DOE contracts:
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Systematic uncertainty of off-shell corrections and higher-twist contribution in DIS at large x

Matteo Cerutti

CTEQ-JLab Collaboration

**A. Accardi, I. Fernando, X. Jing, S. Li, J. Owens, S. Park,
C.E. Keppel, W. Melnitchouk, P. Monaghan**

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April 10, 2024

Motivations

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We have to deal with Deuterium target at large-x

Deuterium: nuclear smearing

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Nuclear impulse approximation

Melnitchouk, Schreiber, Thomas, PRD 49 (1994)
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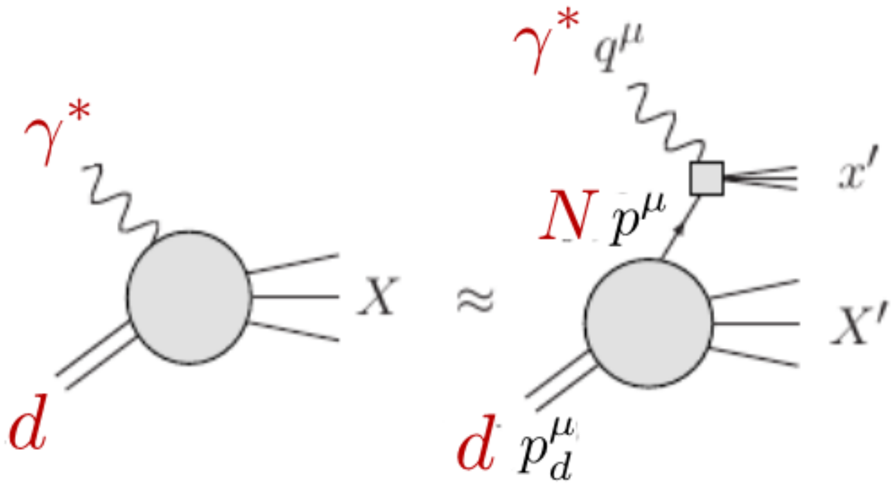

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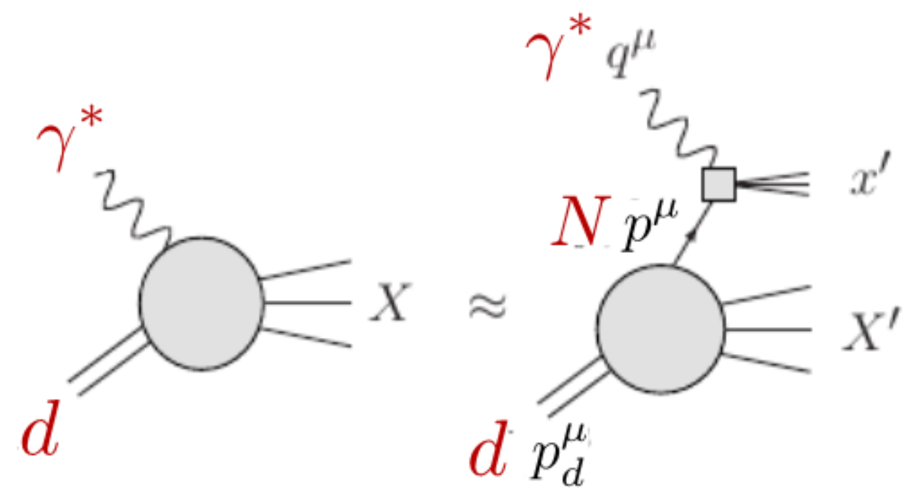

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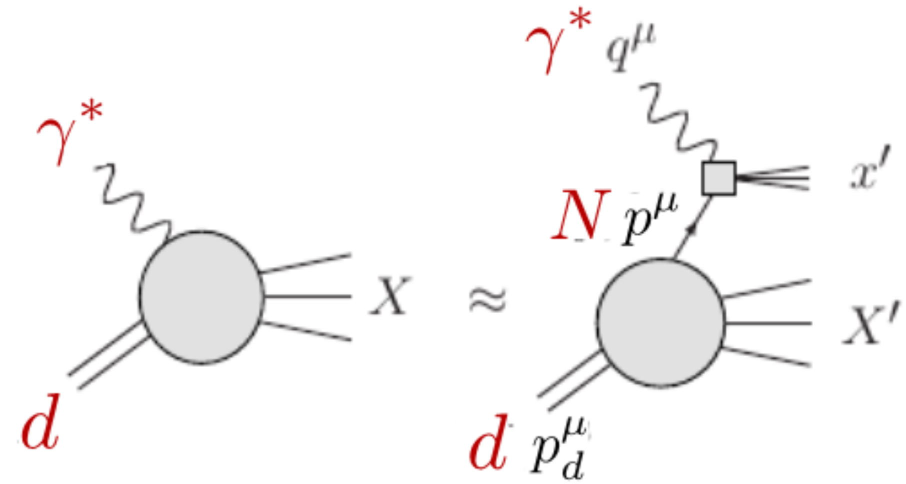

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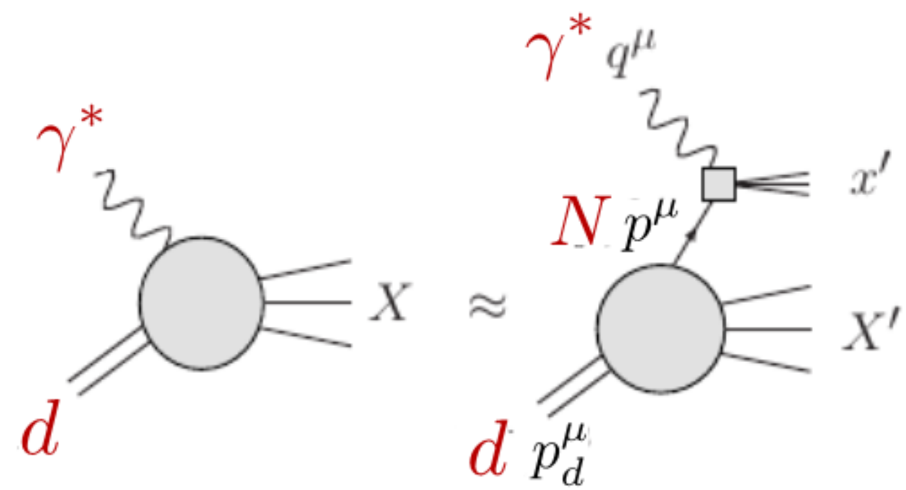

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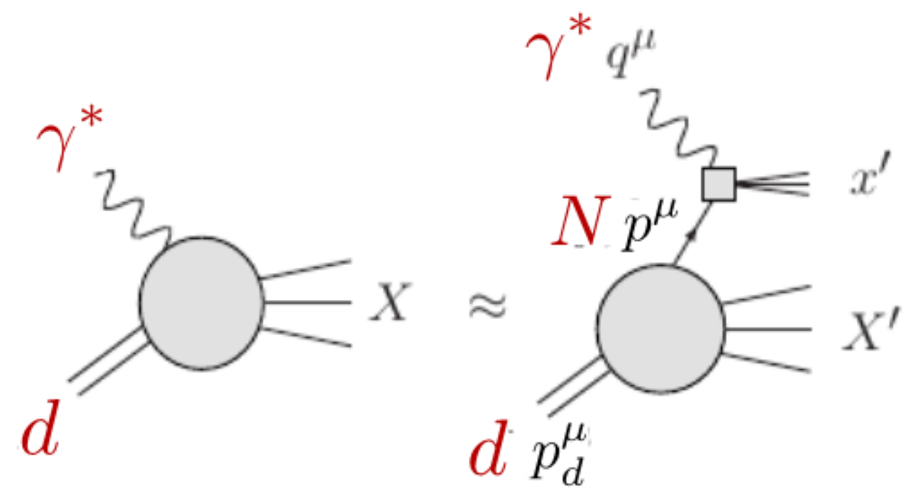

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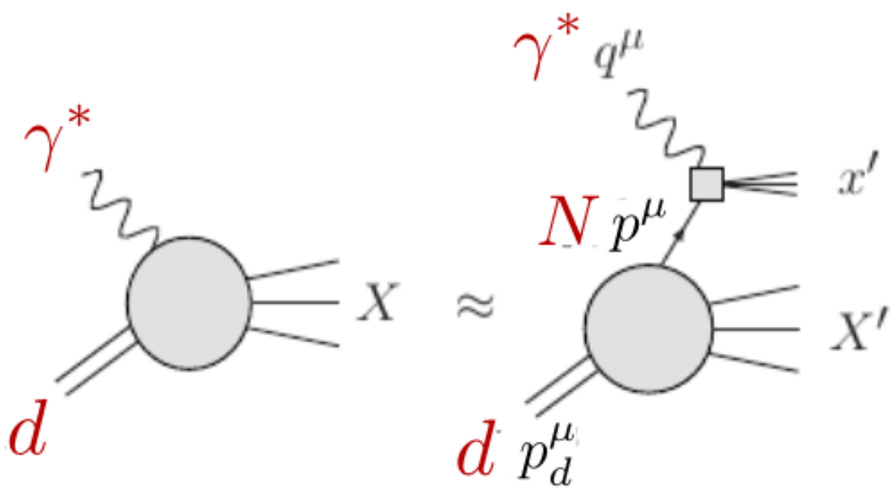

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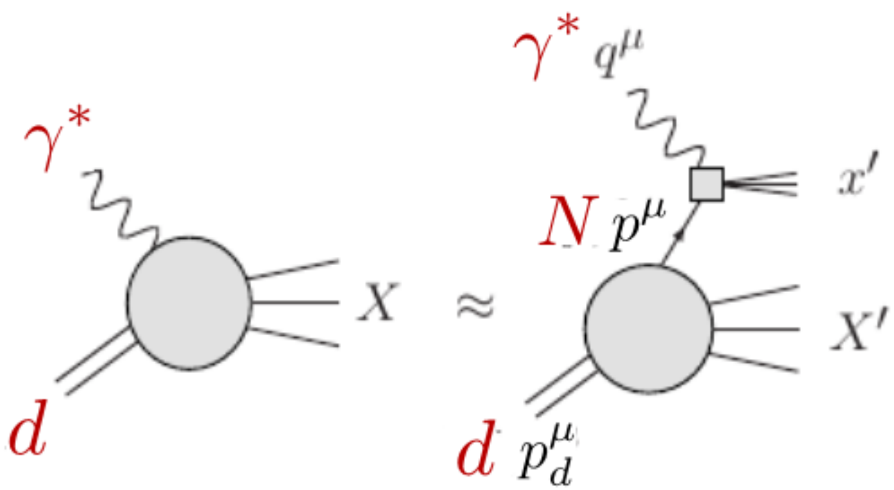

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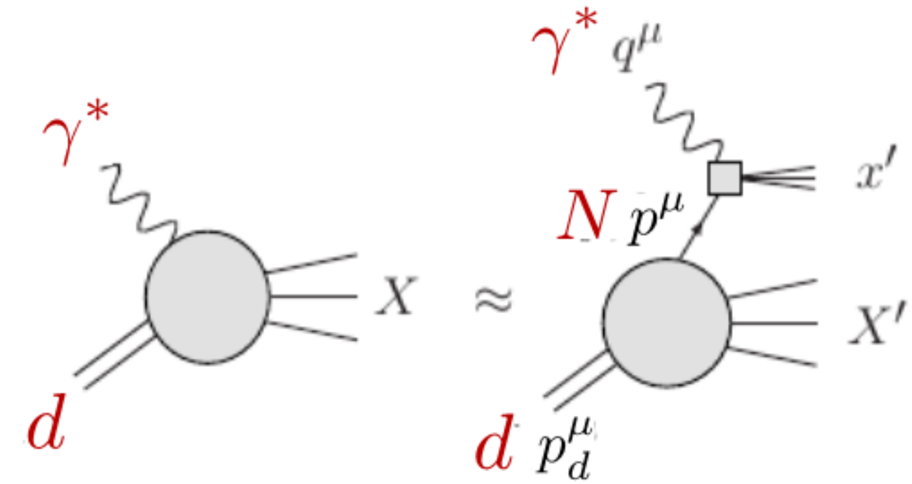

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Structure function

of a bound, off-shell nucleon

Deuterium: off-shell corrections

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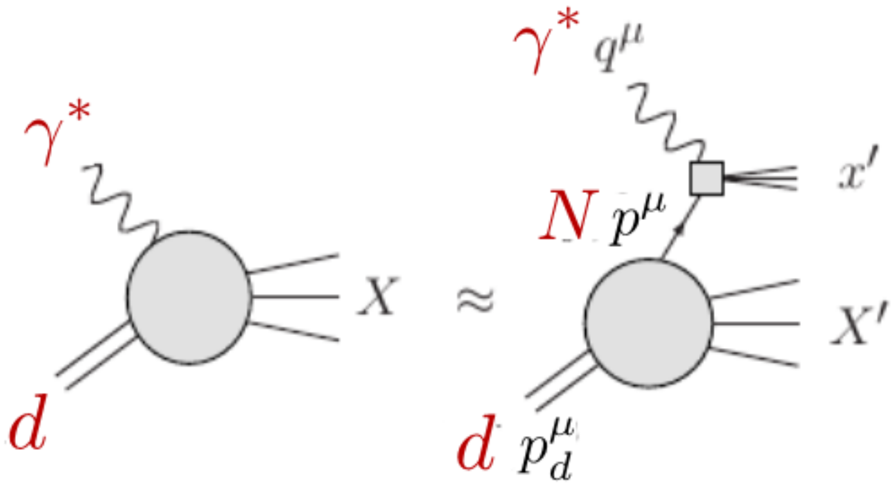
Bound, off-shell nucleon inside the deuteron



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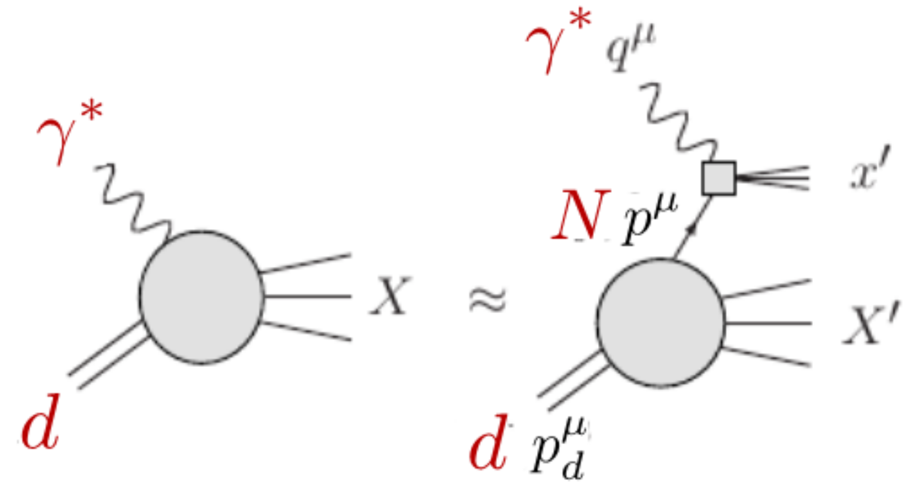

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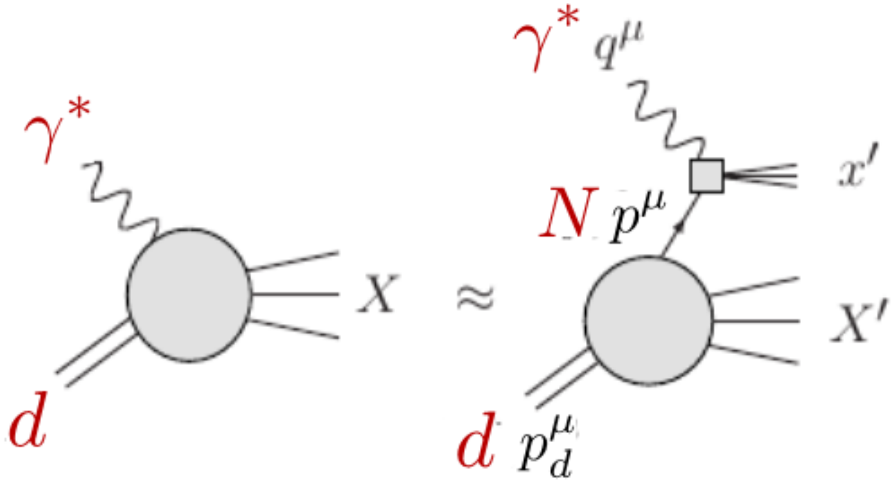

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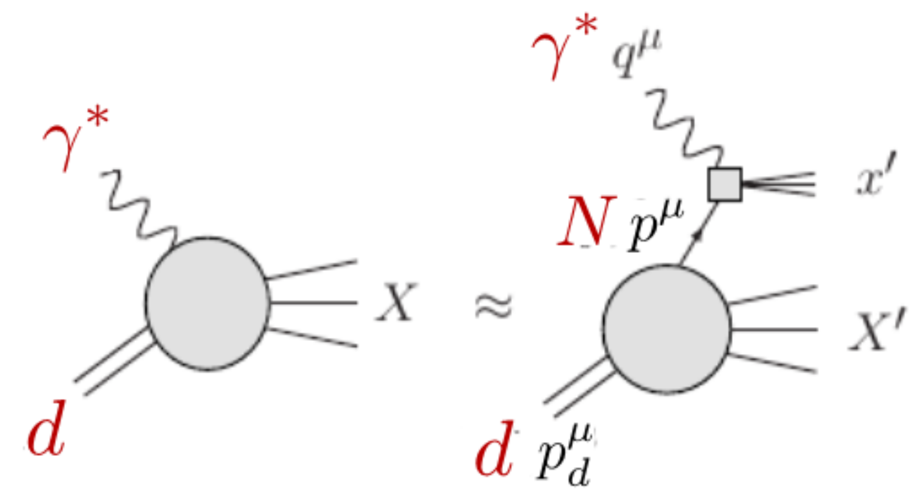

Off-shell expansion (in nucleon virtuality p^2)

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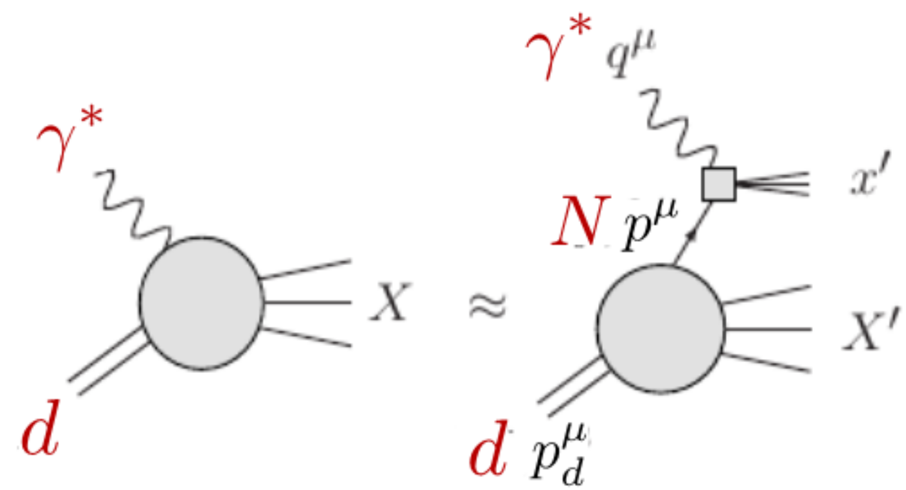

Off-shell expansion (in nucleon virtuality p^2)

parton level

$$q_N(x, Q^2, p^2) = q_N^{\text{free}}(x, Q^2)\left[1 + \frac{p^2 - M^2}{M^2}\delta f(x)\right]$$

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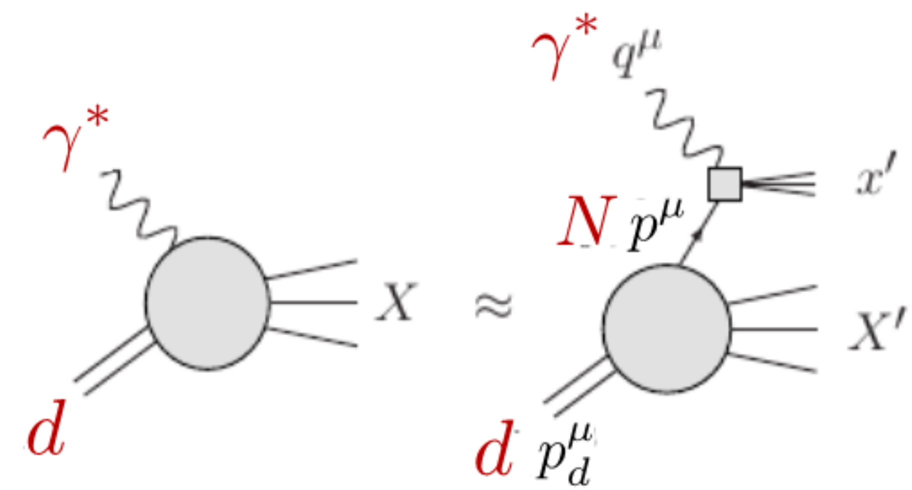

