Partonic collinear structure by quantum computing

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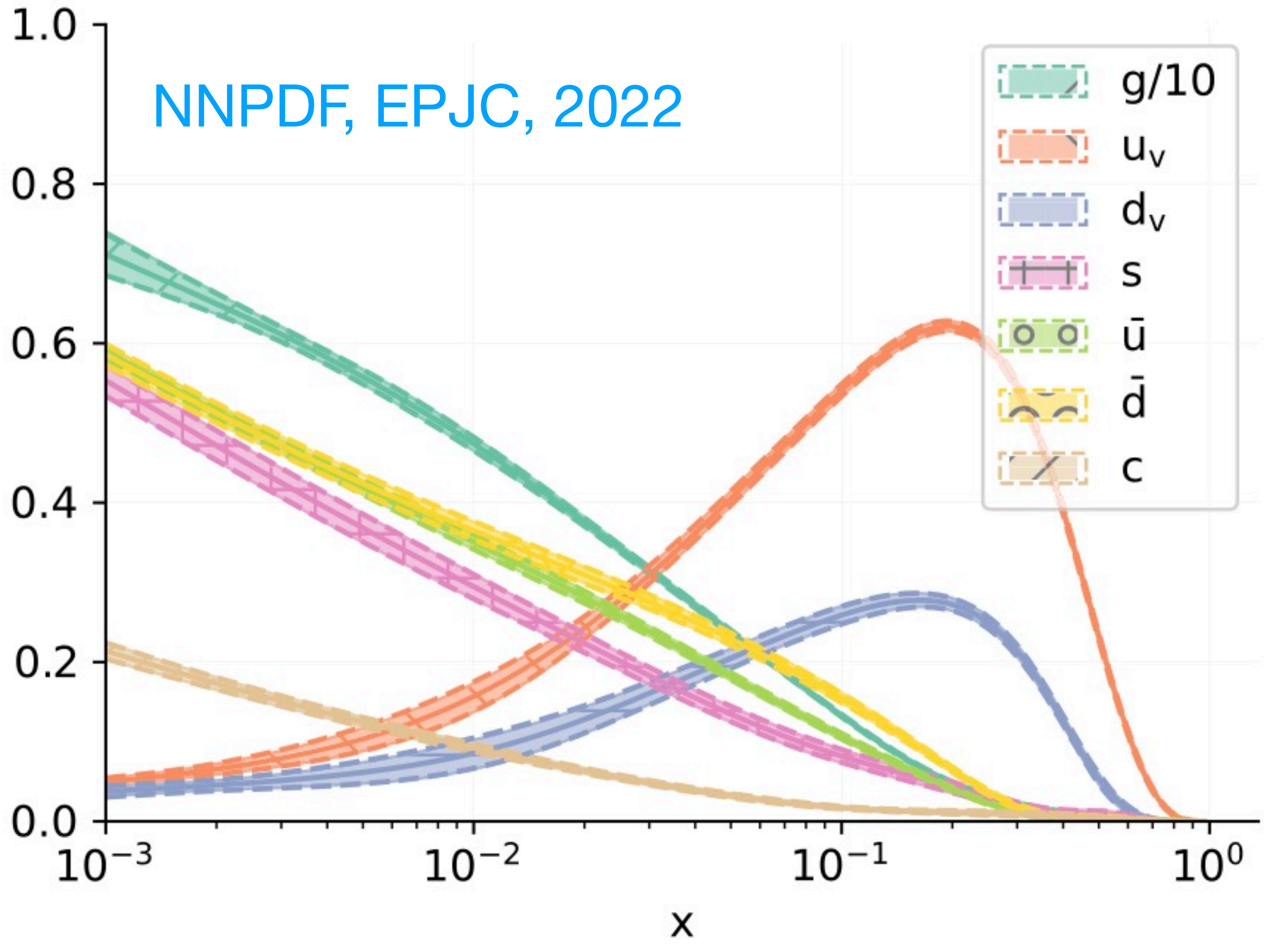
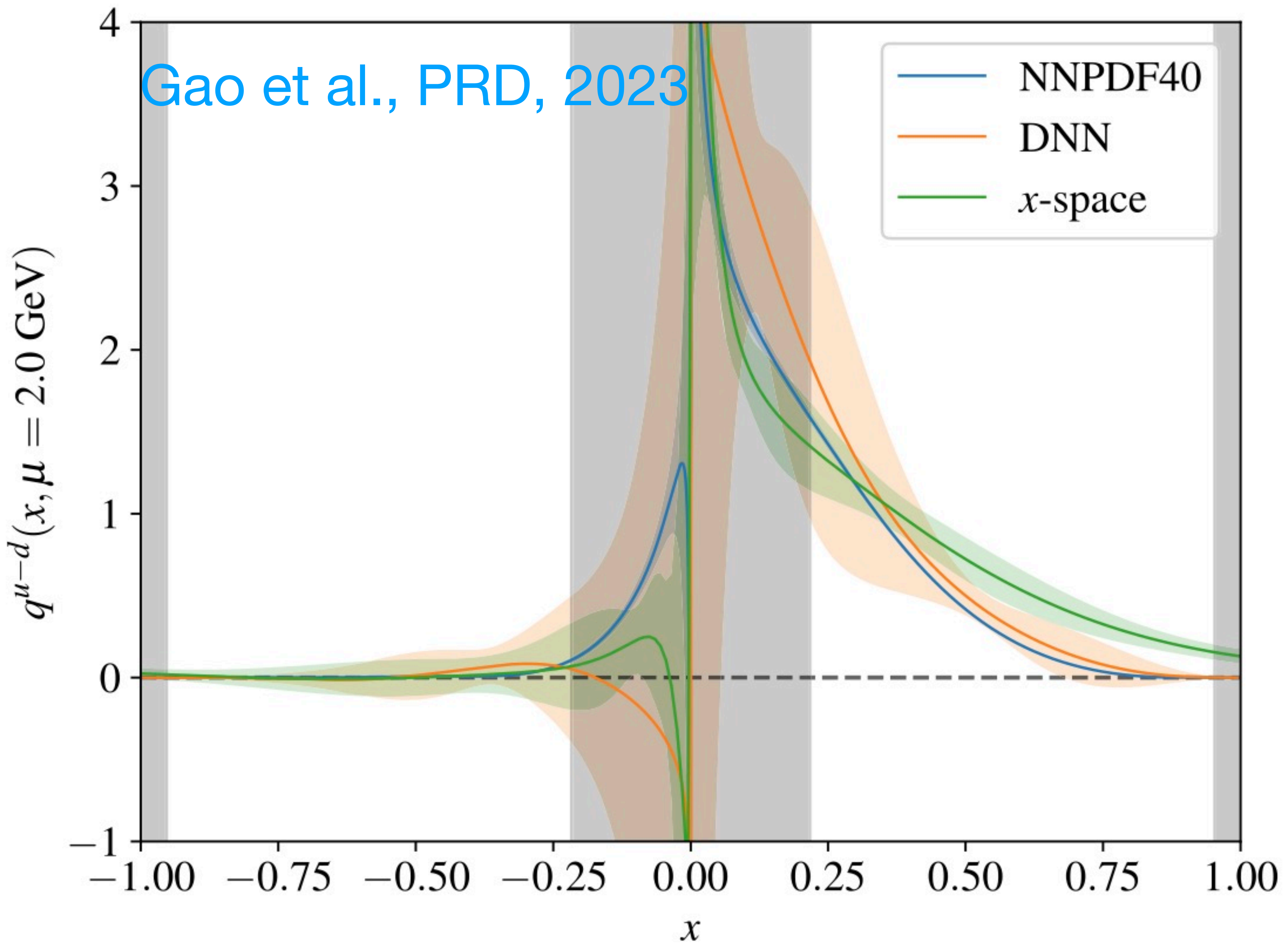
State-of-art determination of collinear PDFs

