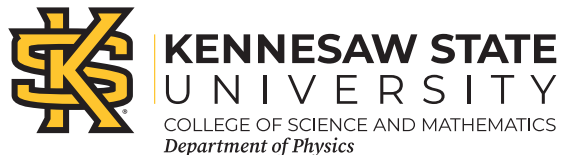


Theoretical predictions for $t\bar{t}W$ cross sections at approximate N³LO

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in collaboration with Chris Foster

- Higher-order soft-gluon corrections
- $t\bar{t}W^+$ and $t\bar{t}W^-$ cross sections
- Top-quark p_T and rapidity distributions



DIS 2024



$t\bar{t}W$ production

observation of $t\bar{t}W$ events at 7, 8, 13 TeV collisions at the LHC

measurements are significantly higher than theoretical predictions

QCD corrections at NLO are large, $\sim 47\%$ at 13.6 TeV

electroweak corrections are smaller but significant

the QCD corrections are dominated by soft-gluon emission

further improvement in theoretical accuracy by the inclusion of higher-order soft-gluon corrections

→ approximate NNLO (aNNLO) and approximate N³LO (aN³LO) predictions

Soft-gluon corrections

They are important for top-quark processes and they approximate known exact results at NLO and NNLO very well

partonic processes $q(p_q) + \bar{q}'(p_{\bar{q}'}) \rightarrow t(p_t) + \bar{t}(p_{\bar{t}}) + W(p_W)$

define $s = (p_q + p_{\bar{q}'})^2$, $t = (p_q - p_t)^2$, $u = (p_{\bar{q}'} - p_t)^2$

we define the threshold variable

$$s_4 = (p_{\bar{t}} + p_W + p_g)^2 - (p_{\bar{t}} + p_W)^2 = s + t + u - m_t^2 - (p_{\bar{t}} + p_W)^2$$

where extra gluon with p_g emitted

At partonic threshold $p_g \rightarrow 0$ and thus $s_4 \rightarrow 0$

Soft corrections $\left[\frac{\ln^k(s_4/m_t^2)}{s_4} \right]_+$ with $k \leq 2n - 1$ for the order α_s^n corrections

Resum these soft corrections for the double-differential cross section

Finite-order expansions \rightarrow no prescription needed or used
(this avoids underestimating the size of the corrections)

Approximate NNLO (aNNLO) and approximate N³LO (aN³LO) predictions
for cross sections and differential distributions

Soft-gluon Resummation

$$d\sigma_{pp \rightarrow t\bar{t}W} = \sum_{q, \bar{q}'} \int dx_a dx_b \phi_{q/p}(x_a, \mu_F) \phi_{\bar{q}'/p}(x_b, \mu_F) d\hat{\sigma}_{q\bar{q}' \rightarrow t\bar{t}W}(s_4, \mu_F)$$

take Laplace transforms $d\tilde{\sigma}_{q\bar{q}' \rightarrow t\bar{t}W}(N) = \int (ds_4/s) e^{-Ns_4/s} d\hat{\sigma}_{q\bar{q}' \rightarrow t\bar{t}W}(s_4)$

and $\tilde{\phi}(N) = \int_0^1 e^{-N(1-x)} \phi(x) dx$ with transform variable N

Then

$$d\tilde{\sigma}_{q\bar{q}' \rightarrow t\bar{t}W}(N) = \tilde{\phi}_{q/q}(N_q, \mu_F) \tilde{\phi}_{\bar{q}'/\bar{q}'}(N_{\bar{q}'}, \mu_F) d\tilde{\sigma}_{q\bar{q}' \rightarrow t\bar{t}W}(N, \mu_F)$$

Refactorization for the cross section

$$d\sigma_{q\bar{q}' \rightarrow t\bar{t}W}(N) = \tilde{\psi}_q(N_q, \mu_F) \tilde{\psi}_{\bar{q}'}(N_{\bar{q}'}, \mu_F) \text{tr} \left\{ H_{q\bar{q}' \rightarrow t\bar{t}W} \tilde{S}_{q\bar{q}' \rightarrow t\bar{t}W} \left(\frac{\sqrt{s}}{N\mu_F} \right) \right\}$$

