Complete 1-loop study of exclusive J/ψ and Υ photoproduction with full GPD evolution

DIS 2024

Grenoble, France

Saad Nabeebaccus

IJCLab

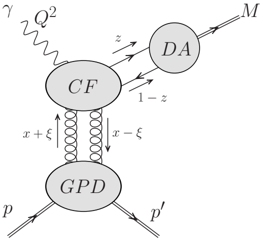
April 9, 2024

Based on work in progress with Chris Flett, Jean-Philippe Lansberg, Maxim Nefedov, Pawel Sznajder and Jakub Wagner

Exclusive quarkonium photoproduction and GPDs

Factorisation at the *amplitude* level:

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz \, H(x)\phi(z)C(x,z)$$

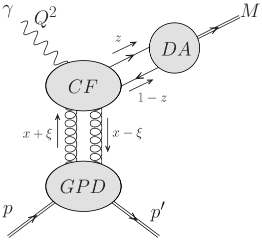


$H(x)$: Generalised parton distribution (GPD)
$\phi(z)$: Distribution amplitude (DA)
$C(x,z)$: Coefficient function (CF)

Exclusive quarkonium photoproduction and GPDs

Factorisation at the *amplitude* level:

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz\, H(x)\phi(z)C(x,z)$$



$H(x)$: Generalised parton distribution (GPD)
$\phi(z)$: Distribution amplitude (DA)
$C(x,z)$: Coefficient function (CF)

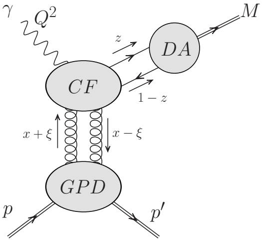
x: *Average* momentum fraction of nucleon carried by the partons

- $H(x,\xi,t)$: Generalised (3-dimensional) parton distribution

ξ: *Longitudinal* momentum fraction *transferred* to hard part

t: momentum difference squared of nucleons

Exclusive quarkonium photoproduction and GPDs



Factorisation at the *amplitude* level:

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz \, H(x) \phi(z) C(x,z)$$

$H(x)$: Generalised parton distribution (GPD)
$\phi(z)$: Distribution amplitude (DA)
$C(x,z)$: Coefficient function (CF)

x: *Average* momentum fraction of nucleon carried by the partons

- $H(x,\xi,t)$: Generalised (3-dimensional) parton distribution

  ξ: *Longitudinal* momentum fraction *transferred* to hard part

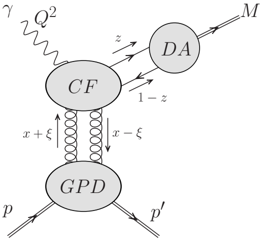
  t: momentum difference squared of nucleons

- GPDs reduce to PDFs in the forward limit:

  $H_q(x,0,0;\mu_F) = q(x;\mu_F)$, $H_g(x,0,0;\mu_F) = xg(x;\mu_F)$

Exclusive quarkonium photoproduction and GPDs

Factorisation at the *amplitude* level:



$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz\, H(x)\phi(z) C(x,z)$$

$H(x)$: Generalised parton distribution (GPD)
$\phi(z)$: Distribution amplitude (DA)
$C(x,z)$: Coefficient function (CF)

x: *Average* momentum fraction of nucleon carried by the partons

- $H(x,\xi,t)$: Generalised (3-dimensional) parton distribution

  ξ: *Longitudinal* momentum fraction *transferred* to hard part

  t: momentum difference squared of nucleons

- GPDs reduce to PDFs in the forward limit:

  $H_q(x,0,0;\mu_F) = q(x;\mu_F)$, $H_g(x,0,0;\mu_F) = xg(x;\mu_F)$

- No all-order proof of factorisation but *NLO* result indicates that it works

[D. Ivanov, A. Schafer, L. Szymanowski, G. Krasnikov: hep-ph/0401131]

Leading order amplitude

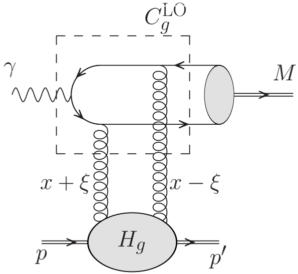
- Exclusive J/ψ photoproduction probes gluon GPDs only at LO.

Leading order amplitude

- Exclusive J/ψ photoproduction probes gluon GPDs only at LO.
- Employ *static limit* (NRQCD):
  $\Longrightarrow \phi(z) \sim \delta(z - 1/2)$.

Leading order amplitude

- Exclusive J/ψ photoproduction probes gluon GPDs only at LO.
- Employ *static limit* (NRQCD): $\Longrightarrow \phi(z) \sim \delta(z - 1/2)$.



$$\mathcal{A} = \epsilon_\gamma^\mu \epsilon_M^{*\nu} \mathcal{T}^{\mu\nu}$$

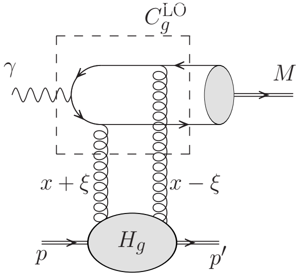
$$\mathcal{T}_{\rm LO}^{\mu\nu} = -g_\perp^{\mu\nu} \int_{-1}^{1} \frac{dx}{x} \left[C_g^{\rm LO}\left(\frac{\xi}{x}\right) \frac{H_g(x,\xi,\mu_F)}{x} \right]$$

$$C_g^{\rm LO}\left(\frac{\xi}{x}\right) = \frac{F_{\rm LO}}{\left[1 + \frac{\xi}{x} - i\delta\,\mathrm{sgn}(x)\right]\left[1 - \frac{\xi}{x} + i\delta\,\mathrm{sgn}(x)\right]}$$

$$F_{\rm LO} = 4\pi\alpha_s e e_q \frac{2T_F}{N_c} \left(\frac{\left\langle \mathcal{O}\left[{}^3S_1^{[1]}\right] \right\rangle}{3m_c^3} \right)^{\frac{1}{2}}, \qquad \xi = \frac{M^2}{2W_{\gamma p}^2 - M^2} \sim \frac{M^2}{2W_{\gamma p}^2}$$

Leading order amplitude

- Exclusive J/ψ photoproduction probes gluon GPDs only at LO.
- Employ *static limit* (NRQCD): $\Longrightarrow \phi(z) \sim \delta(z - 1/2)$.