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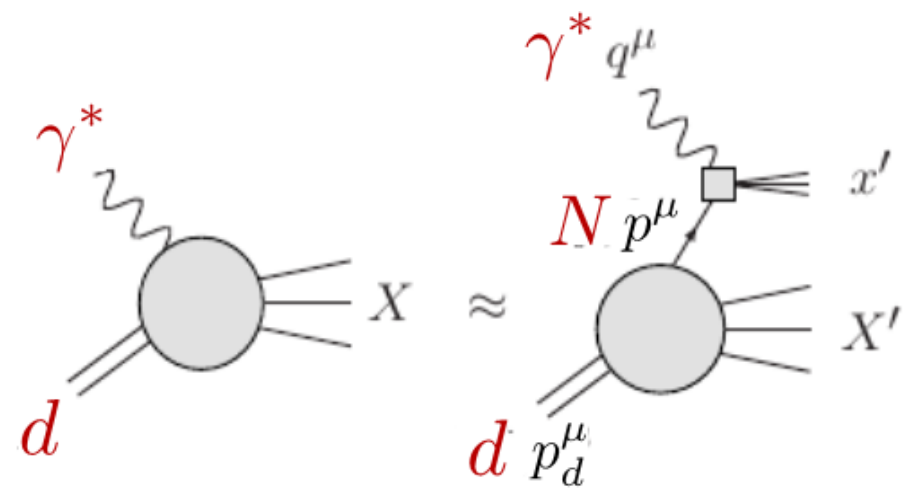

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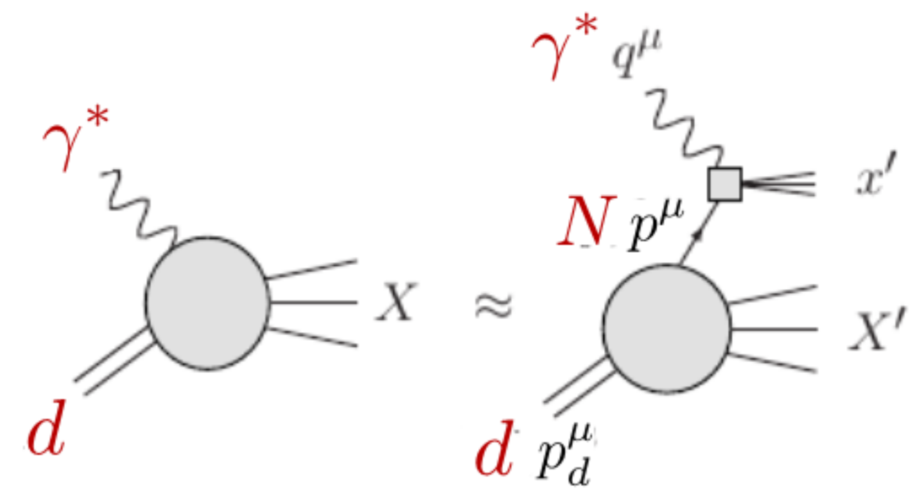

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Off-shell function
(To be fitted)

Latest results from QCD fits in CJ framework

CJ15 fit

Accardi, Brady, et al., PRD 93 (2016)

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CJ15 fit

$$\delta f^N = C(x-x_0)(x-x_1)(1+x_0-x)$$

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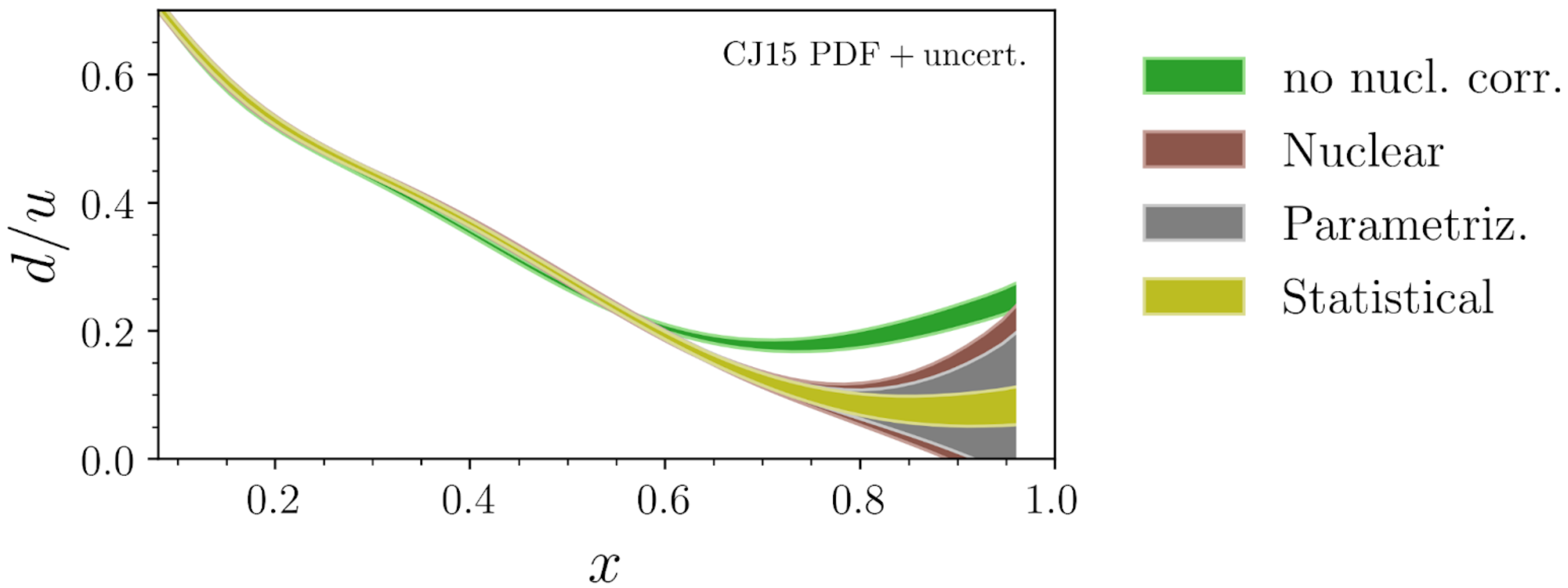

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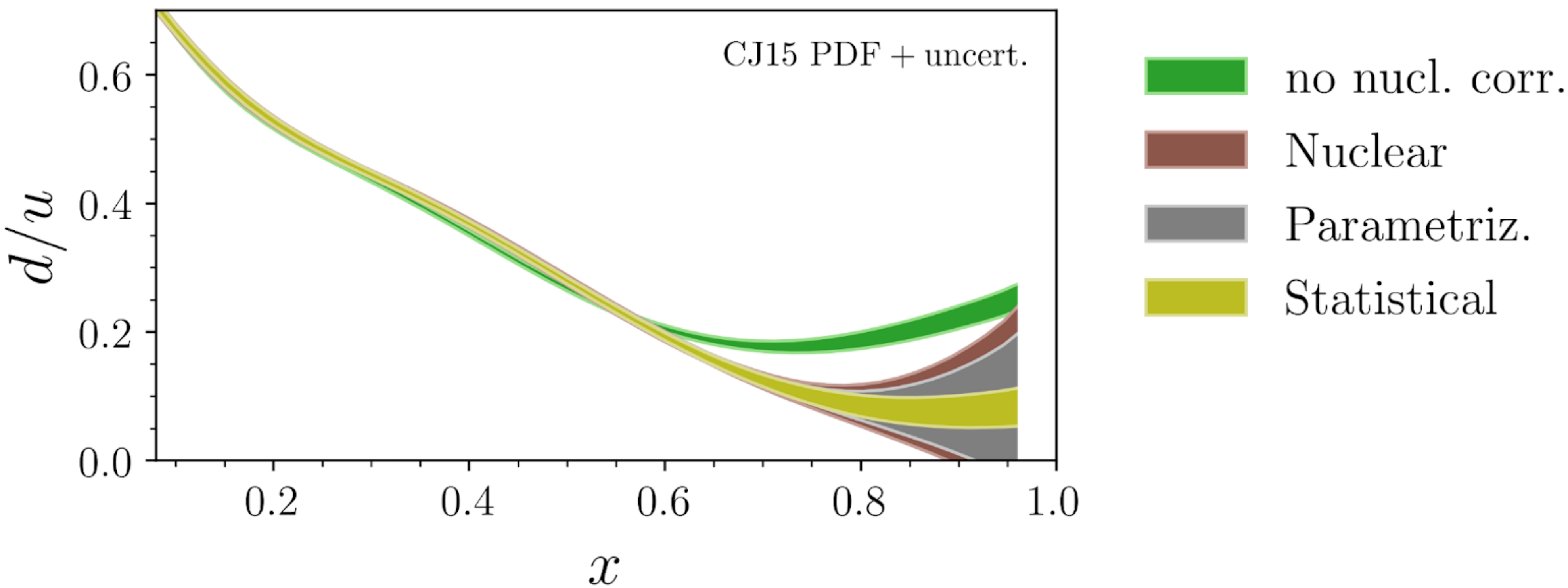

The inclusion of nuclear corrections is impacting the output of the fit

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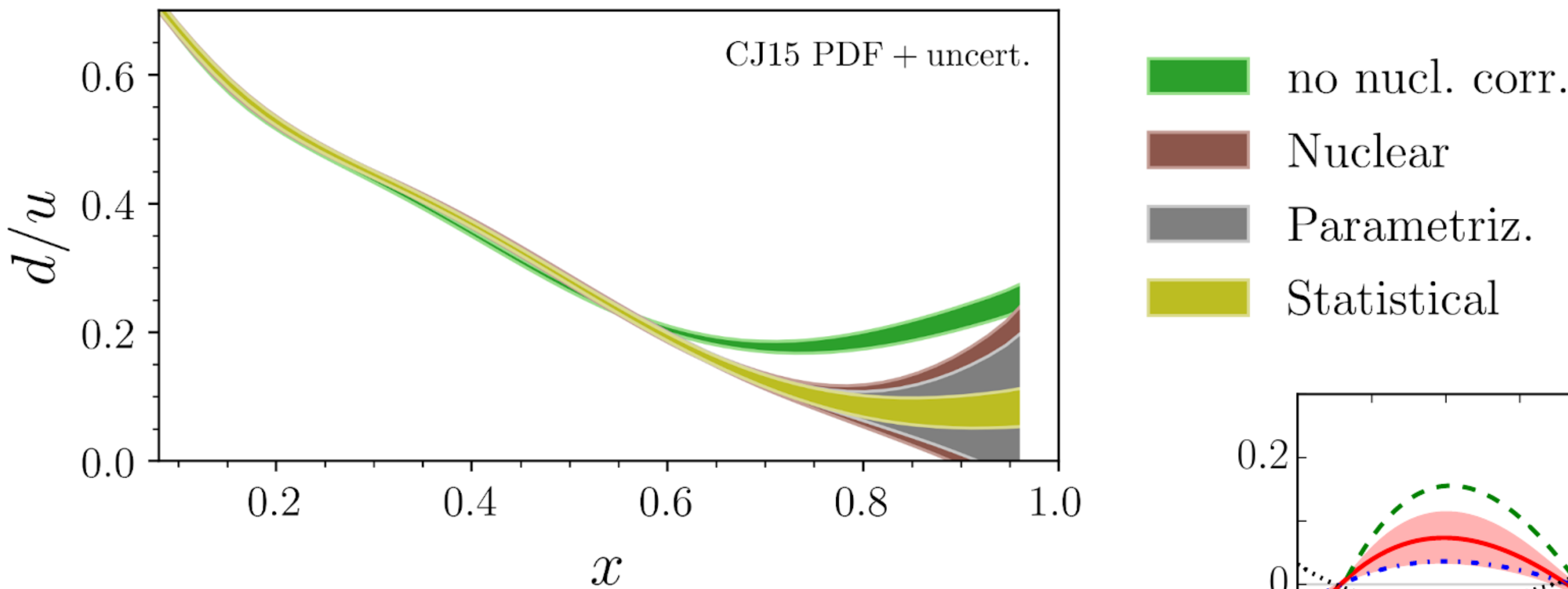

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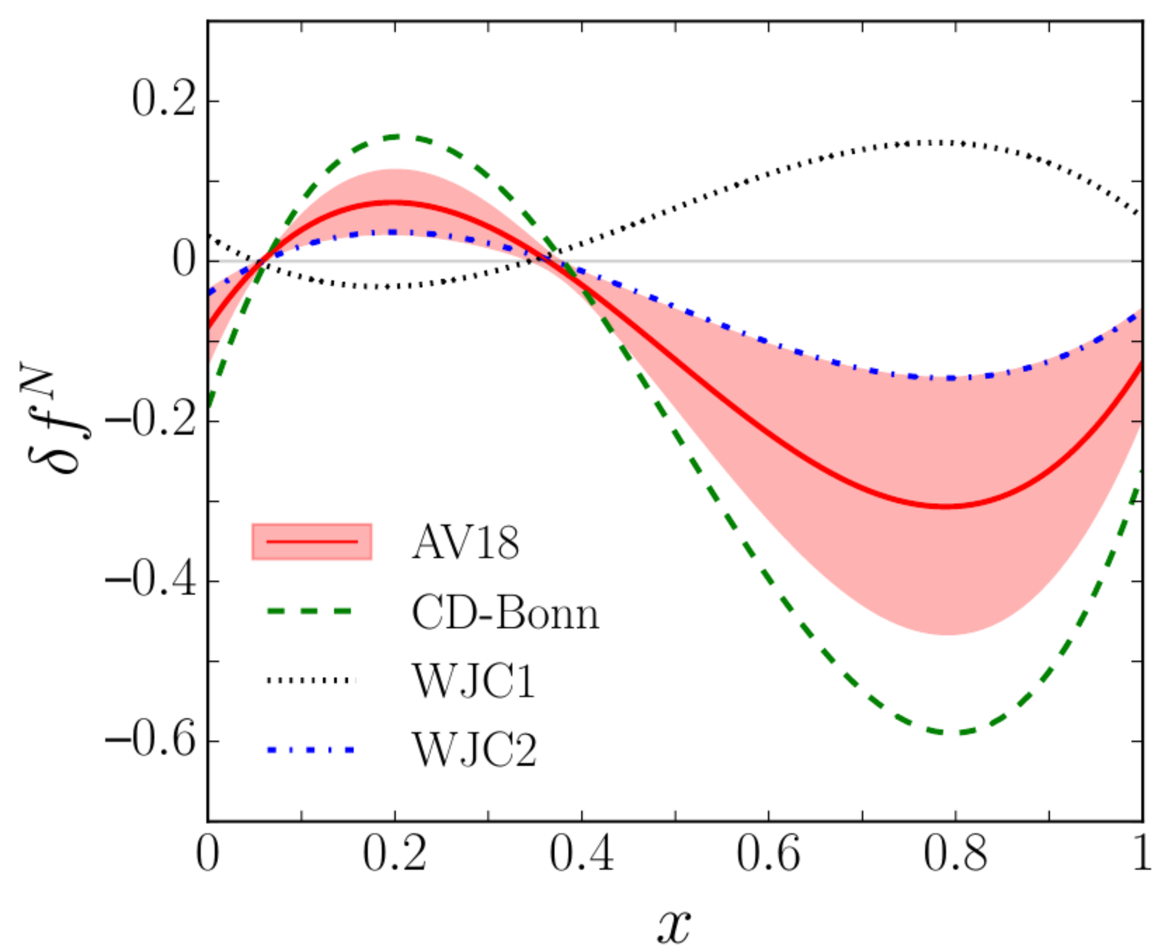

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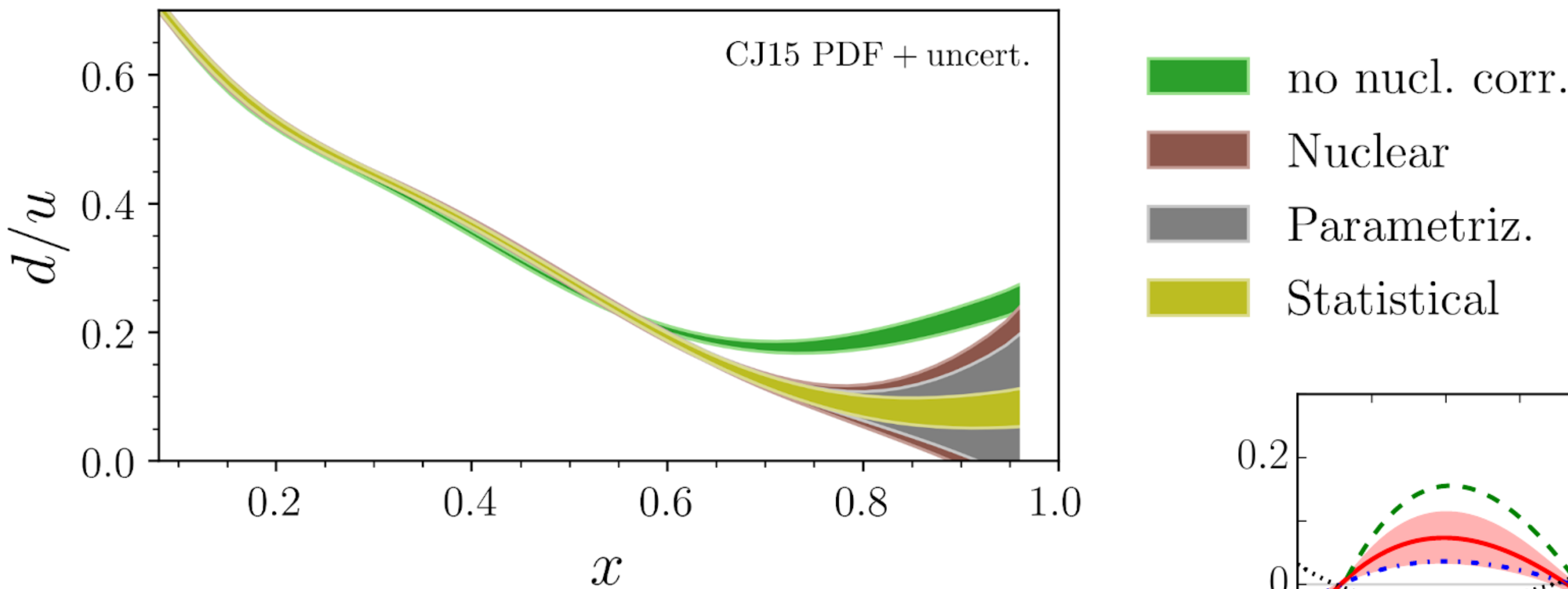

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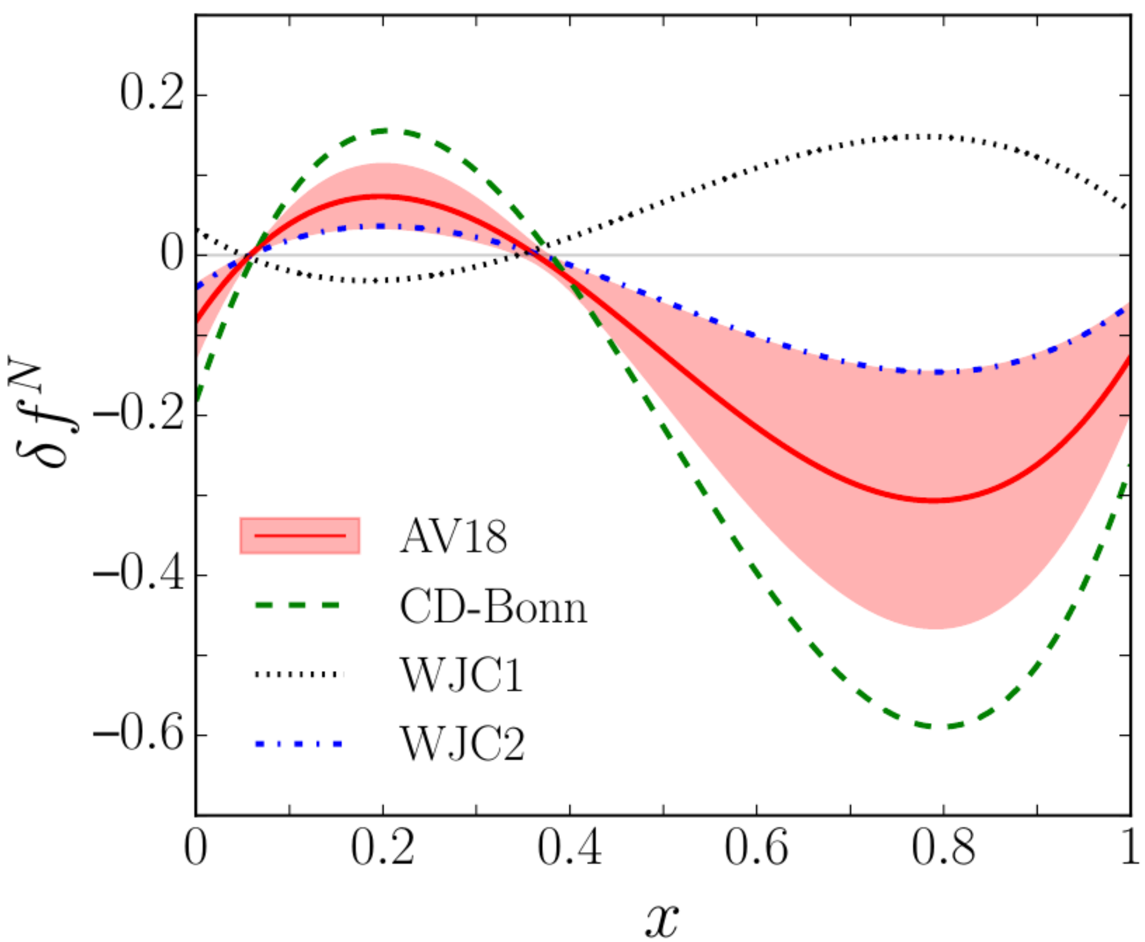
The most of the recent nuclear potentials does not introduce a bias on the fit



