MSHT PDF updates.

Robert Thorne

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University College London

With Tom Cridge and Lucian Harland-Lang

MSHT PDFs - a variety of updates.

- A first set of PDF at approximate N^3LO, i.e. aN^3LO - brief reminder.
- aN^3LO (and LO) PDFs with QED corrections and the photon PDF.
- Comparison of global fits using either inclusive jet or alternatively dijet LHC data.
- A study of the best-fit $\alpha_S(M_Z^2)$ at aN^3LO, and interplay of jet/dijet data on the strong coupling.

Also dedicated studies on (different aspects of) methodologies and relationship to uncertainties by Harland-Lang, Reader – tomorrow morning.

aN^{3}LO PDFs (J. McGowan, T. Cridge, L. Harland-Lang, RT)

Leading source of uncertainties is from from Missing Higher Orders in perturbation theory. Numerous sources of this for e.g structure functions, i.e. splitting functions

$$\boldsymbol{P}(x,\alpha_s) = \alpha_s \boldsymbol{P}^{(0)}(x) + \alpha_s^2 \boldsymbol{P}^{(1)}(x) + \alpha_s^3 \boldsymbol{P}^{(2)}(x) + \alpha_s^4 \boldsymbol{P}^{(3)}(x) + \dots \,,$$

but also heavy flavour transition matrix elements and cross-sections (coefficient functions)

$$F_2(x,Q^2) = \sum\nolimits_{\alpha\in\{H,q,g\}} \Big(C_{q,\alpha}^{\mathrm{VF},\; n_f+1} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2) \\ + C_{H,\alpha}^{\mathrm{VF},\; n_f+1} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2) \Big)\,,$$

Current knowledge is up to NNLO, with full higher orders unknown.

Already lots of progress in calculating features at N^{3}LO [2-13]. Since PDFs appeared also [14-18]

N^3LO - What do we know?

Zero-mass structure function N^3LO coefficient functions are known [2].

Some information from leading terms in the small x and large x regime [3-12], e.g.

$$\boldsymbol{P}_{qg}^{(3)}(x) \rightarrow \frac{C_A^3}{3\pi^4}\left(\frac{82}{81} + 2\zeta_3\right)\frac{1}{2}\frac{\ln^2 1/x}{x} + \rho_{qg}\,\frac{\ln 1/x}{x},$$

Some numerical constraints (Low-integer Mellin moments) [3-12], and intuition from lower orders and expectations from perturbation theory.

Splitting Functions at aN^3LO – N_m Mellin moments and small-x constraints can be used to define

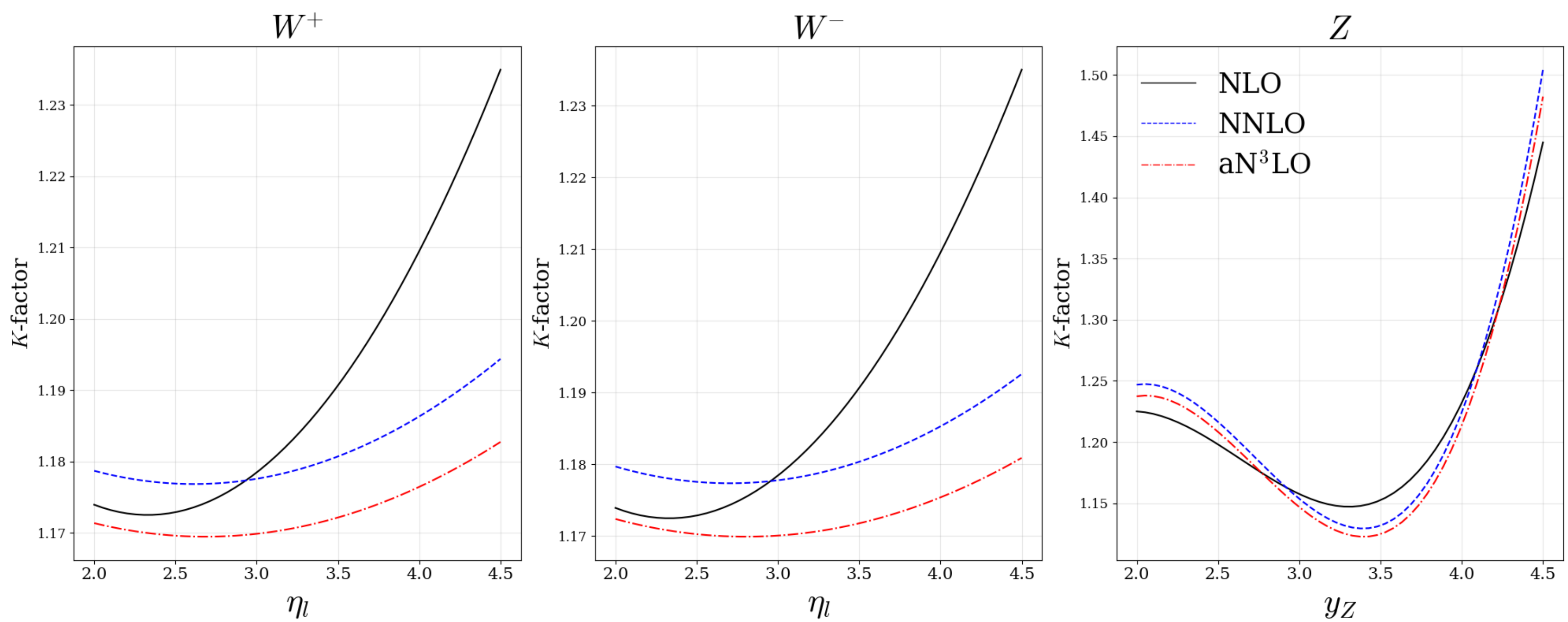
$$F(x) = \sum_{i=1}^{N_m} A_i f_i(x) + f_e(x).$$

Choose a set of relevant functions and solve for A_i.

Very little about many cross-sections (K-factors). Parameterise the N^3LO K-factor as a superposition of both NNLO and NLO K-factors.

$$K(y) = 1 + \frac{\alpha_s}{\pi} D(y) + \left(\frac{\alpha_s}{\pi}\right)^2 E(y) + \left(\frac{\alpha_s}{\pi}\right)^3 F(y) + \mathcal{O}(\alpha_s^4).$$

$$K^{\mathrm{N^3LO/LO}} = K^{\mathrm{NNLO/LO}} \left(1 + \alpha_s^3 \hat{a}_1 \frac{\mathcal{N}^2}{\pi} D + \alpha_s^3 \hat{a}_2 \frac{\mathcal{N}}{\pi^2} E\right).$$



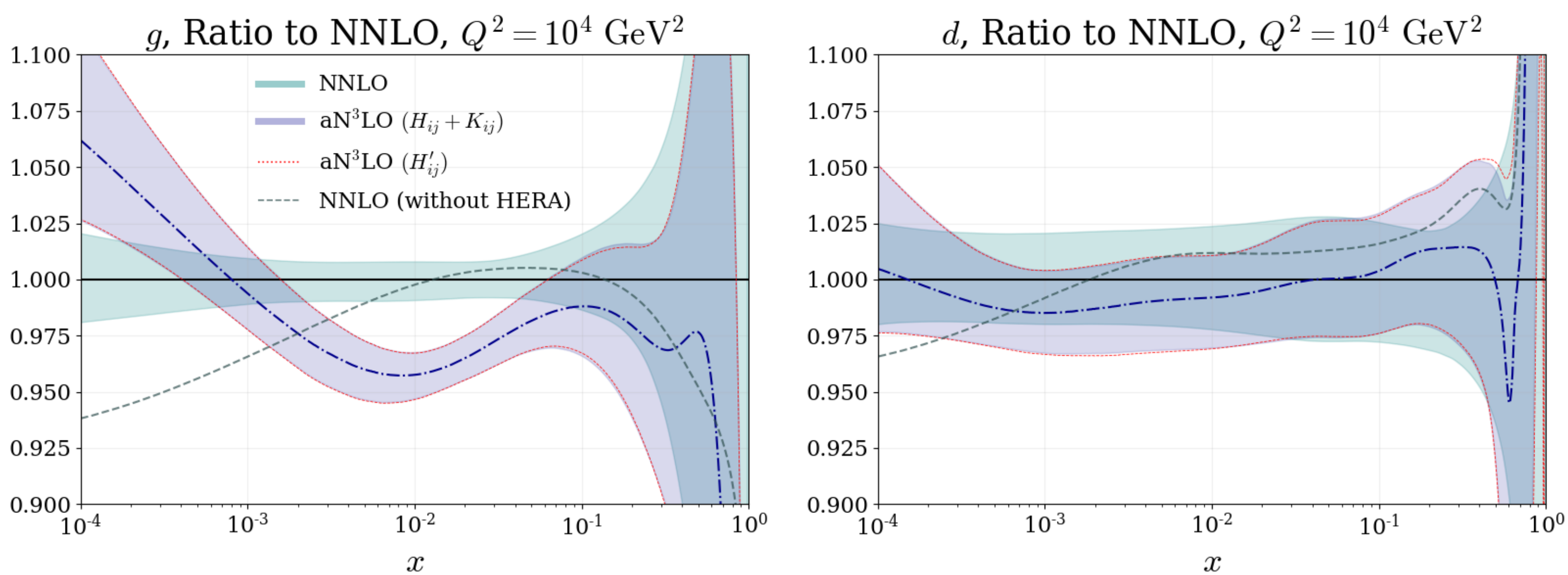
Calculations of N^3LO Drell Yan production now exist [19-21].

Global Fit Quality at aN^{3}LO

The overall χ^2 follows the general trend one may expect from perturbation theory.