$$\mathcal{A} = \epsilon_\gamma^\mu \epsilon_M^{*\nu} \mathcal{T}^{\mu\nu}$$

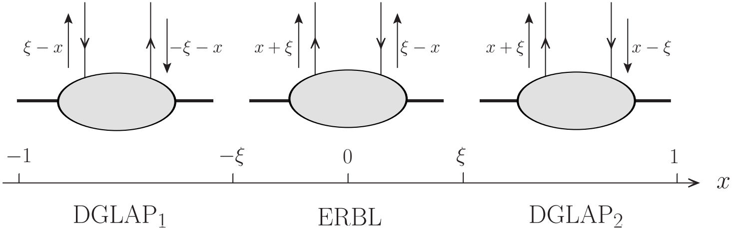
$$\mathcal{T}_{\mathrm{LO}}^{\mu\nu} = -g_\perp^{\mu\nu} \int_{-1}^{1} \frac{dx}{x} \left[C_g^{\mathrm{LO}} \left(\frac{\xi}{x} \right) \frac{H_g(x, \xi, \mu_F)}{x} \right]$$

$$C_g^{\mathrm{LO}} \left(\frac{\xi}{x} \right) = \frac{F_{\mathrm{LO}}}{\left[1 + \frac{\xi}{x} - i\delta \, \mathrm{sgn}(x) \right] \left[1 - \frac{\xi}{x} + i\delta \, \mathrm{sgn}(x) \right]}$$

$$F_{\mathrm{LO}} = 4\pi\alpha_s e e_q \frac{2T_F}{N_c} \left(\frac{\left\langle \mathcal{O} \left[{}^3S_1^{[1]} \right] \right\rangle}{3m_c^3} \right)^{\frac{1}{2}}, \qquad \xi = \frac{M^2}{2W_{\gamma p}^2 - M^2} \sim \frac{M^2}{2W_{\gamma p}^2}$$

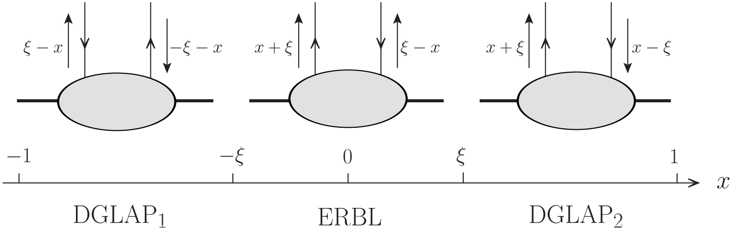
Large $W_{\gamma p}$ (small x in inclusive physics) $\leftrightarrow$ *small ξ*

Imaginary part of amplitude
DGLAP and ERBL regions

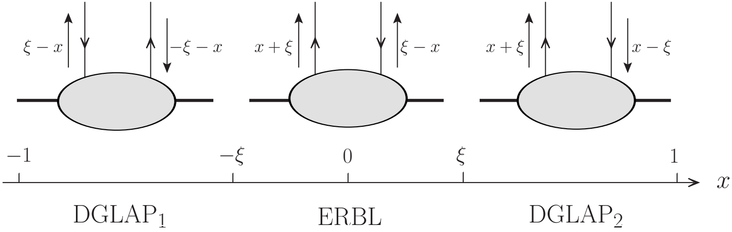


- Evolution equations different in ERBL/DGLAP regions.

Imaginary part of amplitude
DGLAP and ERBL regions



- Evolution equations different in ERBL/DGLAP regions.
- ERBL region shrinks as $W_{\gamma p}$ increases.



For LO amplitude:

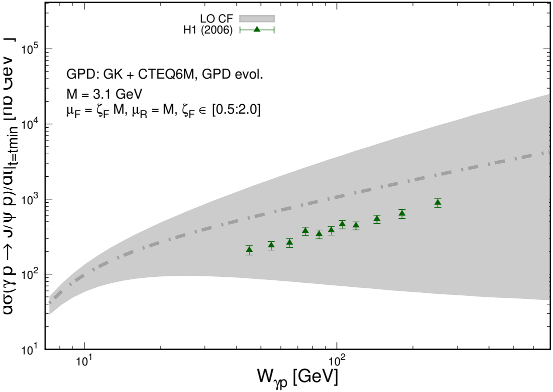
- Picks up *imaginary part* at $x = \pm\xi$.

$$\mathrm{Im}\, C_g^{\mathsf{LO}}\left(\frac{\xi}{x}\right) = -\pi \frac{F_{LO}}{2}\left[\delta\left(\frac{\xi}{x} - 1\right) + \delta\left(\frac{\xi}{x} + 1\right)\right]$$

$$\mathrm{Im}\, \mathcal{T}^{\mu\nu}_{\mathrm{LO}} = \pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} H_g(\xi, \xi)$$

- Otherwise, amplitude fully real (principal value contribution).

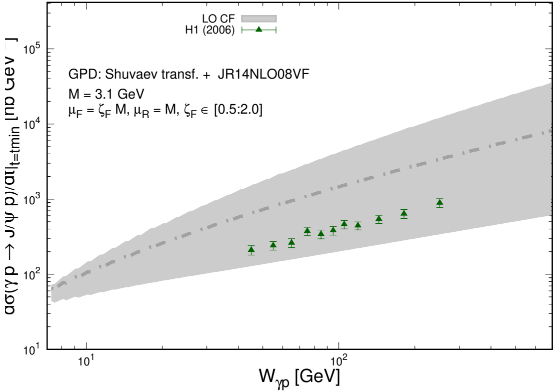
LO cross section



GPDs used are based on the *Goloskokov-Kroll model* [hep-ph/0611290]; fit parameters are based on CTEQ6M as the input PDFs to construct the GPDs.

Full LO evolution of GPDs is performed.

LO cross section

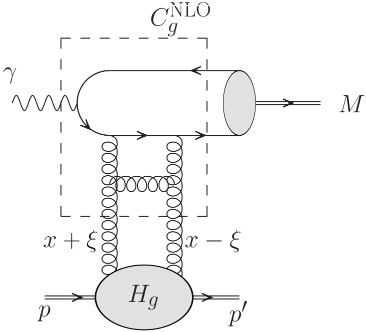


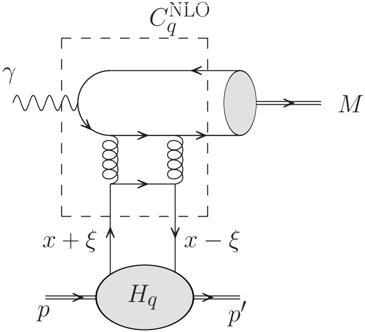
GPDs based on *Shuvaev transform*, using JR14 PDF set as the input.

Only PDFs are evolved.

NLO amplitude

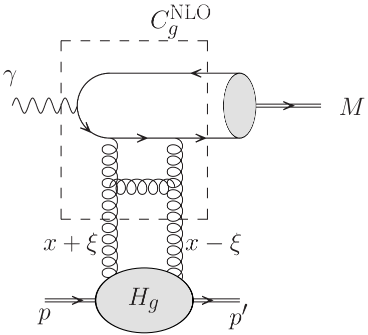
NLO amplitude has contributions from *both* quark and gluon GPDs:





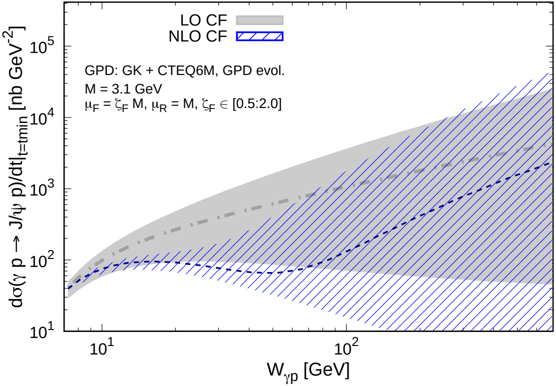
NLO amplitude

NLO amplitude has contributions from *both* quark and gluon GPDs:



Imaginary part comes fully from the *DGLAP region* ($\xi \leq |x| \leq 1$)

NLO cross section



GPDs used are based on the *Goloskokov-Kroll model* [hep-ph/0611290]; fit parameters are based on CTEQ6M as the input PDFs to construct the GPDs.