Experimental data based global analysis



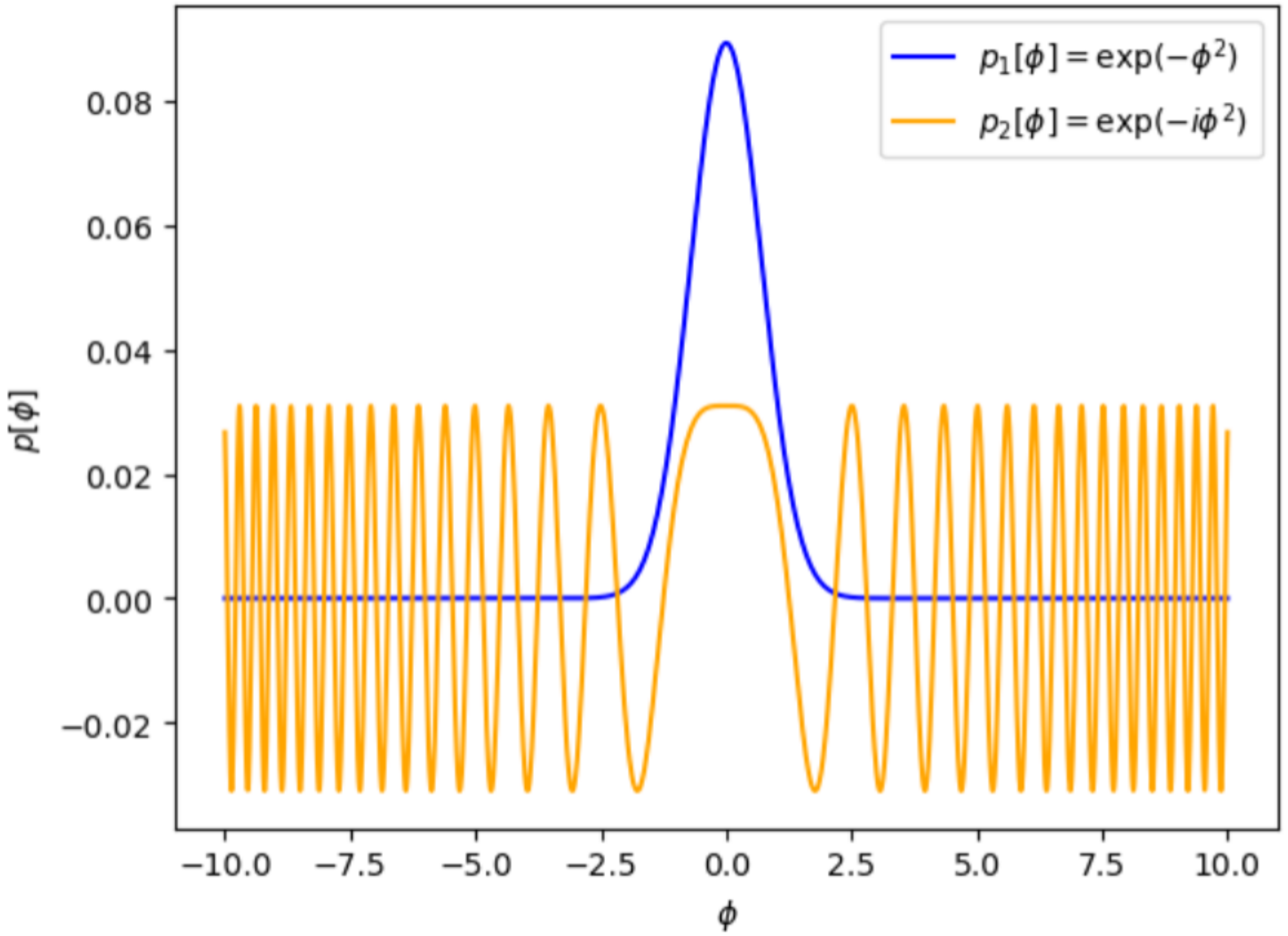
Talk by Lucian Harland-Lang on Monday

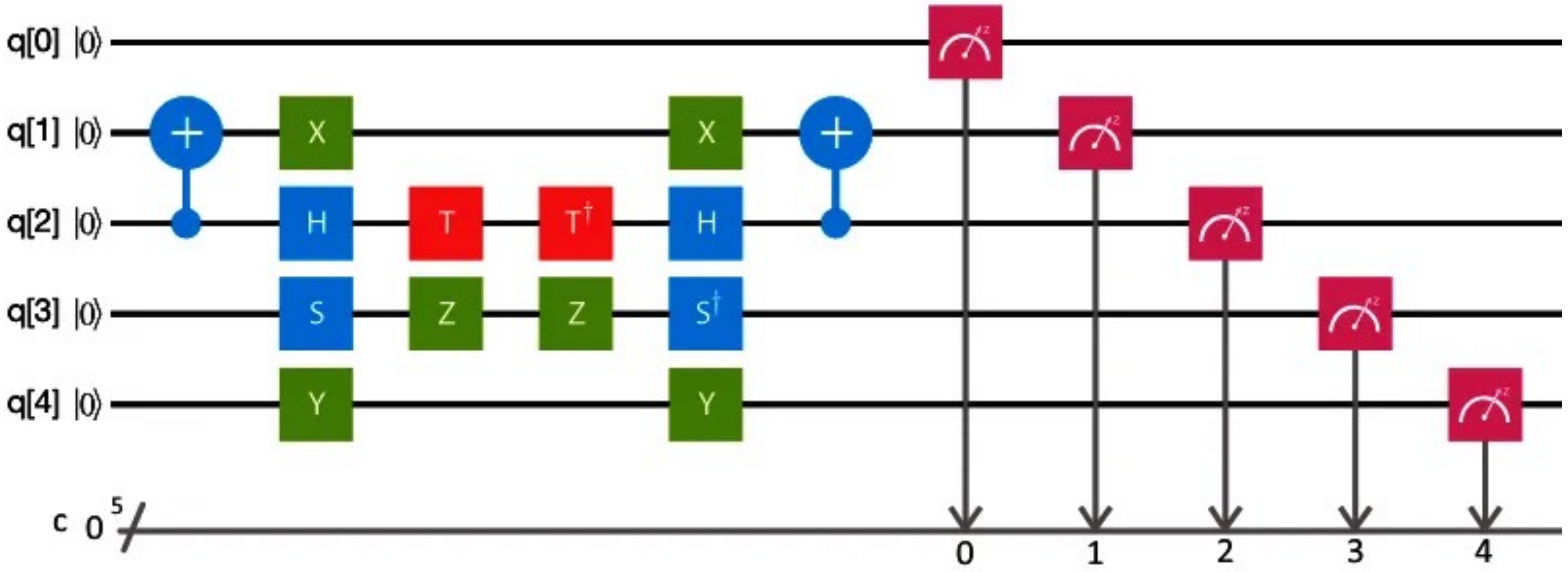
Equal time lattice calculation



Talk by Huey-Wen Lin on Friday

A new method for first principle calculation





Sign problem in Monte Carlo sampling of distribution and real time evolution

$$\langle \mathcal{O} \rangle = \frac{\int D\phi(x) e^{-S_E[\phi]} \mathcal{O}}{\int D\phi(x) e^{-S_E[\phi]}}$$

quantum computing: each gate is the result of time evolution

$$U(t) | \psi \rangle = e^{-it(H_0 + H_1 + \ldots)} | \psi \rangle$$

Simulate hadron partonic structure on quantum computer

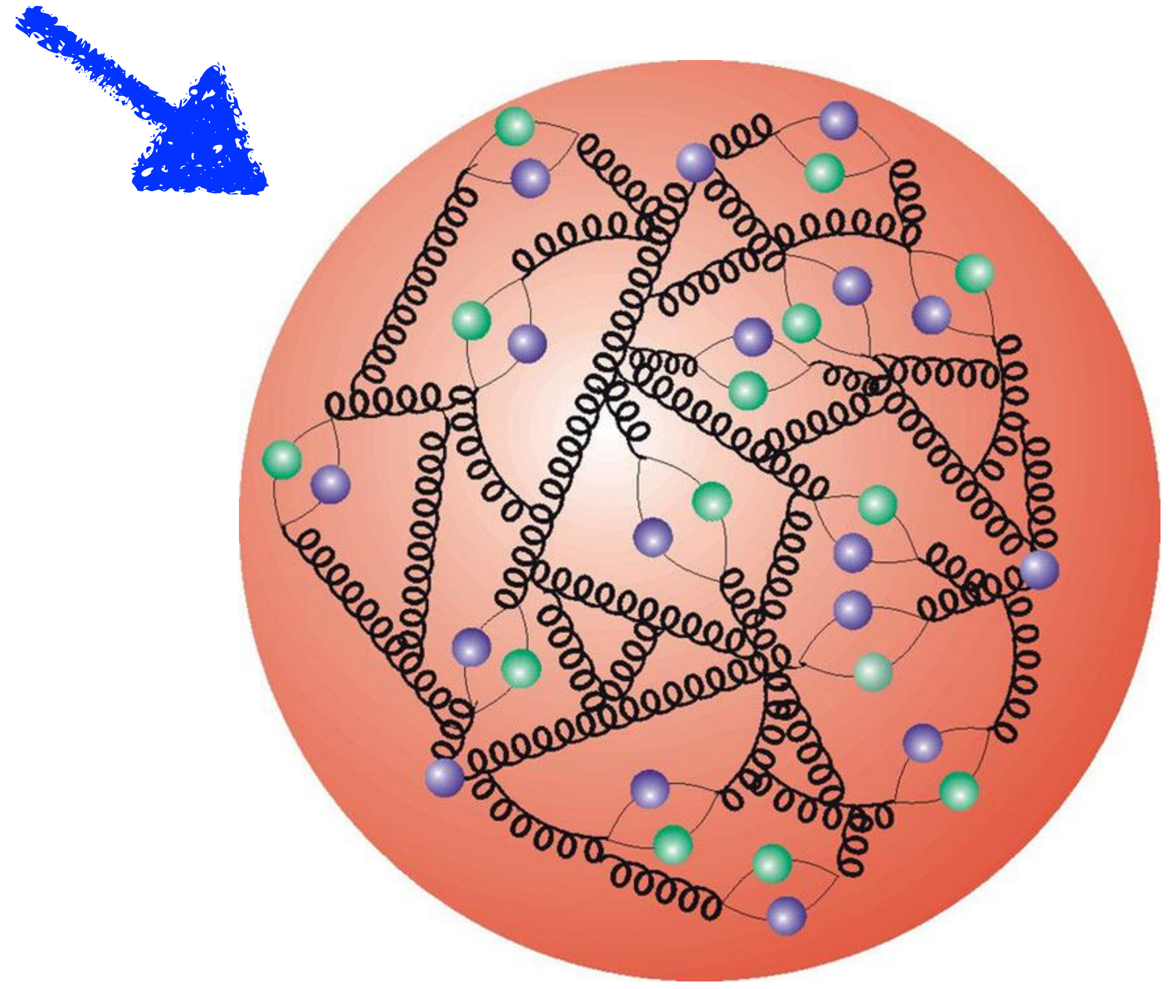
✦ Operator definition of quark PDF

$$\sigma \sim f_A \otimes H \otimes D$$

$$f_{q/p}(x) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle p | \bar{\psi}(0) \frac{\gamma^+}{2} \mathscr{W}(0,y^-) \psi(y^-) | p \rangle$$

$$y^- = (y_0 - y_3)/\sqrt{2}$$

real time correlation function



✦ PDFs are extremely challenge to simulate directly in Euclidean lattice calculation, due to multidimensional oscillating integral.