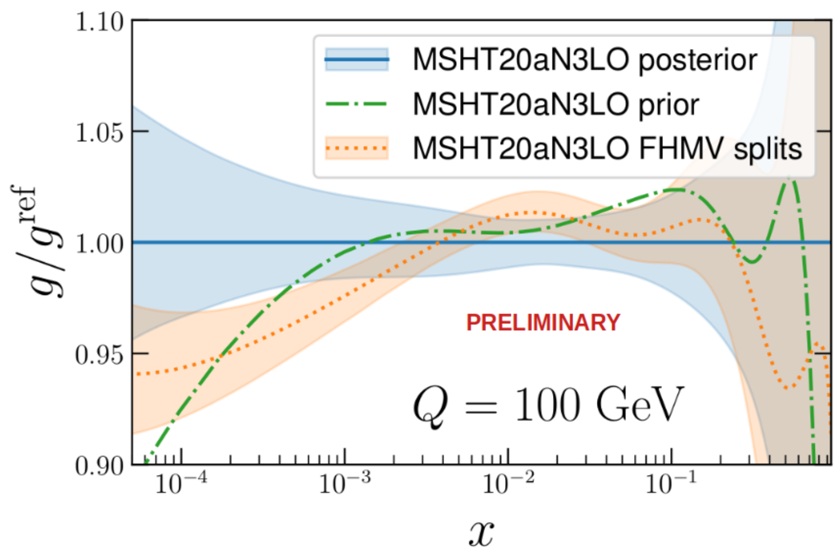
	LO	NLO	NNLO	aN^{3}LO
$\chi^2_{N_{pts}}$	2.57	1.33	1.17	1.14

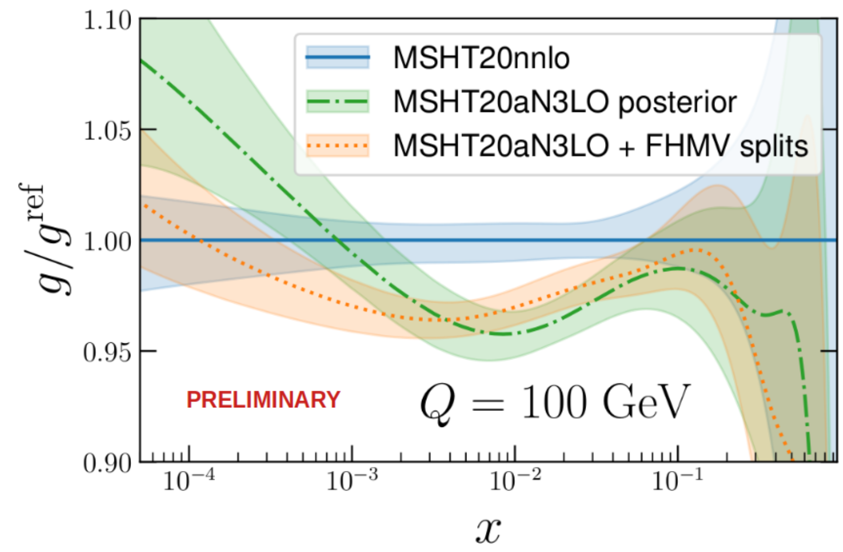
Evidence that including aN^{3}LO has reduced tensions between small and large-x.



The gluon is enhanced at small-x due to the large logarithms present at higher orders. Light quarks enhanced slightly at high x.

Effect of MSHT fits with improved [14-16] splitting functions.





Note - no uncertainties used for improved splitting functions - only central value. Now almost exclusively at small x.

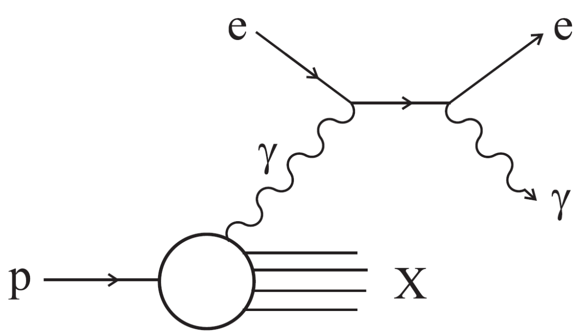
$\chi^2 \sim 50$ worse than before (over 100 lower than NNLO) very largely at small x - would improve at some level once uncertainty accounted for.

Use of (central value of) improved aN^{3}LO splitting functions changes aN^{3}LO gluon a little compared to published MSHT PDFs, raising 1.5% near $x = 0.01$.

Main features of aN^{3}LO comparison to NNLO remain the same.

aN^{3}LO and LO PDFs with QED corrections 2312.07665

At the level of accuracy we are now approaching it is important to account for electroweak corrections. For a consistent treatment we need PDFSs which incorporate QED into the evolution, i.e. the inclusion of the photon PDF $\gamma(x, Q^2)$ [22-24].



$$\frac{\partial \gamma(x,Q^2)}{\partial \log Q^2} = \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\left(P_{\gamma\gamma}\otimes\gamma + \sum_1 e_i^2 P_{\gamma q}\otimes q_i\right)$$

Put on truly quantitative footing in LUXqed photon PDF [25]. Relates photon to structure functions, and uncertainty of at most a few percent.

$$x f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)}\int_x^1 \frac{dz}{z}\left\{\int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2}\alpha^2(Q^2)\left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2}\right)F_2(x/z,Q^2) - z^2 F_L\left(\frac{x}{z},Q^2\right)\right] - \alpha^2(\mu^2) z^2 F_2\left(\frac{x}{z},\mu^2\right)\right\}, \quad (6)$$

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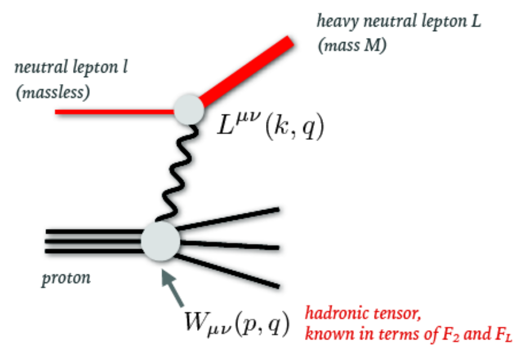
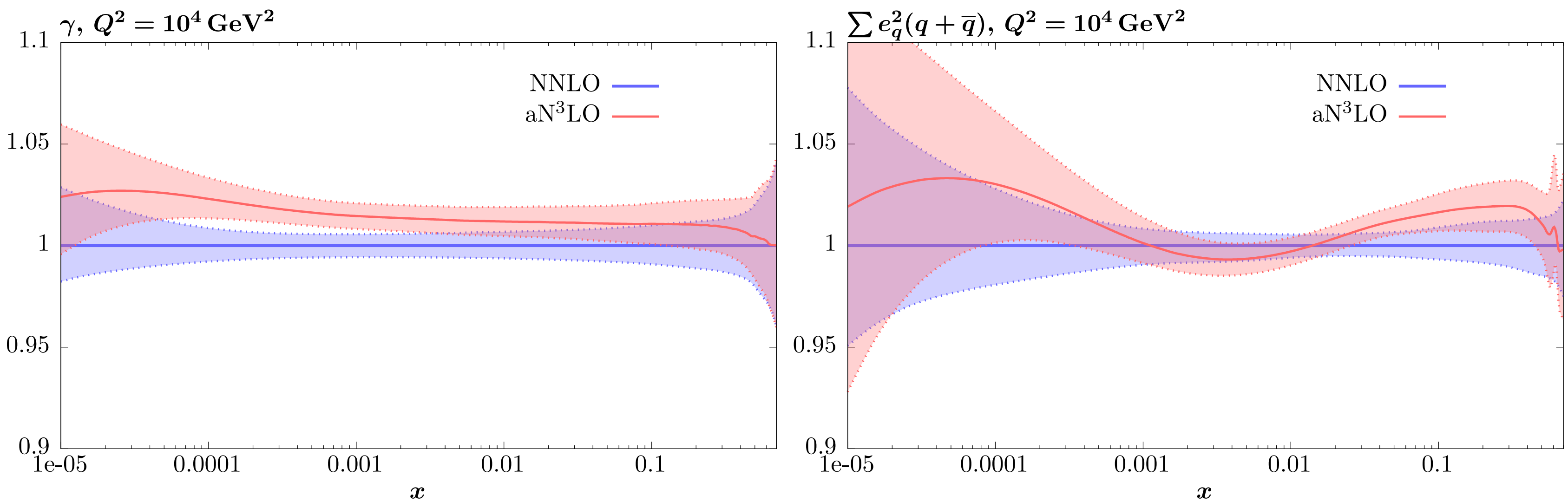


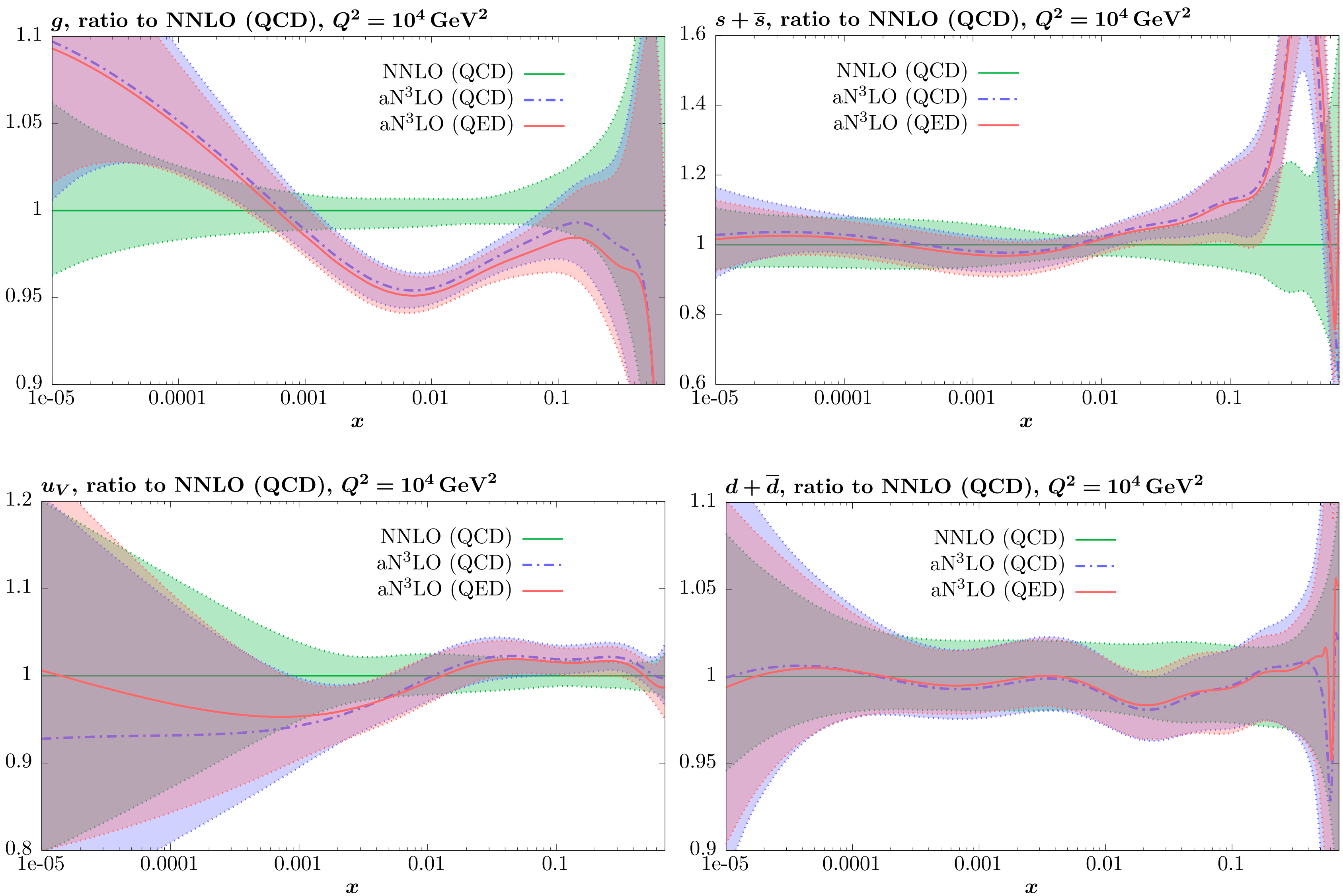
We saw slight deterioration in QED corrected fits at lower orders (photon takes PDF momentum). Now largely eliminated at aN^{3}LO.

