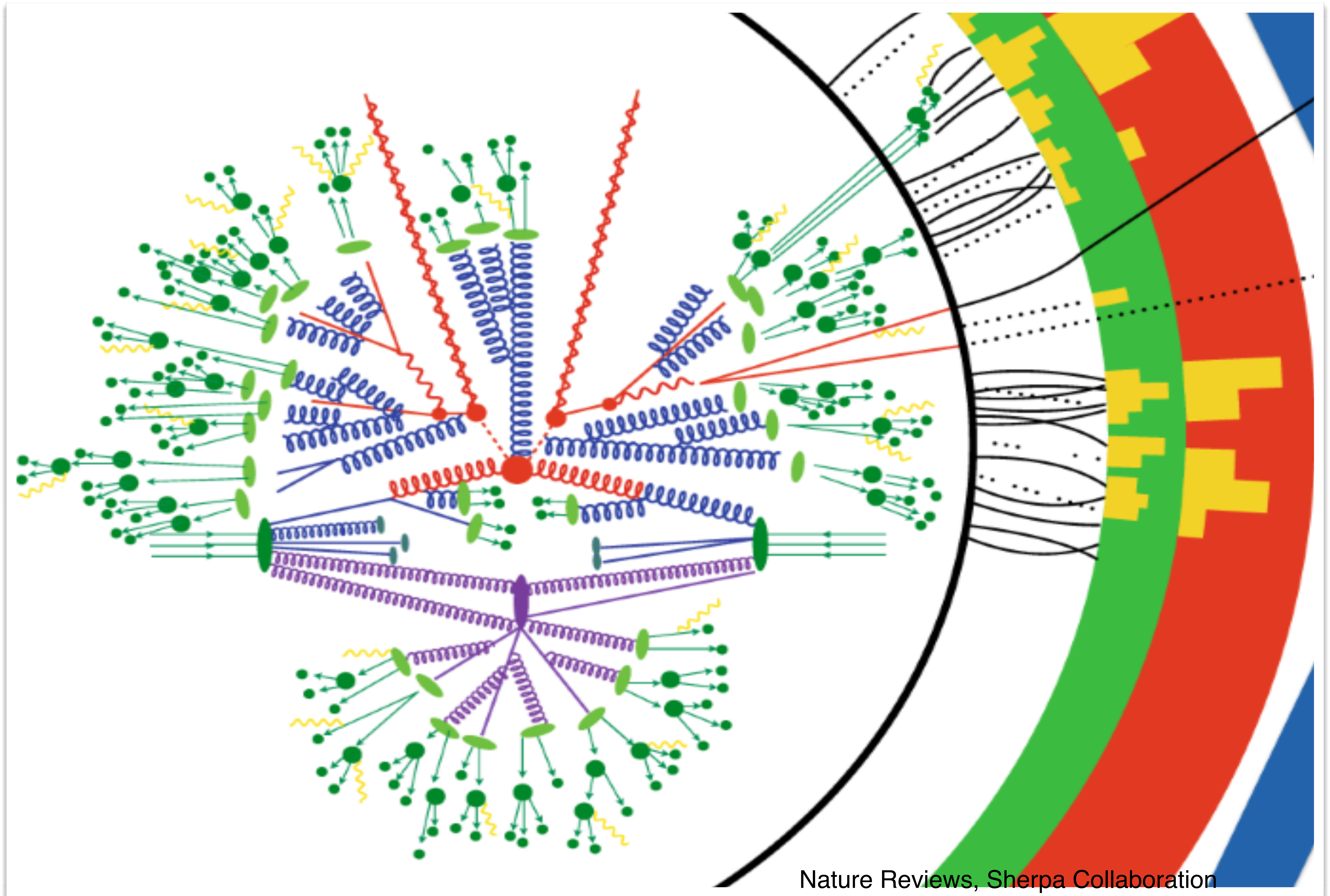
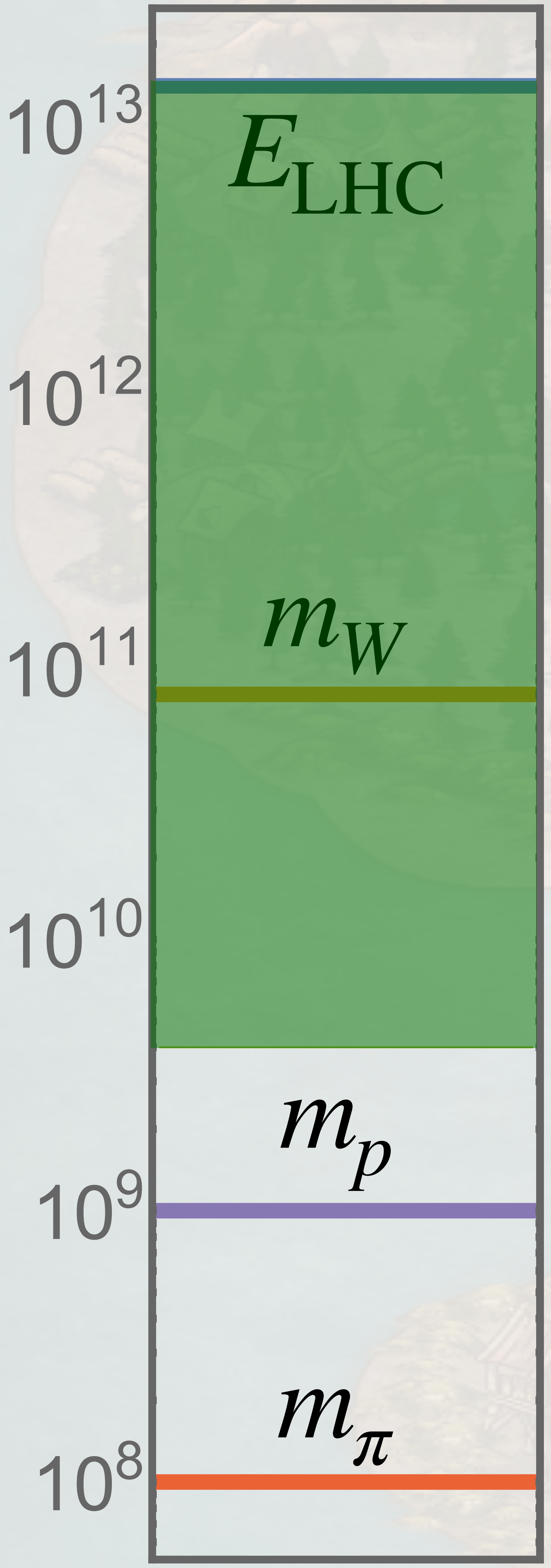