✦ QC can naturally simulate real-time dynamics.

✦ We are far from QCD Quantum Supremacy, start from a toy model for proof of concept study

Talk by Christian Bauer on Friday

Simulate hadron partonic structure on quantum computer

✦ A toy model - 1+1D NJL (Gross, Neveu, 1974), no gauge field

$$\mathcal{L} = \bar{\psi}_\alpha (i\gamma^\mu \partial_\mu - m_\alpha)\psi_\alpha + g(\bar{\psi}_\alpha \psi_\alpha)^2$$

$$f(x) = \int dz^- e^{-ixM_h z^-} \langle h | \bar{\psi}(z^-)\gamma^+ \psi(0) | h \rangle = \int dz^- e^{-ixM_h z^-} \langle h | e^{iHz} \bar{\psi}(0, -z) e^{-iHz} \gamma^+ \psi(0) | h \rangle$$

✦ Challenges in quantum computing

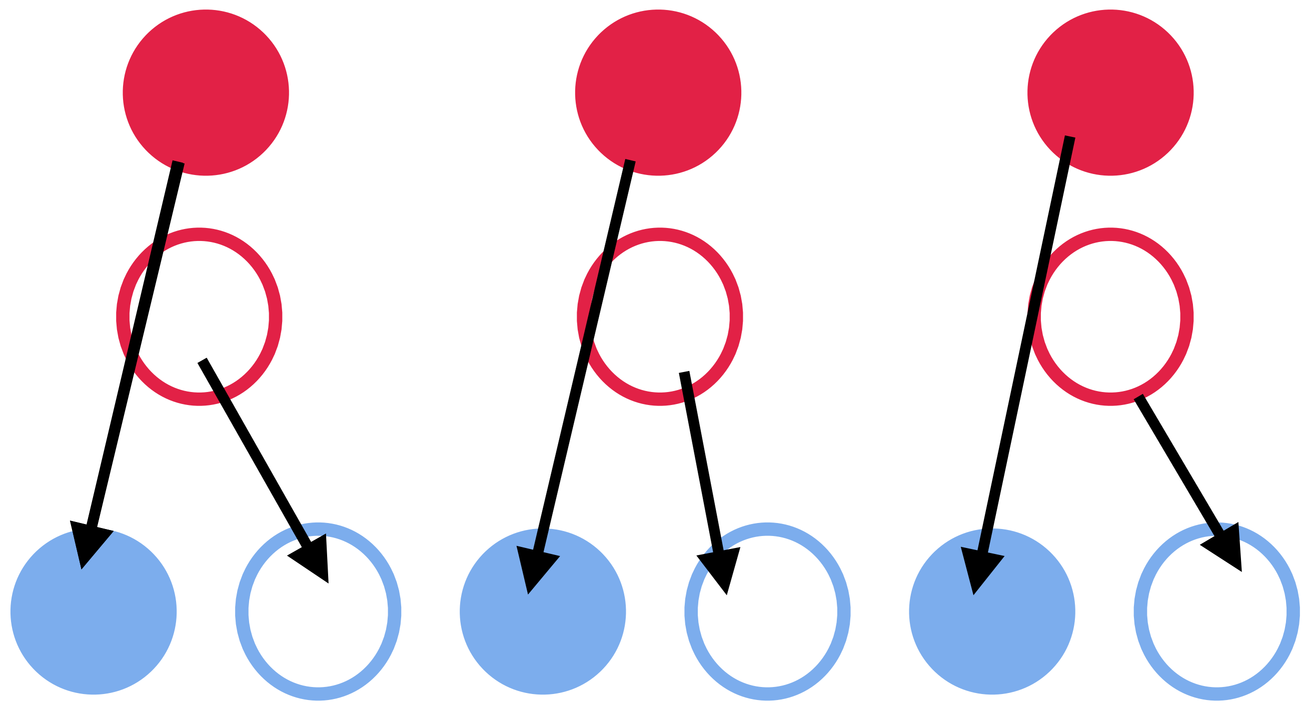
- Map QFT to qubits+gates system
- Prepare the external hadronic state $| h \rangle$
- Evaluate the real-time dynamical correlation function
- Measurement of final observable

Simulate hadron partonic structure on quantum computer

✦ Quantum field to qubits+gates

$$\mathscr{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g(\bar{\psi}\psi)^2$$

- Discretization: staggered fermion, put different fermion components, flavors on different sites



$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \phi_{2n} \\ \phi_{2n+1} \end{pmatrix}$$

- Jordan-Wigner transformation

$$\phi_n = \prod_{i<n} Z_i (X + iY)_n$$

- Discretized PDF:

$$f(x) \rightarrow \sum_{i,j} \sum_{z} \frac{1}{4\pi} e^{-ixM_h z} \langle h \,|\, e^{iHz} \phi^{\dagger}_{-2z+i} e^{-iHz} \phi_j \,|\, h \rangle$$

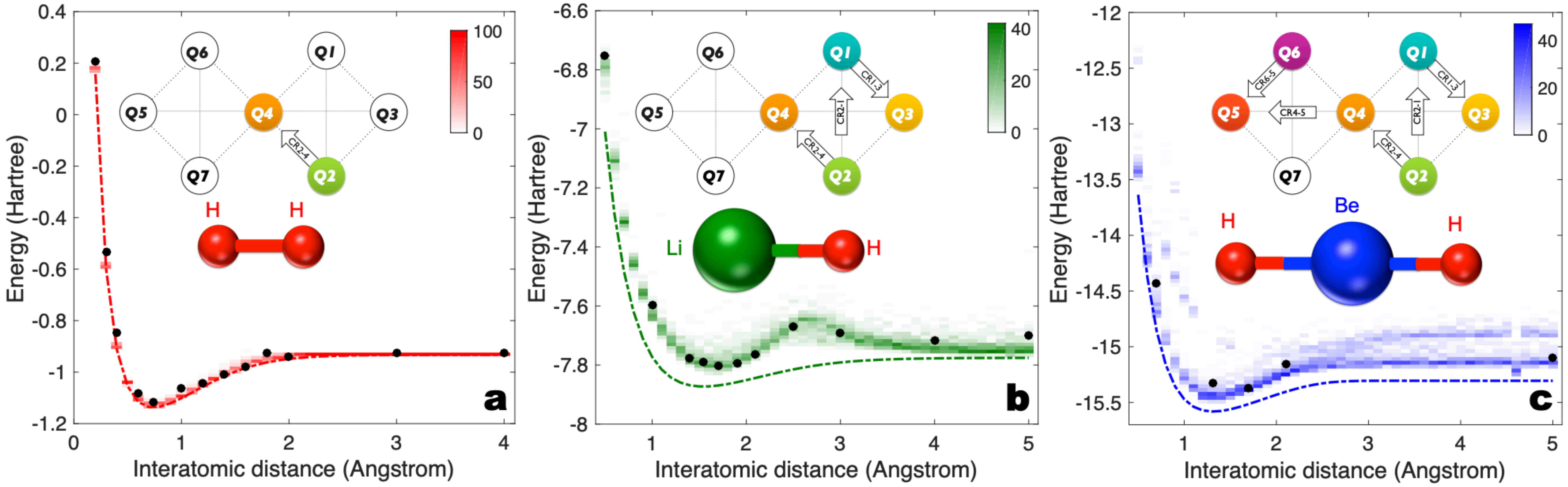
$$H = H_1 + H_2 + H_3 + H_4 \qquad H_1 = \sum_{n=\mathrm{even}} \frac{1}{4} \left[X_n Y_{n+1} - Y_n X_{n+1} \right]$$

Simulate hadron partonic structure on quantum computer

✦ Hadron state preparation - VQE

- Hadron states are the eigenstates of the Hamiltonian with certain quantum numbers.
- Prepare the state by variational quantum eigensolver (VQE) 2103.08505 + …
- VQE is a hybrid method involves both classical and quantum computers

Potential energy surfaces Nature 549, 242 (2017)



show its power in quantum chemistry

Simulate hadron partonic structure on quantum computer

✦ Hadron state preparation - VQE

Li et al (QuNu), PRD (letter, 2022)