$\psi_q, \psi_{\bar{q}'}$ → collinear emission from incoming partons

$H_{q\bar{q}' \rightarrow t\bar{t}W}$ is hard function → short distance

$S_{q\bar{q}' \rightarrow t\bar{t}W}$ is soft function → noncollinear soft gluons

Thus

$$d\tilde{\sigma}_{q\bar{q}' \rightarrow t\bar{t}W}(N) = \frac{\tilde{\psi}_{q/q}(N_q, \mu_F) \tilde{\psi}_{\bar{q}'/\bar{q}'}(N_{\bar{q}'}, \mu_F)}{\tilde{\phi}_{q/q}(N_q, \mu_F) \tilde{\phi}_{\bar{q}'/\bar{q}'}(N_{\bar{q}'}, \mu_F)} \text{tr} \left\{ H_{q\bar{q}' \rightarrow t\bar{t}W} \tilde{S}_{q\bar{q}' \rightarrow t\bar{t}W} \left(\frac{\sqrt{s}}{N\mu_F} \right) \right\}$$

$S_{q\bar{q}' \rightarrow t\bar{t}W}$ satisfies the renormalization group equation

$$\left(\mu_R \frac{\partial}{\partial \mu_R} + \beta(g_s) \frac{\partial}{\partial g_s} \right) S_{q\bar{q}' \rightarrow t\bar{t}W} = -\Gamma_{S q\bar{q}' \rightarrow t\bar{t}W}^\dagger S_{q\bar{q}' \rightarrow t\bar{t}W} - S_{q\bar{q}' \rightarrow t\bar{t}W} \Gamma_{S q\bar{q}' \rightarrow t\bar{t}W}$$

Soft anomalous dimension $\Gamma_{S q\bar{q}' \rightarrow t\bar{t}W}$ controls the evolution of the soft function which gives the exponentiation of logarithms of N

Renormalization group evolution \rightarrow resummation

$$\begin{aligned} d\tilde{\sigma}_{q\bar{q}' \rightarrow t\bar{t}W}^{\text{resum}}(N) &= \exp \left[\sum_{i=q, \bar{q}'} E_i(N_i) \right] \exp \left[\sum_{i=q, \bar{q}'} 2 \int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu} \gamma_{i/i}(N_i) \right] \\ &\times \text{tr} \left\{ H_{q\bar{q}' \rightarrow t\bar{t}W}(\alpha_s(\sqrt{s})) \bar{P} \exp \left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu} \Gamma_{S q\bar{q}' \rightarrow t\bar{t}W}^\dagger(\alpha_s(\mu)) \right] \right. \\ &\quad \left. \times \tilde{S}_{q\bar{q}' \rightarrow t\bar{t}W} \left(\alpha_s \left(\frac{\sqrt{s}}{N} \right) \right) P \exp \left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu} \Gamma_{S q\bar{q}' \rightarrow t\bar{t}W}(\alpha_s(\mu)) \right] \right\} \end{aligned}$$

$H_{q\bar{q}' \rightarrow t\bar{t}W}$ and $\tilde{S}_{q\bar{q}' \rightarrow t\bar{t}W}$ and $\Gamma_{S q\bar{q}' \rightarrow t\bar{t}W}$ are 2×2 matrices

choose color tensor basis of s -channel singlet and octet exchange

$$c_1^{q\bar{q}' \rightarrow t\bar{t}W} = \delta_{ab}\delta_{12}, \quad c_2^{q\bar{q}' \rightarrow t\bar{t}W} = T_{ba}^c T_{12}^c$$

