



# Systematic uncertainty of off-shell corrections and higher-twist contribution in DIS at large $x$

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**CTEQ-JLab Collaboration**

**A. Accardi, I. Fernando, X. Jing, S. Li, J. Owens, S. Park,  
C.E. Keppel, W. Melnitchouk, P. Monaghan**

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**We have to deal with Deuterium target at large- $x$**



# Deuterium: nuclear smearing

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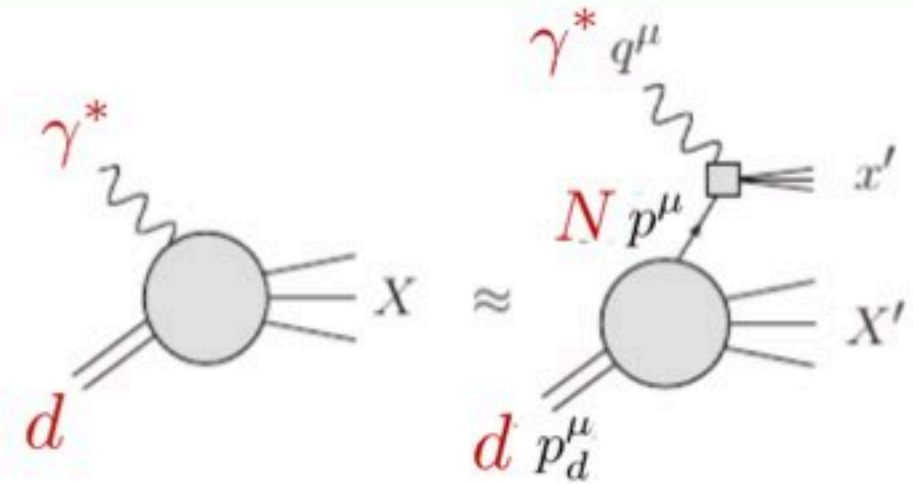
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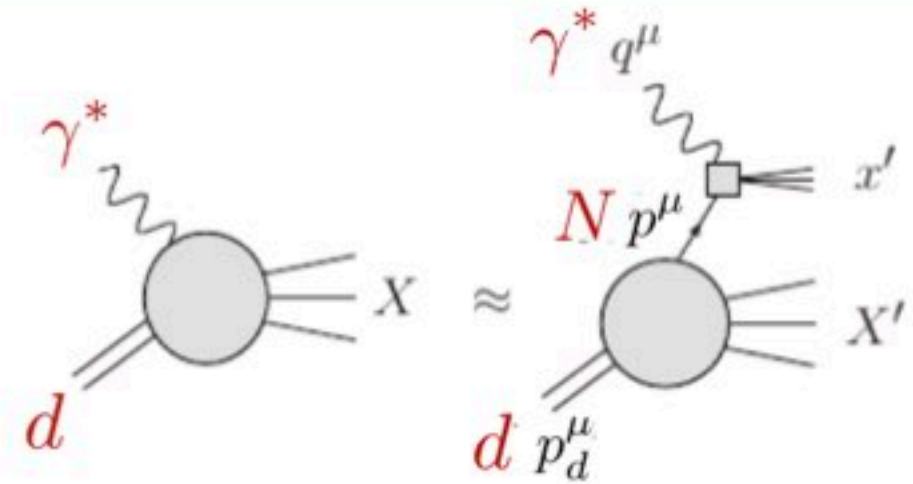
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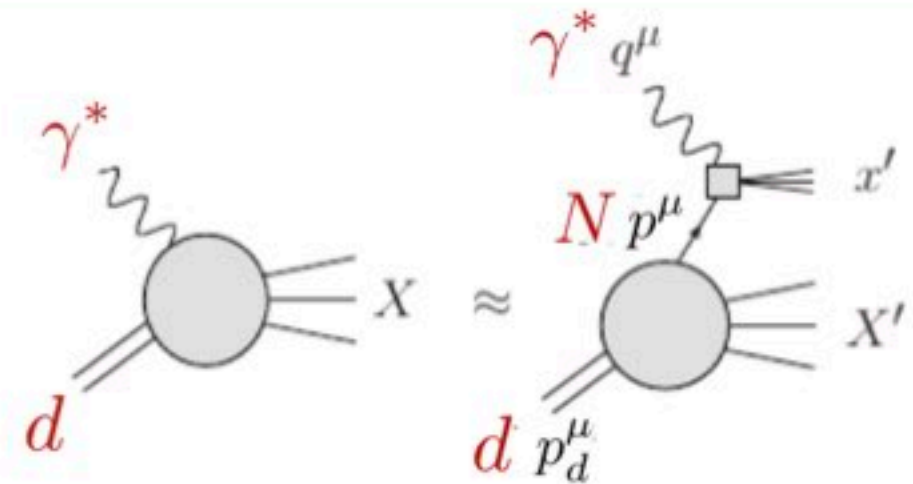
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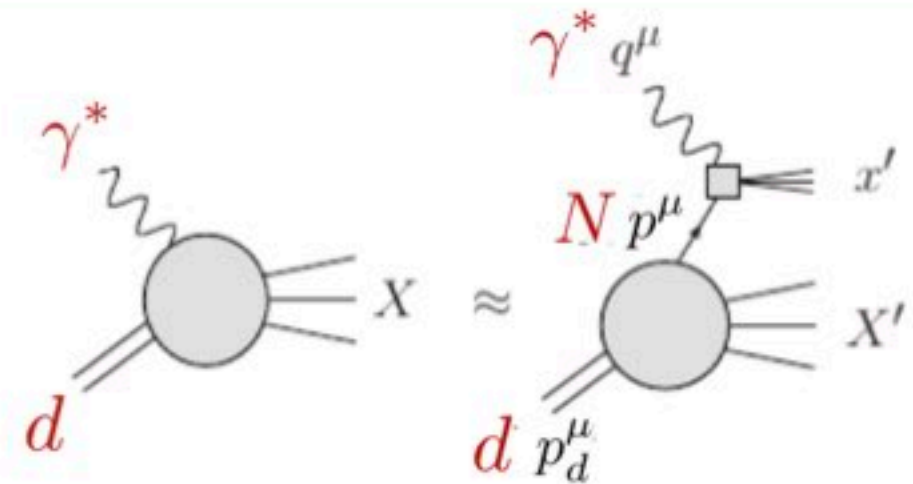
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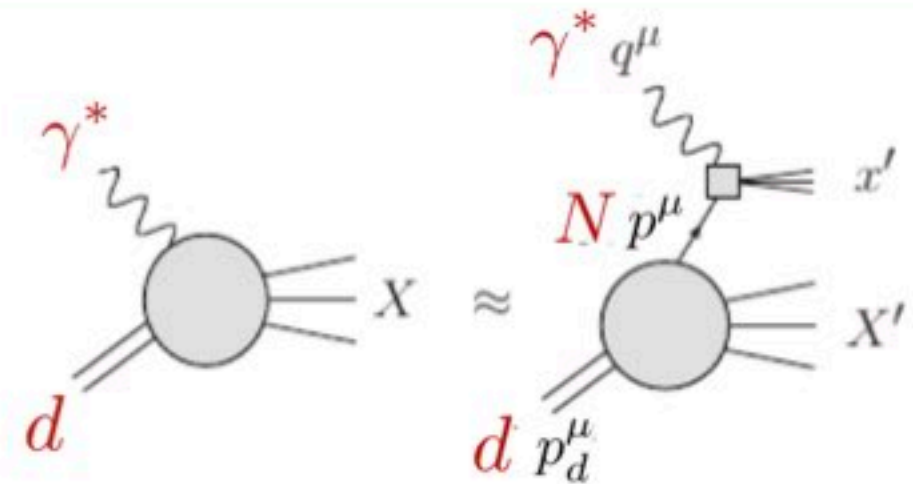
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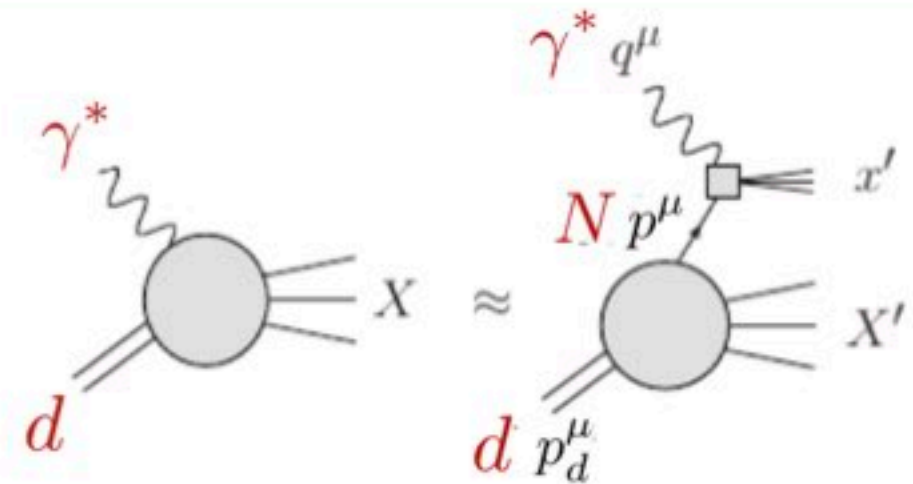
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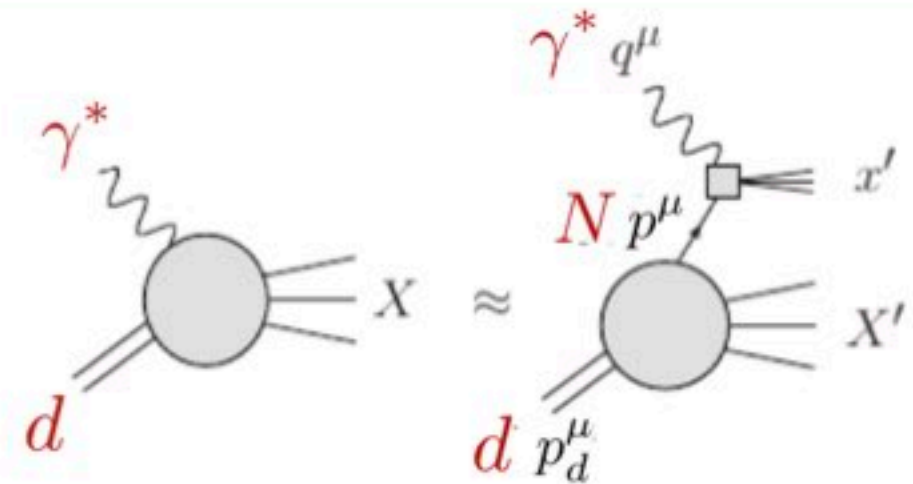
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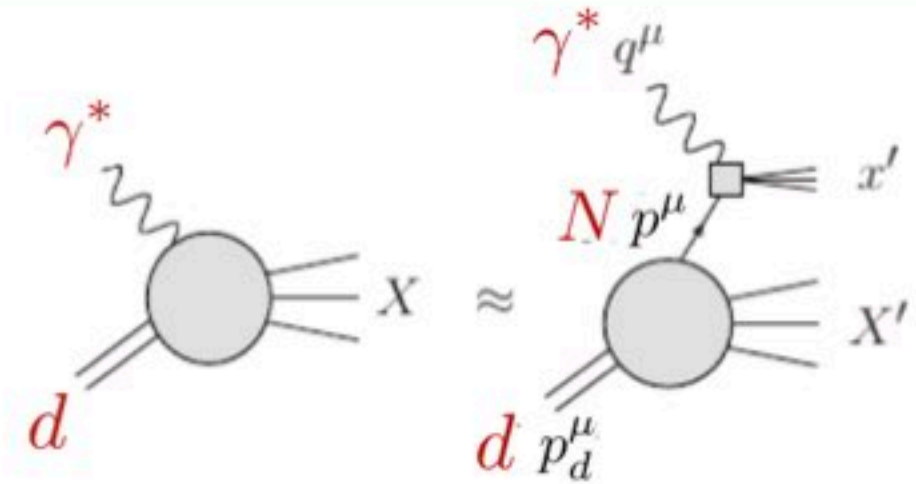
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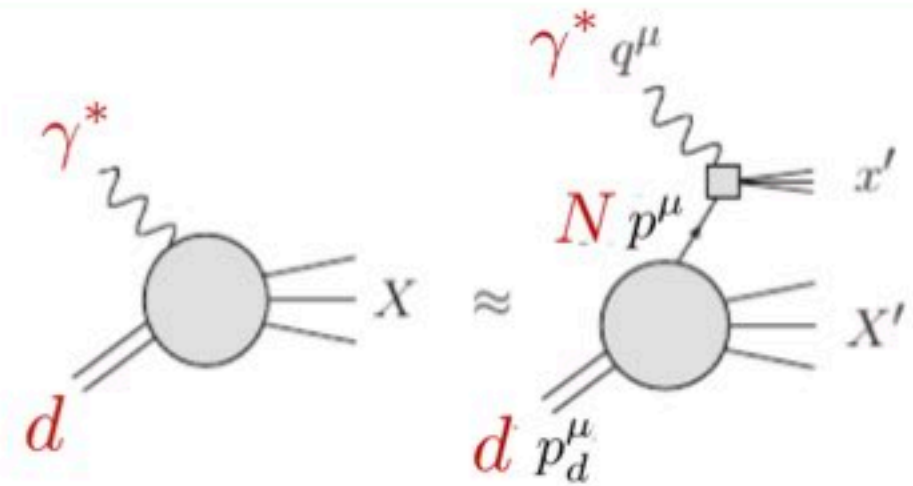
## Structure function

*of a bound, off-shell nucleon*

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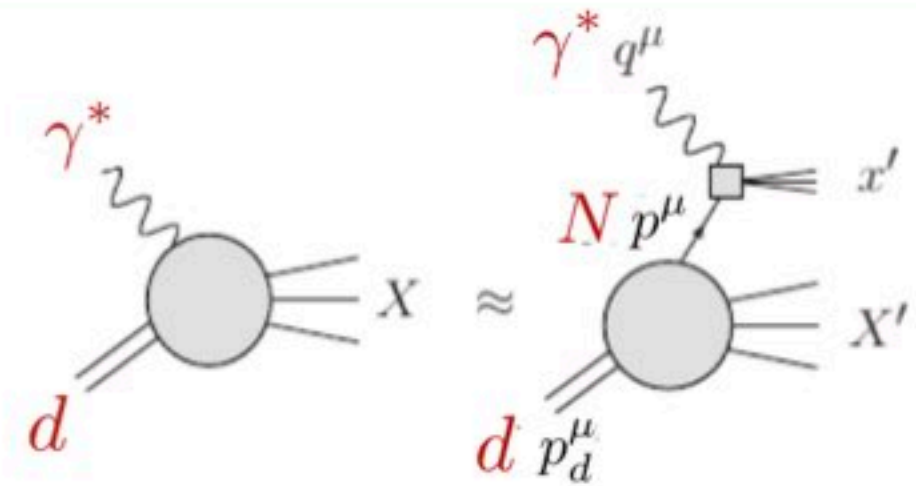
Bound, off-shell nucleon inside the deuteron



# Deuterium: off-shell corrections

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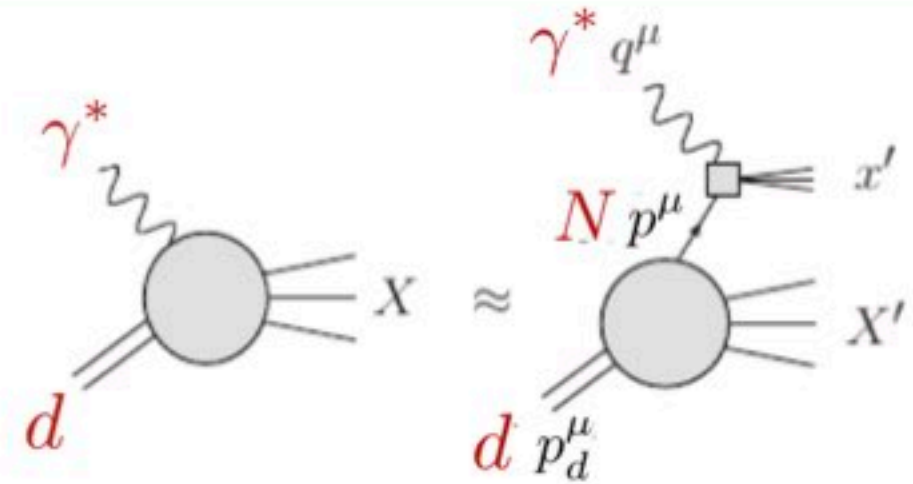


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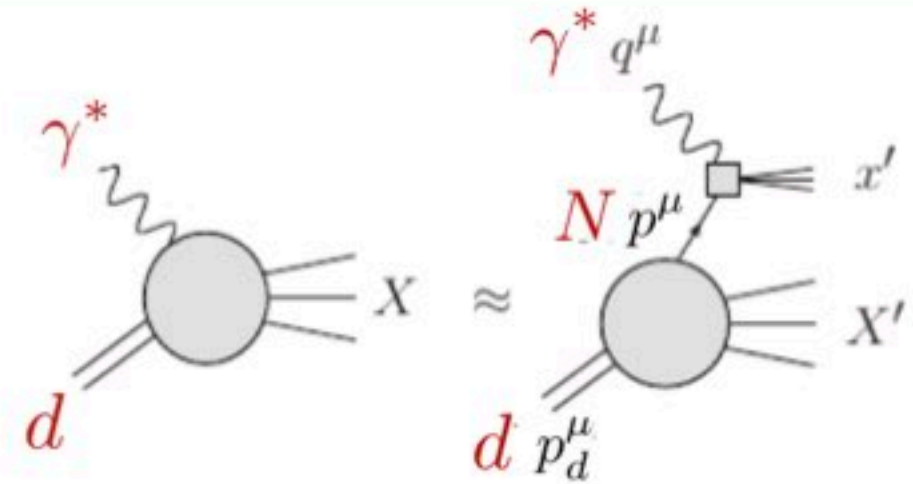


