Evolution of structure functions at NLO without PDFs

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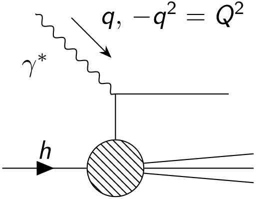
- The novelty of our work:
  - Momentum space
  - Full three-flavor basis at NLO
- Continuation for LO physical basis 2304.06998

Straightforward example with only two observables



$$F_i(x, Q^2) = \sum_j C_{F_i f_j}(Q^2, \mu^2) \otimes f_j(\mu^2),$$

where $F_i = F_2, F_{\mathrm{L}}/\frac{\alpha_{\mathrm{s}}}{2\pi}$, and $f_j = \Sigma, g$

Quark singlet:
$\Sigma(x, \mu^2) = \sum_q^{n_{\mathrm{f}}} \left[q(x, \mu^2) + \overline{q}(x, \mu^2)\right]$, $n_{\mathrm{f}} = 3$
Gluon PDF: $g(x, \mu^2)$

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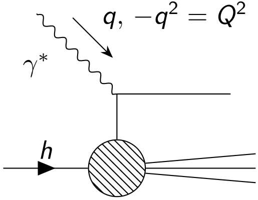

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First step: invert the linear mapping (difficult because $f \otimes g = \int_x^1 \frac{\mathrm{d}z}{z} f(z) g\left(\frac{x}{z}\right)$)

$f_j(\mu^2) = \sum_i C^{-1}_{F_i f_j}(Q^2, \mu^2) \otimes F_i(Q^2) + \mathcal{O}(\alpha_{\mathrm{s}}^2)$

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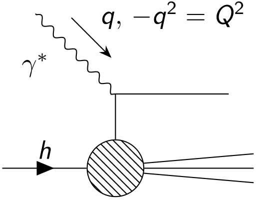

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DGLAP evolution in physical basis

$$\frac{\mathrm{d}F_i(x, Q^2)}{\mathrm{d}\log(Q^2)} = \sum_j \frac{\mathrm{d}C_{F_i f_j}(Q^2, \mu^2)}{\mathrm{d}\log(Q^2)} \otimes f_j(\mu^2)$$

$$= \sum_j \frac{\mathrm{d}C_{F_i f_j}(Q^2, \mu^2)}{\mathrm{d}\log(Q^2)} \otimes \sum_k C^{-1}_{F_k f_j}(Q^2, \mu^2) \otimes F_k(Q^2) + \mathcal{O}(\alpha_\mathrm{s}^3)$$

Scheme and scale dependence at NLO

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$\mathcal{P}_{ij}$'s determined by:

- Splitting functions
- Coefficient functions
  $\longrightarrow$ The scheme and scale dependence exactly cancels between these two

Inverting the gluon PDF at NLO

Simple case without quarks

Invert $g(x)$ from $\widetilde{F}_{\rm L} = C^{(1)}_{F_{\rm L}g} \otimes g + \frac{\alpha_{\rm s}(Q^2)}{2\pi} C^{(2)}_{F_{\rm L}g} \otimes g$ $\qquad$ $\widetilde{F}_{\rm L}(x, Q^2) \equiv \frac{2\pi}{\alpha_{\rm s}(Q^2)} \frac{F_{\rm L}(x,Q^2)}{x}$

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Define differential operator $\hat{P}(x) \equiv \frac{1}{8 T_{\mathrm{R}} n_{\mathrm{f}} \bar{e}_q^2} \left[x^2 \frac{\mathrm{d}^2}{\mathrm{d}x^2} - 2x \frac{\mathrm{d}}{\mathrm{d}x} + 2 \right]$

Notice $g(x) = \hat{P}(x) \left[C^{(1)}_{F_{\mathrm{L}}g} \otimes g \right]$

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Get $C^{(1)}_{F_{\mathrm{L}}g} \otimes g$ from $\widetilde{F}_{\mathrm{L}}$: $\quad C^{(1)}_{F_{\mathrm{L}}g} \otimes g = \widetilde{F}_{\mathrm{L}} - \frac{\alpha_{\mathrm{s}}(Q^2)}{2\pi} C^{(2)}_{F_{\mathrm{L}}g} \otimes g$

