Precise phenomenology at the LHC: state of art in perturbative QCD

Luca Buonocore

31th International Workshop on Deep Inelastic Scattering

Grenoble - 8-12 April 2024

Precise phenomenology at the LHC: state of art in perturbative QCD

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Personal perspective and selection of topics!

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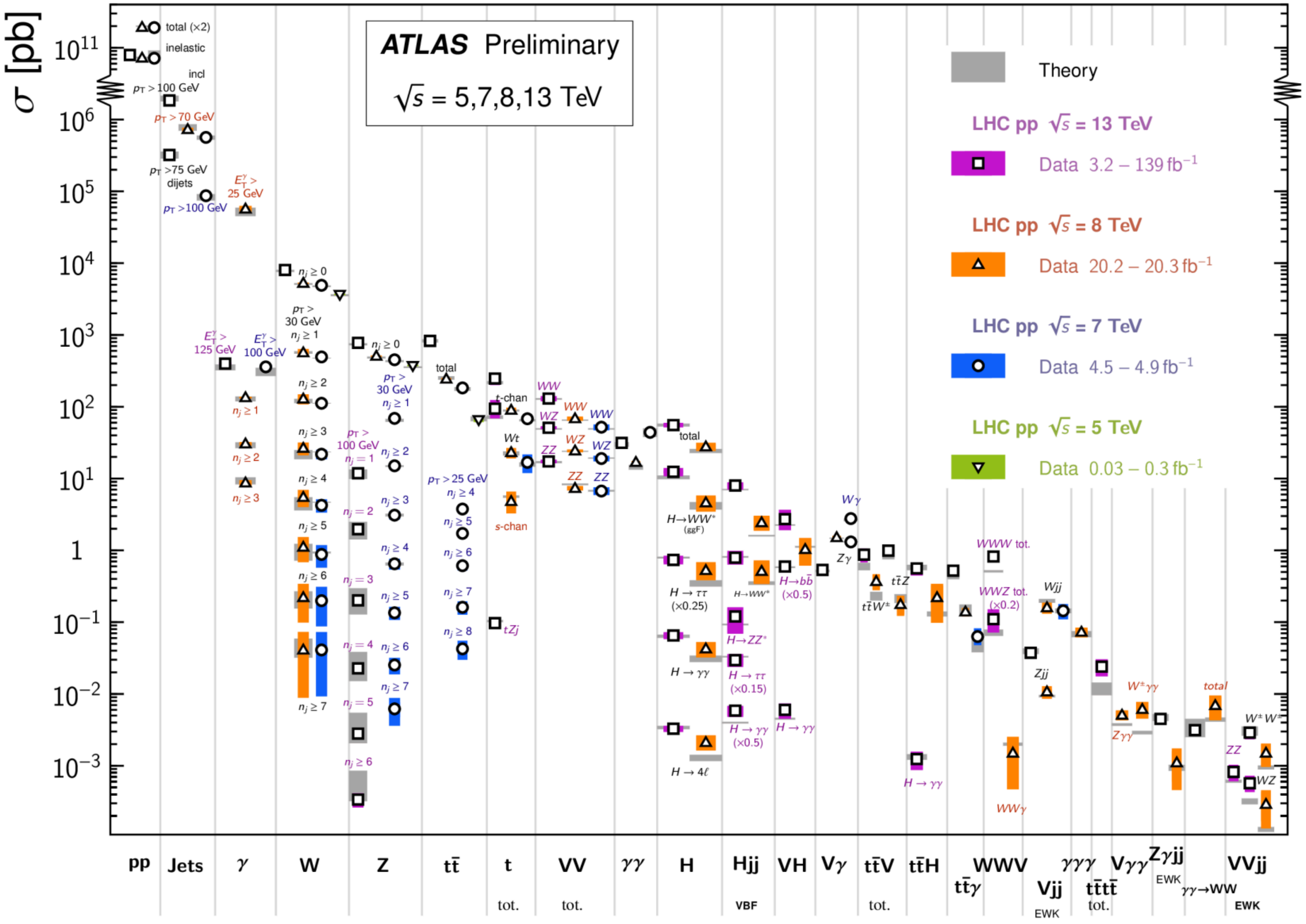
Introduction

With Higgs's discovery, SM is complete

- No free parameters
- Fully predictive theory

Precision era @ LHC

- astonishing measurements of many SM processes spanning across several order of magnitudes
- so far, agreement with accurate theoretical predictions
- great opportunity for advancing our (experimental and theory) understanding and possibly uncover NP



[ATL-PHYS-PUB-2022-009, February 2022]

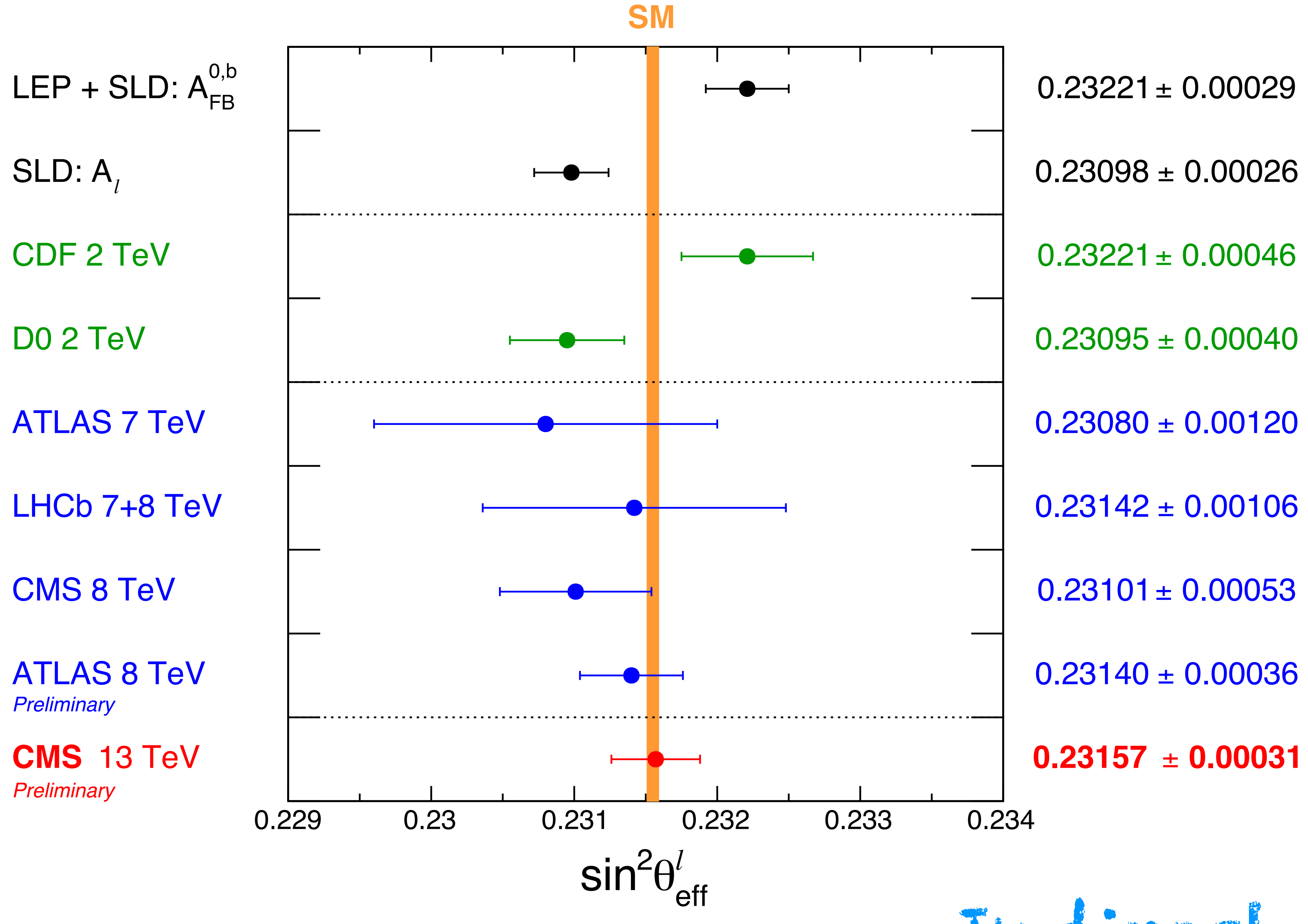
Introduction: role of precision in BSM searches

- Measurements of EW precision observables at LHC are becoming competitive with LEP/SLD results
- Control of higher-order radiative corrections crucial for parameter extraction from data
- Sensitivity of precision tests of SM consistency to NP

$$\delta\mathcal{O} \sim Q^2/\Lambda_{NP}^2$$

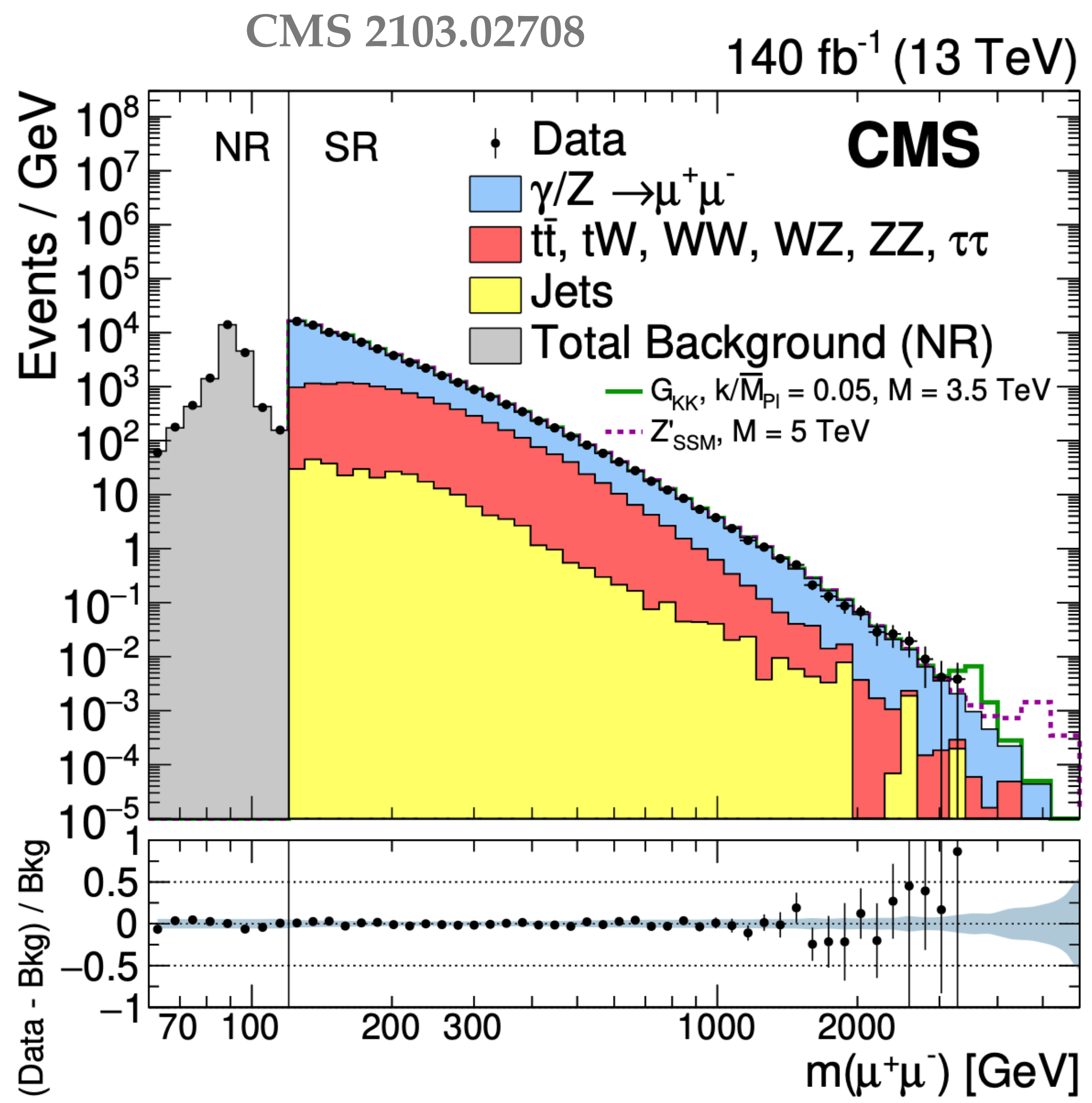
$$\%\ \text{accuracy at EW} \Longrightarrow \text{scale } \Lambda_{NP} \sim \text{TeV}$$

CMS-PAS-SMP-22-010



Indirect Searches

Direct Searches



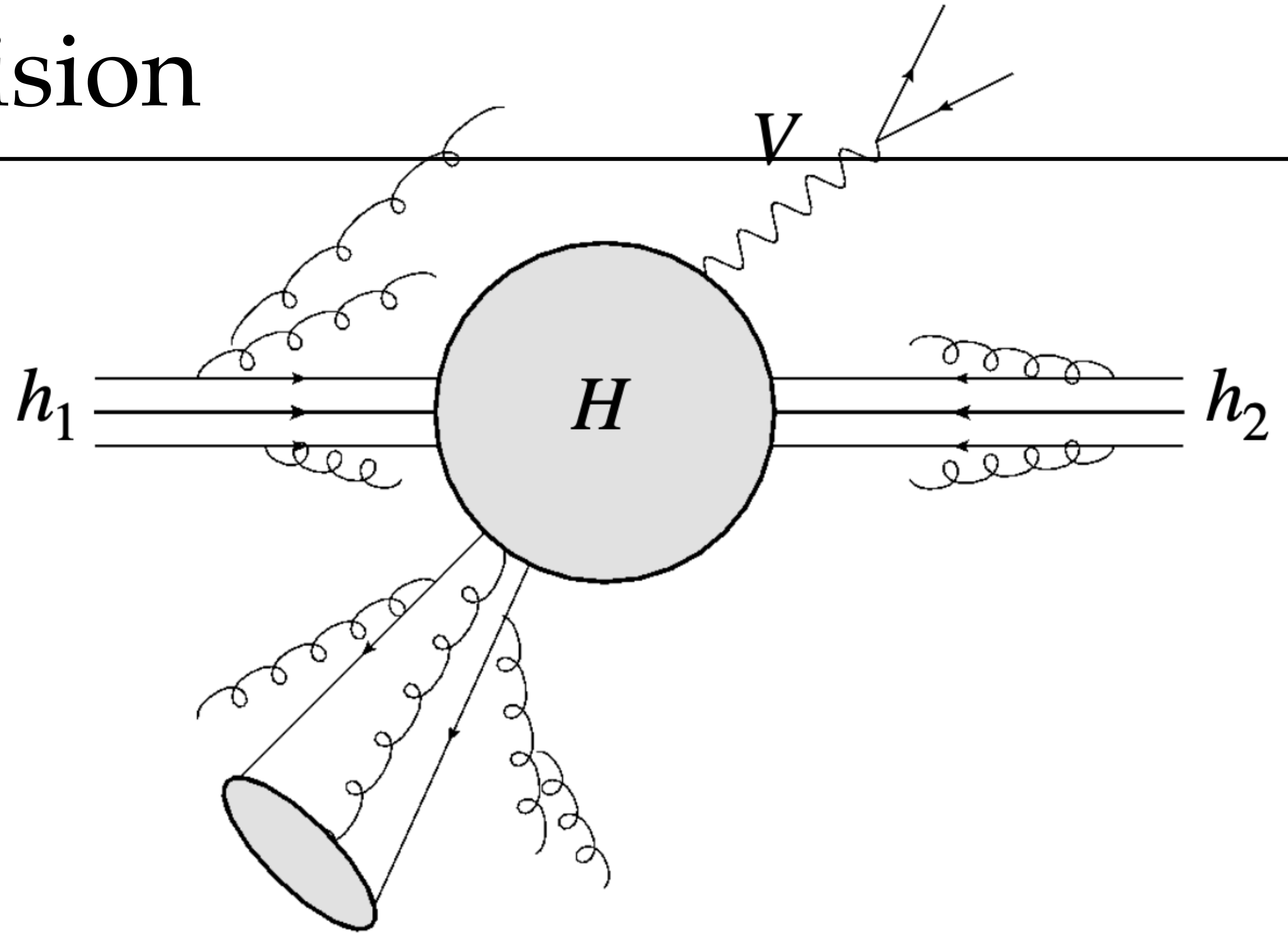
mass window [GeV]	stat. unc. 140fb^{-1}	stat. unc. 3ab^{-1}
$600 < m_{\mu\mu} < 900$	1.4%	0.2%
$900 < m_{\mu\mu} < 1300$	3.2%	0.6%

- For bump search of a narrow resonance, little theory needed
- But for **broad resonances** and/or effects in quantum loop corrections, control of the SM background is required!

Introduction: anatomy of a hadron-hadron collision

talk by Harland-Lang (and many others WG1)

- PDFs uncertainties are becoming a relevant component of the error budget
- Path to (sub-) percent level accuracy: methodology, higher orders, theory uncertainties, flavour, QED



Energy

Collinear factorisation

Hard scale

$$\sigma(h_1 + h_2 \to V + X) = \sum_{ab} \int dx_1 dx_2 f_{a/h_1}(x_1, \mu_F) f_{b/h_2}(x_2, \mu_F) \hat{\sigma}_{ab \to V+X}(\hat{s}, \mu_R) + \mathcal{O}\left(\frac{\Lambda^p}{Q^p}\right)$$

parton branching evolution

RESUMMATION/PARTON SHOWER

perturbative QFT

transition to hadrons

Hadronic scale $\Lambda \lesssim 1$ GeV

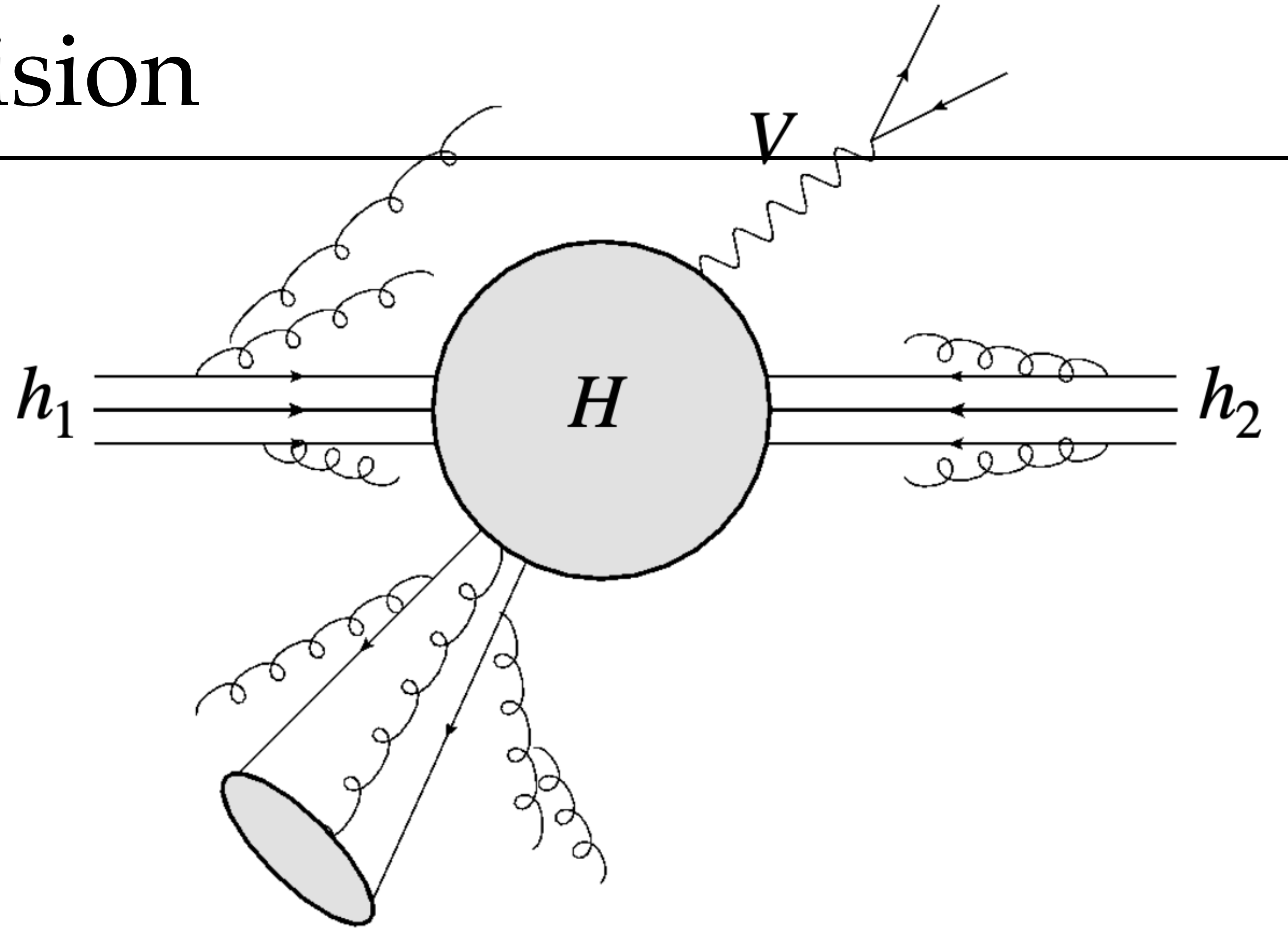
Hadronization

Multiparticle interactions

Underlying event

non-perturbative QFT

Introduction: anatomy of a hadron-hadron collision



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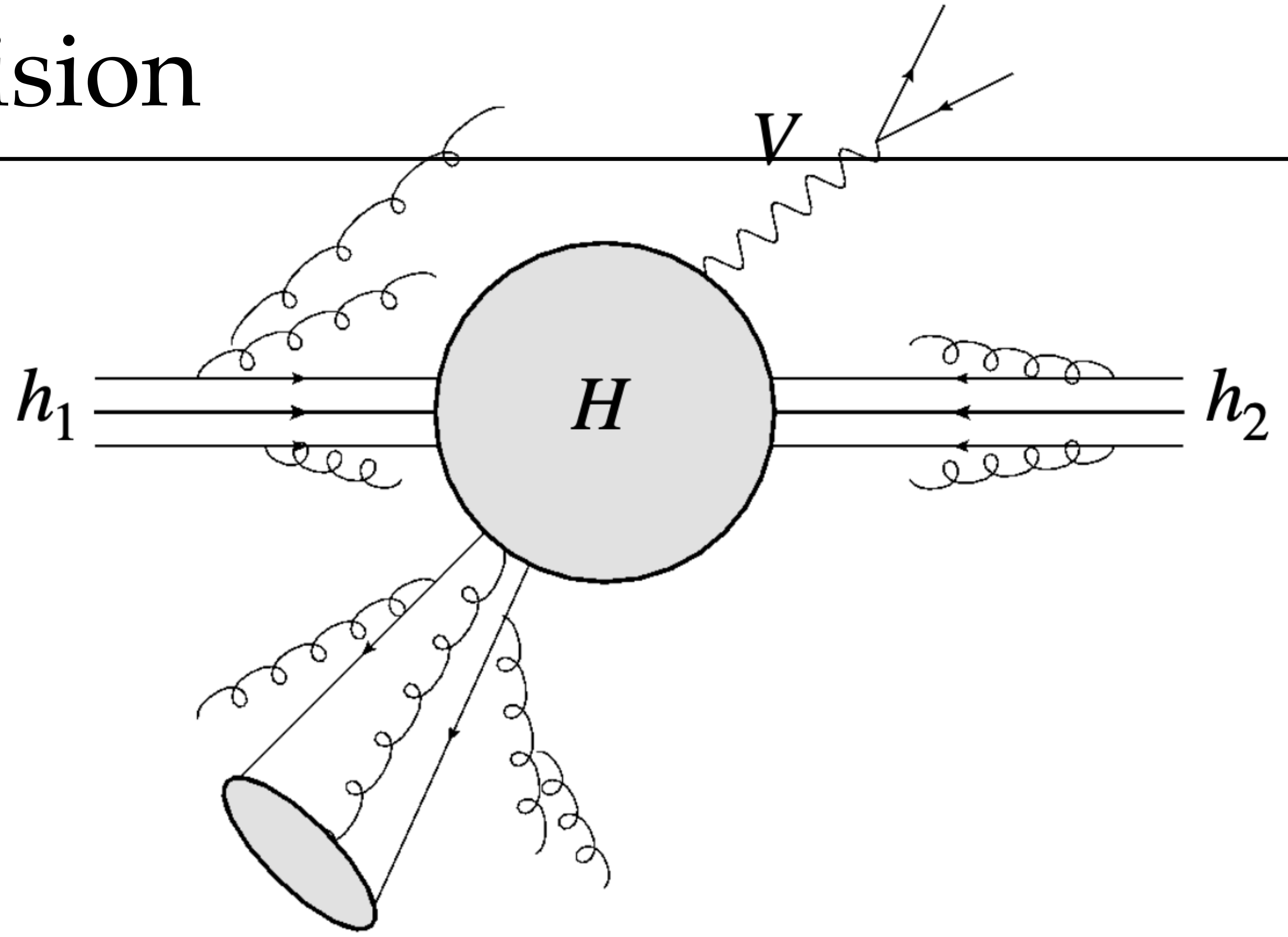
non-perturbative QFT

- $p = 1$ would be problematic for standard candles as Drell-Yan production: $\mathcal{O}(\lesssim 1\%)$
- Renormalon calculations within the large-n_f model exclude linear power corrections in DY

[Ferrario, Limatola, Nason '20, + Caola, Melnikov, '21]

talk by Ozcelik (WG4) for top physics

Introduction: anatomy of a hadron-hadron collision



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parton branching evolution

RESUMMATION/PARTON SHOWER

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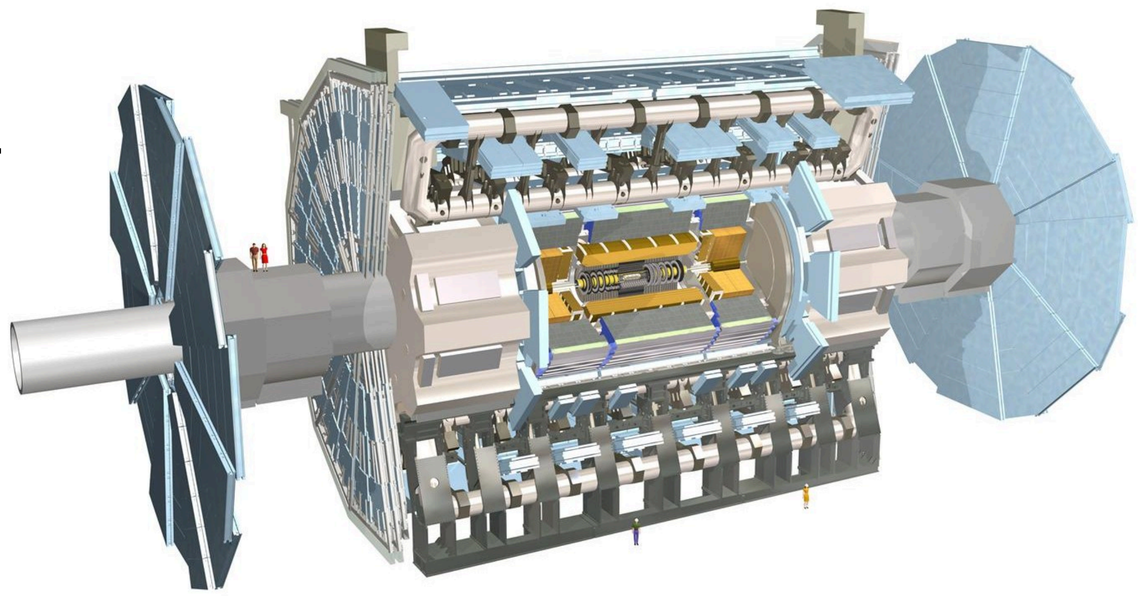
Underlying event

Elementary partonic cross section can be computed in perturbation theory

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \frac{\alpha_S}{2\pi}\hat{\sigma}^{(1)}_{ab} + \left(\frac{\alpha_S}{2\pi}\right)^2 \hat{\sigma}^{(2)}_{ab} + \ldots$$

$\mathcal{O}(100\%)$	$\mathcal{O}(20\%)$	$\mathcal{O}(5\%)$
LO	NLO	NNLO

Introduction: how do we do the calculation?