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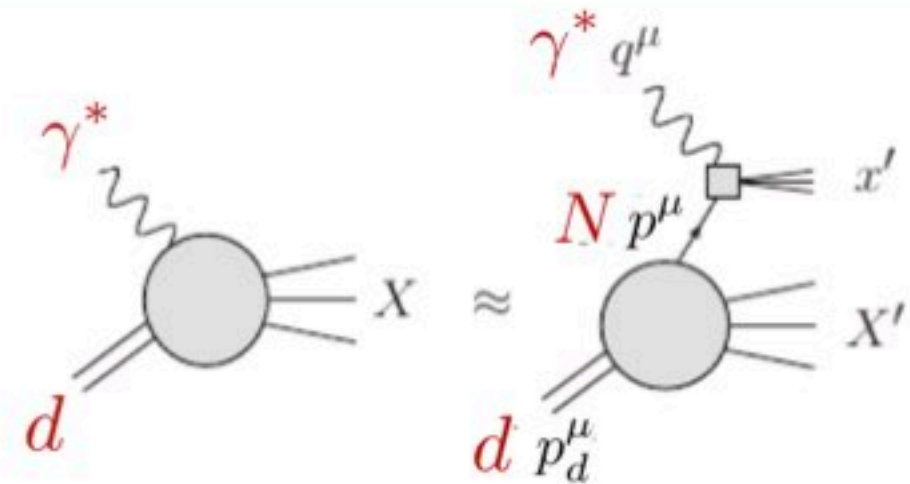
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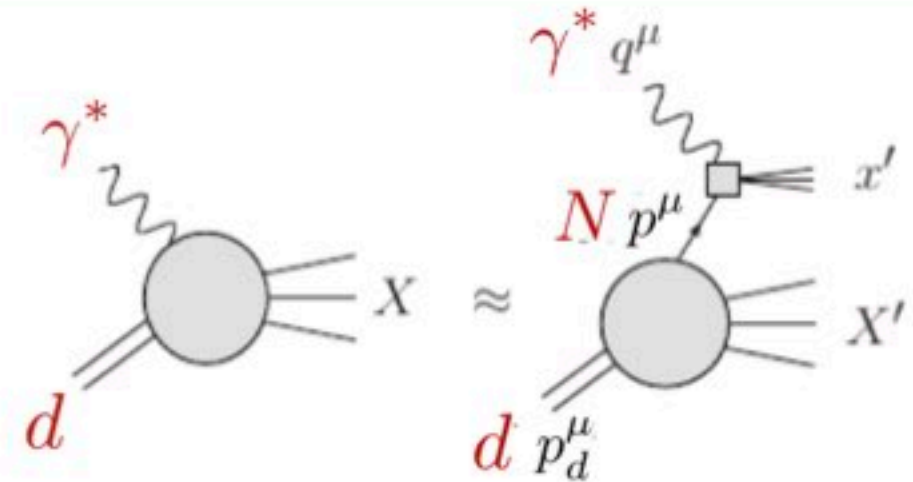
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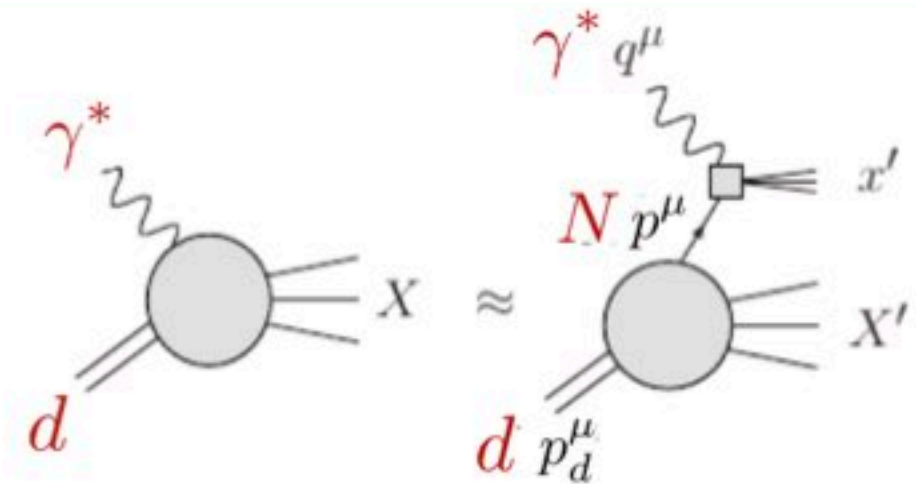


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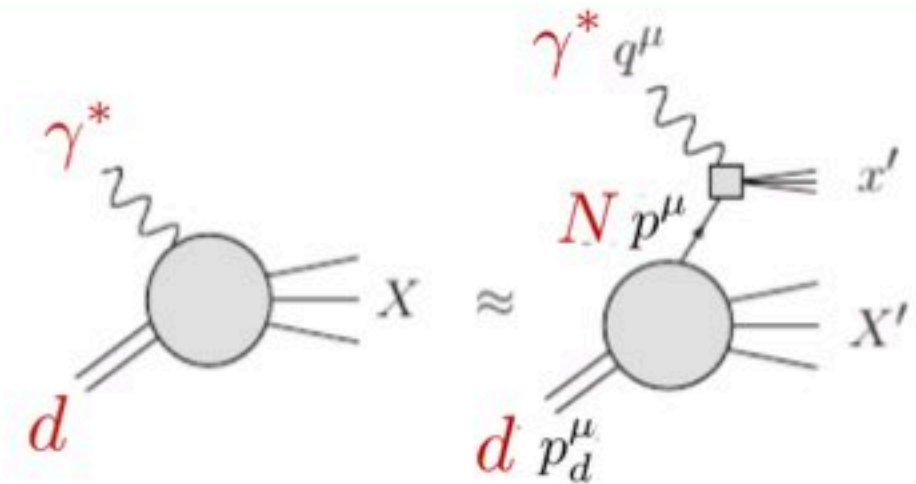
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Off-shell function

(To be fitted)

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Accardi, Brady, et al., PRD 93 (2016)

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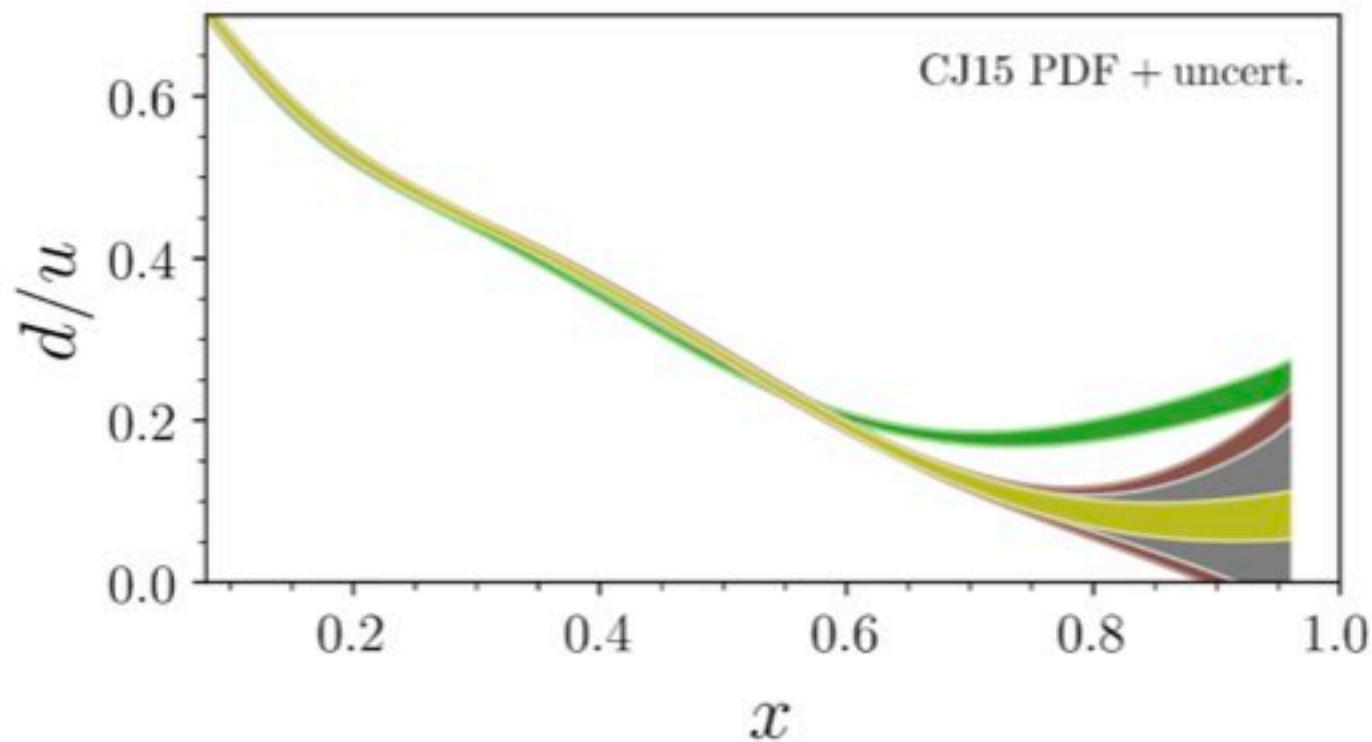
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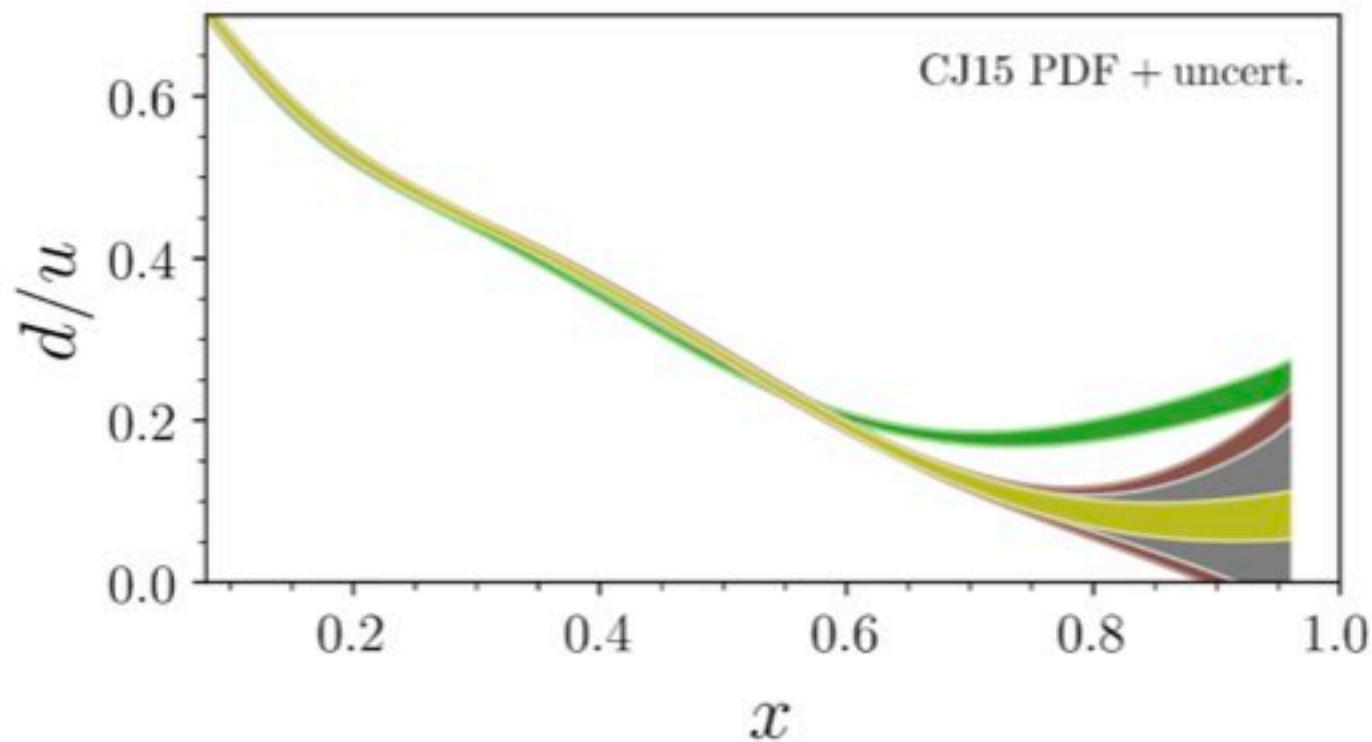
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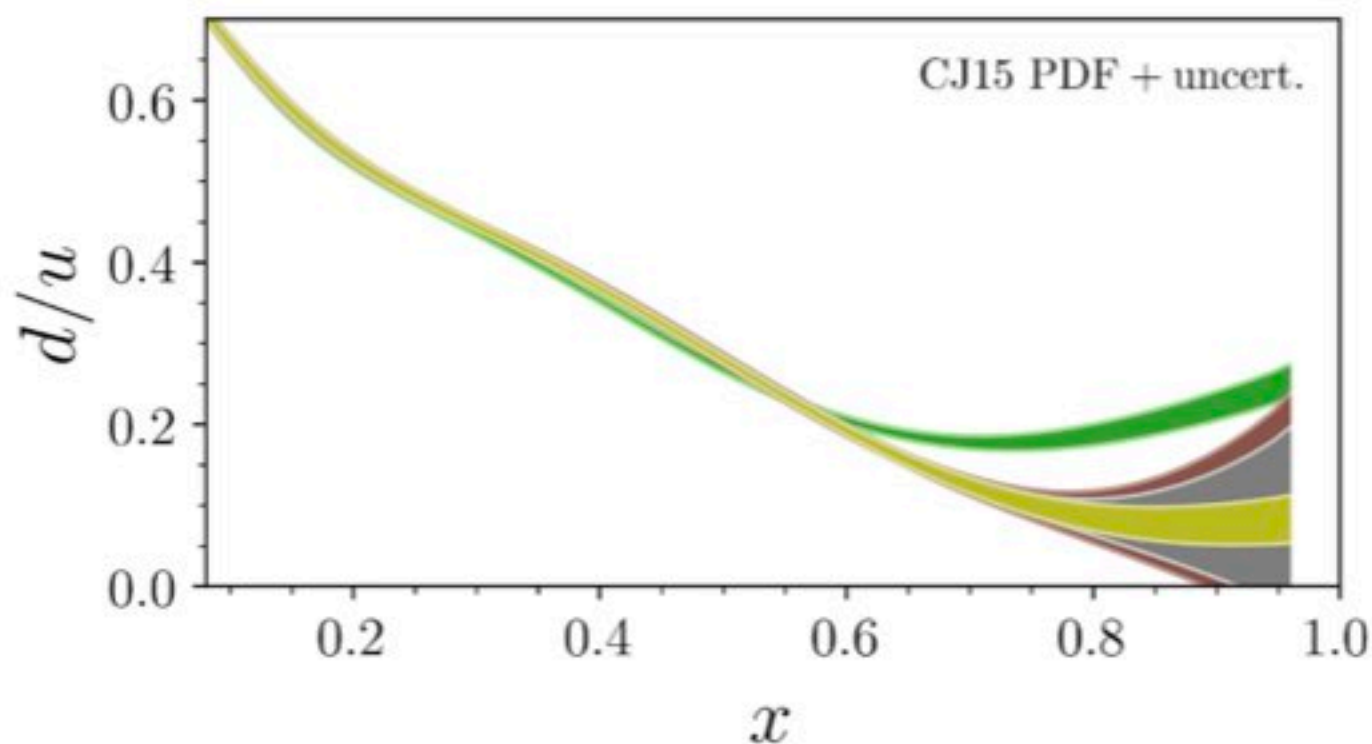
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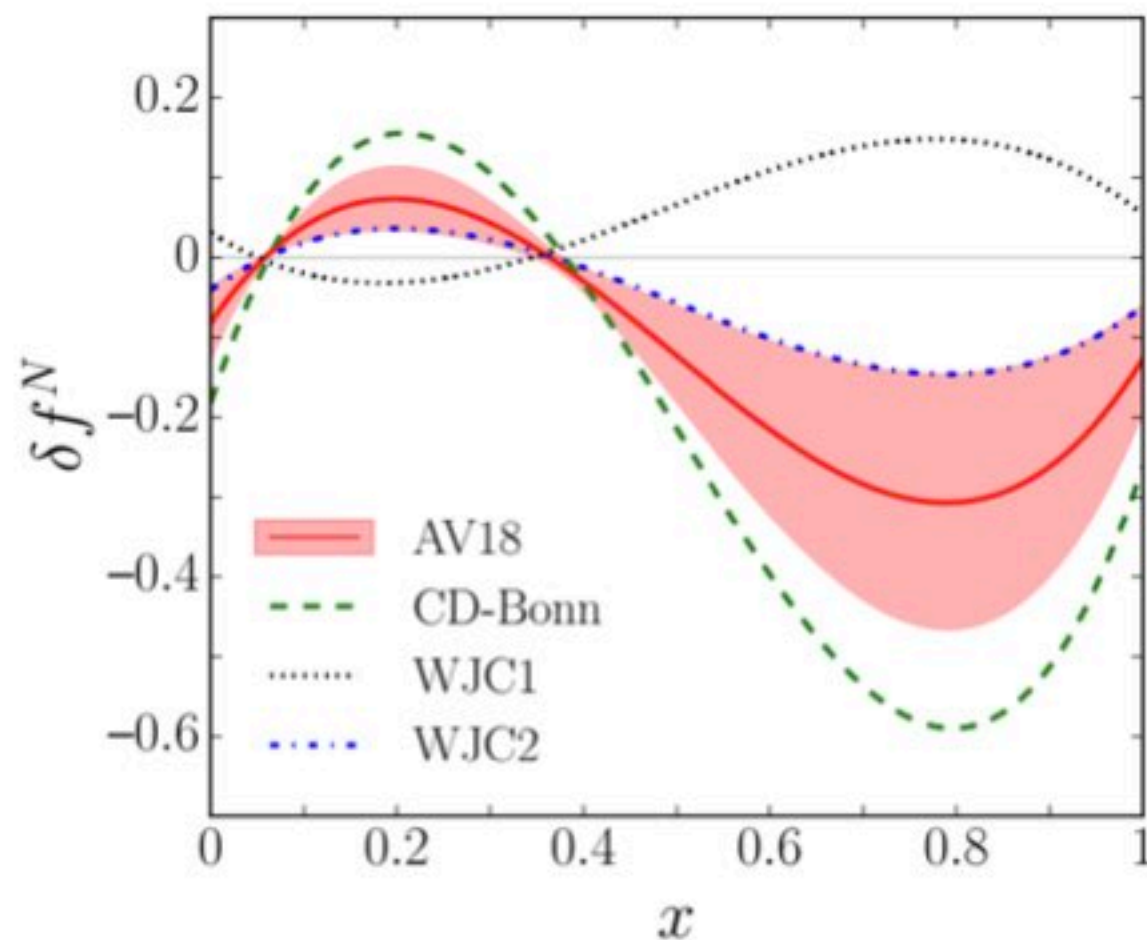
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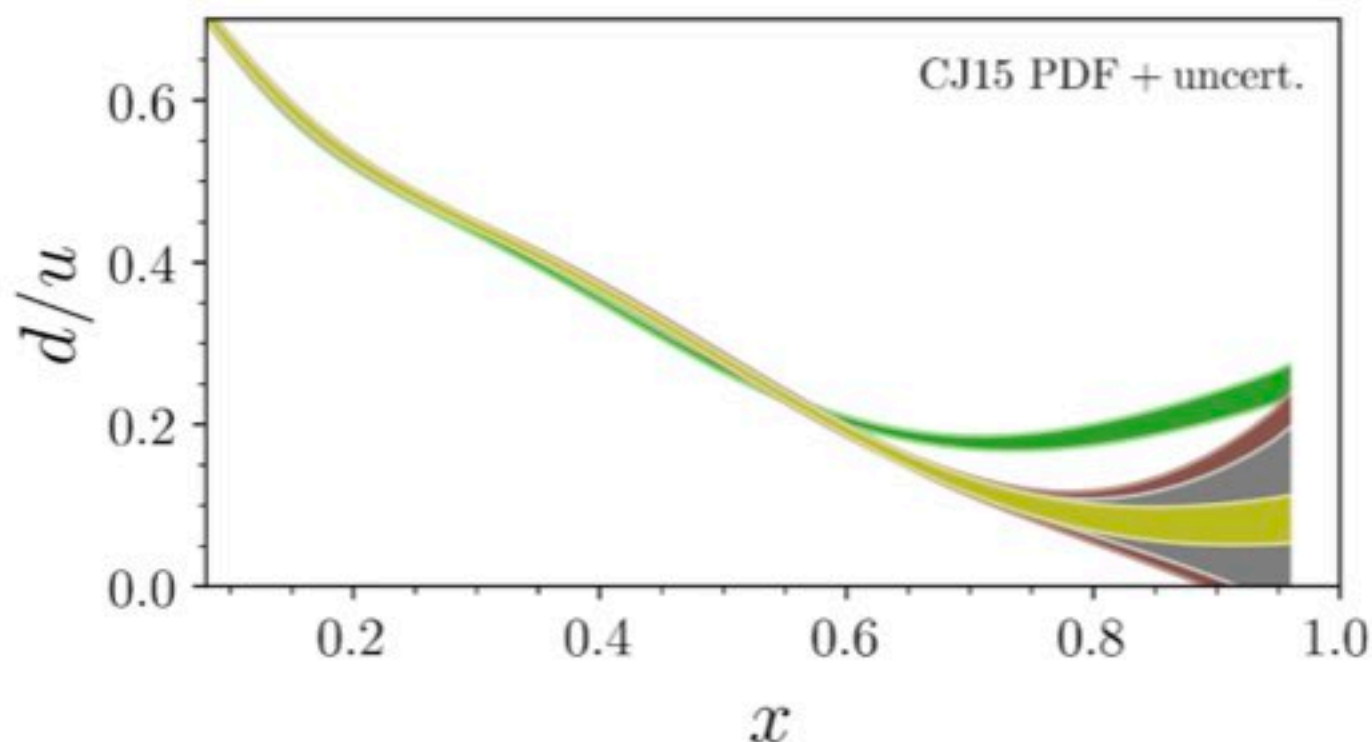
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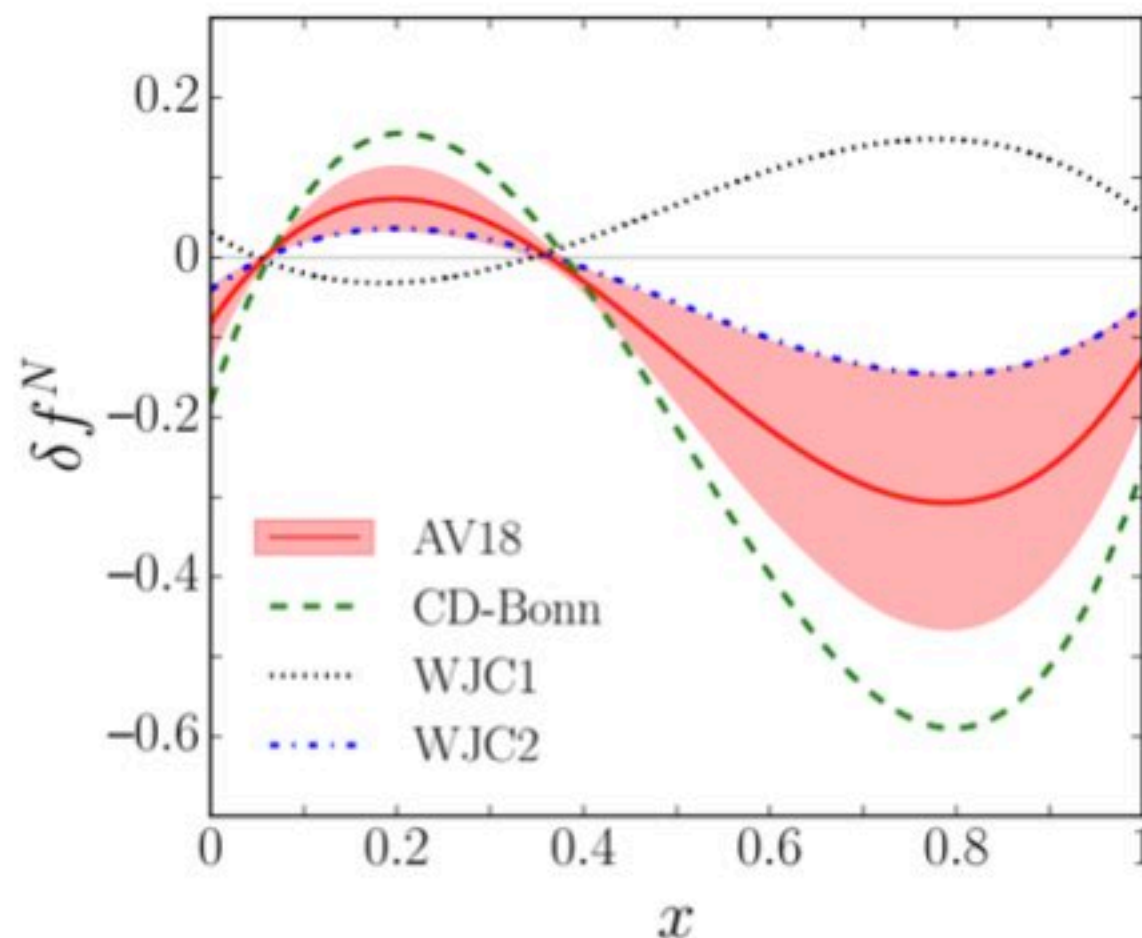
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The most of the recent nuclear potentials does not introduce a bias on the fit