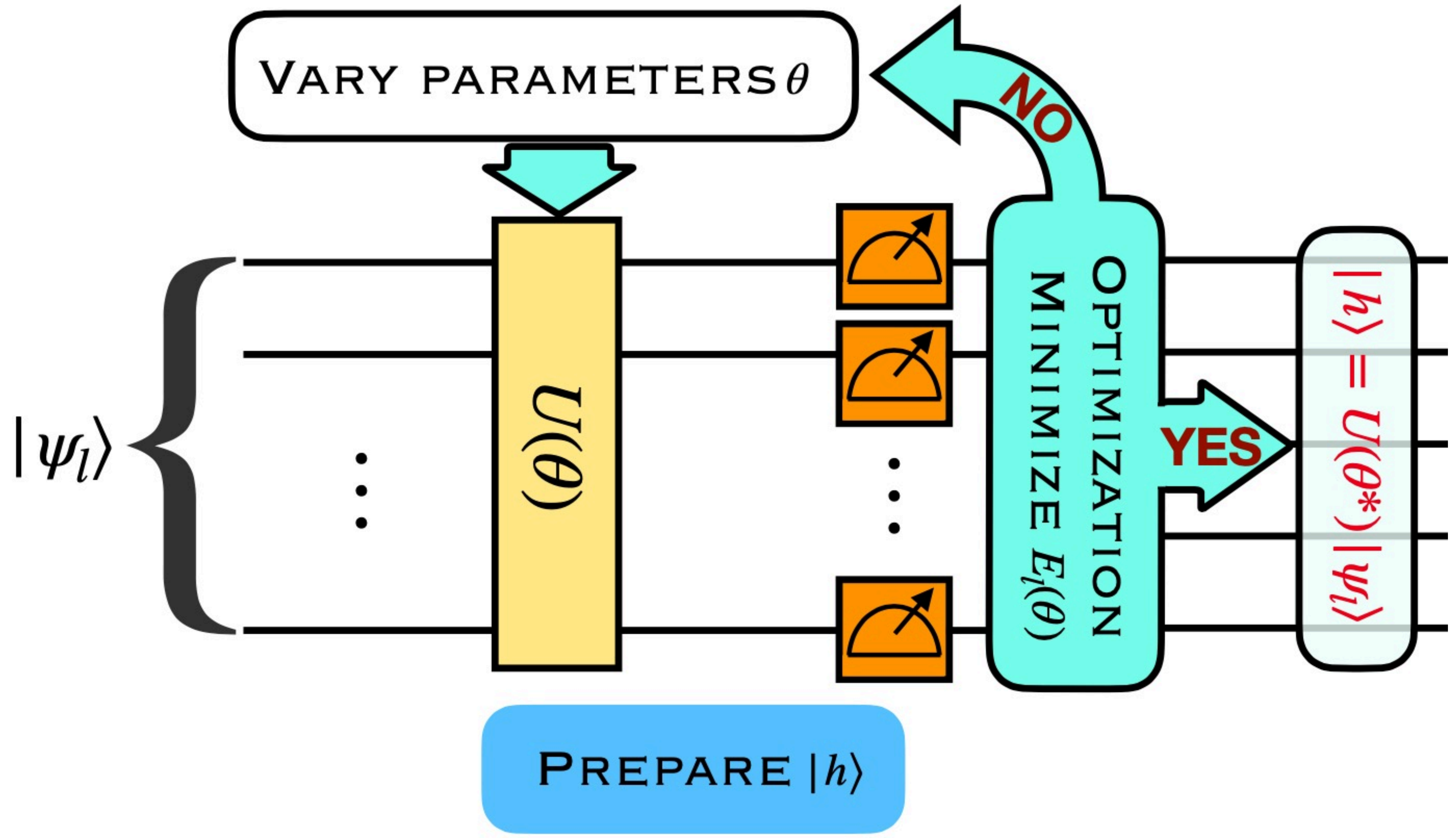
I. Construct a trial hadronic state $|\psi_{lk}\rangle$, and a symmetry-preserving unitary operator $U(\theta)$

II. The k-th state with quantum number l
$|\psi_{lk}(\theta)\rangle = U(\theta)|\psi_{lk}\rangle$

III. Optimization for hadronic state, minimize the cost function (PRL 113, 020505)

$$E_l(\theta) = \sum_{i=1}^{k} w_{li}\langle\psi_{li}(\theta)|H|\psi_{li}(\theta)\rangle$$

IV. $|h\rangle = U(\theta^*)|\psi_{lk}\rangle$, θ^* is the optimized parameter set



Step II is carried out on quantum computer, all the others are computed on classical machine

Simulate hadron partonic structure on quantum computer

✦ Hadron state preparation

- Construct $U(\theta)$: quantum alternating operator ansatz (QAOA)

I. Divide the hamiltonian, each term inherits the symmetries of H, $H = H_1 + H_2 + H_3 + H_4$

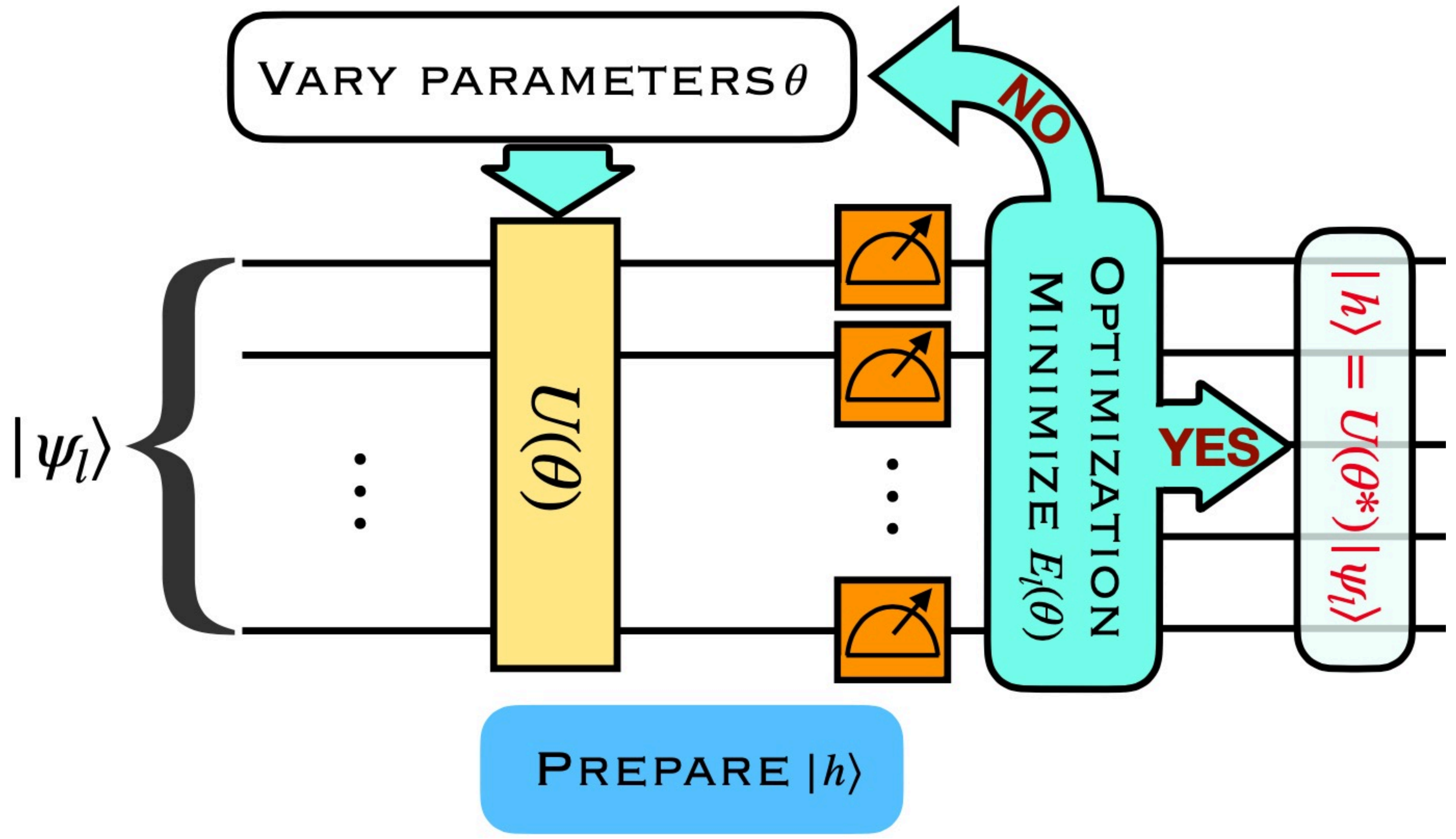
II. $U(\theta)$ consists p layers, each layer evolve H_j with time duration θ_{ij}, $U(\theta) \equiv \prod_{i=1}^{p} \prod_{j=1}^{n} \exp(i\,\theta_{ij} H_j)$