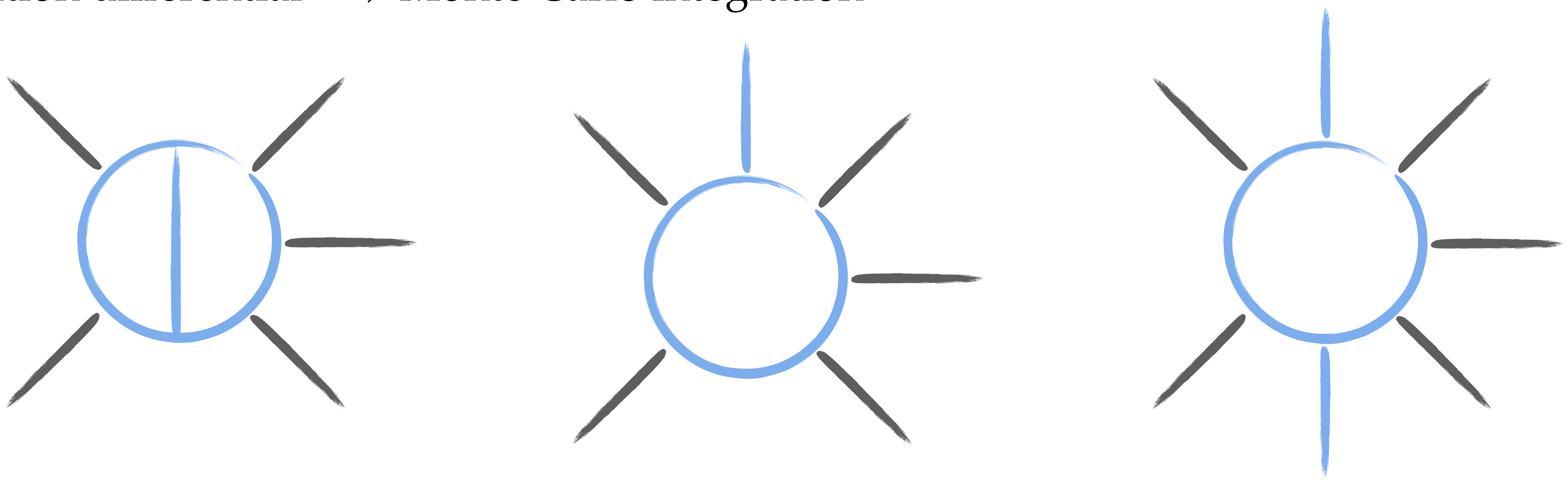


- Basics: integration of matrix elements over phase space
- At each order, more loops & more legs (virtual and real corrections)
- Fiducial volume: keep the calculation differential $\Longrightarrow$ Monte Carlo integration

matrix elements

$$\sigma \sim \int d\Phi \, |M|^2$$

phase space



Amplitudes (multi-loop Feynman integrals)

Complexity rapidly grows with **number of loops and number of scales** (internal/external)

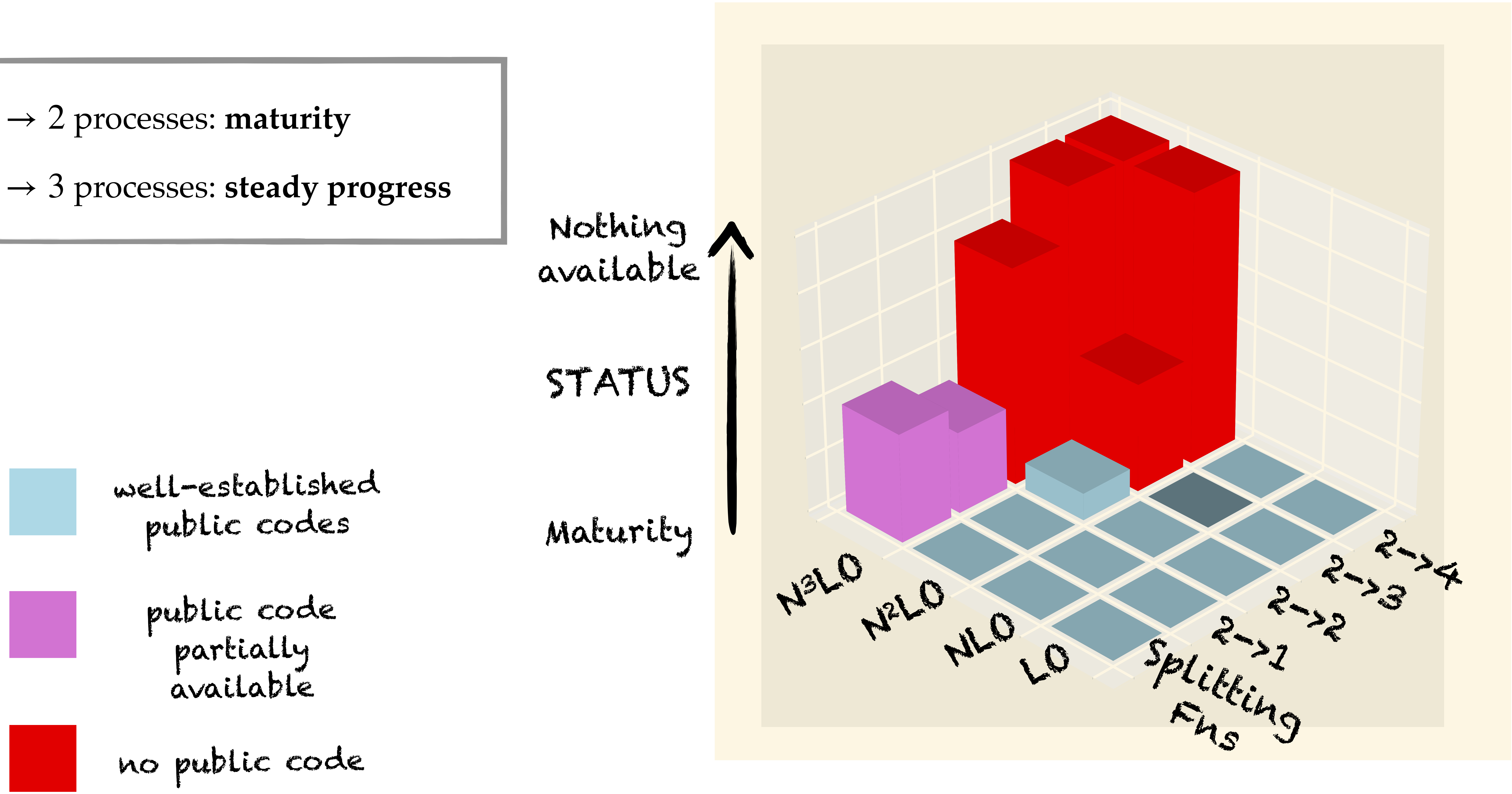
Generally bottleneck of higher-order calculations

Development of **subtraction/slicing** numerical schemes:

- extract analytically IR singularities; complexity grows with the number of external colored legs
- numerical efficiency is challenged by increasing multiplicities

Where Do We Come From? What Are We? Where Are We Going?

- $2 \rightarrow 2$ processes: **maturity**
- $2 \rightarrow 3$ processes: **steady progress**



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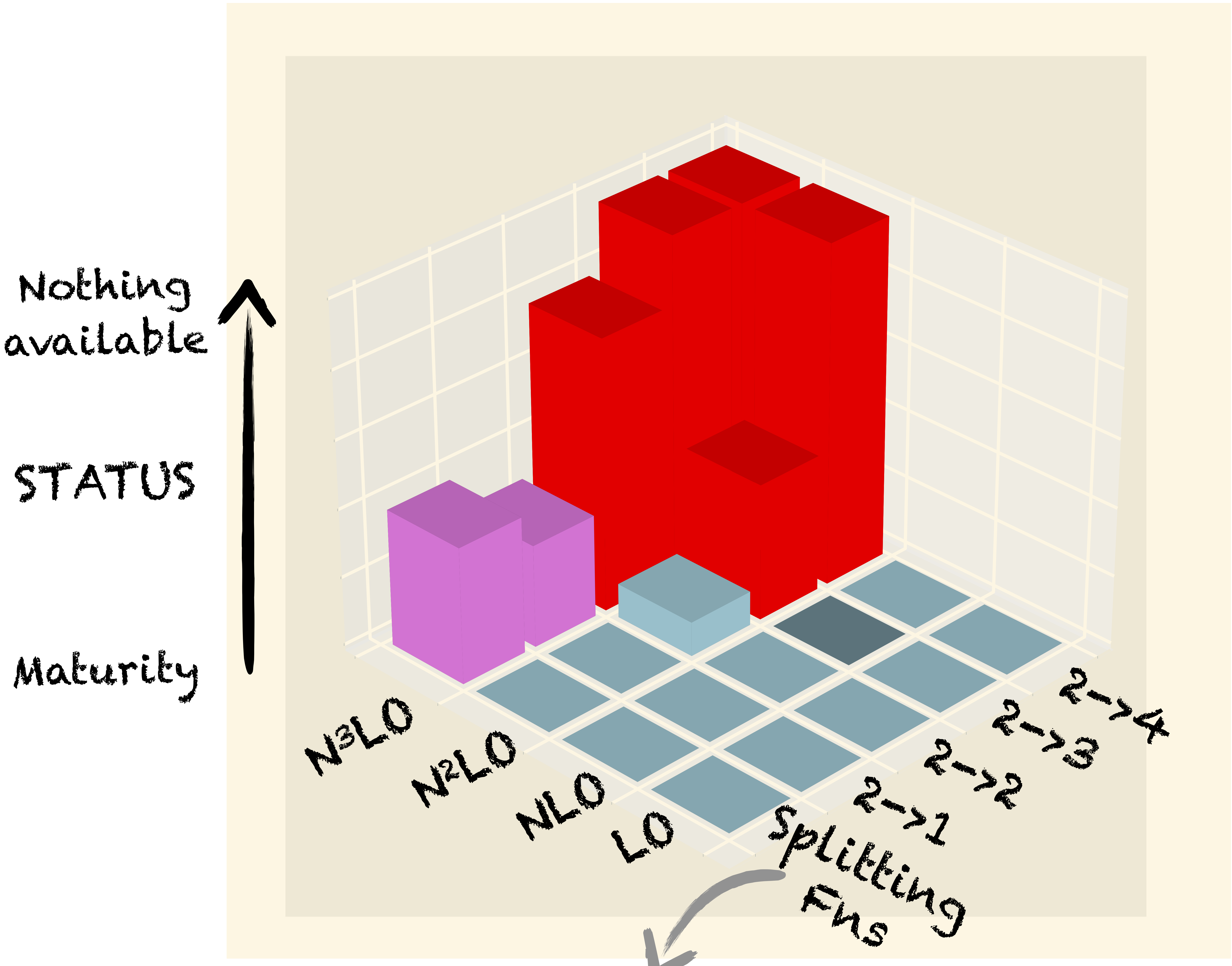
- $2 \to 3$ processes: **steady progress**

well-established public codes

public code partially available

no public code

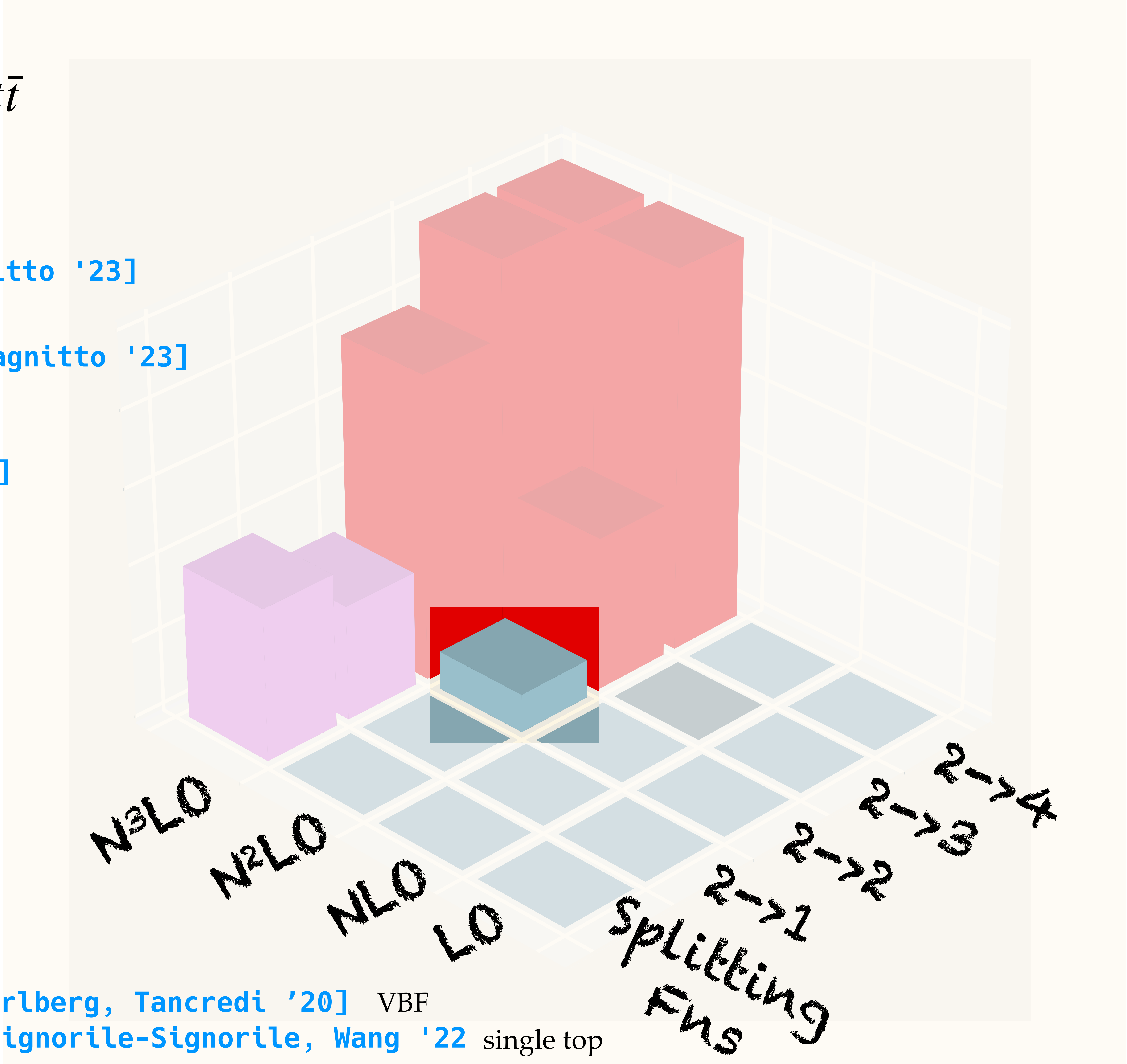


talks by Falcioni and T. Yang (WG1)

Where Do We Come From? What Are We? Where Are We Going?

▷ 2 → 2 processes: **maturity**

Going beyond standard processes: VV (dibosons), $V+j$, $t\bar{t}$



▷ Flavour-jets

Z+b-jet [Gauld, Gehrmann-De Ridder, Glover, Huss, Majer '20]
Z+c-jet [Gauld, Gehrmann-De Ridder, Glover, Huss, Garcia, Stagnitto '23]
W+c-jet [Czakon, Mitov, Pellen, Poncelet '20,'23]
[Gehrmann-De Ridder, Gehrmann, Glover, Huss, Garcia, Stagnitto '23]

▷ Massive final-state

$pp \to WH(H \to b\bar{b})$ [Behring, Bizoń, Caola, Melnikov, Röntsch '20]
$pp \to b\bar{b}$ [Catani, Devoto, Grazzini, Kallweit, Mazzitelli '21]

▷ Identified hadrons / Fragmentation

hadron fragmentation [Czakon, Generet, Mitov, Poncelet '21,'22]

isolated photons [Gehrmann, Schürmann '22, + Chen, Glover, Höfer, Huss '22]

▷ Mixed QCD-EW corrections

NC-Drell Yan. [Bonciani et al '21] [Buccioni et al '22]

▷ Beyond approximation

non-factorizable corrections [Liu, Melnikov, Penin '19][Dreyer, Karlberg, Tancredi '20] VBF
[Brønnum-Hansen, Melnikov, Quarroz, Signorile-Signorile, Wang '22 single top

Higgs beyond HTL [Czakon, Harlander, Klappert, Niggetiedt '20]
[Czakon, Eschment, Niggetiedt, Poncelet, Schellenberger '23]

Where Do We Come From? What Are We? Where Are We Going?

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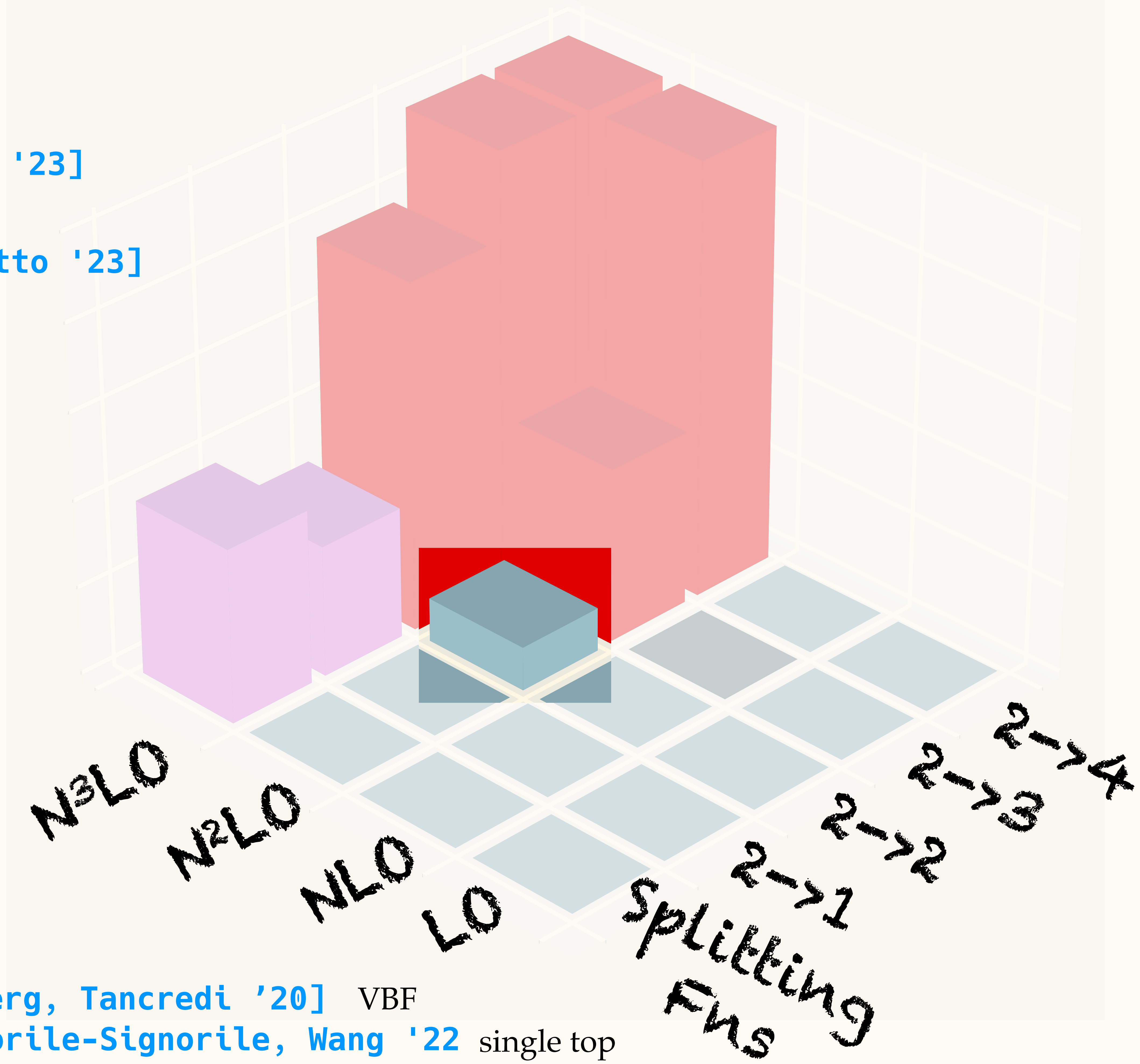
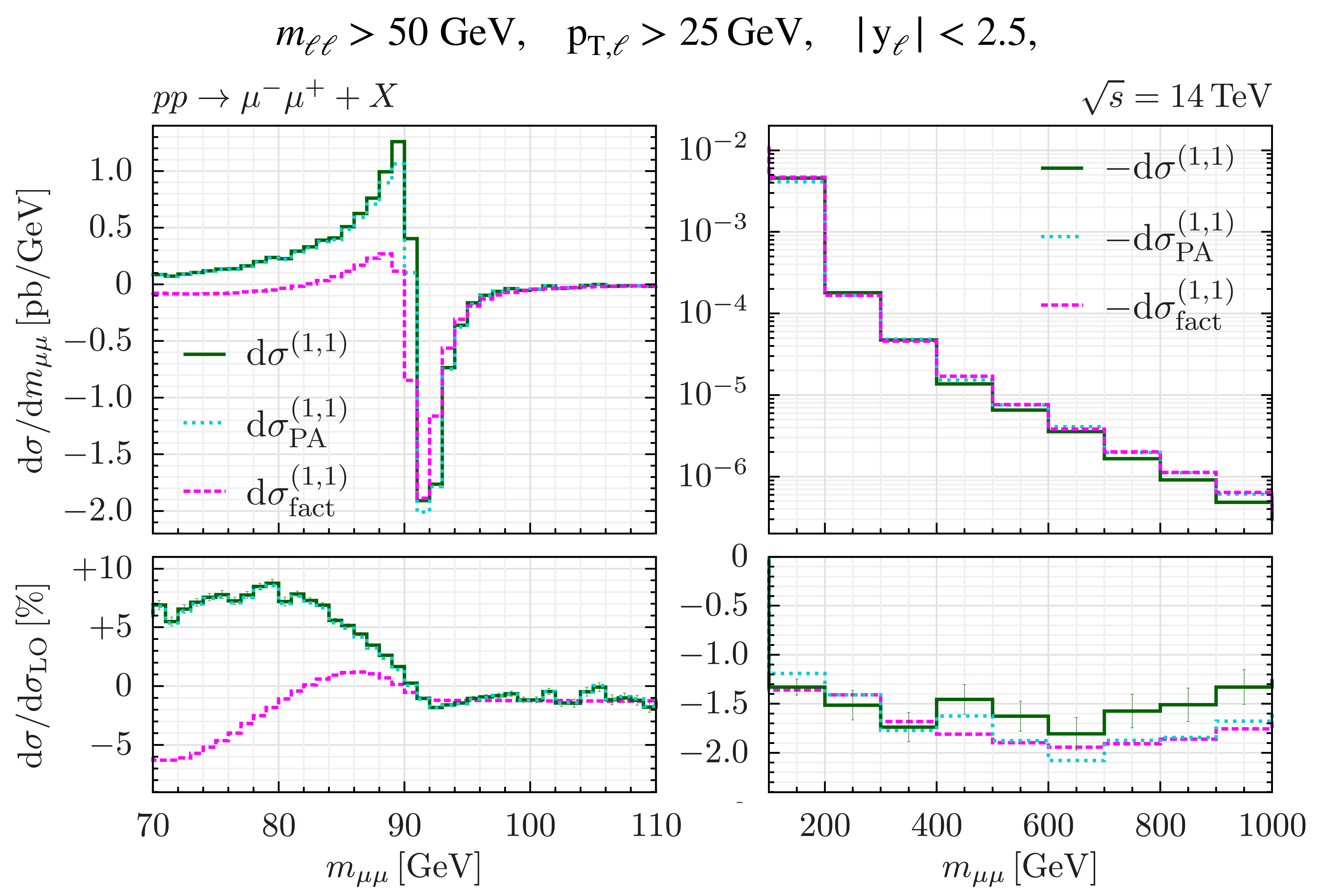


Beyond “standard” 2 → 2 calculations: mixed QCD-EW corrections

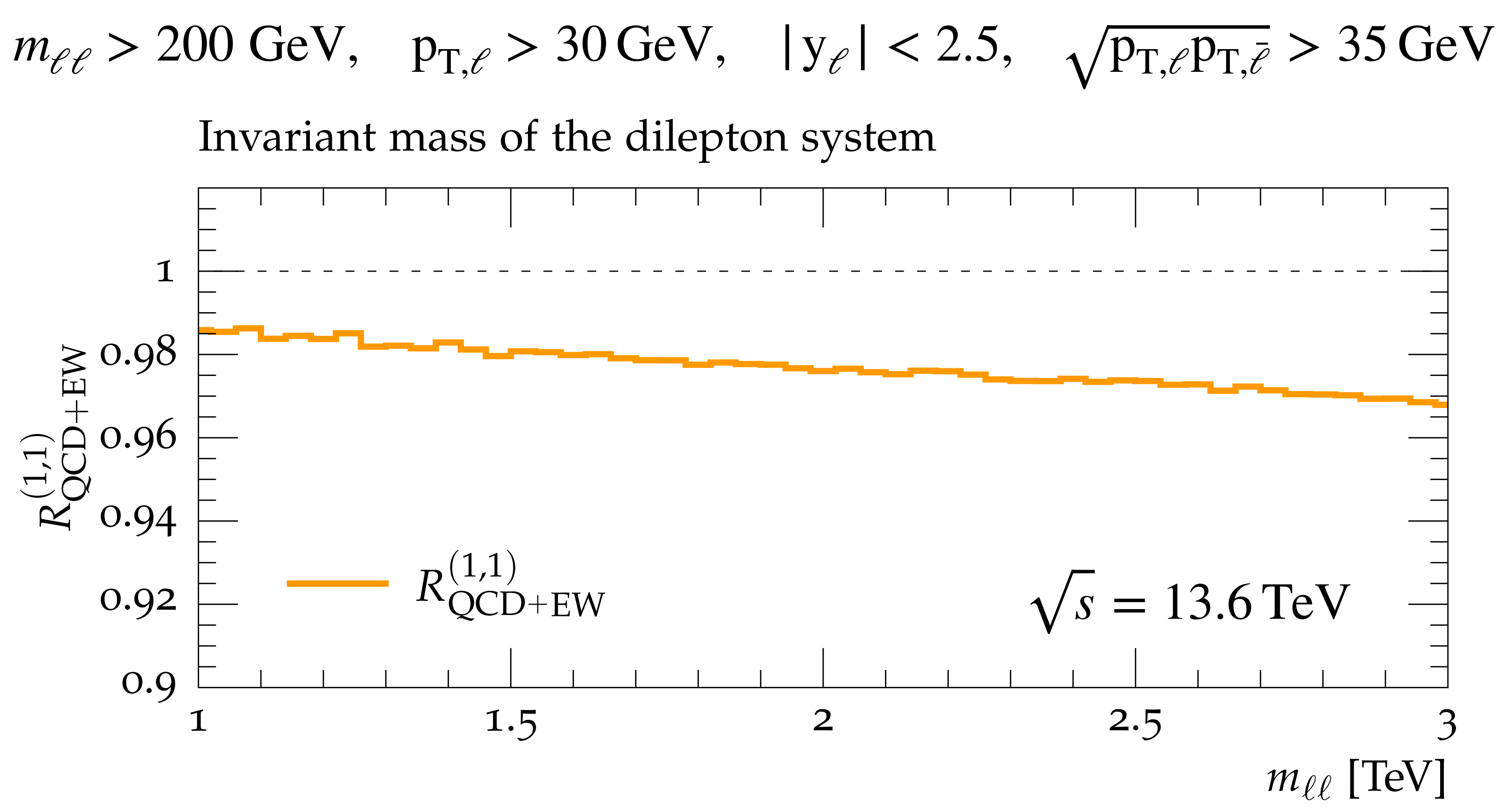
NC current Drell-Yan

Bare muons (massive calculation)
[Bonciani, LB, Grazzini, Kallweitt, Rana, Tramontano, Vicini ’21]

Impact at large invariant masses (massless leptons)
[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melinokov, Röntsch, Signorile-Signorile et al ’22]



- The Drell-Yan is the cornerstone of the precision physics program at the LHC (m_W, $\sin\theta_W$, α_S extractions)
- **Negligible?**
  mixed QCD-EW parametrically of similar importance as N^{3}LO in QCD
- **Factorized ansatz?**
  is a multiplicative combination of QCD and EW justified?



Non-negligible impact at high invariant masses

But well described by the product of QCD and EW (large Sudakov log) corrections

Beyond "standard" 2 → 2 calculations: Identified particles / fragmentation

Hadron fragmentation

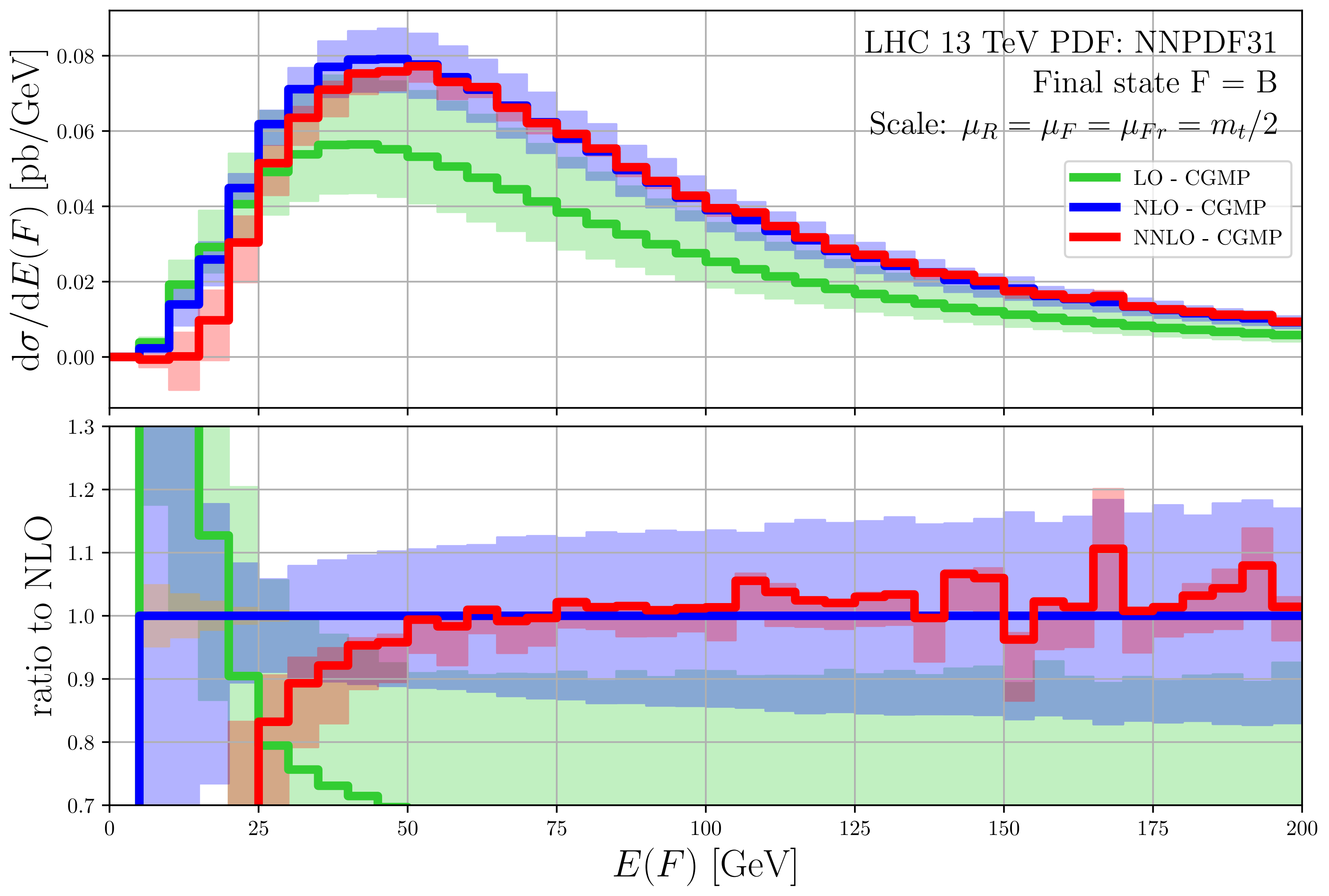
B hadron production in $t\bar{t}$ events
[Czakon, Generet, Mitov, Poncelet '21,'22]

- Investigate b-fragmentation in a **high-purity, high-statistics** environment
- Fragmentation functions fitted at NNLO exploiting e^+e^- data
  see also [Bonino, Cacciari, Stagnitto '23]
- Consider observables **sensitive to the top mass** m_t

Application to **open bottom production**

talk by T. Generet (WG4)

Different approach: **NNLO+PS**
[Mazzitelli, Ratti, Wiesemann, Zanderighi '23]

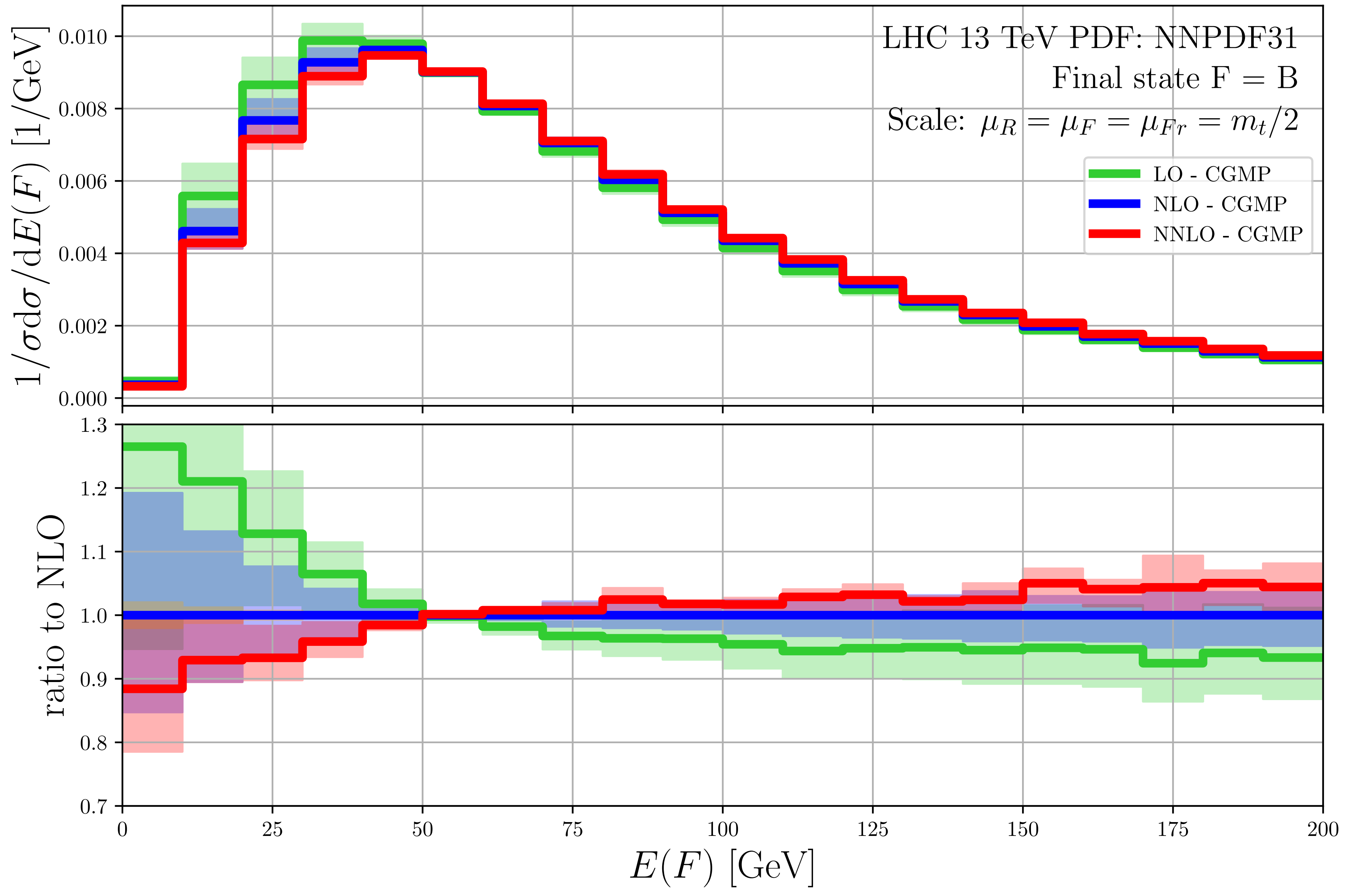


- Norm.
- no jet selection requirments

Value at the peak sensitive to top mass

$$E_{\max}(m_t) = a\, m_t + b$$

It gets shifted including high-order corrections!