# Latest results from QCD fits in CJ framework

## CJ22 fit

Accardi, Jing, Owens et al., PRD 107 (2023)

# Latest results from QCD fits in CJ framework

## **CJ22 fit**

Same off-shell parameterization

More flexible parameterization of sea quarks (NuSea and SeaQuest data)

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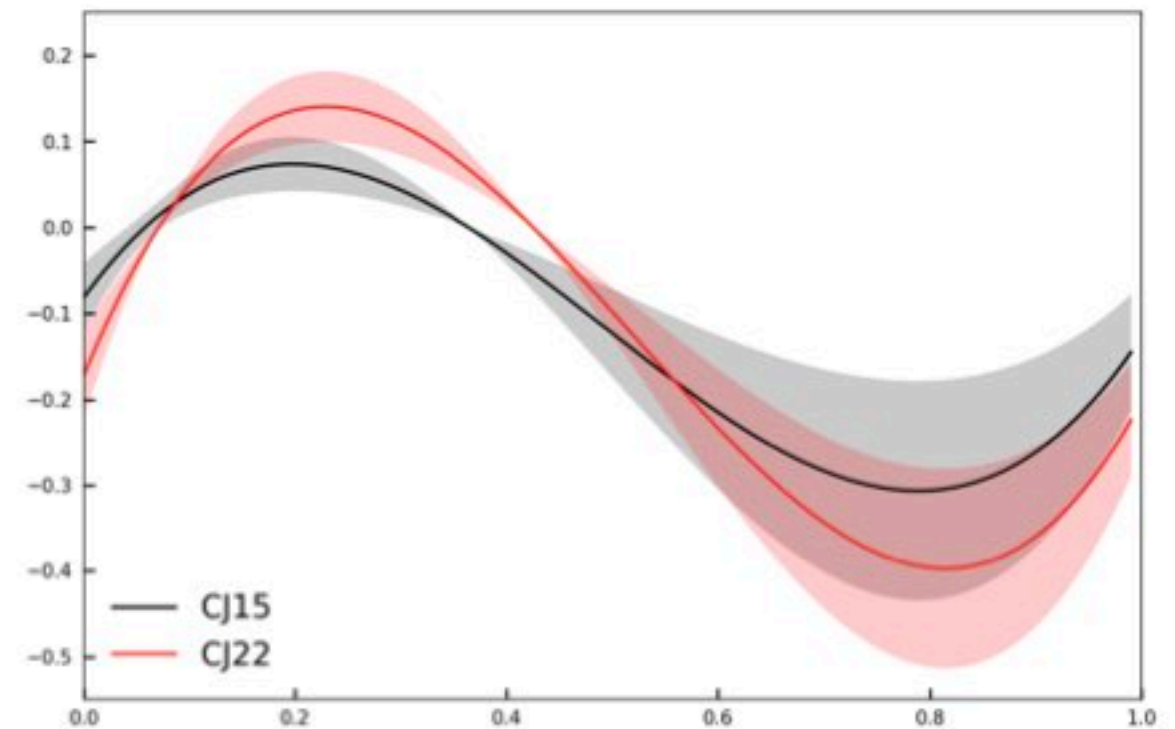
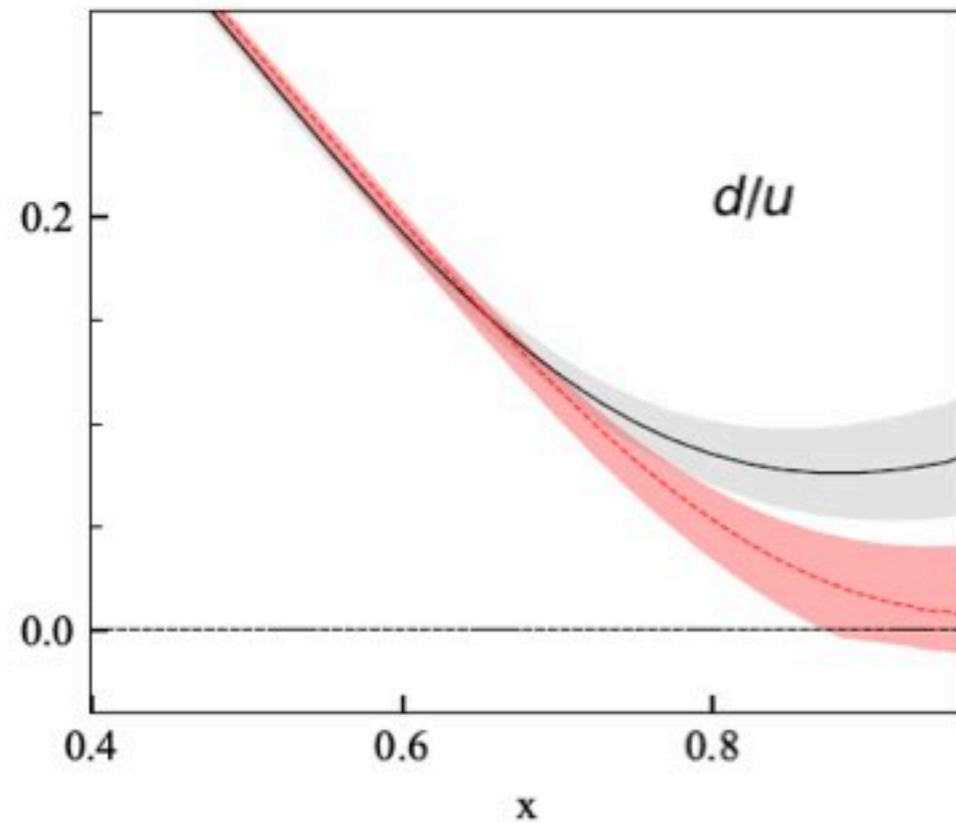
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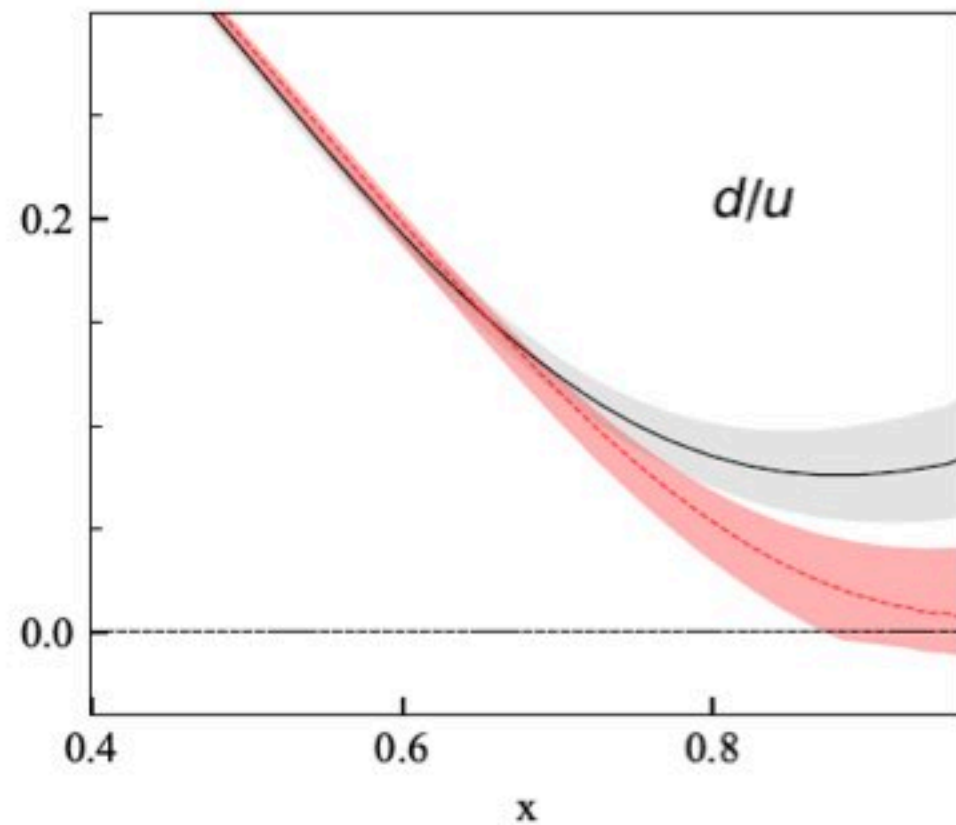
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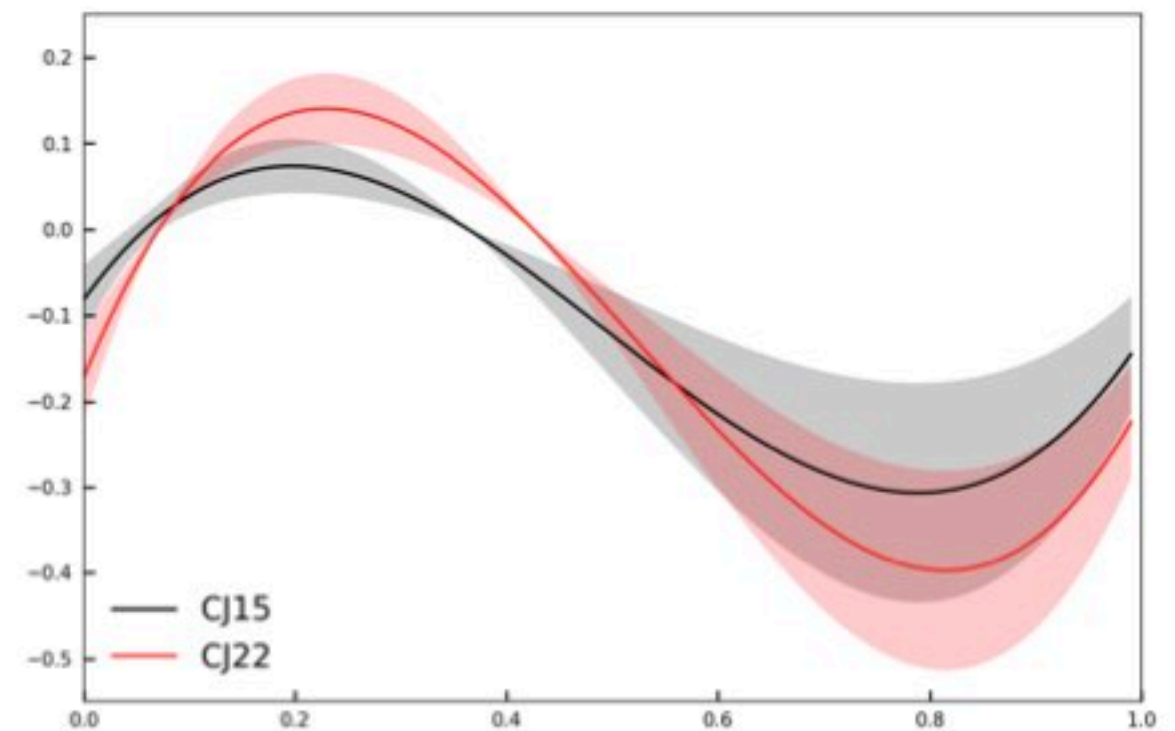
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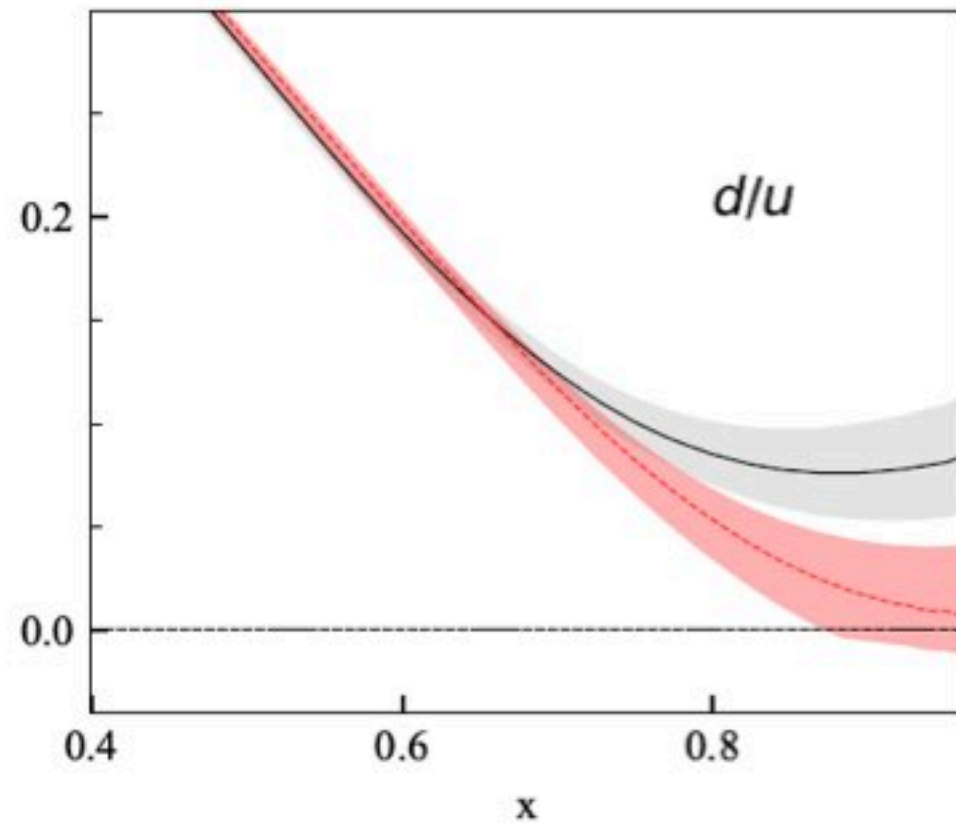
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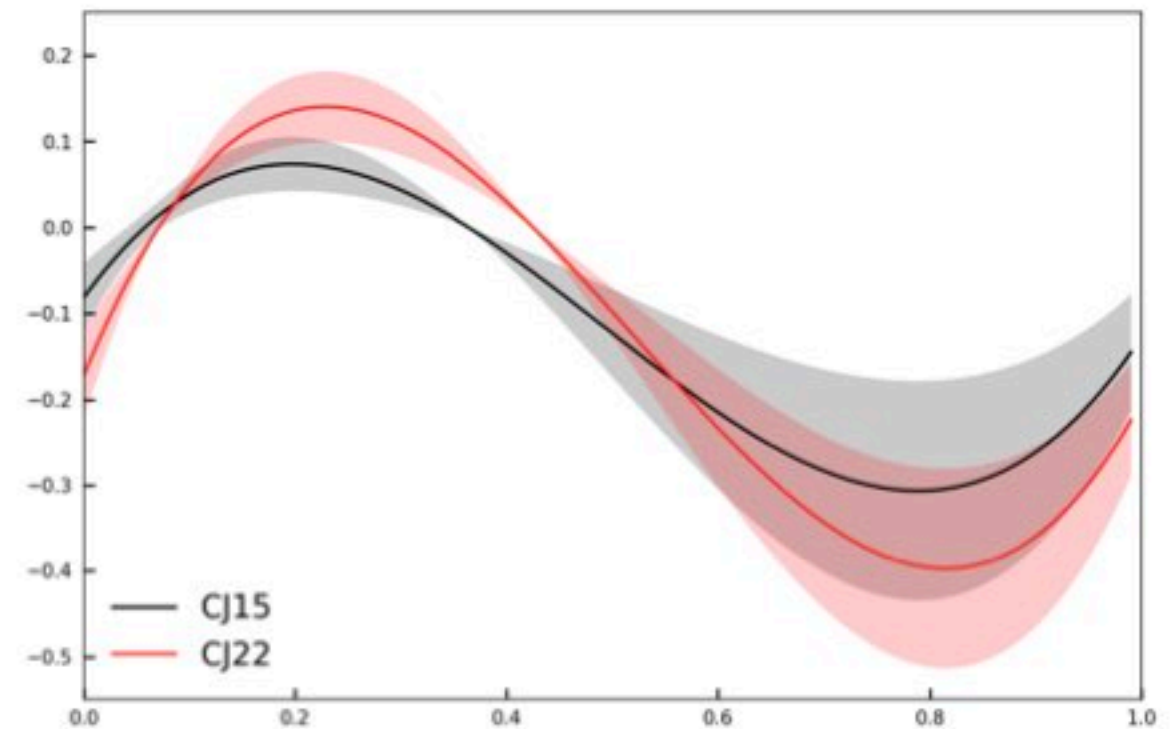
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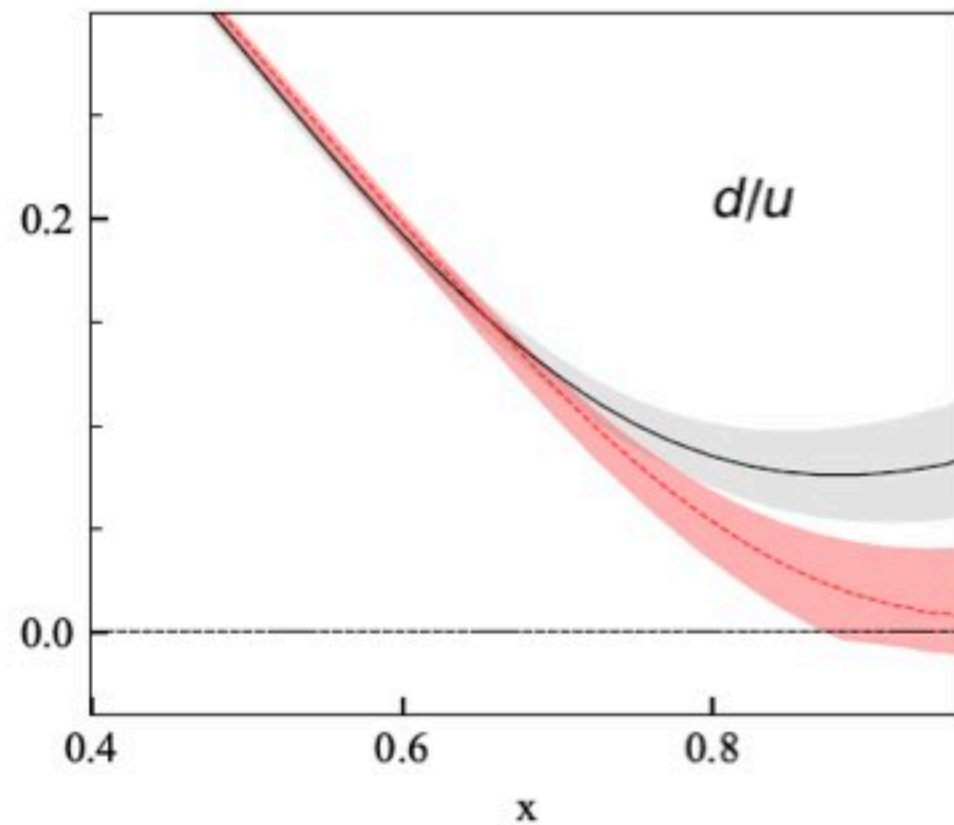
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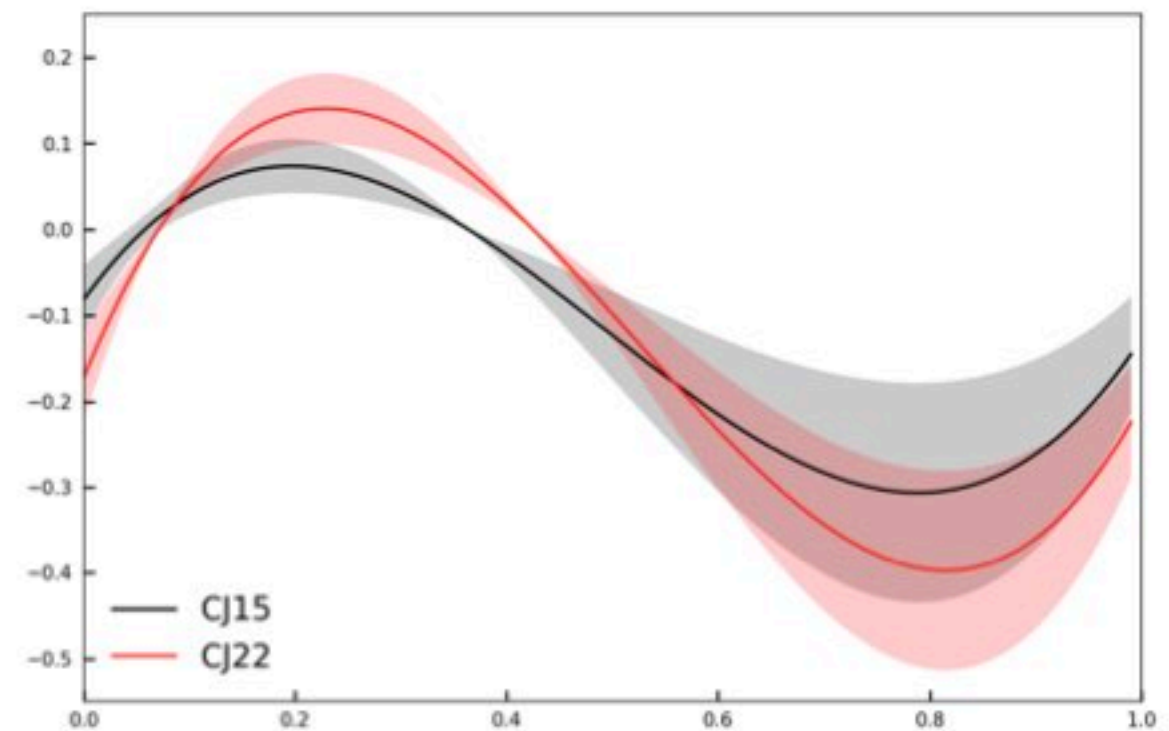
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Difference on  $d/u$  is absorbed in something else