III. Prepare the input reference states for QAOA

$$|\psi_{\Omega,1}\rangle = |010101\ldots01\rangle$$ → Naive vacuum

$$|\psi_{\Omega,2}\rangle = \frac{1}{\sqrt{N/2}}\Big(|\underline{10}01,\ldots,01\rangle + |01\underline{10},\ldots,01\rangle + \ldots + |0101,\ldots,\underline{10}\rangle \Big)$$

"quark pair" excitation



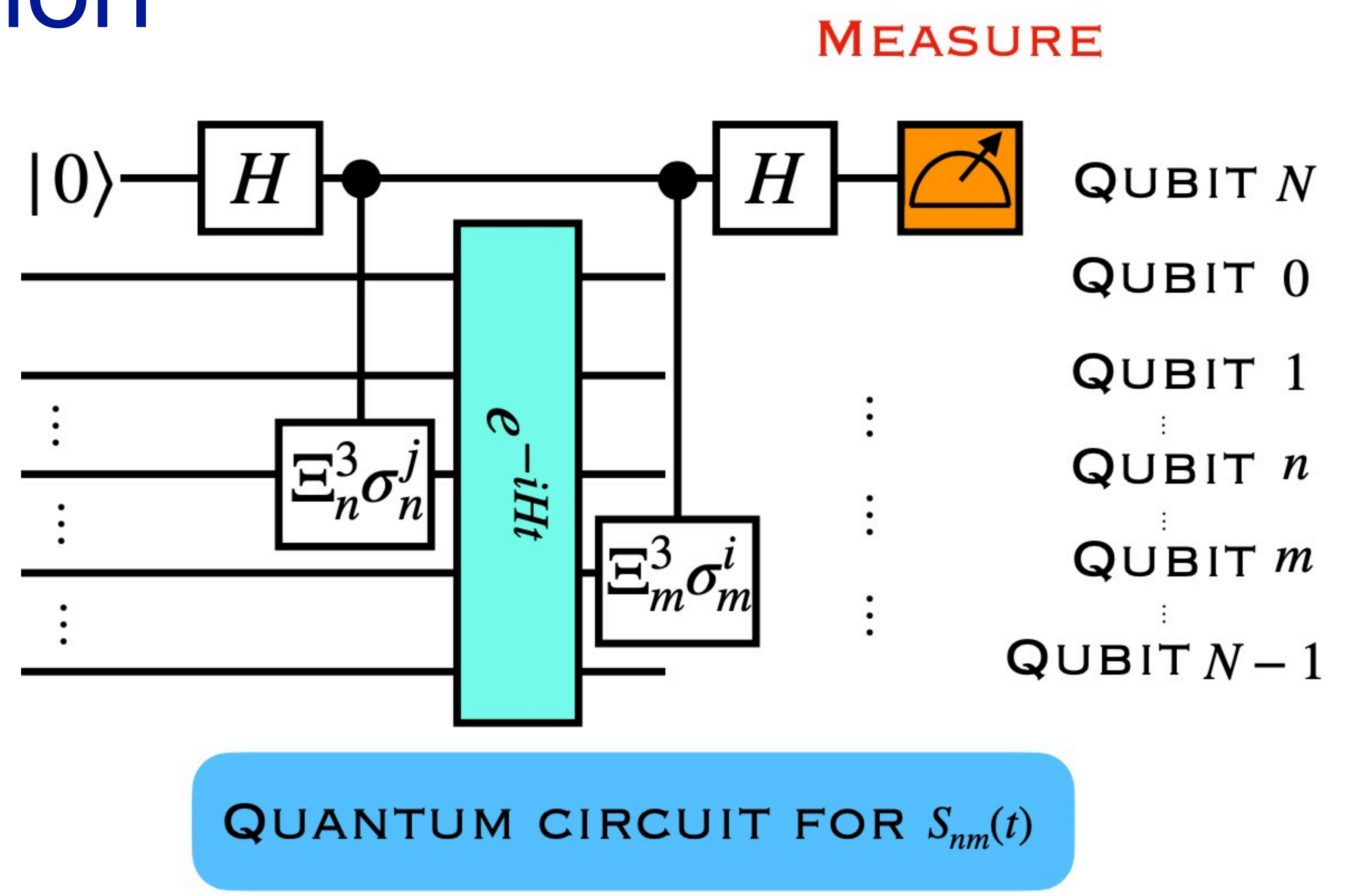
Simulate hadron partonic structure on quantum computer

Pedernales et al, PRL. 113, 020505 (2014)

✦ Evaluate the real-time dynamical correlation function

$$S_{mn}(t) = \langle h | e^{iHt} \Xi_m^3 \sigma_m^i e^{-iHt} \Xi_n^3 \sigma_n^j | h \rangle$$

PDFs can be written as a sum of such correlation functions

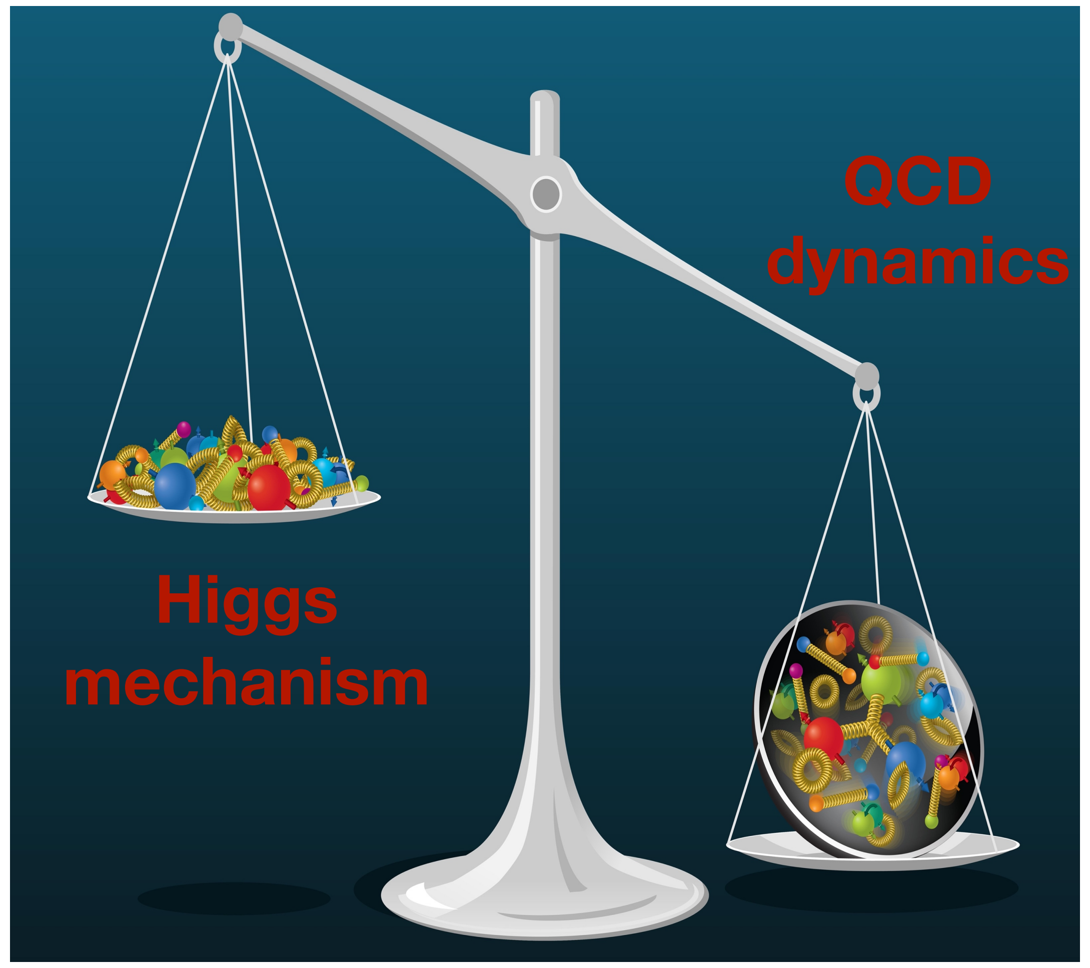


✦ Measure the observable with one auxiliary qubit

Measure the ancillary qubit on X (Y) basis to get the real (imaginary) part of $S_{mn}(t)$

- $|\alpha\rangle_a |0\rangle_b \rightarrow \frac{\sqrt{2}}{2} |\alpha\rangle_a (|0\rangle_b + |1\rangle_b) \rightarrow |\phi\rangle \equiv \frac{\sqrt{2}}{2}(|\alpha\rangle_a |0\rangle_b + \hat{O} |\alpha\rangle_a |1\rangle_b)$
- $\langle \phi | I_a \otimes X_b | \phi \rangle = \frac{1}{2} + Re(\langle \alpha | \hat{O} | \alpha \rangle)$
- $\langle \phi | I_a \otimes Y_b | \phi \rangle = \frac{1}{2} - Im(\langle \alpha | \hat{O} | \alpha \rangle)$

Numerical results from quantum computing



✦ Measurement of hadron mass

$$M_h = \langle h | H | h \rangle - \langle \Omega | H | \Omega \rangle$$

g	0.2	0.4	0.6	0.8	1.0
$M_{h,\mathrm{QC}}a$	1.002	1.810	2.674	3.534	4.352
$M_{h,\mathrm{NUM}}a$	1.001	1.801	2.659	3.509	4.342