The four matrix elements of $\Gamma_{S q\bar{q}' \rightarrow t\bar{t}W}$ are at one loop

$$\begin{aligned} \Gamma_{11 q\bar{q}' \rightarrow t\bar{t}W}^{(1)} &= \Gamma_{\text{cusp}}^{(1)}, \quad \Gamma_{12 q\bar{q}' \rightarrow t\bar{t}W}^{(1)} = \frac{C_F}{2N_c} \Gamma_{21 q\bar{q}' \rightarrow t\bar{t}W}^{(1)}, \quad \Gamma_{21 q\bar{q}' \rightarrow t\bar{t}W}^{(1)} = \ln \left(\frac{(t - m_t^2)(t' - m_t^2)}{(u - m_t^2)(u' - m_t^2)} \right), \\ \Gamma_{22 q\bar{q}' \rightarrow t\bar{t}W}^{(1)} &= \left(1 - \frac{C_A}{2C_F} \right) \left[\Gamma_{\text{cusp}}^{(1)} + 2C_F \ln \left(\frac{(t - m_t^2)(t' - m_t^2)}{(u - m_t^2)(u' - m_t^2)} \right) \right] + \frac{C_A}{2} \left[\ln \left(\frac{(t - m_t^2)(t' - m_t^2)}{s m_t^2} \right) - 1 \right] \end{aligned}$$

where $\Gamma_{\text{cusp}}^{(1)} = -C_F (L_{\beta_t} + 1)$ is the one-loop QCD massive cusp anomalous dimension,

with $L_{\beta_t} = (1 + \beta_t^2)/(2\beta_t) \ln[(1 - \beta_t)/(1 + \beta_t)]$ and $\beta_t = \sqrt{1 - 4m_t^2/s'}$,
 $s' = (p_t + p_{\bar{t}})^2$, $t' = (p_{\bar{q}'} - p_{\bar{t}})^2$, $u' = (p_q - p_{\bar{t}})^2$

At two loops

$$\begin{aligned} \Gamma_{11 q\bar{q}' \rightarrow t\bar{t}W}^{(2)} &= \Gamma_{\text{cusp}}^{(2)}, \quad \Gamma_{12 q\bar{q}' \rightarrow t\bar{t}W}^{(2)} = \left(K_2 - C_A N_2^{\beta_t} \right) \Gamma_{12 q\bar{q}' \rightarrow t\bar{t}W}^{(1)}, \quad \Gamma_{21 q\bar{q}' \rightarrow t\bar{t}W}^{(2)} = \left(K_2 + C_A N_2^{\beta_t} \right) \Gamma_{21 q\bar{q}' \rightarrow t\bar{t}W}^{(1)}, \\ \Gamma_{22 q\bar{q}' \rightarrow t\bar{t}W}^{(2)} &= K_2 \Gamma_{22 q\bar{q}' \rightarrow t\bar{t}W}^{(1)} + \left(1 - \frac{C_A}{2C_F} \right) \left(\Gamma_{\text{cusp}}^{(2)} - K_2 \Gamma_{\text{cusp}}^{(1)} \right) + \frac{1}{4} C_A^2 (1 - \zeta_3) \end{aligned}$$

where $K_2 = C_A(67/36 - \zeta_2/2) - 5n_f/18$ with n_f the number of light-quark flavors,

$$N_2^{\beta_t} = \frac{1}{4} \ln^2 \left(\frac{1 - \beta_t}{1 + \beta_t} \right) + \frac{(1 + \beta_t^2)}{8\beta_t} \left[\zeta_2 - \ln^2 \left(\frac{1 - \beta_t}{1 + \beta_t} \right) - \text{Li}_2 \left(\frac{4\beta_t}{(1 + \beta_t)^2} \right) \right]$$