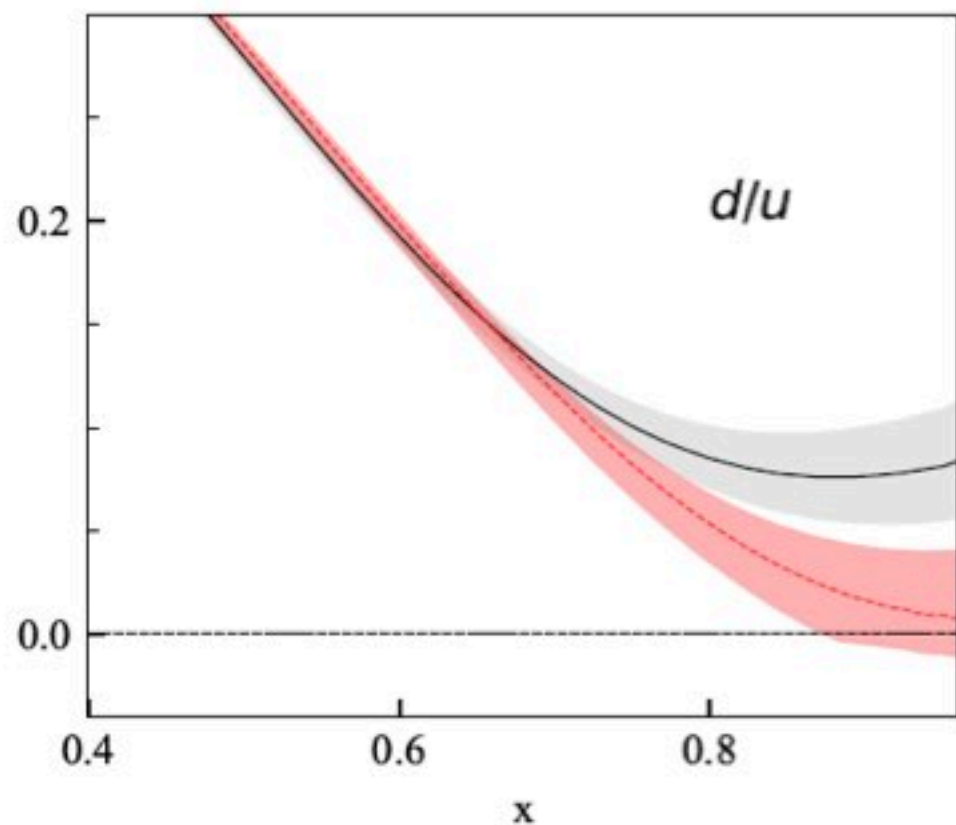
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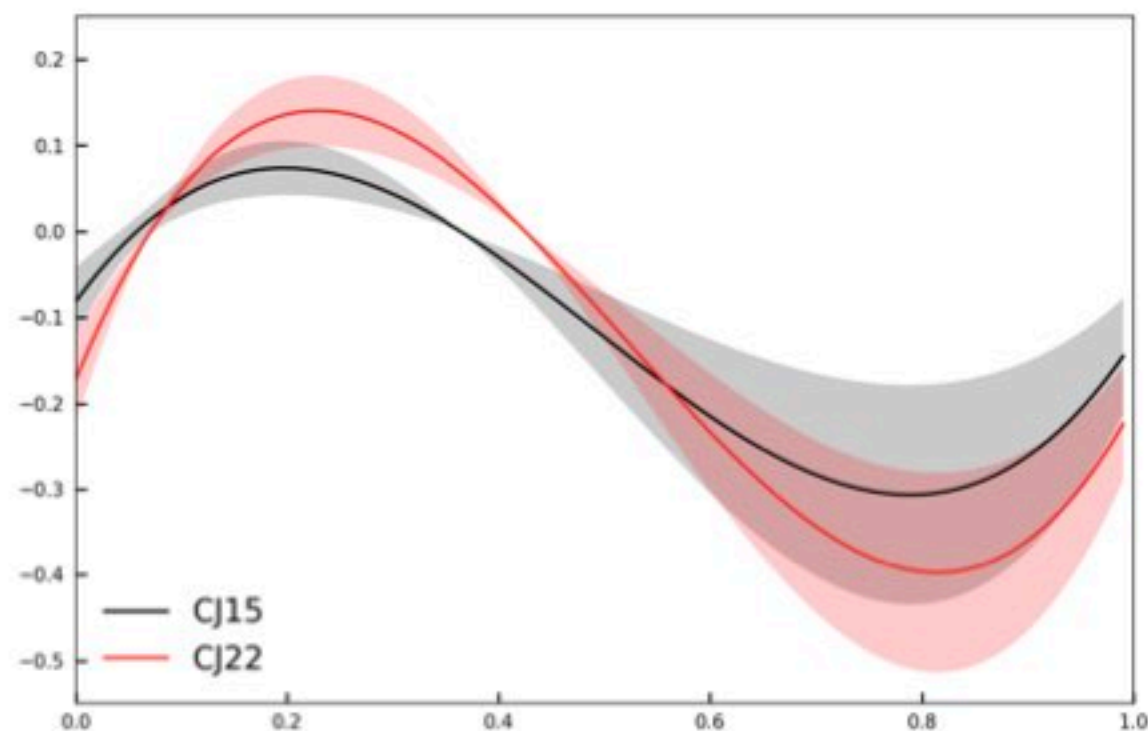
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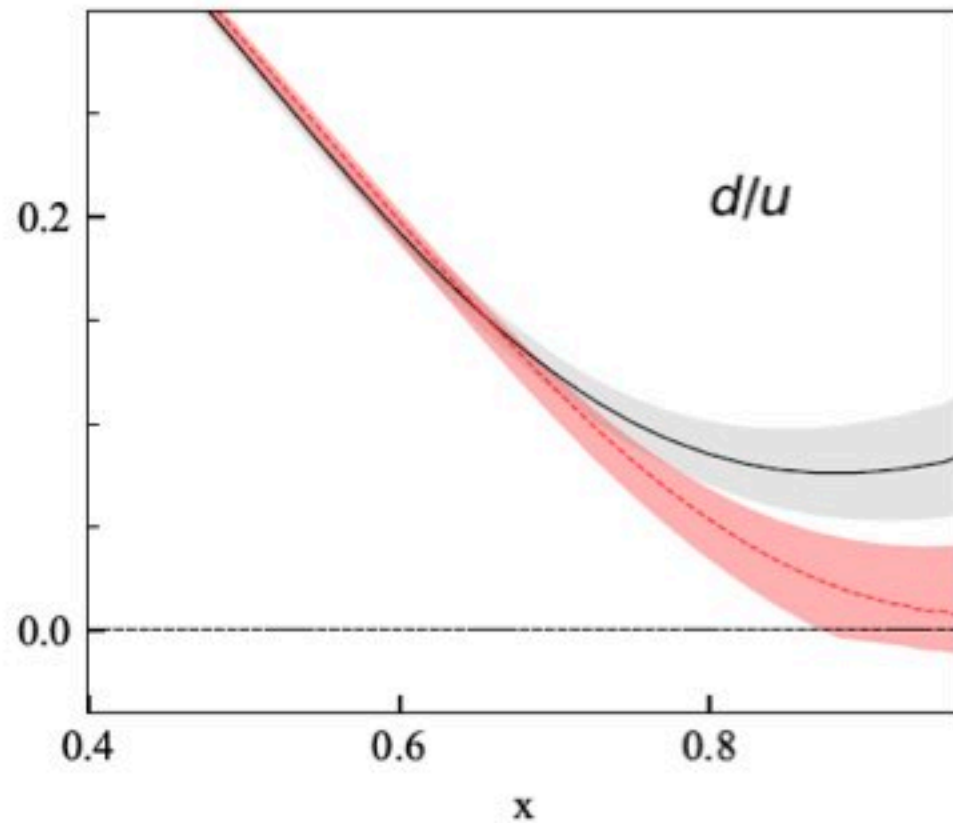
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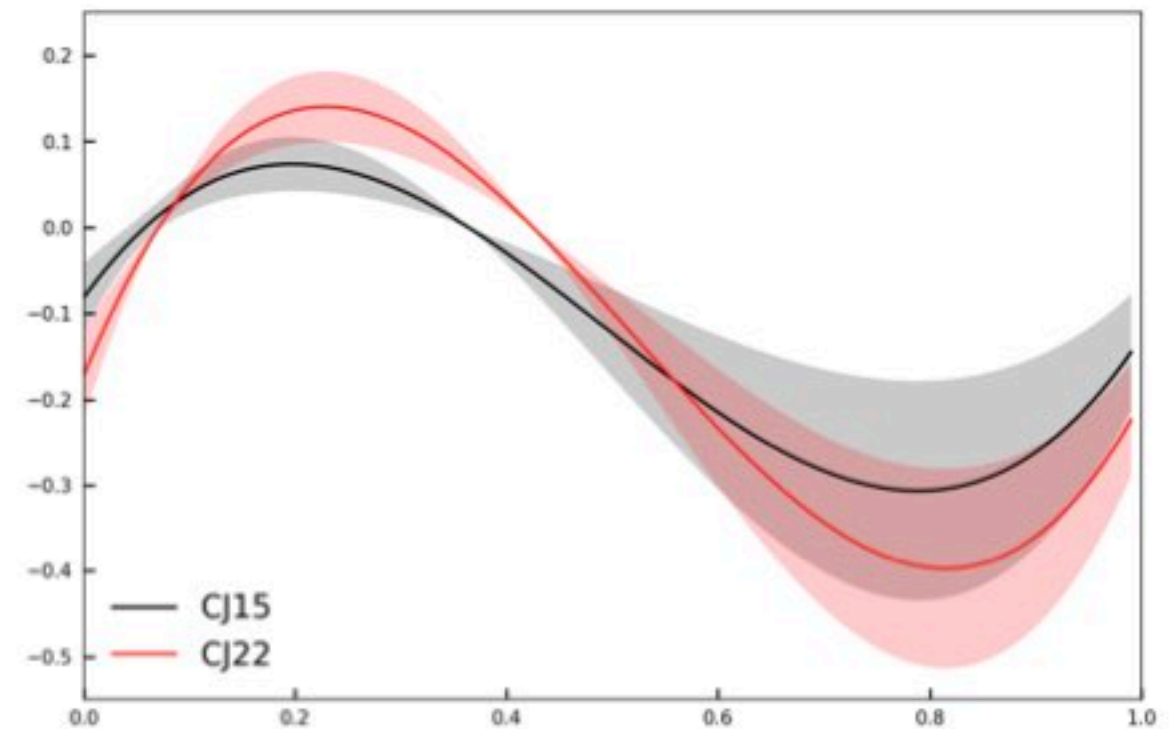
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Is the model for off-shell correction enough flexible?