Full LO evolution of GPDs is performed.

Origin of problem for NLO cross section

$$\mathcal{T}^{\mu\nu}_{\text{NLO}} \supset i\pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} \Bigg[H_g(\xi,\xi) + \frac{\alpha_s(\mu_R) C_A}{\pi} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 \frac{dx}{x} H_g(x,\xi)$$

$$+ \frac{\alpha_s(\mu_R) C_A}{\pi} \frac{C_F}{C_A} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 dx \left(H_q(x,\xi) - H_q(-x,\xi)\right) \Bigg]$$

$H_g(x,\xi) \sim \mathrm{const}$, as $x \to \xi$ for small ξ

$\Longrightarrow$ appearance of $\ln \xi$ (high-energy logs).

Origin of problem for NLO cross section

$$\mathcal{T}^{\mu\nu}_{\text{NLO}} \supset i\pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} \left[H_g(\xi,\xi) + \frac{\alpha_s(\mu_R) C_A}{\pi} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 \frac{dx}{x} H_g(x,\xi) \right.$$
$$\left. + \frac{\alpha_s(\mu_R) C_A}{\pi} \frac{C_F}{C_A} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 dx \left(H_q(x,\xi) - H_q(-x,\xi)\right) \right]$$

$H_g(x,\xi) \sim \mathrm{const}$, as $x \to \xi$ for small ξ
$\implies$ appearance of $\ln \xi$ (high-energy logs).

Same thing happens in the quark case, since
$H_q^{(+)}(x,\xi) \equiv H_q(x,\xi) - H_q(-x,\xi) \sim \frac{1}{x}$ as $x, \xi \to 0$

Origin of problem for NLO cross section

$$\mathcal{T}^{\mu\nu}_{\text{NLO}} \supset i\pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} \left[H_g(\xi,\xi) + \frac{\alpha_s(\mu_R) C_A}{\pi} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 \frac{dx}{x} H_g(x,\xi) \right.$$

$$\left. + \frac{\alpha_s(\mu_R) C_A}{\pi} \frac{C_F}{C_A} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 dx \left(H_q(x,\xi) - H_q(-x,\xi)\right) \right]$$

$H_g(x,\xi) \sim \text{const}$, as $x \to \xi$ for small ξ
$\implies$ appearance of $\ln\xi$ (high-energy logs).

Same thing happens in the quark case, since
$H_q^{(+)}(x,\xi) \equiv H_q(x,\xi) - H_q(-x,\xi) \sim \frac{1}{x}$ as $x, \xi \to 0$

Large $\ln\xi$ contributions are purely imaginary and come from the DGLAP region ($\xi < |x| < 1$).

Origin of problem for NLO cross section

$$\mathcal{T}^{\mu\nu}_{\text{NLO}} \supset i\pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} \left[H_g(\xi,\xi) + \frac{\alpha_s(\mu_R) C_A}{\pi} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 \frac{dx}{x} H_g(x,\xi) \right.$$
$$\left. + \frac{\alpha_s(\mu_R) C_A}{\pi} \frac{C_F}{C_A} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 dx \left(H_q(x,\xi) - H_q(-x,\xi)\right) \right]$$

$H_g(x,\xi) \sim \mathrm{const}$, as $x \to \xi$ for small ξ
$\implies$ appearance of $\ln \xi$ (high-energy logs).

Same thing happens in the quark case, since
$H_q^{(+)}(x,\xi) \equiv H_q(x,\xi) - H_q(-x,\xi) \sim \frac{1}{x}$ as $x, \xi \to 0$

Large $\ln \xi$ contributions are purely imaginary and come from the DGLAP region ($\xi < |x| < 1$).

Opposite sign to LO for $\mu_F > M/2$.

Why large scale uncertainties present?

In the DGLAP evolution of low ξ GPDs, the probability of emitting a new gluon is *strongly enhanced* by the large value of $\ln \xi$.

Why large scale uncertainties present?

In the DGLAP evolution of low ξ GPDs, the probability of emitting a new gluon is *strongly enhanced* by the large value of $\ln \xi$.

In contrast, the NLO coefficient function allows for the emission (and reabsorption) of *only one gluon*.

Why large scale uncertainties present?

In the DGLAP evolution of low ξ GPDs, the probability of emitting a new gluon is *strongly enhanced* by the large value of $\ln \xi$.

In contrast, the NLO coefficient function allows for the emission (and reabsorption) of *only one gluon*.

$\Longrightarrow$ we cannot expect compensation between the contributions coming from the GPD and the coefficient function as we vary the scale μ_F .

Why large scale uncertainties present?

In the DGLAP evolution of low ξ GPDs, the probability of emitting a new gluon is *strongly enhanced* by the large value of $\ln \xi$.

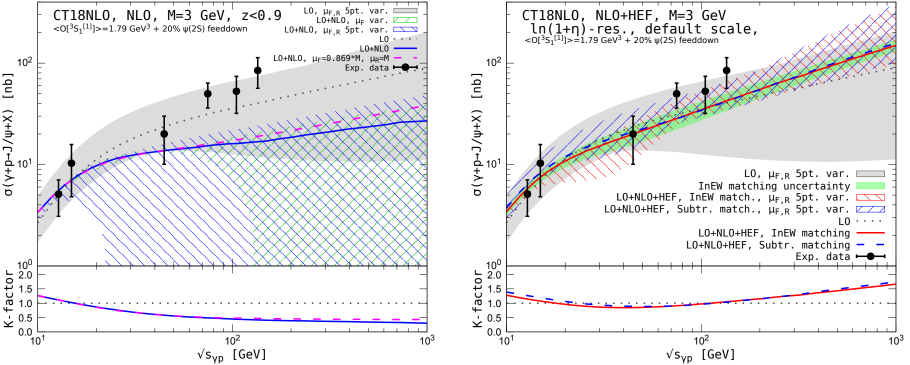
In contrast, the NLO coefficient function allows for the emission (and reabsorption) of *only one gluon*.

$\Longrightarrow$ we cannot expect compensation between the contributions coming from the GPD and the coefficient function as we vary the scale μ_F .

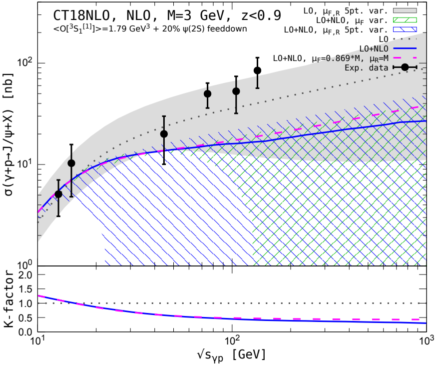
$\Longrightarrow$ Hints towards a solution through *resummation* of these logarithms...

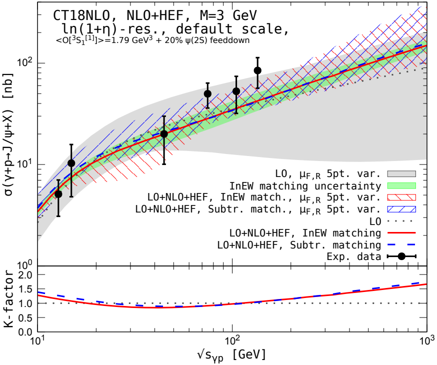
Instabilities in the inclusive case

Solution through resummation



In J-P. Lansberg, M. Nefedov, M. Ozcelik [2112.06789, 2306.02425], instabilities in the total inclusive photoproduction cross sections of pseudoscalar quarkonia and vector S-wave quarkonia are *cured by resumming the high-energy logarithms*.