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Plug in $g(x) = \hat{P}(x) \widetilde{F}_{\mathrm{L}}(x) + \mathcal{O}\left(\alpha_{\mathrm{s}}(Q^2)\right)$ to the right hand side

$$g(x) = \hat{P}(x) \widetilde{F}_{\mathrm{L}}(x) - \frac{\alpha_{\mathrm{s}}(Q^2)}{2\pi} \hat{P}(x) \left[C^{(2)}_{F_{\mathrm{L}}g} \otimes \hat{P} \widetilde{F}_{\mathrm{L}} \right] + \mathcal{O}\left(\alpha_{\mathrm{s}}^2(Q^2)\right)$$

Comparison with conventional DGLAP evolution

Physical basis evolution

- Renormalization scheme in $\alpha_s(\mu_r^2)$
- Perturbative truncation
  $\longrightarrow$ sum rule not exact
- Parametization of observable quantities

Evolution with PDFs

- Factorization scheme and scale
- Renormalization scheme in $\alpha_s(\mu_r^2)$
- Easy to enforce an exact sum rule
- Parametization of non-observable quantities

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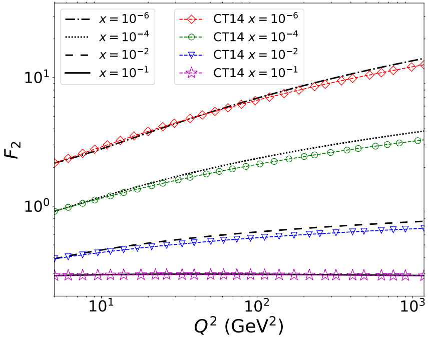
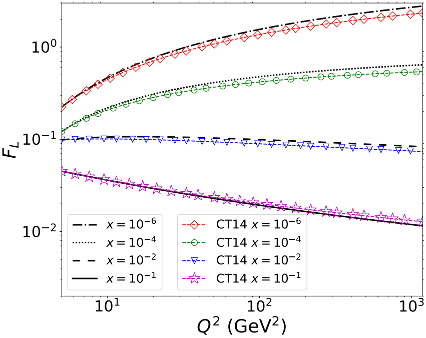


F_2 CT14 NLO



F_{L} CT14 NLO

Differences in values from:

- uncertainty in PDFs from factorization scheme and scale dependence (error band not shown)
- perturbative truncation

Six observable basis (work in progress)

- Full three-flavor basis: $u, \bar{u}, d, \bar{d}, s = \bar{s}$, and g
  $\longrightarrow$ Need six linearly independent DIS structure functions

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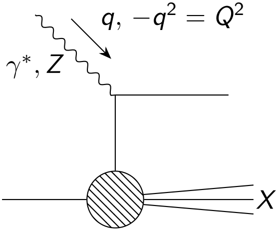
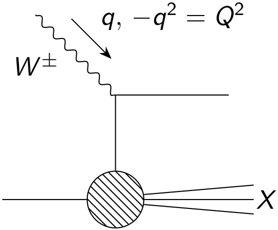
- We choose the NLO structure functions:





Neutral current γ^*, Z

- γ^* exhange $\rightarrow F_2$ and F_{L}
- Z boson exhange $\rightarrow F_3$

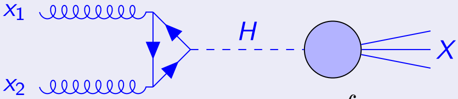
Charged current $W^\pm$

W^- exhange $\rightarrow F_2^{\mathrm{W}^-}$, $F_3^{\mathrm{W}^-}$, and $F_{2c}^{\mathrm{W}^-}$

(Claculations done, numerics in construction)

Cross sections in terms of physical basis

Example of Higgs production by gluon fusion



$$\sigma(p + p \longrightarrow \mathrm{H} + X) = \int \mathrm{d}x_1 \mathrm{d}x_2 g(x_1, \mu) g(x_2, \mu) \hat{\sigma}_{gg \to H+X}(x_1, x_2, \frac{m_H^2}{\mu^2}),$$

where m_H is the Higgs mass, $g(x_1, \mu)$ and $g(x_2, \mu)$ are the gluon PDFs

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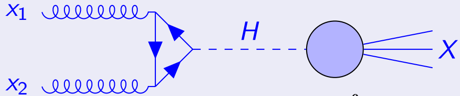