and

$$\begin{aligned}
\Gamma_{\text{cusp}}^{(2)} = & K_2 \Gamma_{\text{cusp}}^{(1)} + C_F C_A \left\{ \frac{1}{2} + \frac{\zeta_2}{2} + \frac{1}{2} \ln^2 \left(\frac{1 - \beta_t}{1 + \beta_t} \right) \right. \\
& + \frac{(1 + \beta_t^2)}{4\beta_t} \left[\zeta_2 \ln \left(\frac{1 - \beta_t}{1 + \beta_t} \right) - \ln^2 \left(\frac{1 - \beta_t}{1 + \beta_t} \right) + \frac{1}{3} \ln^3 \left(\frac{1 - \beta_t}{1 + \beta_t} \right) - \text{Li}_2 \left(\frac{4\beta_t}{(1 + \beta_t)^2} \right) \right] \\
& + \frac{(1 + \beta_t^2)^2}{8\beta_t^2} \left[-\zeta_3 - \zeta_2 \ln \left(\frac{1 - \beta_t}{1 + \beta_t} \right) - \frac{1}{3} \ln^3 \left(\frac{1 - \beta_t}{1 + \beta_t} \right) - \ln \left(\frac{1 - \beta_t}{1 + \beta_t} \right) \text{Li}_2 \left(\frac{(1 - \beta_t)^2}{(1 + \beta_t)^2} \right) \right. \\
& \left. \left. + \text{Li}_3 \left(\frac{(1 - \beta_t)^2}{(1 + \beta_t)^2} \right) \right] \right\}
\end{aligned}$$

is the two-loop massive cusp anomalous dimension in QCD

These expressions are very similar to those for $t\bar{t}$ production via quark-antiquark annihilation (same color structure)

$t\bar{t}W$ production

Expansions of the resummed cross section to fixed order

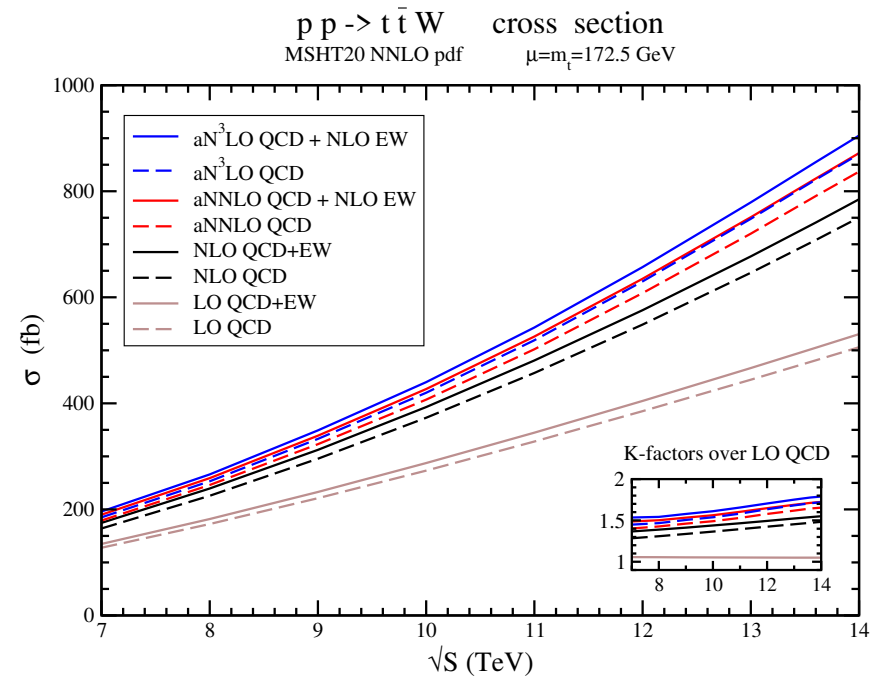
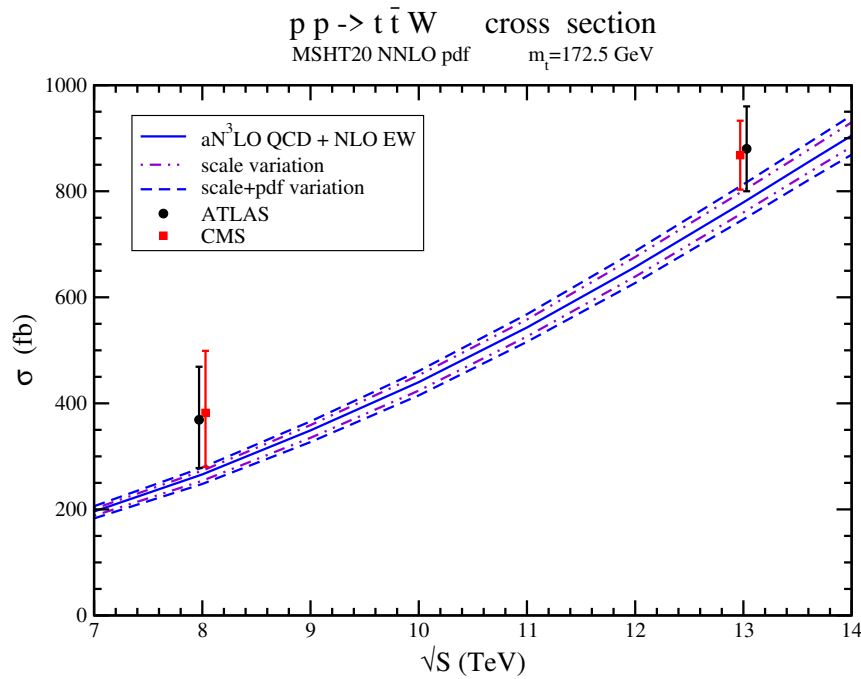
NLO expansions closely approximate exact NLO results
for total cross sections and top-quark p_T and rapidity distributions