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KP-like model

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$C, x_0$  and  $x_1$  fitted



$$x_1 \simeq x_0$$

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KP-like model

Kulagin and Petti, NPA 765 (2006)

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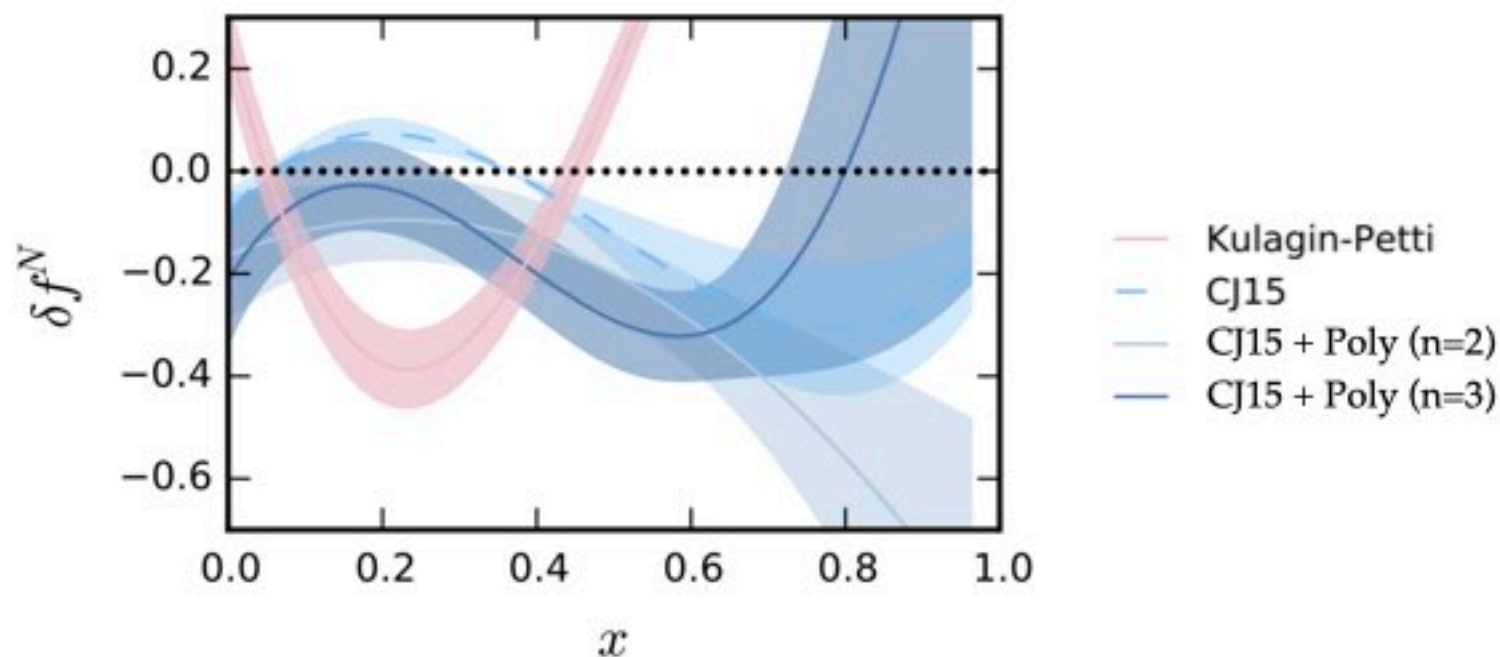
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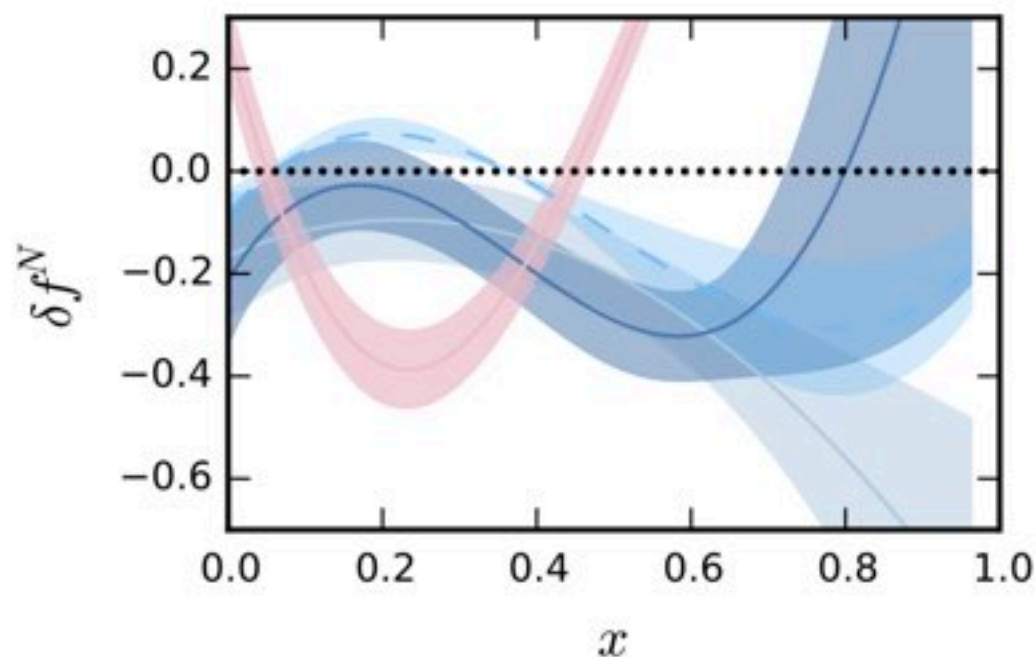
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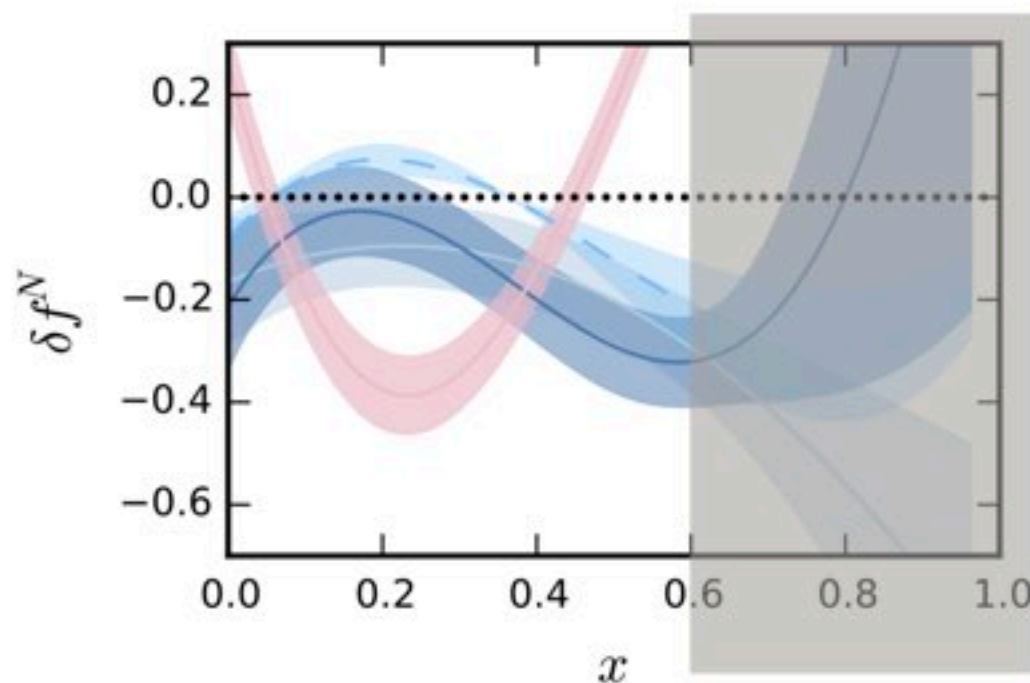
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Constrain power of CJ15  
dataset only up to  $x = 0.6$



# Higher-Twist function

**Higher Twist correction**



# Higher-Twist function

## Higher Twist correction

### Multiplicative

$$F_2(x, Q^2) = F_2^{LT}(x, Q^2) \left( 1 + \frac{C(x)}{Q^2} \right)$$

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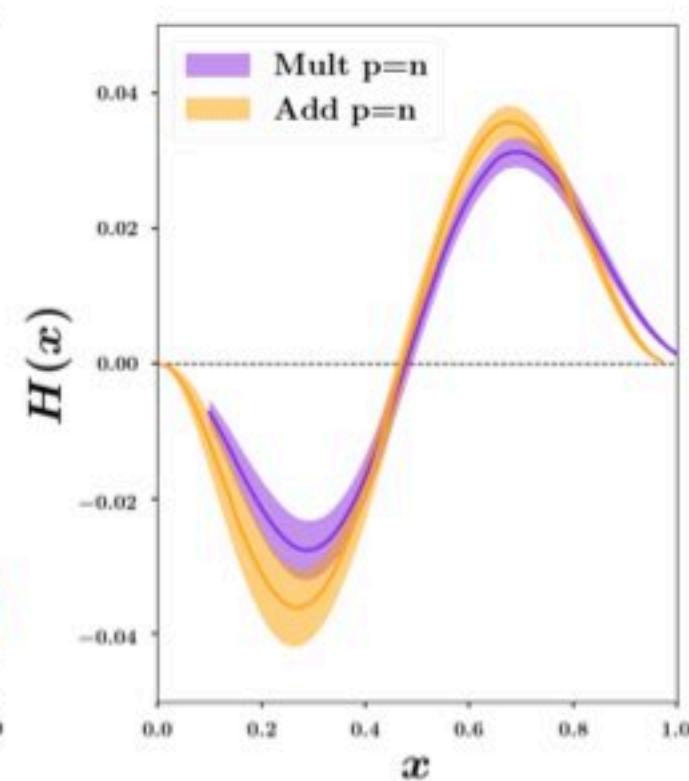
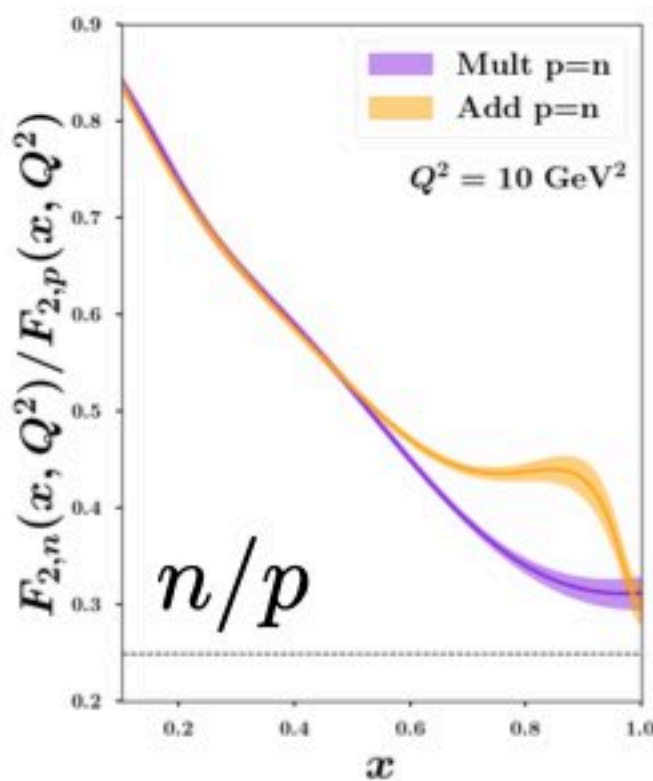
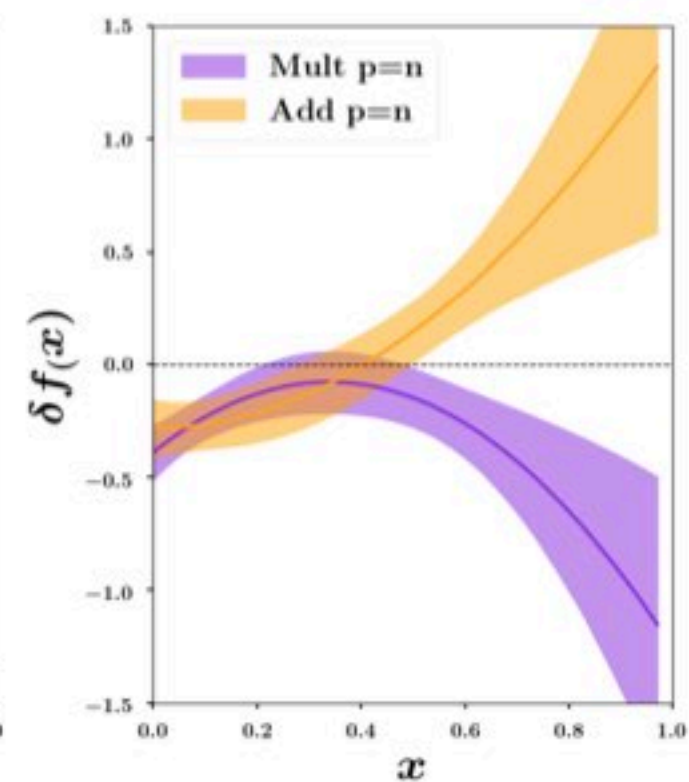
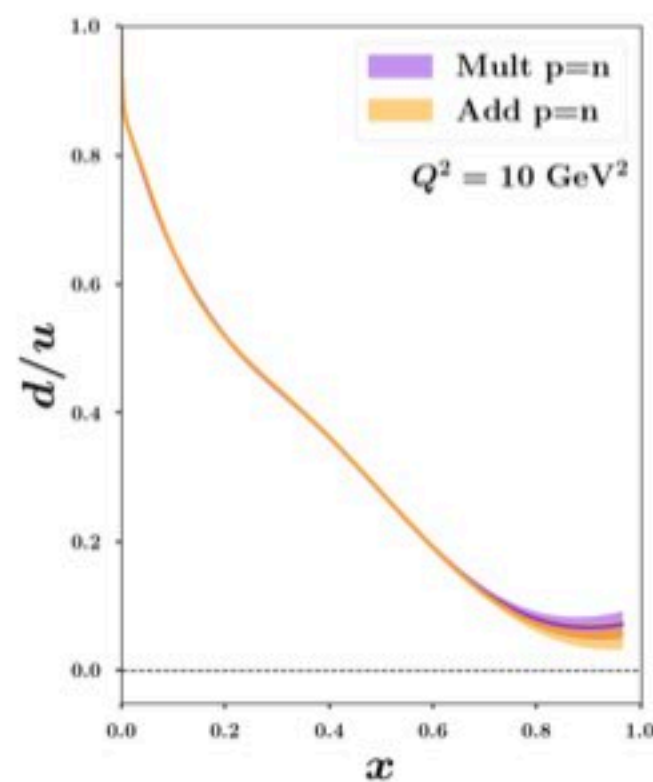
same as Add

**Bias not present!**



# Results in the CJ fitting framework

## Case 1: isospin symmetry



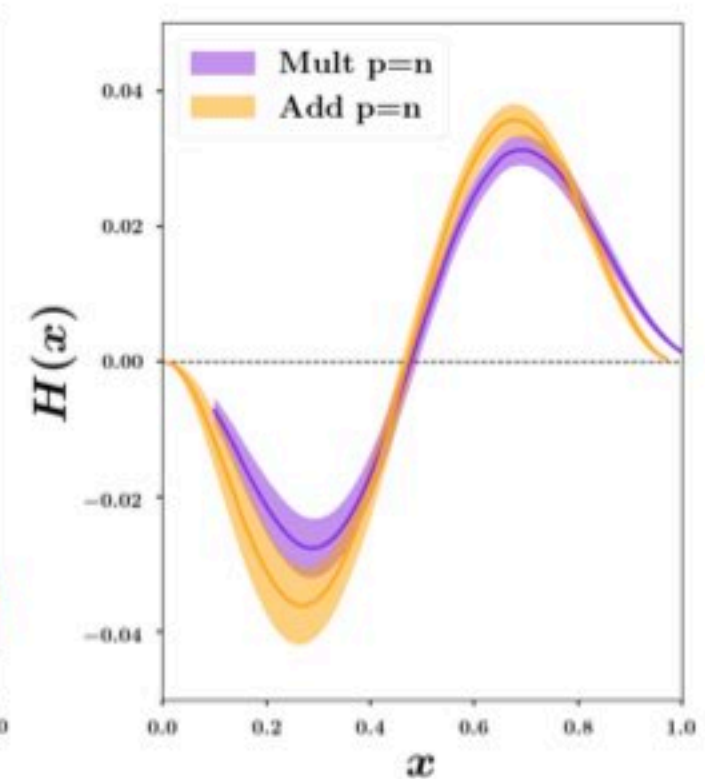
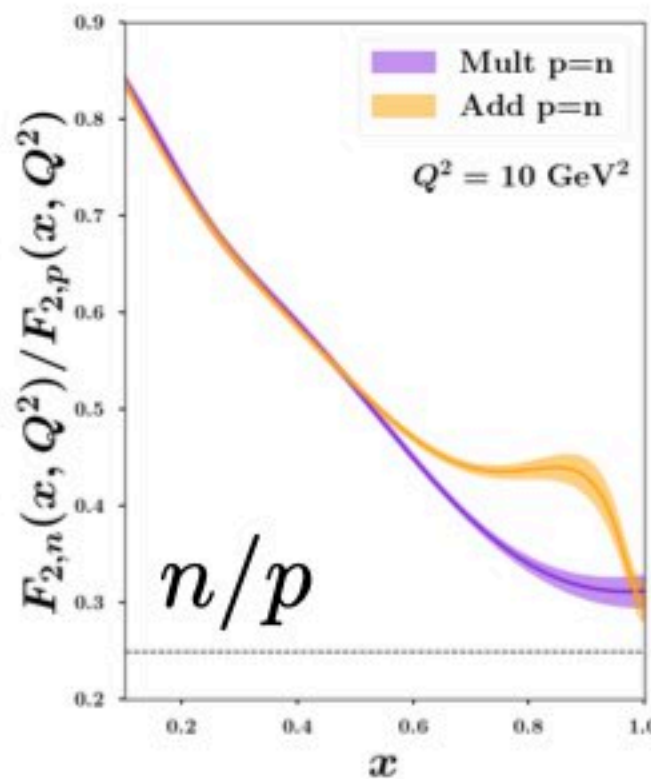
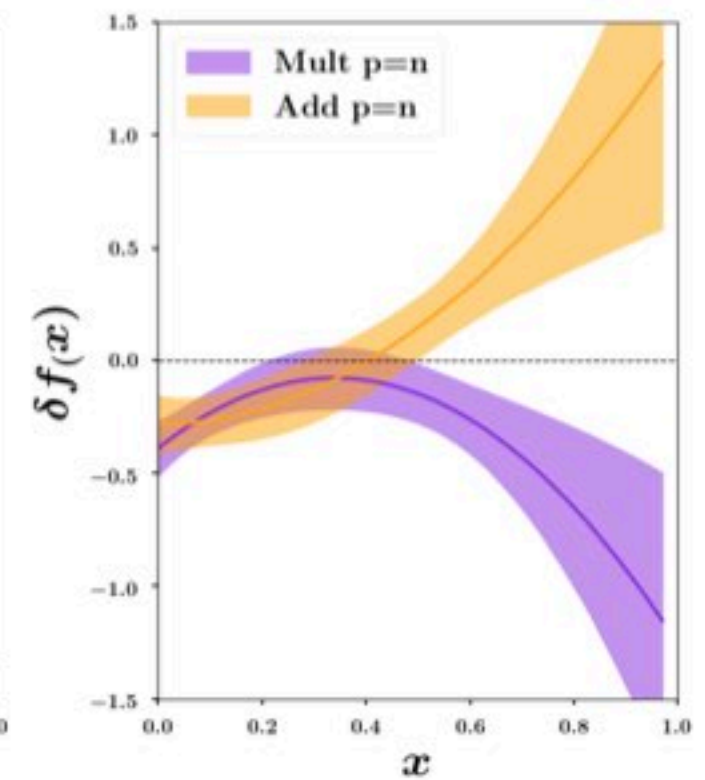
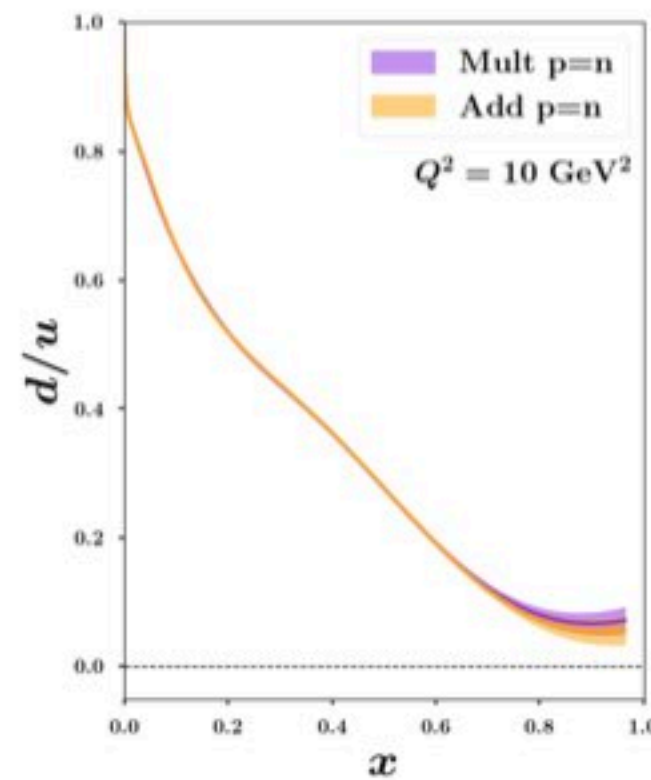
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Unnaturally large  $n/p$