- Global fit quality:

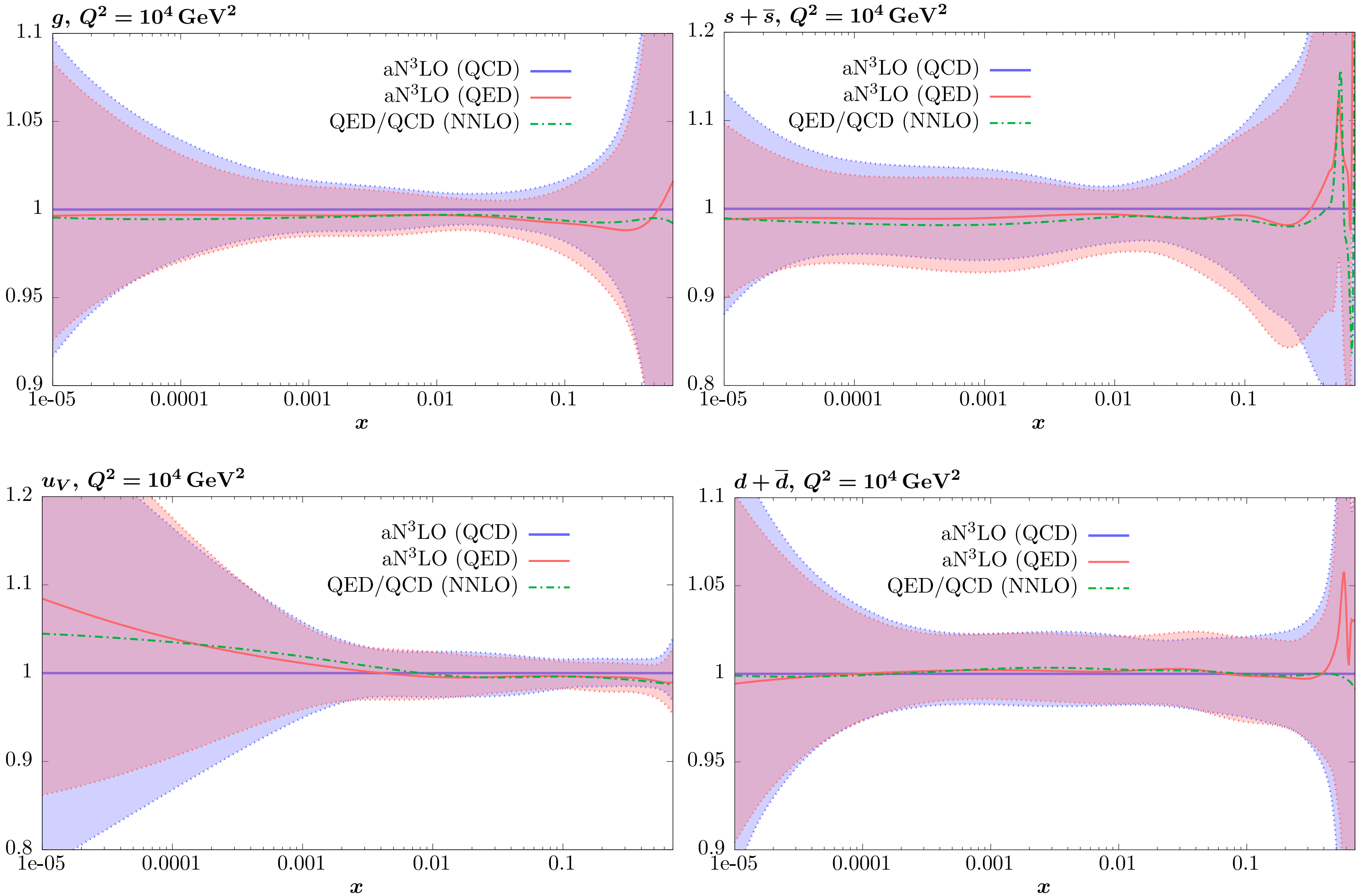
	$\chi^2/N_{\rm pt}$ aN^{3}LO (QED)	$\Delta\chi^2_{\rm aN^3LO}$ QED-QCD	$\Delta\chi^2_{\rm NNLO}$ QED-QCD	$\Delta\chi^2_{\rm QCD,QED}$ aN^{3}LO-NNLO
Total	5323.6/4534	(+3.6)	(+17.3)	(-209.3, -223.1)

The photon PDF is a couple of percent bigger at high Q^2 at aN^{3}LO – simply due to increased quarks and structure function.



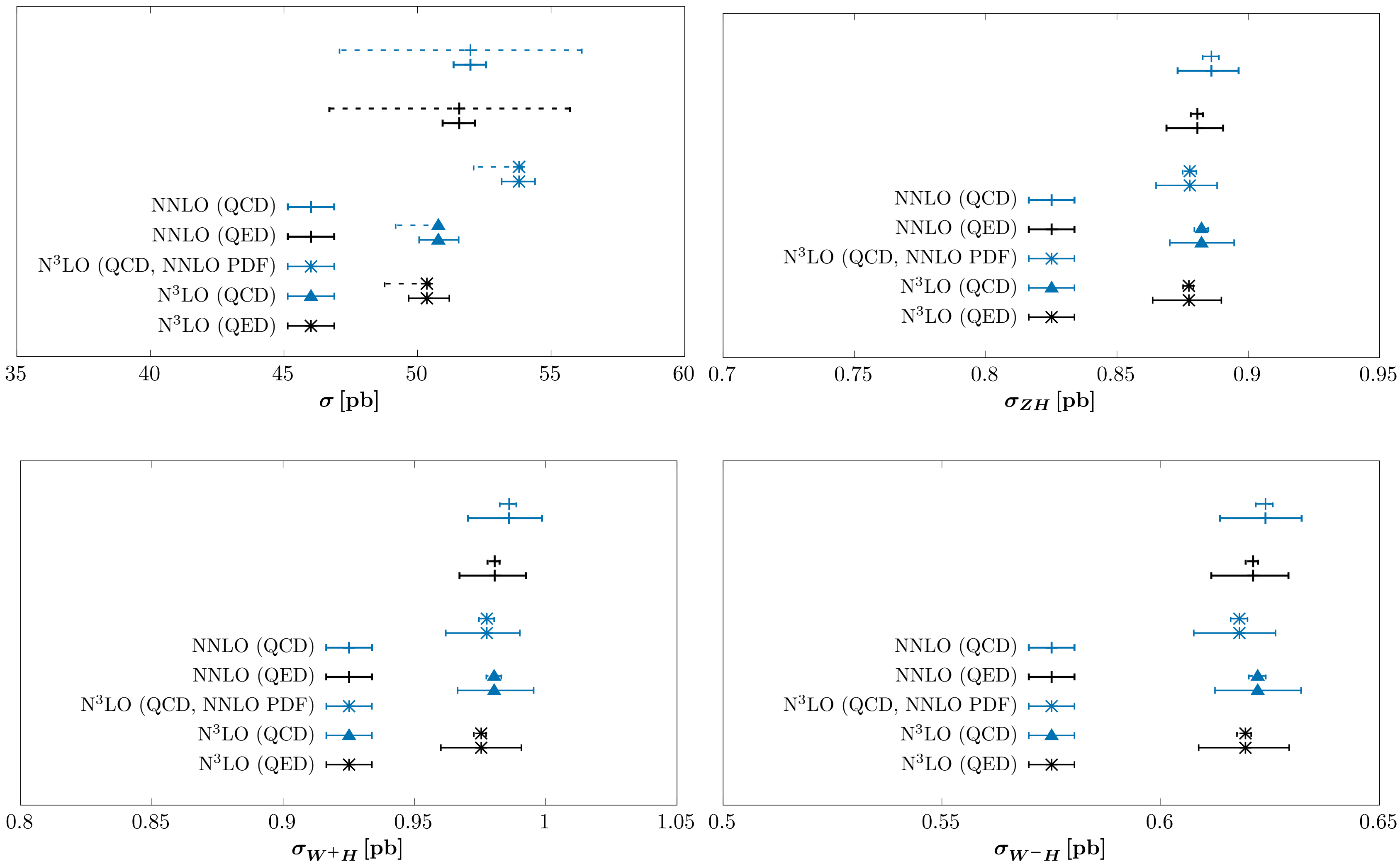


Change in PDFs due to QED much smaller than from NNLO $\rightarrow$ aN^{3}LO, and well within PDF uncertainties.



Relative change in quark/gluon PDFs similar at N^{3}LO to NNLO, i.e. slightly greater radiation of very high-x quarks and reduction in gluon due to photon momentum.

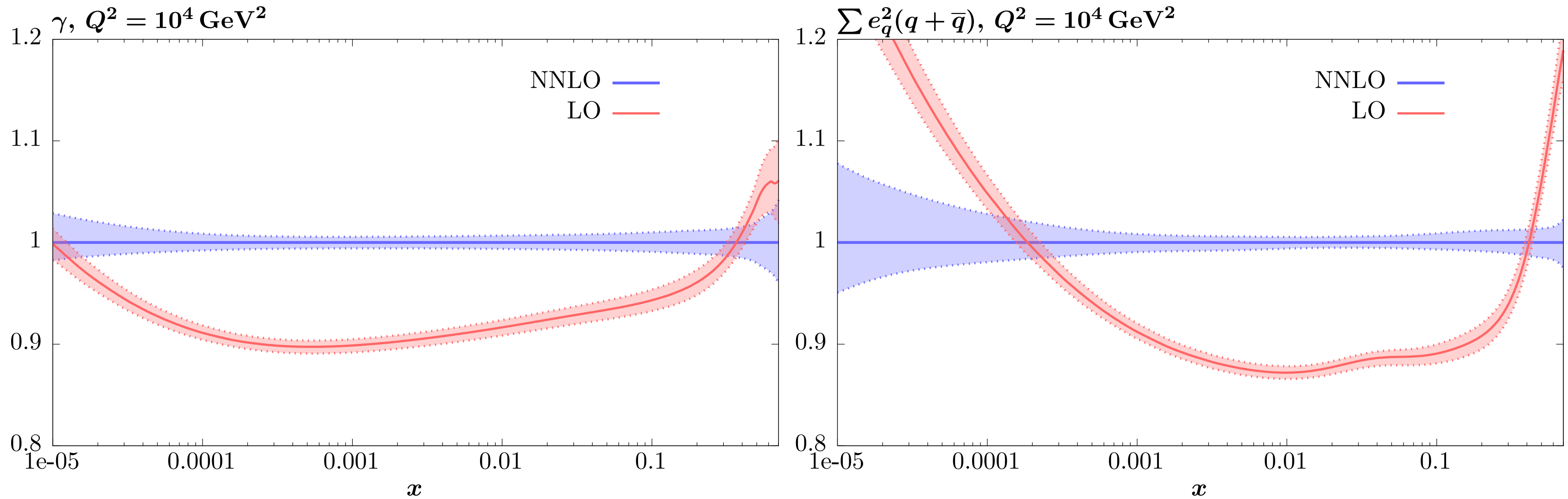
Benchmark cross-section comparisons.



Again, changes in cross sections due to QED similar to that at NNLO. Generally a slight reduction.

Photon PDF at LO.

Potentially useful in some MC generators (requested).