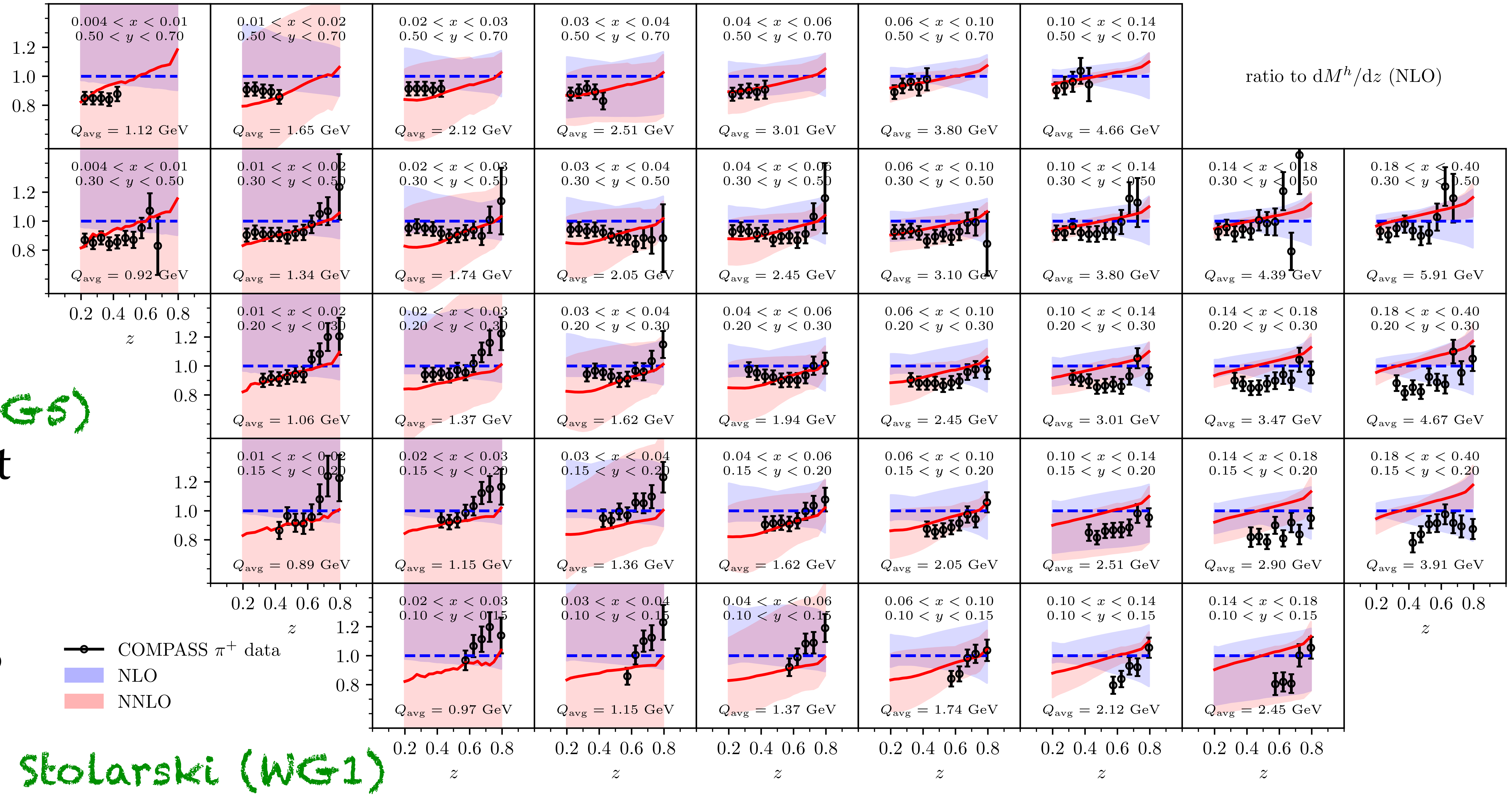


Beyond "standard" $2 \to 2$ calculations: Identified particles / fragmentation

Hadron fragmentation

Semi-inclusive DIS (SIDIS)
[Goyal, Moch, Pathak, Rana, Ravindran '23]
[Bonino, Gehrmann, Stagnitto '24]

- SIDIS as a probe of quark-to-hadron FF
- Results for single-inclusive π^+ production
- Substantial reduction of theory uncertainty and improved description of COMPASS data at NNLO



Concerning unpolarized calculation in DIS

Polarised DIS @ NNLO

[Borsa, de Florian, Pedron '22]

Progress on polarised NNLO PDFs

talks by Hekhorn and Chiefa (WG5)

Progress on the IR factorization of the relevant polarized amplitudes

talk by Löchner (WG5)

Small-x evolution in polarised DIS sand SIDIS

talk by Tawabutr (WG2)

talk by Stolarski (WG1) on new COMPASS results

Beyond “standard” 2 → 2 calculations: Identified particles / fragmentation

Hadron fragmentation

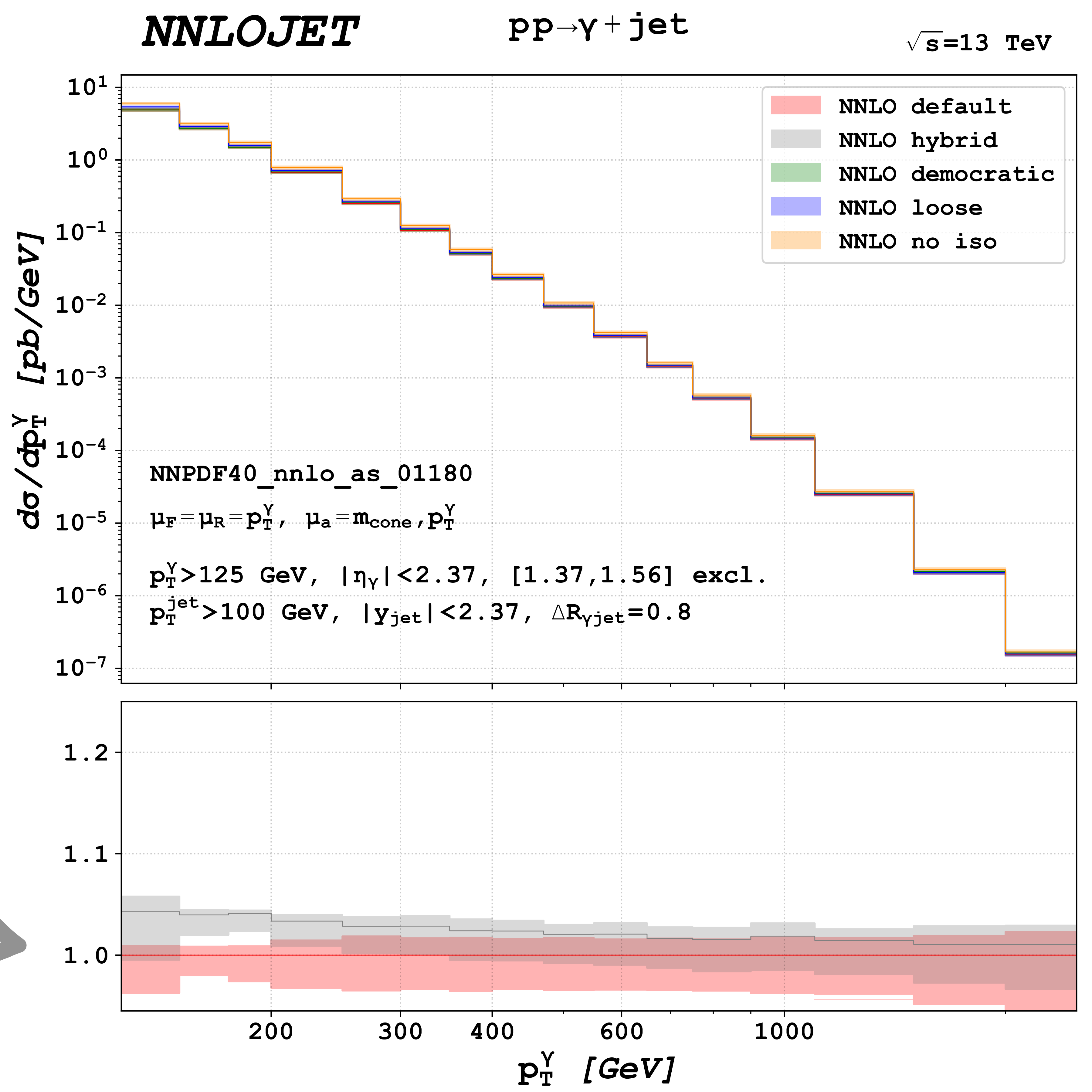
Semi-inclusive DIS (SIDIS)
[Goyal, Moch, Pathak, Rana, Ravindran ’23]
[Bonino, Gehrmann, Stagnitto ’24]

- SIDIS as a probe of quark-to-hadron FF
- Results for sir
- Substantial re description of

Photon fragmentation

Realistic photon isolation in $pp \to \gamma + (j) + X$
[Gehrmann, Schürmann ’22]
[Chen, Gehrmann, Glover Höfer, Huss, Schürmann, ’22]

it matters @ NNLO!

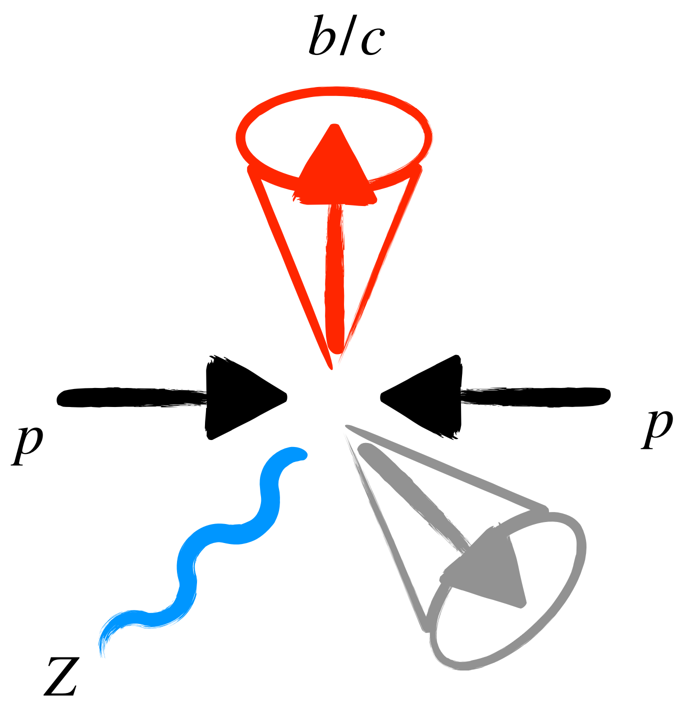


- **Isolation** is required to select **prompt photon**
- **Idealized** (smooth or hybrid) **isolation** is usually used in theory predictions to remove / reduce the NP fragmentation component
- Photon FFs loosely constrained from data (from e^+e^-)
  Is a **hadron collider measurement** possible?

Beyond “standard” 2 → 2 calculations: flavor jets!

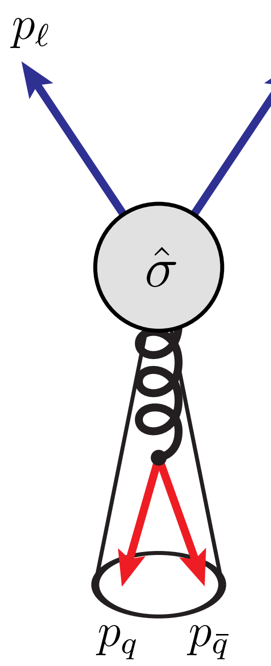
Flavoured jets ubiquitous

Higgs Physics	Top Physics	Bosons + HF	F-j + $E_{T,\mathrm{miss}}$
coupling to b	PDFs,α_S,BSM	PDFs,α_S,Bkg	BSM

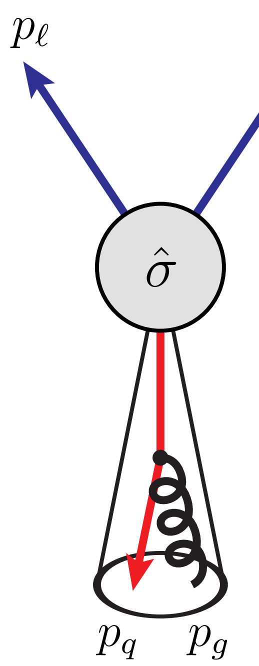


- Experimental definition: naive ant-k_T algorithm + flavour tagging

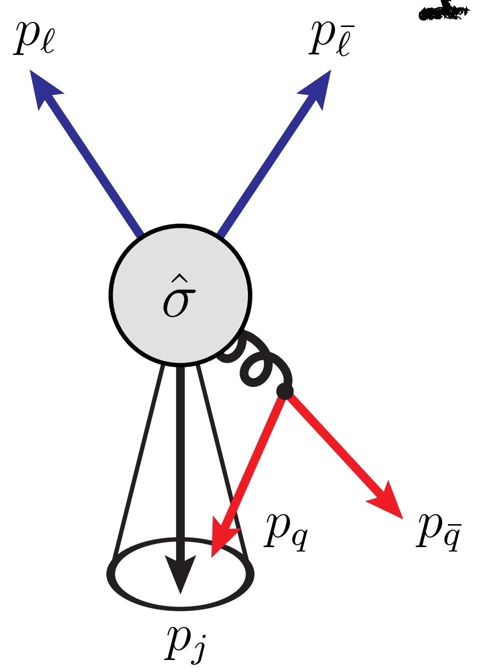
IRC unsafe!



Collinear
TH: no flavour!
EXP: most likely, flavoured!

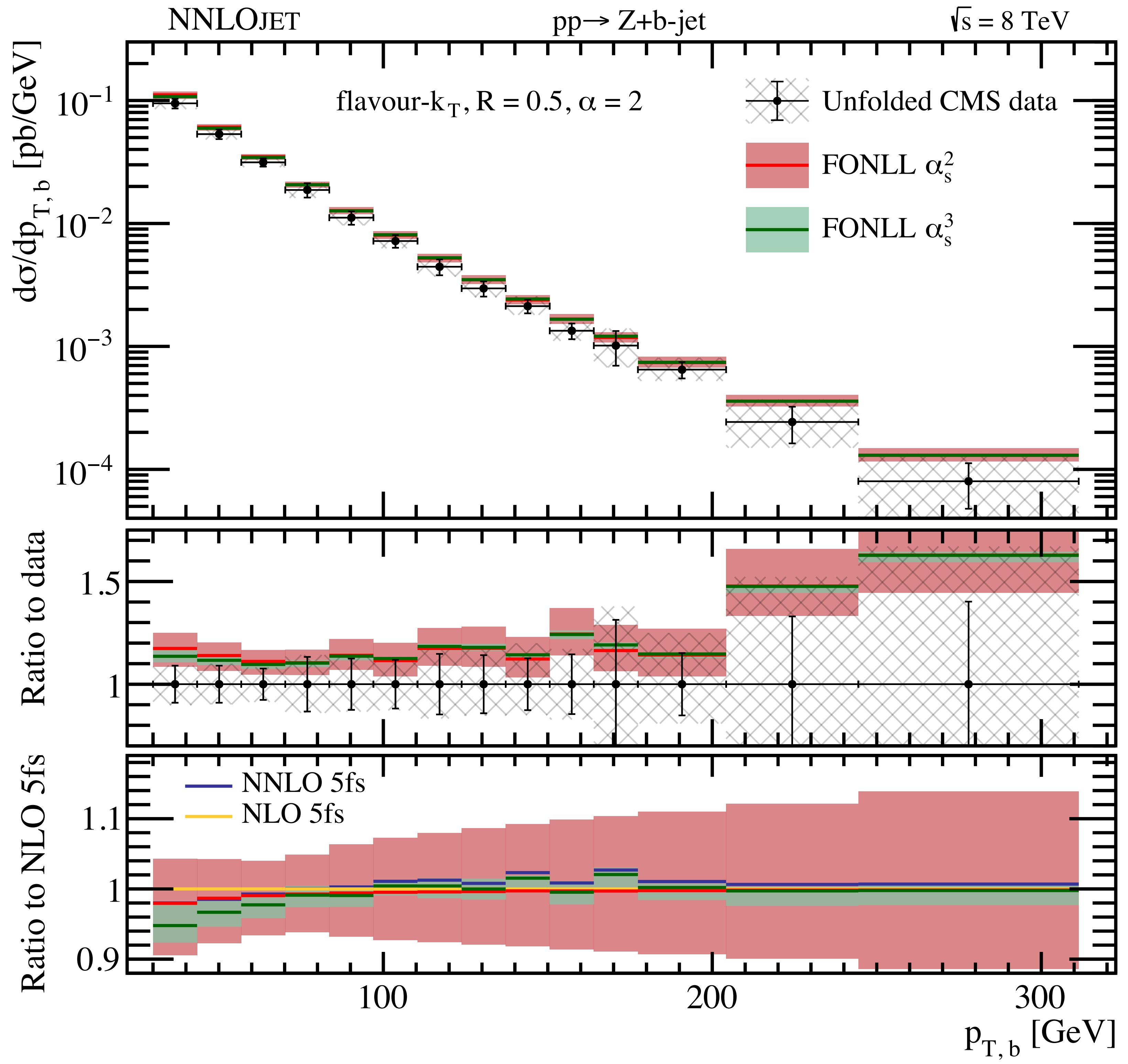


Collinear
TH: flavoured!
EXP: depends on flavoured hadron selection



Soft
TH: no flavour!
EXP: depends on flavoured hadron selection

[Gauld, Gehrmann-De Ridder, Glover, Huss, Majer, ’20]



- First proposal: **flavoured k_T algorithm**

  [Banfi, Salam, Zanderighi ’06]

Required parton level information that is not experimentally accessible

Based on k_T algorithm kinematics

Sizeable unfolding corrections $\sim 12\,\%$ $Z + b$ jet (estimated at with NLO+PS)

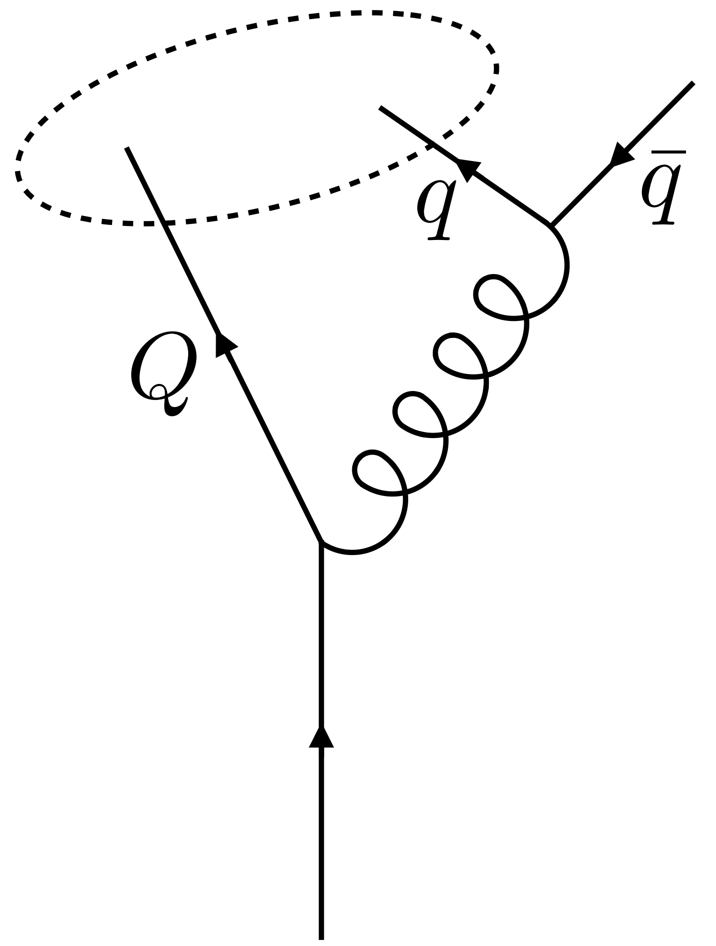
Beyond "standard" 2 → 2 calculations: flavor jets!

Renewed interest in flavor tagging

Apply **Soft Drop** to remove soft quarks

No unfolding needed

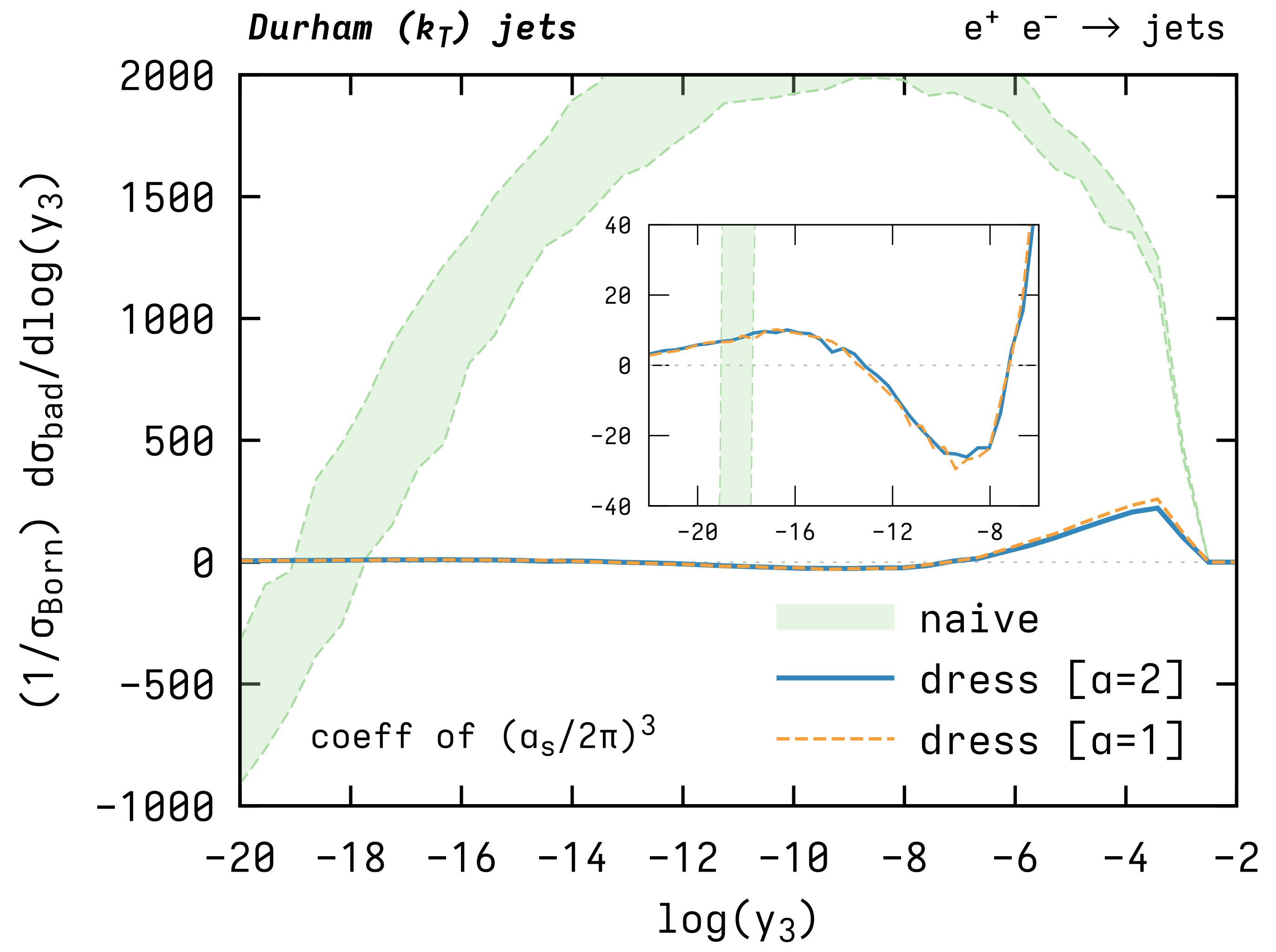
Requires reclustering with JADE (issue with IRC safety beyond NNLO)



[Caletti, Larkoski, Marzani, Reichelt '22]

Assign a **flavour dressing** to jets reconstructed with any IRC flavour-blind jet algorithms

Requires flavour information of many particles in the event

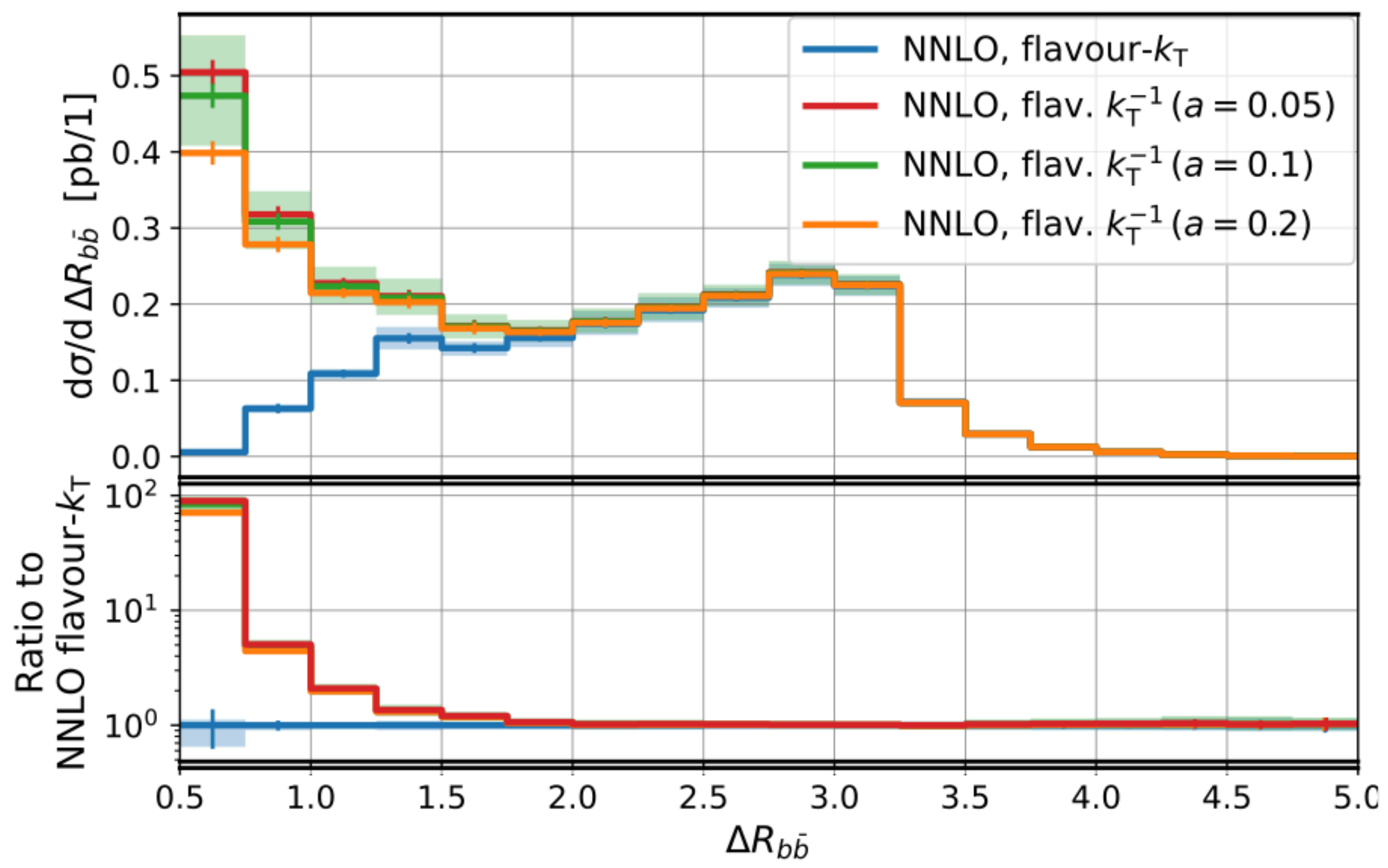


[Gauld, Huss, Stagnitto '22]

Make **anti-k_T algorithm flavour-aware**
Cluster soft $f\bar{f}$ pairs first

$$d_{ij}^{(F)} = d_{ij} \times \begin{cases} \mathcal{S}_{ij}, & \text{if } i \text{ and } j \text{ have opposite flavours} \\ 1, & \text{otherwise} \end{cases}$$

$$\mathcal{S}_{ij} = 1 - \theta(1-\kappa)\cos\left(\frac{\pi}{2}\kappa\right), \qquad \kappa = \frac{1}{a}\frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,max}^2}$$



[Czakon, Mitov, Poncelet '22]

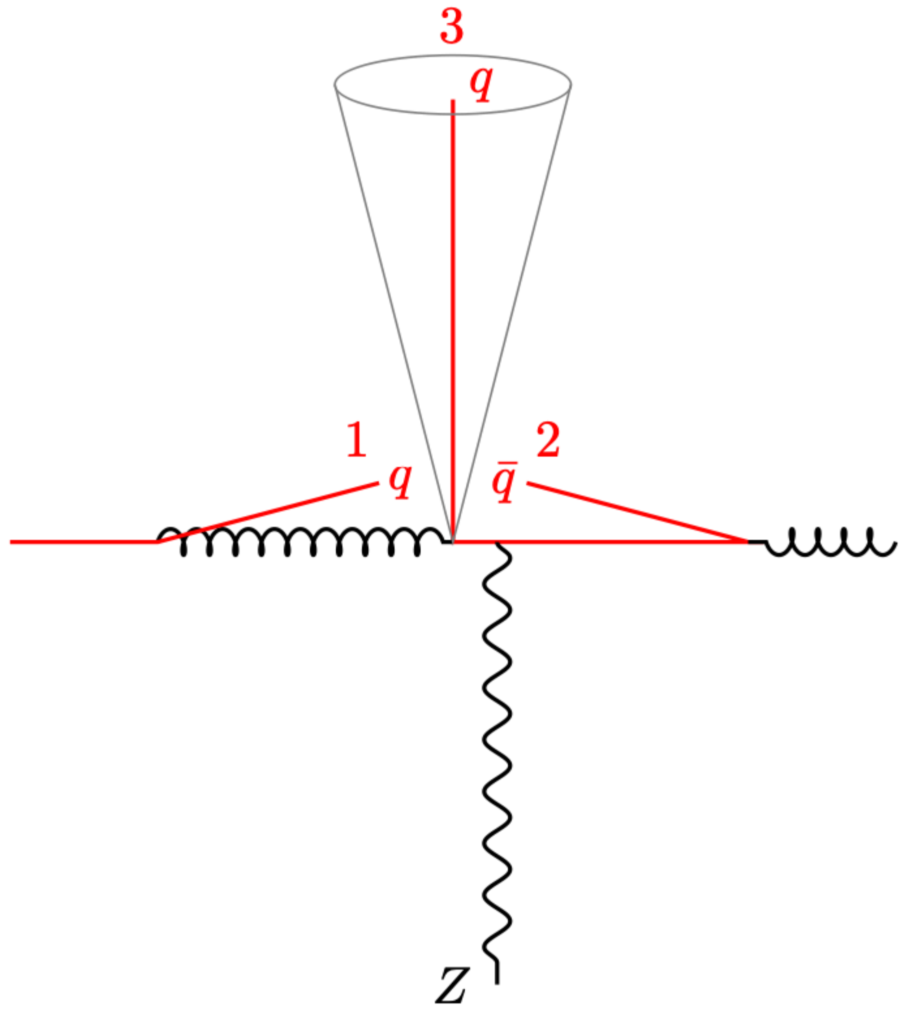
Beyond "standard" $2 \to 2$ calculations: flavor jets!