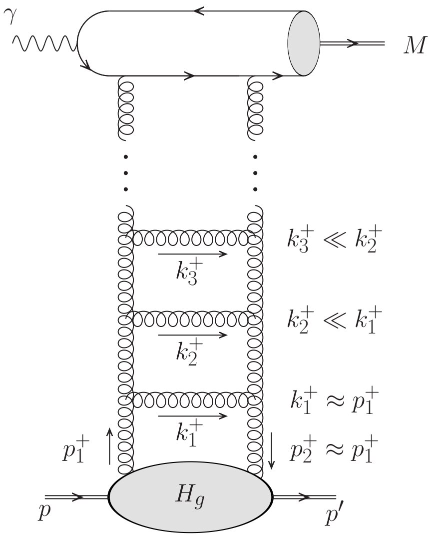




In J-P. Lansberg, M. Nefedov, M. Ozcelik [2112.06789, 2306.02425], instabilities in the total inclusive photoproduction cross sections of pseudoscalar quarkonia and vector S-wave quarkonia are *cured by resumming the high-energy logarithms*.

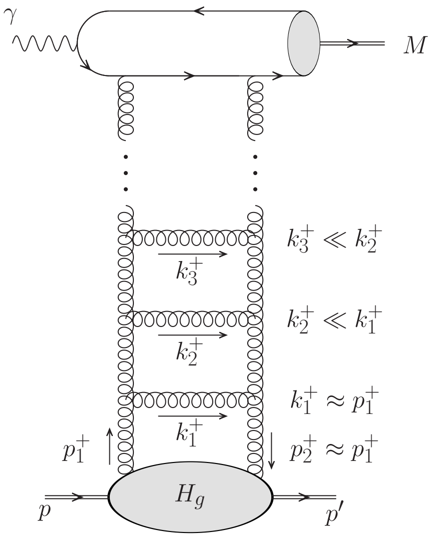
$\Longrightarrow$ See Maxim Nefedov's talk in WG2

Multiple gluon emissions:
BFKL ladder and resummation



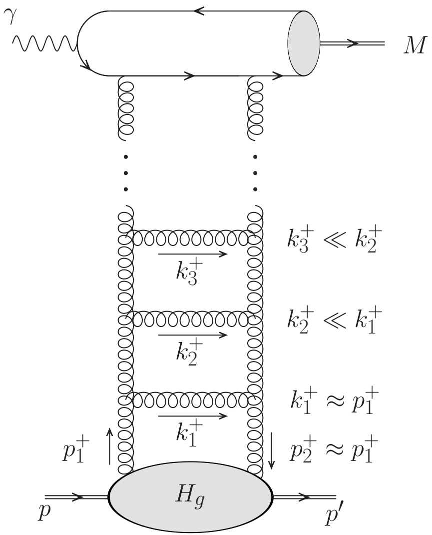
- Logarithms are generated by emission of gluons, with strong ordering in $+$ lightcone momentum.

Multiple gluon emissions:
BFKL ladder and resummation



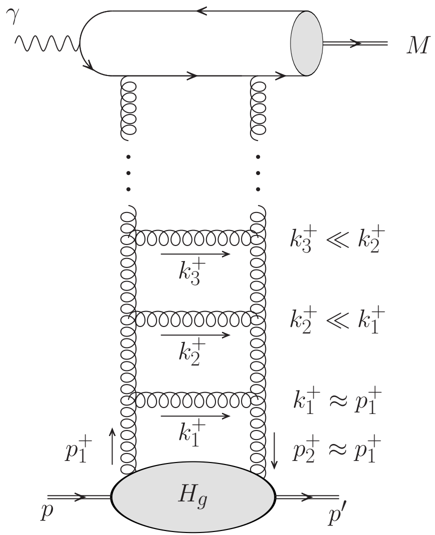
- Logarithms are generated by emission of gluons, with strong ordering in + lightcone momentum.
- They become large at high energies, and need to be *resummed*.

Multiple gluon emissions:
BFKL ladder and resummation



- Logarithms are generated by emission of gluons, with strong ordering in + lightcone momentum.
- They become large at high energies, and need to be *resummed*.
- We implement a *resummation* of these BFKL-type logs, consistent with fixed-order evolution of GPD:

Multiple gluon emissions:
BFKL ladder and resummation



- Logarithms are generated by emission of gluons, with strong ordering in + lightcone momentum.
- They become large at high energies, and need to be *resummed*.
- We implement a *resummation* of these BFKL-type logs, consistent with fixed-order evolution of GPD: $\implies$ *Doubly-logarithmic approximation (DLA)*

Implementation of resummation: $C^{\mathrm{CF}} \to C^{\mathrm{HEF}}$

$$C_g^{\mathrm{HEF}}\left(\frac{\xi}{x}\right) = \frac{-i\pi\hat{\alpha}_s F_{\mathrm{LO}}}{2\left|\frac{\xi}{x}\right|}\sqrt{\frac{L_\mu}{L_x}}\left\{ I_1\left(2\sqrt{L_x L_\mu}\right) - 2\sum_{k=1}^{\infty}\mathrm{Li}_{2k}(-1)\left(\frac{L_x}{L_\mu}\right)^k I_{2k-1}\left(2\sqrt{L_x L_\mu}\right)\right\},$$

where $L_\mu = \ln[M^2/(4\mu_F^2)]$ and $L_x = \hat{\alpha}_s \ln\left|\frac{x}{\xi}\right|$.

Implementation of resummation: $C^{\text{CF}} \to C^{\text{HEF}}$

$$C_g^{\text{HEF}}\left(\frac{\xi}{x}\right) = \frac{-i\pi\hat{\alpha}_s F_{\text{LO}}}{2\left|\frac{\xi}{x}\right|}\sqrt{\frac{L_\mu}{L_x}}\left\{ I_1\left(2\sqrt{L_x L_\mu}\right) - 2\sum_{k=1}^{\infty}\text{Li}_{2k}(-1)\left(\frac{L_x}{L_\mu}\right)^k I_{2k-1}\left(2\sqrt{L_x L_\mu}\right)\right\},$$

where $L_\mu = \ln[M^2/(4\mu_F^2)]$ and $L_x = \hat{\alpha}_s \ln\left|\frac{x}{\xi}\right|$.

This yields, when expanded in α_s,

$$C_g^{\text{HEF}}\left(\frac{\xi}{x}\right) = \frac{-i\pi F_{\text{LO}}}{2}\left(\underbrace{\delta\left(\left|\frac{\xi}{x}\right| - 1\right) + \frac{\hat{\alpha}_s}{\left|\frac{\xi}{x}\right|}\ln\left(\frac{M^2}{4\mu_F^2}\right)}_{\to C_g^{\text{asy.}}} + \frac{\hat{\alpha}_s^2}{\left|\frac{\xi}{x}\right|}\ln\frac{1}{\left|\frac{\xi}{x}\right|}\left[\frac{\pi^2}{6} + \frac{1}{2}\ln^2\left(\frac{M^2}{4\mu_F^2}\right)\right] + \dots\right)$$

- First two terms in α_s match the fixed-order CF computation at small x.