Considerably smaller than at higher orders. Due to reduced high-Q^2 structure function due to intrinsically smaller quark evolution at LO.

Other PDFs at LO change much less than uncertainties under addition of QED, and less systematically than at higher orders due to fit difficulties.

Jet, Dijet and Zp_T data at aN^{3}LO 2312.12505 - accepted

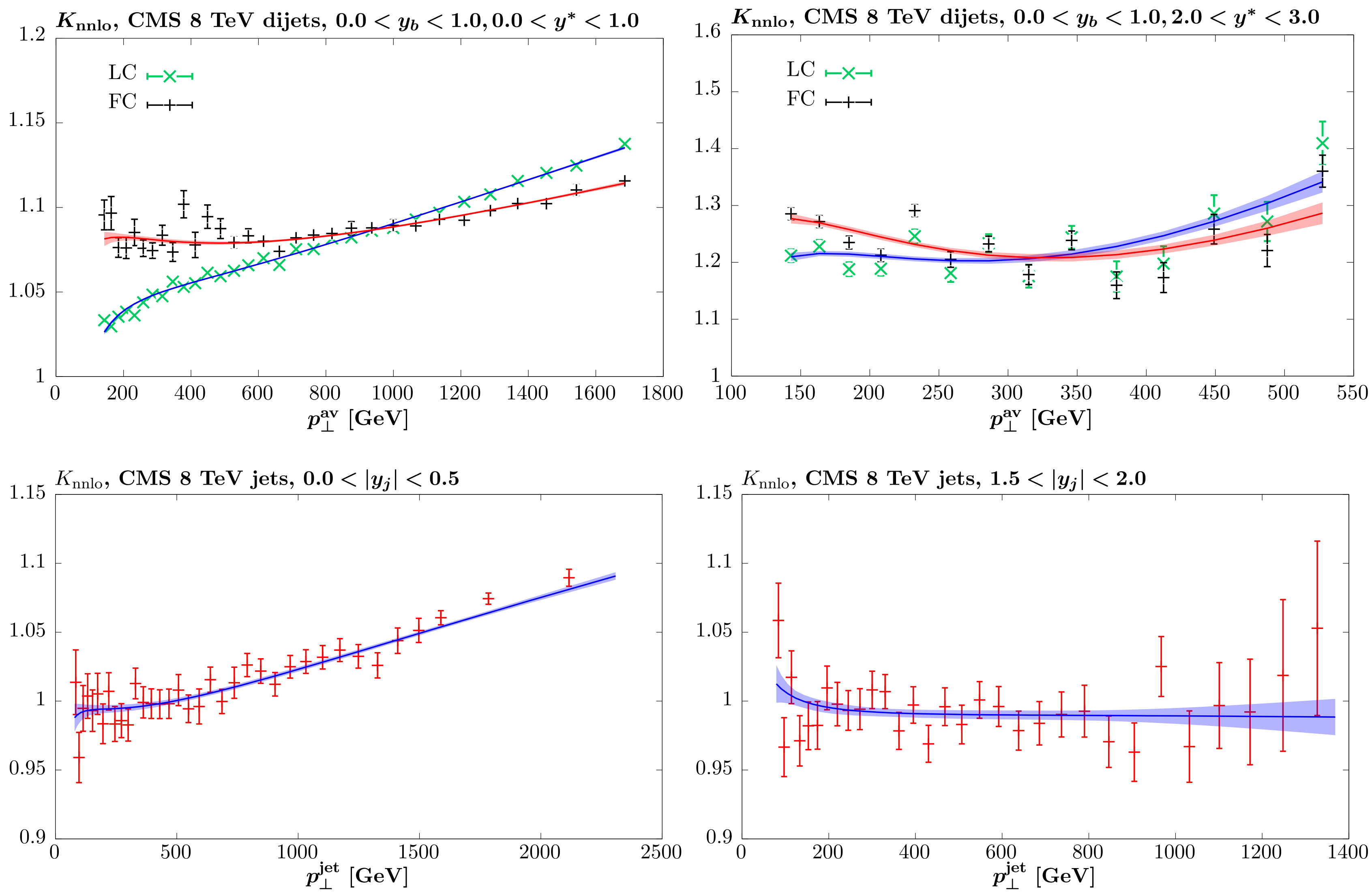
- Focussing on Run-I data (i.e. current PDF fits): $d^2\sigma/dp_\perp dy$
  - Inclusive jets: $0.0 < |y| < 2.5 - 3.0$
    - ★ CMS 2.76 TeV: 81 points — 5.43 pb^{-1} — $74 < p_\perp < 592$ GeV
    - ★ CMS 7 TeV: 158 points — 5.0 fb^{-1} — $74 < p_\perp < 2500$ GeV
    - ★ CMS 8 TeV: 174 points — 19.7 fb^{-1} — $60 < p_\perp < 1300$ GeV
    - ★ ATLAS 7 TeV: 140 points — 4.5 fb^{-1} — $100 < p_\perp < 2000$ GeV
    - ★ **ATLAS 8 TeV**: 171 points — 20.2 fb^{-1} — $70 < p_\perp < 2500$ GeV

  $\longrightarrow$ ***724*** points in total, v.s. ~4500 in global MSHT fit (inc.).

- Dijets:
  - ★ ATLAS 7 TeV: 90 points — 4.5 fb^{-1} — $d^2\sigma/dm_{jj}d|y_{\max}|$ $0.26 < m_{jj} < 5.04$ TeV
  - ★ CMS 7 TeV: 54 points — 5.0 fb^{-1} — $d^2\sigma/dm_{jj}d|y^*|$ $0.25 < m_{jj} < 4.48$ TeV
  - ★ CMS 8 TeV: 122 points — 19.7 fb^{-1} — $d^3\sigma/dp_{\perp,avg}dy_b dy^*$ $143 < p_{\perp,avg} < 1638$ GeV

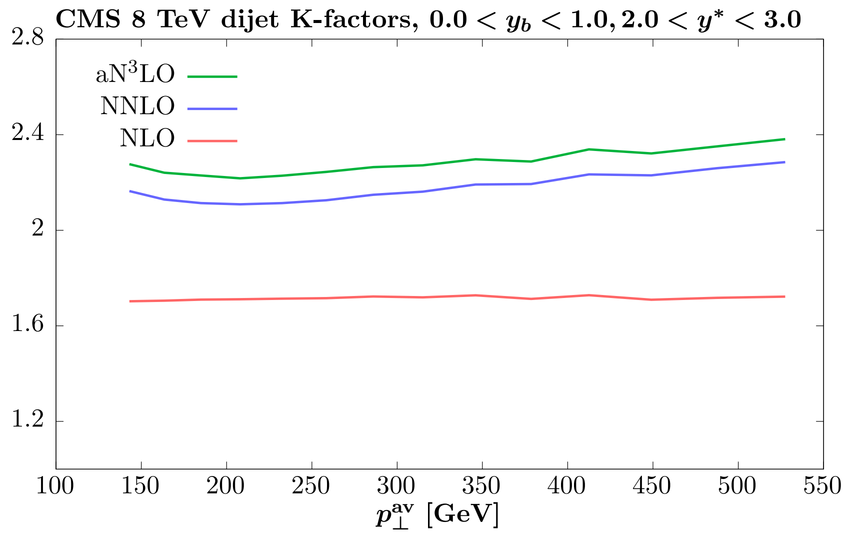
  $\longrightarrow$ **266** points in total, v.s. ~4000 in global MSHT fit (inc.).

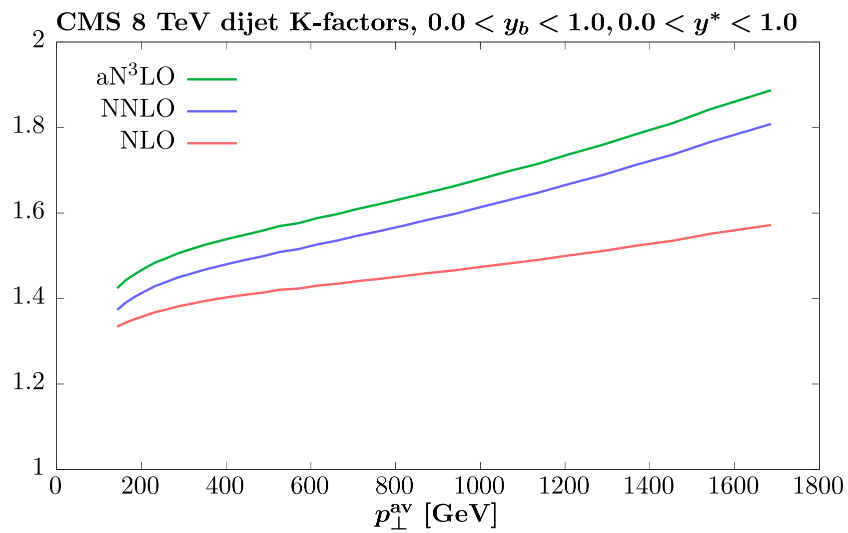
We convert K-factor points into smooth functions with uncertainties.



Investigate full colour where available. Clearly different from leading colour approx.

We find N^{3}LO K-factors by fitting nuisance parameters related to lower-order corrections.





Results show convergence in large perturbative corrections.

		NNLO			aN^3LO			aN^3LO (K_{nnlo})	
	N_{pts}	No jets/dijets	Jets	Dijets	No jets/dijets	Jets	Dijets	Jets	Dijets
ATLAS 7 TeV jets [39]	140	1.60	**1.54**	1.64	1.72	**1.46**	1.54	**1.56**	1.44
CMS 7 TeV jets [40]	158	1.39	**1.29**	1.54	1.51	**1.32**	1.34	**1.33**	1.10
ATLAS 8 TeV jets [44]	171	2.02	**1.96**	1.92	2.03	**1.90**	1.94	**1.93**	1.83
CMS 8 TeV jets [41]	174	1.80	**1.83**	1.85	1.86	**1.80**	1.74	**1.90**	2.06
Total (jets)	643	1.71	**1.67**	1.75	1.79	**1.63**	1.65	**1.69**	1.63
ATLAS 7 TeV dijets [27]	90	1.08	1.09	**1.05**	1.13	1.13	**1.12**	1.13	**1.12**
CMS 7 TeV dijets [28]	54	1.51	1.64	**1.44**	1.47	1.47	**1.40**	1.48	**1.42**
CMS 8 TeV dijets [29]	122	1.22	1.47	**1.22**	1.06	1.01	**0.86**	0.90	**0.98**
Total (dijets)	266	1.23	1.38	**1.21**	1.19	1.14	**1.06**	1.10	**1.12**
CMS 2.76 TeV jets [56]	81	**1.28**	**1.25**	**1.32**	**1.34**	**1.37**	**1.32**	**1.33**	**1.42**
ATLAS 8 TeV Z p_T [35]	104	**1.75**	**1.87**	**1.66**	**0.99**	**1.04**	**1.05**	**1.37**	**1.24**
Differential $t\bar{t}$ [57-60]	54	**1.23**	**1.10**	**1.26**	**1.11**	**1.06**	**1.09**	**1.06**	**1.17**
Total	-	**1.15**	**1.22**	**1.15**	**1.09**	**1.17**	**1.09**	**1.19**	**1.11**