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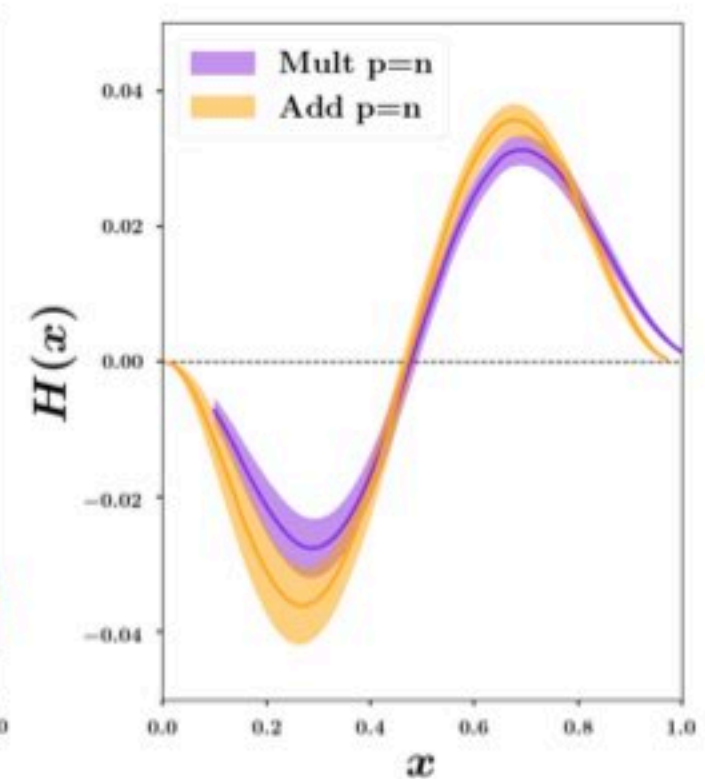
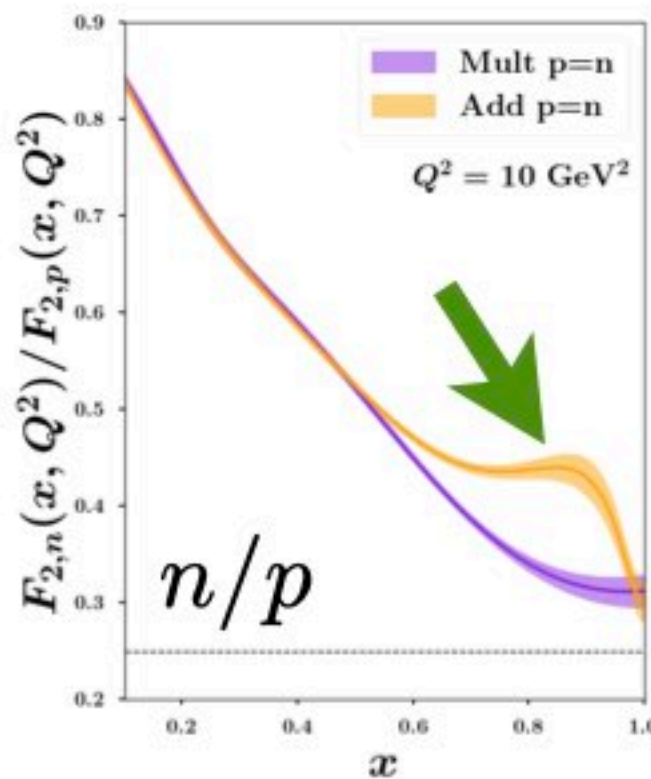
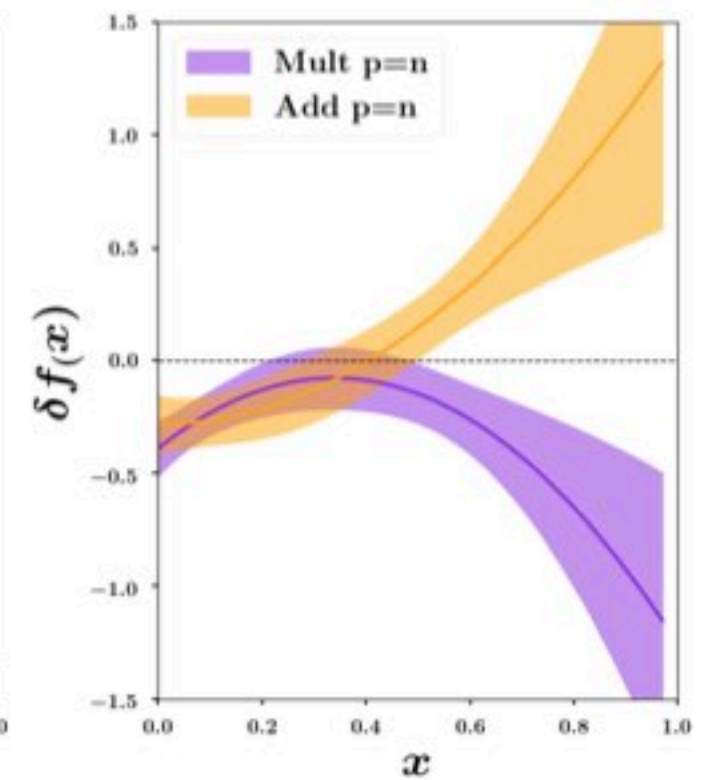
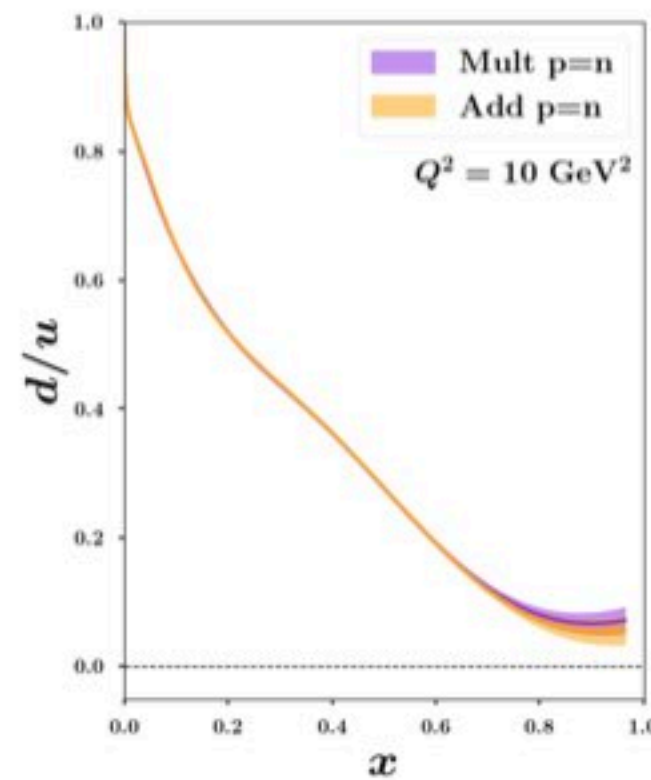
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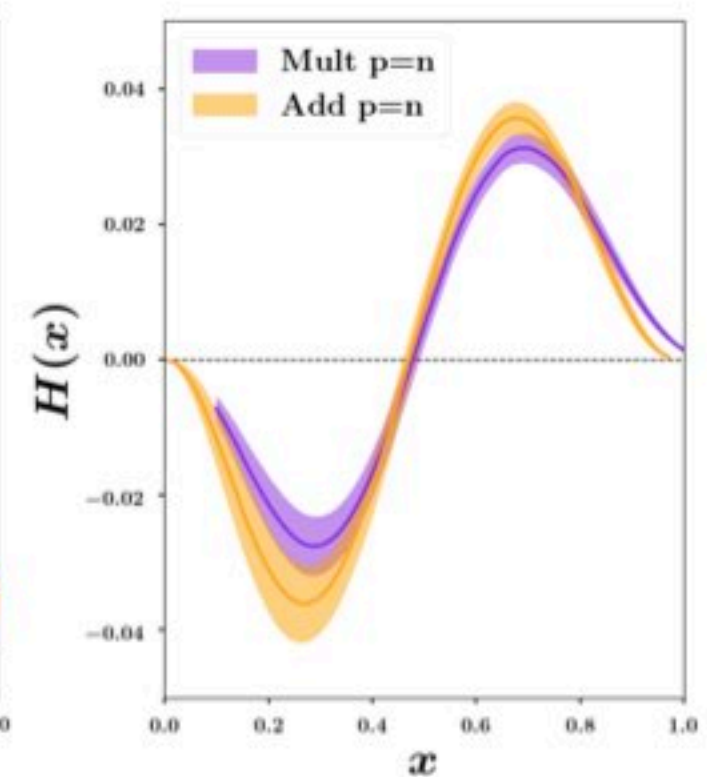
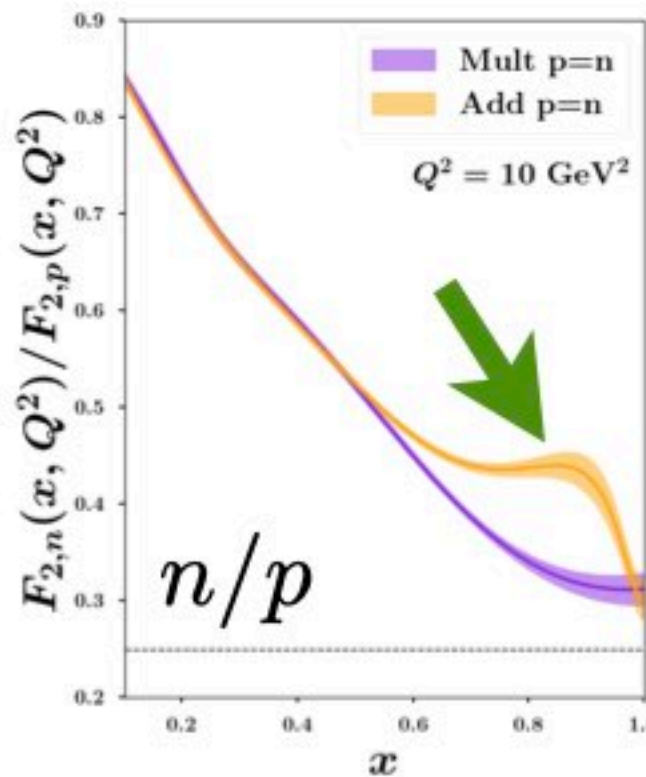
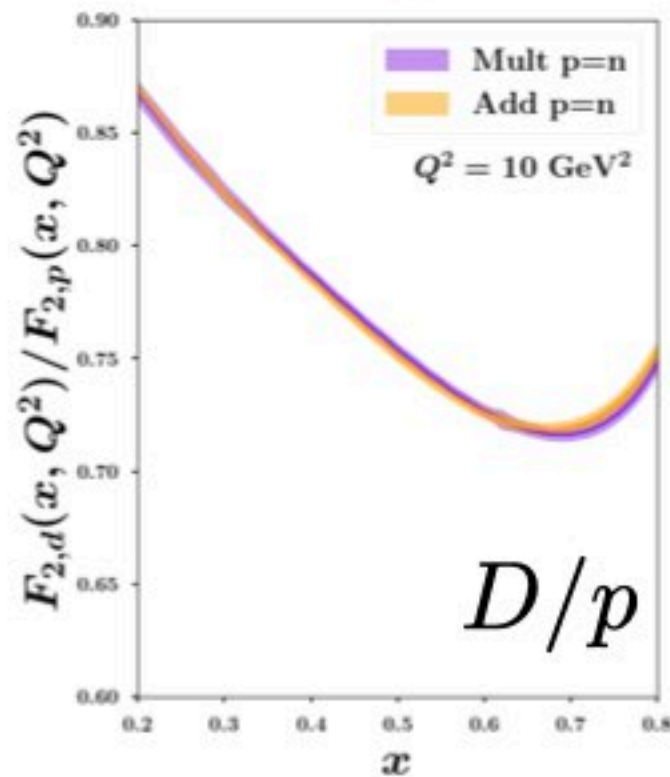
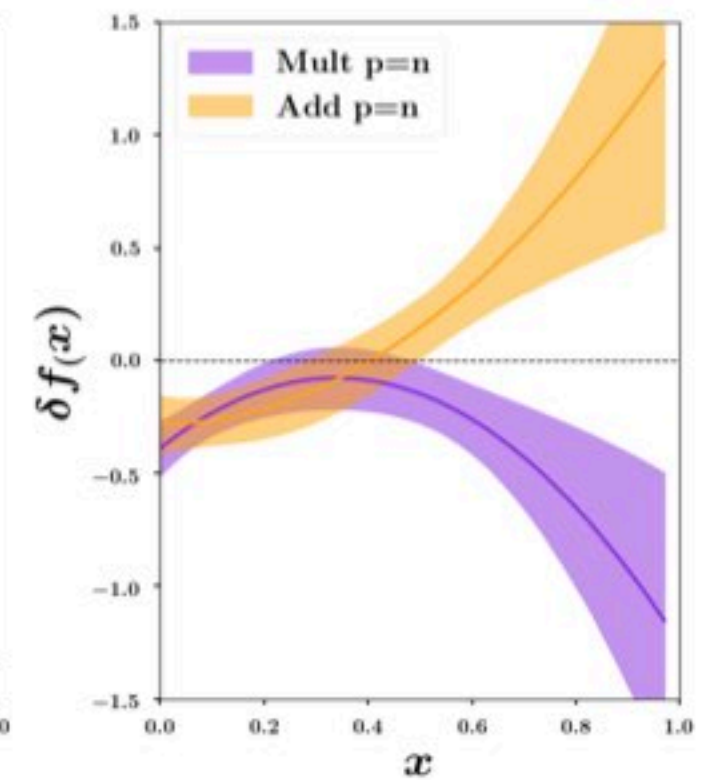
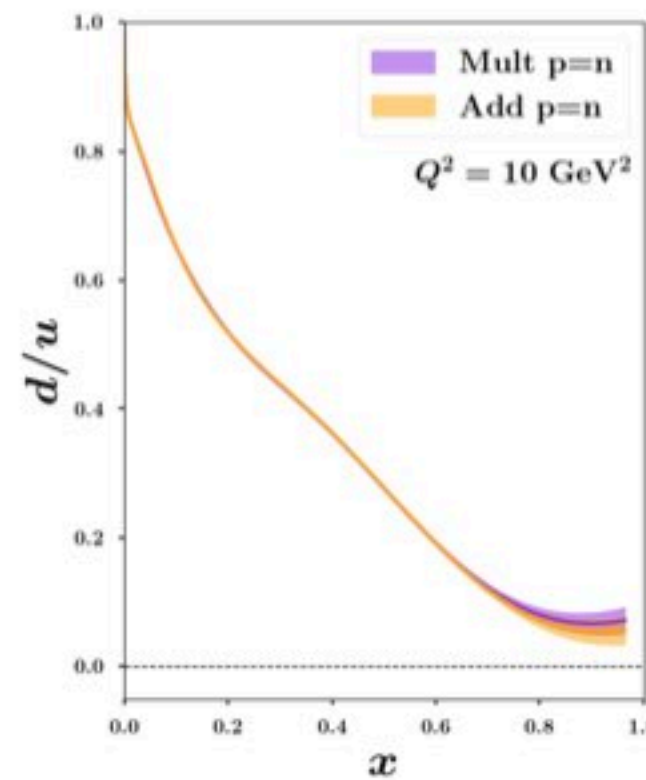
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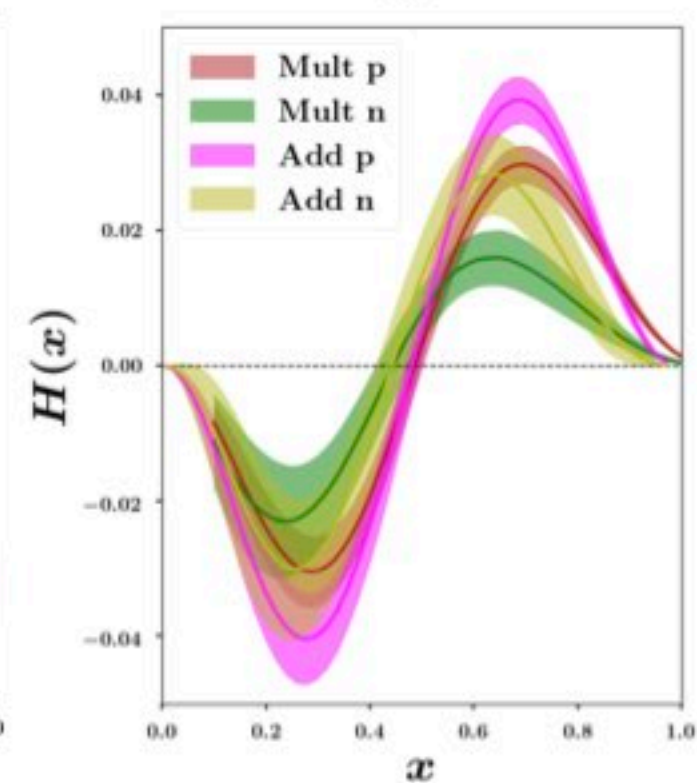
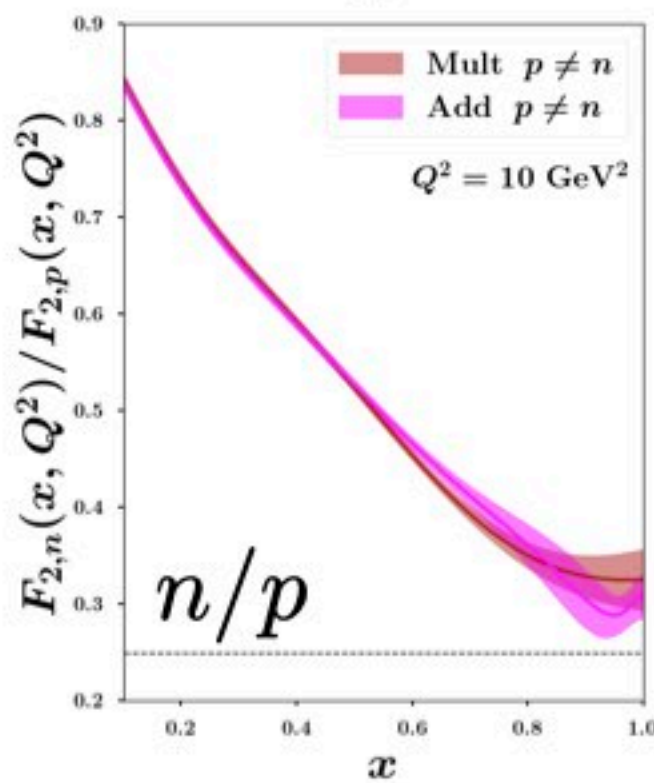
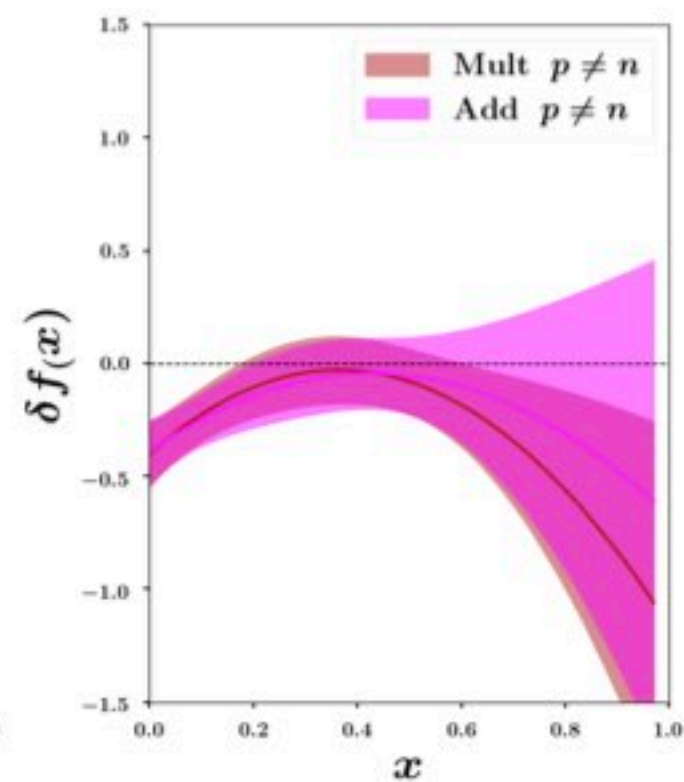
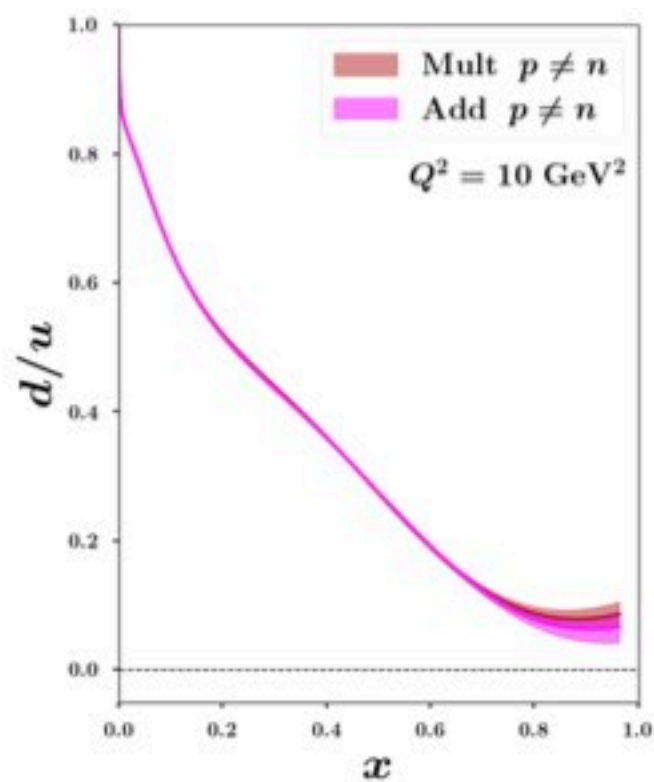
Off-shell compensates  $n/p$  bias