$N = 12$

$ma = 0.2$

- Considering the current limitations of using real quantum devices, the results are generated using a classical simulation of the quantum circuit
- Measure the mass of the lowest-lying ud-like hadron in NJL model with 2 flavors, QAOA has good accuracy
- For small quark mass, the dominant contribution comes from the interaction rather than the quark masses
- For $ma = 0.8$, the quark masses are dominant

Numerical results from quantum computing

✦ quark PDF of the lowest-lying zero-charge hadron

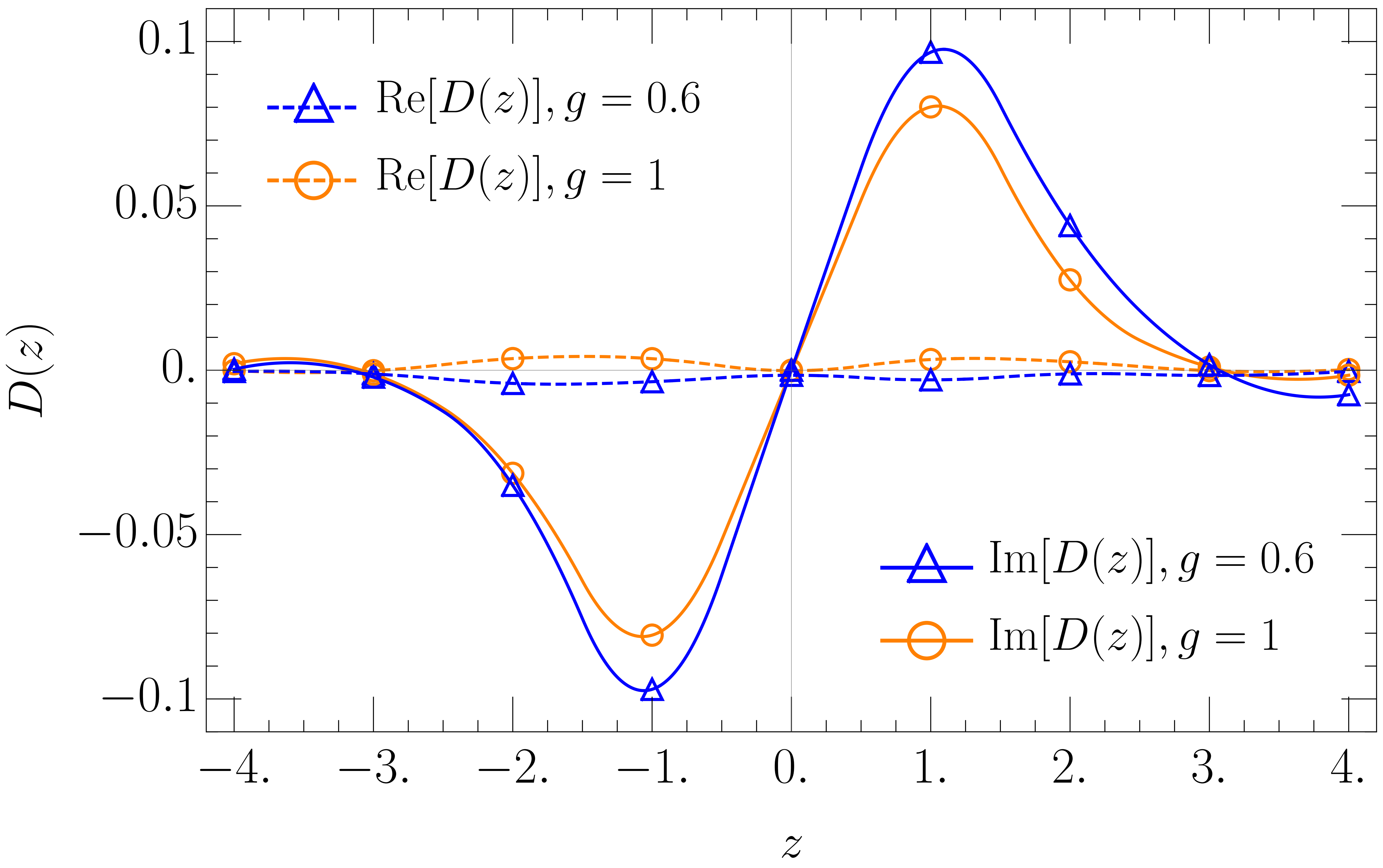
- quark PDF in position space

$$ma = 0.8 \qquad N = 18 \qquad n_f = 1$$

- The real part is consistent with 0

$$f_q(x) = f_{\bar{q}}(x) = -f_q(-x)$$

- The bound state behavior

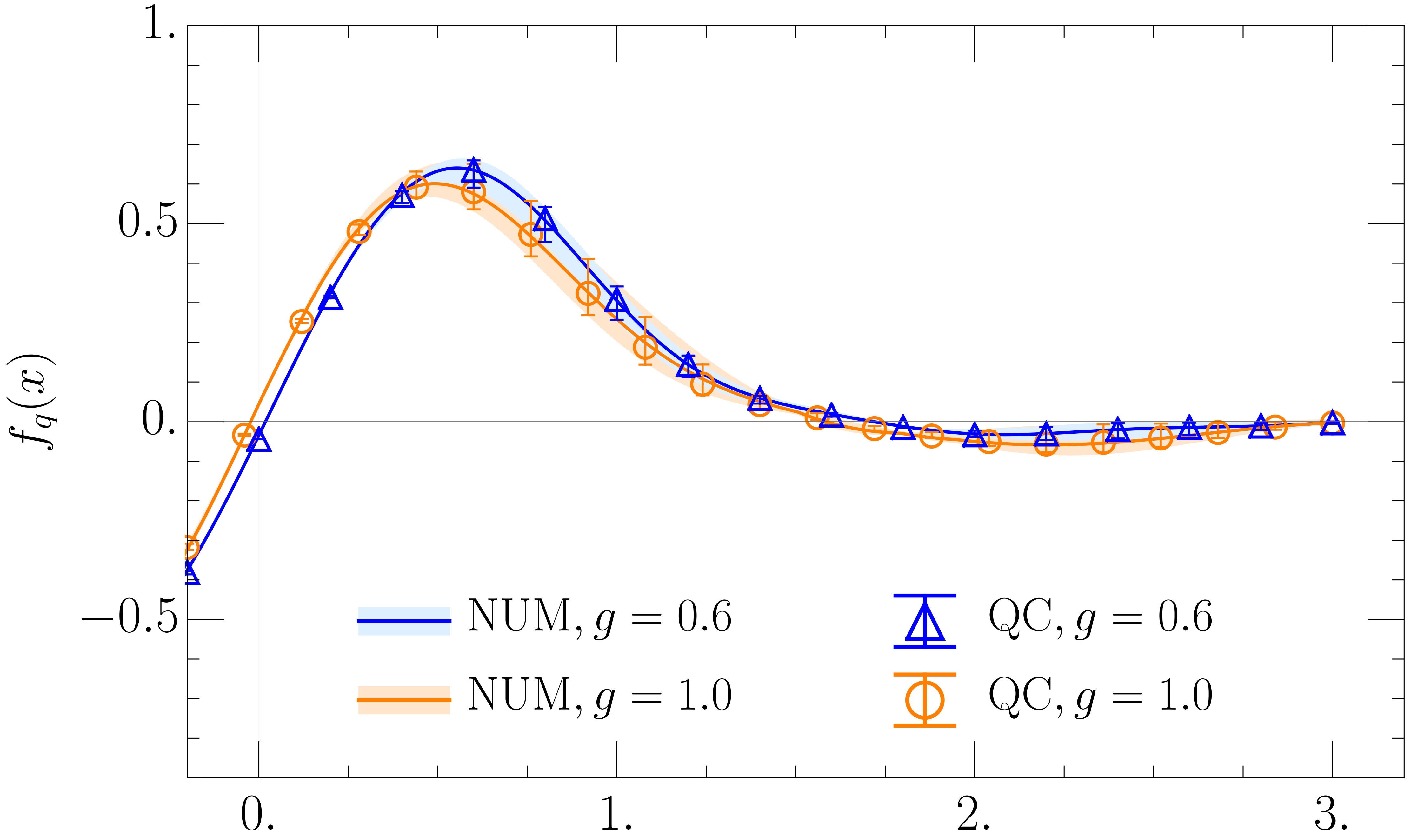


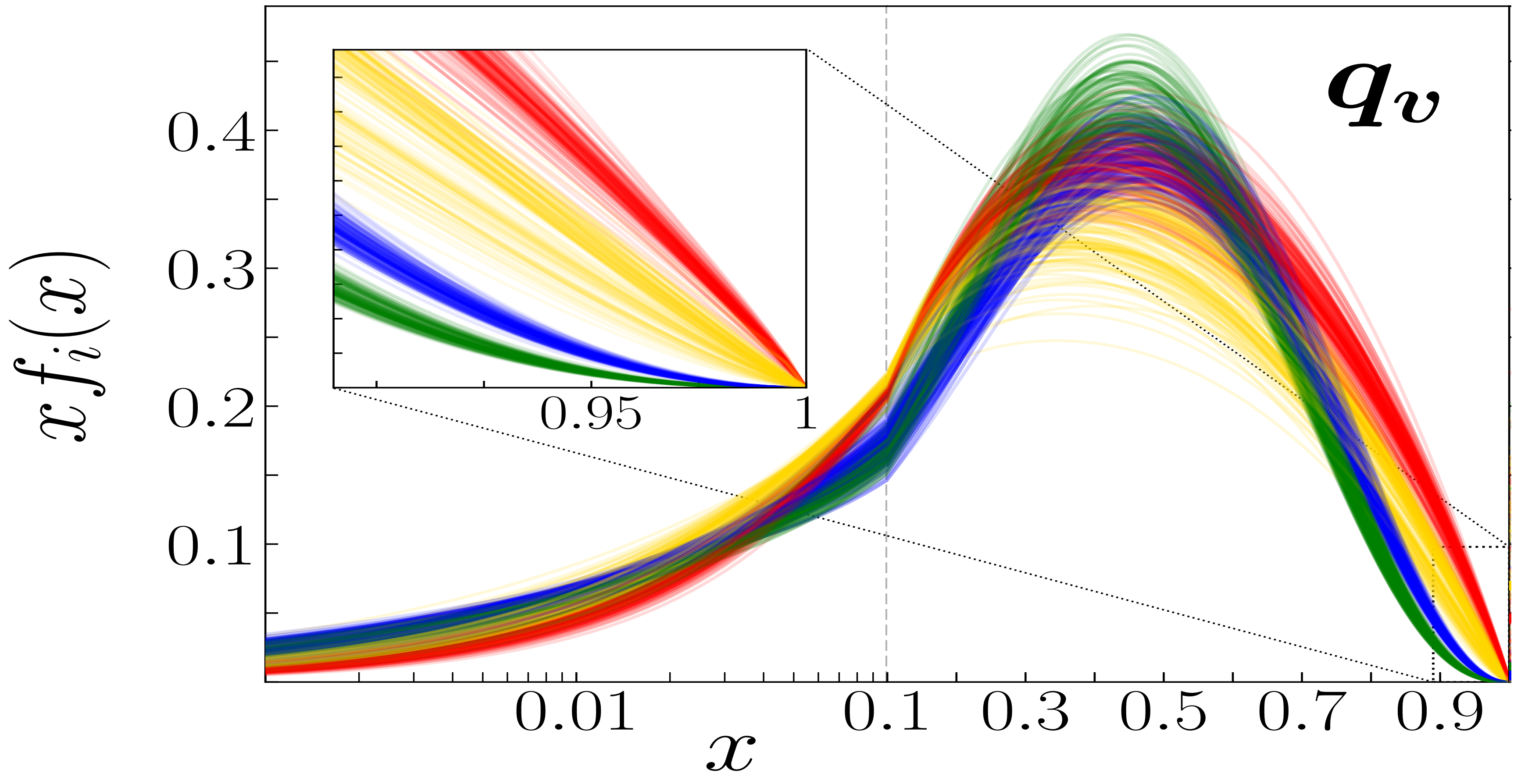
Numerical results from quantum computing

Li et al (QuNu), PRD (letter, 2022)

✦ quark PDF of the lowest-lying zero-charge hadron