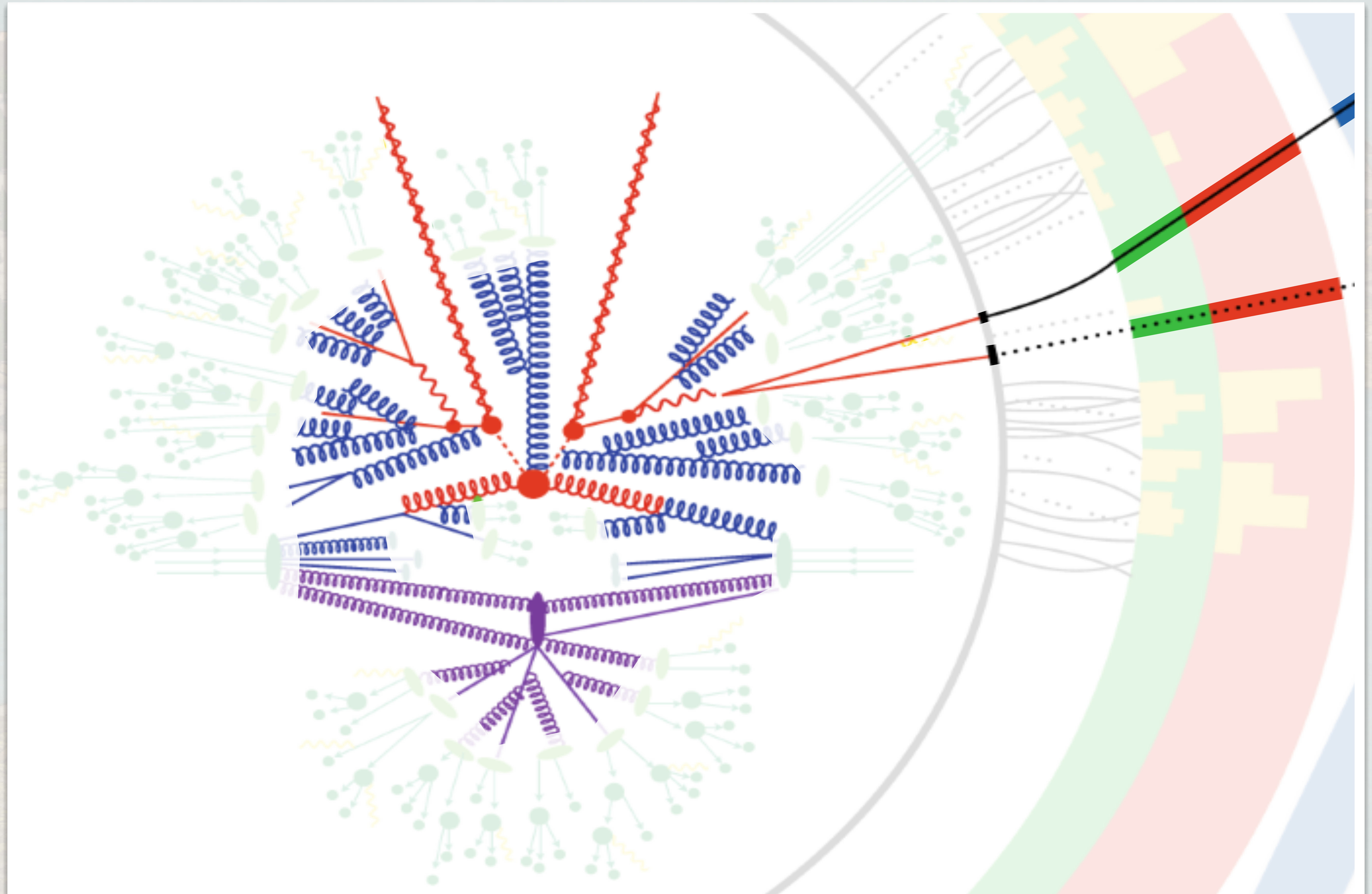


$$N = \left(\frac{E_{\max}}{E_{\min}}\right)^3$$


10^{13}
$E_{\rm LHC}$
10^{12}
m_W
10^{11}
10^{10}
m_p
10^{9}
m_π
10^{8}


Nature Reviews, Sherpa Collaboration





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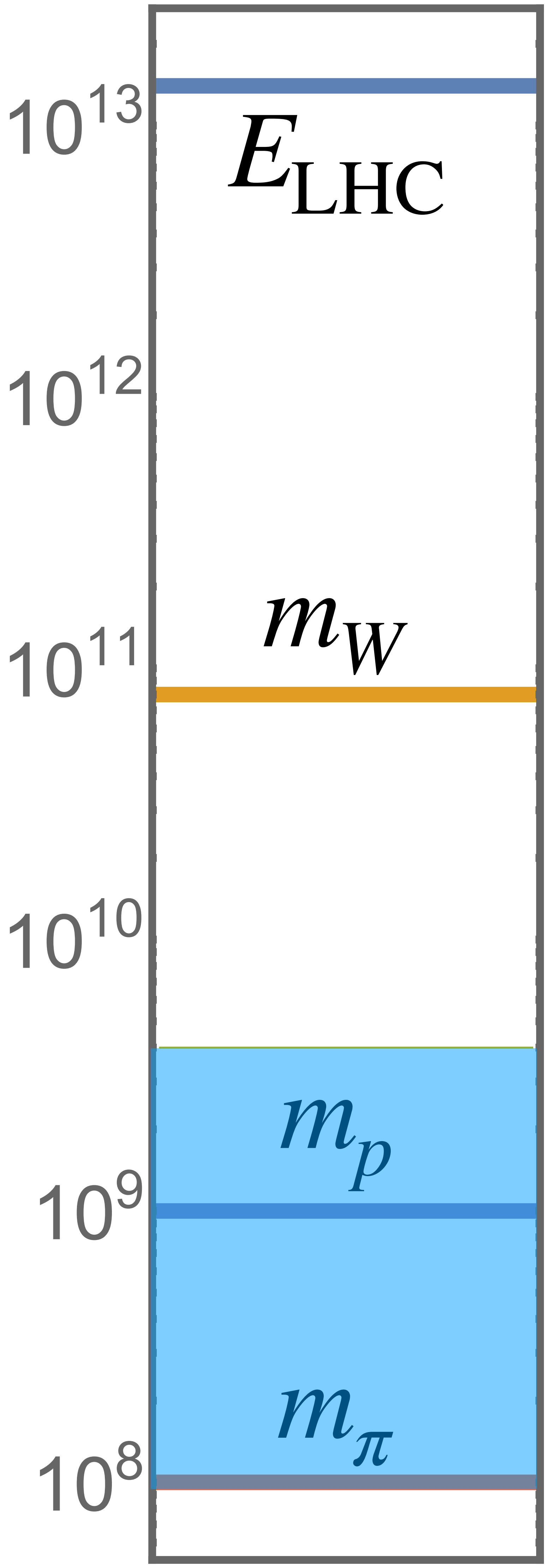