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CJ22 fit

Accardi, Jing, Owens et al., PRD 107 (2023)

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Same off-shell parameterization

More flexible parameterization of sea quarks (NuSea and SeaQuest data)

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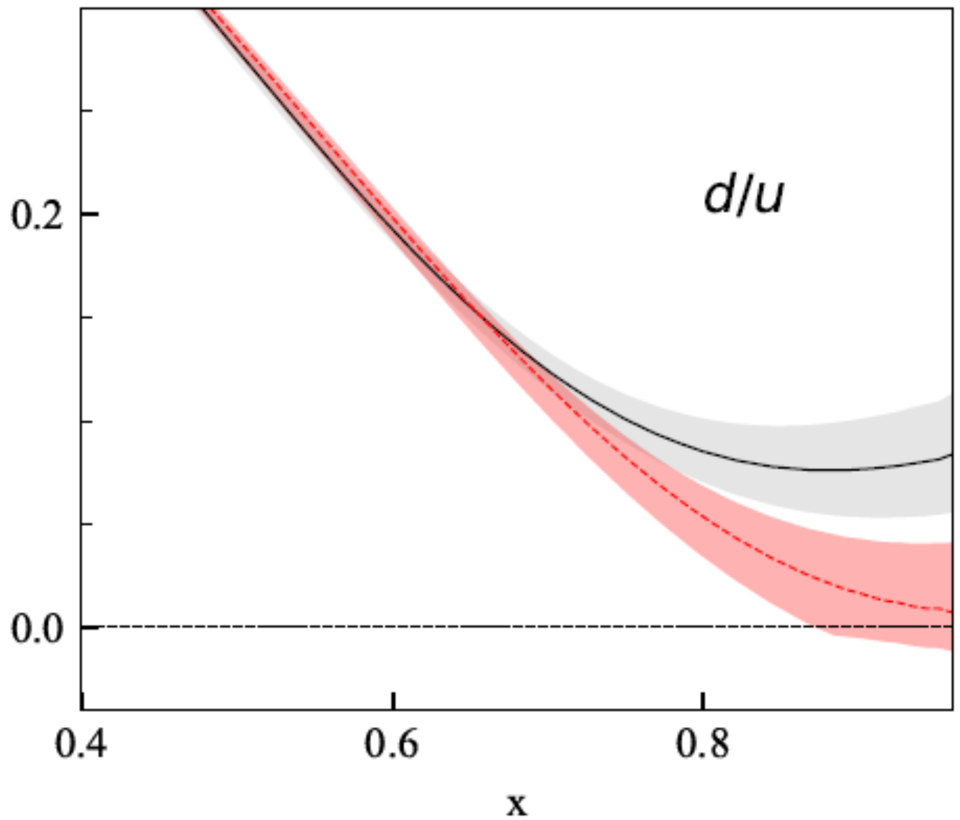

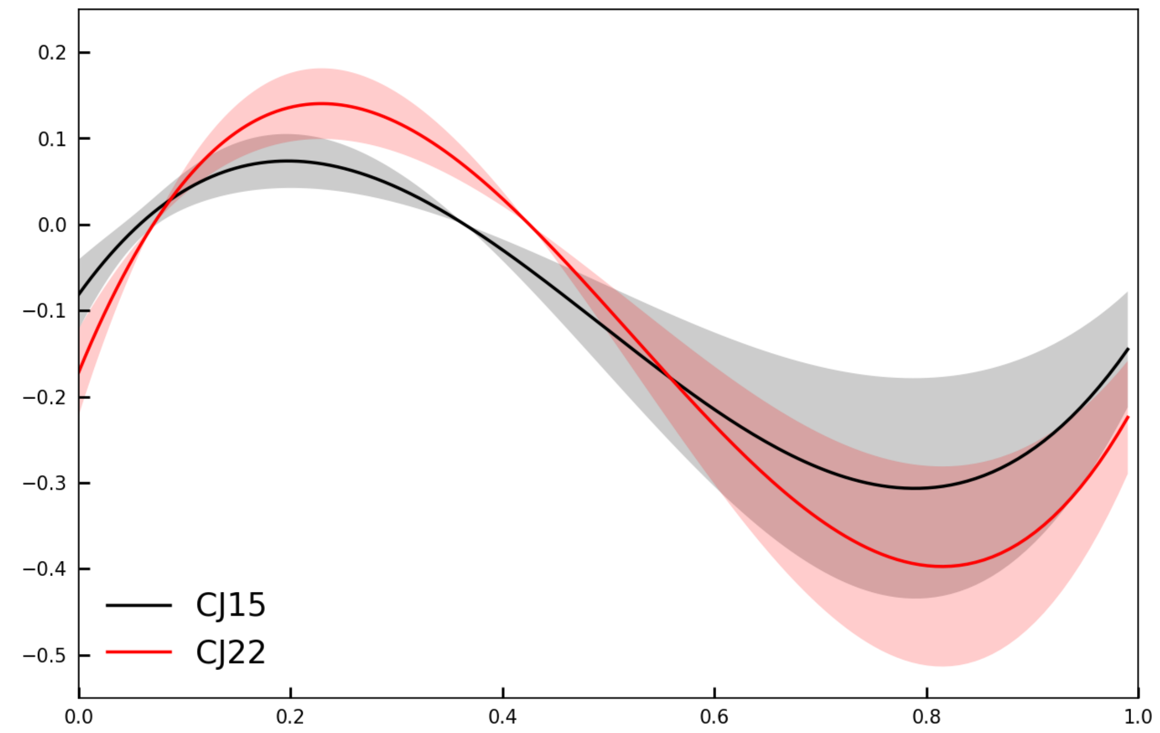

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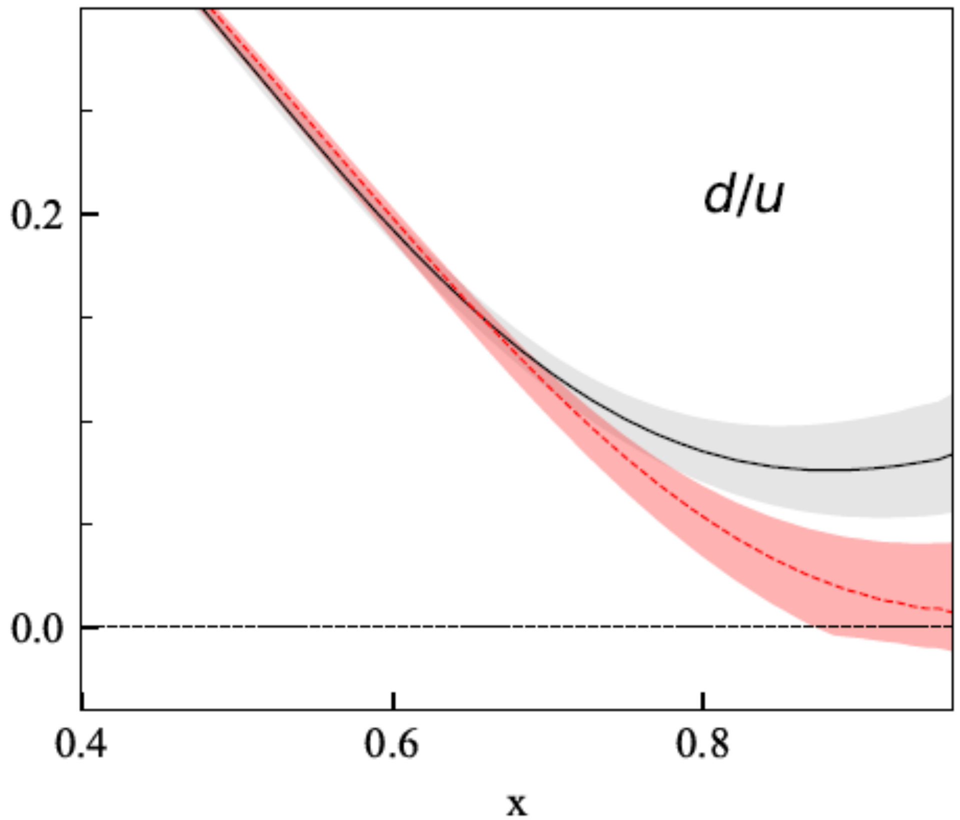

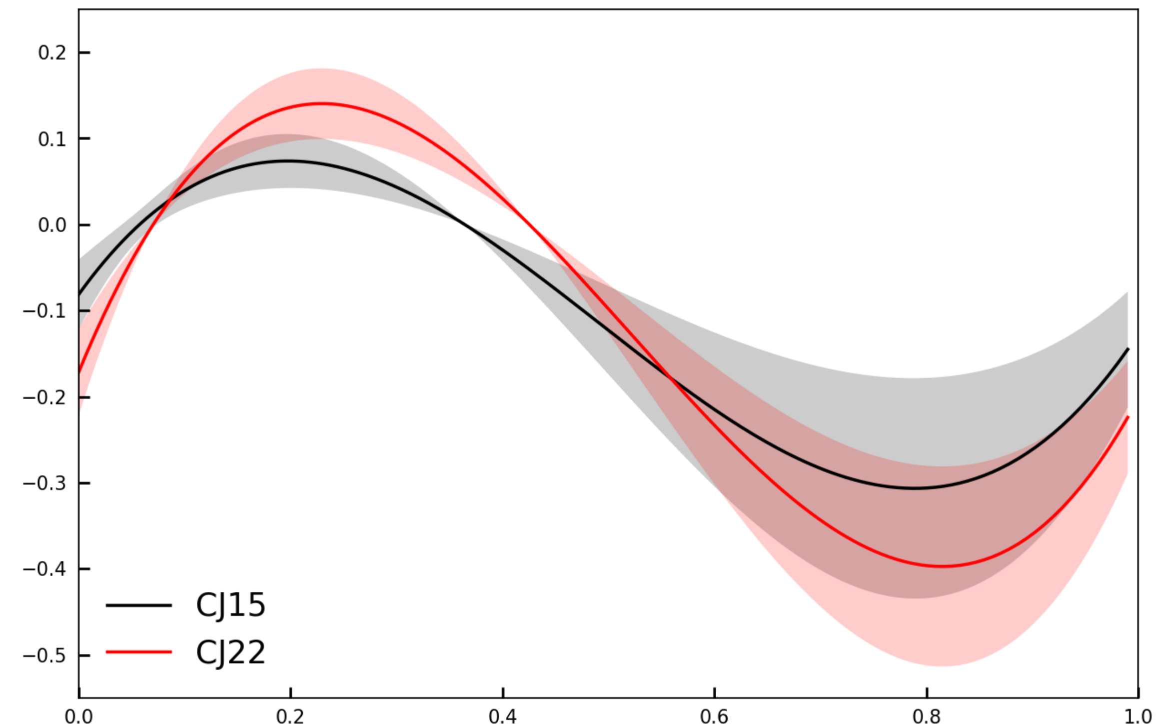

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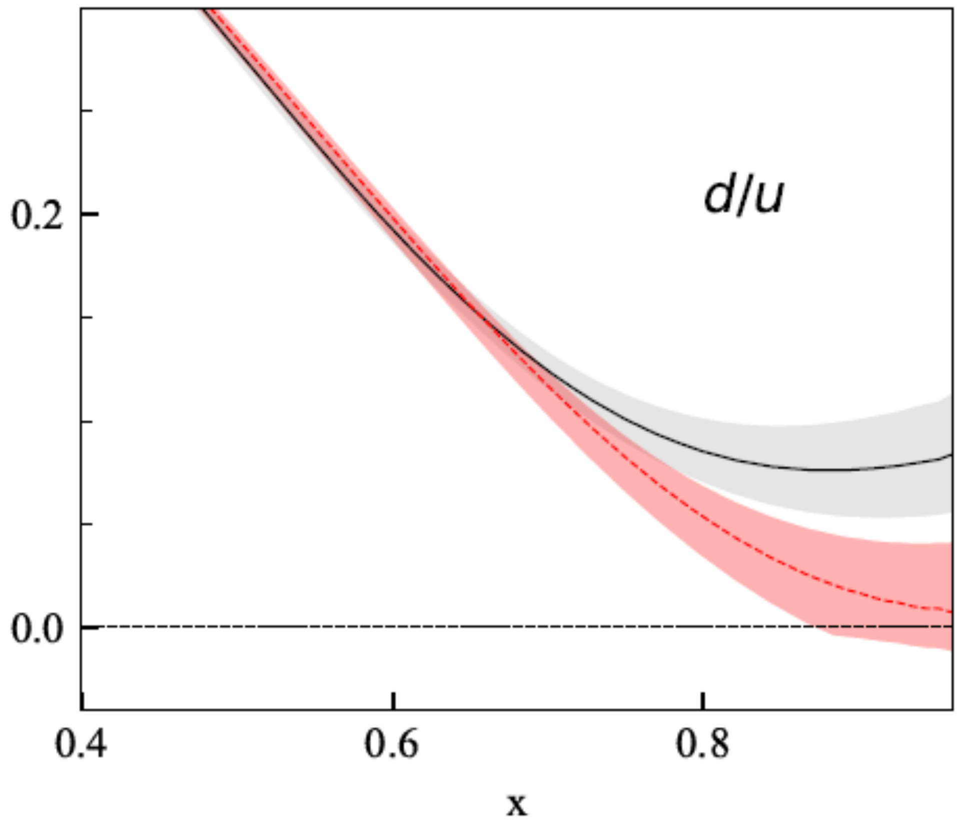

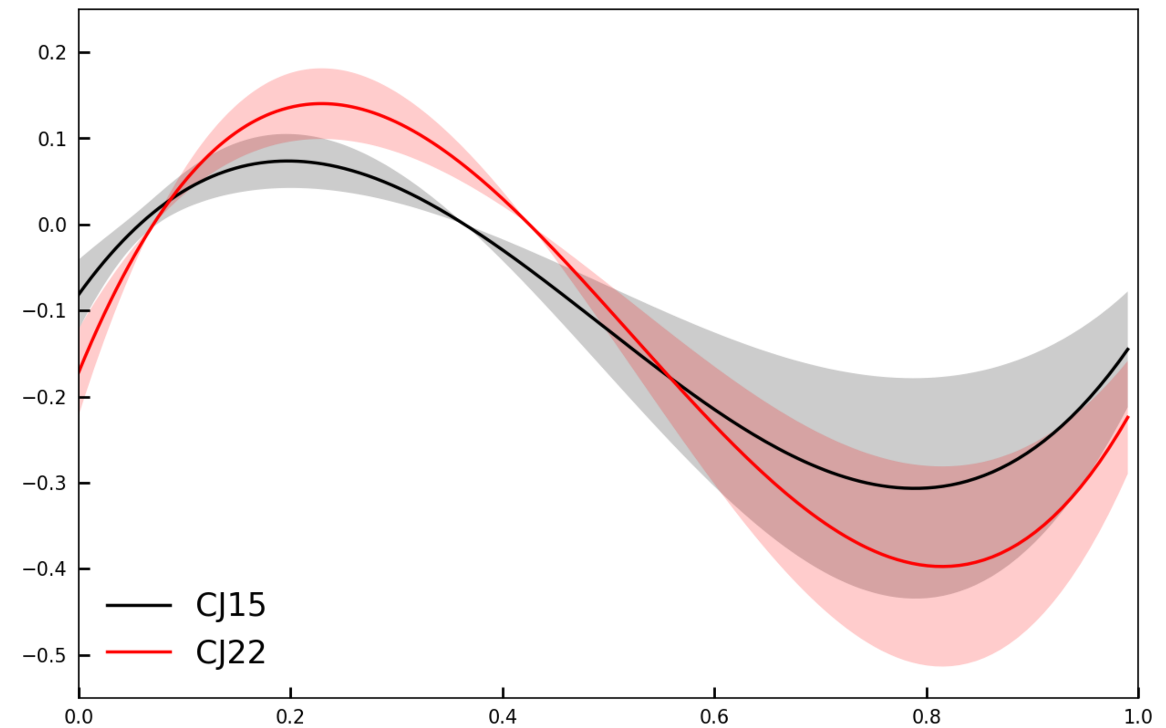

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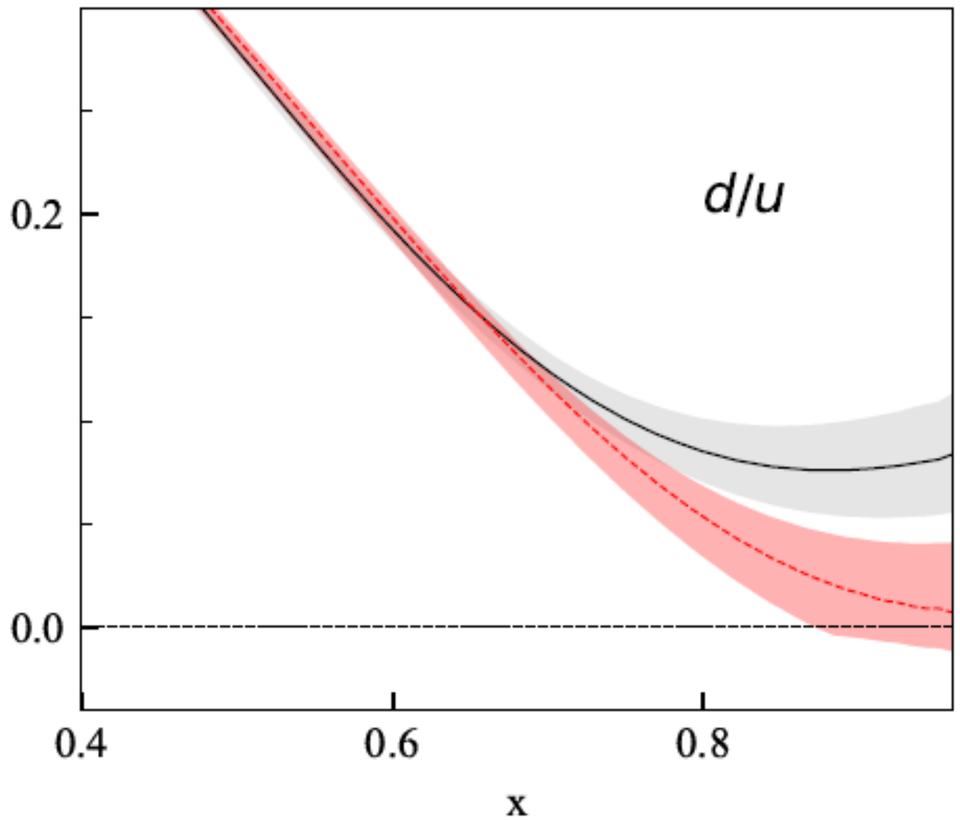

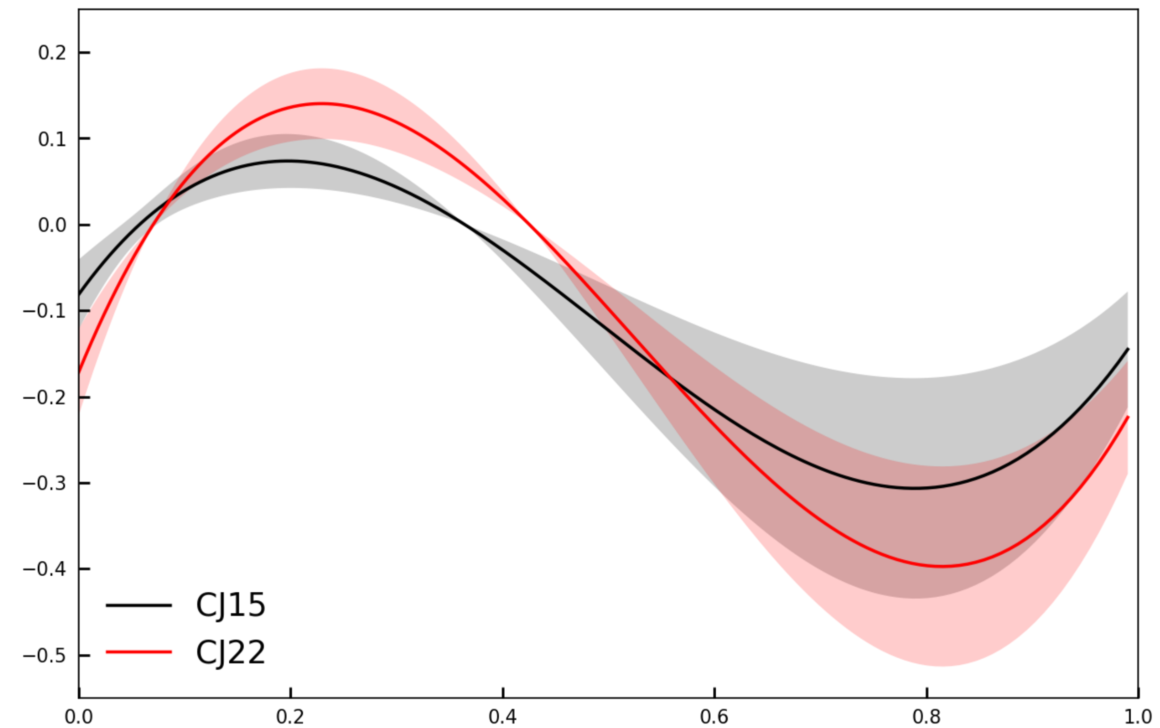

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Difference on d/u is absorbed in something else