Implementation of resummation: $C^{\text{CF}} \to C^{\text{HEF}}$

$$C_g^{\text{HEF}}\left(\frac{\xi}{x}\right) = \frac{-i\pi\hat{\alpha}_s F_{\text{LO}}}{2\left|\frac{\xi}{x}\right|}\sqrt{\frac{L_\mu}{L_x}}\left\{I_1\left(2\sqrt{L_x L_\mu}\right) - 2\sum_{k=1}^{\infty}\text{Li}_{2k}(-1)\left(\frac{L_x}{L_\mu}\right)^k I_{2k-1}\left(2\sqrt{L_x L_\mu}\right)\right\},$$

where $L_\mu = \ln[M^2/(4\mu_F^2)]$ and $L_x = \hat{\alpha}_s \ln\left|\frac{x}{\xi}\right|$.

This yields, when expanded in α_s,

$$C_g^{\text{HEF}}\left(\frac{\xi}{x}\right) = \frac{-i\pi F_{\text{LO}}}{2}\Bigg(\underbrace{\delta\left(\left|\frac{\xi}{x}\right| - 1\right) + \frac{\hat{\alpha}_s}{\left|\frac{\xi}{x}\right|}\ln\left(\frac{M^2}{4\mu_F^2}\right)}_{\to C_g^{\text{asy.}}} + \frac{\hat{\alpha}_s^2}{\left|\frac{\xi}{x}\right|}\ln\frac{1}{\left|\frac{\xi}{x}\right|}\left[\frac{\pi^2}{6} + \frac{1}{2}\ln^2\left(\frac{M^2}{4\mu_F^2}\right)\right] + \dots\Bigg)$$

- First two terms in α_s match the fixed-order CF computation at small x.
- *Cannot fix the scale at NNLO to get rid of all the $1/\xi$-enhanced contributions*

Implementation of resummation: $C^{\rm CF} \to C^{\rm HEF}$

$$C_g^{\rm HEF}\left(\frac{\xi}{x}\right) = \frac{-i\pi\hat{\alpha}_s F_{\rm LO}}{2\left|\frac{\xi}{x}\right|}\sqrt{\frac{L_\mu}{L_x}}\left\{ I_1\left(2\sqrt{L_x L_\mu}\right) - 2\sum_{k=1}^{\infty} {\rm Li}_{2k}(-1)\left(\frac{L_x}{L_\mu}\right)^k I_{2k-1}\left(2\sqrt{L_x L_\mu}\right)\right\},$$

where $L_\mu = \ln[M^2/(4\mu_F^2)]$ and $L_x = \hat{\alpha}_s \ln\left|\frac{x}{\xi}\right|$.

This yields, when expanded in α_s,

$$C_g^{\rm HEF}\left(\frac{\xi}{x}\right) = \frac{-i\pi F_{\rm LO}}{2}\Bigg(\underbrace{\delta\left(\left|\frac{\xi}{x}\right| - 1\right) + \frac{\hat{\alpha}_s}{\left|\frac{\xi}{x}\right|}\ln\left(\frac{M^2}{4\mu_F^2}\right)}_{\to C_g^{\rm asy.}} + \frac{\hat{\alpha}_s^2}{\left|\frac{\xi}{x}\right|}\ln\frac{1}{\left|\frac{\xi}{x}\right|}\left[\frac{\pi^2}{6} + \frac{1}{2}\ln^2\left(\frac{M^2}{4\mu_F^2}\right)\right] + \dots\Bigg)$$

- First two terms in α_s match the fixed-order CF computation at small x.
- *Cannot fix the scale at NNLO to get rid of all the $1/\xi$-enhanced contributions*

Quark coefficient function:

$$C_q^{\rm HEF}\left(\frac{\xi}{x}\right) = \frac{2C_F}{C_A}C_g^{\rm HEF}\left(\frac{\xi}{x}\right),$$

Matching

We use *subtractive matching*:

$$
\begin{aligned}
C_{g,q}^{\text{match.}}\left(\frac{\xi}{x}\right) &= C_{g,q}^{\text{NLO CF}}\left(\frac{\xi}{x}\right) - C_{g,q}^{\text{asy.}}\left(\frac{\xi}{x}\right) + C_{g,q}^{\text{HEF}}\left(\frac{\xi}{x}\right), \\
C_{g}^{\text{asy.}}\left(\frac{\xi}{x}\right) &= \frac{C_A}{2C_F} C_{q}^{\text{asy.}}\left(\frac{\xi}{x}\right) \\
&= \frac{-i\pi F_{\mathrm{LO}}}{2}\left[\delta\left(\left|\frac{\xi}{x}\right| - 1\right) + \frac{\hat{\alpha}_s}{\left|\frac{\xi}{x}\right|}\ln\left(\frac{M^2}{4\mu_F^2}\right)\right].
\end{aligned}
$$

- $C_{g}^{\text{asy.}}\left(\frac{\xi}{x}\right)$: first two terms in the α_s expansion of $C_{g}^{\text{HEF}}\left(\frac{\xi}{x}\right)$.

Matching

We use *subtractive matching*:

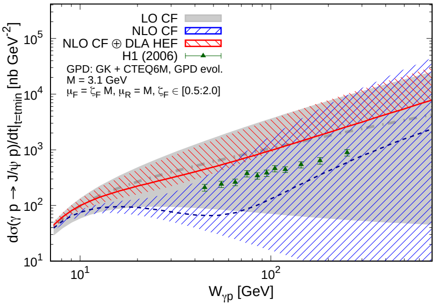
$$C_{g,q}^{\text{match.}}\left(\frac{\xi}{x}\right) = C_{g,q}^{\text{NLO CF}}\left(\frac{\xi}{x}\right) - C_{g,q}^{\text{asy.}}\left(\frac{\xi}{x}\right) + C_{g,q}^{\text{HEF}}\left(\frac{\xi}{x}\right),$$

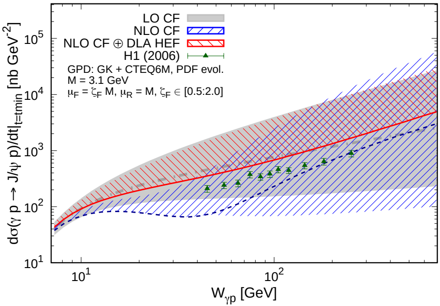
$$C_{g}^{\text{asy.}}\left(\frac{\xi}{x}\right) = \frac{C_A}{2C_F} C_{q}^{\text{asy.}}\left(\frac{\xi}{x}\right)$$

$$= \frac{-i\pi F_{\text{LO}}}{2}\left[\delta\left(\left|\frac{\xi}{x}\right| - 1\right) + \frac{\hat{\alpha}_s}{\left|\frac{\xi}{x}\right|}\ln\left(\frac{M^2}{4\mu_F^2}\right)\right].$$

- $C_g^{\text{asy.}}\left(\frac{\xi}{x}\right)$: first two terms in the α_s expansion of $C_g^{\text{HEF}}\left(\frac{\xi}{x}\right)$.
- Matching performed before x-integration.