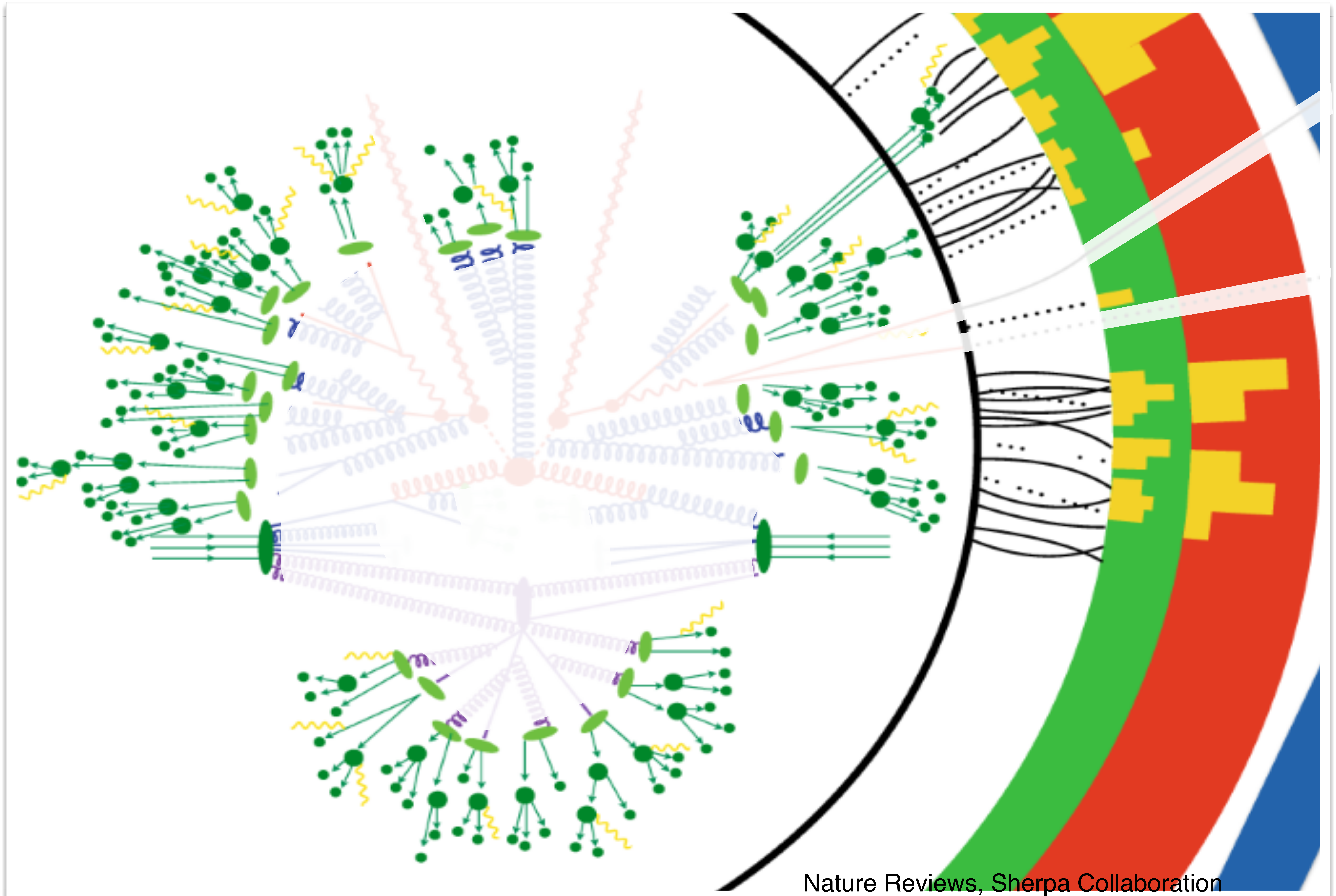

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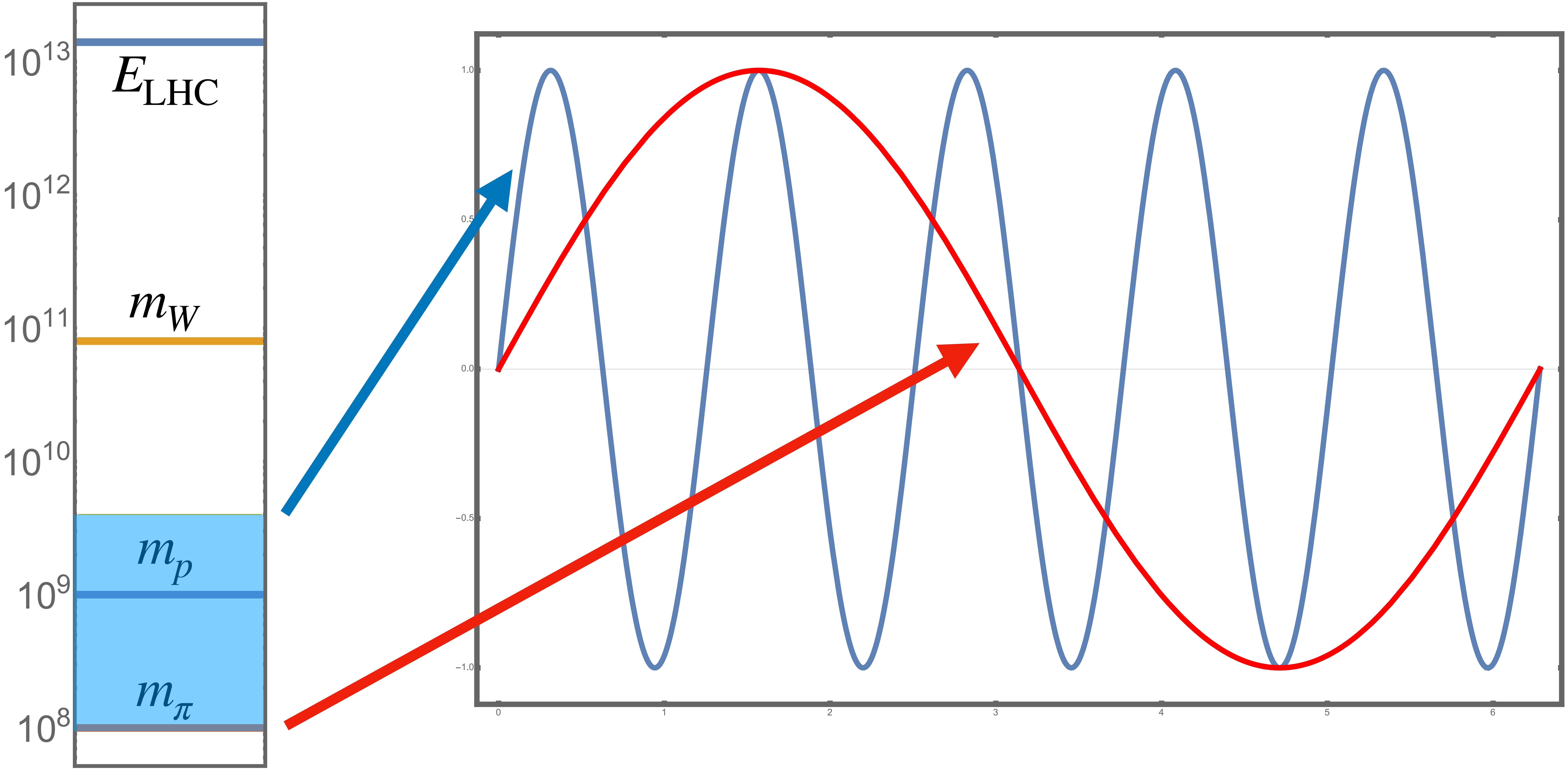