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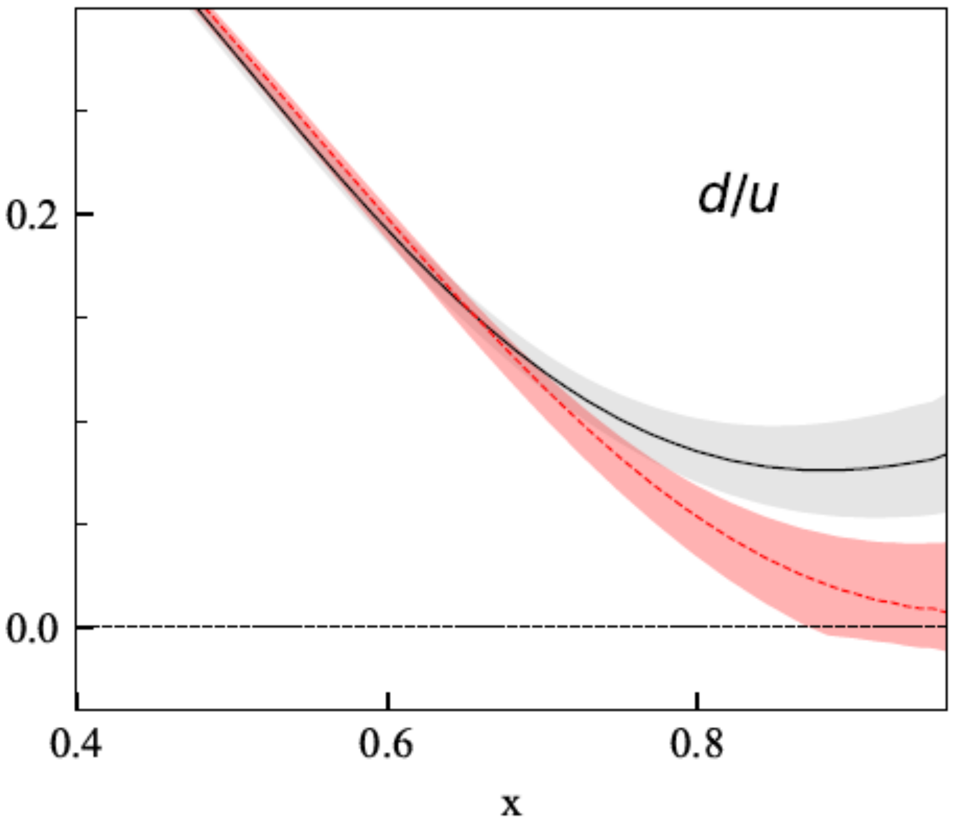

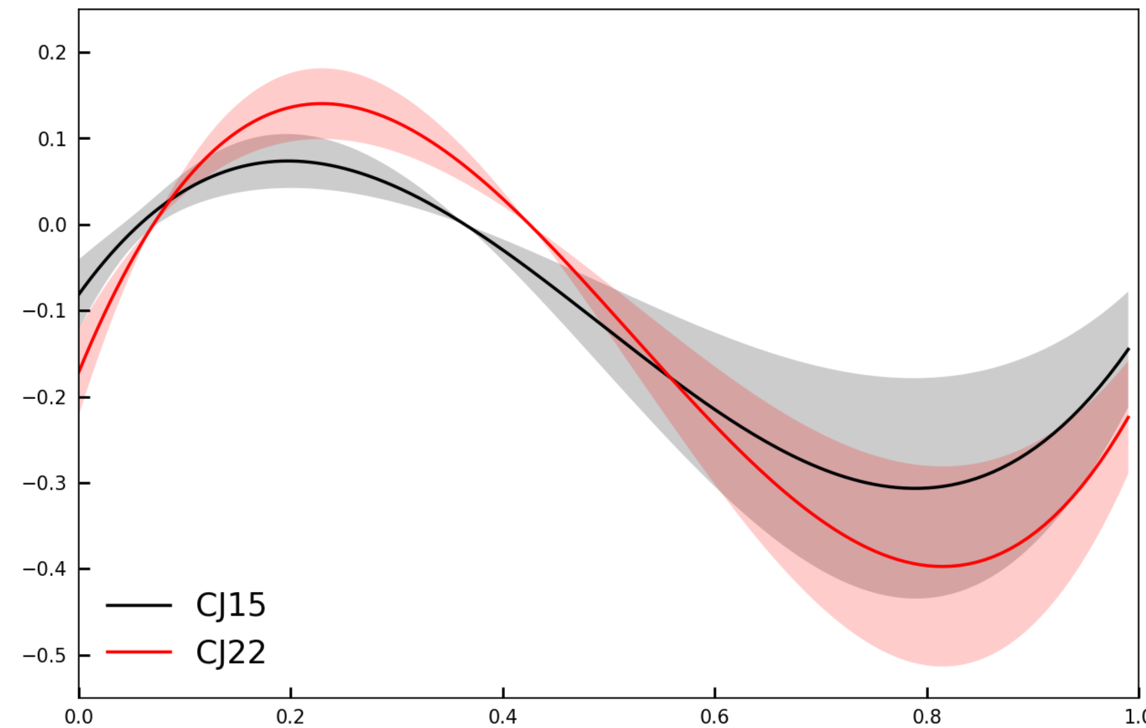

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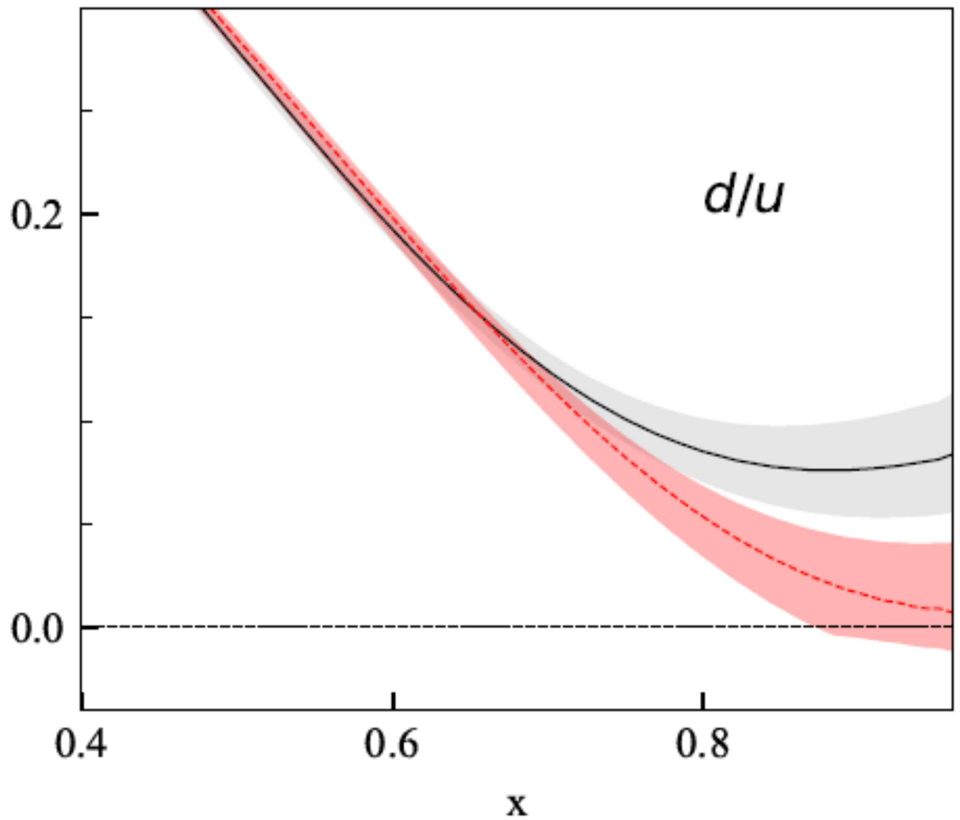

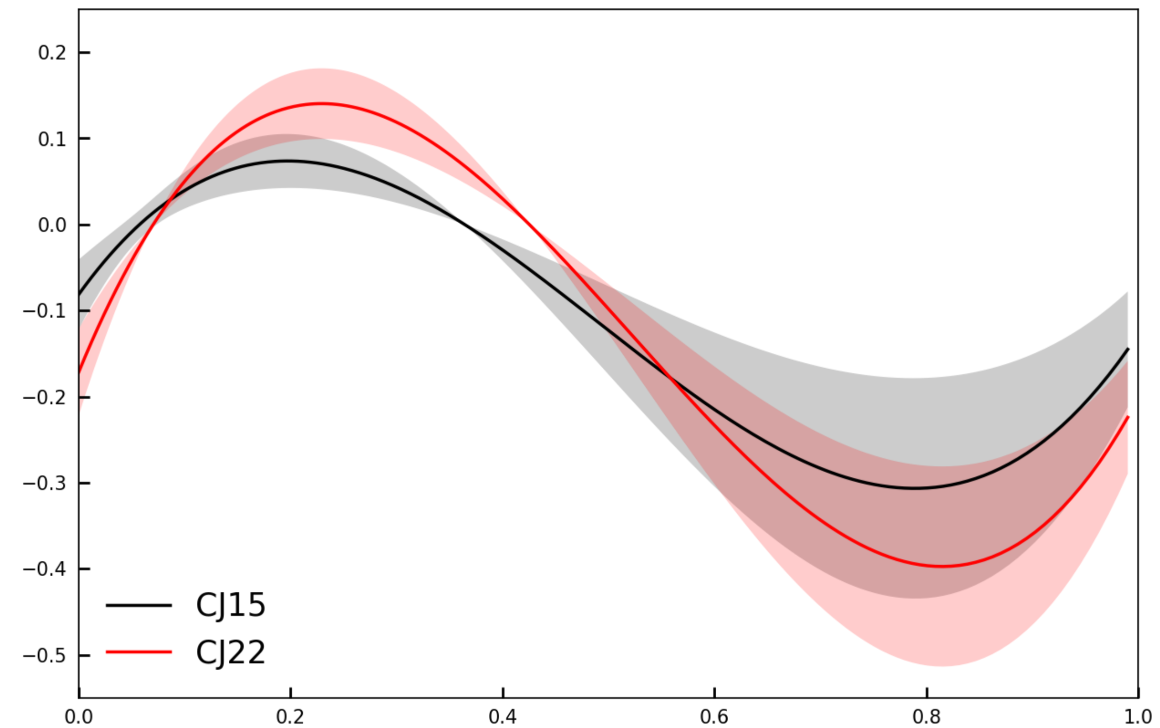

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Difference on d/u is absorbed in something else ⟶ **Higher Twist**

Is the model for off-shell correction enough flexible?

Polynomial off-shell function

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$$\delta f^N \ = \ C(x - x_0)(x - x_1)(1 + x_0 - x)$$

KP-like model

Kulagin and Petti, NPA 765 (2006)

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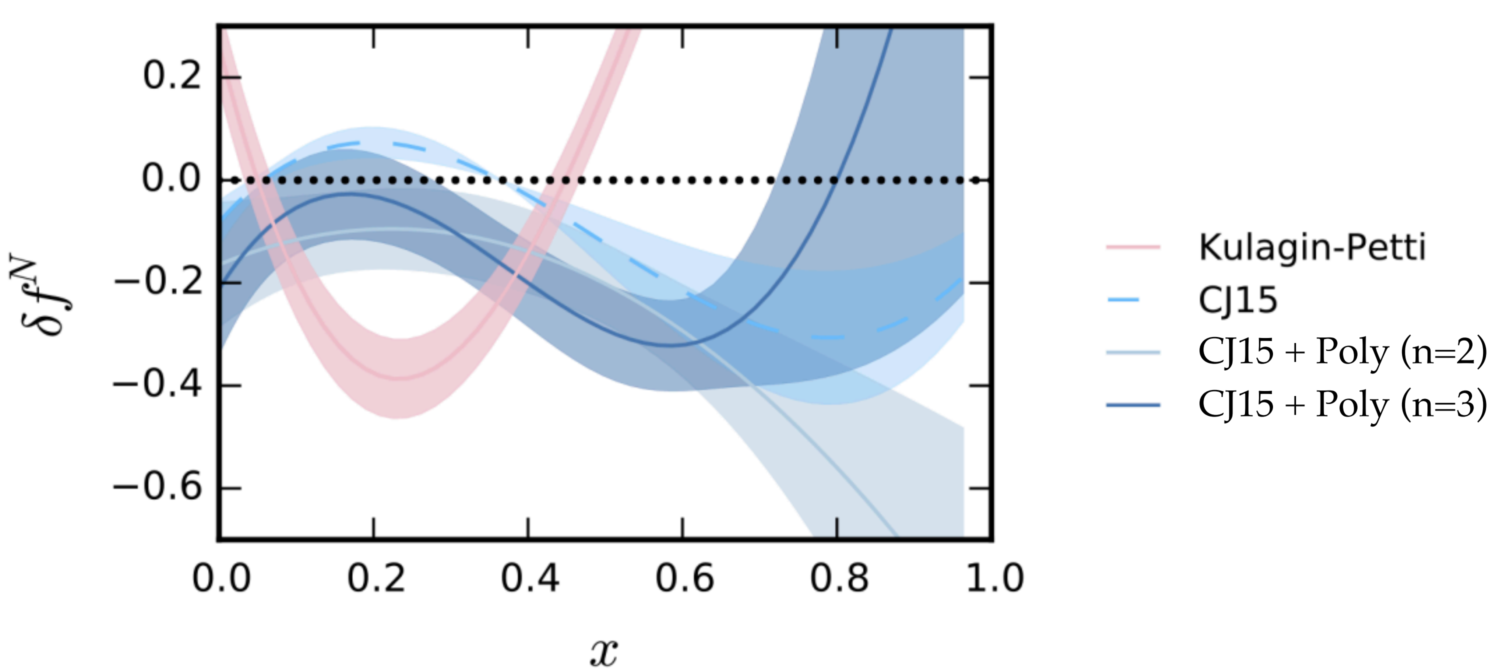

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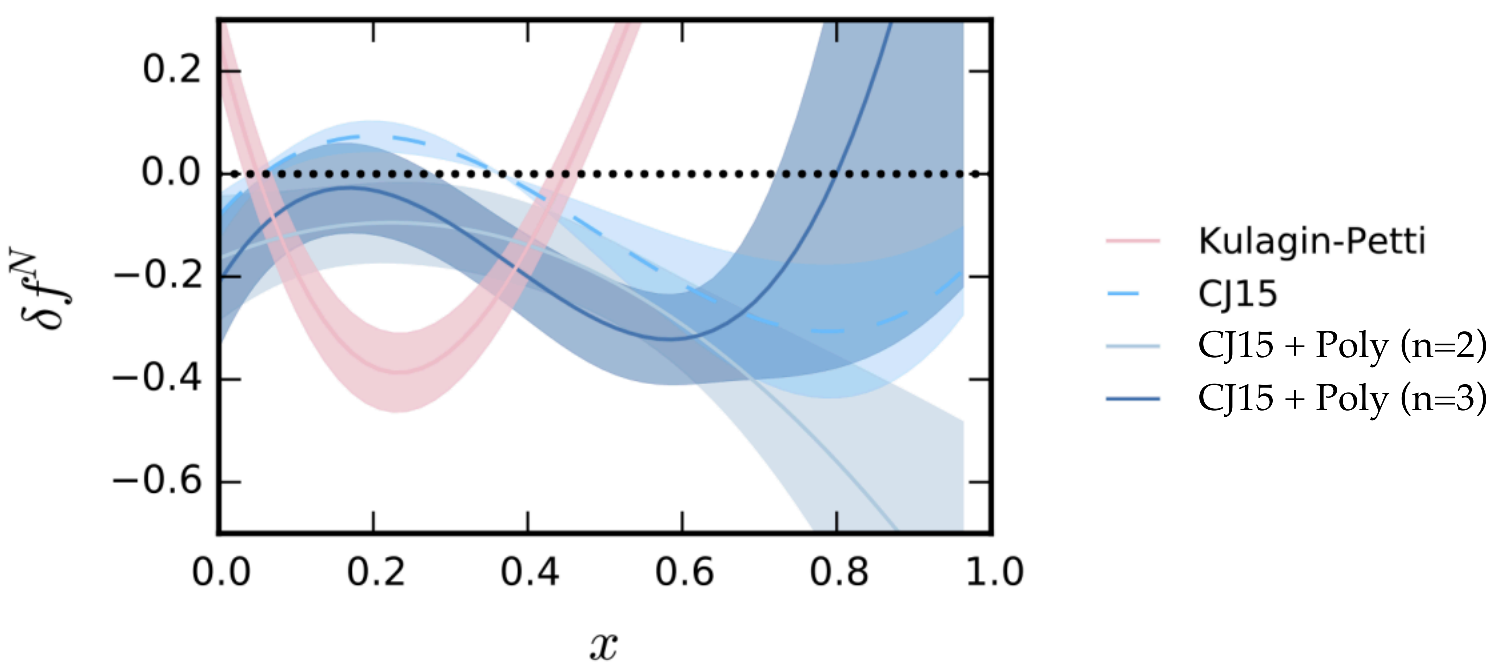

Better agreement with the data w/o imposing nodes a priori to the off-shell function

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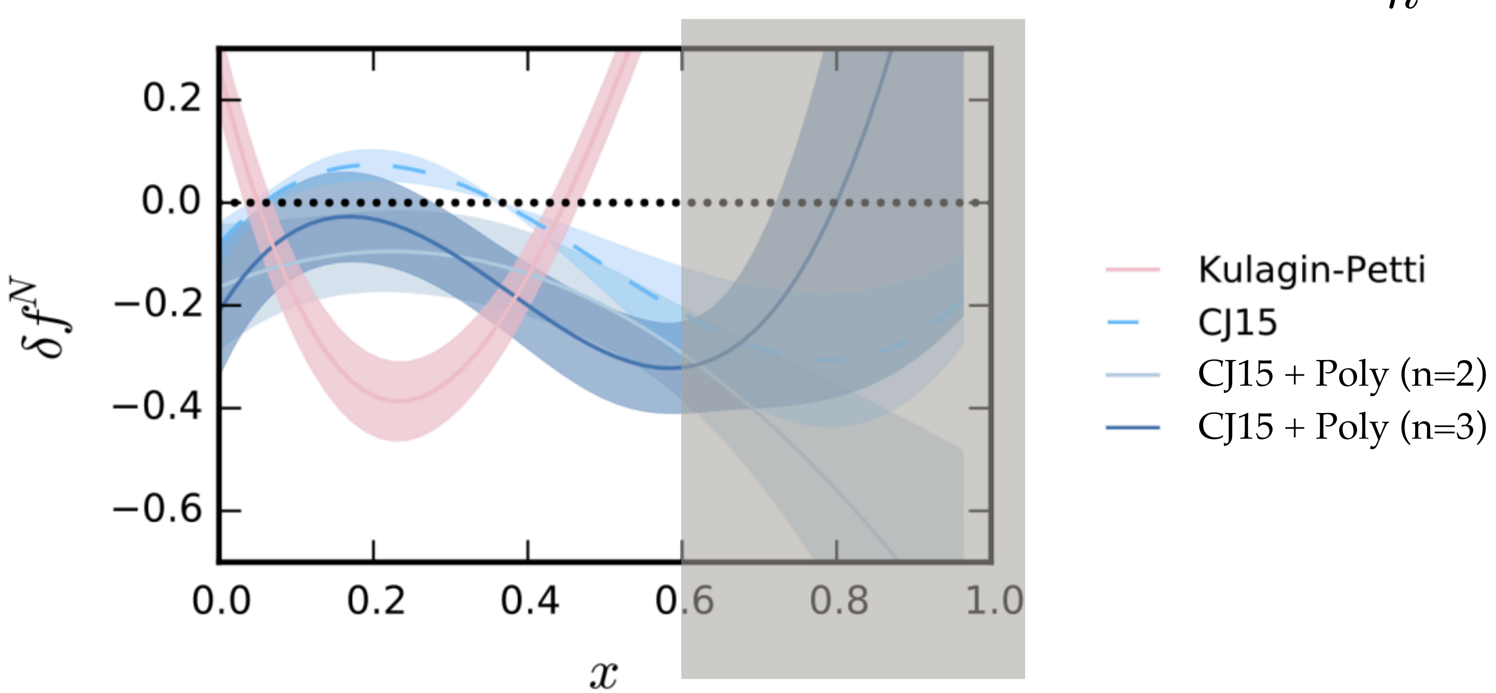