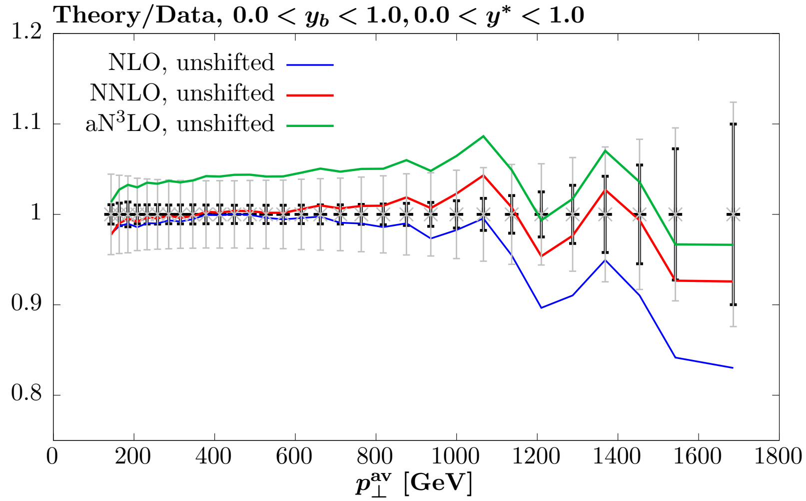
		NLO	
	N_{pts}	Jets	Dijets
ATLAS 7 TeV jets [39]	140	**1.60**	1.83
CMS 7 TeV jets [40]	158	**1.37**	1.81
ATLAS 8 TeV jets [44]	171	**2.25**	2.34
CMS 8 TeV jets [41]	174	**1.66**	1.92
Total (jets)	643	**1.73**	1.98
ATLAS 7 TeV dijets [27]	90	1.51	**1.12**
CMS 7 TeV dijets [28]	54	2.24	**1.70**
CMS 8 TeV dijets [29]	122	7.84	**5.27**
Total (dijets)	266	4.56	**3.14**
Total	-	**1.35**	**1.42**

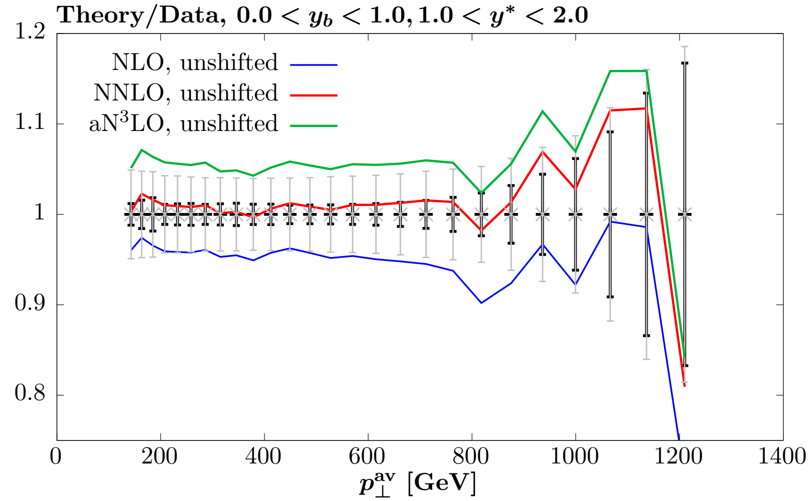
Dijet fit at NLO very poor.

Fit quality to dijet data at NNLO and aN^3LO shows an improvement from inclusive jet data.

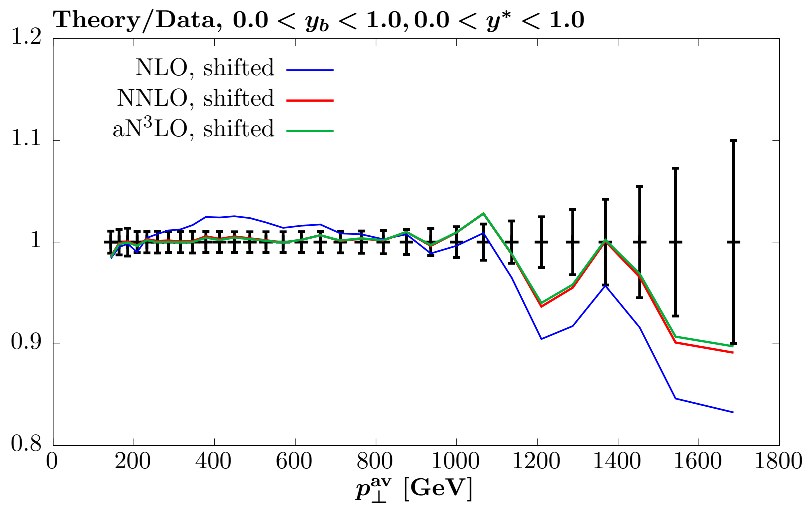
Dijet $\rightarrow$ much better fit to Z p_T data, worse fit to top data.

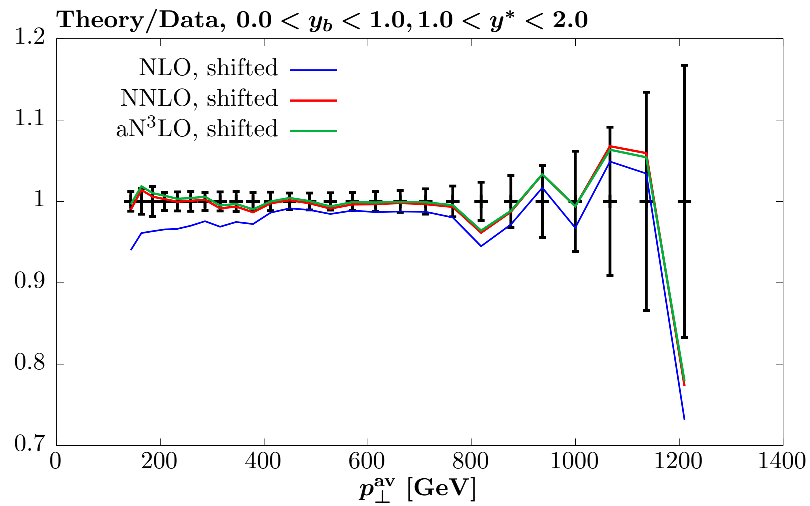
Difficult to appreciate fit quality by comparing theoretical predictions to experimental data without applying shifts corresponding to best fit of correlated systematic uncertainty parameters.



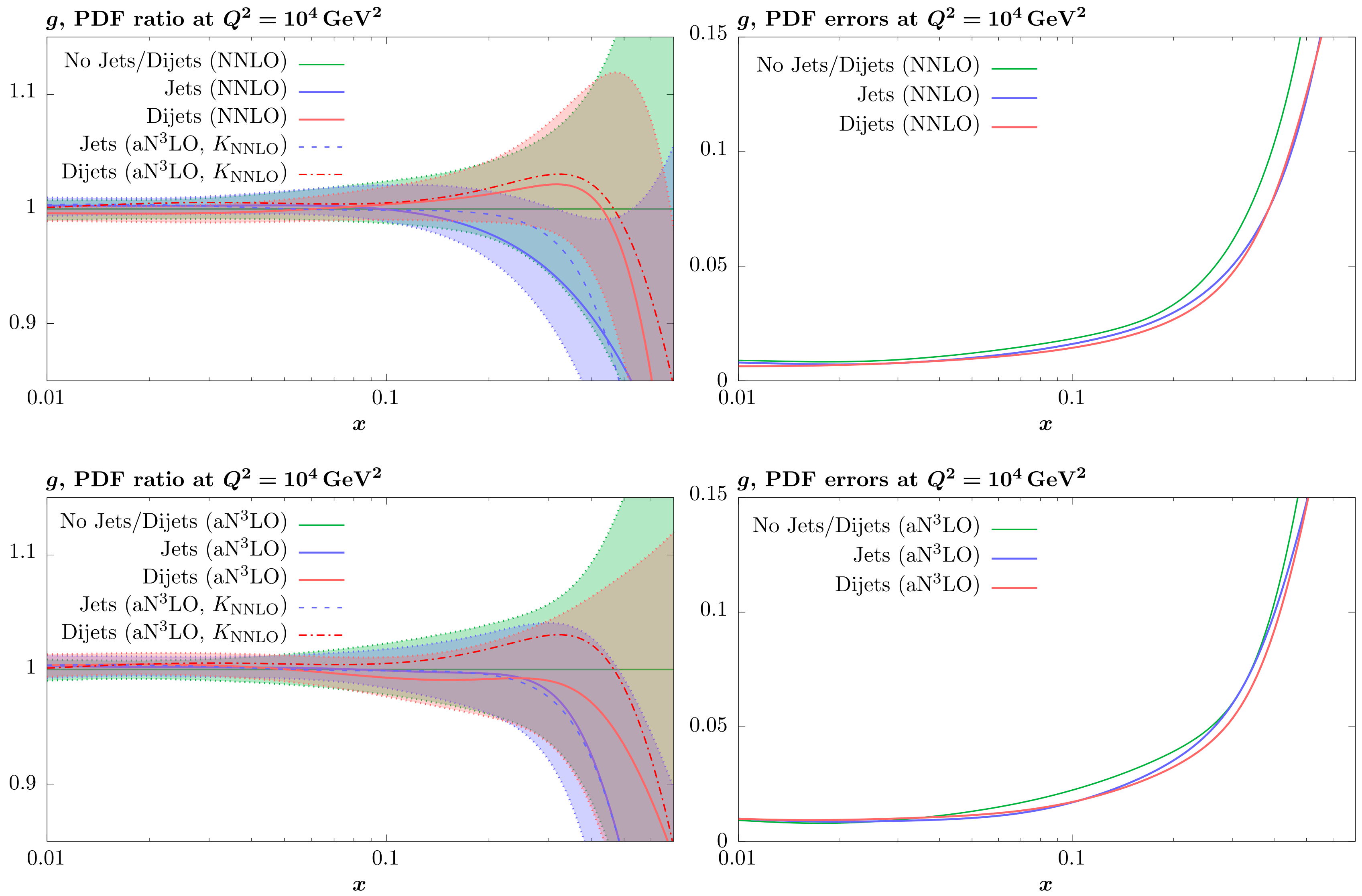


With shifts applied see that at NLO the shape as a function of p_T is incorrect.