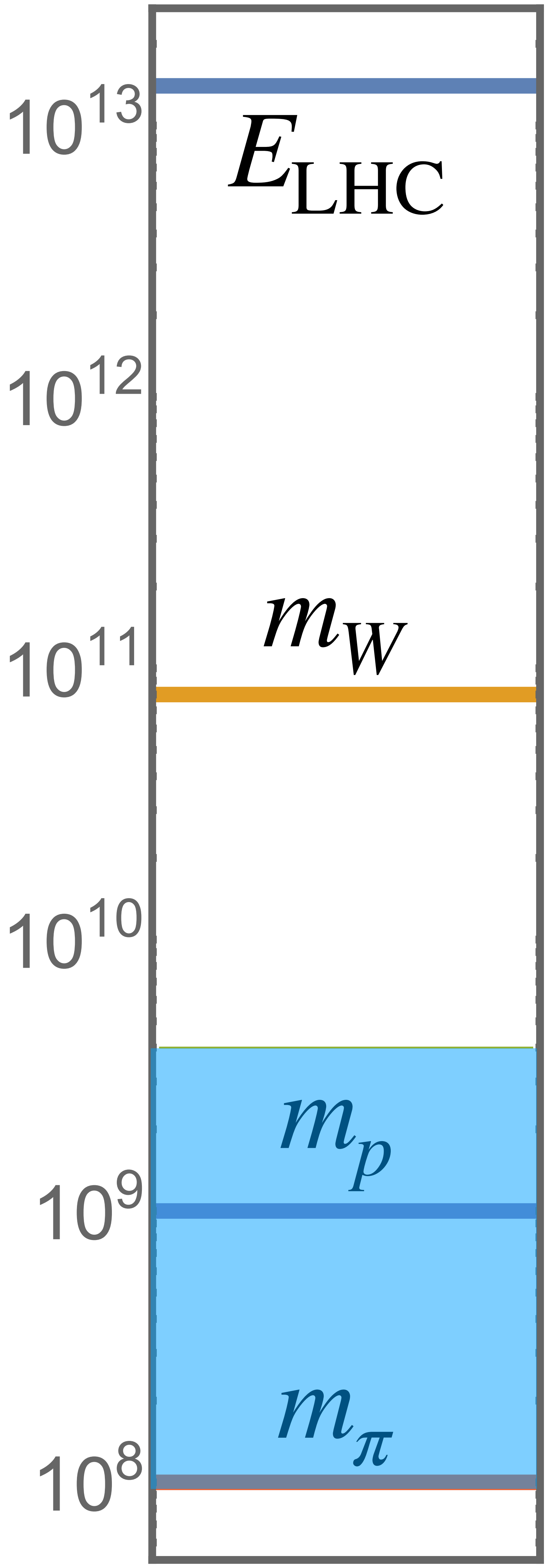
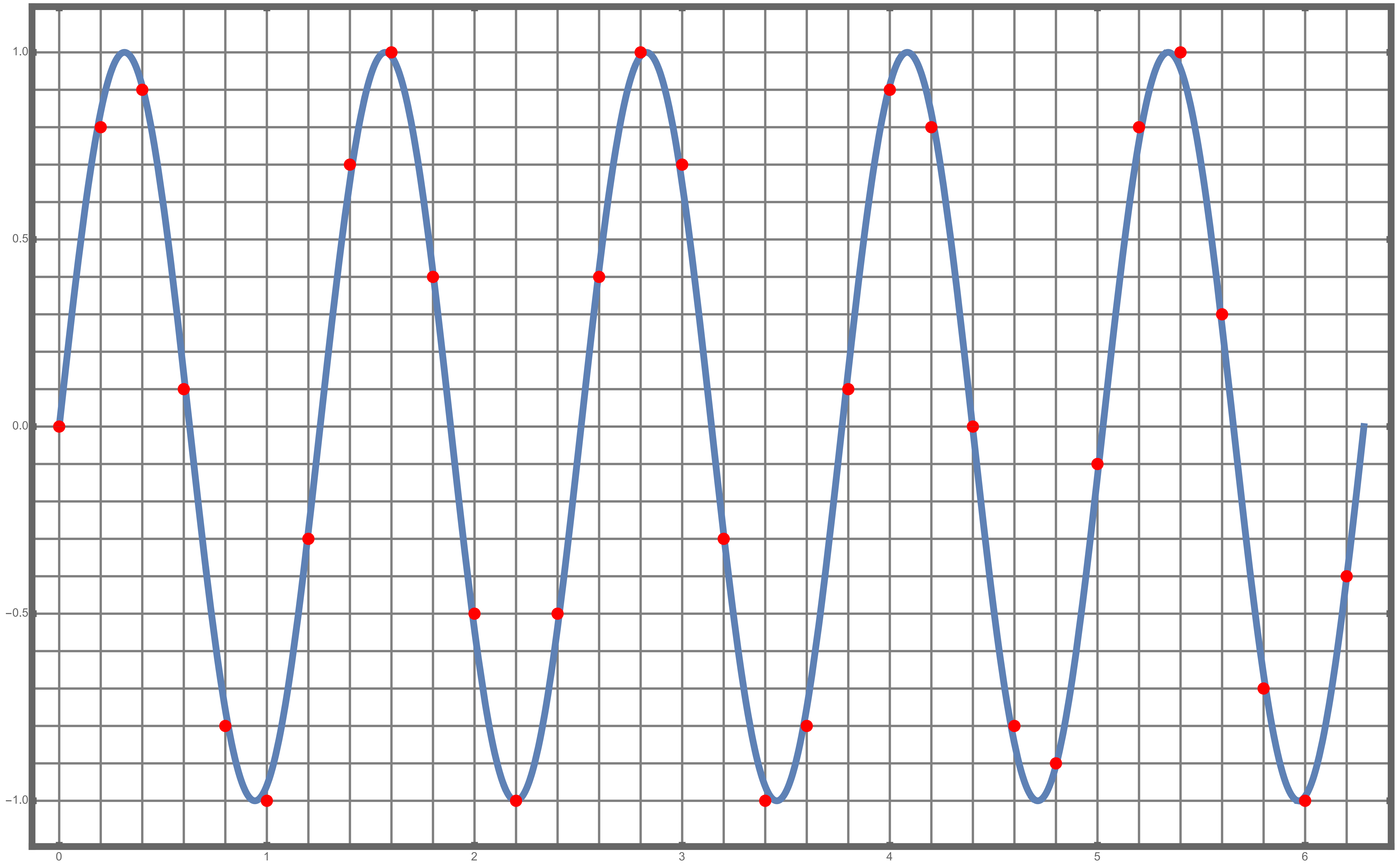
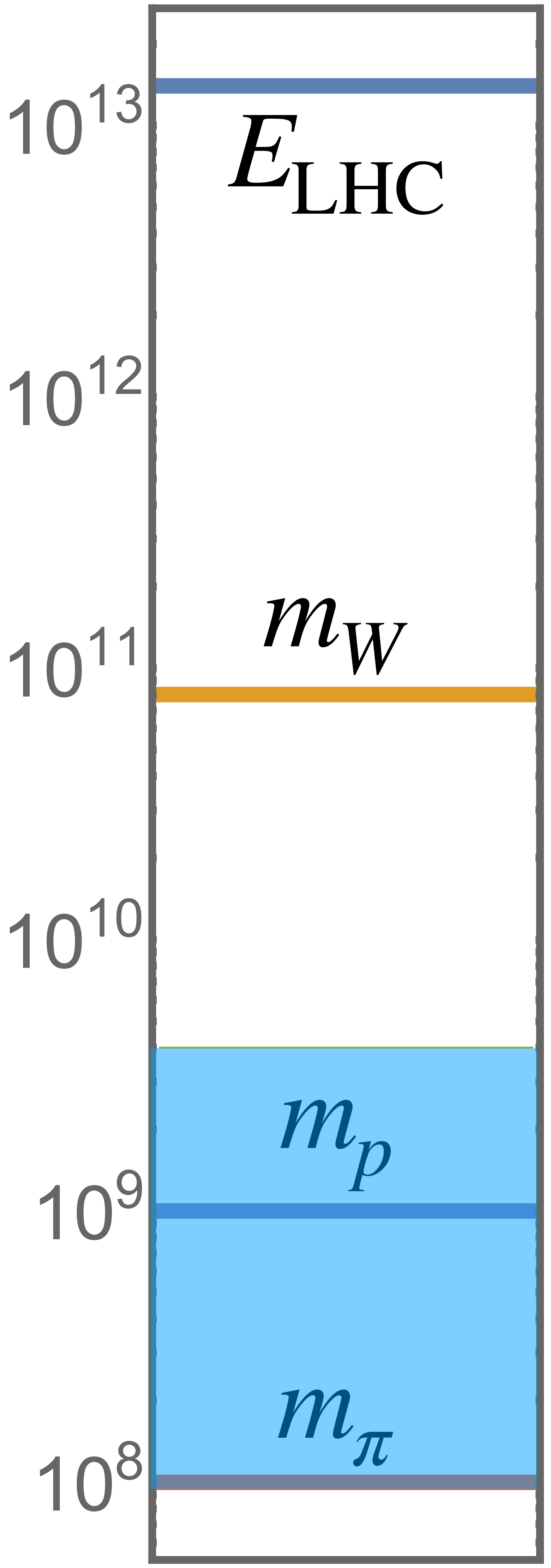
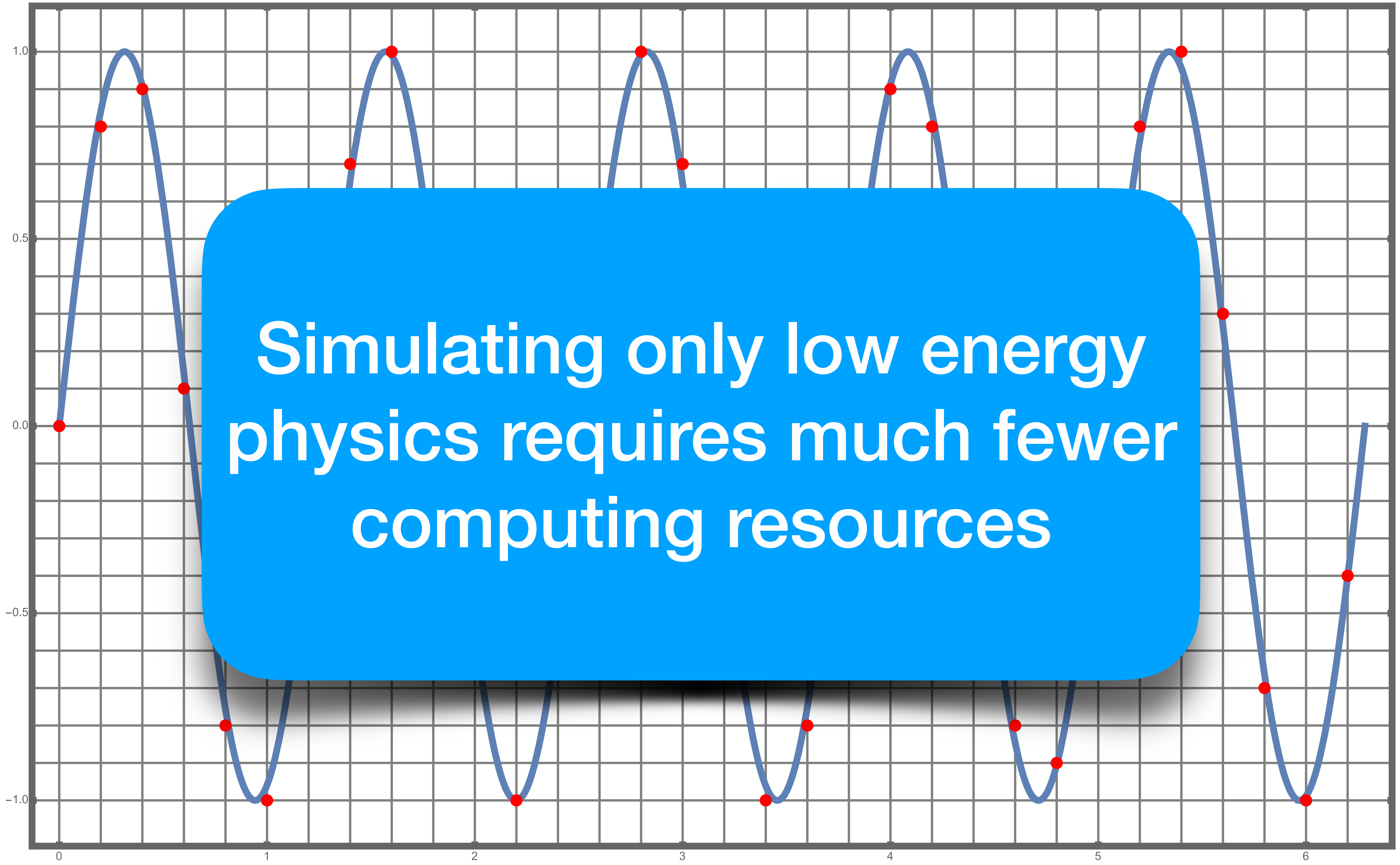


