

Evolution of structure functions at NLO without PDFs

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- Structure functions will be measured at Electron-Ion Collider (EIC)

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 - | Parametrize non-observable quantities
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- Physical basis set of linearly independent DIS observables
- DGLAP evolution of observables in a physical basis
 - | Avoiding the problems with PDFs
 - | More straightforward to compare to experimental data

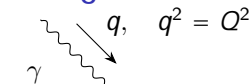
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- The novelty of our work:
 - | Momentum space
 - | Full three-flavor basis at NLO
- Continuation for LO physical basis [2304.06998](#)

Straightforward example with only two observables



$$q, \quad q^2 = Q^2$$

$$F_i(x, Q^2) = \sum_j C_{F_i f_j}(Q^2, \mu^2) f_j(\mu^2),$$

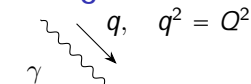
where $F_i = F_2, F_L/\frac{\alpha_s}{2\pi}$, and $f_j = , g$

Quark singlet:

$$f_q(x, \mu^2) = \sum_q^{n_f} [q(x, \mu^2) + \bar{q}(x, \mu^2)], \quad n_f = 3$$

Gluon PDF: $g(x, \mu^2)$

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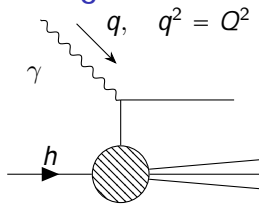
$$q(x, \mu^2) = \sum_q^{n_f} [q(x, \mu^2) + \bar{q}(x, \mu^2)], \quad n_f = 3$$

Gluon PDF: $g(x, \mu^2)$

First step: invert the linear mapping (difficult because $f = \int_x^1 \frac{dz}{z} f(z) g(\frac{x}{z})$)

$$f_j(\mu^2) = \sum_i C_{F_i f_j}^{-1}(Q^2, \mu^2) F_i(Q^2) + O(\alpha_s^2)$$

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DGLAP evolution in physical basis

$$\begin{aligned} \frac{dF_i(x, Q^2)}{d \log(Q^2)} &= \sum_j \frac{dC_{F_i f_j}(Q^2, \mu^2)}{d \log(Q^2)} f_j(\mu^2) \\ &= \sum_j \frac{dC_{F_i f_j}(Q^2, \mu^2)}{d \log(Q^2)} \sum_k C_{F_k f_j}^{-1}(Q^2, \mu^2) F_k(Q^2) + O(\alpha_s^3) \end{aligned}$$

Scheme and scale dependence at NLO

DGLAP evolution in physical basis:

$$\begin{aligned}\frac{dF_i(x, Q^2)}{d \log(Q^2)} &= \sum_j \frac{dC_{F_i f_j}(Q^2, \mu^2)}{d \log(Q^2)} \sum_k C_{F_k f_j}^{-1}(Q^2, \mu^2) F_k(Q^2) + O(\alpha_s^3) \\ &= \sum_k P_{ik} F_k(Q^2) + O(\alpha_s^3)\end{aligned}$$

Kernels P_{ik} are independent of the factorization scheme and scale

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Kernels P_{ik} are independent of the factorization scheme and scale

P_{ij} 's determined by:

- Splitting functions
 - Coefficient functions
- ! The scheme and scale dependence exactly cancels between these two

Inverting the gluon PDF at NLO

Simple case without quarks

$$\text{Invert } g(x) \text{ from } \tilde{F}_L = C_{F_L g}^{(1)} g + \frac{\alpha_s(Q^2)}{2\pi} C_{F_L g}^{(2)} g \quad \tilde{F}_L(x; Q^2) = \frac{2\pi}{\alpha_s(Q^2)} \frac{F_L(x, Q^2)}{x}$$

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$$\text{Define differential operator } \hat{P}(x) = \frac{1}{8T_R n_f e_q^2} \left[x^2 \frac{d^2}{dx^2} + 2x \frac{d}{dx} + 2 \right]$$

$$\text{Notice } g(x) = \hat{P}(x) \left[C_{F_L g}^{(1)} g \right]$$

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$$\text{Get } C_{F_L g}^{(1)} g \text{ from } \tilde{F}_L: \quad C_{F_L g}^{(1)} g = \tilde{F}_L - \frac{\alpha_s(Q^2)}{2\pi} C_{F_L g}^{(2)} g$$