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Plug in the gluon PDF in physical basis: $g(x,\mu^2) = \sum_j C_{jg}^{-1}(Q^2,\mu^2) \otimes F_j(Q^2)$

where $F_j = F_2, F_{\mathrm{L}}/\frac{\alpha_{\mathrm{s}}}{2\pi}, F_3, F_2^{\mathrm{W}^-}, F_3^{\mathrm{W}^-}, F_{2\mathrm{c}}^{\mathrm{W}^-}$

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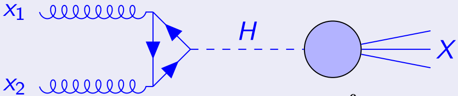

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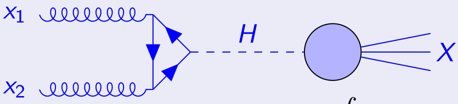

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Harland-Lang and Thorne 1811.08434: explicit μ dependence vanishes and terms $\log\left(Q^2/m_H^2\right)$ are left behind
$\longrightarrow$ no need to choose relation between μ and Q or m_H

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- Scheme dependence of PDFs starts to play part at NLO in α_{s}
  $\longrightarrow$ By using the NLO physical basis, we are able to avoid scheme dependence
- What next:
  - Study how LHC cross sections, e.g. Drell-Yan, are expressed in physical basis
  - Obtain physical basis including also heavy quarks

Backup: Inverting the gluon PDF

Gluon PDF in mellin space

$$g(n) = \frac{1}{C^{(1)}_{F_{\mathrm{L}}g}(n)} \left[\frac{1}{\bar{e}_q^2} \widetilde{F}_{\mathrm{L}}(n) - C^{(1)}_{F_{\mathrm{L}}\Sigma}(n)\Sigma(n) \right]$$

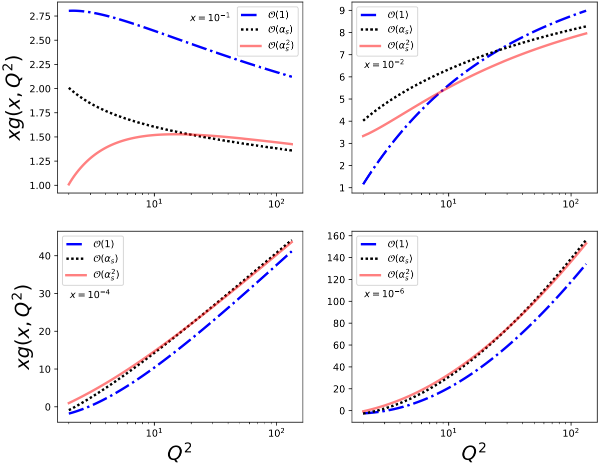
$$\frac{1}{C^{(1)}_{F_{\mathrm{L}}g}(n)} = \frac{1}{8\, T_{\mathrm{R}}\, z_0^n} \int_0^1 \mathrm{d}z z^{n+2} \delta''(z - z_0),$$

where $z_0 \in]0,1[$ is an arbitrary constant that cancels in final result.

$$\begin{aligned} g(x,Q^2) = & \int_x^1 \frac{\mathrm{d}z}{z} \delta(1-z) \left\{ \frac{C_{\mathrm{F}}}{4\, T_{\mathrm{R}}\, n_{\mathrm{f}} \bar{e}_q^2} \left[\frac{x}{z} \frac{\mathrm{d}}{\mathrm{d}\frac{x}{z}} - 2 \right] \frac{F_2\left(\frac{x}{z}, Q^2\right)}{\frac{x}{z}} \right. \\ & \left. + \frac{2\pi}{\alpha_{\mathrm{s}}(Q^2)} \frac{1}{8\, T_{\mathrm{R}}\, n_{\mathrm{f}} \bar{e}_q^2} \left[\frac{x^2}{z^2} \frac{\mathrm{d}^2}{\mathrm{d}\left(\frac{x}{z}\right)^2} - 2\frac{x}{z} \frac{\mathrm{d}}{\mathrm{d}\frac{x}{z}} + 2 \right] \frac{F_{\mathrm{L}}\left(\frac{x}{z}, Q^2\right)}{\frac{x}{z}} \right\} \\ \equiv & \frac{1}{n_{\mathrm{f}} \bar{e}_q^2} \left\{ C_{g\widetilde{F'}_2} \otimes \widetilde{F'}_2 + C_{g\widetilde{F}_2} \otimes \widetilde{F}_2 + C_{g\widetilde{F''}_{\mathrm{L}}} \otimes \widetilde{F''}_{\mathrm{L}} + C_{g\widetilde{F'}_{\mathrm{L}}} \otimes \widetilde{F'}_{\mathrm{L}} + C_{g\widetilde{F}_{\mathrm{L}}} \otimes \widetilde{F}_{\mathrm{L}} \right\} \end{aligned}$$