- Testing IRC safety to all orders in perturbation theory is a highly non-trivial task

New proposal for a flavour-aware jet-clustering algorithm IRC safe up to $\mathcal{O}(\alpha_S^6)$, thanks to the development of a dedicated **testing framework** [Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler 2023]

- On-going comparisons among different proposals (LesHouches '23) and exchanges with experimentalists

Example of IRC issue in flavour anti k_T



Configuration with two collinear initial-state emissions

Expectation: the algorithm should assign particle 1 and particle 2 to the beams leaving untouched the proto-jet 3 BUT

1. particle 1 and particle 2 cluster together!

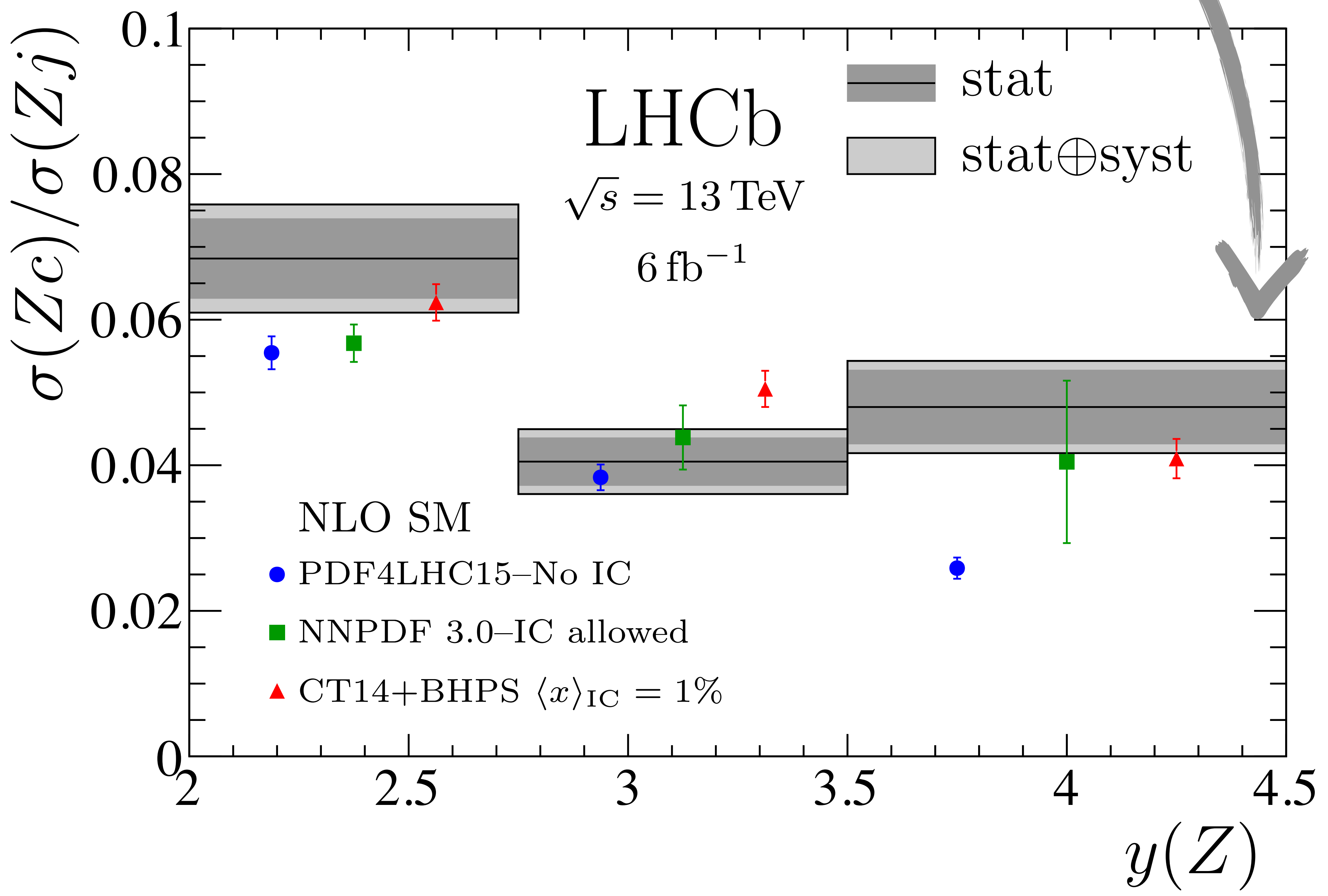
2. Flavourless protojet (12) can be, then, clustered with protojet 3, changing substantially its momentum

Beyond “standard” 2 → 2 calculations: flavour jets!

Flavour jet(s) for PDFs

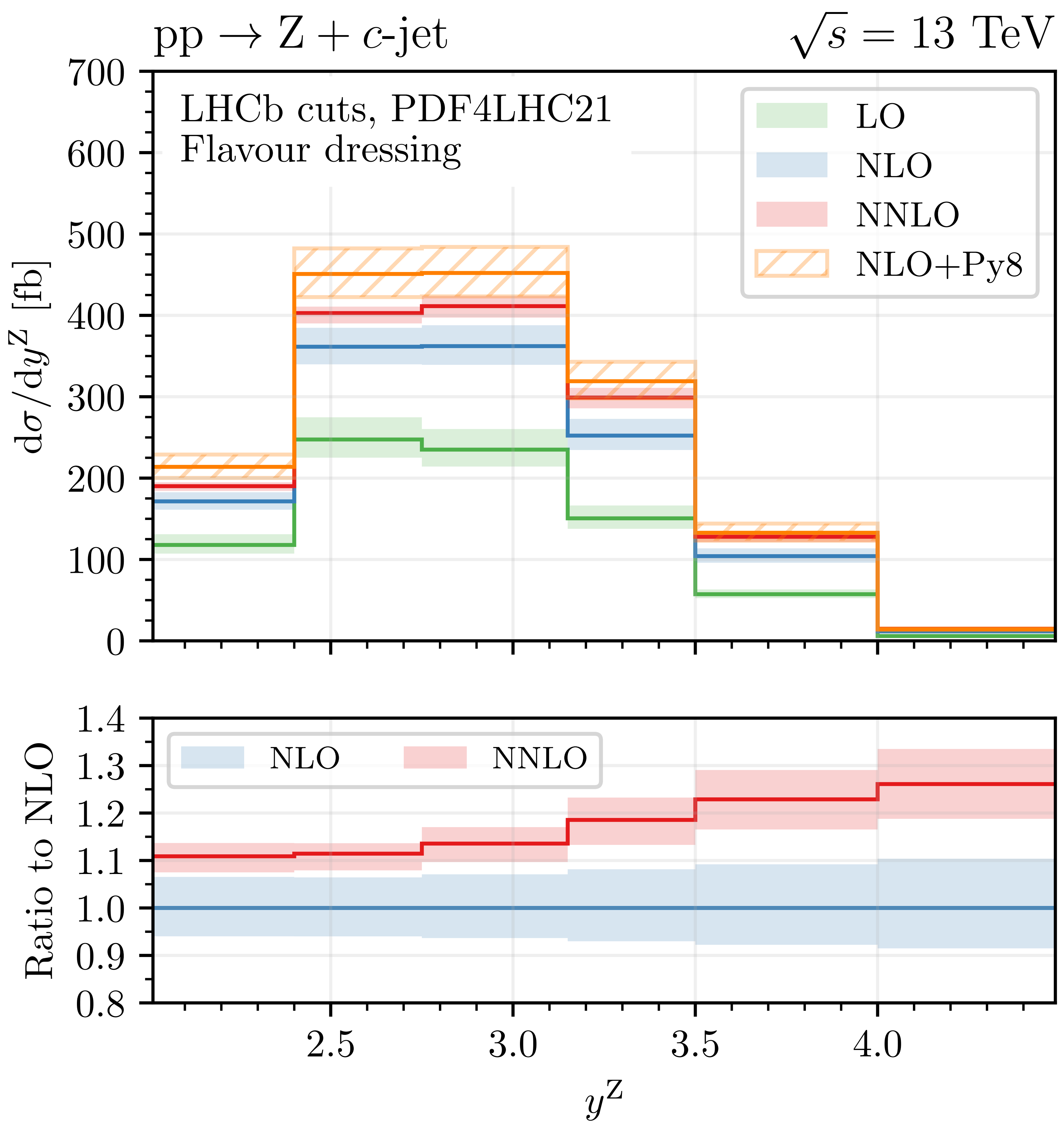
$pp \to Z + c + X$ at LHCb
[Gauld, Gehrmann-De Ridder, Glovel, Huss, Rodriguez Garcia, Stagnitto, ’23]

- LHCb forward kinematics: sensitivity to a possible **non-perturbative charm** component
[LHCb, Phys.Rev.Lett. 128 (2022) 8, 082001]
[NNPF, Nature 608 (2022)]
[Guzzi et al ’22]



NNLO

- +10-20% corrections
- outside NLO bands
- non trivial shapes
- NLO+PS provides a decent description



- Results obtained with the **flavour dressing algorithm**
Cannot be directly compared with current data

(correction factors applied in very recent ATLAS analysis [CERN-EP-2024-081])

Where Do We Come From? What Are We? Where Are We Going?

▷ $2 \to 3$ processes: **steady progress**

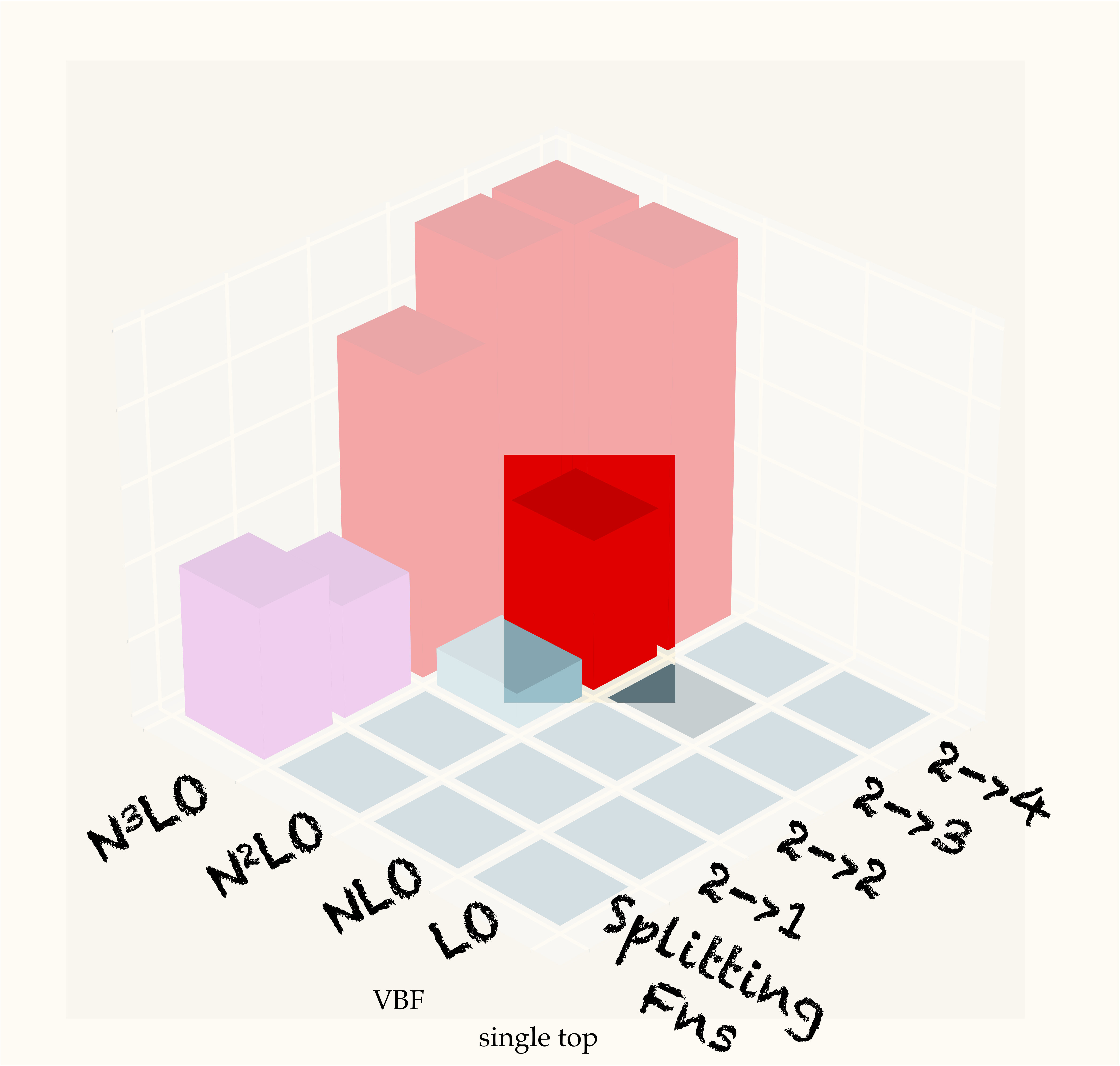
▷ $pp \to \gamma\gamma\gamma$
[Chawdhry, Czakon, Mitov, Poncelet '19]
[Kallweit, Sotnikov, Wiesemann '20]

▷ $pp \to \gamma\gamma j$
[Chawdhry, Czakon, Mitov, Poncelet '21]

▷ $pp \to jjj$
[Czakon, Mitov, Poncelet '21]

▷ $pp \to b\bar{b}W$ (massless bottom)
[Hartanto, Poncelet, Popescu, Zoia '22]

▷ $pp \to \gamma jj$
[Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia '23]



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- $pp \to \gamma\gamma\gamma$
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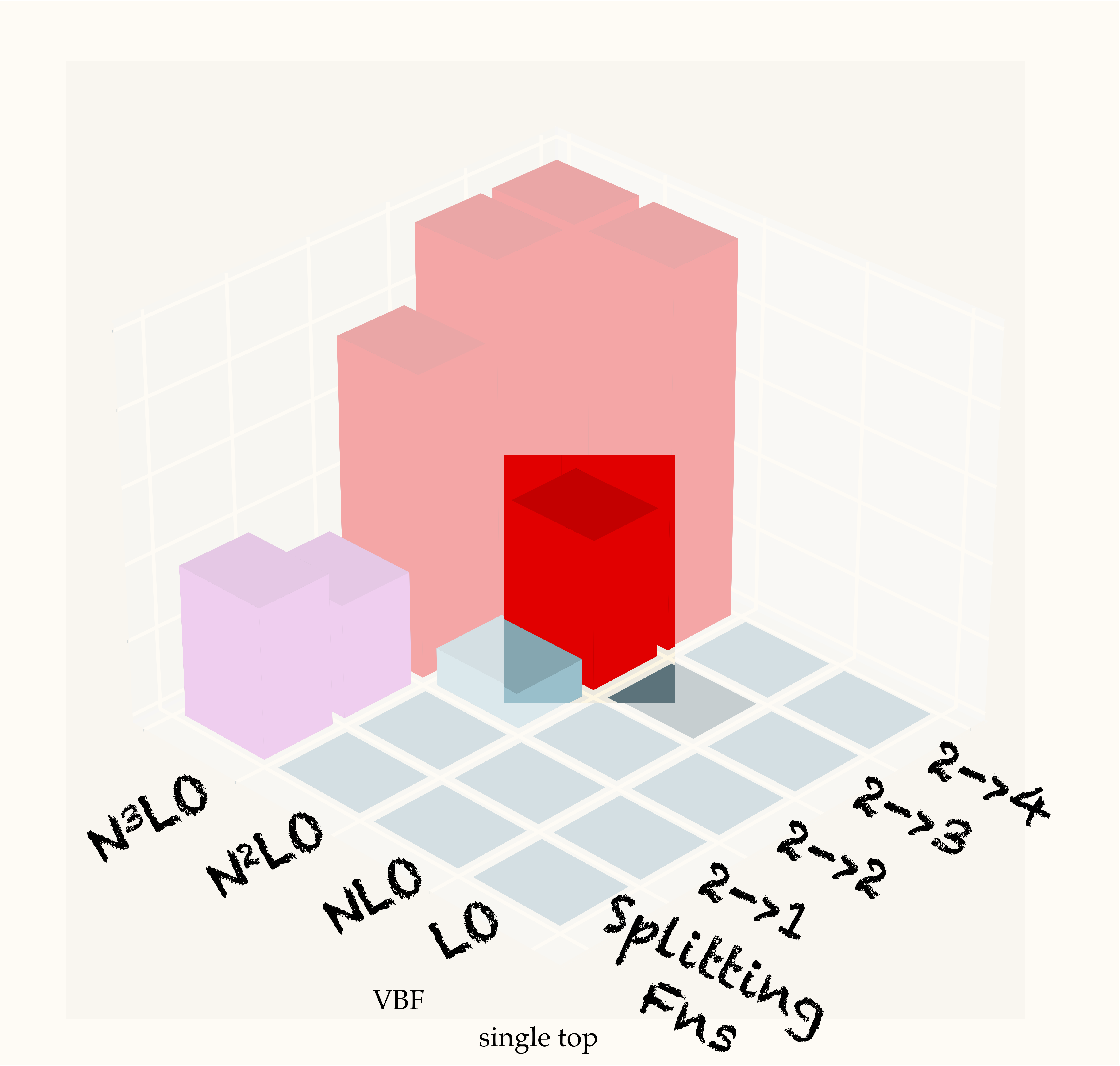
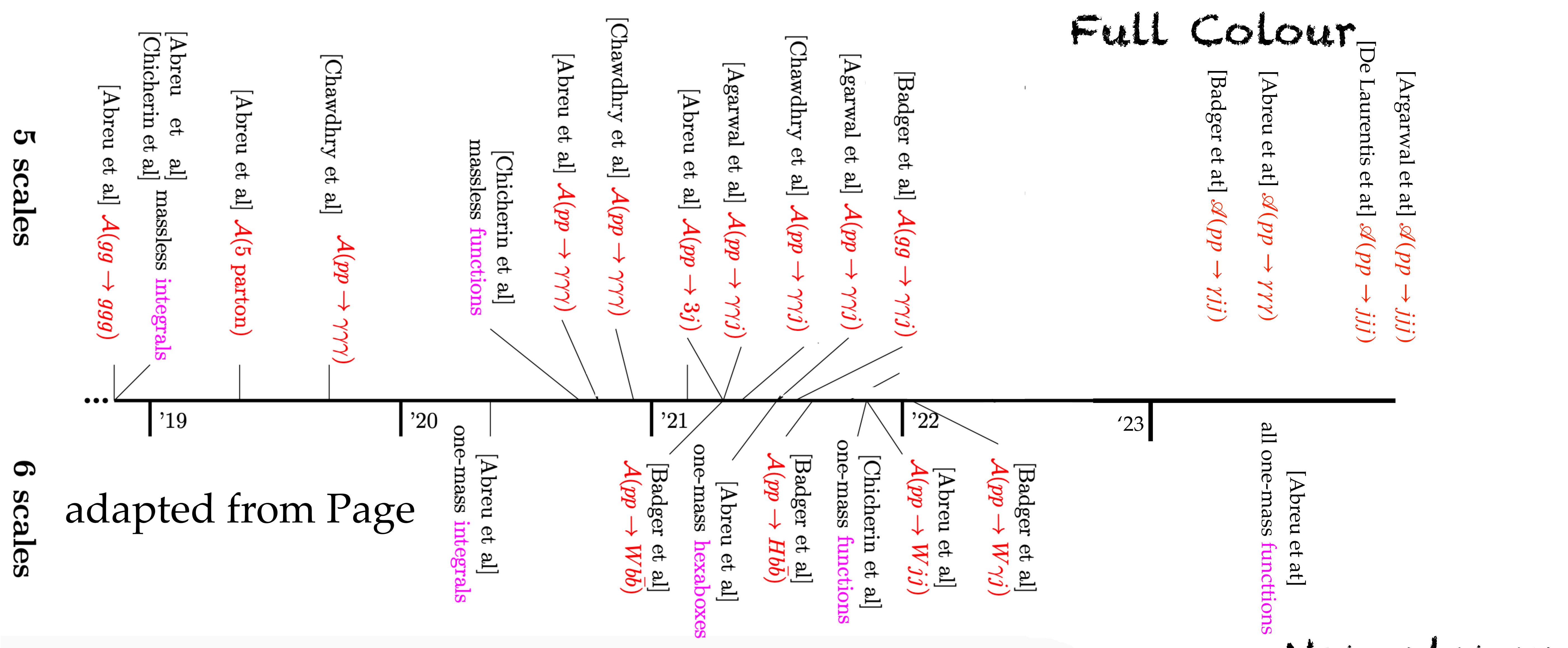
- $pp \to b\bar{b}W$ (massless bottom)
  [Hartanto, Poncelet, Popescu, Zoia '22]
- $pp \to \gamma jj$
  [Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia '23]

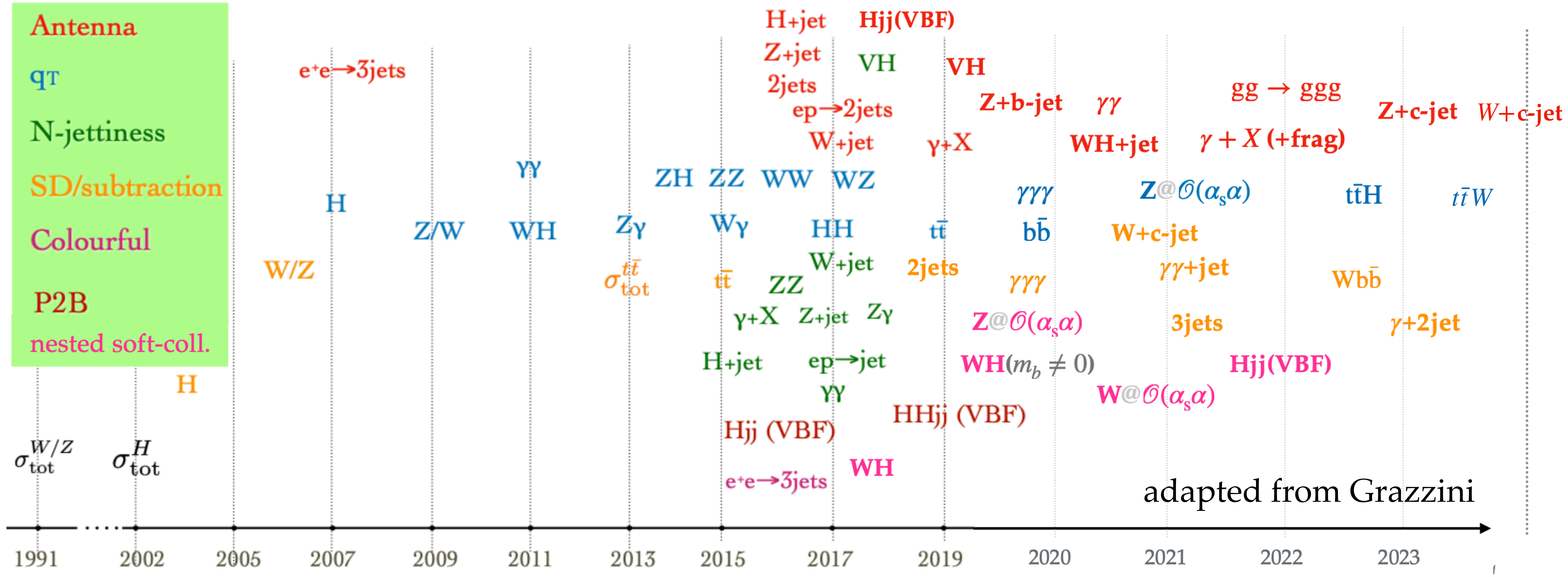


2 → 3 calculations

- Outstanding progress in amplitude calculation
  - Differential equations and canonical basis [Henn '13]
  - Efficient evaluation of 5-point basis function (pentagons)
  - Finite fields for rational reconstruction [Peraro '16] [von Manteuffel, Schabinger '14]
  - AMFlow [Liu, Ma, Wang '18]
  - …



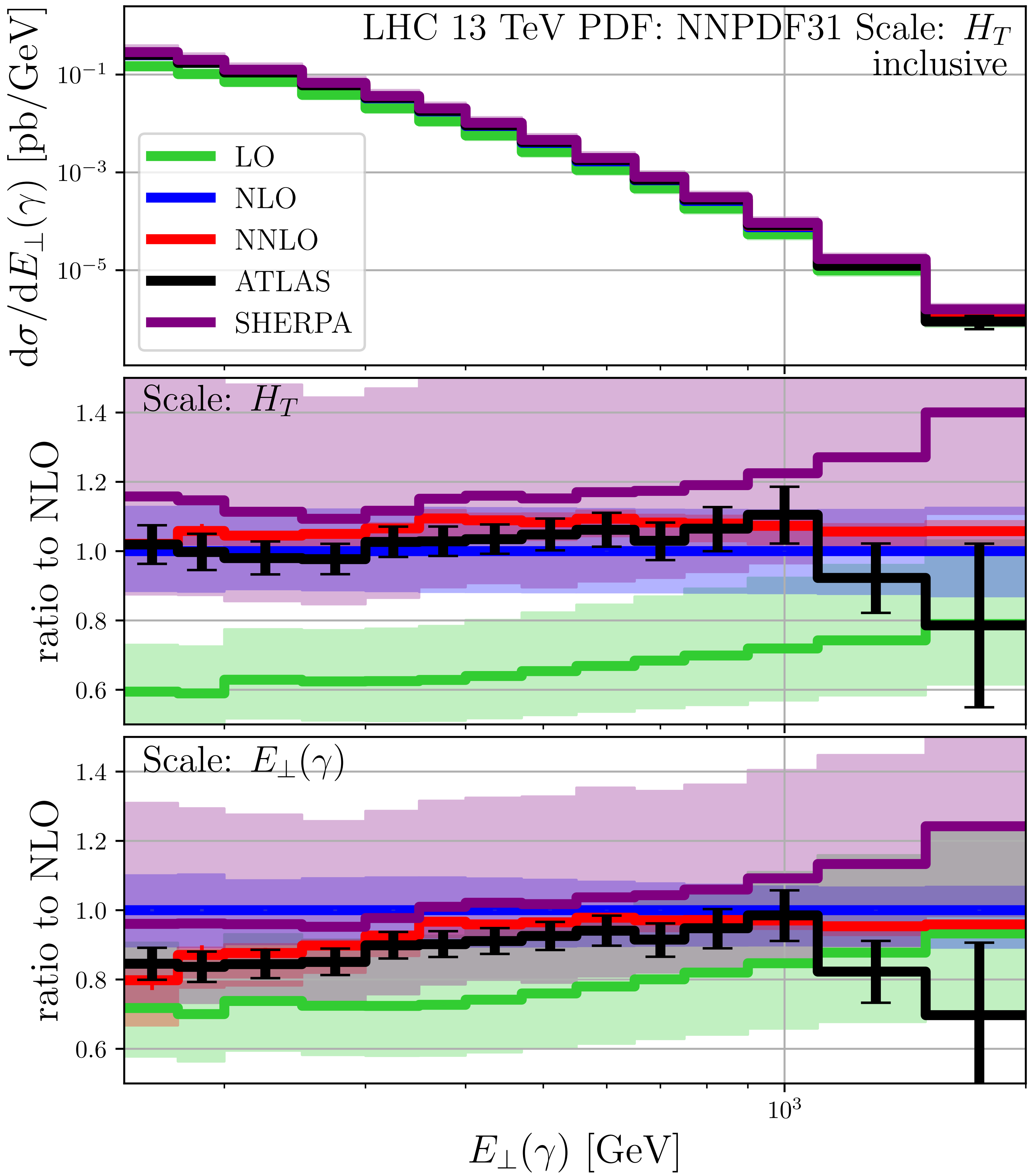
Subtraction/Slicing



Progress in amplitude calculations finds immediate phenomenological applications

$2 \rightarrow 3$ calculations: first calculation with exact full colour 2-loop amplitude

$pp \rightarrow \gamma + 2j + X$
[Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia '23]



- Access to angular correlations between photon and jets
- Access to different kinematic regimes (enhance direct component, high- or low-z fragmentation)
- Background to BSM searches

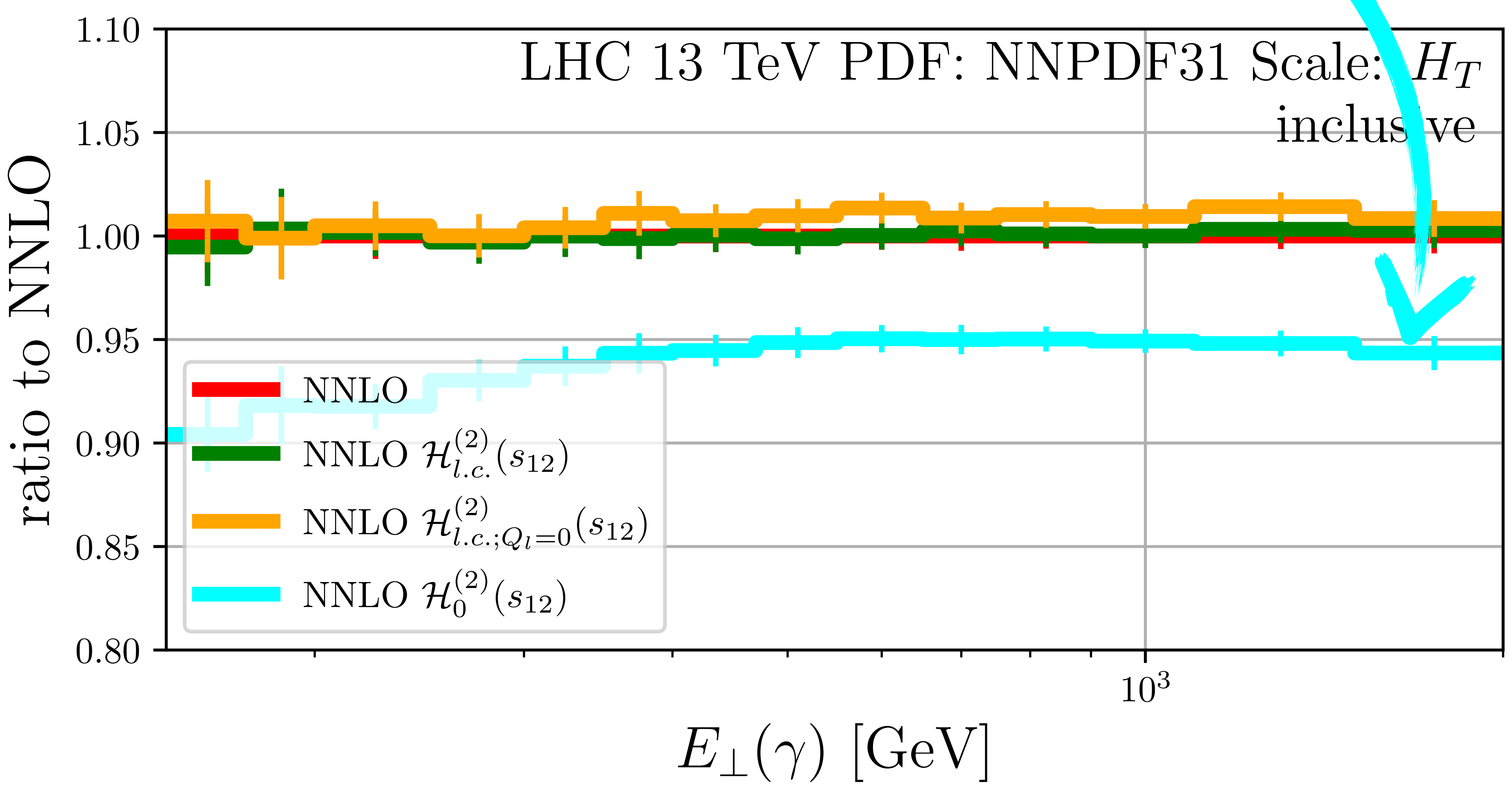
Interesting comparison with **ATLAS data** and with SHERPA predictions (which include double real matrix elements through jet merging)