Stabilisation after resummation

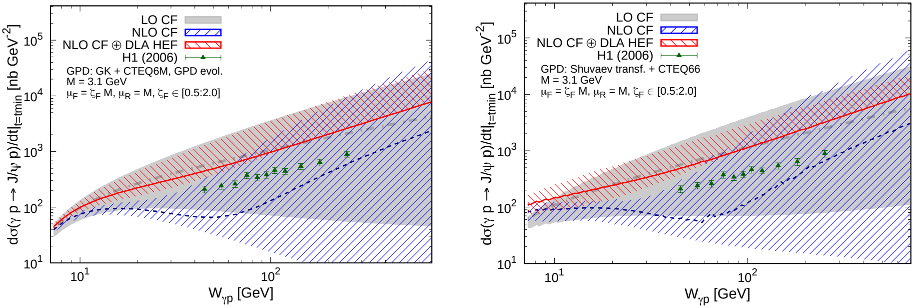




- Left: GK model based on CTEQ6M PDF input, *full LO evolution of GPDs*.
- Right: GK model based on CTEQ6M PDF input, *evolution of input PDF only*.

Results

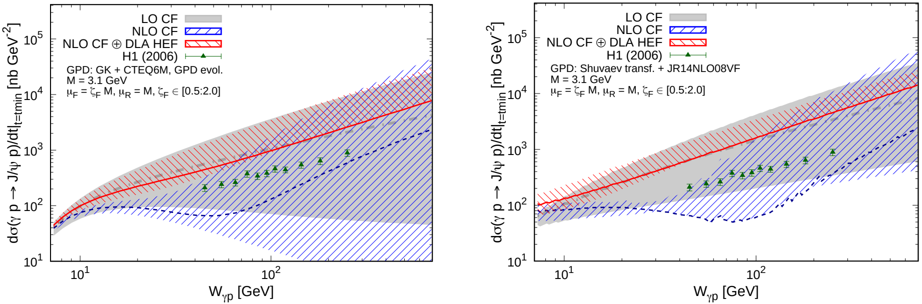
Stabilisation after resummation



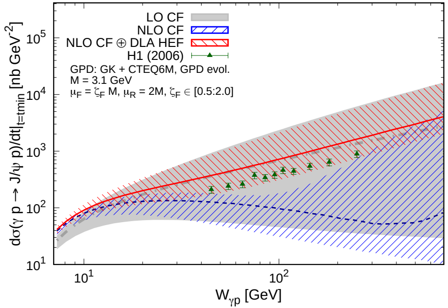
- Left: GK model based on CTEQ6M PDF input, *full LO evolution of GPDs*.
- Right: Shuvaev transform, CTEQ66 PDF input, *evolution of input PDF only*

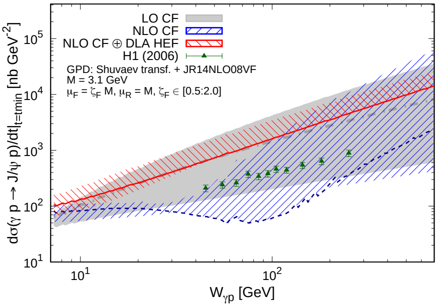
Results

Stabilisation after resummation



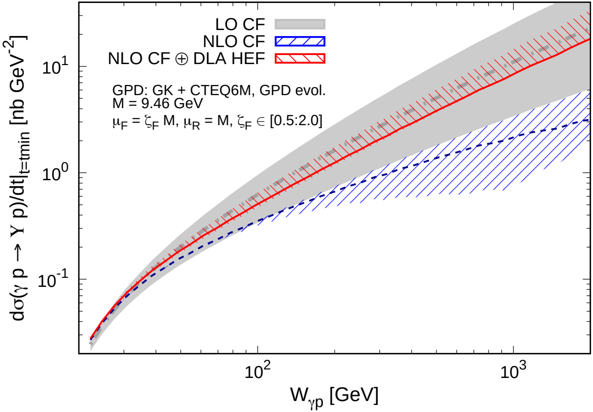
- Left: GK model based on CTEQ6M PDF input, *full LO evolution of GPDs*.
- Right: Shuvaev transform, JR14 PDF input, *evolution of input PDF only*





- Left: GK model based on CTEQ6M PDF input, *full LO evolution of GPDs*, $\mu_R = 2M$.
- Right: Shuvaev transform, JR14 PDF input, *evolution of input PDF only*

Results
Exclusive Υ photoproduction



- GK model based on CTEQ6M PDF input, *full LO evolution of GPDs*

Conclusion

- Exclusive J/ψ photoproduction at increasing $W_{\gamma p}$ suffers from *perturbative instabilities at NLO*.

Conclusion

- Exclusive J/ψ photoproduction at increasing $W_{\gamma p}$ suffers from *perturbative instabilities at NLO*.
- Similar situation to inclusive J/ψ photoproduction and hadroproduction.

Conclusion

- Exclusive J/ψ photoproduction at increasing $W_{\gamma p}$ suffers from *perturbative instabilities at NLO*.
- Similar situation to inclusive J/ψ photoproduction and hadroproduction.
- *Scale fixing* provides a patch ("partial resummation") at NLO, but *insufficient* beyond NLO.

Conclusion

- Exclusive J/ψ photoproduction at increasing $W_{\gamma p}$ suffers from *perturbative instabilities at NLO*.
- Similar situation to inclusive J/ψ photoproduction and hadroproduction.
- *Scale fixing* provides a patch ("partial resummation") at NLO, but *insufficient* beyond NLO.
- We performed a high-energy resummation of the large logarithms of $\ln \xi$ matched to the NLO CF results (no new ingredients).

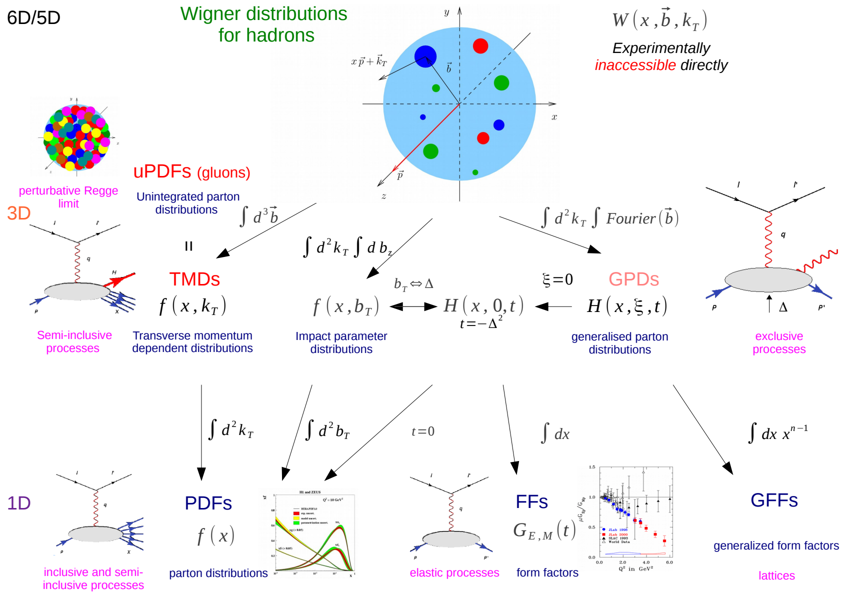
Conclusion

- Exclusive J/ψ photoproduction at increasing $W_{\gamma p}$ suffers from *perturbative instabilities at NLO*.
- Similar situation to inclusive J/ψ photoproduction and hadroproduction.
- *Scale fixing* provides a patch ("partial resummation") at NLO, but *insufficient* beyond NLO.
- We performed a high-energy resummation of the large logarithms of $\ln \xi$ matched to the NLO CF results (no new ingredients).
- Like in the inclusive case, the matched NLO+HEF results are *stable* and agree with data with the (large) theoretical uncertainties.