NNLO expansions (aNNLO) are consistent with (almost exact) NNLO results
for total cross sections

aN³LO is the state of the art

electroweak corrections are also included

Cross sections for $t\bar{t}W$ production



large K -factors

improved agreement with data at aN^3LO

$t\bar{t}W$ cross sections

$t\bar{t}W$ cross sections (fb) in pp collisions at the LHC					
σ in fb	7 TeV	8 TeV	13 TeV	13.6 TeV	14 TeV
LO QCD	128 ⁺³⁹ ₋₂₈	172 ⁺⁵¹ ₋₃₆	445 ⁺¹¹⁴ ₋₈₄	481 ⁺¹²¹ ₋₉₀	506 ⁺¹²⁶ ₋₉₄
LO QCD+EW	135 ⁺⁴¹ ₋₂₉	182 ⁺⁵³ ₋₃₈	467 ⁺¹¹⁹ ₋₈₈	505 ⁺¹²⁷ ₋₉₄	531 ⁺¹³² ₋₉₈
NLO QCD	164 ⁺¹³ ₋₁₇	226 ⁺²⁰ ₋₂₃	646 ⁺⁸³ ₋₇₄	708 ⁺⁹⁴ ₋₈₂	750 ⁺¹⁰¹ ₋₈₈
NLO QCD+EW	175 ⁺¹² ₋₁₇	239 ⁺¹⁹ ₋₂₃	677 ⁺⁸⁰ ₋₇₄	741 ⁺⁹⁰ ₋₈₂	785 ⁺⁹⁷ ₋₈₈
aNNLO QCD	179 ⁺⁶ ₋₁₀	246 ⁺⁹ ₋₁₅	720 ⁺²⁹ ₋₄₃	791 ⁺³² ₋₄₇	837 ⁺³⁴ ₋₅₀
aNNLO QCD + NLO EW	190 ⁺⁶ ₋₁₀	259 ⁺⁹ ₋₁₅	751 ⁺²⁷ ₋₄₃	824 ⁺²⁹ ₋₄₇	872 ⁺³¹ ₋₅₀
aN ³ LO QCD	185 ⁺⁵ ₋₈	253 ⁺⁷ ₋₁₂	748 ⁺²⁴ ₋₁₉	822 ⁺²⁶ ₋₂₀	870 ⁺²⁸ ₋₂₁
aN ³ LO QCD + NLO EW	196 ⁺⁵ ₋₈	266 ⁺⁷ ₋₁₂	779 ⁺²² ₋₁₉	855 ⁺²³ ₋₂₀	905 ⁺²⁵ ₋₂₁