- Good agreement between quantum computing and numerical diagonalization
- The non-vanishing contributions in the $x > 1$ are partly due to the finite volume effect
- We observe the expected peak around $x = 0.5$ and qualitative agreement with pion PDFs





JAM Collaboration, PRL, 2021

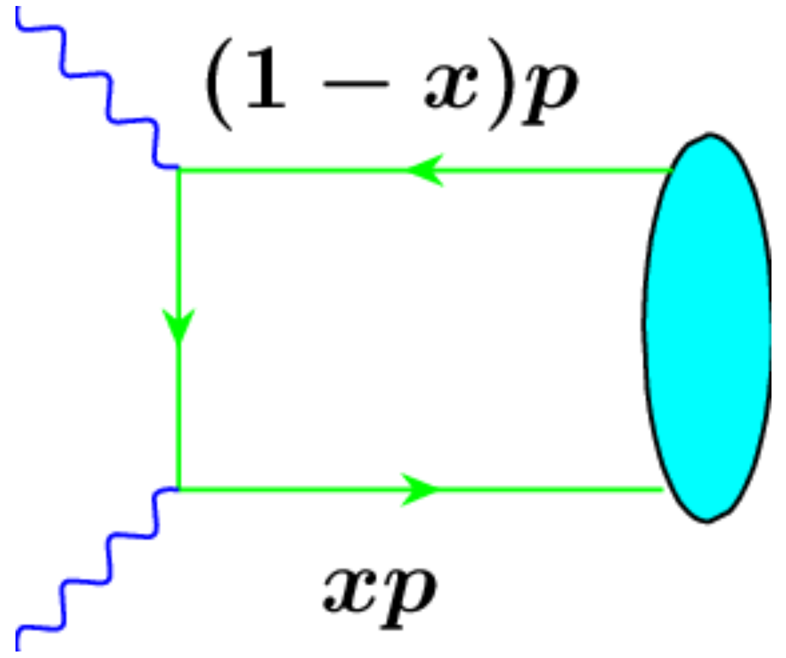
Quantum computing for LCDA

- ✦ LCDA - light cone distribution amplitude, describes the probability amplitude to find a pair of valence $q\bar{q}$ inside a meson

- ✦ LCDA is an essential ingredient in exclusive high-energy QCD processes, e.g. form factor in the process $\gamma^*\gamma \to \pi^0$

$$F(Q^2) = f_\pi \int_0^1 dx\, T_H(x, Q^2; \mu)\phi_\pi(x; \mu) + \mathcal{O}(\Lambda^2_{\mathrm{QCD}}/Q^2)$$

$$\phi(x) = \frac{1}{f} \int dz e^{-i(x-1)n\cdot Pz} \langle \Omega | \bar{\psi}(zn)\gamma^+\psi(0) | h(P) \rangle$$



- ✦ The current knowledge on LCDA is limited, mainly on models and lattice calculations