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$$\text{Define differential operator } \hat{P}(x) = \frac{1}{8T_R n_f e_q^2} \left[x^2 \frac{d^2}{dx^2} - 2x \frac{d}{dx} + 2 \right]$$

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Notice $g(x) = \hat{P}(x) \left[C_{F_L g}^{(1)} g \right]$

Get $C_{F_L g}^{(1)} g$ from \tilde{F}_L : $C_{F_L g}^{(1)} g = \tilde{F}_L - \frac{\alpha_s(Q^2)}{2\pi} C_{F_L g}^{(2)} g$

$$g(x) = \hat{P}(x) \left[\tilde{F}_L(x) - \frac{\alpha_s(Q^2)}{2\pi} C_{F_L g}^{(2)} g \right]$$

Plug in $g(x) = \hat{P}(x)\tilde{F}_L(x) + O(\alpha_s(Q^2))$ to the right hand side

$$g(x) = \hat{P}(x)\tilde{F}_L(x) - \frac{\alpha_s(Q^2)}{2\pi} \hat{P}(x) \left[C_{F_L g}^{(2)} \hat{P}\tilde{F}_L \right] + O(\alpha_s^2(Q^2))$$

Comparison with conventional DGLAP evolution

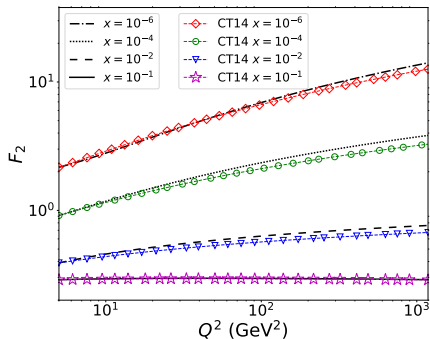
Physical basis evolution

- Renormalization scheme in $\alpha_s(\mu_r^2)$
- Perturbative truncation
! sum rule not exact
- Parametization of observable quantities

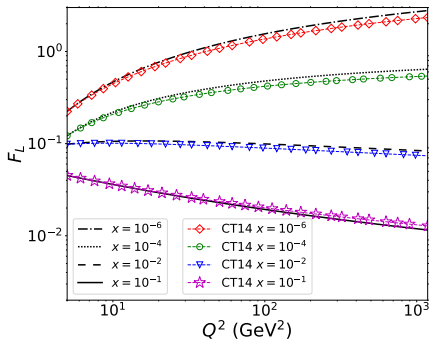
Evolution with PDFs

- Factorization scheme and scale
- Renormalization scheme in $\alpha_s(\mu_r^2)$
- Easy to enforce an exact sum rule
- Parametization of non-observable quantities

Comparison with conventional DGLAP evolution



F_2 CT14 NLO



F_L CT14 NLO

Differences in values from:

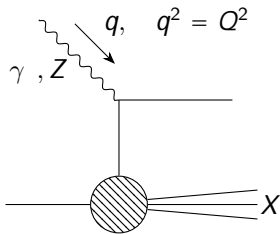
- uncertainty in PDFs from factorization scheme and scale dependence (error band not shown)
- perturbative truncation

Six observable basis (work in progress)

- Full three-flavor basis: $u, u, d, d, s = s$, and g
 - ! Need six linearly independent DIS structure functions

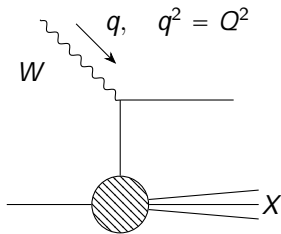
Six observable basis (work in progress)

- Full three-flavor basis: $u, u, d, d, s = s$, and g
! Need six linearly independent DIS structure functions
- We choose the NLO structure functions:



Neutral current γ, Z

- γ exchange ! F_2 and F_L
- Z boson exchange ! F_3



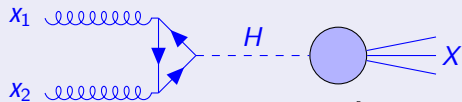
Charged current W

W exchange ! F_2^W , F_3^W , and F_{2c}^W

(Calculations done, numerics in construction)