- $\mathcal{O}(1-10\%)$ NNLO corrections
- Much-improved description of data
- Reduced scale uncertainties
- Default SHERPA predictions: poor description of data with large uncertainties

$2 \to 3$ calculations: first calculation with exact full colour 2-loop amplitude

$pp \to \gamma + 2j + X$
[Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia '23]

Put to zero the 2-loop finite remainder



- Access to angular correlations between photon and jets
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- Background to BSM searches

Impact of **Leading Colour Approximation**

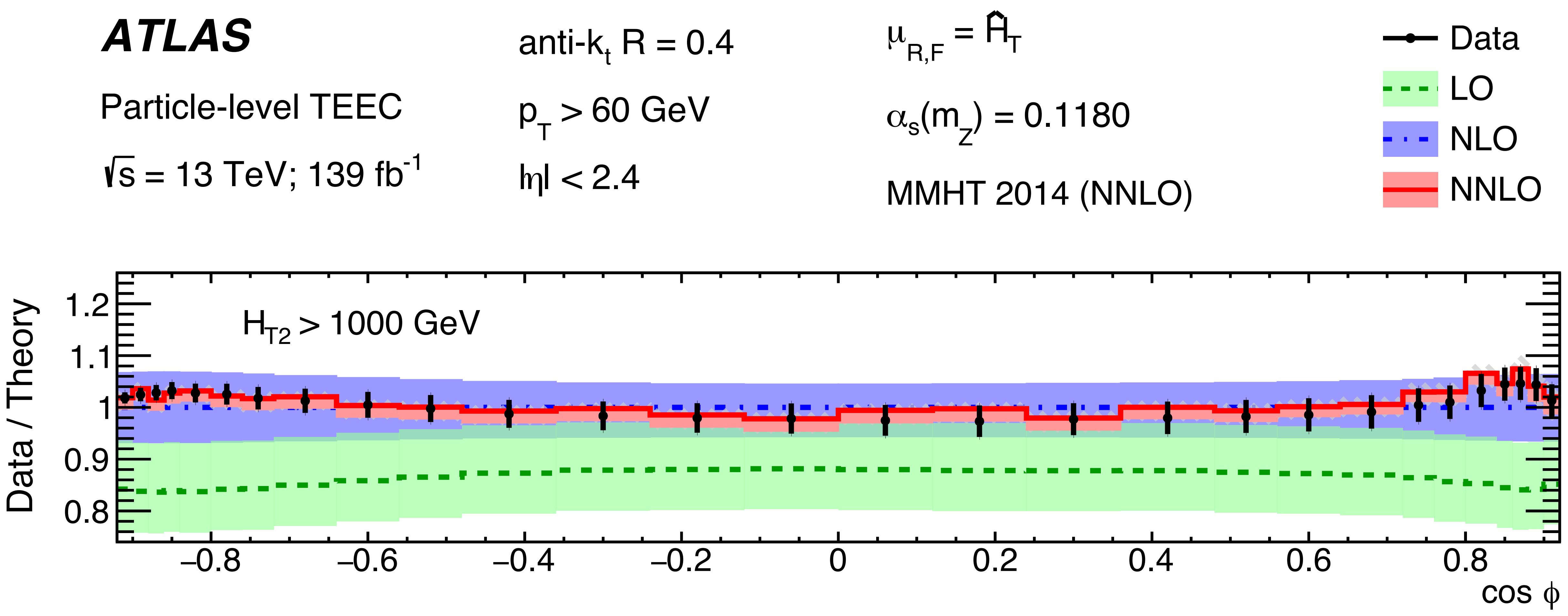
- 2-loop finite remainder represents a **small fraction**, $\mathcal{O}(5-10\%)$, of NNLO cross section
- **Negligle** size of subleading color corrections, $\mathcal{O}(\lesssim 1\%)$, compared to scale uncertainties

To be verified on a process-by-process basis: relative deviations between LC and FC amplitudes can be sizeable ($35-50\,\%$, as observed for the tri-photon amplitudes [Abreu, De Laurentis, Ita, Klinkert, Page, Sotnikov '23]) with a small impact on the NNLO cross section because the finite remainder represent only a small fraction of the NNLO correction

$2 \to 3$ calculations: transverse energy–energy correlations in multijet events

$pp \to jjj + X$

[Czakon, Mitov, Poncelet ’21]
[ATLAS Collab. JHEP 07 (2023) 85]

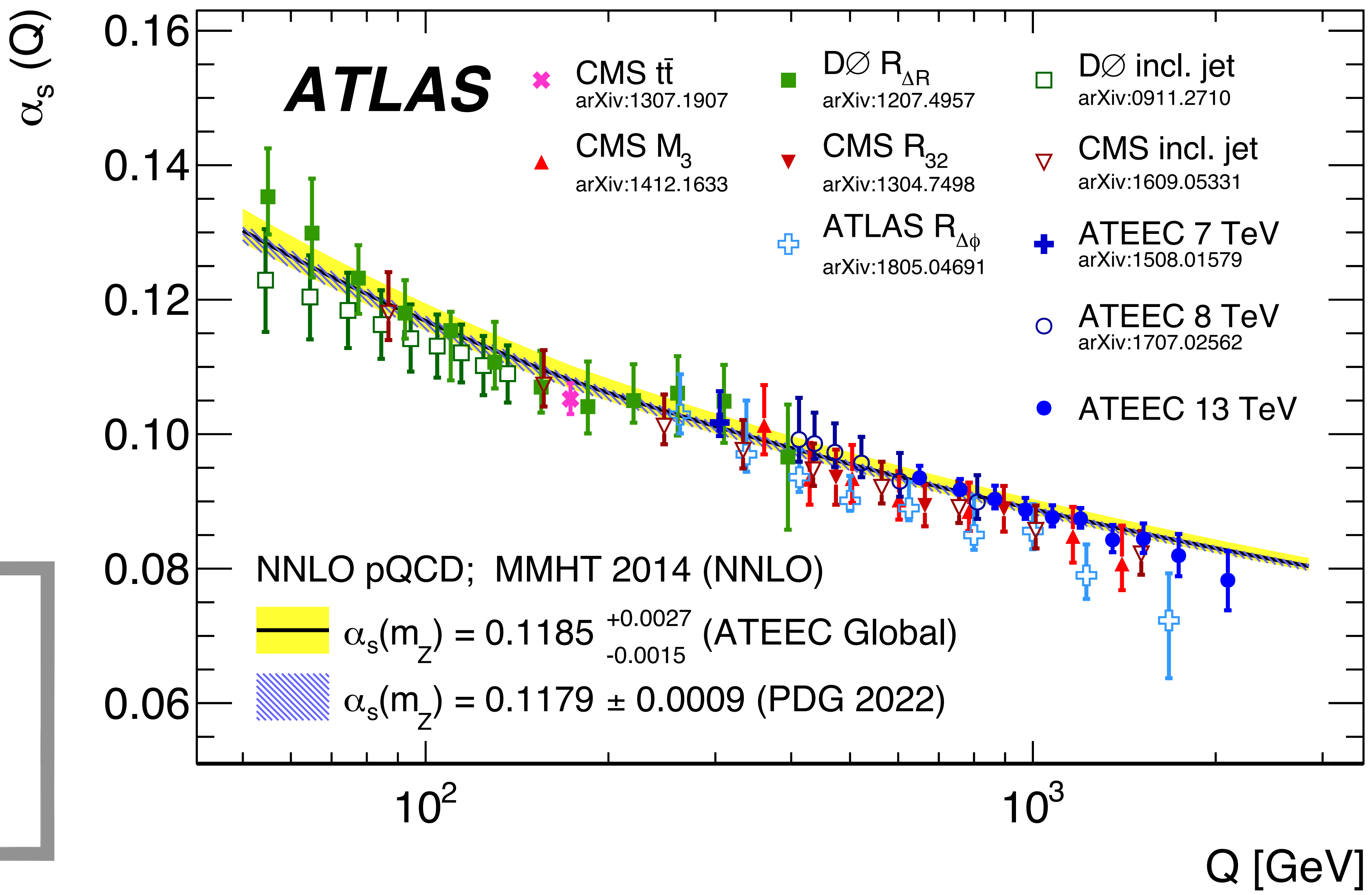


TEEC: transverse-energy-weighted distribution of the azimuthal differences between jet pairs in the final state

$$\frac{1}{\sigma}\frac{d\Sigma}{d\cos\phi} = \frac{1}{N}\sum_{A=1}^{N}\sum_{i,j}\frac{E_{Ti}^{A}E_{Tj}^{A}}{\left(\sum_k E_{Tk}^{A}\right)^2}\delta(\cos\phi - \cos(\phi_i - \phi_j))$$

$$\text{ATEEC} = \frac{1}{\sigma}\frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\phi}\bigg|_{\phi} - \frac{1}{\sigma}\frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\phi}\bigg|_{\pi-\phi}$$ TEEC **asymmetry**

- sensitive to strong the coupling!



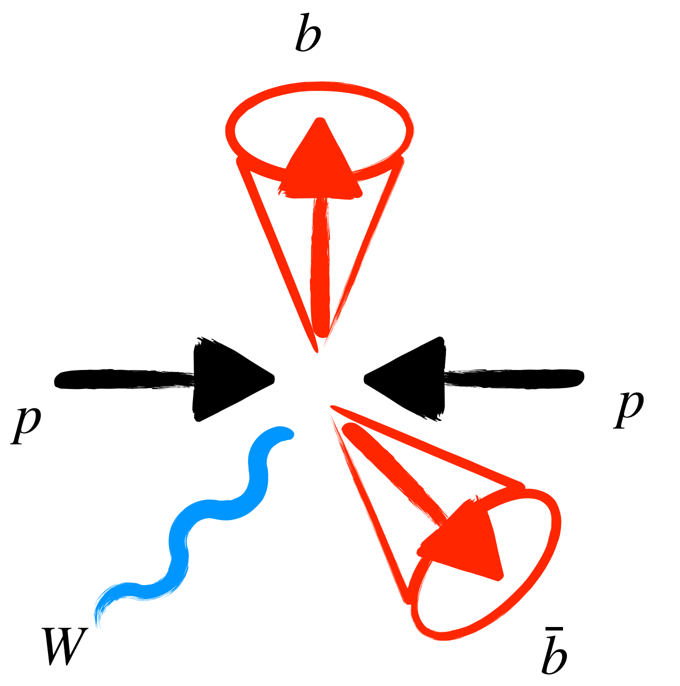
- Spectacular data description
- Much-reduced scale uncertainties
- But **monster calculation**: $\mathcal{O}(100\,M)$ CPU hours!

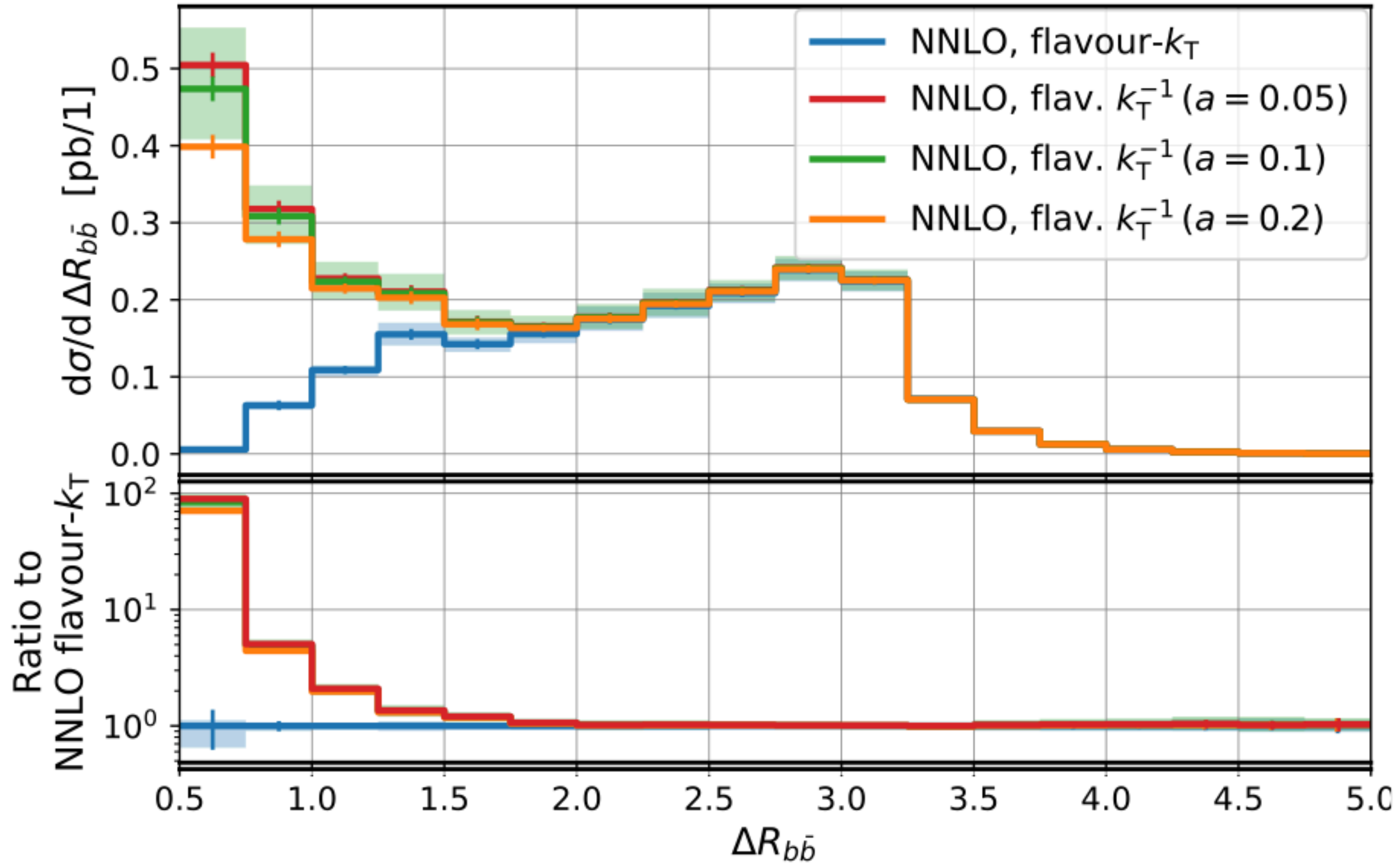
$$\alpha_S(m_Z) = 0.1175 \pm 0.0006\,(\text{exp})^{+0.0034}_{-0.0017}\,(\text{th}) \quad \text{TEEC}$$
$$\alpha_S(m_Z) = 0.1185 \pm 0.0009\,(\text{exp})^{+0.0025}_{-0.0012}\,(\text{th}) \quad \text{ATEEC}$$

$2 \to 3$ calculations: first calculation with an extra mass

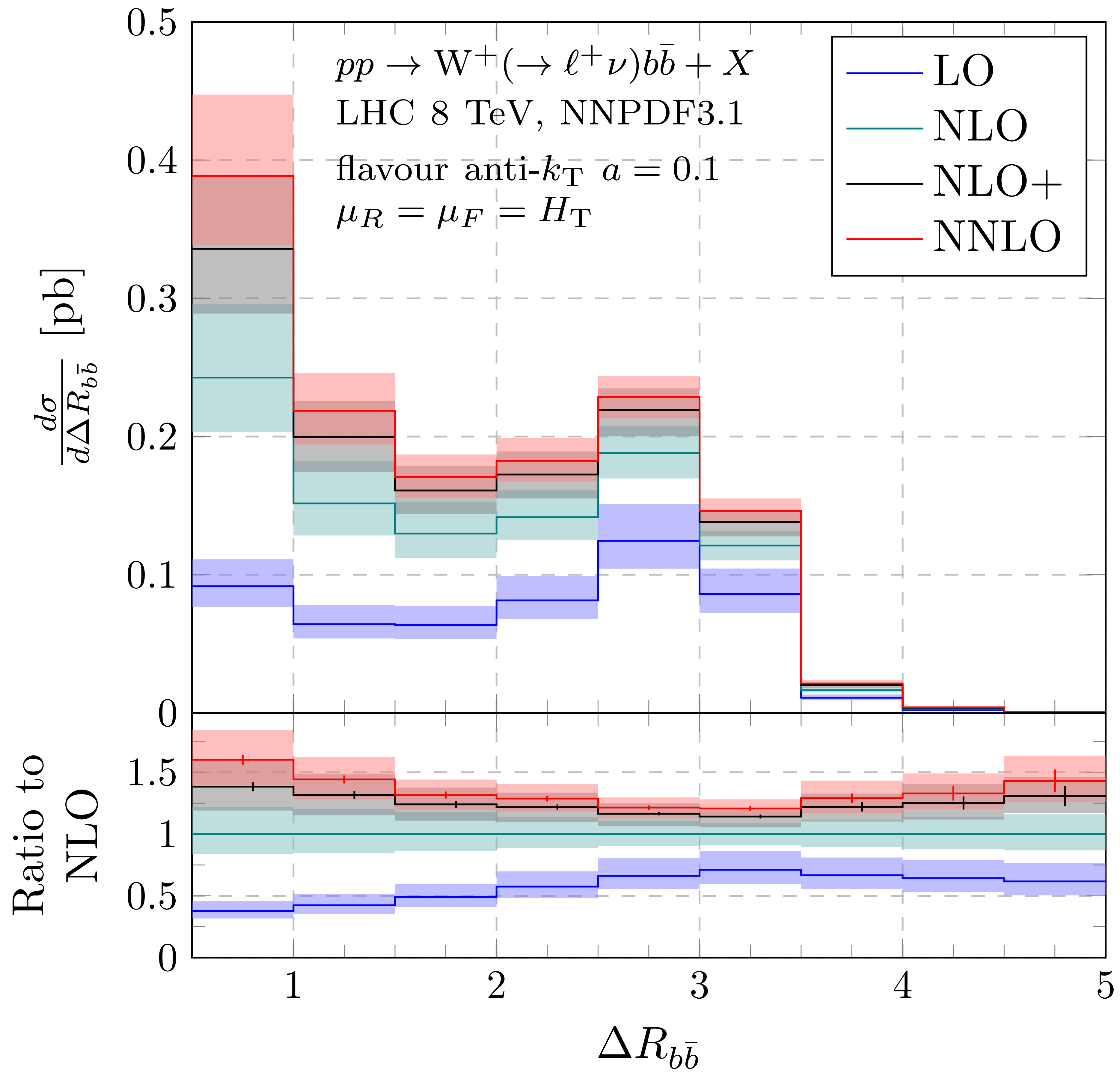
$pp \to b\bar{b}W + X$
[Hartanto, Poncelet, Popescu, Zoia '22]

- Background to $H \to b\bar{b}$
- Test of pQCD
- flavour jets!





First phenomenological application of the flavour anti-k_T algorithm



- Sizeable positive NNLO corrections
- NLO+: NLO-merged calculation (include $b\bar{b}Wj$ at NLO) captures a substantial part of the NNLO corrrections

Where Do We Come From? What Are We? Where Are We Going?

▷ $2 \to 3$ processes: **steady progress**

Going beyond: associated production of heavy quarks and a heavy boson

Missing ingredient : 2-loop virtual amplitude

Find a smart approximation of (only) the finite remainder only!

▷ $pp \to b\bar{b}W + X$ with massive bottom

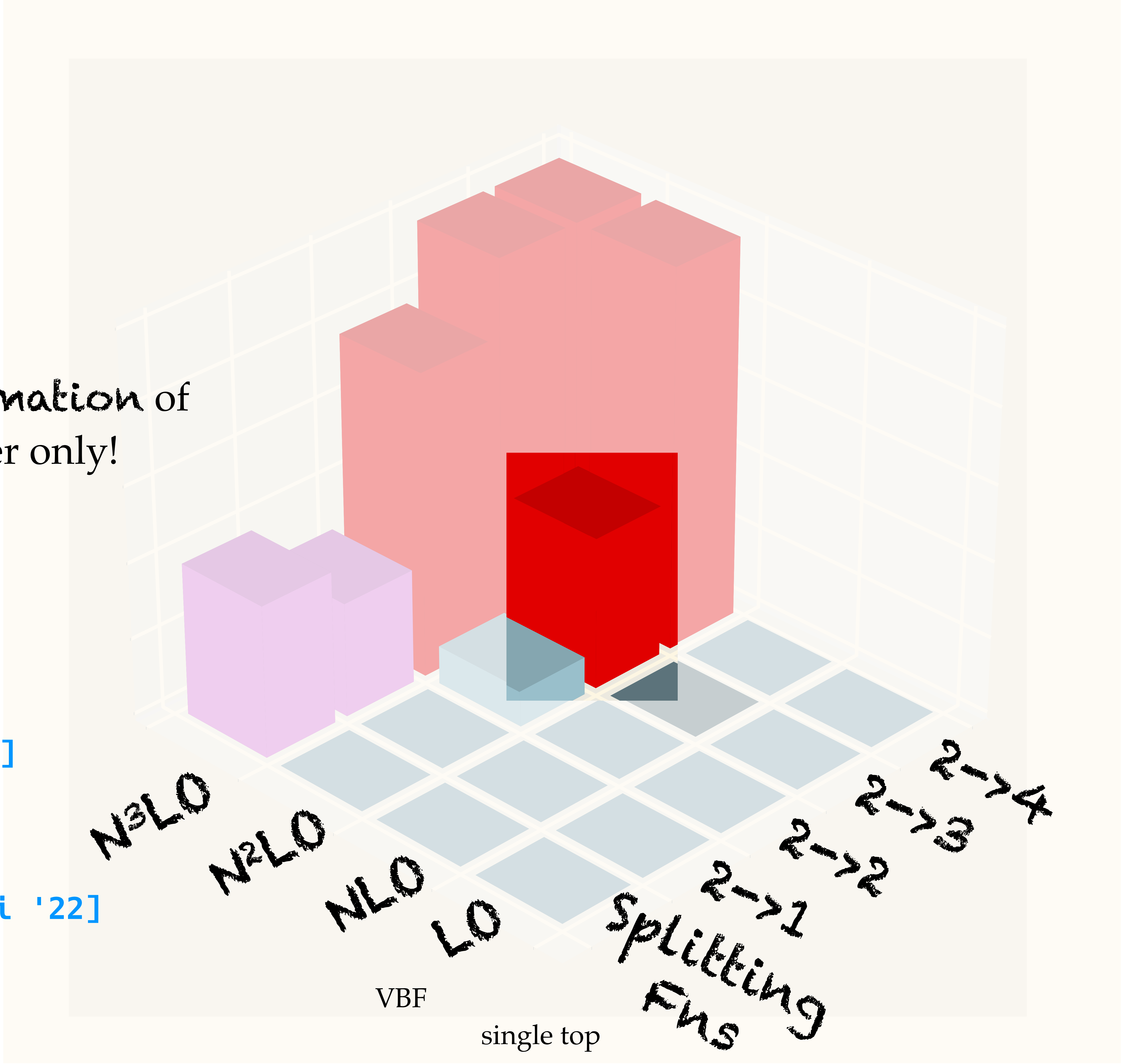
[LB, Devoto, Kallweit, Mazzitelli, Rottoli, Savoini '22]

▷ $pp \to t\bar{t}H + X$

[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini '22]

▷ $pp \to t\bar{t}W + X$

[LB, Devoto, Grazzini, Kallweit, Mazzitelli, Rottoli, Savoini '22]



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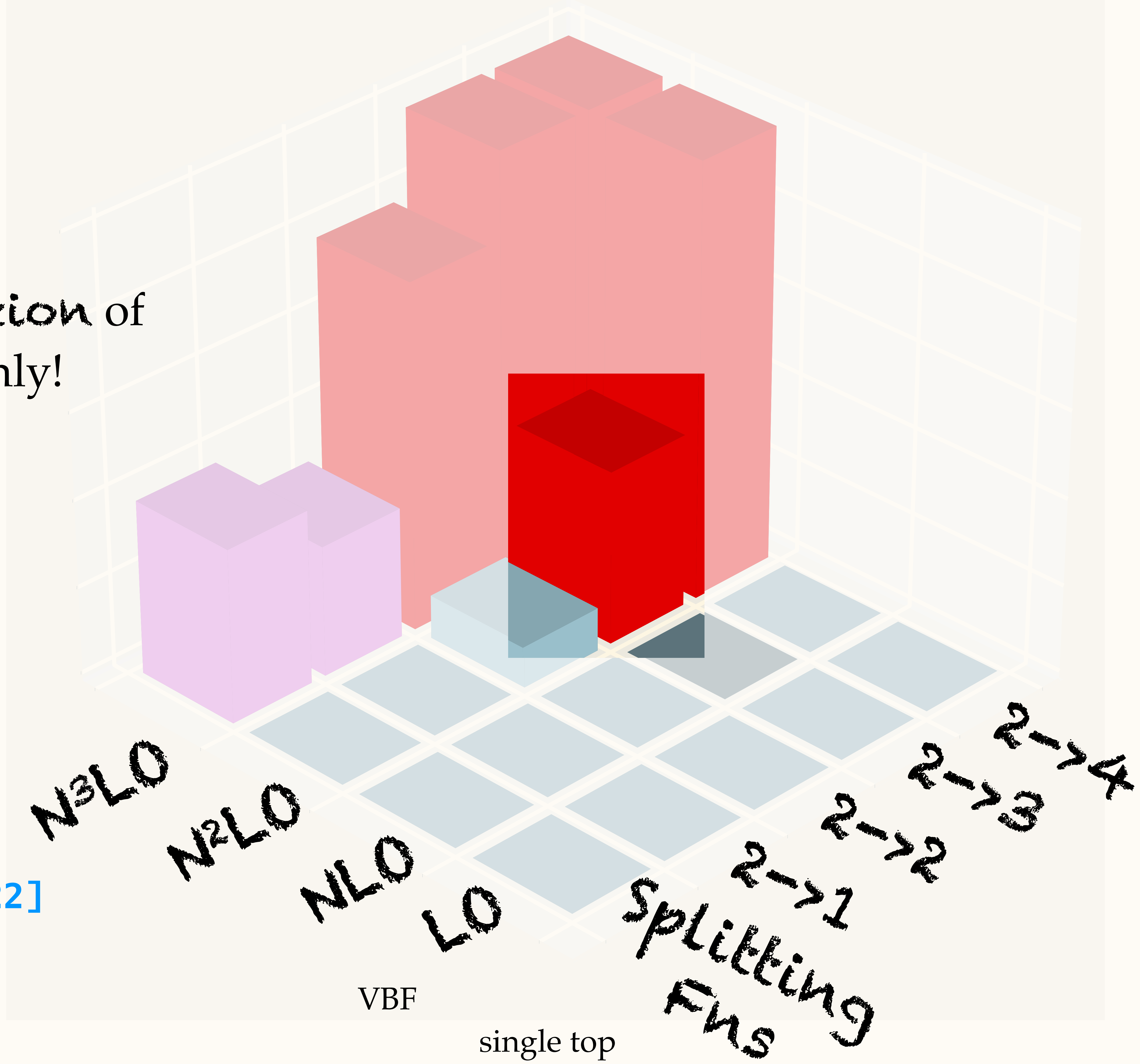
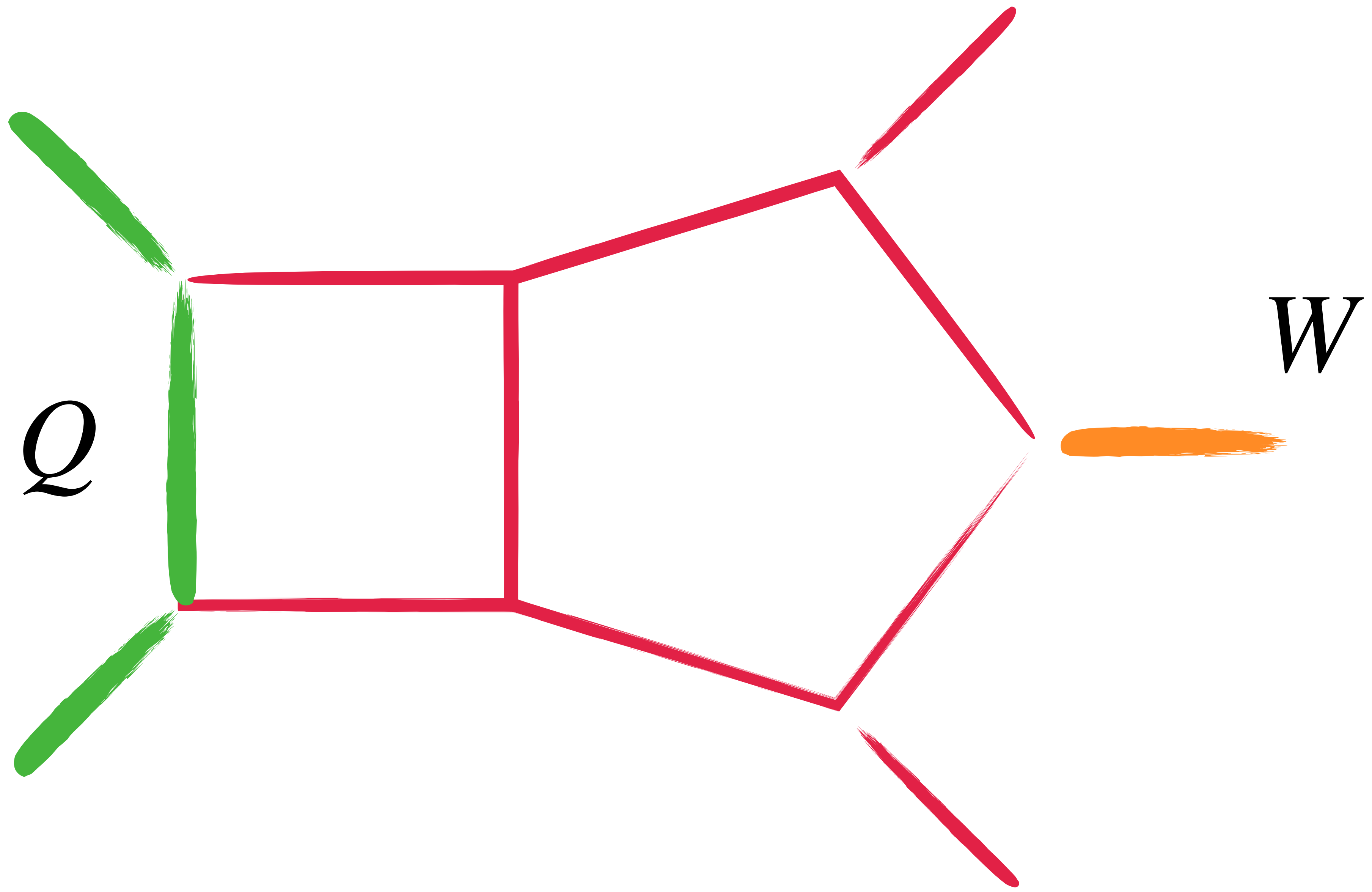
Exploit other approximations based on soft-gluon resummation

talk by Kidonakis (WG4)



Two-loop virtual amplitude: approximation in the ultra-relativistic limit



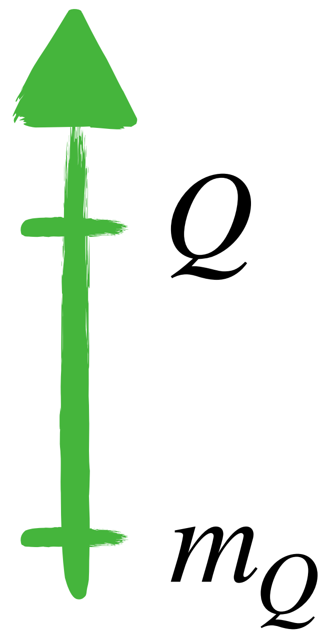
Leading color 5-point amplitude with 1 massive particle current state of the art, more masses out of reach!