10^{13}
$E_{\rm LHC}$
10^{12}
m_W
10^{11}
10^{10}
m_p
10^{9}
m_π
10^{8}

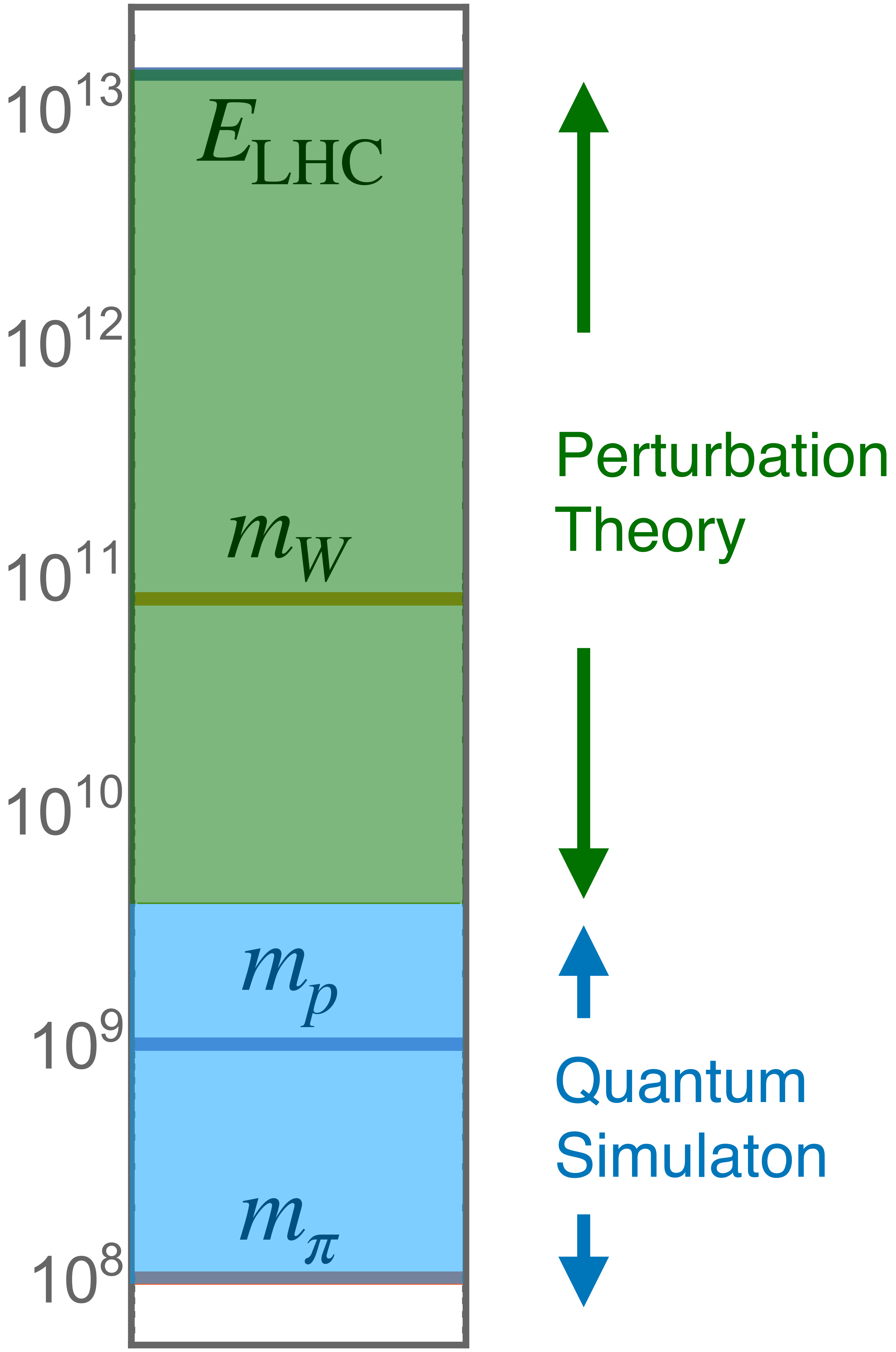

10^{13}
E_{LHC}
10^{12}
m_W
10^{11}
10^{10}
m_p
10^{9}
m_π
10^{8}


1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
4
5
6





Simulating only low energy physics requires much fewer computing resources



Both calculations can be combined using EFT techniques (SCET in this case)

PHYSICAL REVIEW D, VOLUME 63, 114020

An effective field theory for collinear and soft gluons: Heavy to light decays

Christian W. Bauer,[1] Sean Fleming,[2] Dan Pirjol,[1] and Iain W. Stewart[1]

[1]*Physics Department, University of California at San Diego, La Jolla, California 92093*
[2]*Physics Department, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213*

(Received 30 November 2000; published 7 May 2001)

We construct the Lagrangian for an effective theory of highly energetic quarks with energy Q, interacting with collinear and soft gluons. This theory has two low energy scales, the transverse momentum of the collinear particles, $p_\perp$, and the scale $p_\perp^2/Q$. The heavy to light currents are matched onto operators in the effective theory at one loop and the renormalization group equations for the corresponding Wilson coefficients are solved. This running is used to sum Sudakov logarithms in inclusive $B\rightarrow X_s\gamma$ and $B\rightarrow X_u l\bar{\nu}$ decays. We also show that the interactions with collinear gluons preserve the relations for the soft part of the form factors for heavy-to-light decays found by Charles *et al*. [Phys. Rev. D **60**, 014001 (1999)], establishing these relations in the large energy limit of QCD.

PHYSICAL REVIEW LETTERS **127,** 212001 (2021)

Simulating Collider Physics on Quantum Computers Using Effective Field Theories

Christian W. Bauer[*] and Benjamin Nachman[†]
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Marat Freytsis[‡]
NHETC, Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854, USA
and Physics Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

(Received 18 May 2021; accepted 12 October 2021; published 18 November 2021)

Simulating the full dynamics of a quantum field theory over a wide range of energies requires exceptionally large quantum computing resources. Yet for many observables in particle physics, perturbative techniques are sufficient to accurately model all but a constrained range of energies within the validity of the theory. We demonstrate that effective field theories (EFTs) provide an efficient mechanism to separate the high energy dynamics that is easily calculated by traditional perturbation theory from the dynamics at low energy and show how quantum algorithms can be used to simulate the dynamics of the low energy EFT from first principles. As an explicit example we calculate the expectation values of vacuum-to-vacuum and vacuum-to-one-particle transitions in the presence of a time-ordered product of two Wilson lines in scalar field theory, an object closely related to those arising in EFTs of the standard model of particle physics. Calculations are performed using simulations of a quantum computer as well as measurements using the IBMQ Manhattan machine.

Find Theory Formulation for SU(3)

Quantum Simulations Research

There are many different parts of the theory that need to be worked out when formulating a Hamiltonian lattice gauge theory

1. How to formulate a lattice theory that reproduces SU(3) in the limit of vanishing lattice spacing

   - Whether to add any additional expansions in the theory

2. What basis to choose for the Hilbert space
3. How to implement gauge invariance
4. How to truncate the theory (how to choose a discrete set of field values)

Goal is a Hamiltonian Lattice theory that reproduces QCD in continuum limit

A Trailhead for Quantum Simulation of SU(3) Yang-Mills Lattice Gauge Theory in the Local Multiplet Basis

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Part 1: What lattice Hamiltonian to use in the without any truncation.

In this case the Kogut-Susskind Hamiltonian is used

$$\hat{H} = \frac{g^2}{2a^{d-2}} \sum_{b,\text{links}} |\hat{\mathbf{E}}^{(b)}|^2 + \frac{1}{2a^{4-d}g^2} \sum_{\text{plaquettes}} \left[6 - \hat{\Box}(\mathbf{x}) - \hat{\Box}^\dagger(\mathbf{x}) \right]$$

Part 2: How to represent a basis to choose for the Hilbert space

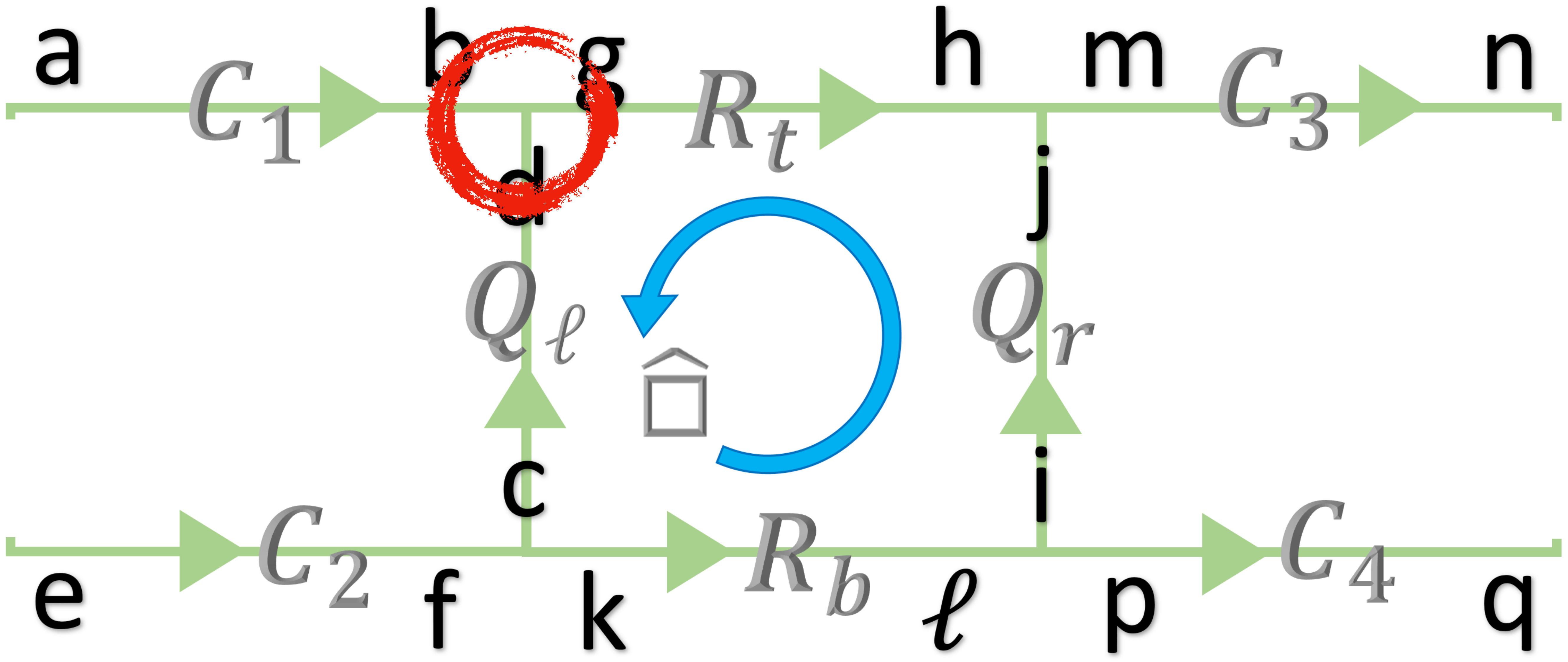
In this case a basis in representation of SU(3) was chosen in which electric Hamiltonian is diagonal