Cross sections in terms of physical basis

Example of Higgs production by gluon fusion

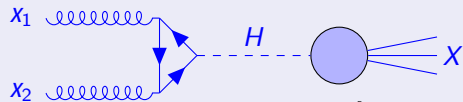


$$\sigma(p + p \rightarrow H + X) = \int dx_1 dx_2 g(x_1; \mu) g(x_2; \mu) \hat{\sigma}_{gg \rightarrow H+X}(x_1; x_2; \frac{m_H^2}{2});$$

where m_H is the Higgs mass, $g(x_1; \mu)$ and $g(x_2; \mu)$ are the gluon PDFs

Cross sections in terms of physical basis

Example of Higgs production by gluon fusion



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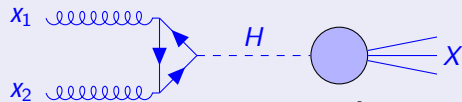
where m_H is the Higgs mass, $g(x_i; \mu^2)$ and $\hat{\sigma}_{gg \rightarrow H+X}(x_1, x_2; \frac{m_H^2}{2})$ are the **gluon PDFs**

Plug in the gluon PDF in physical basis: $g(x, \mu^2) = \sum_j C_{jg}^{-1}(Q^2, \mu^2) F_j(Q^2)$

where $F_j = F_2; F_L = \frac{\alpha_s}{2\pi}; F_3; F_2^W; F_3^W; F_{2c}^W$

Cross sections in terms of physical basis

Example of Higgs production by gluon fusion



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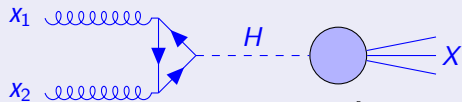
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Cross sections in terms of physical basis

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Harland-Lang and Thorne 1811.08434: explicit μ dependence vanishes and terms $\log(Q^2/m_H^2)$ are left behind

! no need to choose relation between μ and Q or m_H

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 F_2 , F_L , F_3 , F_2^W , F_3^W , and F_{2c}^W
- Scheme dependence of PDFs starts to play part at NLO in α_s
! By using the NLO physical basis, we are able to avoid scheme dependence
- What next:
 - | Study how LHC cross sections, e.g. Drell-Yan, are expressed in physical basis
 - | Obtain physical basis including also heavy quarks

Backup: Inverting the gluon PDF

Gluon PDF in mellin space

$$g(n) = \frac{1}{C_{F_L g}^{(1)}(n)} \left[\frac{1}{e_q^2} \tilde{F}_L(n) - C_{F_L}^{(1)}(n) g(n) \right]$$

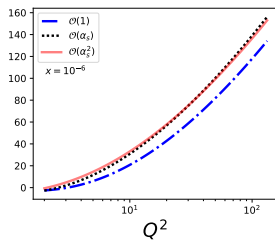
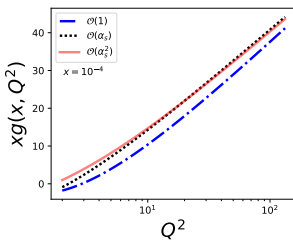
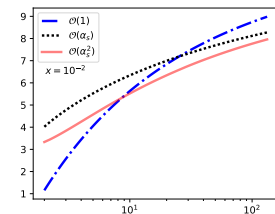
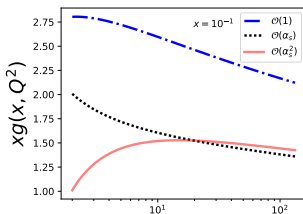
$$\frac{1}{C_{F_L g}^{(1)}(n)} = \frac{1}{8 T_R Z_0^n} \int_0^1 dz z^{n+2} \delta^{(0)}(z - z_0),$$

where $z_0 \in]0, 1[$ is an arbitrary constant that cancels in final result.