[Badger, Hartanto, Zoia, 2021]
[Abreu, Febres Cordero, Ita, Klinkert, Page, Sotnikov, 2021]

Smart idea: look for reliable approximation(s) based on **factorisation theorems**

In some kinematical regimes, the amplitude "**factorises**" into a *calculable factor* and a *simpler (available) amplitude*

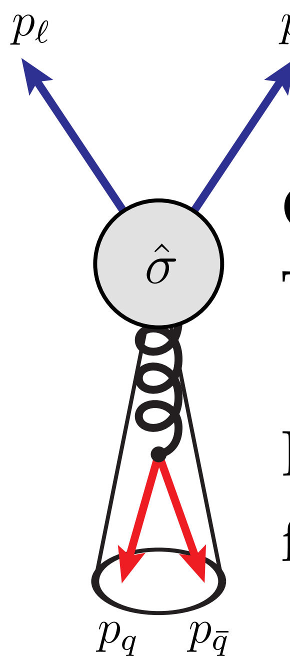
- the mass of the heavy quark is negligible compared to its energy and other relevant hard scales (ultra relativistic quarks)
  massification



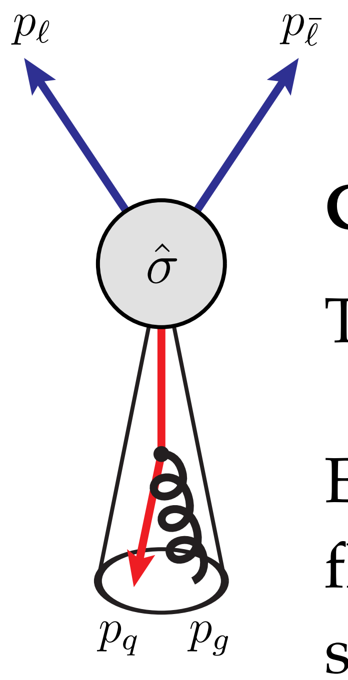
$2 \to 3$ calculations: flavour jets with massive regulator

(Heavy) Flavour jet(s)

$pp \to b\bar{b}W + X$ with massive bottom
[LB, Devoto, Kallweitt, Mazzitelli, Rottoli, Savoini ’22]

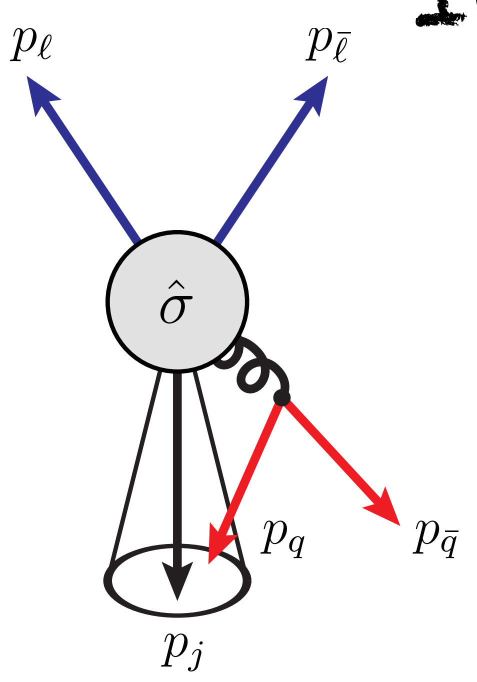


Collinear
TH: no flavour!
EXP: most likely, flavoured!



Collinear
TH: flavoured!
EXP: depends on flavoured hadron selection

IRC unsafe!

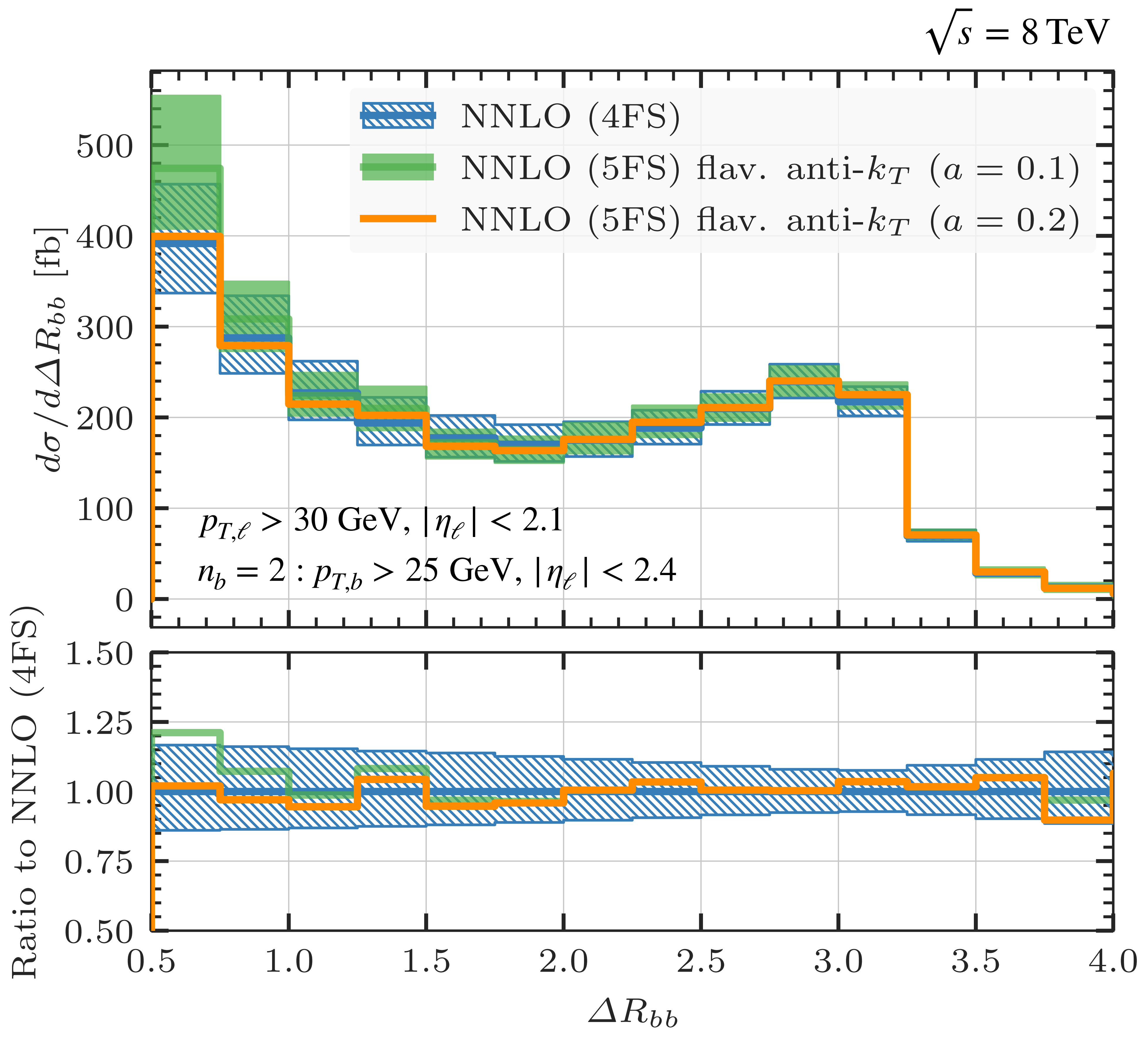


Soft
TH: no flavour!
EXP: depends on flavoured hadron selection

for massive (flavoured) quarks

$\alpha_S \log\left(Q^2/m_Q^2\right)$ $\qquad$ $\alpha_S^2 \log\left(Q^2/m_Q^2\right)$

- Heavy quark mass regulates IRC unsafe configurations
- Logs of the bottom mass are not extremely large
- 2-loop amplitude: “massify” massless amplitude up to $\mathcal{O}(m_Q/Q)$



[Mitov, Moch ’07]

$$|\mathcal{M}^{[p],(m)}> = \prod_i \left[Z_{[i]}\left(\frac{m^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)\right]^{1/2} \times |\mathcal{M}^{[p]}> + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

$$Z_{[i]}\left(\frac{m^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right) = \mathcal{F}^i\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right)\left[\mathcal{F}^i\left(\frac{Q^2}{\mu^2}, 0, \alpha_S(\mu^2), \epsilon\right)\right]^{-1}$$

2 → 3 calculations: flavour jets with massive regulator

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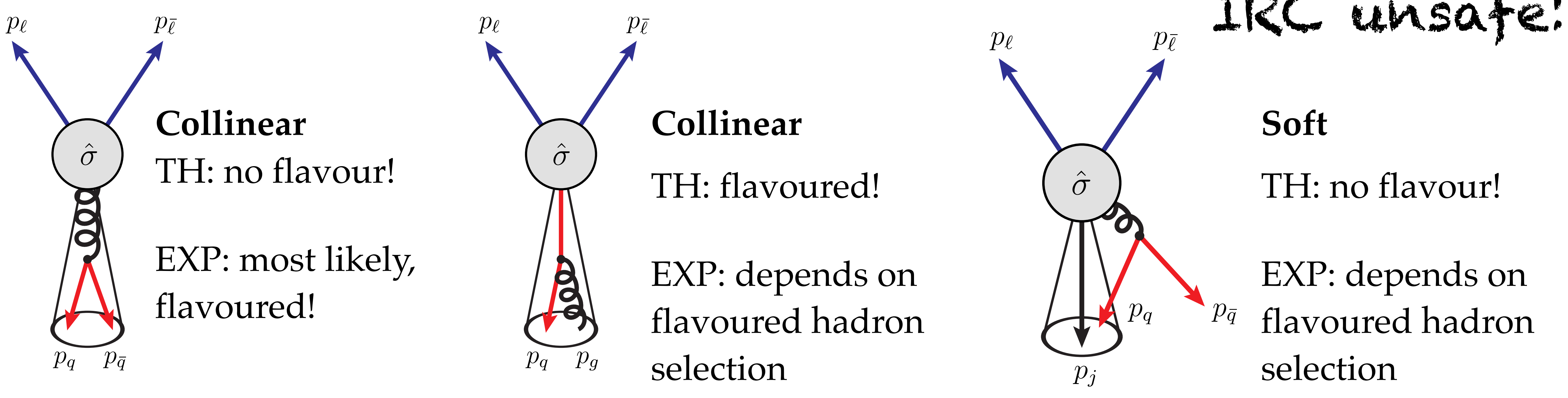
$pp \to b\bar{b}W + X$ with massive bottom
[LB, Devoto, Kallweitt, Mazzitelli, Rottoli, Savoini '22]



IRC unsafe!

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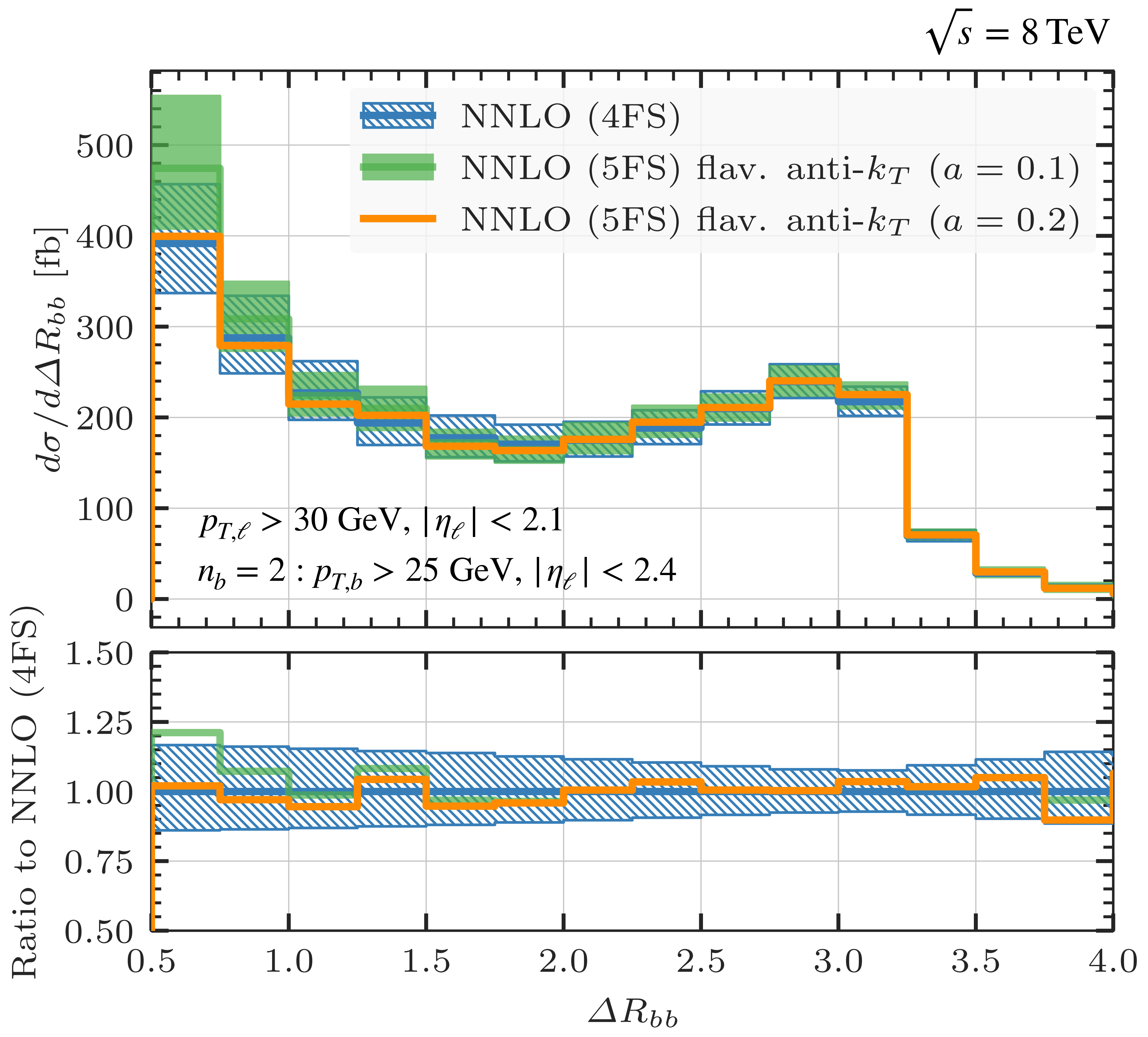
→ $\alpha_S \log\left(Q^2/m_Q^2\right)$ $\alpha_S^2 \log\left(Q^2/m_Q^2\right)$

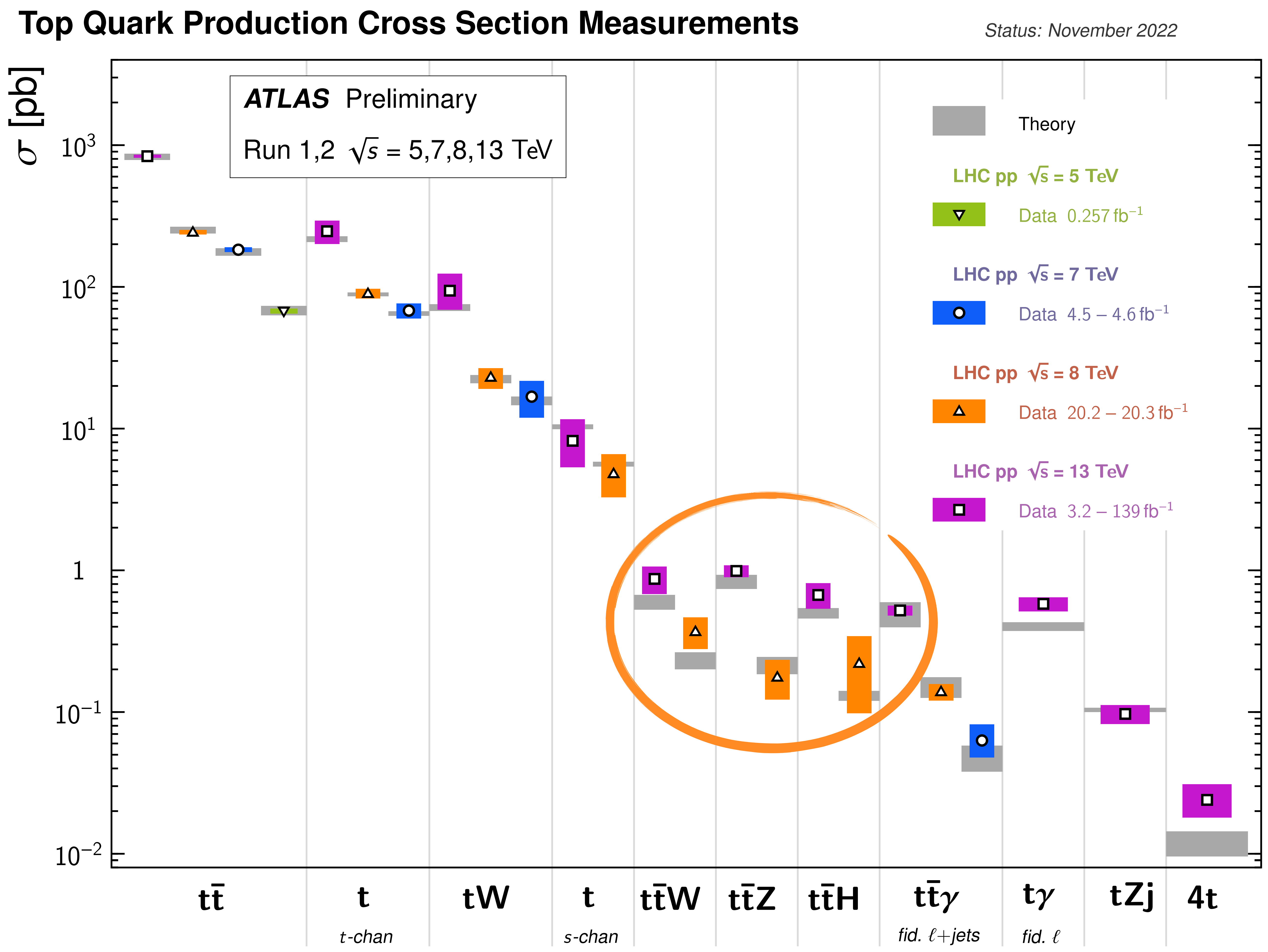


- Heavy quark mass regulates IRC unsafe configurations
- Logs of the bottom mass are not extremely large
- 2-loop amplitude: "massify" massless amplitude up to $\mathcal{O}(m_Q/Q)$

- Remarkable agreement between massless (5FS) and massive calculation (4FS)

$2 \to 3$ calculations: "massive" SM signatures

The production of a top-quark pair together with a EW heavy boson or a Higgs are among **the most massive SM signatures** at hadron colliders

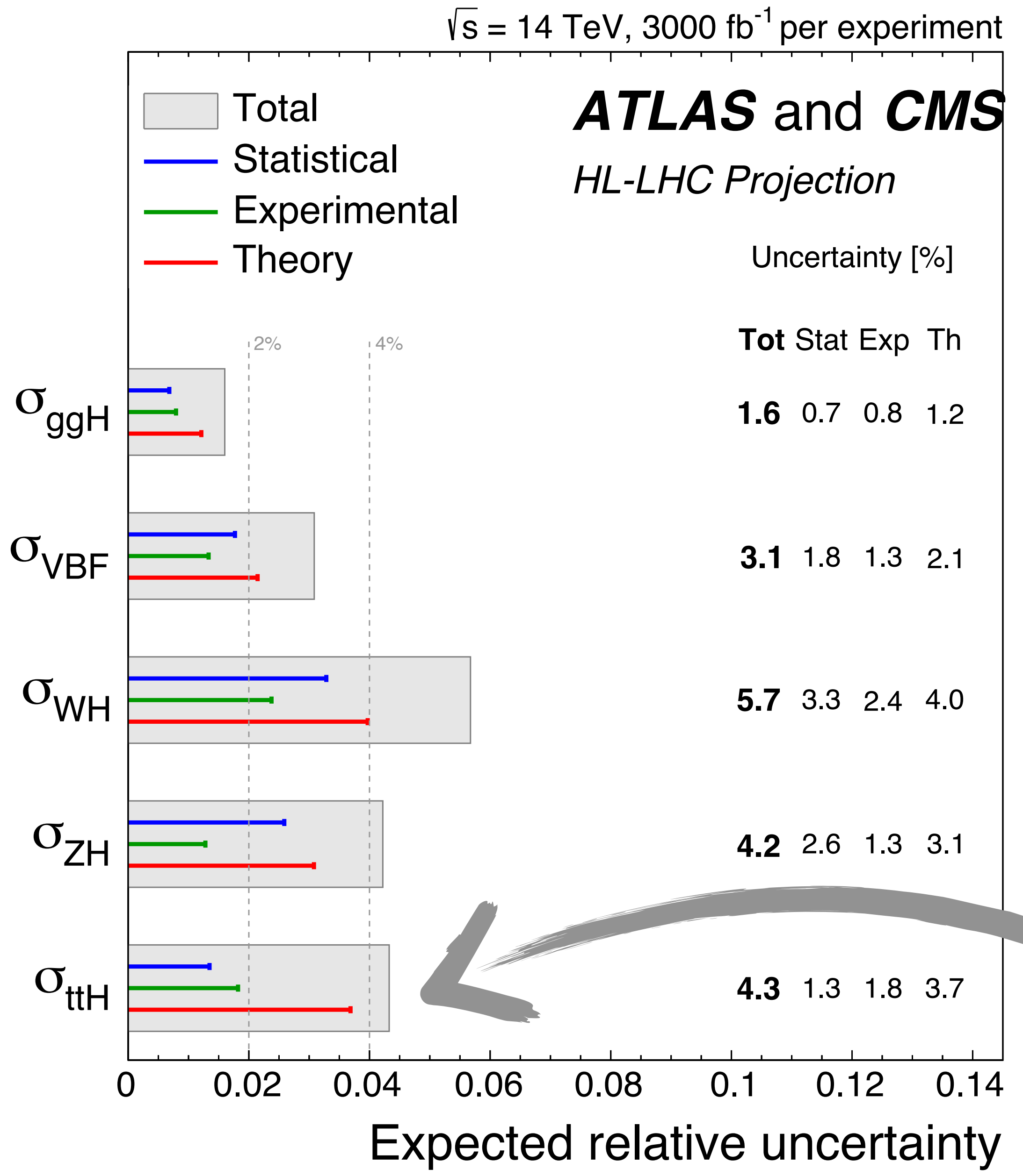


Small cross sections, but already observed and **measured with** $10-20\,\%$ **uncertainties**

Crucial to characterise the top-quark interactions, in particular with the Higgs boson

Talks by Pellen (TH) and Mazumdar (EXP)

$2 \to 3$ calculations: $t\bar{t}H$, a soft Higgs ~~fairy~~-tale



[Report from WG2:
Higgs Physics at the HL-LHC and HE-LHC '19]

Heavy quarks + boson

$pp \to t\bar{t}H + X$

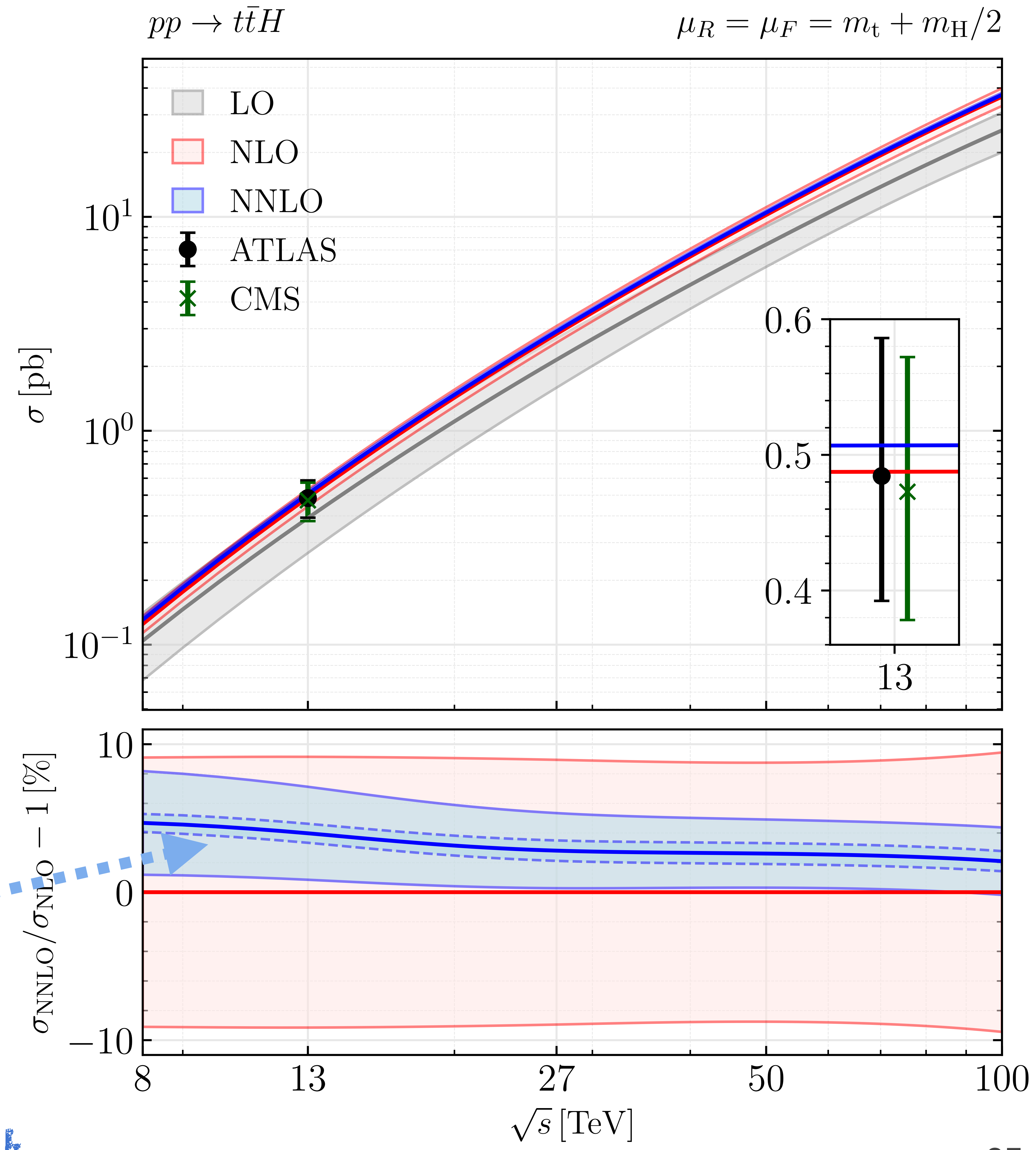
[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini '22]

- $t\bar{t}H$ direct access to the top Yukava y_t
- Measured signal strength at $\mathcal{O}(20\%)$
- Exp. uncertainties at HL-LHC at $\mathcal{O}(2\%)$

Missing 2-loop amplitude

Approximation
Soft Higgs???

$$|\mathcal{M}^{[p,k]}_{t\bar{t}H}> \simeq F(\alpha_s(\mu)R); m_t/\mu_R) \times J(k) \times |\mathcal{M}^{[p]}_{t\bar{t}}>$$



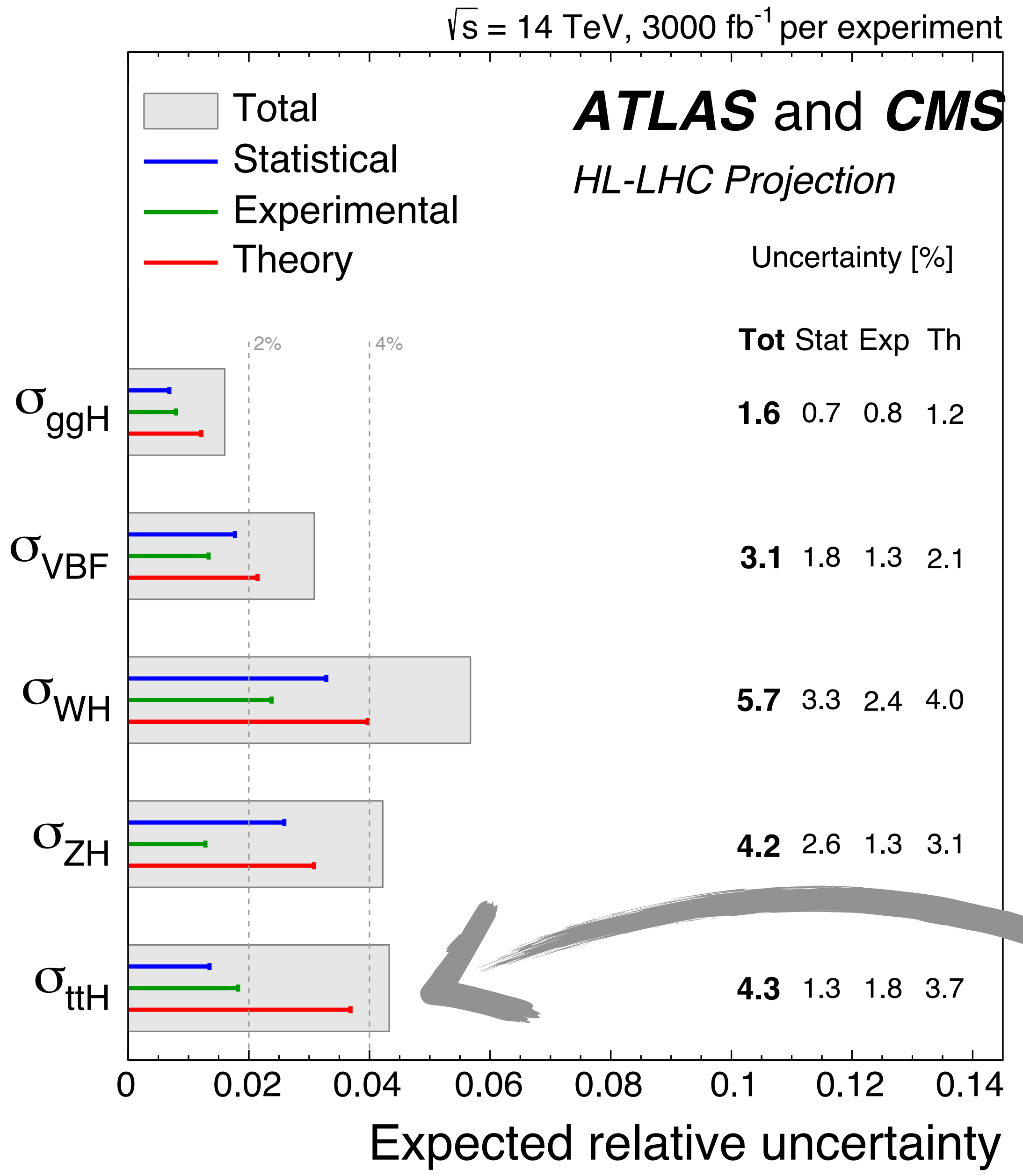
FINAL UNCERTAINTY:

$\pm 0.6\,\%$ on σ_{NNLO}, $\pm 15\,\%$ on $\Delta\sigma_{\mathrm{NNLO}}$

Finite two-loop remainder subdominant

2 → 3 calculations: $t\bar{t}H$, a soft Higgs ~~fairy~~-tale

- Nice perturbative convergence
- TH uncertainties $\mathcal{O}(3\%)$



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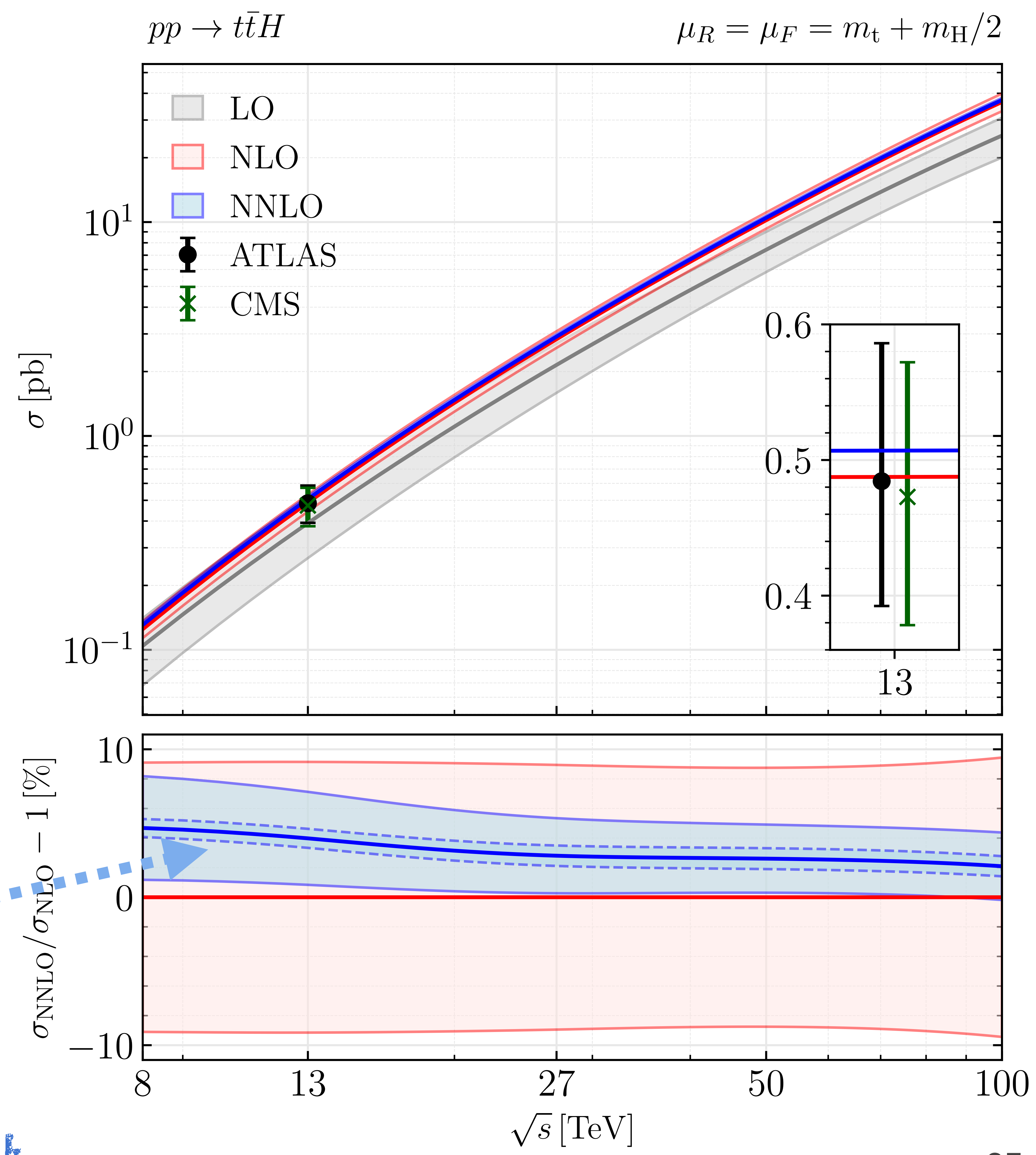
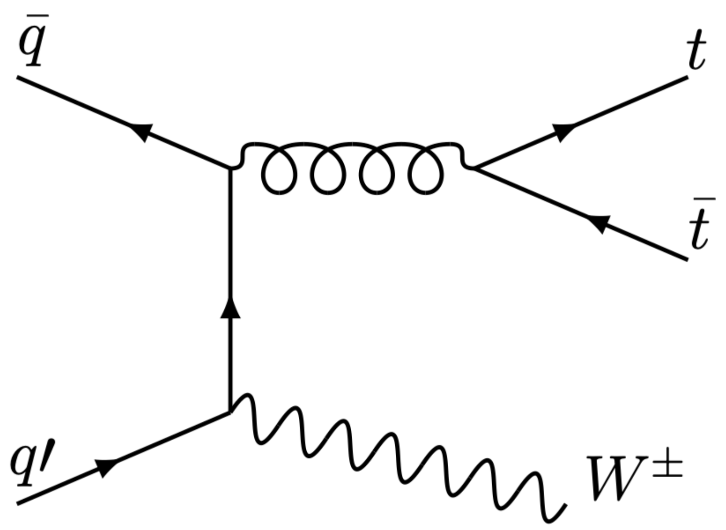
Finite two-loop remainder subdominant



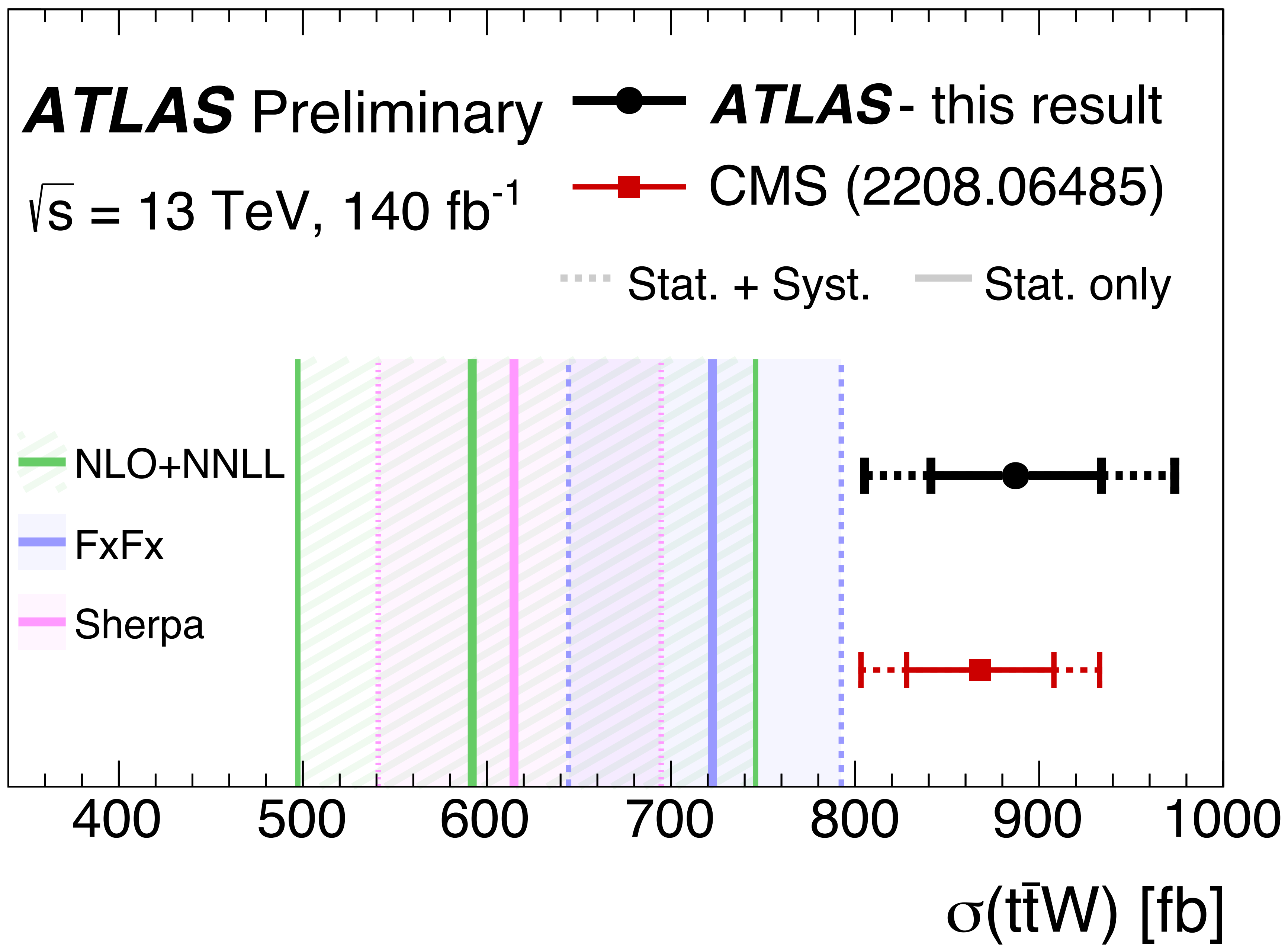
$2 \rightarrow 3$ calculations: $t\bar{t}W$

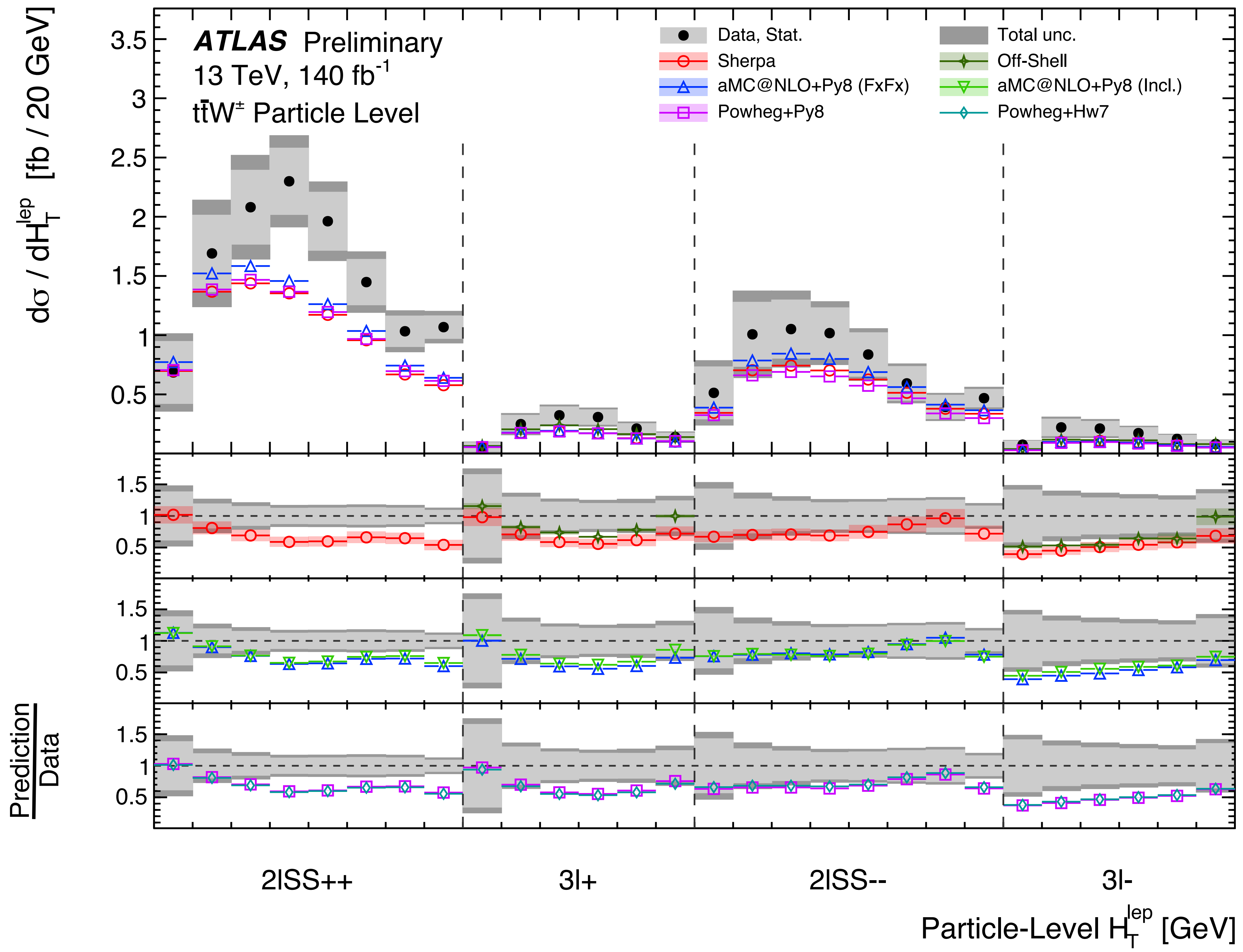


- $pp \rightarrow t(\rightarrow bW^{+}) + \bar{t}(\rightarrow \bar{b}W^{-})W$: irreducible SM source of same sign dilepton pairs
- Background to $t\bar{t}H$ e $t\bar{t}t\bar{t}$

- Measured $t\bar{t}W$ rates by ATLAS and CMS at $\sqrt{s} = 8$ TeV and $\sqrt{s} = 13$ TeV are consistently higher than the SM predictions.
- The most recent measurements confirm this picture with a slight excess at the $1\sigma - 2\sigma$ level

[ATLAS-CONF-2023-019 '23]





$2 \to 3$ calculations: $t\bar{t}W$

Heavy quarks + boson

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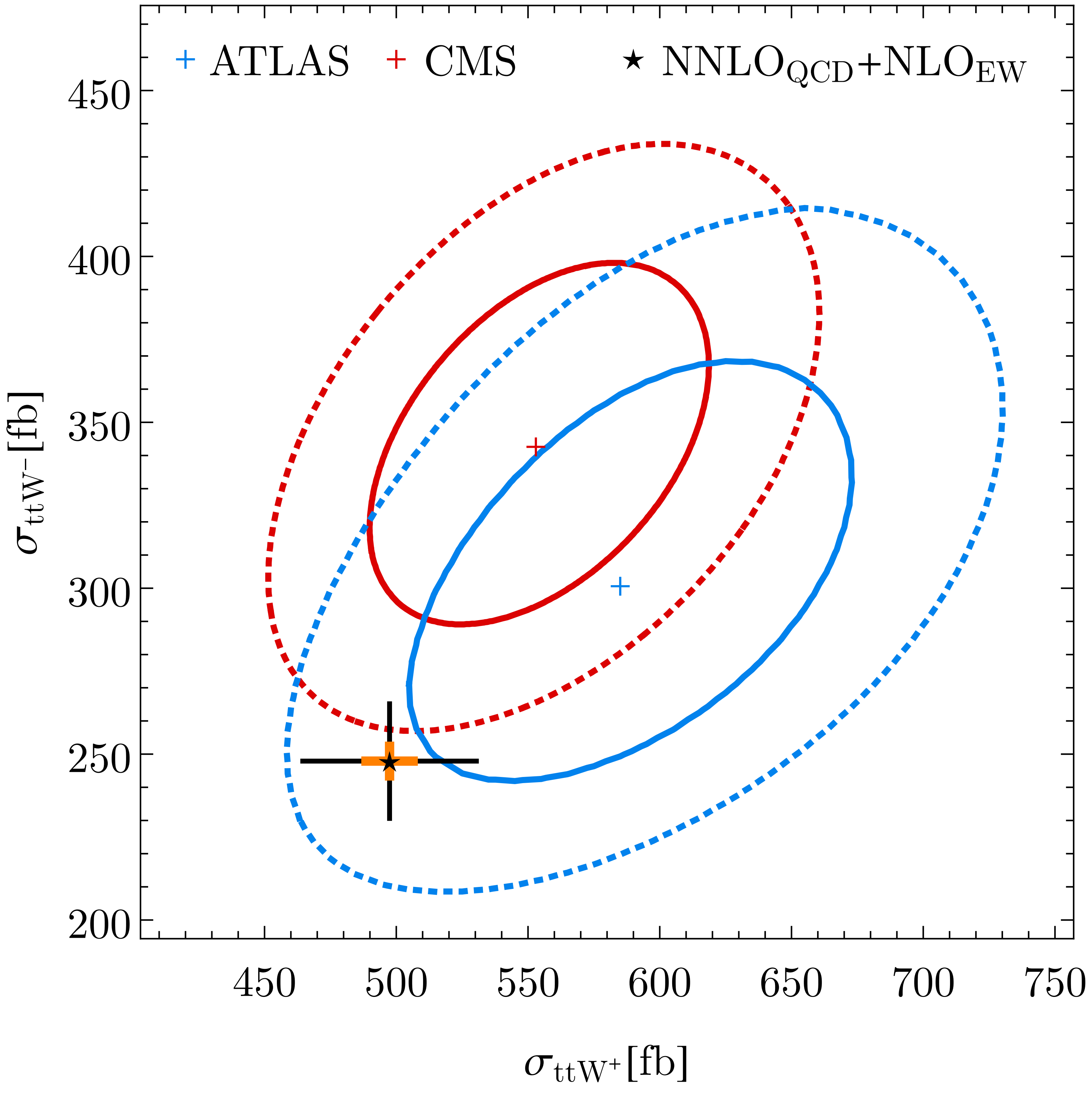
[LB, Devoto, Grazzini, Kallweit, Mazzitelli, Rottoli, Savoini '22]

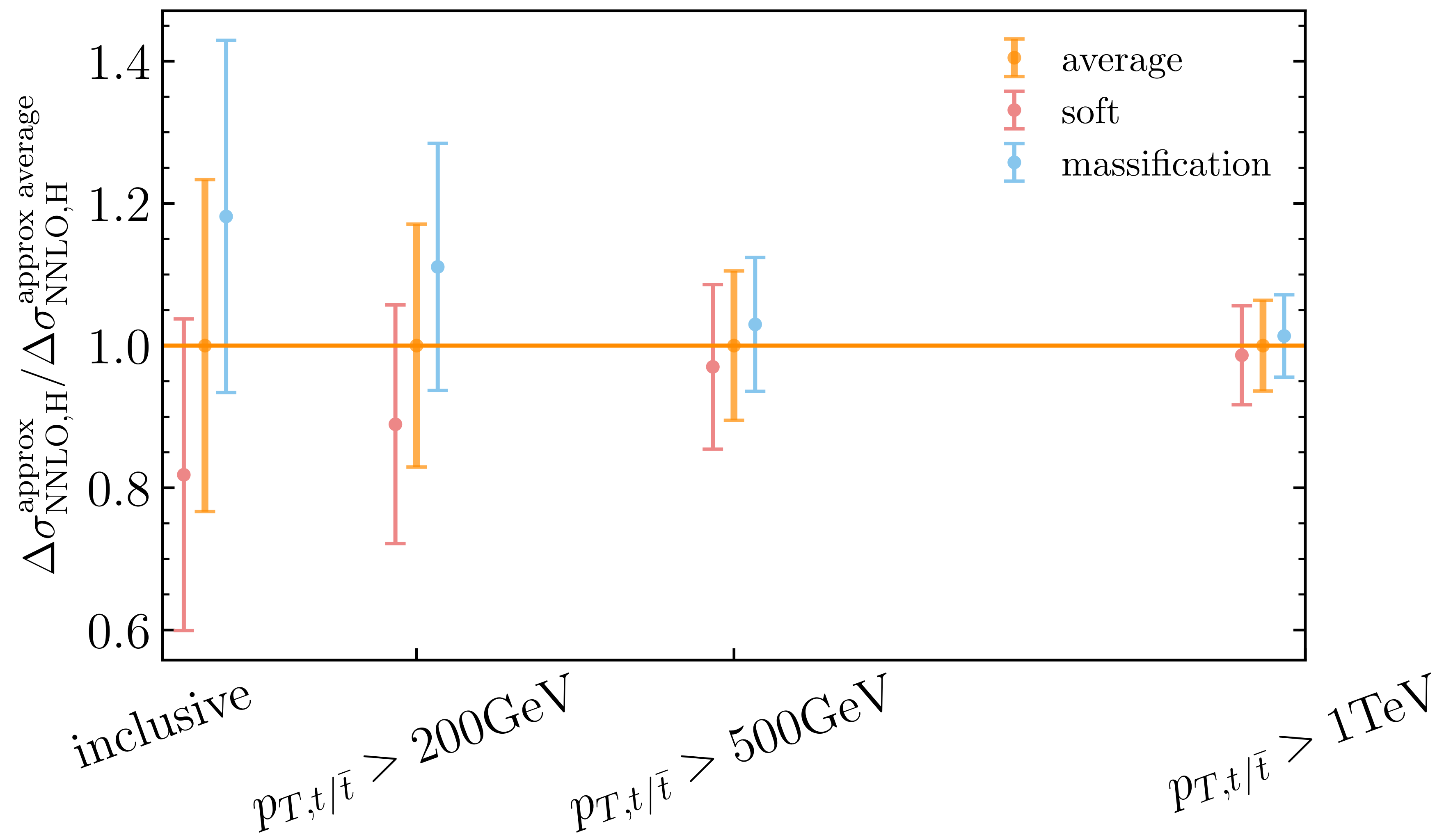


$$\sigma_{t\bar{t}W} = 745.3^{+6.7\%}_{-6.7\%}$$
$$\sigma^{FxFx}_{t\bar{t}W} = 722.3^{+9.7\%}_{-10.8\%}$$

NNLO corrections leads to

- moderately higher rates: $+14 - 15\,\%$
- reduction of perturbative uncertainties

Tension stays at the level of 1σ (ATLAS) - 2σ (CMS)



- **Relatively large impact** of two-loop virtual contribution: 7 % of the NNLO cross section
- **Strategy**: apply soft W and ultra-relativistic top approximations

FINAL UNCERTAINTY:

$\pm 1.8\,\%$ on σ_{NNLO}, $\pm 25\,\%$ on $\Delta\sigma_{\text{NNLO,H}}$

similar to what obtained in recent
$2 \to 3$ in leading colour approximation

Some considerations

- New progress in amplitude/subtraction finds almost immediate application to phenomenology
- Outstanding advancements in scattering amplitudes, especially in understanding the structure of 5-point functions with up to one external mass (planar and non-planar)
- There are mature and effective implementations of subtraction/slicing schemes (antenna, sector decomposition, q_T subtraction, N-jettiness)

Concerning the availability of the NNLO results (for theory/experimental community)

- NNLO calculations are computationally demanding and generally require expert users
  Variations of parameters, PDF uncertainties may be expensive
- So far, only two public codes are available (MCFM and MATRIX) https://mcfm.fnal.gov/ https://matrix.hepforge.org/

Some ideas:

A (not so new) proposal of producing n-tuples of Monte Carlo events for performing different studies without repeating the whole calculation. It is not be most flexible solution https://www.precision.hep.phy.cam.ac.uk/hightea/

Concerning at least PDF uncertainties, produce grids (as PINEAPPL grid) for ultra-efficient evaluation with different PDF members/sets

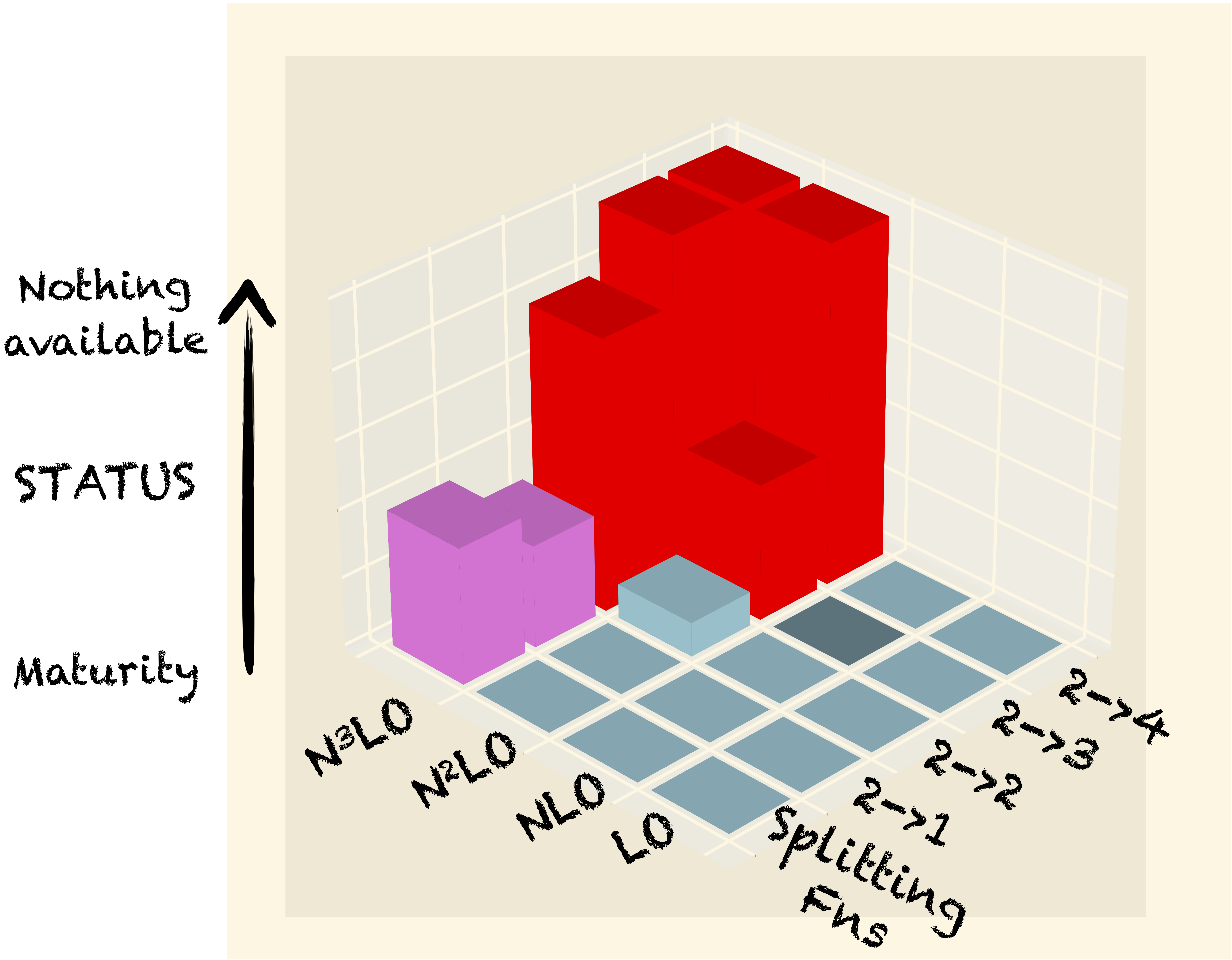
Where Do We Come From? What Are We? Where Are We Going?

- $2 \to 2$ processes: **maturity**
- $2 \to 3$ processes: **steady progress**

well-established public codes

public code partially available

no public code



Backups

Beyond “standard” $2 \to 2$ calculations: mixed QCD-EW corrections

- Complicated **multi-scale** 2-loop amplitudes are the bottleneck

- **Idea:** rely on a *semi-analytical approach* (also for W (+1 internal mass) production? NNLO EW?)