$$\sum_b |\hat{\mathbf{E}}^{(b)}|^2 \, |p, q\rangle = \frac{p^2 + q^2 + pq + 3p + 3q}{3} \, |p, q\rangle$$

$$\dim(p, q) = \frac{(p+1)(q+1)(p+q+2)}{2}$$

Part 3: How to implement gauge invariance

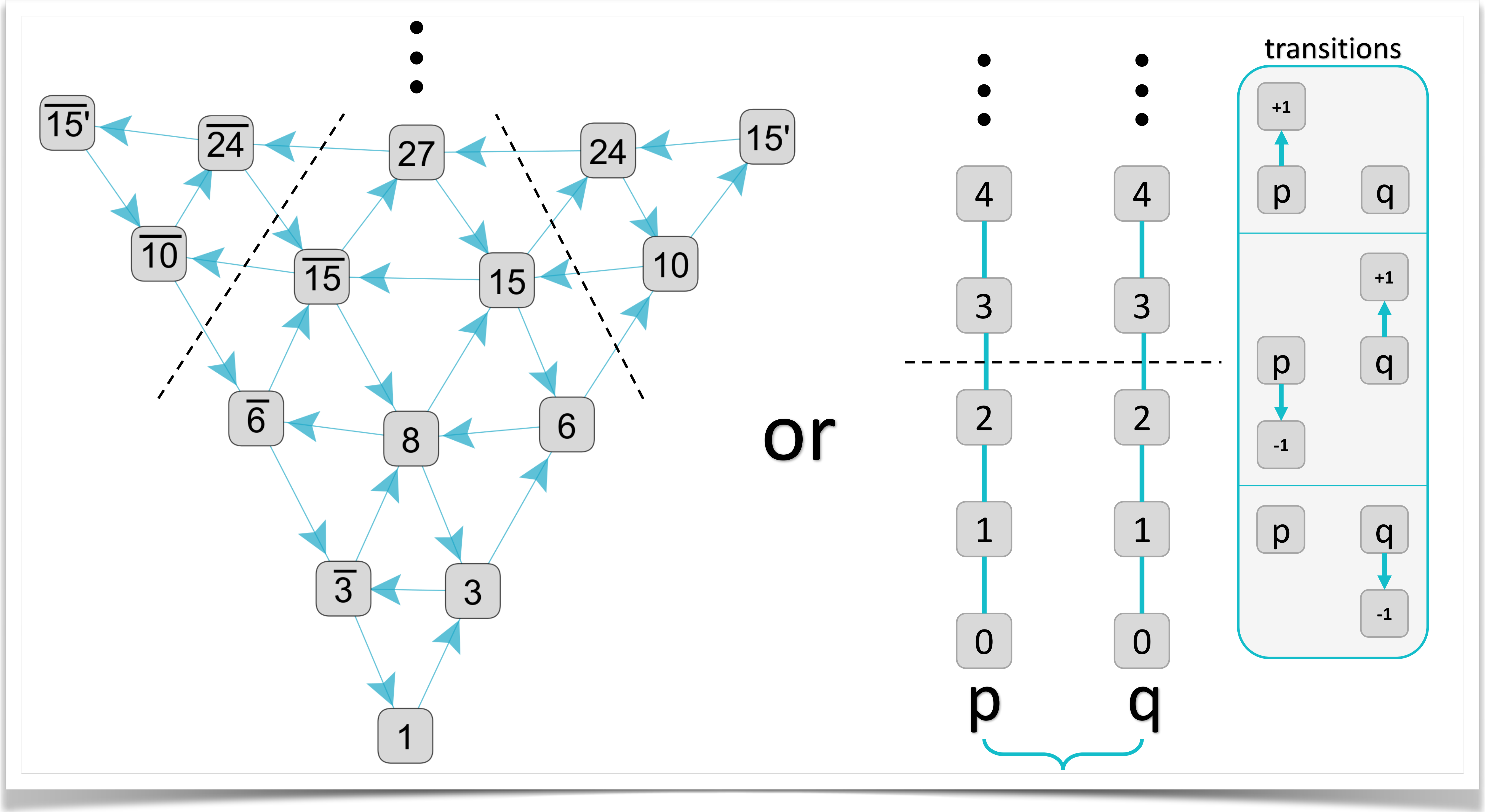
In this case gauge invariance is implemented by requiring that representations satisfy Gauss' law, therefore putting restrictions on each plaquette



$$|\psi_{3pt}\rangle \sim \sum_{b,g,d,\Gamma} \langle \mathbf{C}_1, b, \overline{\mathbf{R}}_t, g|\overline{\mathbf{Q}}_\ell, d\rangle_\Gamma \; |\mathbf{C}_1, a, b\rangle|\mathbf{Q}_\ell, c, d\rangle|\mathbf{R}_t, g, h\rangle$$

Part 4: How to truncate the theory

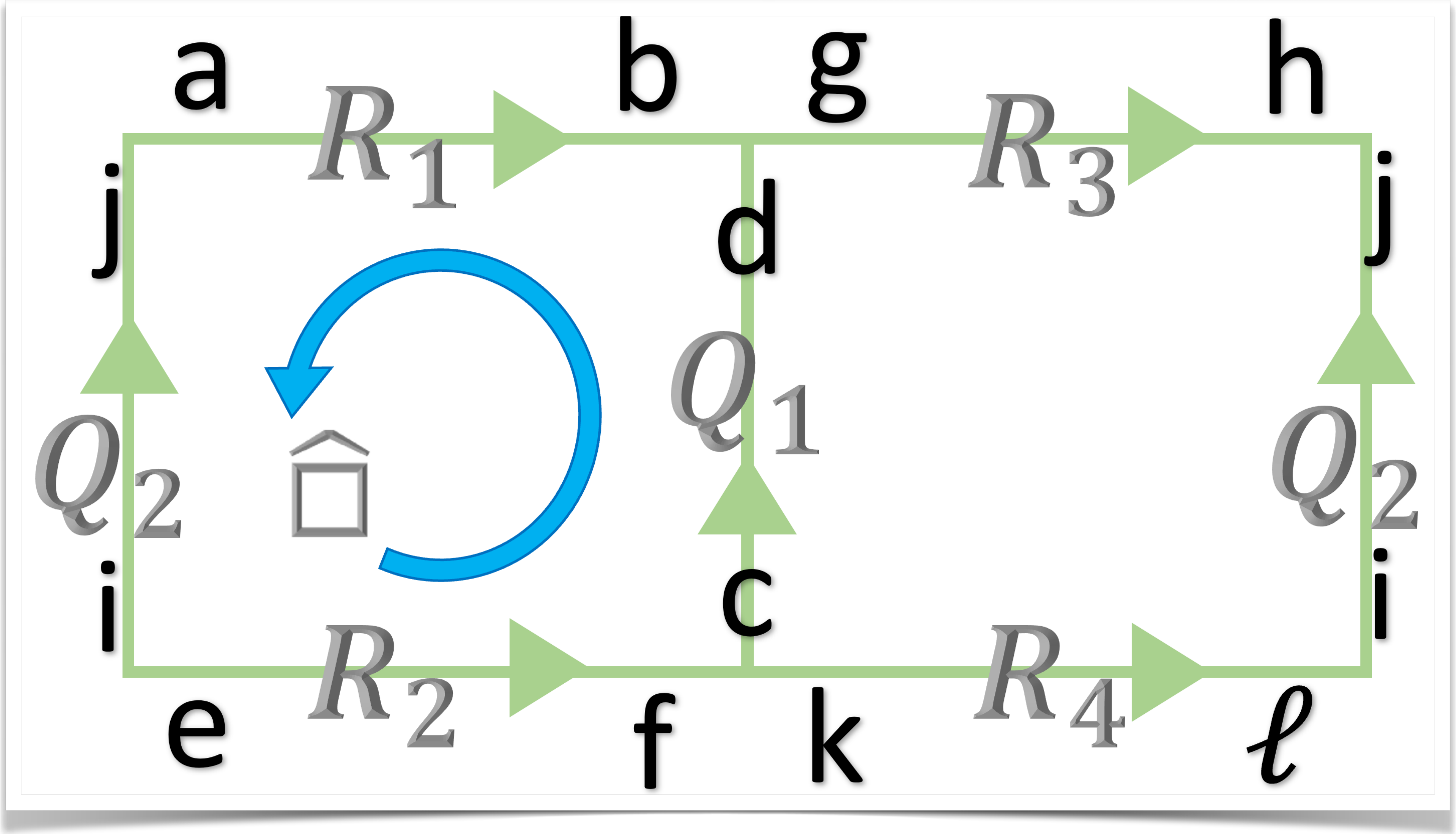
In this case theory is truncated by the maximum allowed p and q values of the representation at each link