At 13.6 TeV

NLO QCD corrections \rightarrow 47%

aNNLO QCD corrections \rightarrow 17%

aN³LO QCD corrections \rightarrow 6%

electroweak NLO corrections \rightarrow 7%

Total aN³LO QCD+NLO EW cross section is 78% bigger than LO QCD

$t\bar{t}W^+$ and $t\bar{t}W^-$ cross sections

$t\bar{t}W^+$ and $t\bar{t}W^-$ cross sections (fb) in pp collisions at the LHC				
σ in fb	$t\bar{t}W^+$ 13 TeV	$t\bar{t}W^+$ 13.6 TeV	$t\bar{t}W^-$ 13 TeV	$t\bar{t}W^-$ 13.6 TeV
LO QCD	299^{+77}_{-57}	322^{+82}_{-60}	146^{+37}_{-28}	159^{+40}_{-30}
LO QCD+EW	313^{+80}_{-59}	337^{+85}_{-63}	154^{+39}_{-29}	168^{+42}_{-31}
NLO QCD	431^{+54}_{-49}	470^{+61}_{-54}	215^{+29}_{-25}	238^{+33}_{-28}
NLO QCD+EW	450^{+51}_{-48}	490^{+58}_{-53}	227^{+28}_{-25}	251^{+32}_{-28}
aNNLO QCD	480^{+19}_{-28}	525^{+21}_{-31}	240^{+10}_{-15}	266^{+11}_{-16}
aNNLO QCD + NLO EW	499^{+17}_{-28}	545^{+19}_{-31}	252^{+10}_{-15}	279^{+10}_{-16}
aN ³ LO QCD	498^{+16}_{-12}	545^{+17}_{-13}	250^{+8}_{-7}	277^{+9}_{-7}
aN ³ LO QCD + NLO EW	517^{+14}_{-12}	565^{+15}_{-13}	262^{+8}_{-7}	290^{+8}_{-7}

the $t\bar{t}W^+$ cross sections are larger than for $t\bar{t}W^-$

but the corrections are slightly bigger for $t\bar{t}W^-$

Comparison with 8 and 13 TeV CMS and ATLAS data

NLO and even aNNLO results are not sufficient
we need aN³LO corrections to describe the data

At 8 TeV, measurements from

CMS: 382_{-102}^{+117} fb

and from

ATLAS: 369_{-91}^{+100} fb

Theoretical prediction is

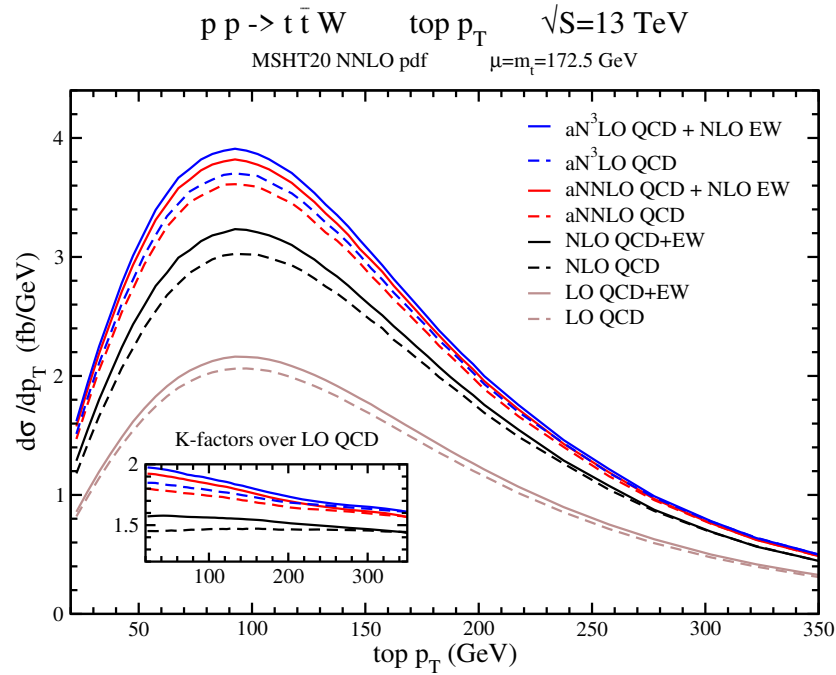
aN³LO QCD + NLO EW: 266_{-12-6}^{+7+6} fb