Better agreement with the data w/o imposing nodes a priori to the off-shell function

Constrain power of CJ15 dataset only up to x = 0.6

Higher-Twist function

Higher Twist correction

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Higher Twist correction

Multiplicative

$$F_2(x, Q^2) = F_2^{LT}(x, Q^2)\left(1 + \frac{\boldsymbol{C}(x)}{Q^2}\right)$$

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Additive

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CJ fits

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Bias in n/p function

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structure function
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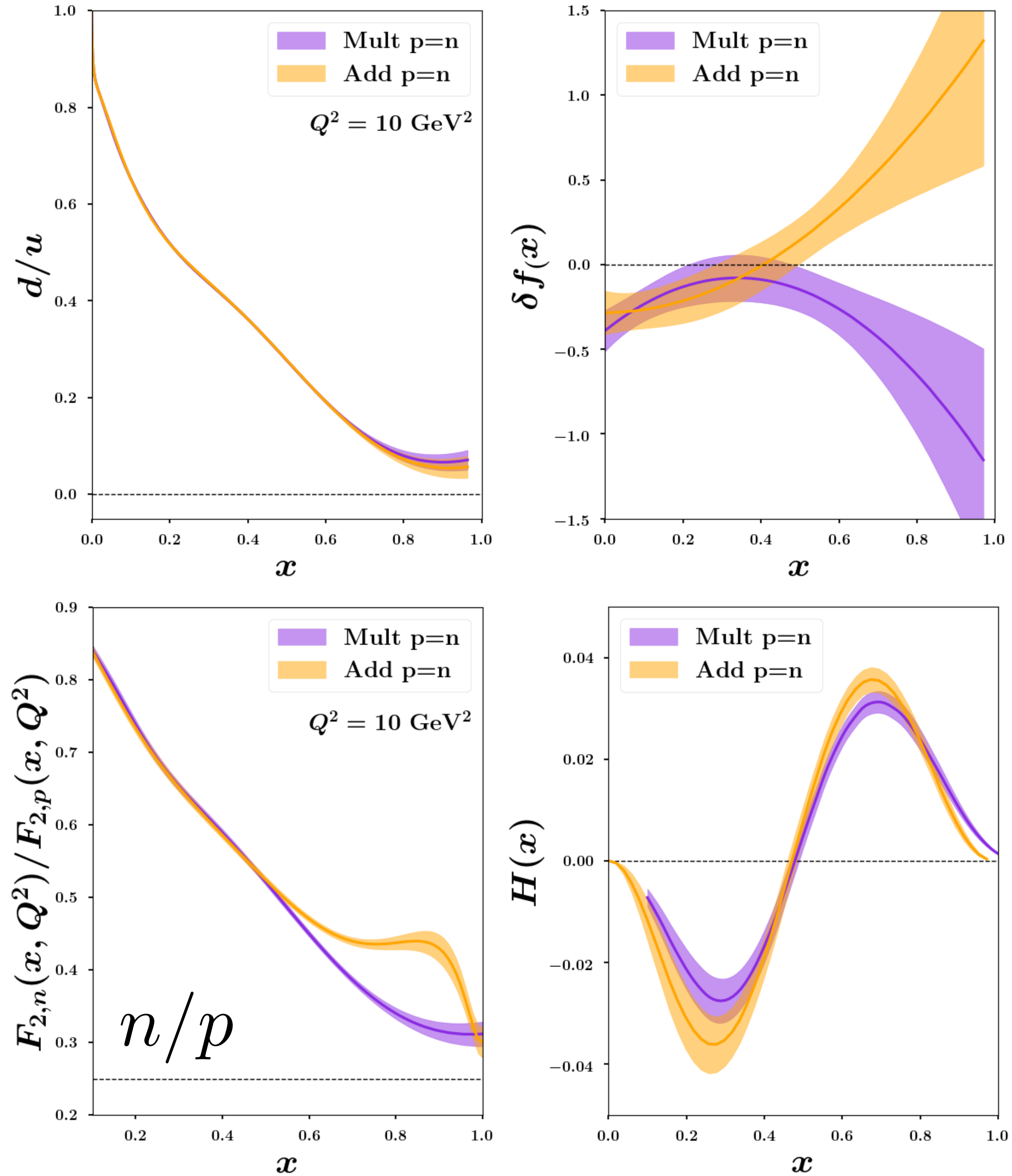
Bias not present!

Results in the CJ fitting framework

Case 1: isospin symmetry



Results in the CJ fitting framework

Case 1: isospin symmetry

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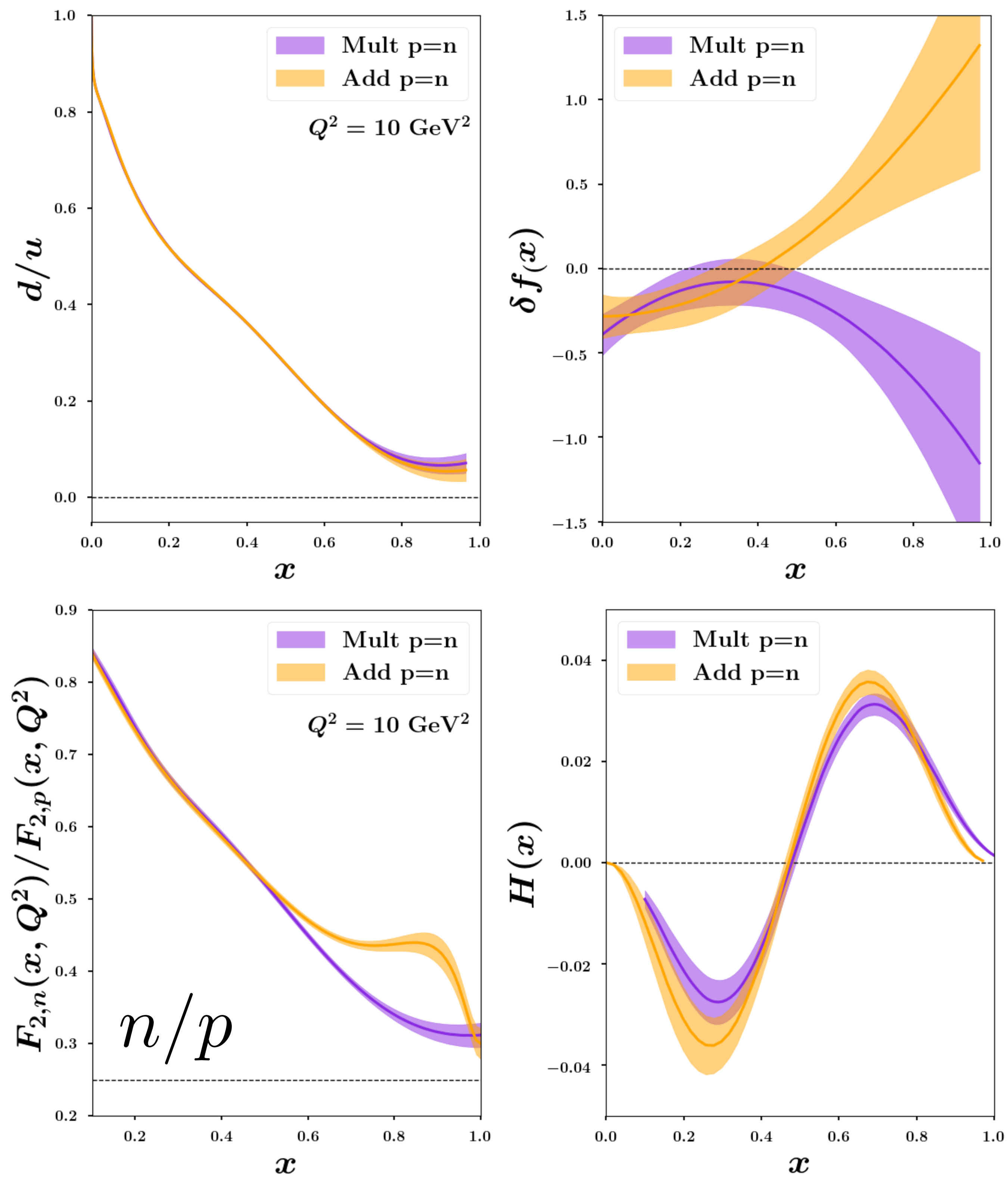
Unnaturally large n/p

BUT smaller d/u than Mult



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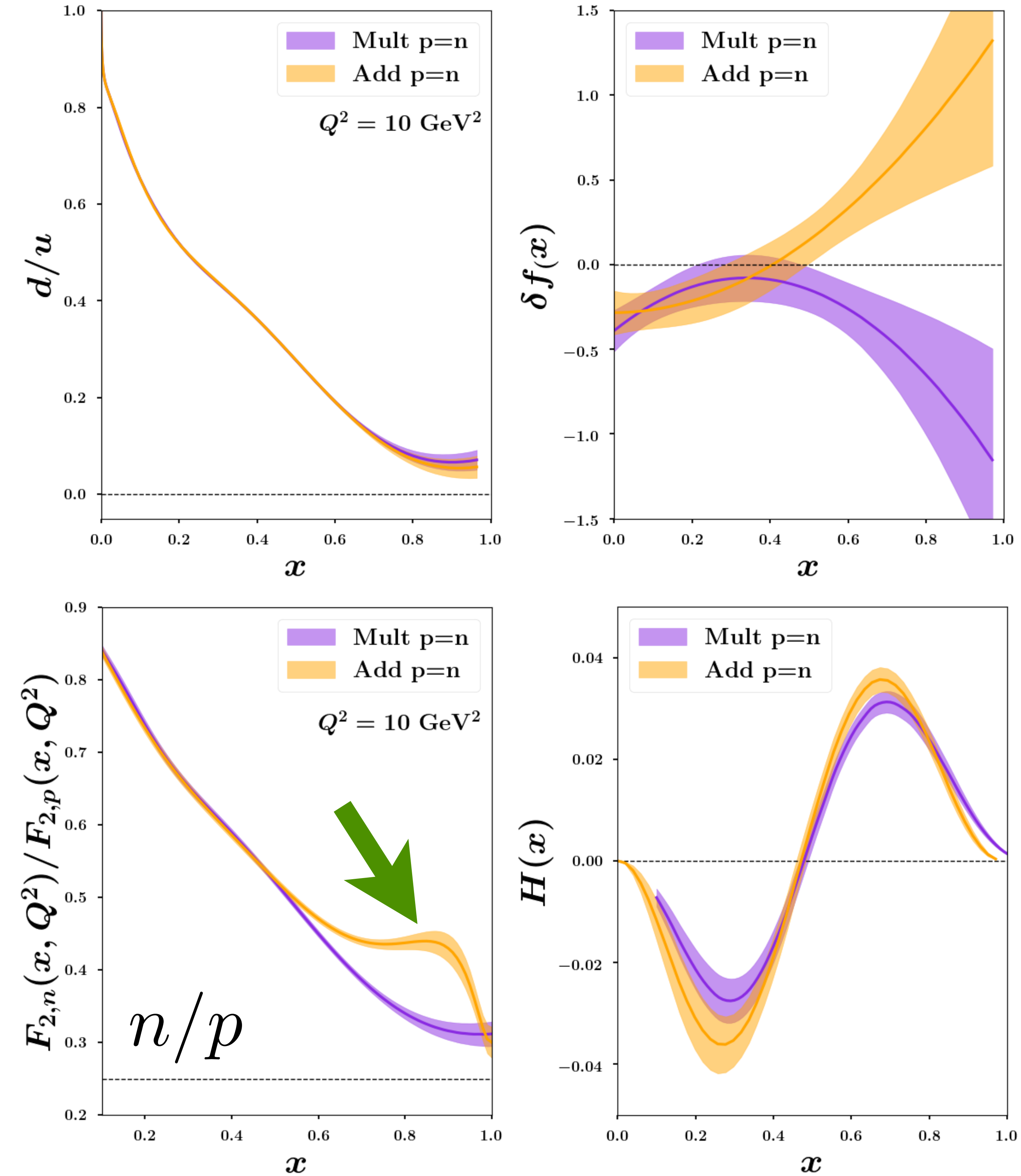

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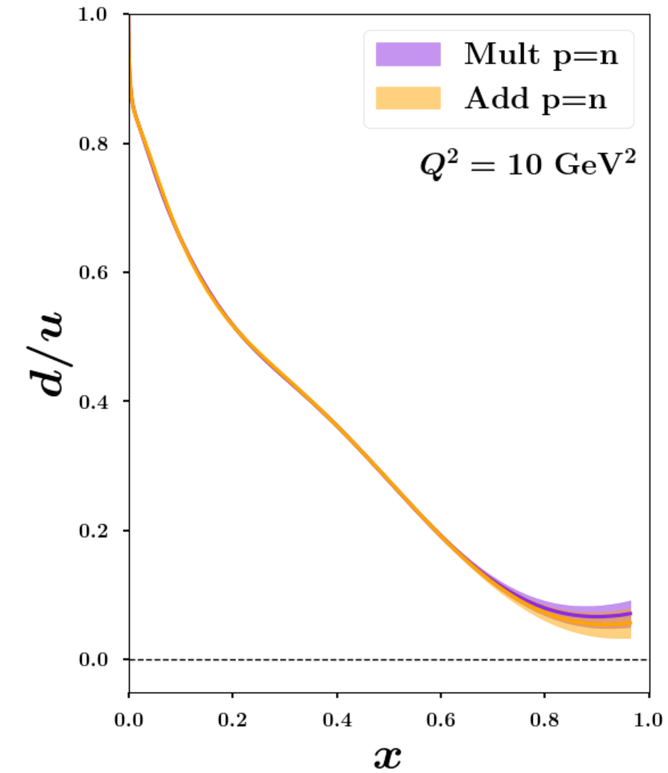
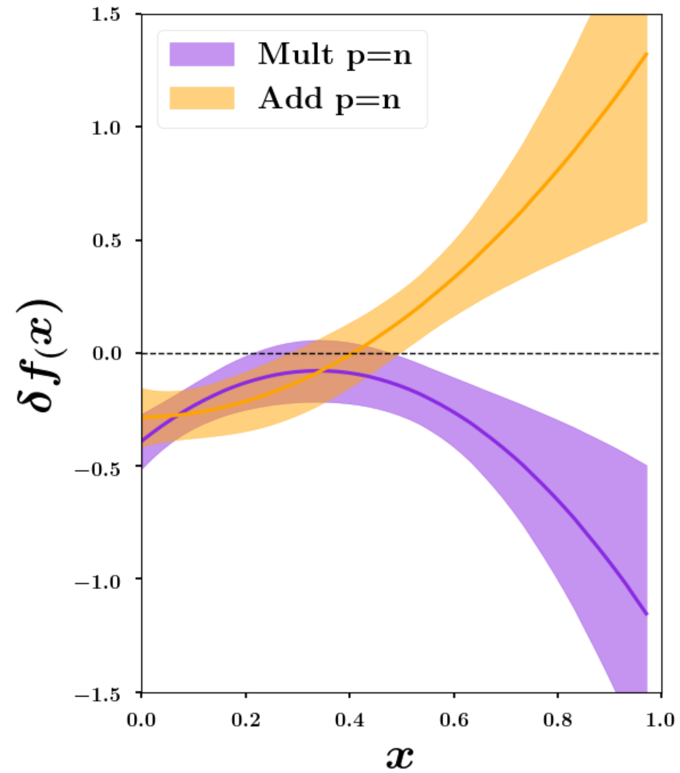
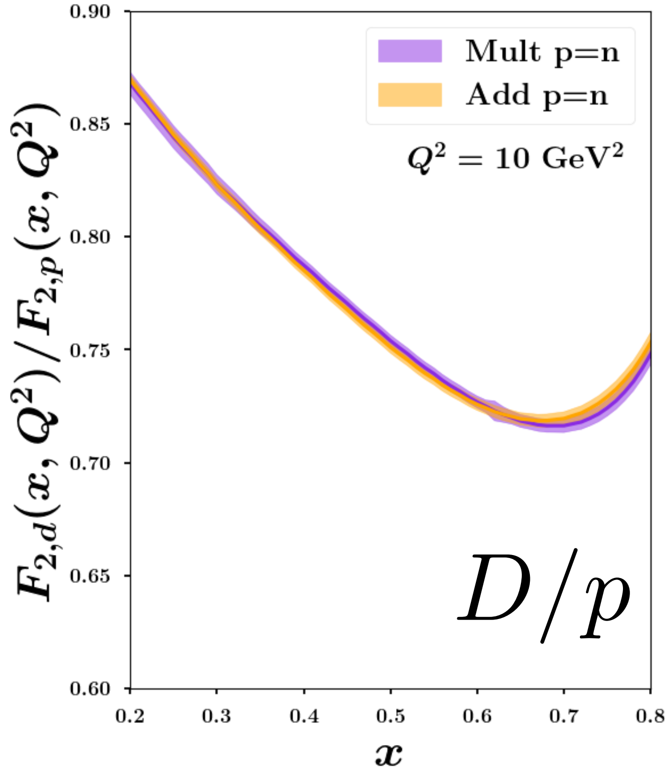
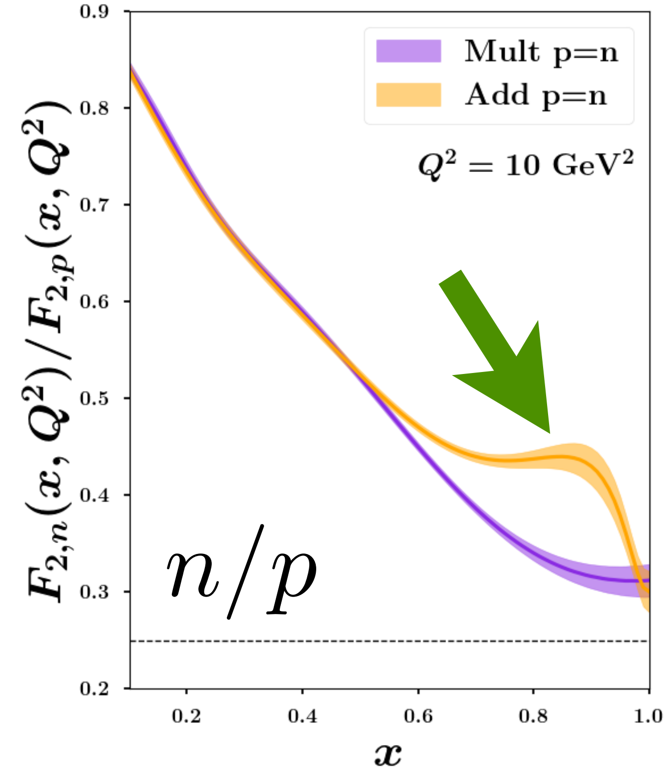
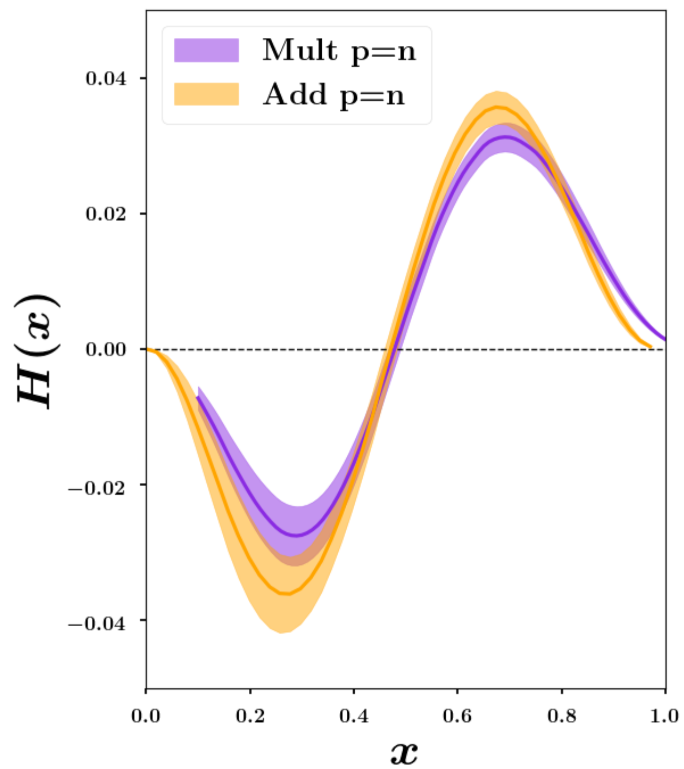
Off-shell compensates n/p bias