With all details in place, one has a theoretical framework. One now needs to work out efficient quantum algorithms and then get results from hardware

Paper presented above was first (and essentially still only one) that could do real SU(3) calculations on quantum hardware

Results could be obtained on a 3x2 lattice



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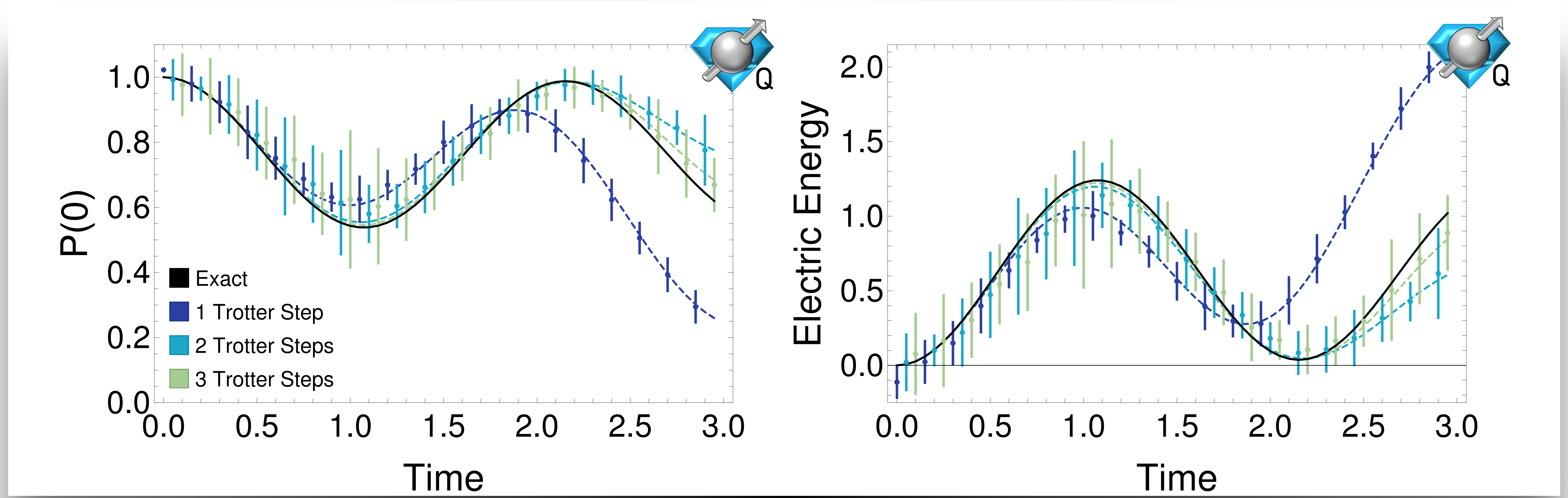


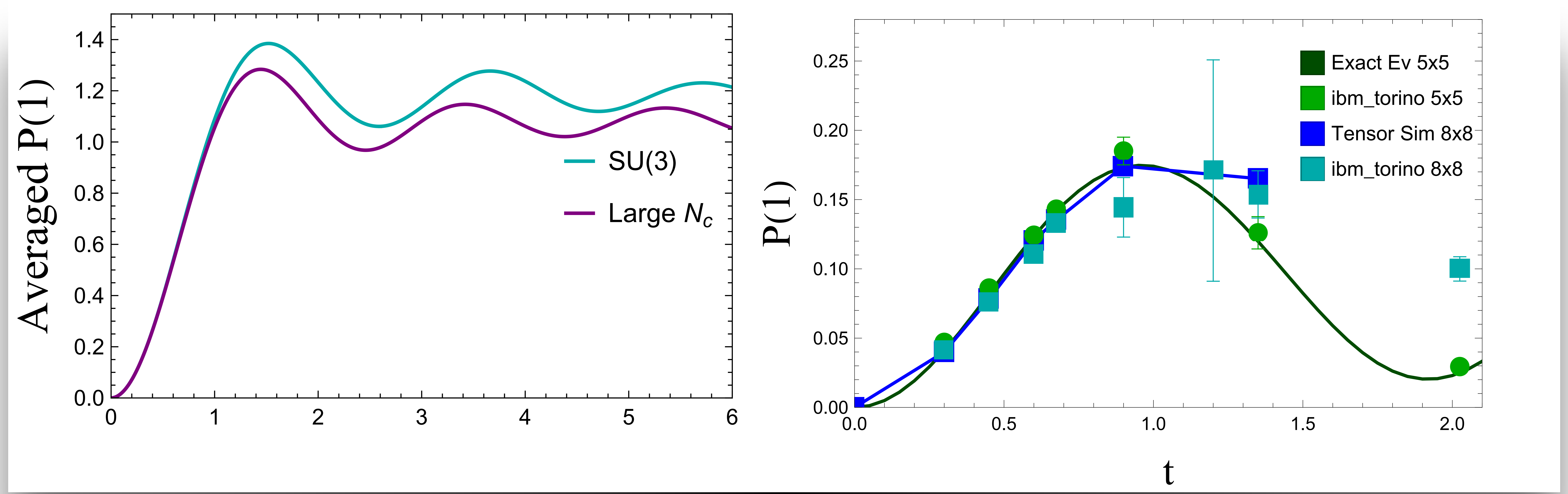
We very recently realized that adding an additional expansion can lead to dramatic simplifications in the lattice theory

1. How to formulate a lattice theory that reproduces SU(3) in the limit of vanishing lattice spacing
   - Whether to add any additional expansions in the theory
2. What basis to choose for the Hilbert space
3. How to implement gauge invariance
4. How to truncate the theory (how to choose a discrete set of field values)
5. …

A $1/N_c$ expansion in QCD is quite standard in many classical applications. Can it help in quantum simulation?

Gives dramatic simplifications on the size of the allowed Hilbert space and dramatically simplifies interactions

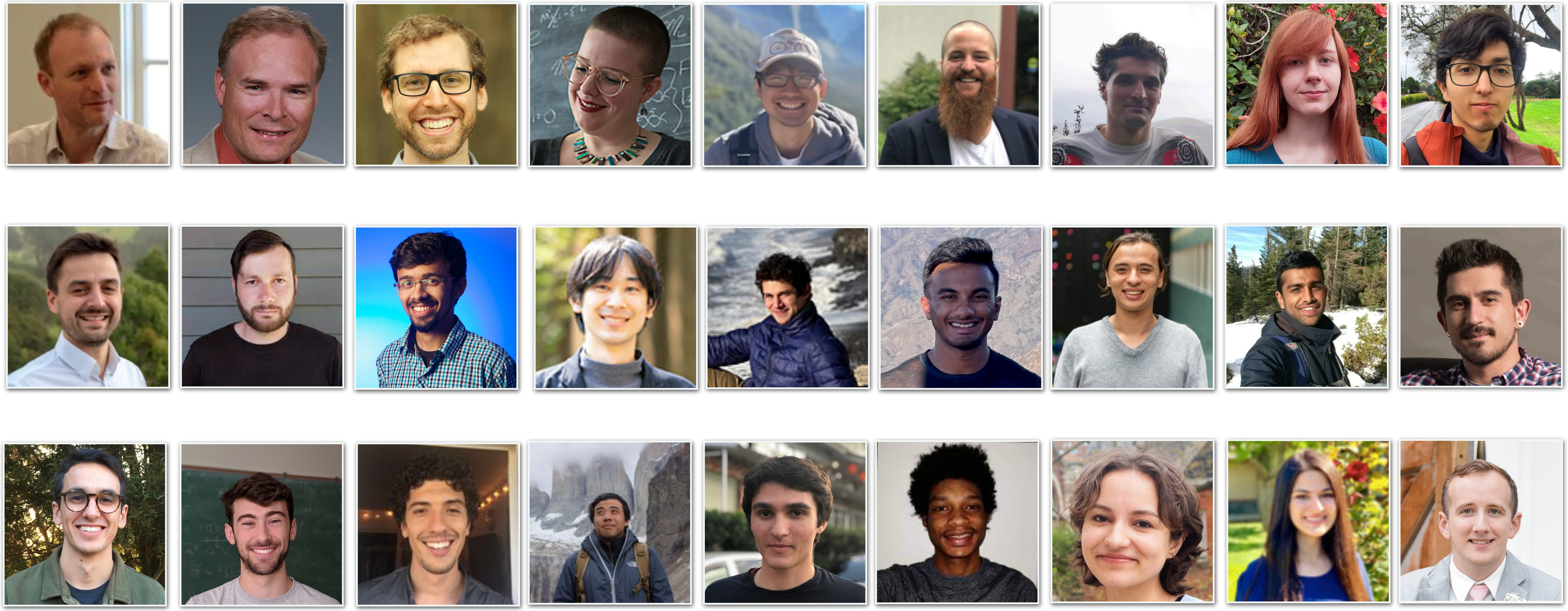
Results obtained on 8x8 lattice (25 times more plaquettes than previous best)