At 13 TeV, CMS finds 868 ± 65 fb with $t\bar{t}W^+$ 553 ± 42 fb and $t\bar{t}W^-$ 343 ± 36 fb
while ATLAS finds 880 ± 80 fb with $t\bar{t}W^+$ 583 ± 58 fb and $t\bar{t}W^-$ 296 ± 40 fb

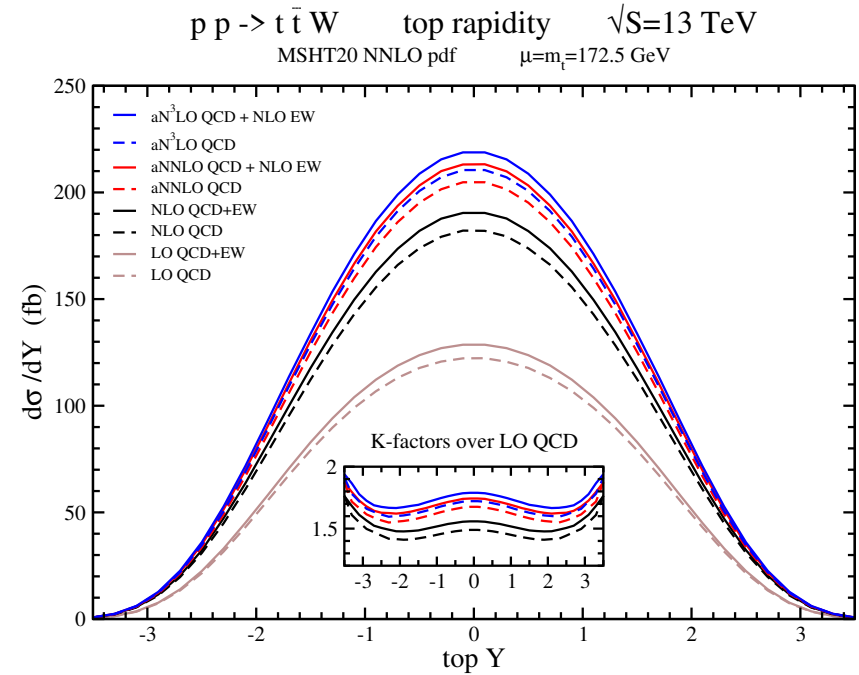
Theoretical prediction is

aN³LO QCD + NLO EW: 779_{-19-13}^{+22+12} fb with $t\bar{t}W^+$ 517_{-12-9}^{+14+8} fb and $t\bar{t}W^-$ 262_{-7-4}^{+8+4} fb