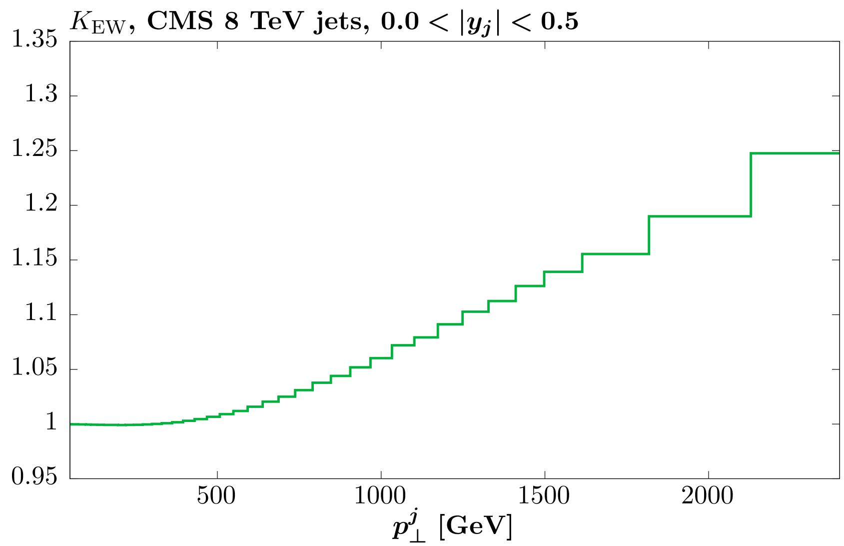
Slightly different pulls on gluon from jet and dijet data. Reduced a little at aN^{3}LO.

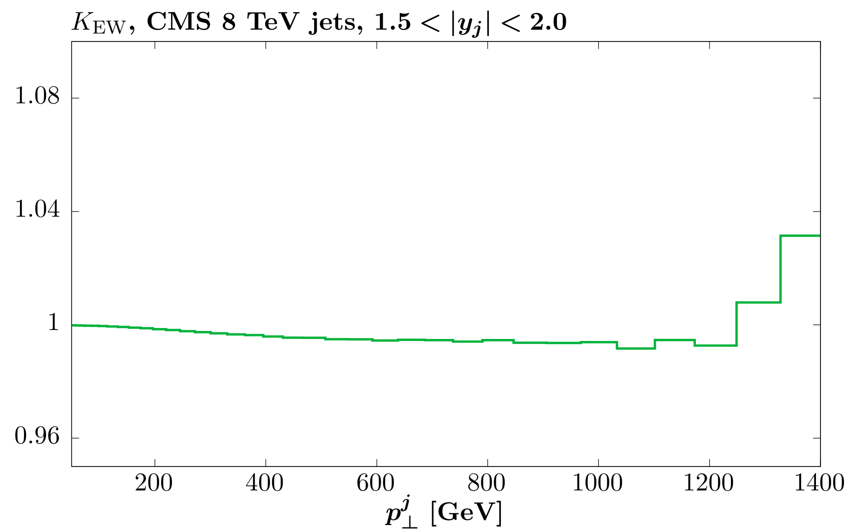


Little difference on uncertainty determination.

Electroweak corrections.

Very similar in form for jet and dijets, i.e. largest at highest p_T.





Improvement in fit quality far more clear with dijet than with inclusive jets.

	N_{pts}	NNLO Default	NNLO No EW.	aN^{3}LO Default	aN^{3}LO No EW
ATLAS 7 TeV jets [39]	140	1.54	1.48	1.46	1.45
CMS 7 TeV jets [40]	158	1.29	1.24	1.32	1.31
ATLAS 8 TeV jets [44]	171	1.96	2.01	1.90	1.92
CMS 8 TeV jets [41]	174	1.83	1.52	1.80	1.60
Total (jets)	643	1.67	1.57	1.63	1.59
Total	4534	1.22	1.20	1.17	1.17

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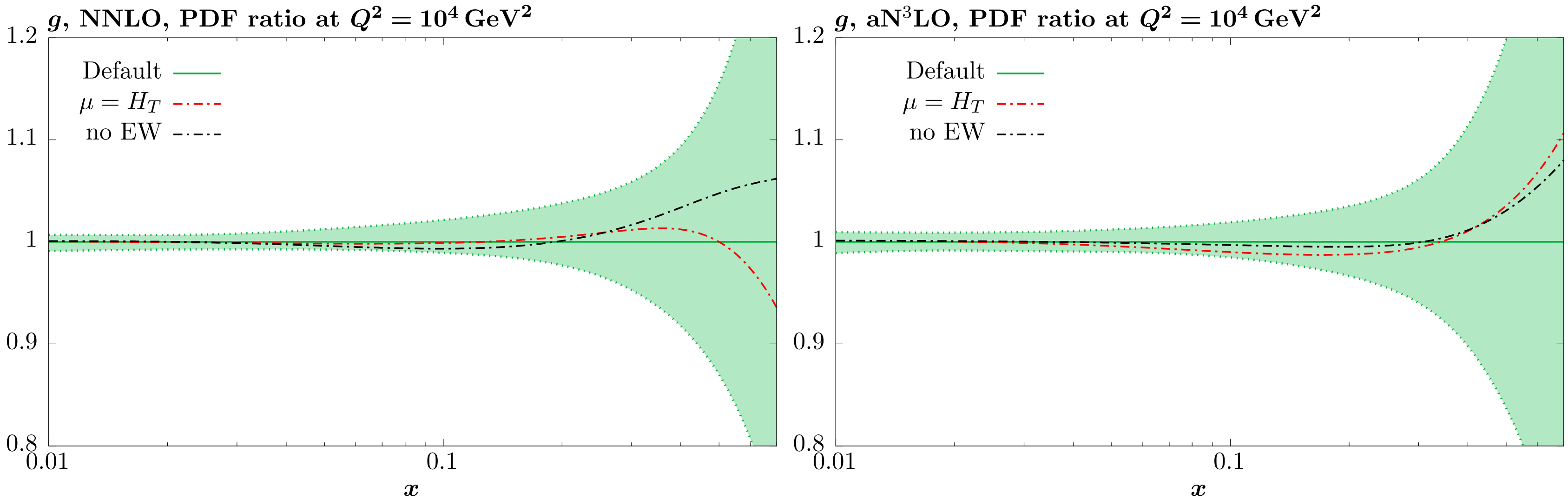
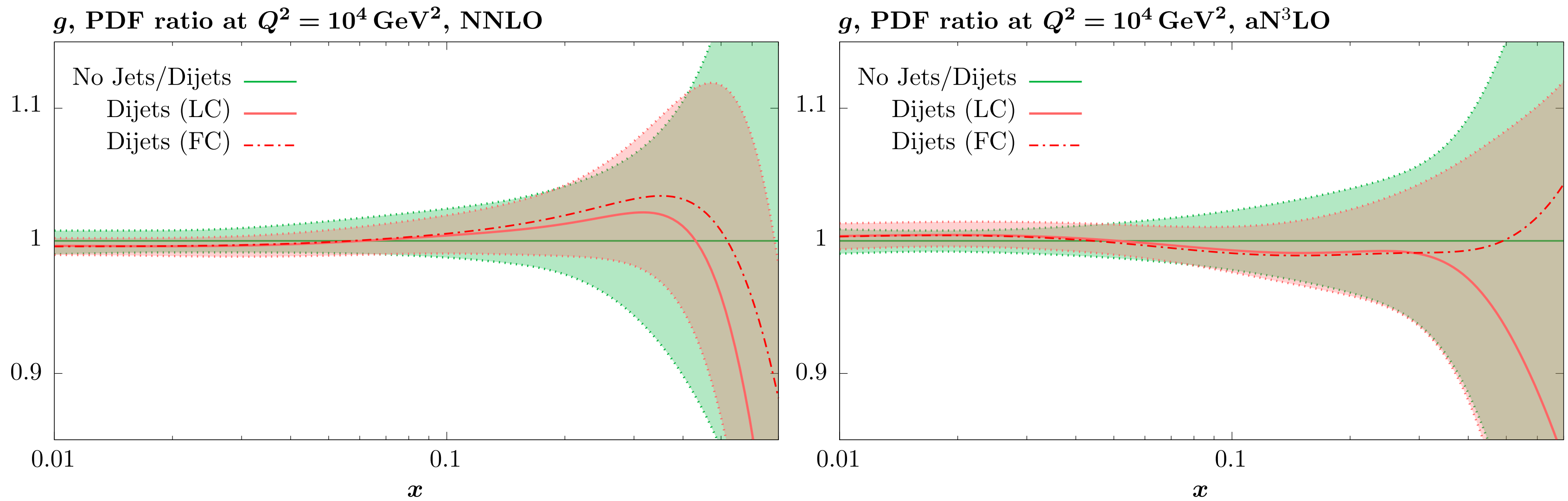
CMS 7 TeV dijets [28]	54	1.44	1.59	1.40	1.56
CMS 8 TeV dijets [29]	122	1.22	1.44	0.86	1.06
Total (dijets)	266	1.21	1.43	1.06	1.29
Total	4157	1.15	1.16	1.09	1.11

Both electroweak corrections and choice of scales have minimal impact on the gluon at both NNLO and N^{3}LO.

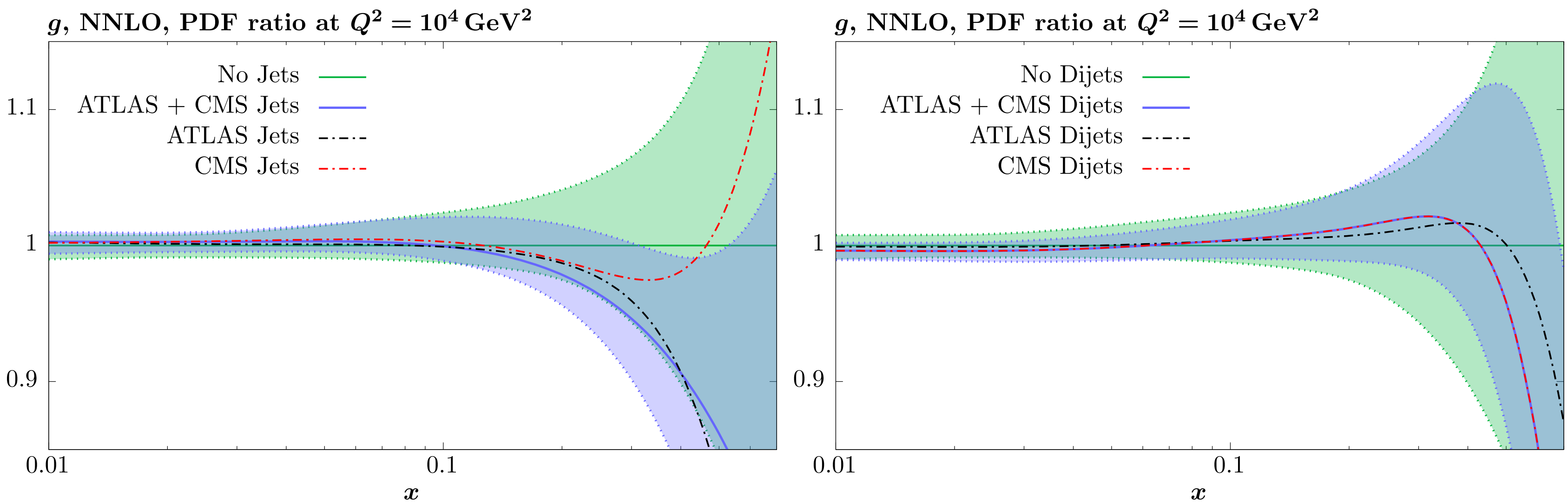


Impact of leading colour corrections on gluon significant, mainly at very high x, but not dramatic.



Similarly, impact on fit quality relatively mild and varies with specific case.