Conclusion

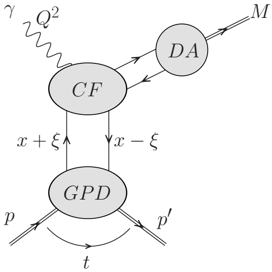
- Exclusive J/ψ photoproduction at increasing $W_{\gamma p}$ suffers from *perturbative instabilities at NLO*.
- Similar situation to inclusive J/ψ photoproduction and hadroproduction.
- *Scale fixing* provides a patch ("partial resummation") at NLO, but *insufficient* beyond NLO.
- We performed a high-energy resummation of the large logarithms of $\ln \xi$ matched to the NLO CF results (no new ingredients).
- Like in the inclusive case, the matched NLO+HEF results are *stable* and agree with data with the (large) theoretical uncertainties.
- The next step is to see how to fit GPD from such data.

BACKUP SLIDES



6D/5D
Wigner distributions for hadrons
$W(x, \vec{b}, k_T)$
Experimentally inaccessible directly
perturbative Regge limit
3D
uPDFs (gluons)
Unintegrated parton distributions
=
$\int d^3\vec{b}$
$\int d^2k_T \int db_z$
$\int d^2k_T \int Fourier(\vec{b})$
TMDs
$f(x, k_T)$
$b_T \Leftrightarrow \Delta$
$f(x, b_T)$
$H(x, 0, t)$
$t = -\Delta^2$
$\xi = 0$
GPDs
$H(x, \xi, t)$
Δ
Semi-inclusive processes
Transverse momentum dependent distributions
Impact parameter distributions
generalised parton distributions
exclusive processes
$\int d^2k_T$
$\int d^2b_T$
$t = 0$
$\int dx$
$\int dx\, x^{n-1}$
1D
PDFs
$f(x)$
FFs
$G_{E,M}(t)$
GFFs
generalized form factors
inclusive and semi-inclusive processes
parton distributions
elastic processes
form factors
lattices

DVMP: γ replaced by $\rho,\ \pi, \cdots$



GPD (soft) $\otimes$ CF (hard) $\otimes$ Distribution Amplitude (soft)

[J. Collins, L. Frankfurt, M. Strikman: hep-ph/9611433]

[A. Radyushkin: hep-ph/9704207]

proofs valid only for some restricted cases

Gluon GPDs at twist 2 [M. Diehl: hep-ph/0307382]

$$F^g = \frac{1}{P^+}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'| G^{+\mu}\left(-\frac{z}{2}\right) G_\mu^{+}\left(\frac{z}{2}\right)|p\rangle\Big|_{z^+=0, z_\perp=0}$$

$$= \frac{1}{2P^+}\left[H^g(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E^g(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

$$\tilde{F}^g = -\frac{i}{P^+}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'| G^{+\mu}\left(-\frac{z}{2}\right) \tilde{G}_\mu^{+}\left(\frac{z}{2}\right)|p\rangle\Big|_{z^+=0, z_\perp=0}$$

$$= \frac{1}{2P^+}\left[\tilde{H}^g(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^g(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2m}u(p)\right]$$

$H^g(x,\xi,t) \xrightarrow{\xi=0, t=0}$ PDF $xg(x)$

$\tilde{H}^g(x,\xi,t) \xrightarrow{\xi=0, t=0}$ polarised PDF $x\Delta g(x)$

Quark GPDs at twist 2 [M. Diehl: hep-ph/0307382]

without helicity flip (chiral-even Γ matrices): 4 chiral-even GPDs: (Note: $\Delta = p' - p$)

$$
\begin{aligned}
F^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+ q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, z_\perp=0} \\
&= \frac{1}{2P^+}\left[H^q(x,\xi,t)\, \bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\, \bar{u}(p')\frac{i\,\sigma^{+\alpha}\Delta_\alpha}{2m} u(p)\right],
\end{aligned}
$$

$$
\begin{aligned}
\tilde{F}^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+\gamma_5\, q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, z_\perp=0} \\
&= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\, \bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\xi,t)\, \bar{u}(p')\frac{\gamma_5\,\Delta^+}{2m} u(p)\right].
\end{aligned}
$$

$H^q \xrightarrow{\xi=0,t=0}$ PDF q $\quad\quad$ $\tilde{H}^q \xrightarrow{\xi=0,t=0}$ polarised PDF Δq

Exclusive J/ψ photoproduction

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz \, H(x)\phi(z) T(x,z)$$

- Factorise further using *NRQCD factorisation*:
  $\implies \phi(z) \sim \delta(z - 1/2)$.
- Amplitude calculated up to NLO: D. Ivanov, A. Schafer, L. Szymanowski, G. Krasnikov [hep-ph/0401131]
  $\implies$ *Collinear factorisation works*
- Also extended to *electroproduction* by C. Flett, J. Gracey, S. Jones, T. Teubner [2105.07657]

Scale fixing?

$$\mathcal{T}^{\mu\nu}_{\mathrm{NLO}} \supset i\pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} \left[H_g(\xi,\xi) + \hat{\alpha}_s \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 \frac{dx}{x} H_g(x,\xi) \right.$$
$$\left. + \hat{\alpha}_s \frac{C_F}{C_A} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 dx \left(H_q(x,\xi) - H_q(-x,\xi)\right) \right]$$

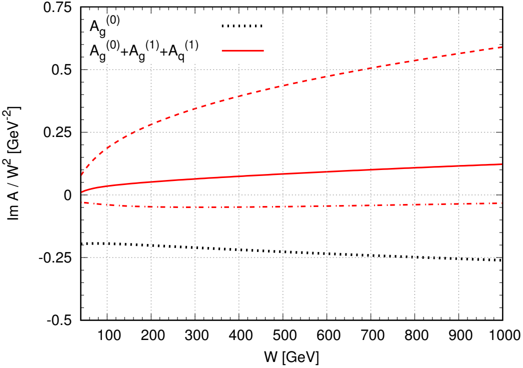
Choose $\mu_F = m_c$. $\implies$ Large $\ln\xi$ terms cancel [S. Jones, A. Martin, M. Ryskin, T. Teubner: 1507.06942].

However, impossible to move all enhanced by powers of $\ln\xi$ contributions from the coefficient function into the GPD (through μ_F evolution)

Big part of NLO correction from the hard coefficient eliminated, *but not from higher order contributions*.