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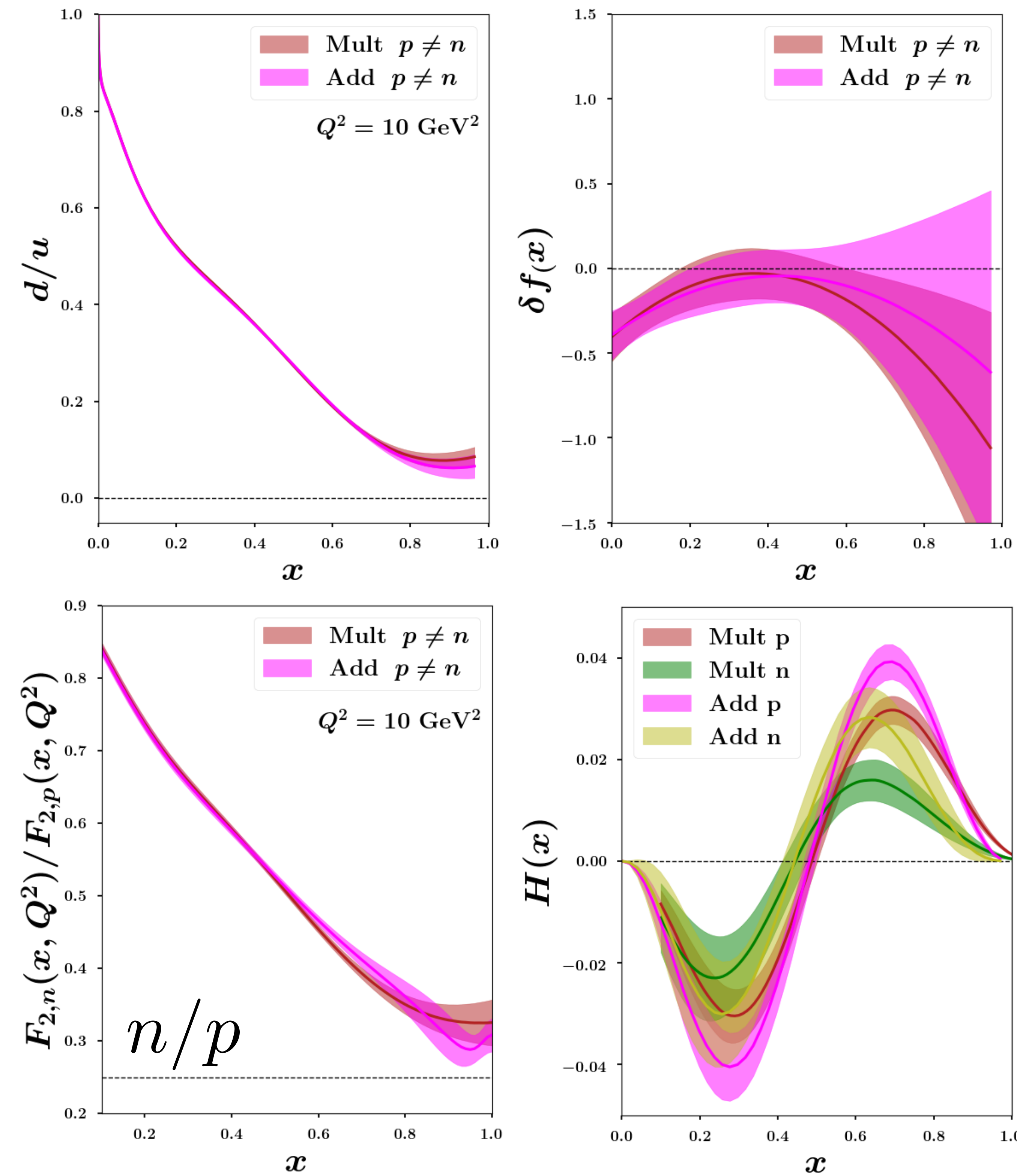
Case 2: isospin breaking

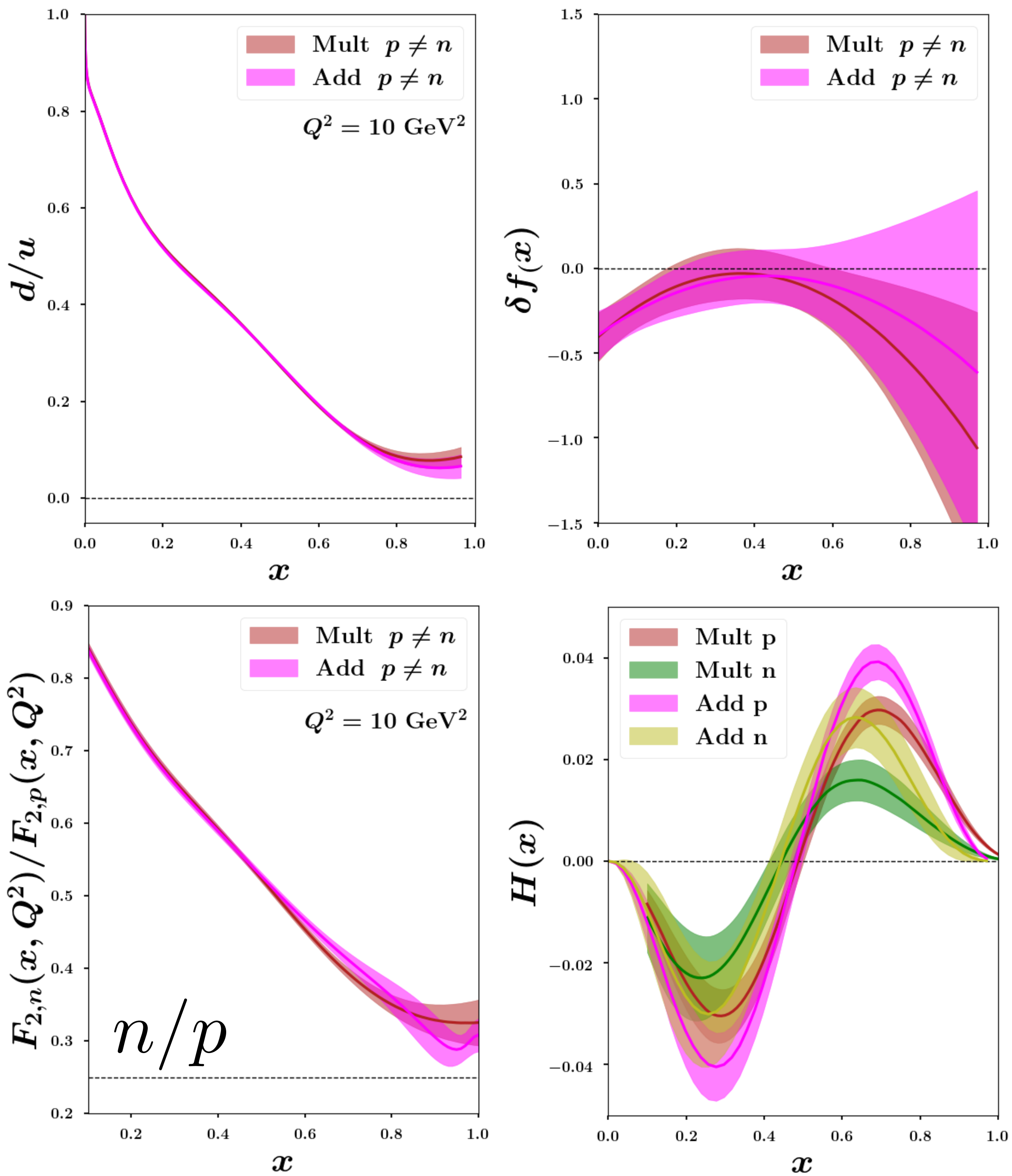


Results in the CJ fitting framework

Case 2: isospin breaking

Compatible n/p

$$H_n(x) \simeq \frac{1}{2} H_p(x)$$



Results in the CJ fitting framework

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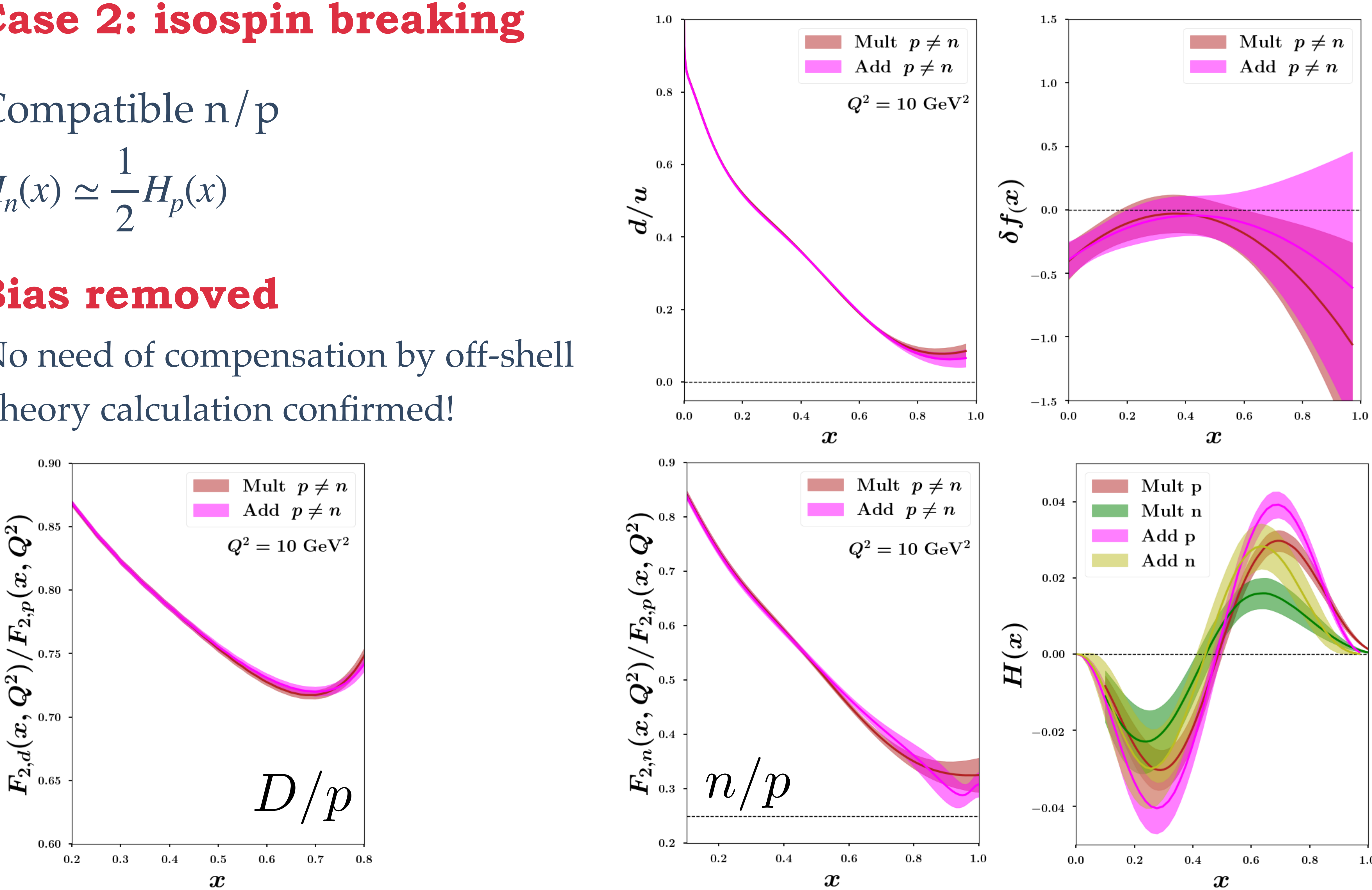
Bias removed

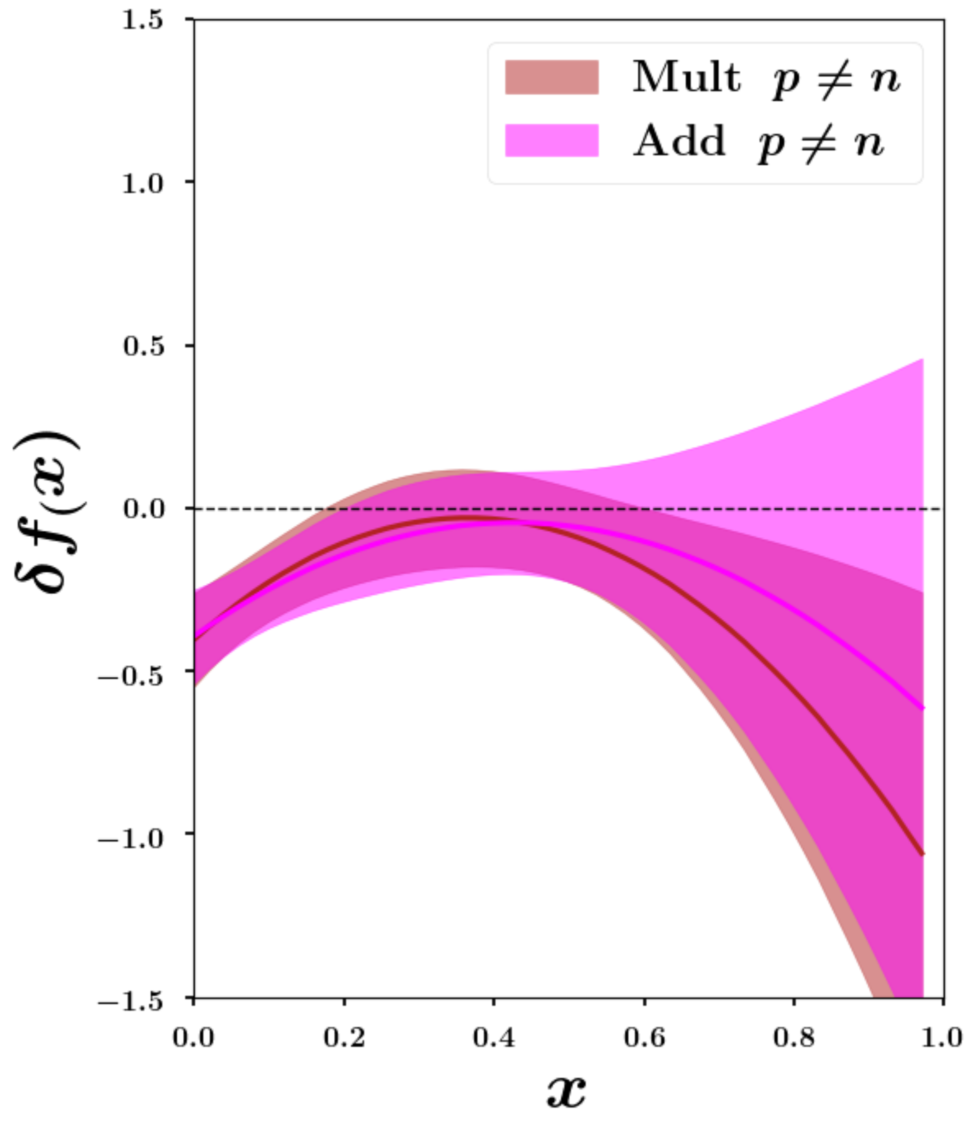
No need of compensation by off-shell

Theory calculation confirmed!



Results in the CJ fitting framework

After removing the bias $\delta f(x) \simeq 0$



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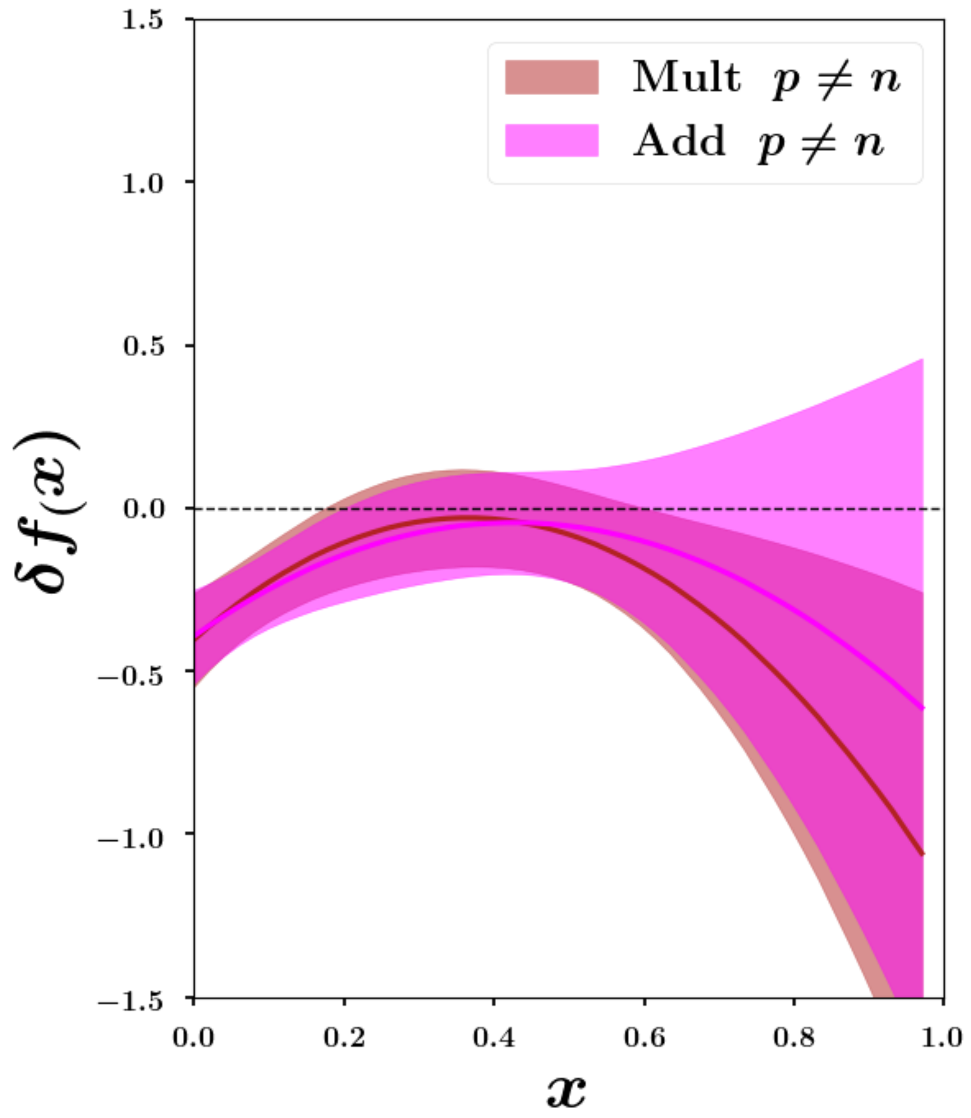

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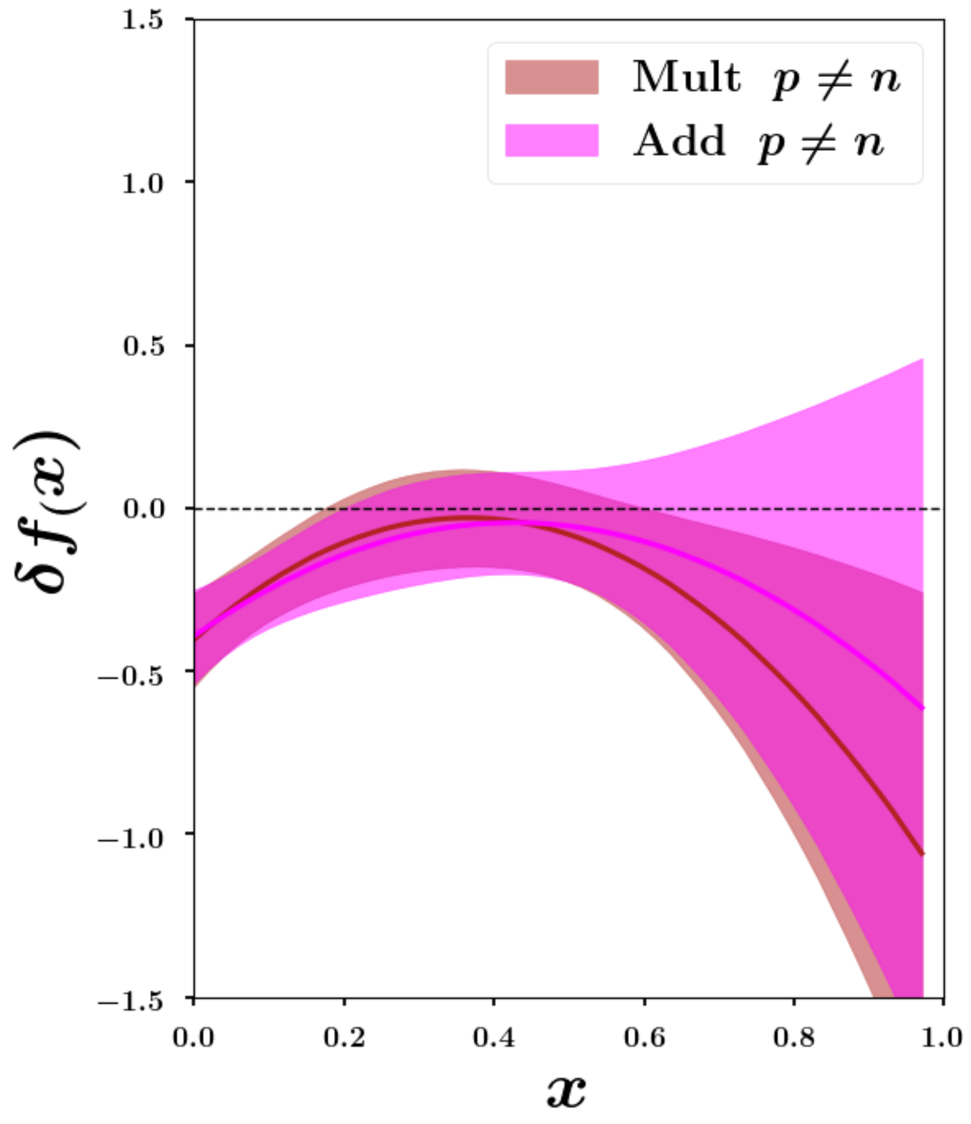