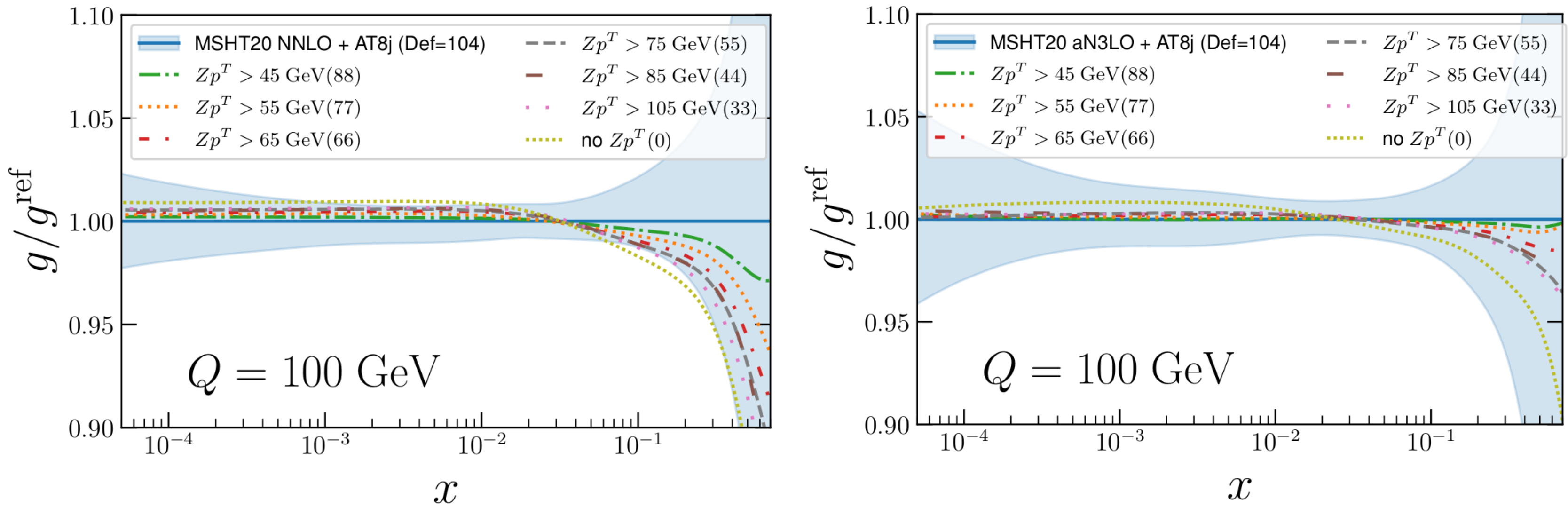
Clearly some mild tension between preferred gluon using either ATLAS or CMS inclusive jet data.



Reduced when using dijet data.

Study of choice of ATLAS Zp_T data.

Raise the lower cut on ATLAS Zp_T data incrementally. Change in gluon distribution is continuous and smooth, though less at aN^{3}LO.

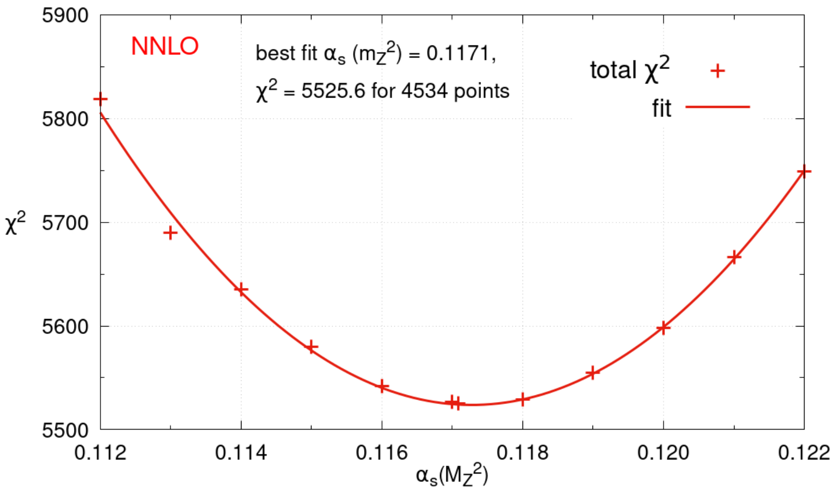


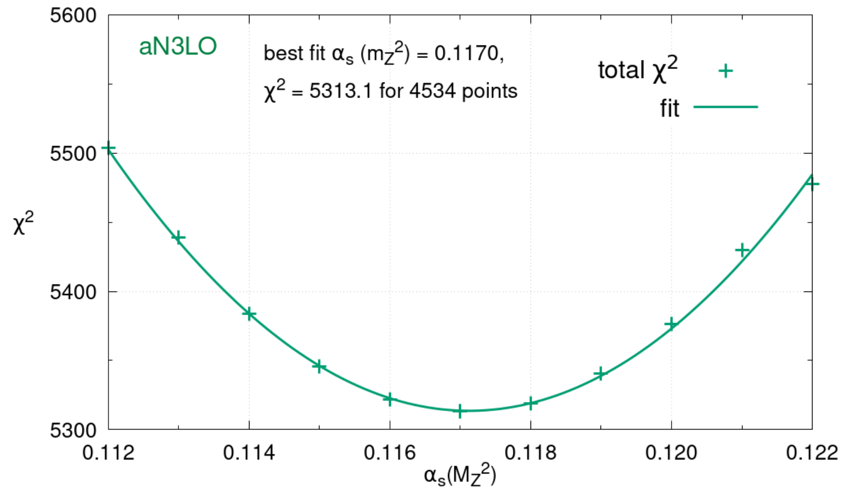
Fit quality also improves slowly and smoothly, again less at aN^{3}LO.

	p_T^{ll} minimum cut (GeV)							p_T^{ll} maximum cut (GeV)
Fit Order	Default (30)	45	55	65	75	85	105	150
NNLO	1.87	1.73	1.72	1.47	1.45	1.47	1.24	1.91
aN^{3}LO	1.04	0.97	1.03	0.86	0.88	0.71	0.83	1.08
N_{pts}	104	88	77	66	55	44	33	82

No sign of impact of resummation/nonperturbative effects strongly impacting normal analysis with $p_T > 30$ GeV.

Best fit value of $\alpha_S(M_Z^2)$ at aN^{3}LO 2404.02964



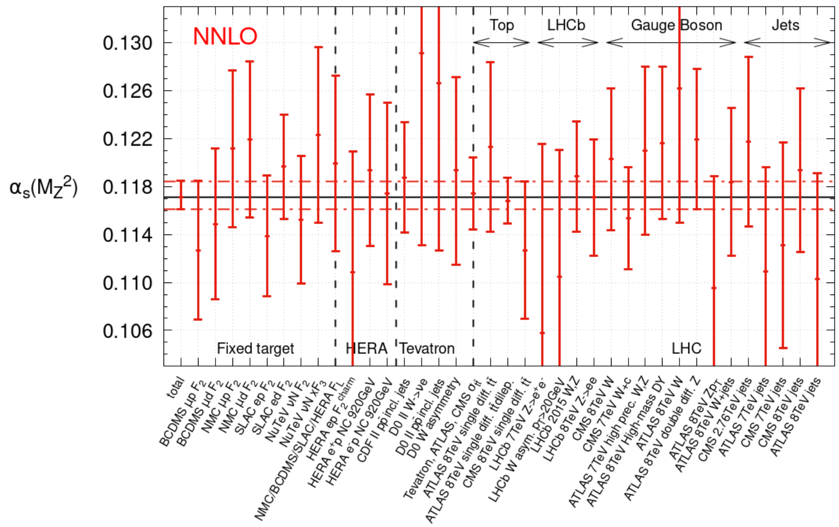


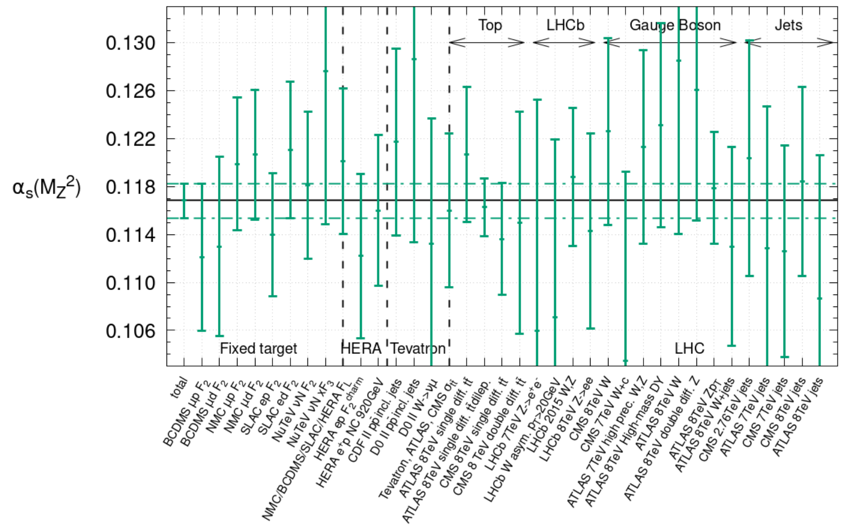
Previously [21] we found at NNLO that $\alpha_S(M_Z^2) = 0.1174 \pm 0.0013$.

Repeat analysis at NNLO with new baseline (ATLAS 8 TeV inclusive jet data) and also at aN^{3}LO.

$$\alpha_S(M_Z^2) = 0.1171 \pm 0.0014 \text{ NNLO}$$

$$\alpha_S(M_Z^2) = 0.1170 \pm 0.0016 \text{ aN}^3\text{LO}$$





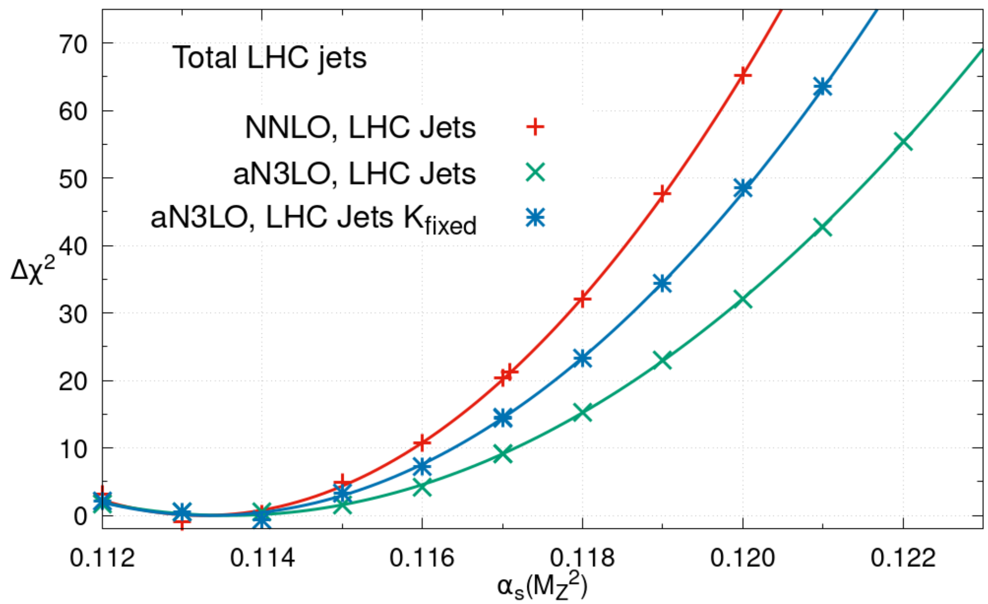
Determine uncertainty by dynamical tolerance procedure, same as for eigenvector uncertainties.