# Results in the CJ fitting framework

## Case 2: isospin breaking

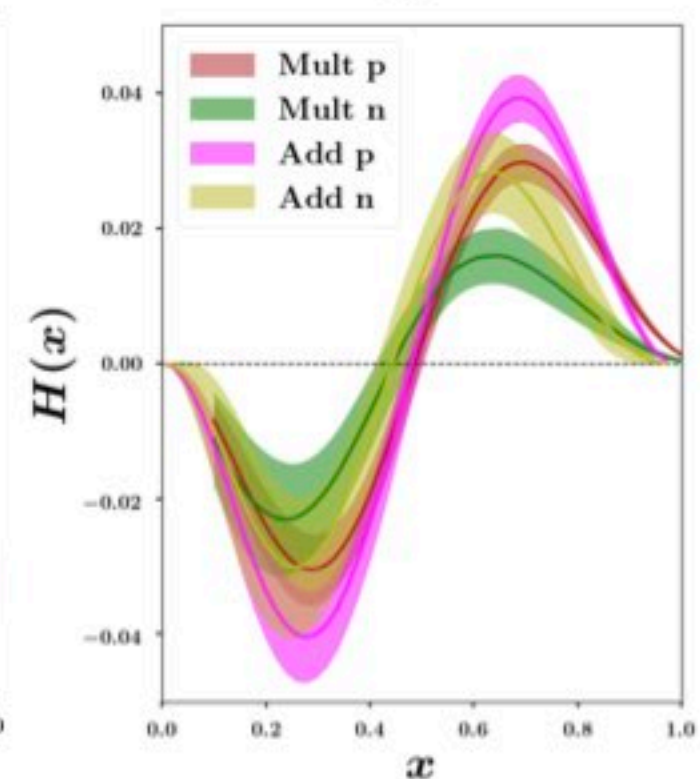
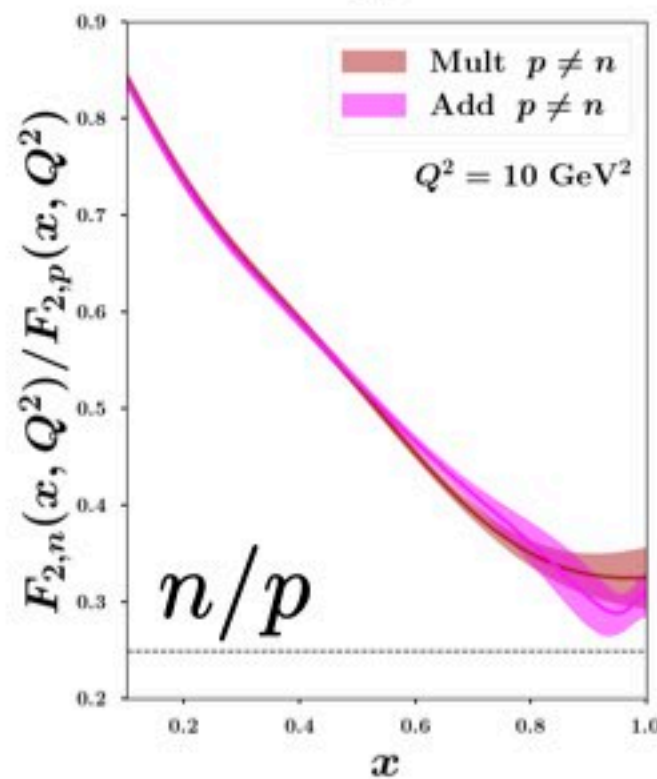
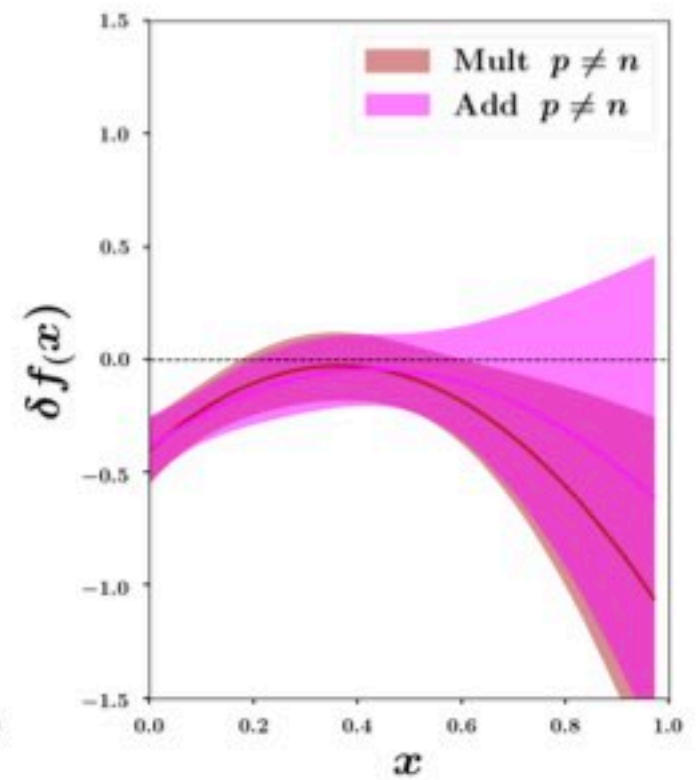
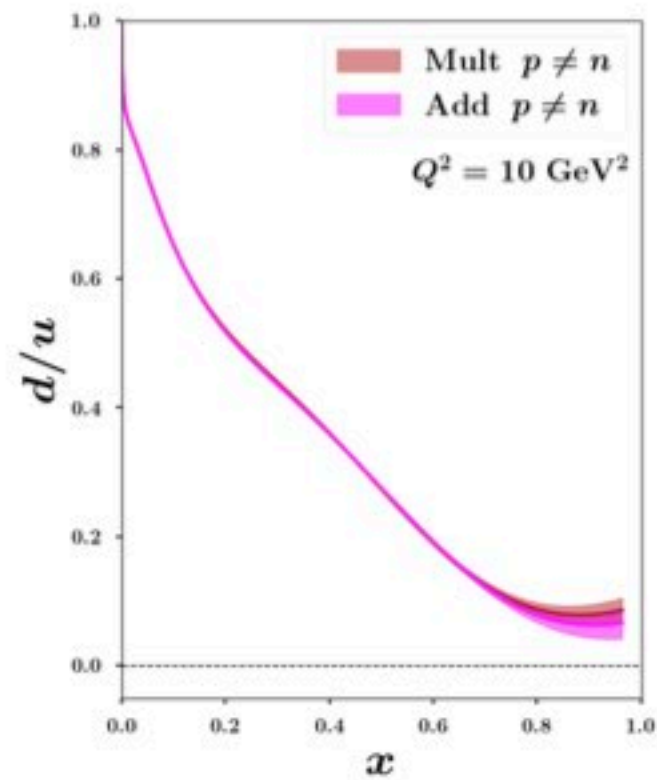


# Results in the CJ fitting framework

## Case 2: isospin breaking

Compatible  $n/p$

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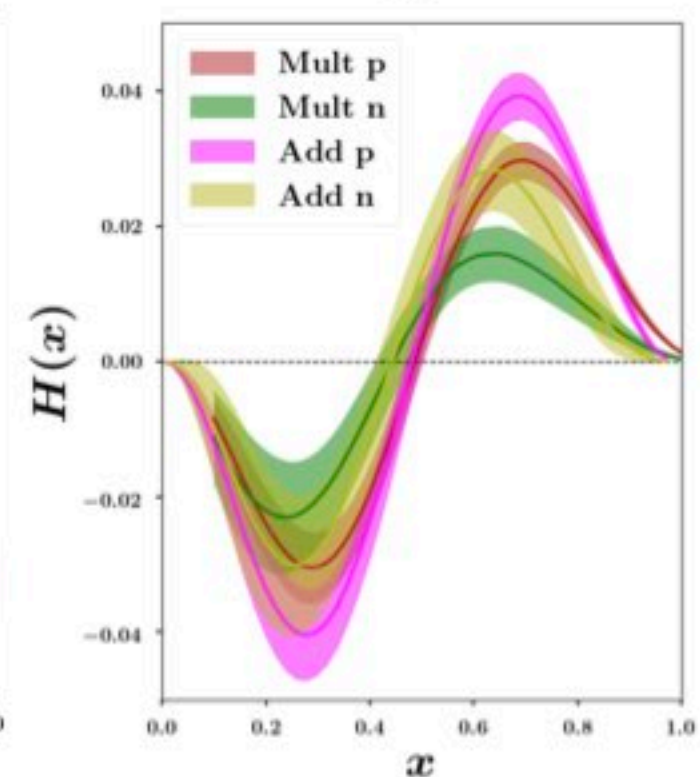
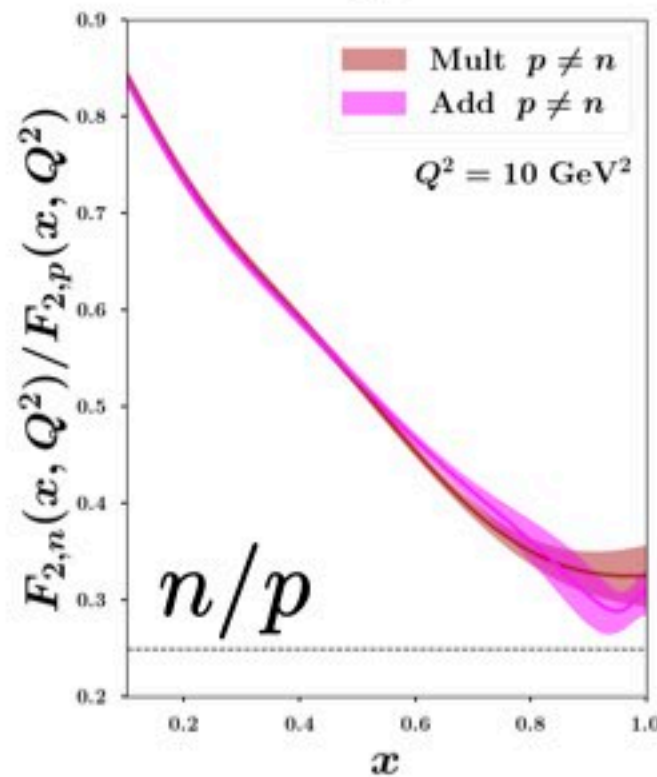
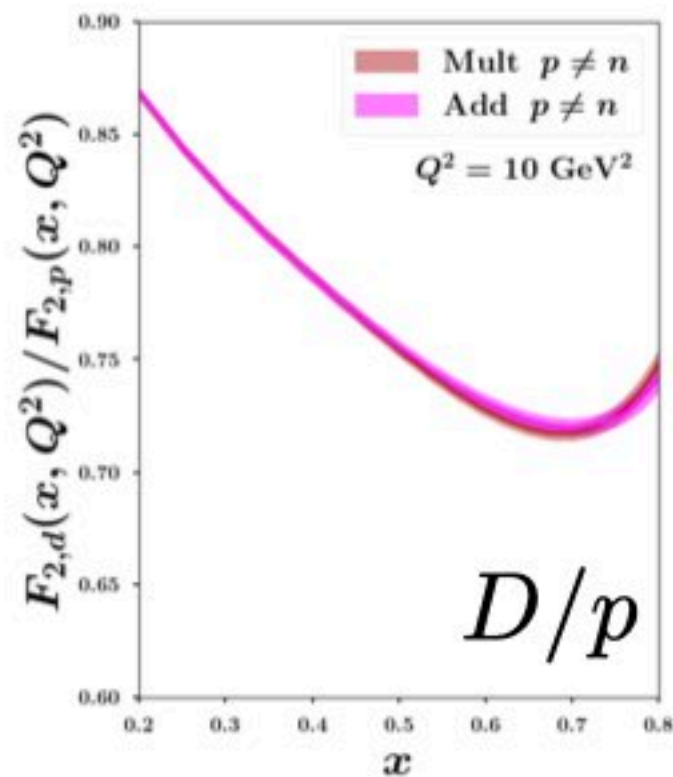
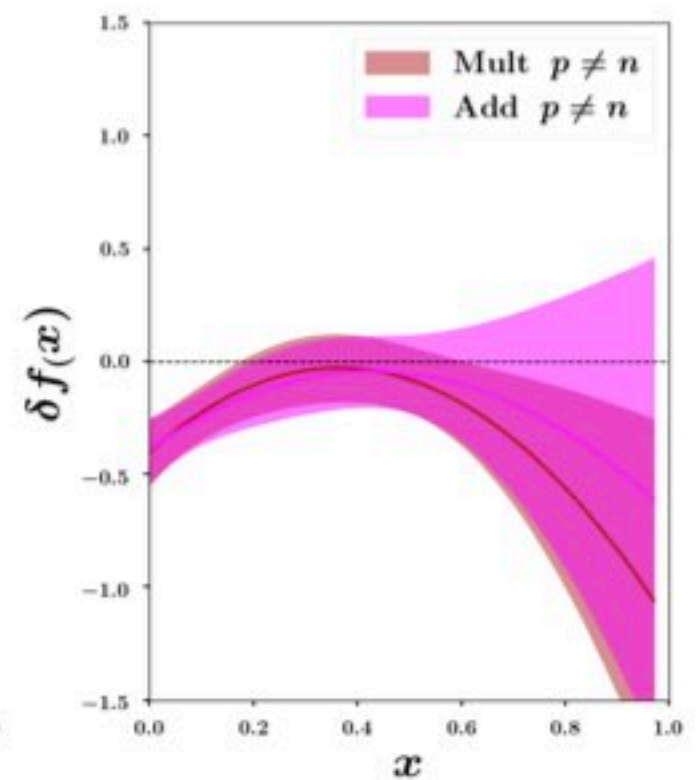
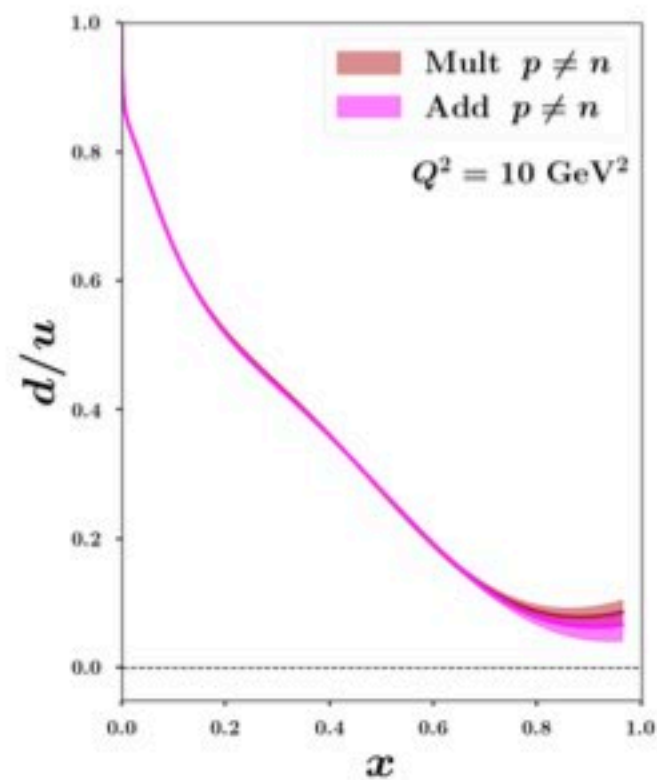
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## Bias removed

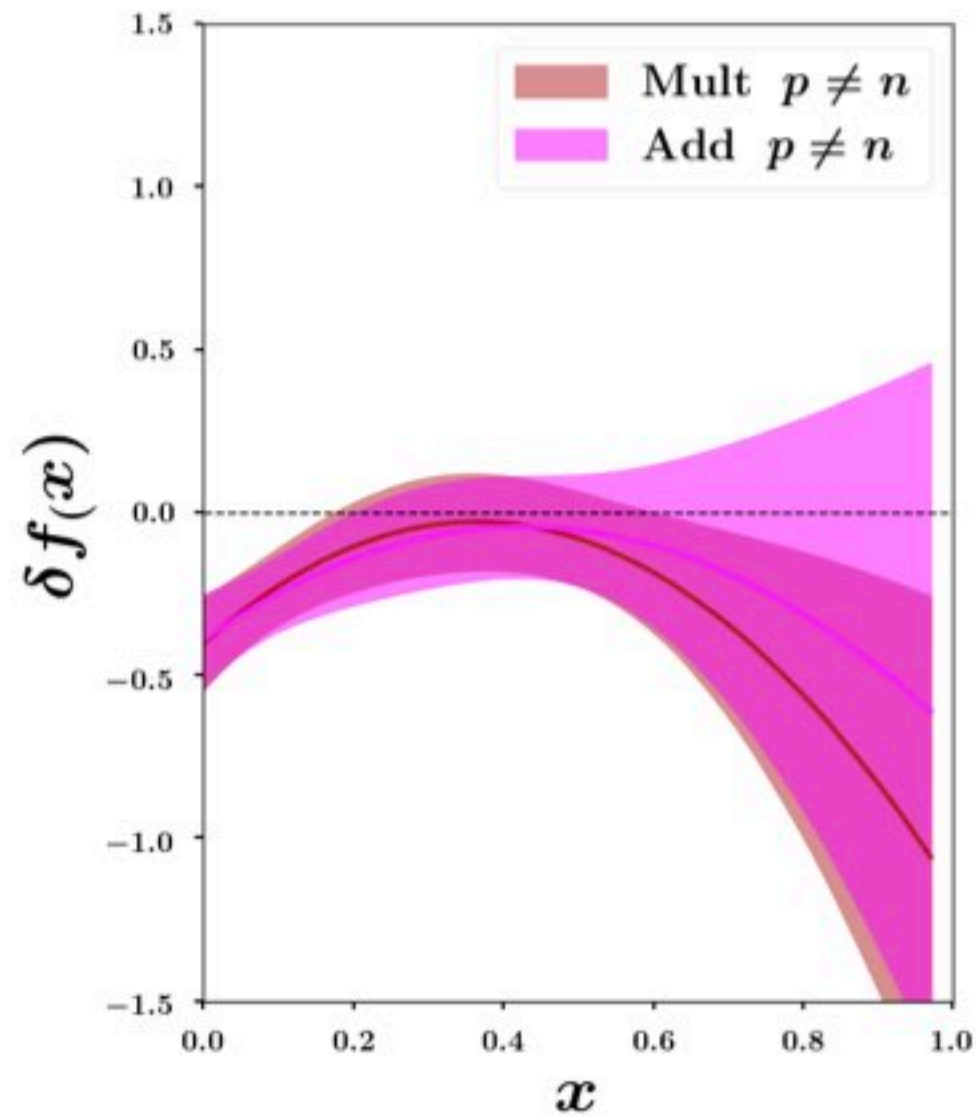
No need of compensation by off-shell  
Theory calculation confirmed!