Top-quark p_T and rapidity distributions in $t\bar{t}W$ production at 13 TeV

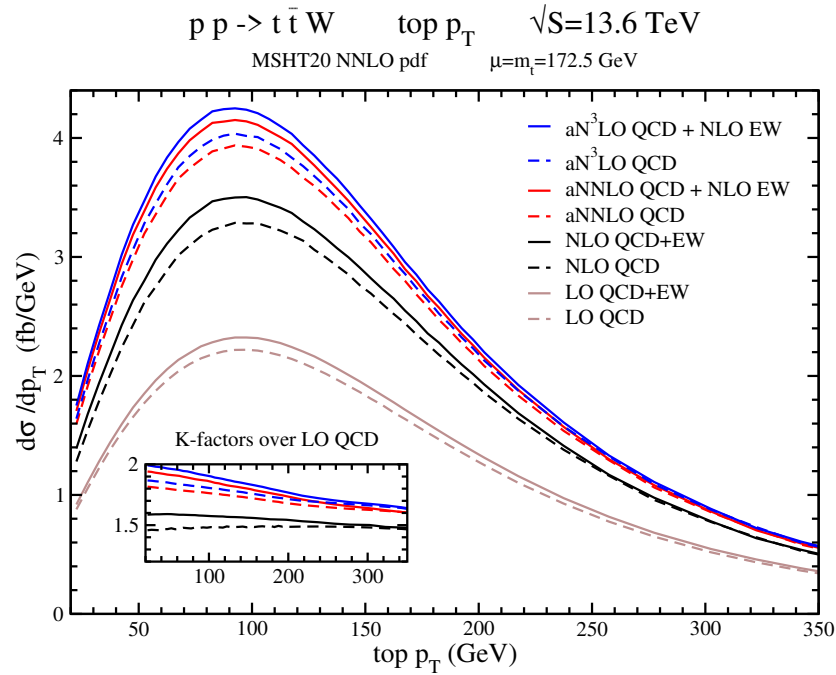


K -factors decrease at larger top p_T

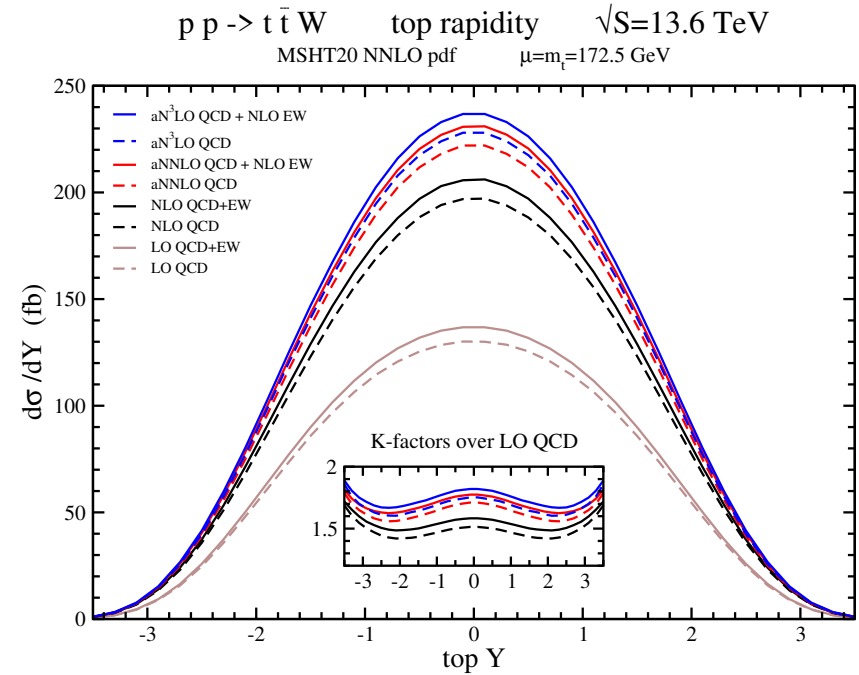


K -factors increase at larger rapidities

Top-quark p_T and rapidity distributions in $t\bar{t}W$ at 13.6 TeV



K -factors decrease at larger top p_T



K -factors increase at larger rapidities

Summary

- $t\bar{t}W$ production
- soft-gluon corrections through aN³LO
- results for total cross sections and differential distributions
- higher-order corrections further enhance and improve the theoretical predictions
- agreement with LHC data within uncertainties