$$g(x; Q^2) = \int_x^1 \frac{dz}{z} (1 - z) \left\{ \frac{C_F}{4 T_R n_f e_q^2} \left[\frac{x}{z} \frac{d}{dz} - 2 \right] \frac{F_2\left(\frac{x}{z}; Q^2\right)}{\frac{x}{z}} \right. \\ \left. + \frac{2}{s(Q^2)} \frac{1}{8 T_R n_f e_q^2} \left[\frac{x^2}{z^2} \frac{d^2}{d\left(\frac{x}{z}\right)^2} - 2 \frac{x}{z} \frac{d}{dz} + 2 \right] \frac{F_L\left(\frac{x}{z}; Q^2\right)}{\frac{x}{z}} \right\} \\ \frac{1}{n_f e_q^2} \left\{ C_{g\tilde{F}_2^0} \tilde{F}_2^0 + C_{g\tilde{F}_2} \tilde{F}_2 + C_{g\tilde{F}^{00}_L} \tilde{F}^{00}_L + C_{g\tilde{F}^0_L} \tilde{F}^0_L + C_{g\tilde{F}_L} \tilde{F}_L \right\}$$

Backup: Gluon PDF in physical basis

NLO structure functions $F_i = \sum_j C_{ij} f_j$, where $F_i = F_2, F_L/\alpha_s$ and $f_j = g, g$
 Structure functions calculated using CT14nlo_NF3

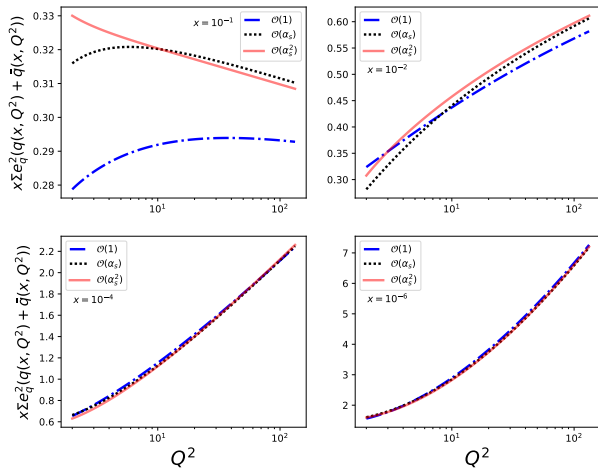


$$xg(x; Q^2) = x \sum_i C_{ig}^{-1} F_i.$$

Perturbative truncation of C_{ig}^{-1} to the orders $0/s$, $1/s$, and $2/s$ is shown.

Backup: Quark singlet in physical basis

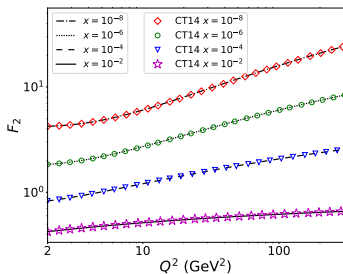
NLO structure functions $F_i = \sum_j C_{ij} f_j$, where $F_i = F_2, F_L/\alpha_s$ and $f_j = \dots, g$
 Structure functions calculated using CT14nlo_NF3



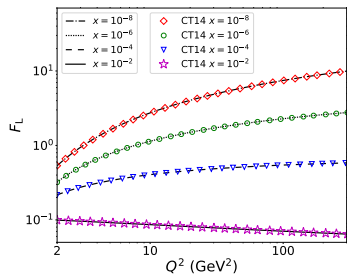
$$x \Sigma e_q^2(q(x; Q^2) + \bar{q}(x; Q^2)) = x \sum_i C_{ig}^{-1} F_i.$$

Perturbative truncation of C_{ig}^{-1} to the orders $\mathcal{O}(1)$, $\mathcal{O}(\alpha_s)$, and $\mathcal{O}(\alpha_s^2)$ is shown.

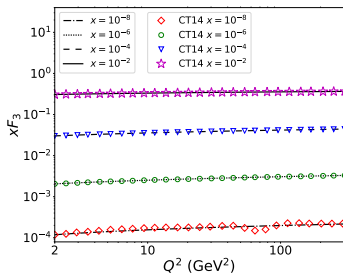
Backup: LO DGLAP evolution



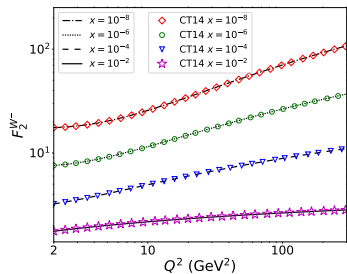
F_2 CT14 LO



F_L CT14 LO



xF_3 CT14 LO



F_2^W CT14 LO