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Need A=3 data to assess flavour dependence of off-shell function

MARATHON data
Adams, et al., PRL 128 (2022)

Other extractions of the off-shell correction

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AKP Alekhin, Kulagin, Petti, PRD 107 (2023)

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Fit to A=3 data: $\delta f_u(x) \neq \delta f_d(x)$

AKP results

AKP Alekhin, Kulagin, Petti, PRD 107 (2023)

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AKP Alekhin, Kulagin, Petti, PRD 107 (2023)

Add HT (p=n) as baseline choice

H_2, H_T parametrized

AKP results

AKP Alekhin, Kulagin, Petti, PRD 107 (2023)

Add HT (p=n) as baseline choice

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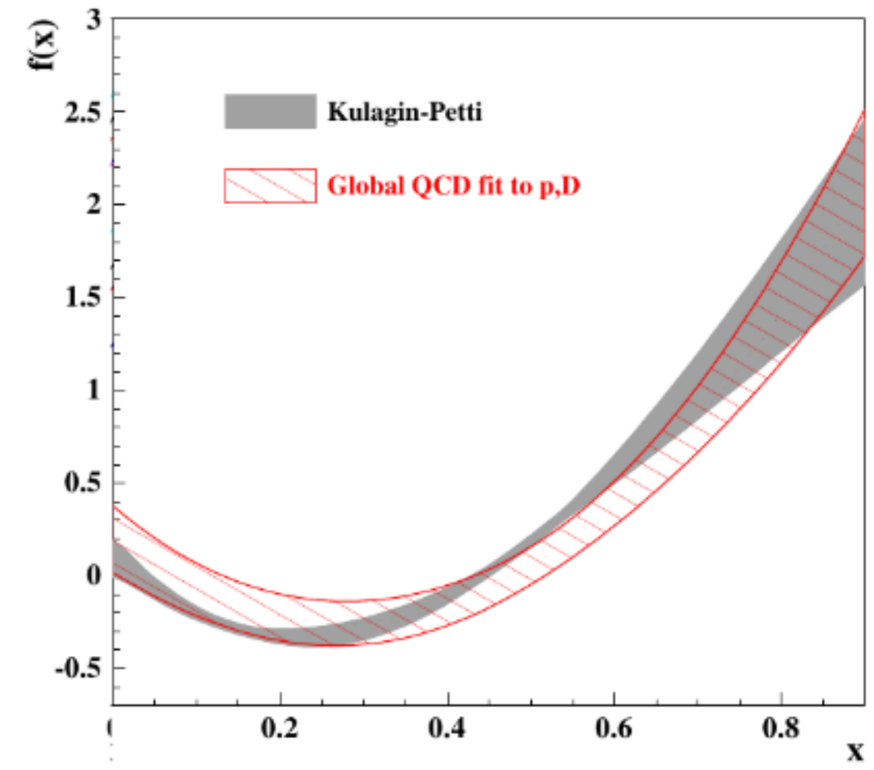

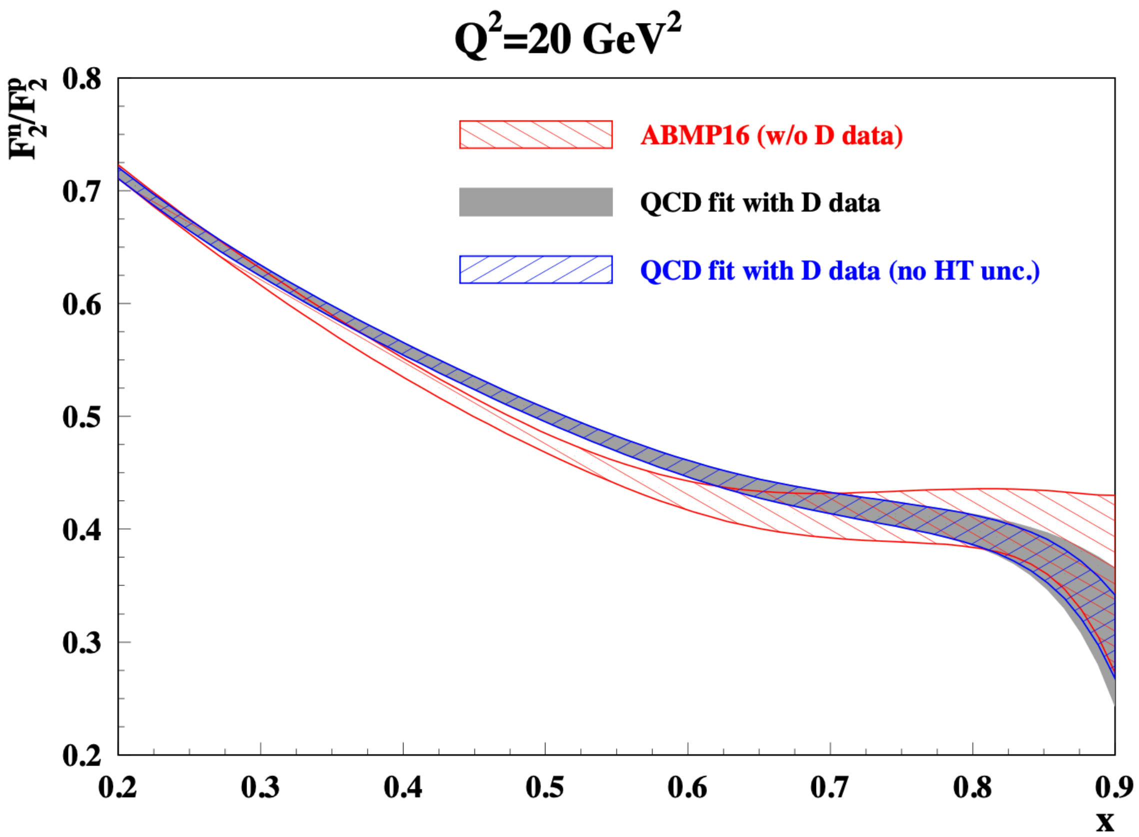

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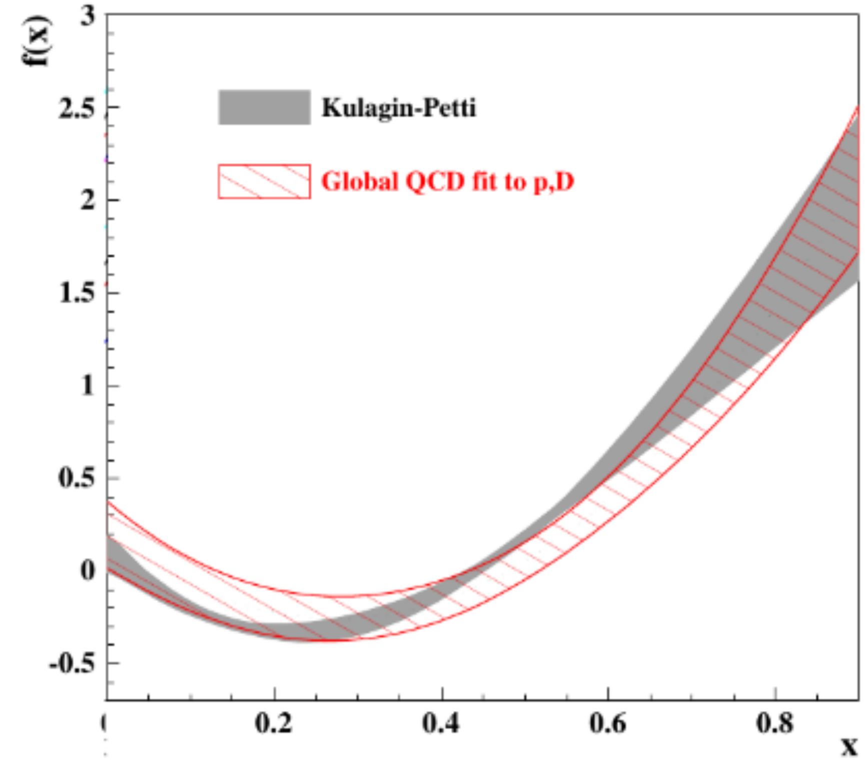

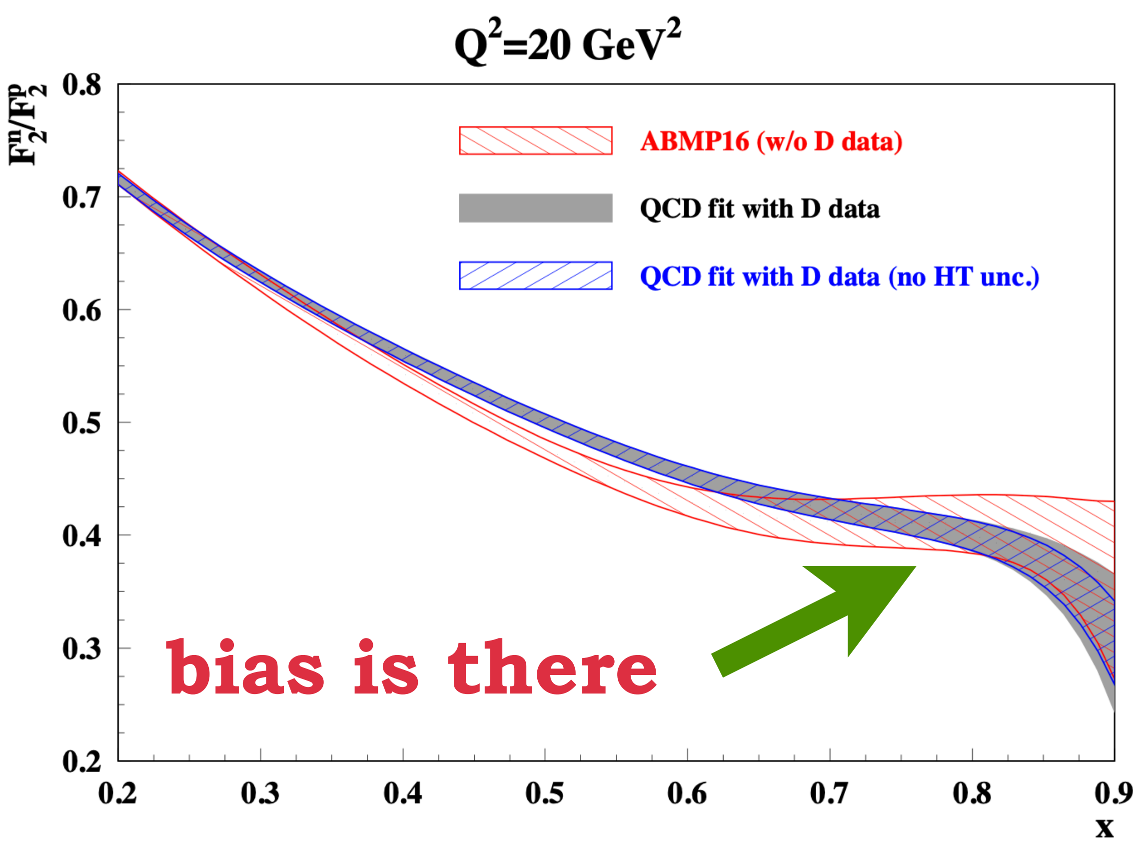

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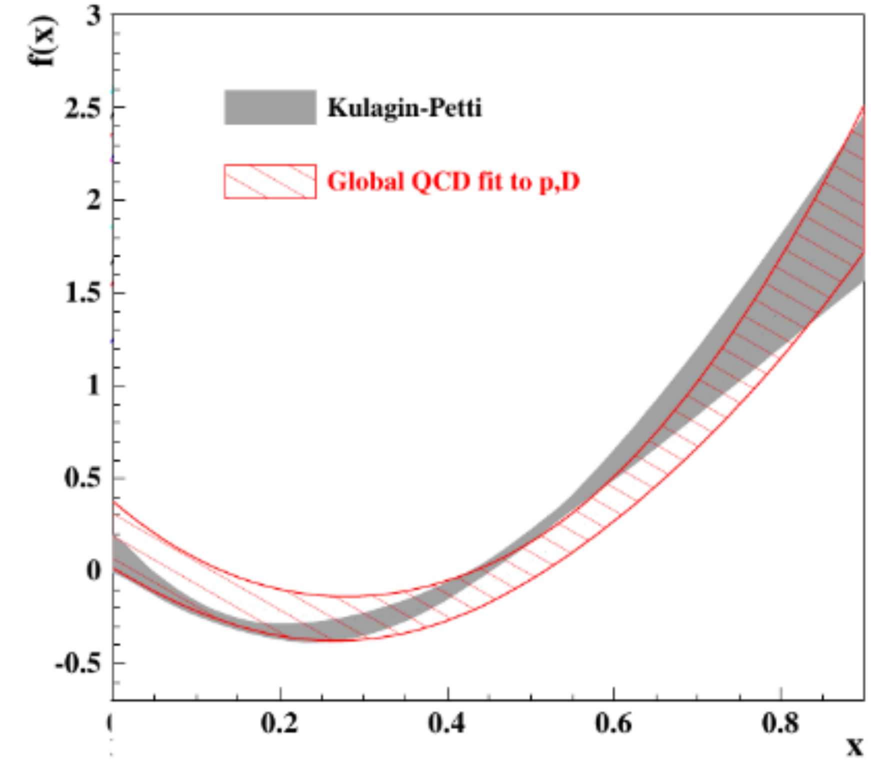
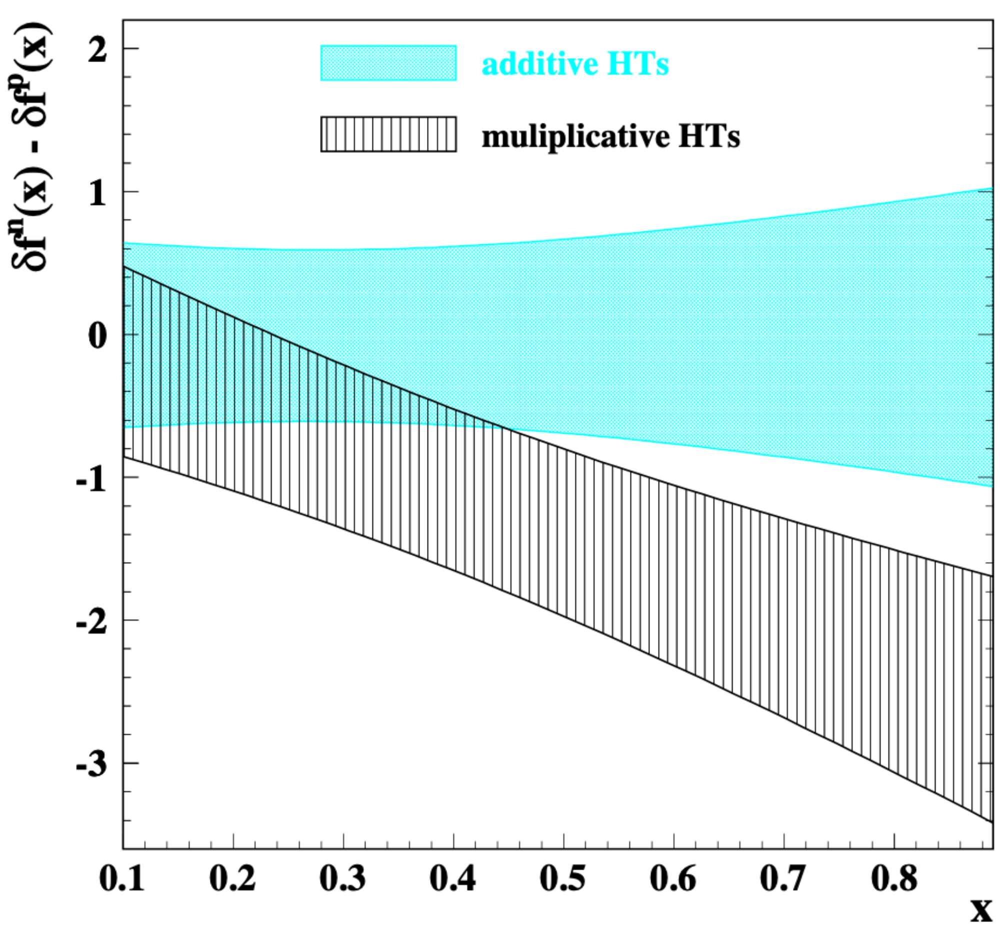
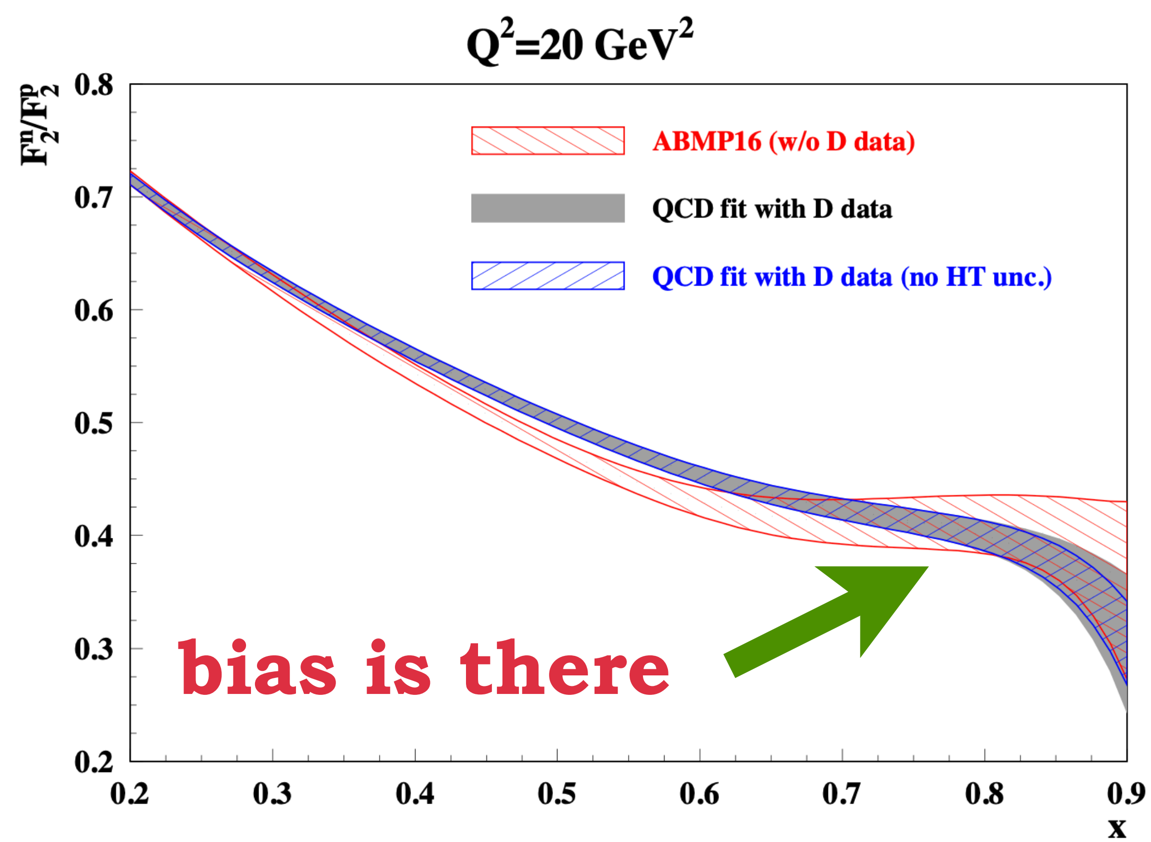
Fit to A=3 data δF_p δF_n







JAM results

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JAM *Fit including A=3 data* δf_u δf_d

JAM Collaboration, PRL 127 (2021)

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Mult HT (p=n) as default choice