Backup: Gluon PDF in physical basis

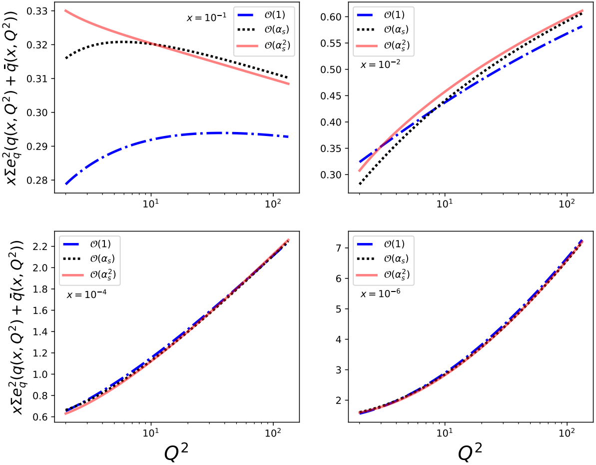
NLO structure functions $F_i = \Sigma_j C_{ij} \otimes f_j$, where $F_i = F_2, F_L/\alpha_s$ and $f_j = \Sigma, g$
Structure functions calculated using CT14nlo_NF3



$xg(x, Q^2) = x\Sigma_i C_{ig}^{-1} \otimes F_i$.
Perturbative truncation of C_{ig}^{-1} to the orders α_s^0, α_s^1, and α_s^2 is shown.

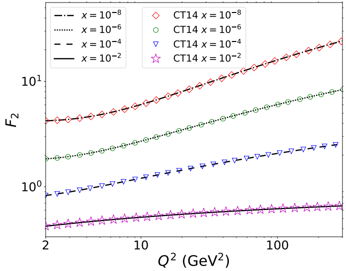
Backup: Quark singlet in physical basis

NLO structure functions $F_i = \Sigma_j C_{ij} \otimes f_j$, where $F_i = F_2, F_L/\alpha_\mathrm{s}$ and $f_j = \Sigma, g$
Structure functions calculated using CT14nlo_NF3

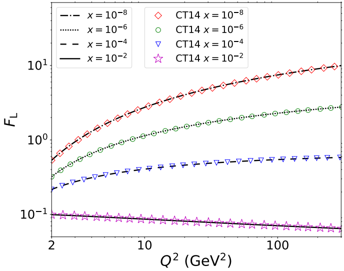


$x\Sigma e_q^2(q(x,Q^2) + \bar{q}(x,Q^2)) = x\Sigma_i C_{ig}^{-1} \otimes F_i.$
Perturbative truncation of C_{ig}^{-1} to the orders α_s^0, α_s^1, and α_s^2 is shown.

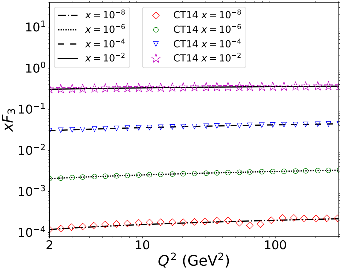
Backup: LO DGLAP evolution



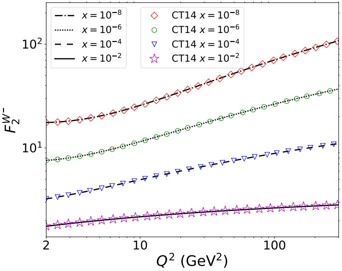
F_2 CT14 LO



F_{L} CT14 LO



xF_3 CT14 LO



$F_2^{\mathrm{W}^-}$ CT14 LO