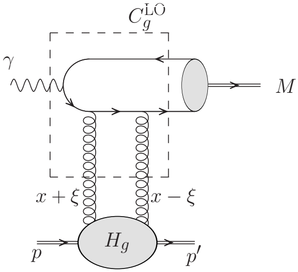
Result after scale-fixing procedure

Plot from S. Jones, A. Martin, M. Ryskin, T. Teubner [1610.02272]



Q_0 subtraction procedure

S. Jones, A. Martin, M. Ryskin, T. Teubner [1610.02272]



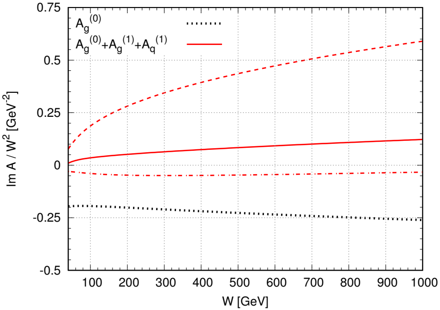
To avoid double counting, exclude the $|l^2| < Q_0^2$ domain whose contribution is already included in the LO term using the input gluon GPD.

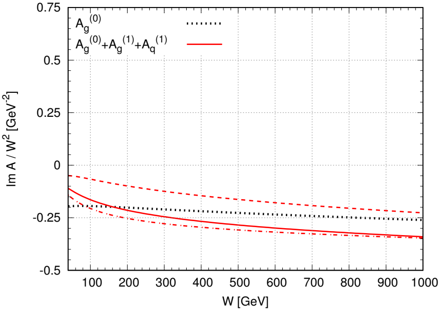
$\Longrightarrow$ Subtract the NLO DGLAP contribution $|l^2| < Q_0^2$ from the NLO $\overline{\mathrm{MS}}$ CF to *avoid double counting* with input GPD at scale Q_0

Typically power suppressed, but sizeable here: $\mathcal{O}(\frac{Q_0^2}{M^2})$

Result after Q_0 subtraction

Plots from S. Jones, A. Martin, M. Ryskin, T. Teubner [1610.02272]





Left: Scale-fixing procedure only

Right: Scale-fixing and Q_0 subtraction

Process-dependent procedure!!

Implementation of high-energy resummation

HEF resummation of LLA contributions $\sim \alpha_s^n \ln^{n-1}(\frac{x}{\xi})$ *at integrand level* to the imaginary part of the $C_g(\frac{\xi}{x})$:

$$C_g^{\text{HEF}}\left(\frac{\xi}{x}\right) = \frac{-i\pi}{2}\frac{F_{\text{LO}}}{\left(\frac{\xi}{x}\right)}\int_0^\infty d\mathbf{q}_T^2\, \mathcal{C}_{gi}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F, \mu_R\right) h(\mathbf{q}_T^2),$$

$$h(\mathbf{q}_T^2) = \frac{M^2}{M^2 + 4\mathbf{q}_T^2}.$$

Implementation of high-energy resummation

Resummation factor, $\mathcal{C}_{gi}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F, \mu_R\right)$ in the *Doubly-Logarithmic Approximation* (DLA) (in order to be consistent with fixed-order evolution of GPD) is given by the Blümlein-Collins-Ellis formula [hep-ph/9506403]

$$\mathcal{C}_{gg}^{(\mathrm{DL})}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F^2, \mu_R^2\right) = \frac{\hat{\alpha}_s}{\mathbf{q}_T^2} \begin{cases} J_0\left(2\sqrt{\hat{\alpha}_s \ln\left(\frac{x}{\xi}\right) \ln\left(\frac{\mu_F^2}{\mathbf{q}_T^2}\right)}\right) & \text{if } \mathbf{q}_T^2 < \mu_F^2, \\ I_0\left(2\sqrt{\hat{\alpha}_s \ln\left(\frac{x}{\xi}\right) \ln\left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)}\right) & \text{if } \mathbf{q}_T^2 > \mu_F^2. \end{cases}$$

$\Longrightarrow$ resums terms scaling like $\left(\alpha_s \ln\left(x/\xi\right) \ln(\mu_F^2/\mathbf{q}_T^2)\right)^n$ to all orders in perturbation theory.

For the quark channel, the resummation factor is given in the DLA by:

$$\mathcal{C}_{gq}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F^2, \mu_R^2\right) = \frac{C_F}{C_A}\left[\mathcal{C}_{gg}\left(\frac{\xi}{x}, \mathbf{q}_T^2, \mu_F^2, \mu_R^2\right) - \delta\left(1 - \frac{\xi}{x}\right)\delta(\mathbf{q}_T^2)\right].$$

Implementation of high-energy resummation

Useful representation in Mellin space:

$$\mathcal{C}_{gg}^{(\mathrm{DL})}(N, \mathbf{q}_T^2, \mu_F^2, \mu_R^2) \quad = \quad R(\gamma_{gg}) \, \frac{\gamma_{gg}}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_{gg}} .$$

γ_{gg} is the solution to the equation

$$\frac{\hat{\alpha}_s}{N} \chi(\gamma_{gg}) = 1, \quad \chi(\gamma) = 2\varphi(1) - \varphi(\gamma) - \varphi(1-\gamma), \quad \varphi(\gamma) = \frac{d \ln \Gamma(\gamma)}{d\gamma}$$

$$\gamma_{gg} = \frac{\hat{\alpha}_s}{N} + \mathcal{O}\left(\frac{\hat{\alpha}_s^4}{N^4} \right), \quad R(\gamma_{gg}) = 1 + \mathcal{O}\left(\hat{\alpha}_s^3 \right)$$

DLA $\Longrightarrow$ *Drop terms in red*: $\gamma_{gg} \to \gamma_N \equiv \frac{\hat{\alpha}_s}{N}$.

Mellin transform maps logarithms $\ln\left(\frac{x}{\xi}\right)$ to the poles at $N = 0$:

$$\frac{x}{\xi} \ln^{k-1}\left(\frac{x}{\xi} \right) \leftrightarrow \frac{(k-1)!}{N^k}.$$

Shuvaev transform

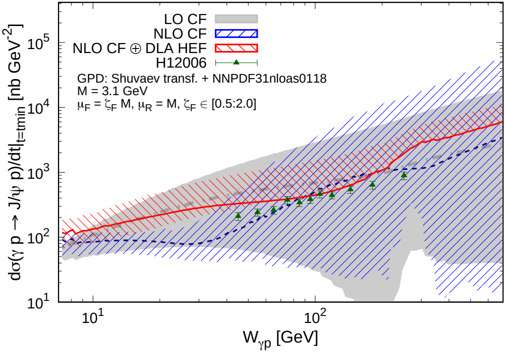
[0812.3558]

$$H_q(x,\xi) \;=\; \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi}\,\mathrm{Im}\int_0^1 \frac{\mathrm{d}s}{y(s)\,\sqrt{1-y(s)x'}}\right] \frac{\mathrm{d}}{\mathrm{d}x'}\left(\frac{q(x')}{|x'|}\right),$$

$$H_g(x,\xi) \;=\; \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi}\,\mathrm{Im}\int_0^1 \frac{\mathrm{d}s(x+\xi(1-2s))}{y(s)\,\sqrt{1-y(s)x'}}\right] \frac{\mathrm{d}}{\mathrm{d}x'}\left(\frac{g(x')}{|x'|}\right),$$

$$y(s) = \frac{4s(1-s)}{x+\xi(1-2s)}\,.$$

Results



Shuvaev transform, NNPDF input, *evolution of input PDF only*