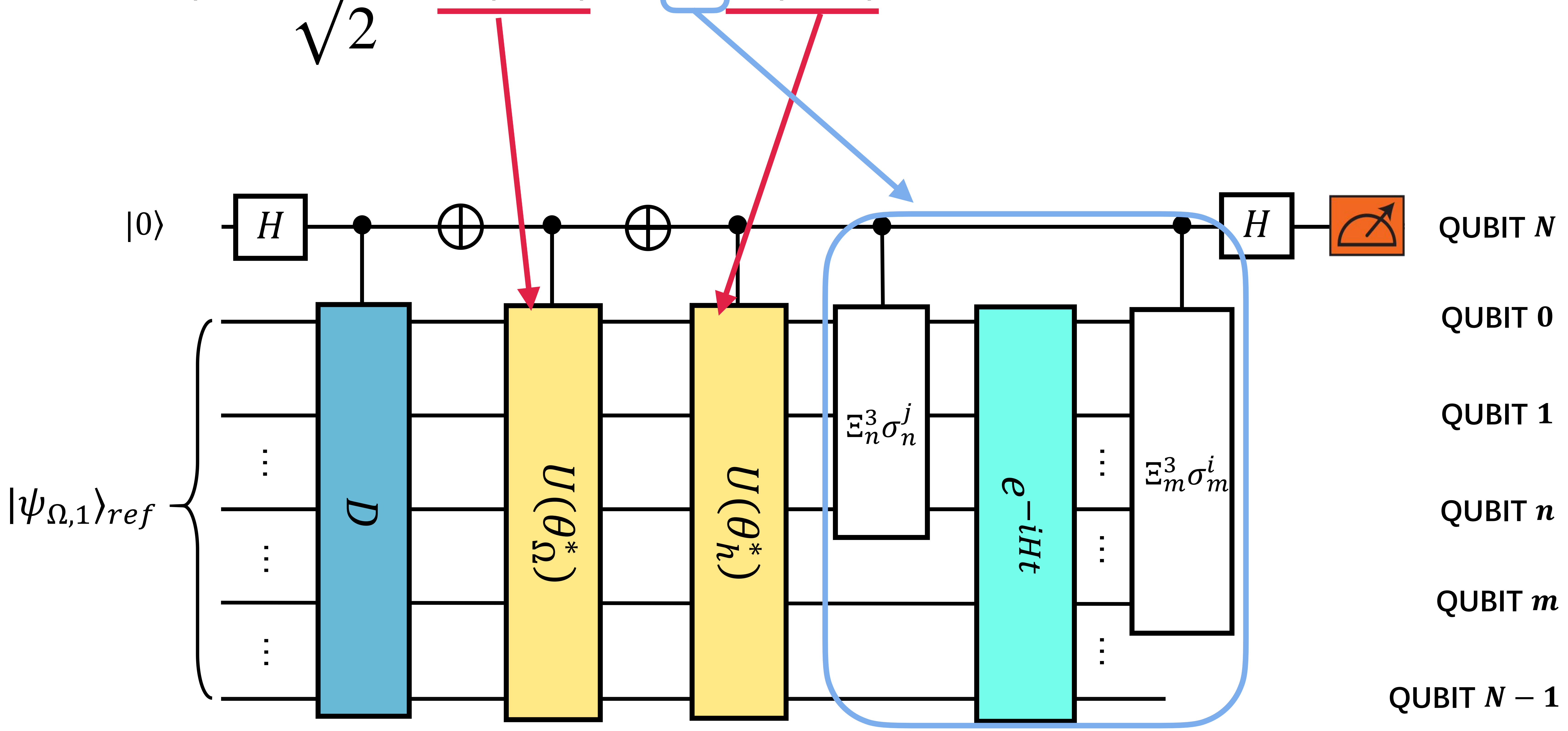
- ✦ First try using quantum computing

LCDA on quantum computer

✦ Quantum circuit

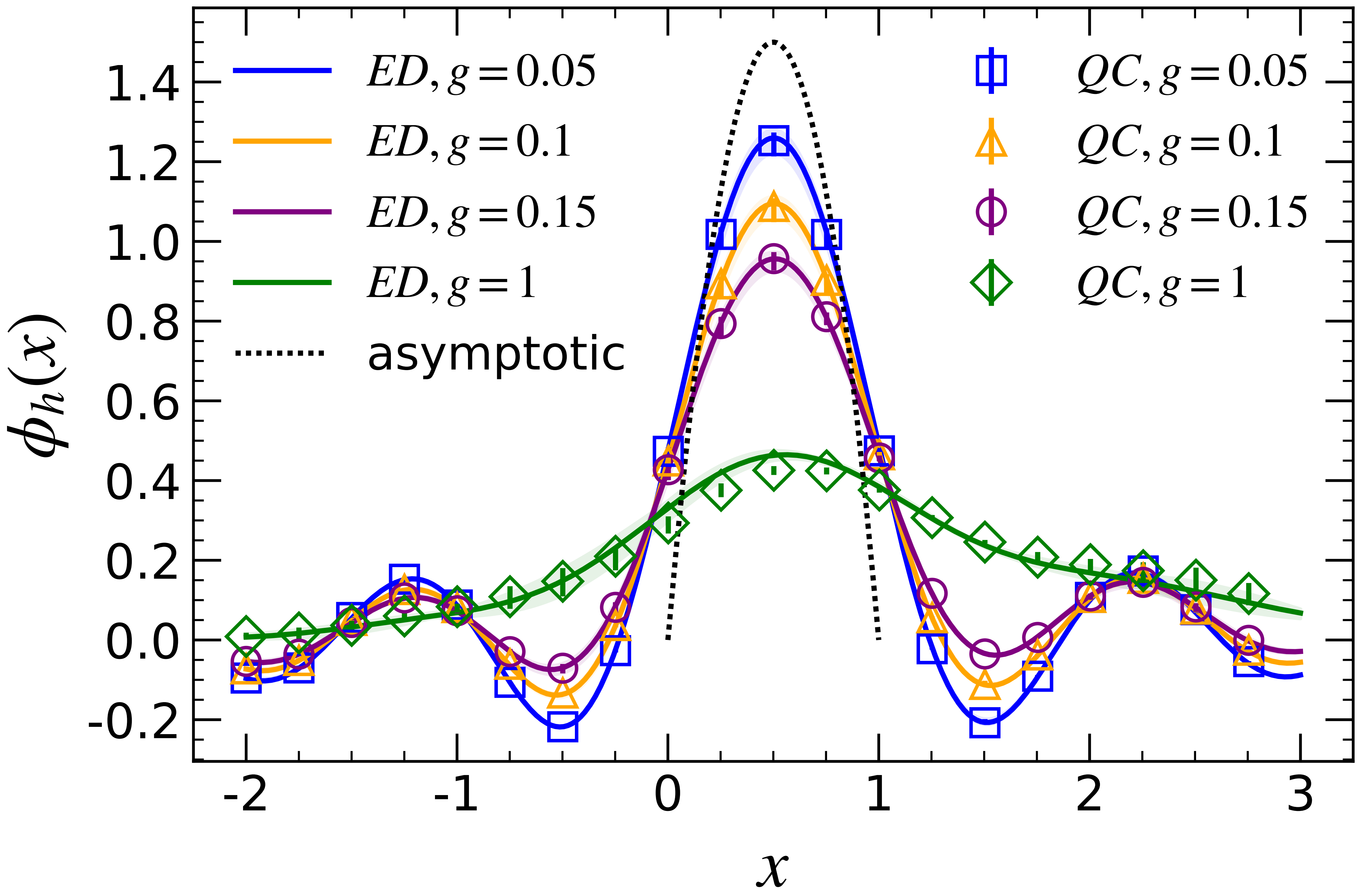
Li et al (QuNu), SCPMA (2023)

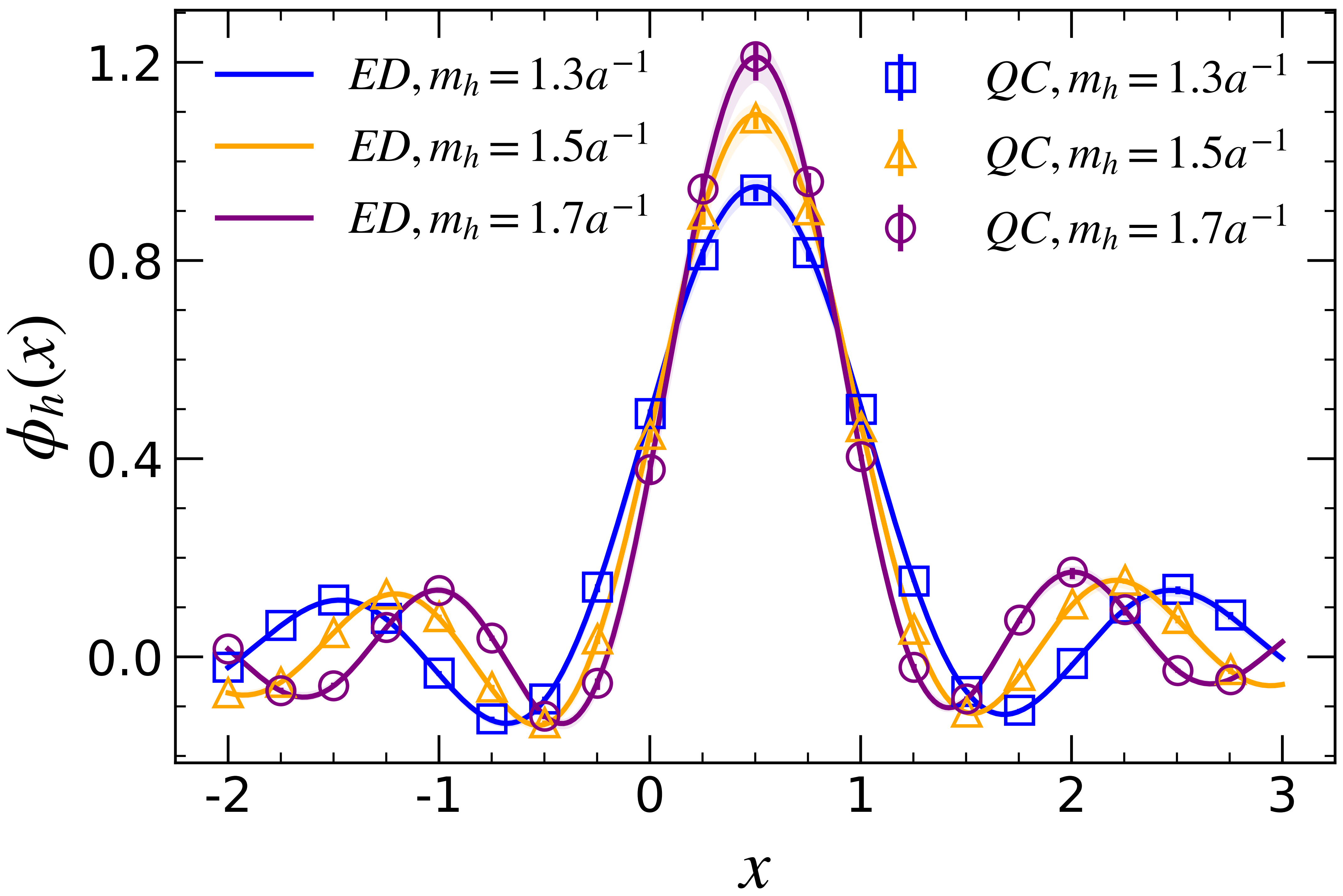
$$|\phi'\rangle = \frac{1}{\sqrt{2}}(|\Omega\rangle|0\rangle + \hat{O}|h\rangle|1\rangle)$$



LCDA on quantum computer

✦ Numerical results





- peak gets narrower with decreasing coupling constant or increasing hadron mass
- Converges to asymptotic result in weak coupling limit

Summary and outlook

- First attempt for direct computing of "PDFs"

  1. Use NJL model as a proof of concept study
  2. parton distribution function and light cone distribution amplitude

- The field is still at early age of development, many more need to be done

  1. gauge field encoding
  2. Extend to higher dimensions for TMDs and spin dependent processes
  3. quantum noise

Thanks for your attention!