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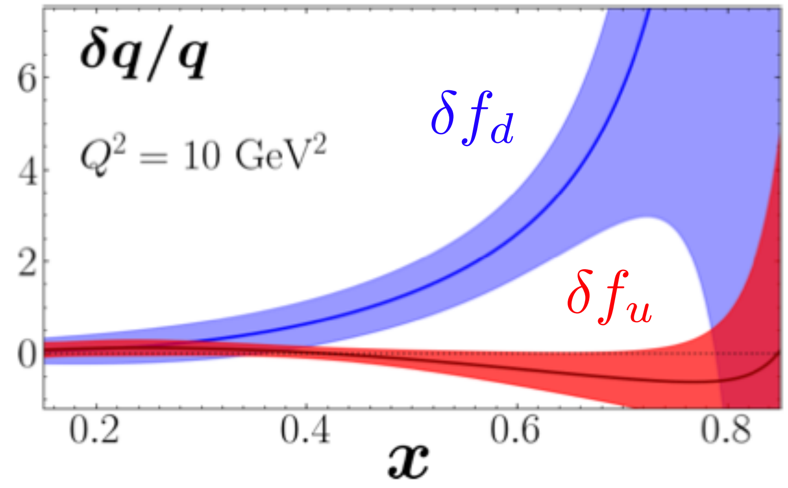

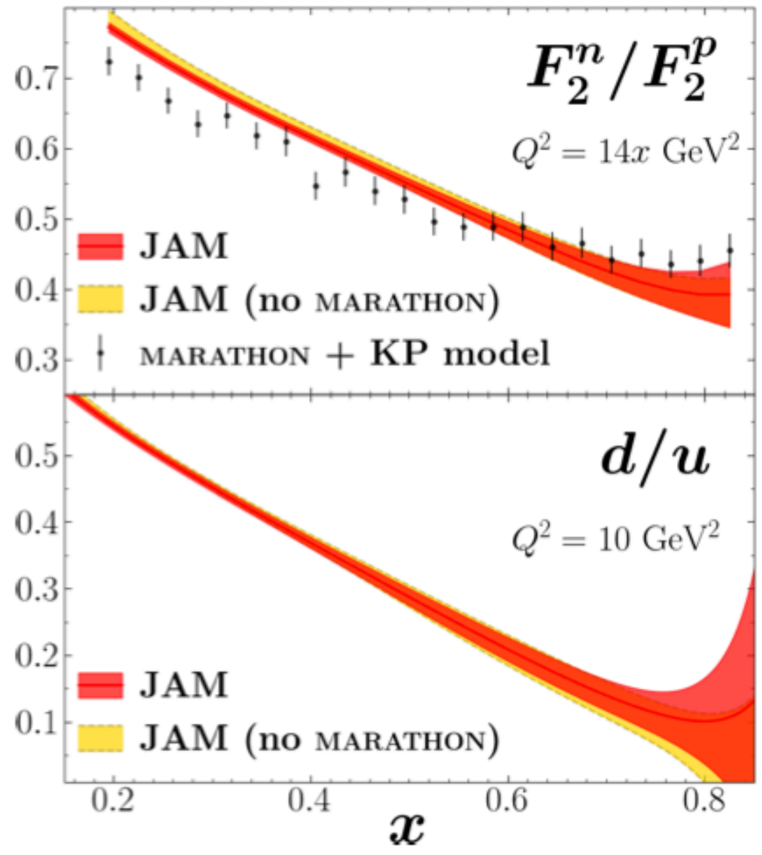

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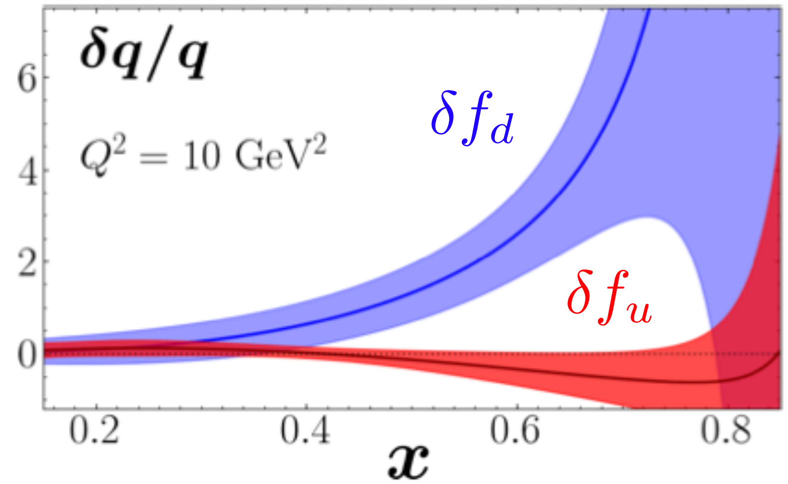

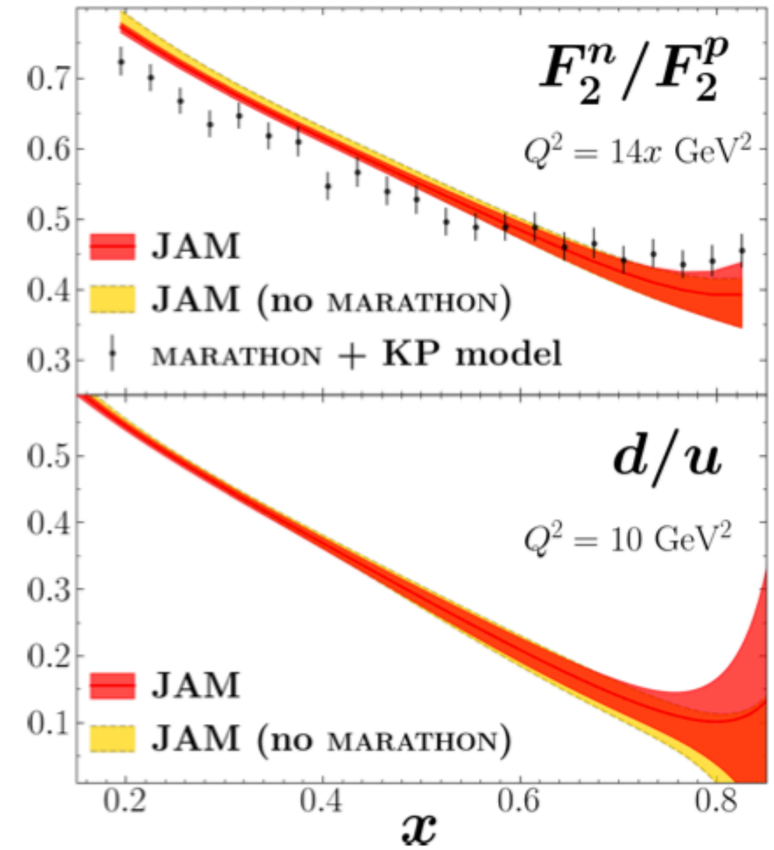

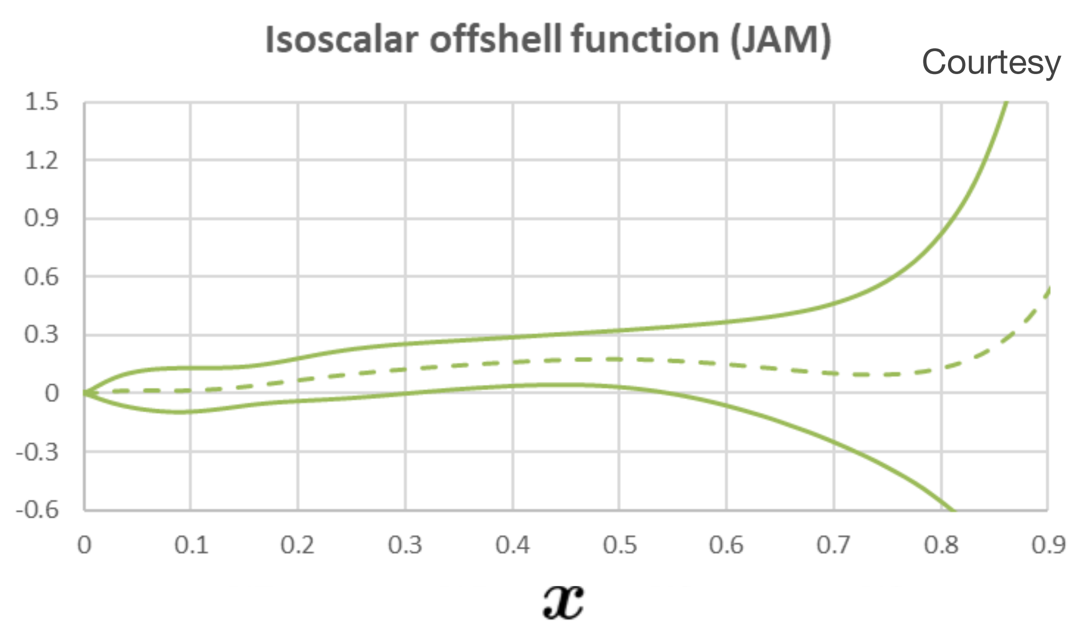

Courtesy of C. Cocuzza

$$\delta f(x)|_{\text{CJ-like}} = \frac{u\delta f_u + d\delta f_d}{u+d}$$

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Experimental data differential on the off-shell proton virtuality $\boldsymbol{p^2}$ would allow us to pin down the off-shell correction in a more clean way

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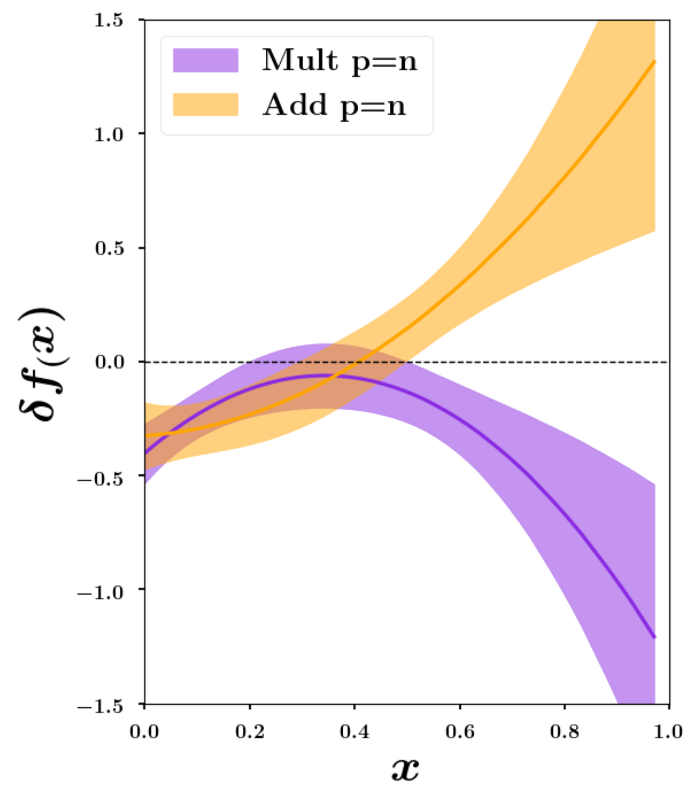
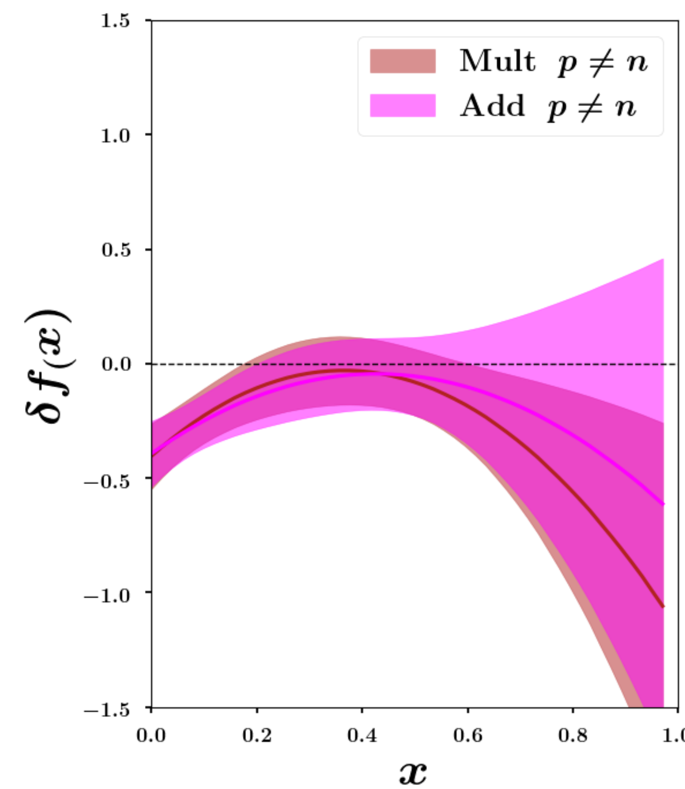
JAM A=3 fit not in agreement with AKP. Average result compatible with CJ

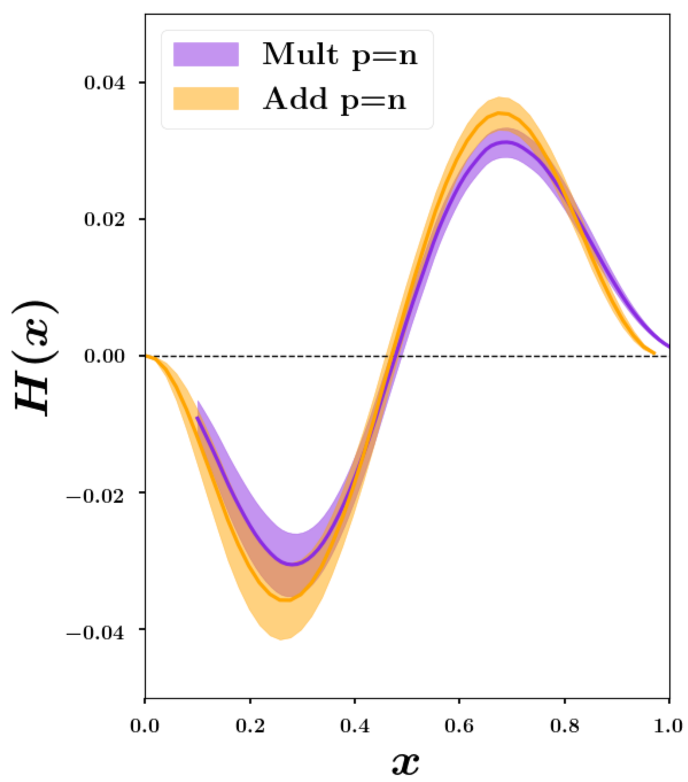
Backup

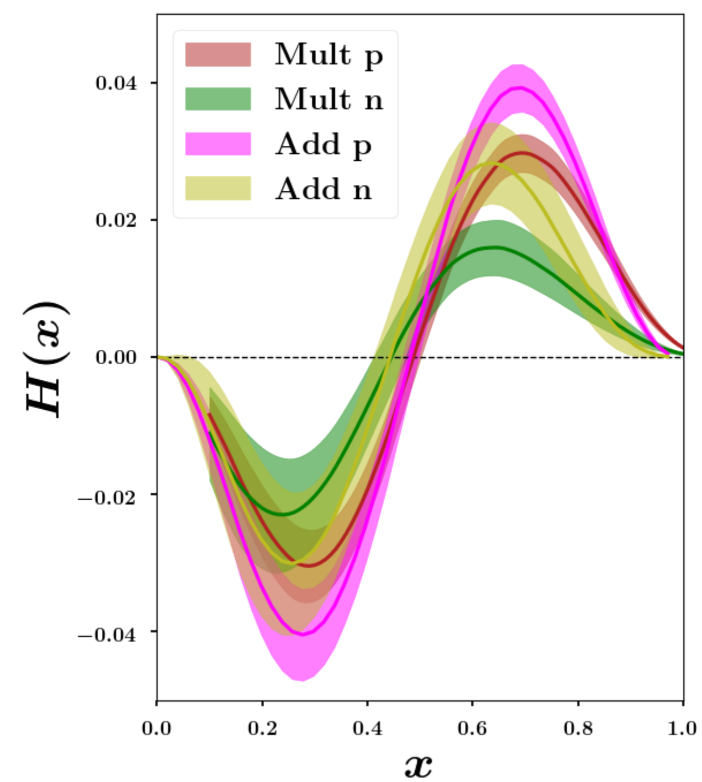
Off-shell table

isospin / higher-twist	Symmetric	Non symmetric
Multiplicative		
Additive		

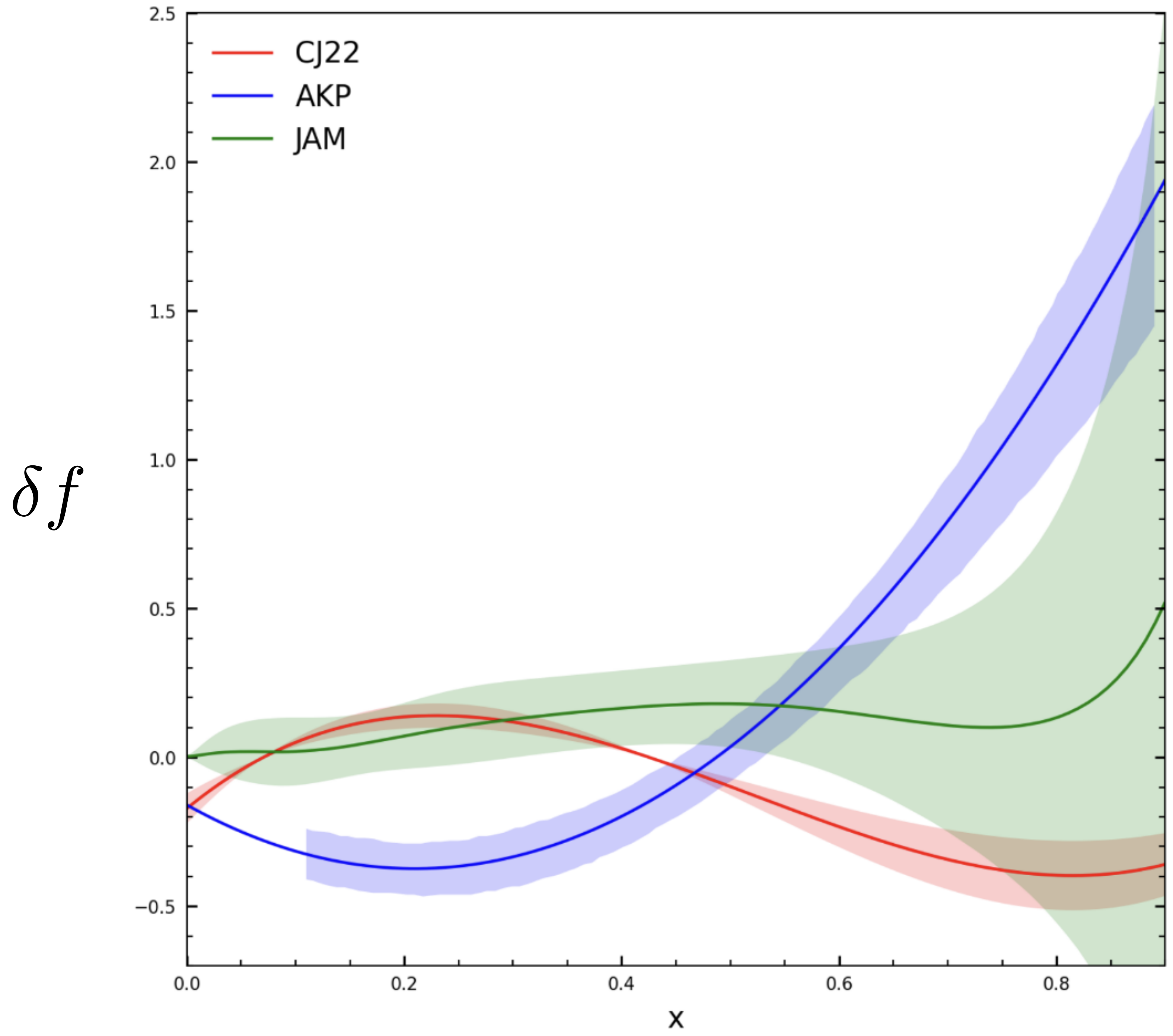
Higher-Twist table

higher-twist \ isospin	Symmetric	Non symmetric
Multiplicative	$\tilde{H} = F_{2,N}(x, Q^2) H_{\text{Mult}}(x)$	$\delta\tilde{H} = F_{2,N}(x, Q^2) \delta H_{\text{Mult}}(x)$
Additive		





AKP vs CJ



Some implementation differences

Theoretical choices →

Corrections (increasing-x) ↓

	KP	AKP	CJ15	AKP-like
shadowing	yes	yes (which one?)	MST x<0.1	(same)
smearing	Paris	AV18	AV18 x>0.1	(same)
pi-cloud	yes	yes	----	----
TMC	GP O(Q4)?	GP O(Q4)??	GP approx.	(same)
HT	H (p=n ??)	H (p=n)	C (p=n)	H & C, p=n & p!=n
HT(x)	??	5 pt. spline	parametrized	parametrized
off-shell	O(p2-M2)	O(p2-M2)	O(p2-M2)	(same)
df(x)	factorized	polyn. 2nd/3rd	factorized + sum rule	polyn. 2nd/3rd
pi thresh.	yes	yes	----	----