Examine fit quality with varying $\alpha_S(M_Z^2)$ for each data set, and find most limiting set in each direction.

Find very similar constraints regarding datasets at each order, though slightly wider bounds at aN^{3}LO on data types with current N^{3}LO K-factors freedom. Better measure of true theoretical uncertainty.

Uncertainty corresponds to $\Delta\chi^2 = 13$ NNLO, $\Delta\chi^2 = 16$ N^{3}LO.

Look in detail at fit quality of inclusive jet data for varying $\alpha_S(M_Z^2)$.

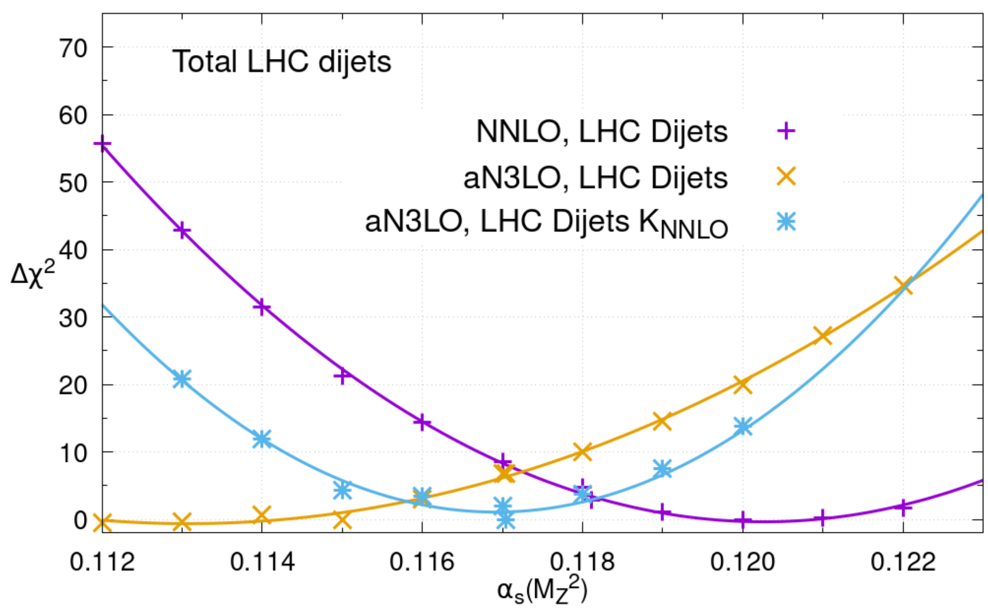


Consistent for minimum between orders.

Width greater at aN^{3}LO, partially due to K-factor freedom.

For dijets best fit value changes.

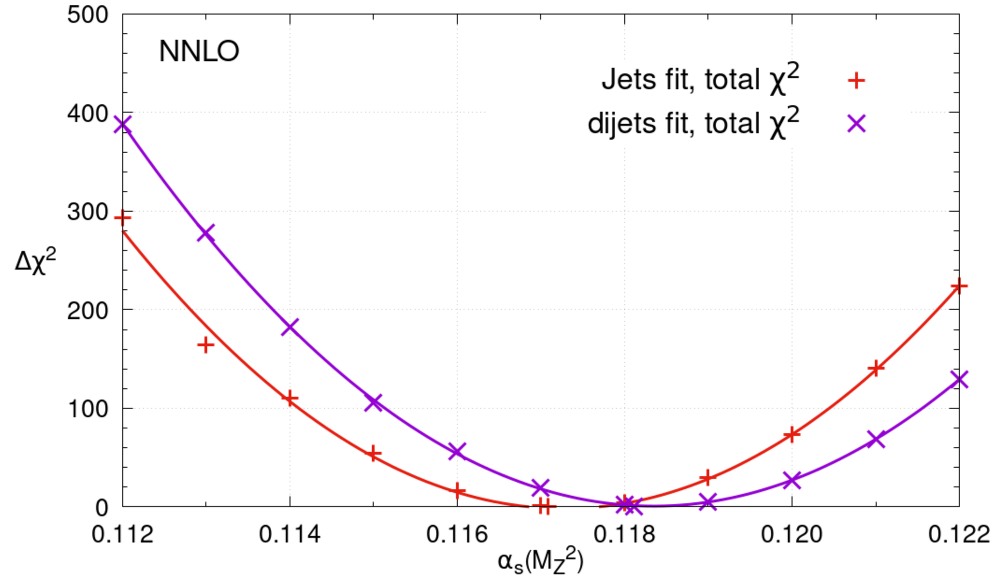
Partially due to K-factor freedom.



For total χ^2 some variation between inclusive jets at dijets at NNLO.

$\alpha_S(M_Z^2)_{\rm dijet} = 0.1181 \pm 0.0012$ NNLO.

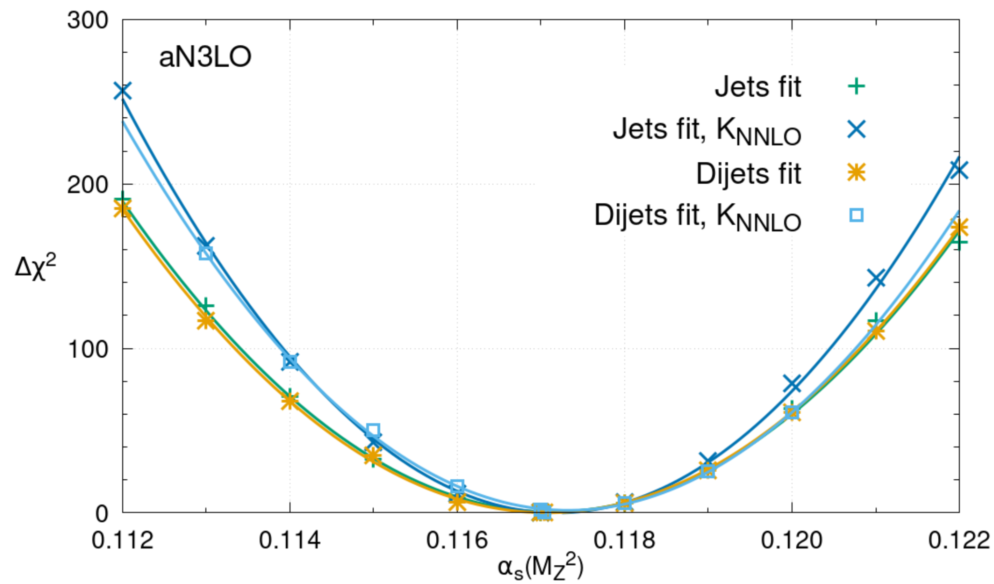
($\alpha_S(M_Z^2)_{\rm jet} = 0.1171 \pm 0.0014$).



At aN^{3}LO much more stability with data choice.

$\alpha_S(M_Z^2)_{\rm dijet} = 0.1170 \pm 0.0013$ aN^{3}LO .

($\alpha_S(M_Z^2)_{\rm jet} = 0.1170 \pm 0.0016$).



Conclusions

Numerous updates associated with MSHT PDFs; aN^{3}LO, QED, study of dijets vs. inclusive jets, best fit $\alpha_S(M_Z^2)$ at aN^{3}LO.

First PDF set at aN^{3}LO. Confirmed main features essentially preserved with more up to date info.

QED effects similar at aN^{3}LO, but fit quality affected less than lower orders. PDFs with QED at LO.

See small but significant effects on gluon using dijets. Overall better fits and consistency using dijets at NNLO and aN^{3}LO. No sign of issues with use of Zp_T data.

Best fit value $\alpha_S(M_Z^2) = 0.1170 \pm 0.0016$ aN^{3}LO (inclusive jets) $\alpha_S(M_Z^2) = 0.1170 \pm 0.0013$ aN^{3}LO (dijets). Better stability at aN^{3}LO, and larger, more accurate uncertainty.

Also various studies on PDF uncertainties (other talks) and on N^{3}LO evolution benchmarking (Thursday).

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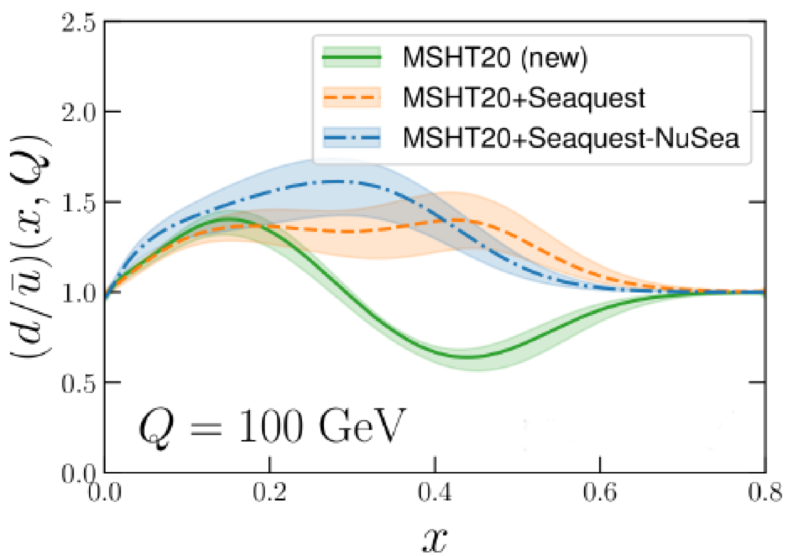
Back-up

New data - Seaquest (NNLO)

Preliminary!

- Seaquest (E906) fixed target DY data - sensitivity to high x q, $\bar{q}$: $\Rightarrow \sigma_D/\sigma_H \sim 1 + \bar{d}/\bar{u}$. Direct measurement of $\bar{d}/\bar{u}$ at high x.
- Various models for $\bar{d}/\bar{u}$ at high x: Pauli blocking, pion cloud, etc.
- Previous questions of NuSea (E866) data preferring $\bar{d} < \bar{u}$ at $x \approx 0.4$.
- Clearly raises high x $\bar{d}/\bar{u}$. Tension with NuSea which pulls it down.



Dataset	N_{pts}	MSHT20	New
Seaquest	6	-	8.2
NuSea	15	9.8	19.0
Total (without Seaquest or NuSea)	4348	5102.3	5112.1

- NuSea χ^2/N_{pts}: $0.65 \rightarrow 1.27$, when Seaquest added.

- Rest of data also worsens in χ^2 by 9 points, with 4.5 in E866 absolute DY (rather than ratio), 4.4 in NMC n/p, 4.3 in DØ W asymmetry.

Slide credit: T. Cridge