WORKFLOW

Feynman diagrams: QGRAF

Dirac Algebra and interference terms: FORM

IBPs: KIRA, LITERED, REDUZE2

Evaluation of MIs

UV renormalisation, subtraction of IR poles

Numerical grid

Treatment of γ_5: Naive anti-commuting γ_5 with reading point prescription and Larin prescription as an independent cross check

MIs with **up to one massive** boson exchange are evaluated analytically
[Bonciani, Di Vita, Matrolia, Schubert, 2016], [Heller, von Manteuffel, and Schabinger, 2020] [Hasan, Schubert, 2020], [R. Bonciani, A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, 2008], [R. Bonciani, A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, 2008], [P. Mastrolia, M. Passera, A. Primo, and U. Schubert, 2017]

5 MIs with **two massive** bosons cannot be easily expressed in terms of GPls
Require an **alternative strategy** (see also [Heller, von Manteuffel, Schabinger (2019)])

Semi-analytical evaluation of tree-loop interference
[Armadillo, Bonciani, Devoto, Rana, Vicini 2022]

- Numerical resolution of differential equations for MIs via **series expansions,** inspired by DiffExp [Hidding (2006)] but extended for **complex masses**
- **Arbitrary number of significant digits** (with analytic boundary condition)
- The method is **general** (applicable to other processes)
- Numerical evaluation of amplitudes takes $\mathcal{O}$(10 min/point) per core

Flavour aware jet algorithms: flavour k_T

[Banfi, Salam, Zanderighi, 2006]

Standard k_T algorithm

$$d_{ij} = \min\left(k_{T,i}^2, k_{T,j}^2\right) R_{ij}^2, \quad d_{iB} = k_{T,i}^2$$

condition 1 automatically satisfied

Flavour aware k_T algorithm (usually $\alpha = 2$):

flavour information available at each step of the clustering procedure

$$d_{ij}^{(F)} = R_{ij}^2 \times \begin{cases} \left[\max\left(k_{T,i}^2, k_{T,j}^2\right)\right]^{\alpha} \left[\min\left(k_{T,i}^2, k_{T,j}^2\right)\right]^{2-\alpha}, & \text{if softer of } i,j \text{ is flavoured} \\ \min\left(k_{T,i}^2, k_{T,j}^2\right), & \text{if softer of } i,j \text{ is flavourless} \end{cases}$$

this ensures condition 2 among final state protojets, as soft flavoured quark-anti-quark pair clusters first

Flavour aware jet algorithms: flavour k_T

[Banfi, Salam, Zanderighi, 2006]

Standard k_T algorithm

$$d_{ij} = \min\left(k_{T,i}^2, k_{T,j}^2\right) R_{ij}^2, \quad d_{iB} = k_{T,i}^2$$

Flavour aware k_T algorithm (usually $\alpha = 2$):
flavour information available at each step of the clustering procedure

Also beam distance problematic:
a soft flavoured parton can be identified as a protojet and removed from the list)

$$d_{iB(\bar{B})}^{(F)} = R_{ij}^2 \times \begin{cases} \left[\max\left(k_{T,i}^2, k_{T,B(\bar{B})}^2\right)\right]^{\alpha} \left[\min\left(k_{T,i}^2, k_{T,B(\bar{B})}^2\right)\right]^{2-\alpha}, & \text{if } i \text{ is flavoured} \\ \min\left(k_{T,i}^2, k_{T,B(\bar{B})}^2\right), & \text{if } i \text{ is flavourless} \end{cases}$$

$$k_{T,B}(y) = \sum_i k_{T,i}\left(\Theta(y_i - y) + \Theta(y - y_i)e^{y_i - y}\right) \qquad k_{T,\bar{B}}(y) = \sum_i k_{T,i}\left(\Theta(y - y_i) + \Theta(y_i - y)e^{y - y_i}\right)$$

Flavour aware jet algorithms: flavour anti-k_T

[Czakon, Mitov, Poncelet, 2022]

Standard anti-k_T algorithm

$$d_{ij} = \min\left(k_{T,i}^{-2}, k_{T,j}^{-2}\right) R_{ij}^2, \quad d_{iB} = k_{T,i}^{-2}$$

does not vanish in the double soft limit

Flavour anti-k_T algorithm

$$d_{ij}^{(F)} = d_{ij} \times \begin{cases} \mathcal{S}_{ij}, & \text{if both } i \text{ and } j \text{ have non-zero flavour of opposite sign} \\ 1, & \text{otherwise} \end{cases}$$

$$\mathcal{S}_{ij} = 1 - \theta(1-\kappa)\cos\left(\frac{\pi}{2}\kappa\right), \qquad \kappa = \frac{1}{a}\frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,max}^2}$$

$$\longrightarrow \quad \mathcal{S}_{ij} \sim E^4 \implies d_{ij}^{(F)} \sim E^2$$

the suppression factor overcompensates the divergent behavior of d_{ij} in the double soft limit

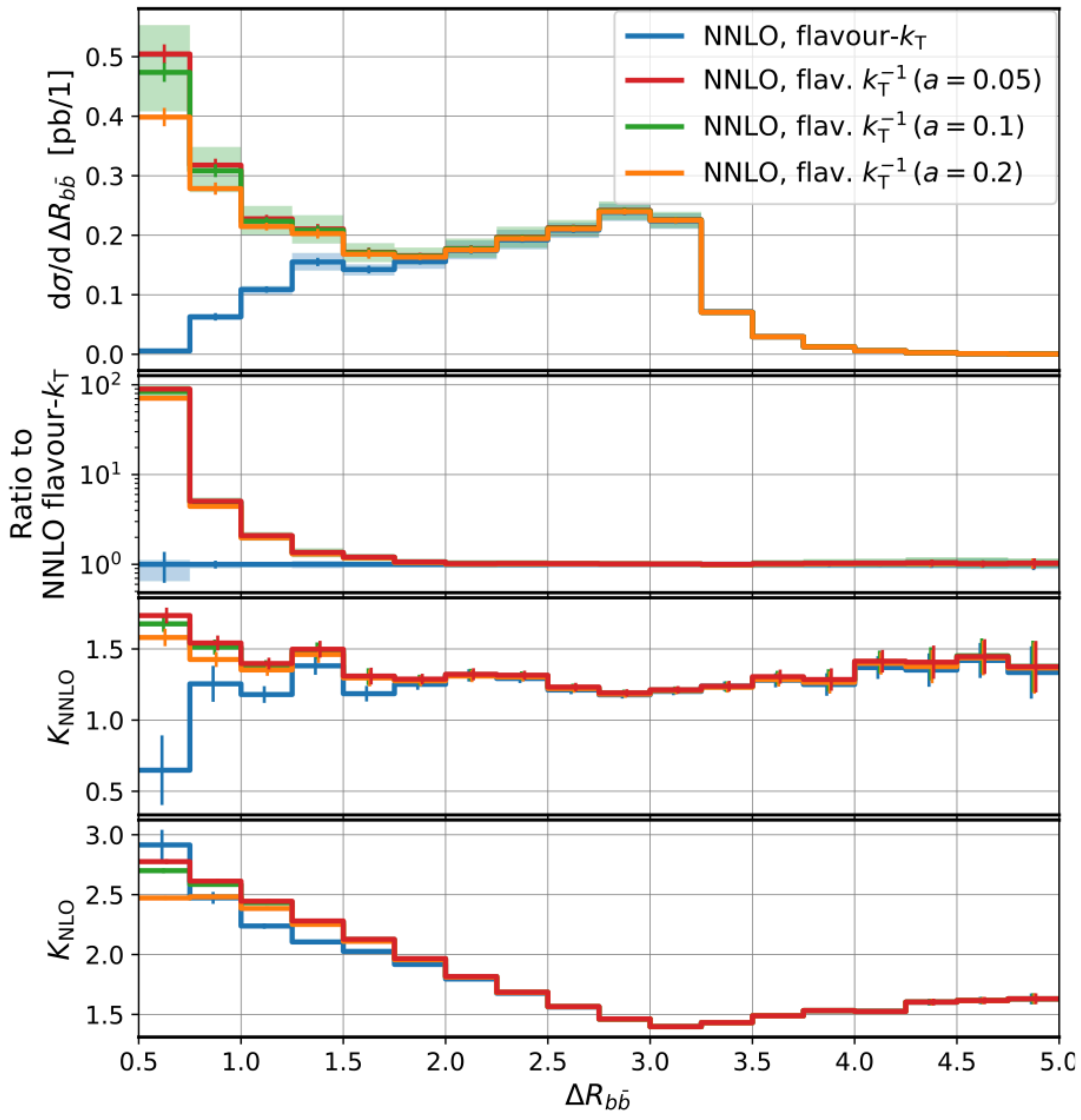
$Wb\bar{b}$ @ NNLO with massless b quarks

[Hartanto, Poncelet, Popescu, Zoia, 2022]

First computation for Wbb @ NNLO with massless b quarks recently performed

But, massless calculations are subject to ambiguities related to flavor tagging

Jet algorithm	σ_{NNLO} [fb]	K_{NNLO}
flavour-k_{T}	$445(5)^{+6.7\%}_{-7.0\%}$	1.23
flavour anti-k_{T} $(a = 0.05)$	$690(7)^{+10.9\%}_{-9.7\%}$	1.38
flavour anti-k_{T} $(a = 0.1)$	$677(7)^{+10.4\%}_{-9.4\%}$	1.36
flavour anti-k_{T} $(a = 0.2)$	$647(7)^{+9.5\%}_{-8.9\%}$	1.33

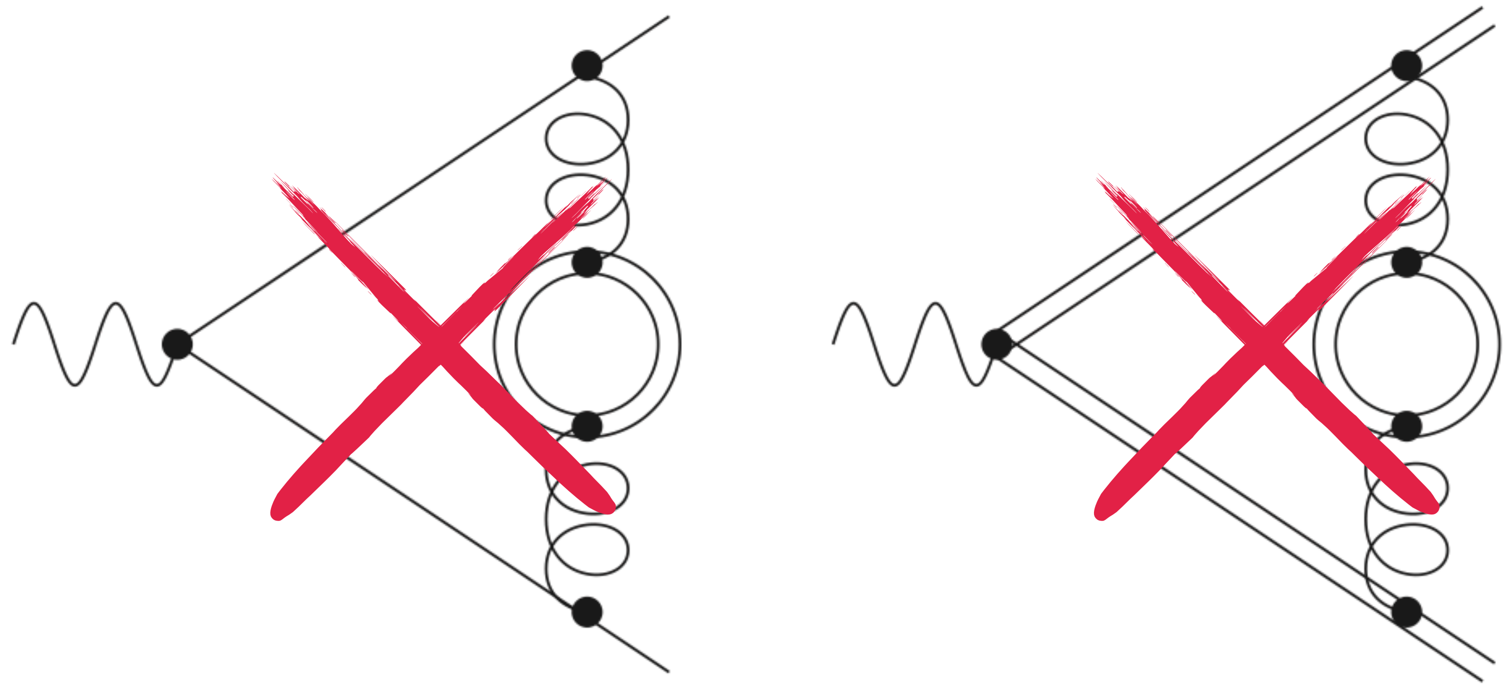


Massification procedure in a nutshell

[Mitov, Moch, 2007]

Caveat: starting from NNLO, heavy quark loop insertions **break** this simple "collinear" factorisation picture

We estimate that they have a negligible impact by inspecting the tree-level emission process of four tops and by removing heavy quark loop diagrams from the real-virtual contribution



Amplitude factorisation in QCD with a **massive** parton of mass $m^2 \ll Q^2$

$$|\mathcal{M}^{[p],(m)}> = \mathcal{J}^{[p]}\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right) \times \mathcal{S}^{[p]}\left(\{k_i\}\frac{Q^2}{\mu^2}, \alpha_S(\mu^2), \epsilon\right) \times |\mathcal{H}^{[p]}> + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

$$\mathcal{J}^{[p]}\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right) = \prod_i \mathcal{J}^i\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right) = \prod_i \left(\mathcal{F}^i\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}\alpha_S(\mu^2), \epsilon\right)\right)^{1/2}$$

space-like massive form factor

Soft approximation

In the limit in which the incoming $q\bar{q}'$ pair emits a soft W, the multi-loop QCD amplitude factorises as

$$|\mathcal{M}^{[p,k]}_{q\bar{q}'\to t\bar{t}W}> \simeq \frac{g}{\sqrt{2}}\left(\frac{p_2\cdot\varepsilon^*(k)}{p_2\cdot k}-\frac{p_1\cdot\varepsilon^*(k)}{p_1\cdot k}\right)\times|\mathcal{M}^{[p]}_{q_L\bar{q}'_R\to t\bar{t}}>$$

Eikonal factor
(analogous to soft photon/gluon)

"reduced" polarised $t\bar{t}$ amplitude

Remarks

- the soft W emission **selects a particular helicity configuration**
- the required NNLO QCD $q\bar{q}' \to t\bar{t}$ amplitude is **available** [Bärnreuther, Czakon, Fiedler, 2013] [Chen, Czakon, Poncelet, 2017] [Mandal, Mastrolia, Ronca, Bobadilla Torres, 2022]
- the use of the formula for a generic phase point required a **momentum mapping**:
  we adopt a recoil scheme in which the momentum of the W is absorbed by the top quark pair preserving the invariant mass of the event

Soft approximation

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Eikonal factor
(analogous to soft photon/gluon)

"reduced" polarised $t\bar{t}$ amplitude

Remarks

- We apply the approximation for estimating the hard-virtual coefficient

$$H^{(n)} = \frac{2\Re < \mathcal{M}^{(n)}_{\text{fin}}|\mathcal{M}^{(0)}>}{|\mathcal{M}^{(0)}|^2}$$

both on numerator and denominator: in this way we are effectively reweighing by the exact LO result!

Comparison with HPPZ: fiducial cross sections

order	$\sigma^{4\mathrm{FS}}$ [fb]	$\sigma^{5\mathrm{FS}}_{a=0.05}$ [fb]	$\sigma^{5\mathrm{FS}}_{a=0.1}$ [fb]	$\sigma^{5\mathrm{FS}}_{a=0.2}$ [fb]
LO	$210.42(2)^{+21.4\%}_{-16.2\%}$	$262.52(10)^{+21.4\%}_{-16.1\%}$	$262.47(10)^{+21.4\%}_{-16.1\%}$	$261.71(10)^{+21.4\%}_{-16.1\%}$
NLO	$468.01(5)^{+17.8\%}_{-13.8\%}$	$500.9(8)^{+16.1\%}_{-12.8\%}$	$497.8(8)^{+16.0\%}_{-12.7\%}$	$486.3(8)^{+15.5\%}_{-12.5\%}$
NNLO	$649.9(1.6)^{+12.6\%}_{-11.0\%}$	$690(7)^{+10.9\%}_{-9.7\%}$	$677(7)^{+10.4\%}_{-9.4\%}$	$647(7)^{+9.5\%}_{-9.4\%}$

Remarks

- The parameter a of the flavour anti k_T algorithm plays a role similar to m_b in our massive calculation
- Uncertainty estimated by varying $a \in [0.05, 0.2]$ amounts to 7 % ; **smaller** uncertainty estimated by varying $m_b \in [4.2, 4.92]$, at the 2% level
- General **agreement within scale variations**, but the massive calculation performed in the 4FS **systematically below due to the different flavour scheme**

Quality of the approximations for $t\bar{t}W$

Observations

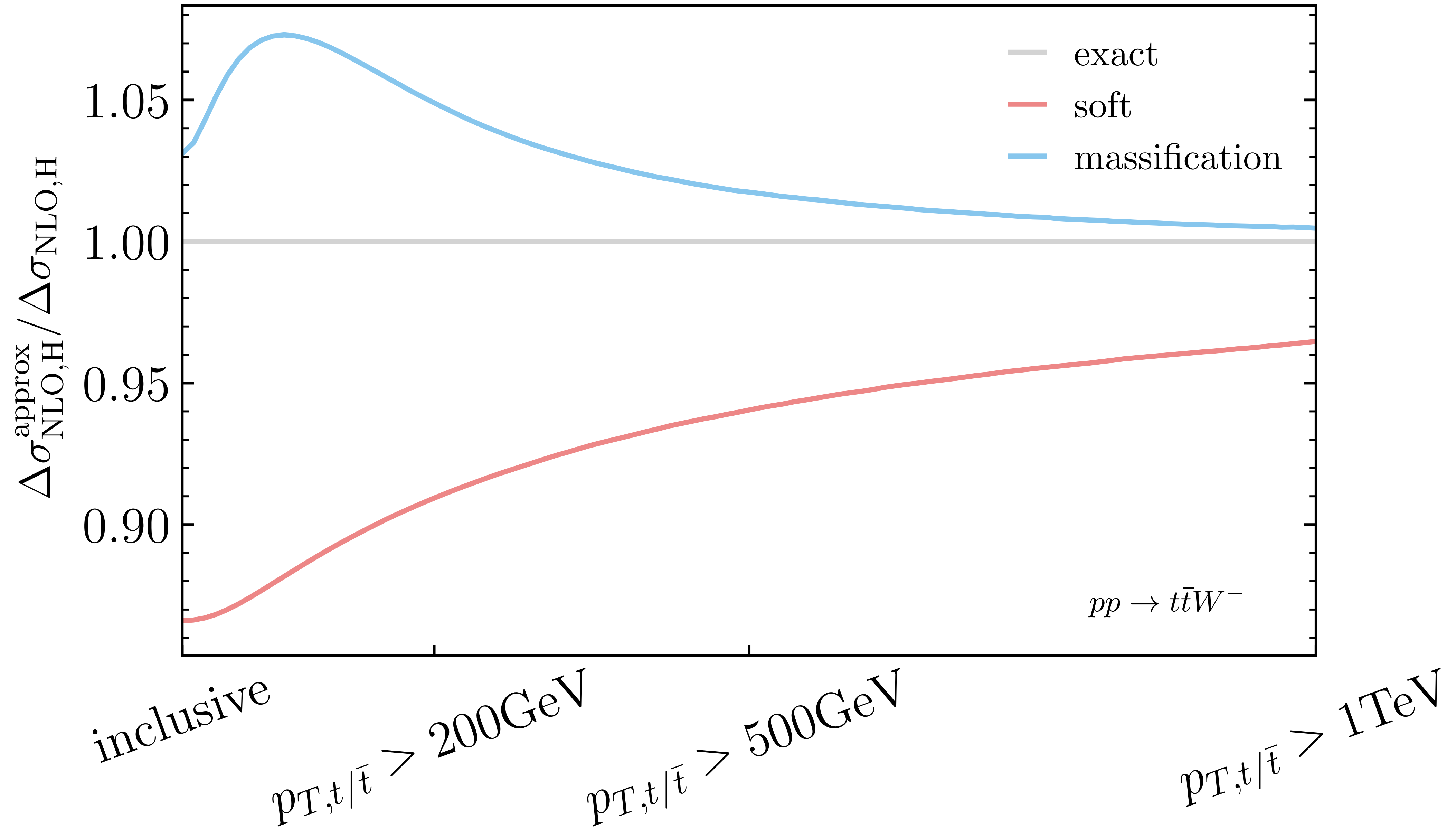
- Soft approximation first applied in $t\bar{t}H$ production: relatively large uncertainty but the corresponding hard virtual contribution represents a small fraction of the full NNLO QCD correction

  but the approximation works better for the $q\bar{q}$ channel!

- massification approach fully justified for $b\bar{b}W$

  does it still work for a very heavy quark as the top?

Analysis at NLO (comparison with the exact result!)



- **Both** approximations **provide a good estimate** of the exact one-loop contribution!
- Clear pattern: soft approximation tends to undershoot the exact result while massification tends to overshoot it
- Convergence in the asymptotic limit for high p_T top quarks where both approximation are expected to work

Quality of the approximations for $t\bar{t}W$

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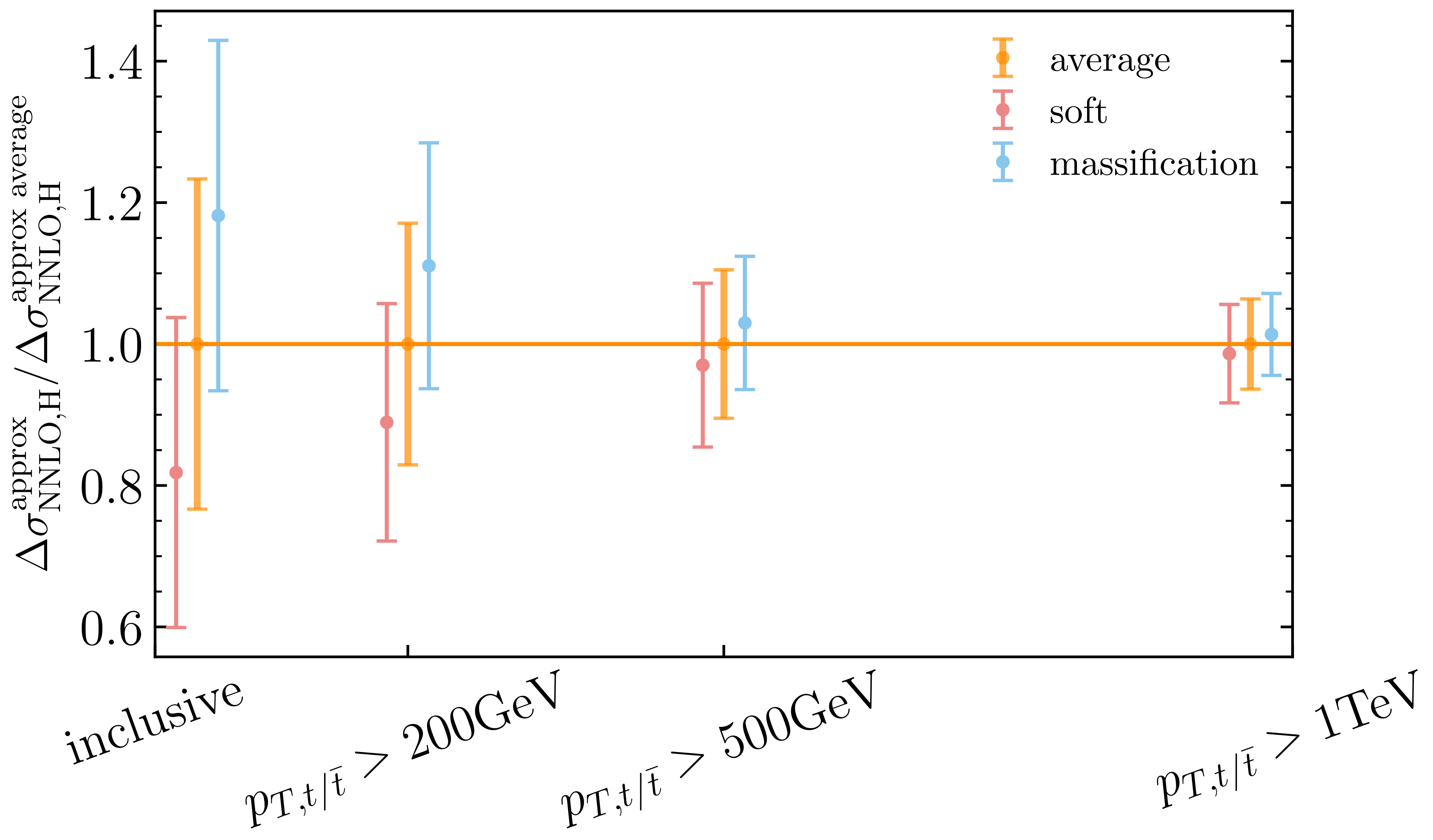
Analysis at NNLO



Best prediction obtained as average of the two with linear combination of uncertainties

Relatively large impact of two-loop virtual contribution: $\sim 7\,\%$ of NNLO cross section

FINAL UNCERTAINTY:

$\pm 1.8\,\%$ on σ_{NNLO}, $\pm 25\,\%$ on $\Delta\sigma_{\text{NNLO,H}}$

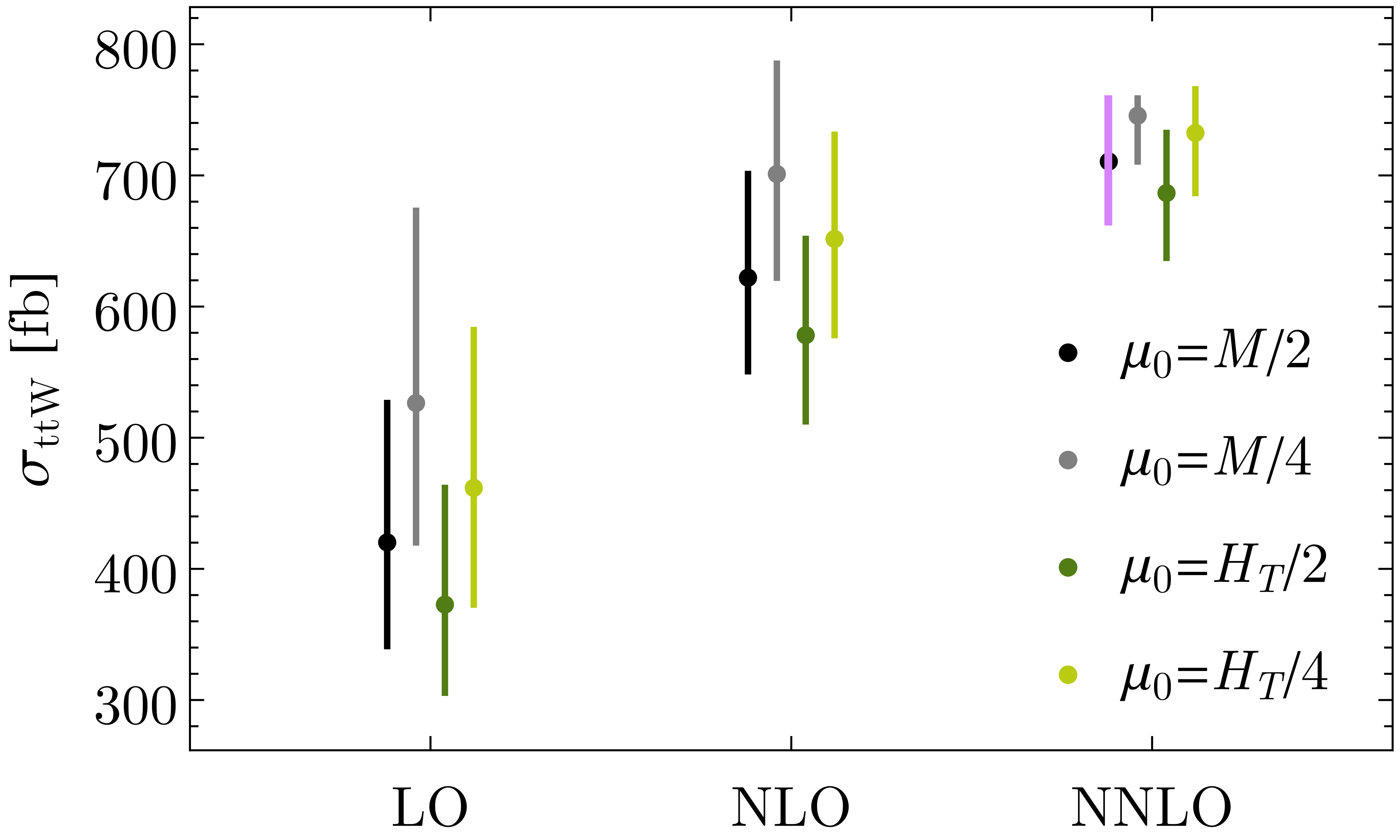
similar to what obtained in recent $2 \to 3$ in leading colour approximation

see e.g. [Abreu, De Laurentis, Ita, Klinkert, Page, Sotnikov 2023]

Scale variations and perturbative uncertainties

We estimate the **perturbative uncertainties** (due to missing higher order corrections) on the basis of

- scale variations
- behaviour of the perturbative series
- different scale choices: $M/2$, $M/4$, $H_T/2$, $H_T/4$
- breakdown of the corrections in different channels



First evidence of the convergence of the perturbative expansion starts at NNLO. Preference for smaller scale choices

Using the predictions with $\mu_0 = M/2$ and **symmetrising its scale uncertainty**, we obtain an interval that almost encompasses also the predictions obtained with $\mu_0 = M/4$ and $\mu_0 = H_T/4$.

$t\bar{t}W$: inclusive cross sections

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
$\mathrm{LO_{QCD}}$	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.2^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
$\mathrm{NLO_{QCD}}$	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
$\mathrm{NNLO_{QCD}}$	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\%$	$2.018^{+1.6\%}_{-1.2\%}$
$\mathrm{NNLO_{QCD}+NLO_{EW}}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\%$	$2.007^{+2.1\%}_{-2.1\%}$
ATLAS	$585^{+6.0\%+8.0\%}_{-5.8\%-7.5\%}$	$301^{+9.3\%+11.6\%}_{-9.0\%-10.3\%}$	$890^{+5.6\%+7.9\%}_{-5.6\%-7.9\%}$	$1.95^{+10.8\%+8.2\%}_{-9.2\%\ \ -6.7\%}$
CMS	$553^{+5.4\%+5.4\%}_{-5.4\%-5.4\%}$	$343^{+7.6\%+7.3\%}_{-7.6\%-7.3\%}$	$868^{+4.6\%+5.9\%}_{-4.6\%-5.9\%}$	$1.61^{+9.3\%+4.3\%}_{-9.3\%-3.1\%}$

Best prediction

Uncertainty associated to the approximation of the 2-loop virtual amplitude

Impact of radiative corrections

- Large positive NLO QCD corrections: $+50\,\%$
- Moderate positive NNLO QCD corrections: $+14 - 15\,\%$
- Relatively sizeable positive corrections from all LO and NLO contributions at $O(\alpha^3)$, $O(\alpha_S^2\alpha^2)$, $O(\alpha\alpha^3)$, $O(\alpha^4)$: $+5\,\%$
- The ratio $\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$ is rather stable and only slightly decreases increasing the perturbative order

$t\bar{t}W$: inclusive cross sections

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
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Best prediction

Uncertainty associated to the approximation of the 2-loop virtual amplitude

Other uncertainties

- PDF uncertainties: $\pm 1.8\ \%$ ($\pm 1.8\ \%$ ratio) computed with new MATRIX+PINEAPPL implementation

[S. Devoto, T. Jezo, S. Kallweit and C. Schwan in preparation]

- α_s uncertainties (half the difference between pdf sets for $\alpha_s(m_Z) = 0.118 \pm 0.001$) $\pm 1.8\ \%$ (negligible for ratio)
- Systematics of the q_T-subtraction method ($r_{\mathrm{cut}} \to 0$ extrapolation) are negligible

Application of soft approximation: $t\bar{t}H$

[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, Savoini, 2022]

In the case of soft H emission, we have a similar factorisation formula (for soft scalars)

$$|\mathcal{M}^{[p,k]}_{t\bar{t}H}> \simeq F(\alpha_s(\mu)R); m_t/\mu_R) \times J(k) \times |\mathcal{M}^{[p]}_{t\bar{t}}>$$

Normalisation correction factor beyond LO factorisation Calculable in perturbation theory

Eikonal factor

Soft H approximation

$$|\mathscr{M}_{t\bar{t}H}^{[p,k]}> \simeq F(\alpha_s(\mu)R); m_t/\mu_R) \times J(k) \times |\mathscr{M}_{t\bar{t}}^{[p]}>$$

$$J(k) = \sum_i \frac{m_t}{v}\frac{m_t}{p_i \cdot k}$$

The perturbative function $F\left(\alpha_S(\mu_R); m_t/\mu_R\right)$ can be extracted from the soft limit of the scalar form factor of the heavy quark

[Bernreuther et al, 2005] [Blümlein et al, 2017]

$$F(\alpha_s(\mu)R); m_t/\mu_R) = 1 + \frac{\alpha_S}{2\pi}(-3C_F)$$
$$+\left(\frac{\alpha_S}{2\pi}\right)^2 \left(\frac{33}{4}C_F^2 - \frac{185}{12}C_F C_A + \frac{13}{6}C)F(n_l+1) - 6C_F\beta_0 \ln\frac{\mu_R^2}{m_t^2}\right) + \mathcal{O}\left(\alpha_S^3\right)$$

Alternatively, it can be derived by using Higgs low-energy theorems

see e.g. [Kniehl, Spira, 1995]

$t\bar{t}H$: quality of the soft H approximation

At LO, the soft H approximation overestimates the exact result by

- gg channel: a factor of 2.3 at $\sqrt{s} = 13$ TeV and a factor of 2 at $\sqrt{s} = 100$ TeV
- $q\bar{q}$ channel: a factor of 1.11 at $\sqrt{s} = 13$ TeV and a factor of 1.06 at $\sqrt{s} = 100$ TeV

	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\text{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\text{NLO,H}}\vert_{\text{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\text{NNLO,H}}\vert_{\text{soft}}$	$-2.980(3)$	2.622(0)	$-239.4(4)$	65.45(1)

At NLO, the approximation performs better than at LO because of the LO re-weighting

$t\bar{t}H$: quality of the soft H approximation & uncertainties

Uncertainties estimates by

- varying the momentum mapping used to absorb the recoil of the H boson
- varying the infrared μ_{IR} subtraction scale at which the $H^{(2)}$ is evaluated from the central value $m_{t\bar{t}H}$ to $m_{t\bar{t}H}/2$ and $2m_{t\bar{t}H}$
  When evaluating $H^{(2)}$ at a subtraction scale different from the central value, we added the contribution stemming from the running from the μ_{IR} to $m_{t\bar{t}H}$ using the exact matrix elements

Uncertainties estimated by multiplying by a **tolerance factor of 3** the deviations found at NLO:
30% for the gg channel and 5% for the $q\bar{q}$ channel.
This encompasses the uncertainties associated to the variations above

Finally uncertainties obtained by combining linearly the gg and the $q\bar{q}$ channel

0.6% on σ_{NNLO}