Quantum computers open the door to perform currenty unattainable simulations

Using Effective Field Theories takes best advantage of quantum hardware

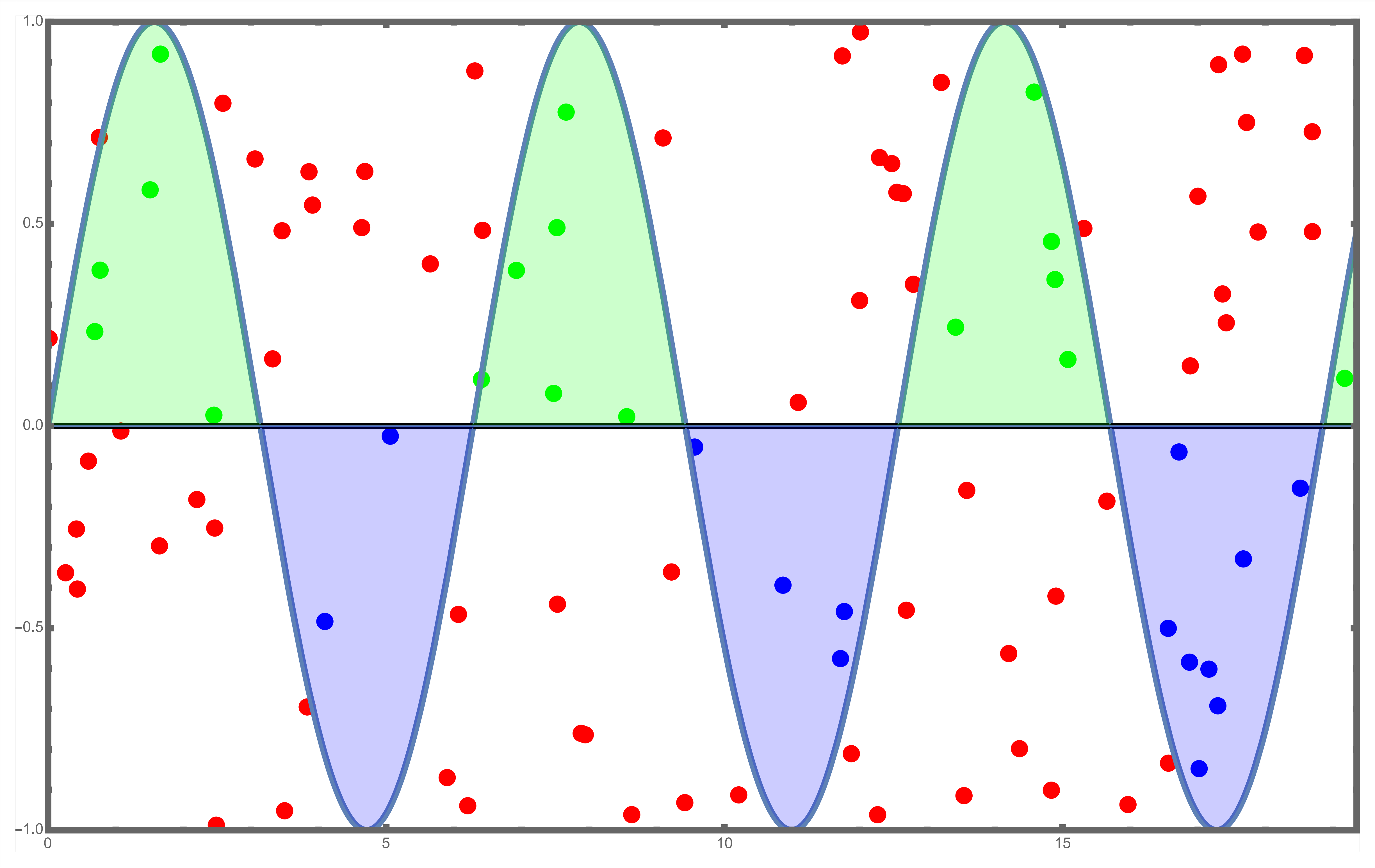
This will open door for exploring the most fundamental forces of the universe



Backup Slides

Trying to integrate a function that is not positive definite using Monte-Carlo techniques is exponentially difficult (so-called sign problem, NP-hard)

$$\int_0^1 \mathrm{d}x_1 \ldots \mathrm{d}x_n \; f(x_1, \ldots x_n) = \int_0^1 \mathrm{d}x_1 \ldots \mathrm{d}x_n \; \left(f_>(x_1, \ldots x_n) - f_<(x_1, \ldots x_n)\right)$$



To have final precision

$$\Delta\left(\int f_< - \int f_>\right) \sim \epsilon$$

need each integral with

$$\Delta\left(\int f_{>,<}\right) \sim \epsilon \times \frac{\int f_< - \int f_>}{\int f_< + \int f_>}$$

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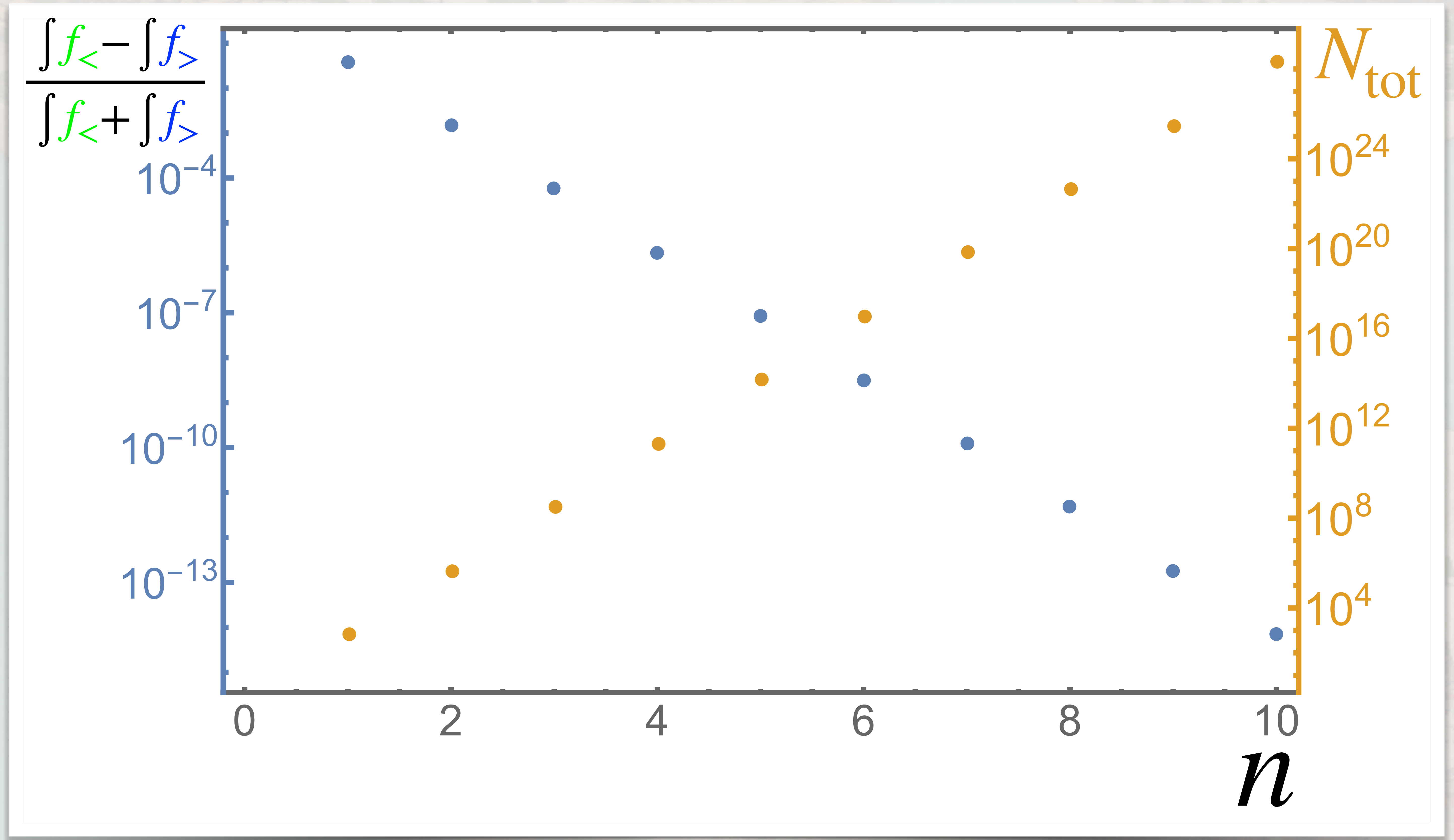

Number of points required is exponential in the size of the system

Impossible to do for systems of interesting size