# Results in the CJ fitting framework

**After removing the bias**

$$\delta f(x) \simeq 0$$

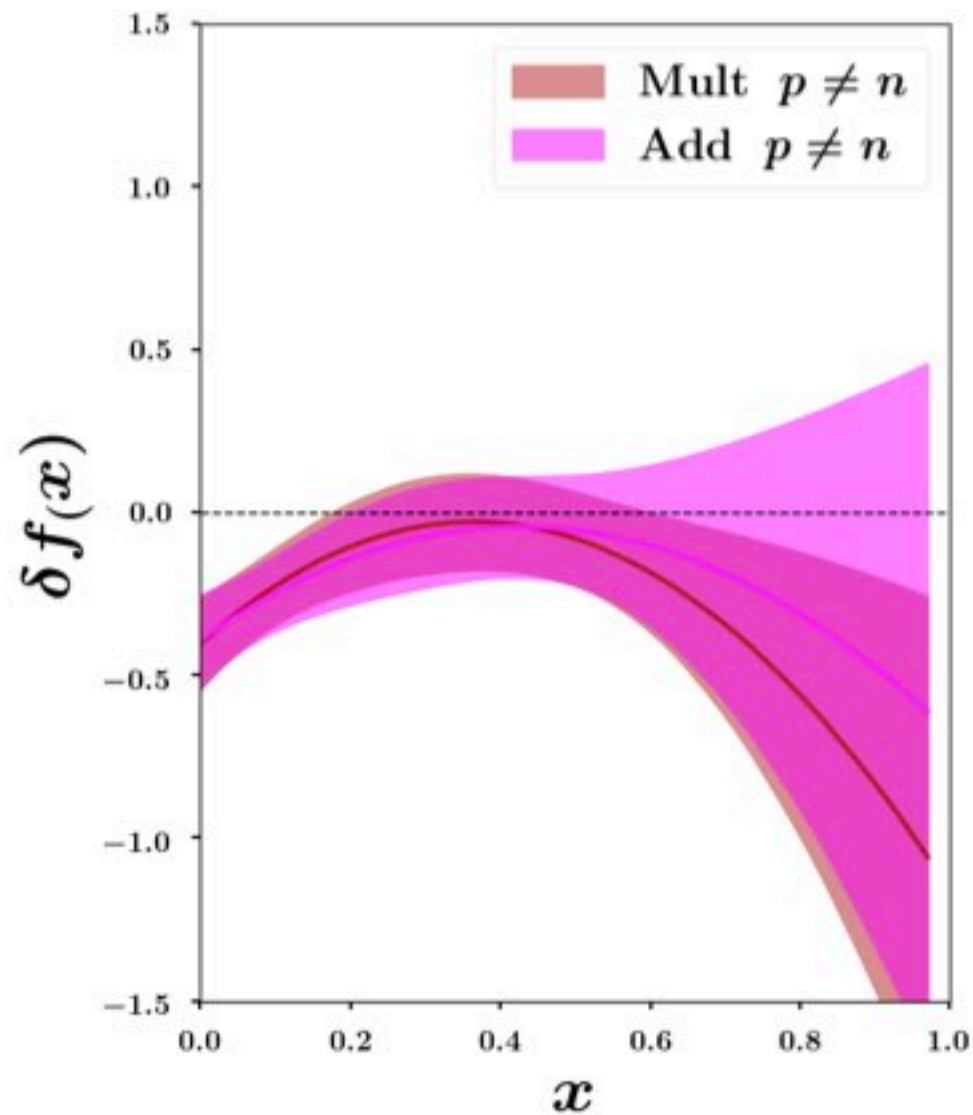




# Results in the CJ fitting framework

**After removing the bias**

$$\delta f(x) \simeq 0$$

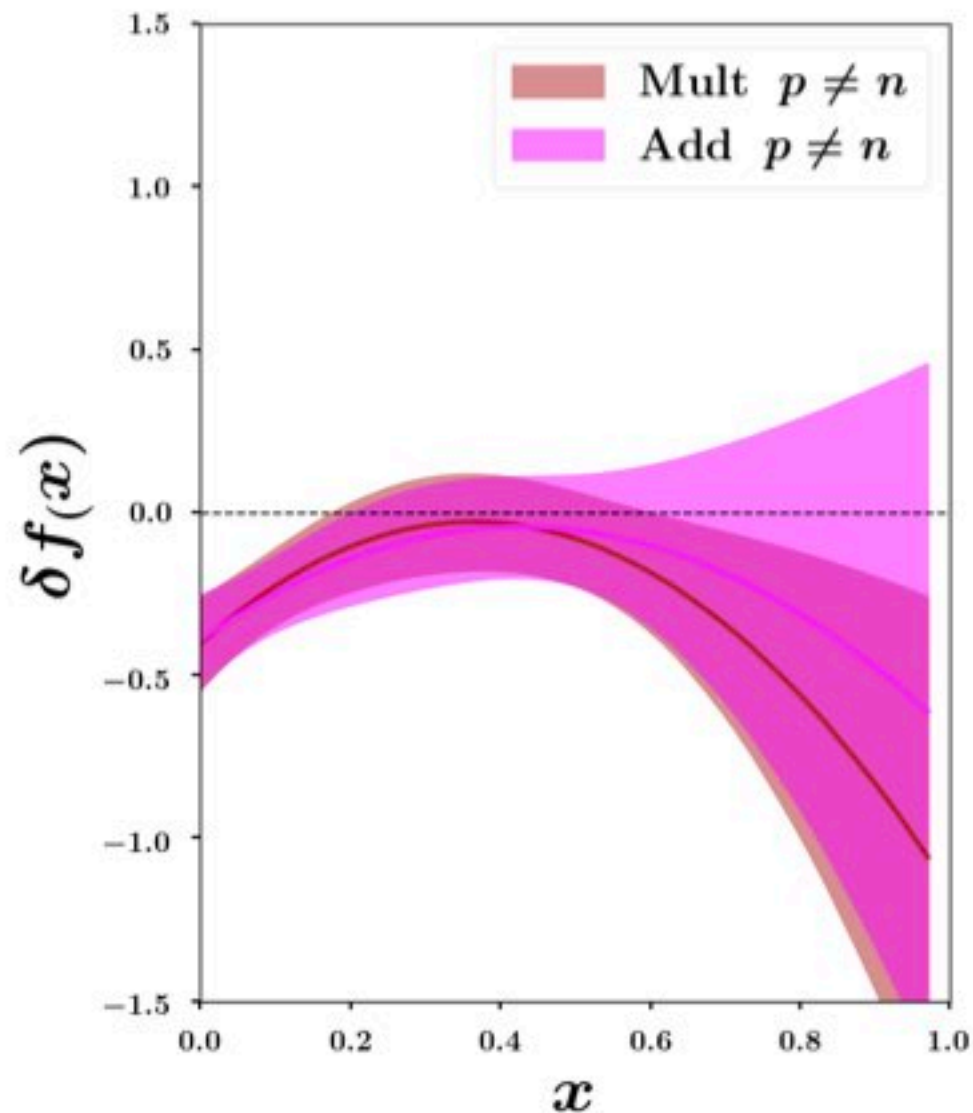


Is the nucleon inside the deuterium  
almost on-shell?

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Need  $A=3$  data to assess flavour  
dependence of off-shell function

MARATHON data  
Adams, et al., PRL 128 (2022)

# Other extractions of the off-shell correction

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**AKP**

Alekhin, Kulagin, Petti, PRD 107 (2023)



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$H_2, H_T$  parametrized

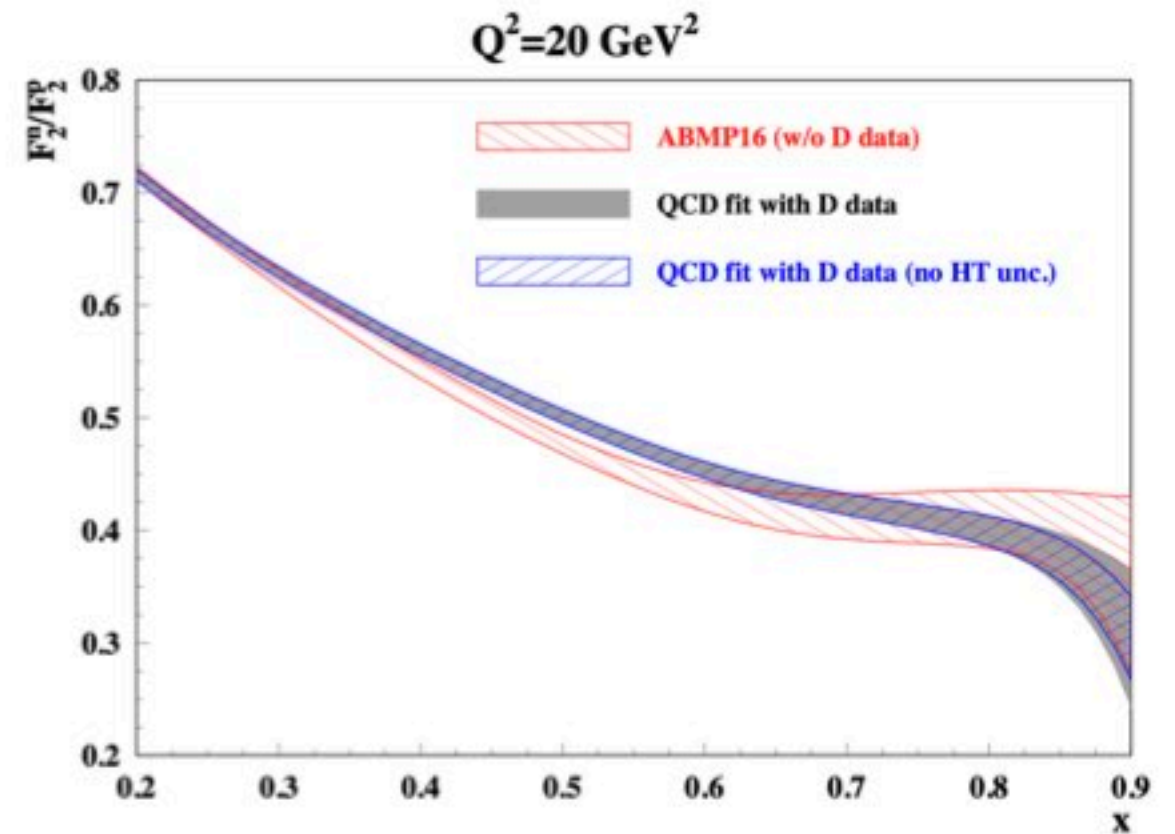
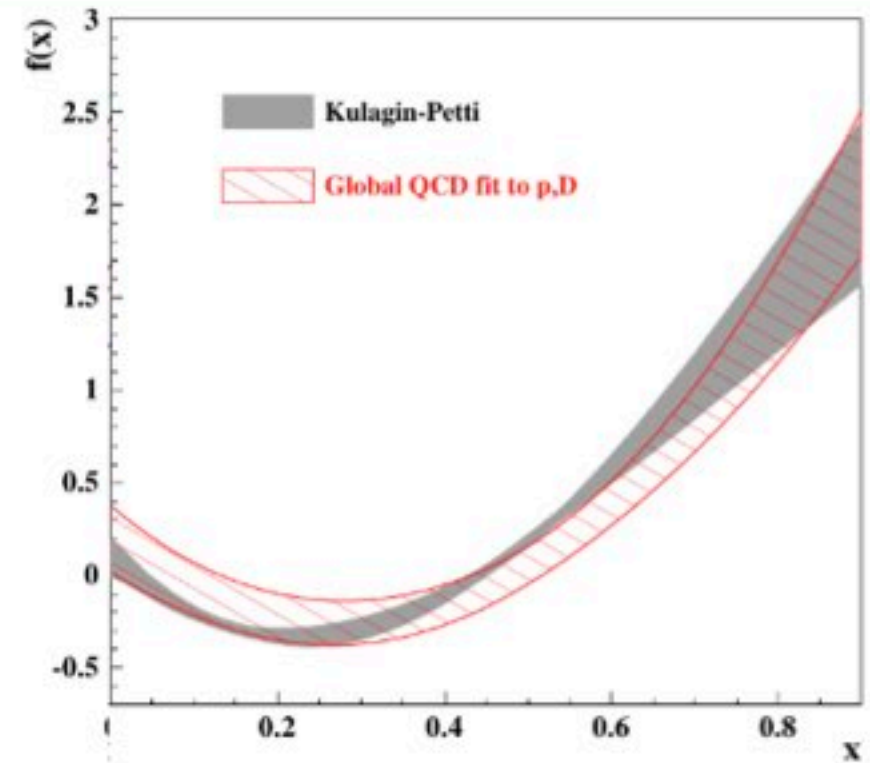
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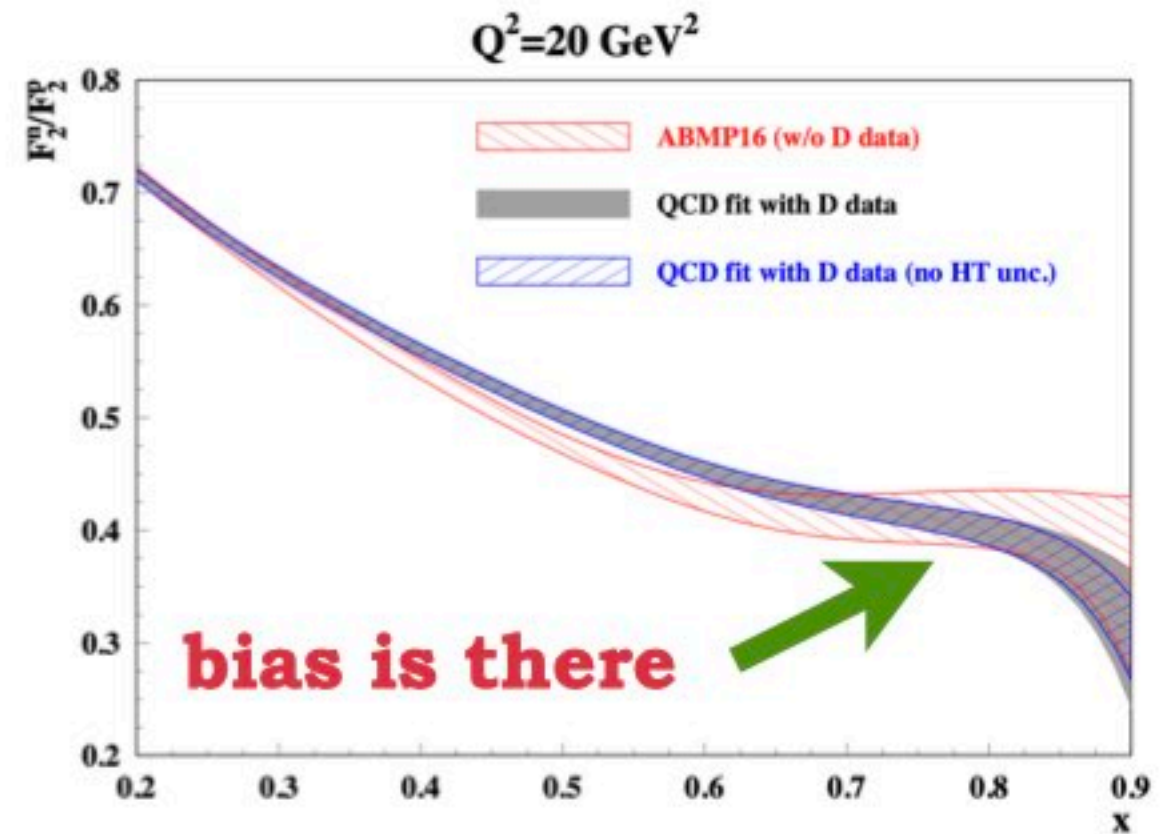
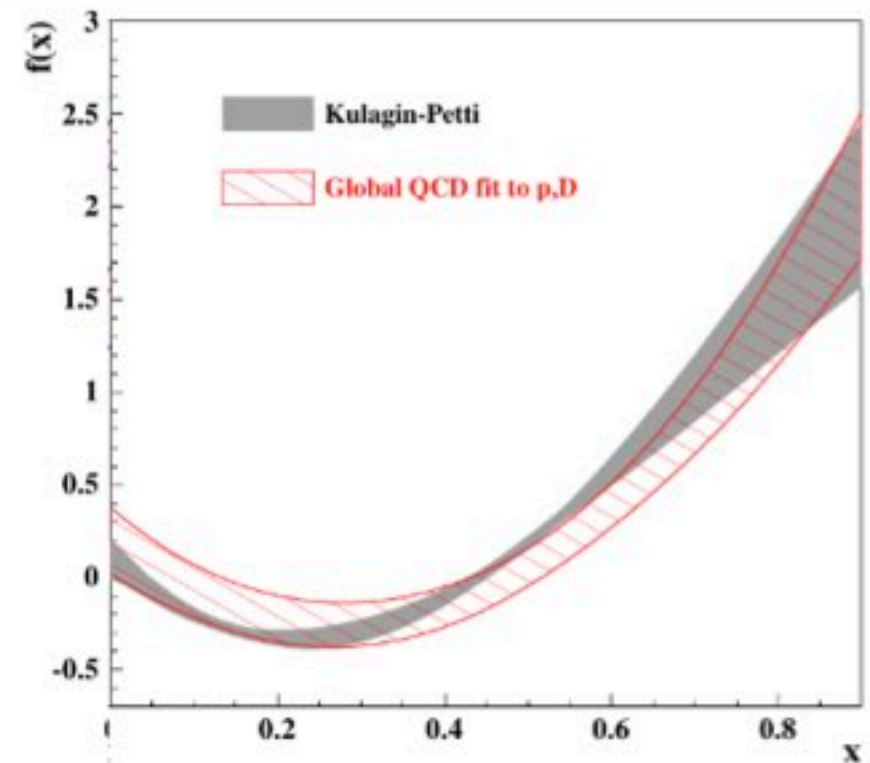
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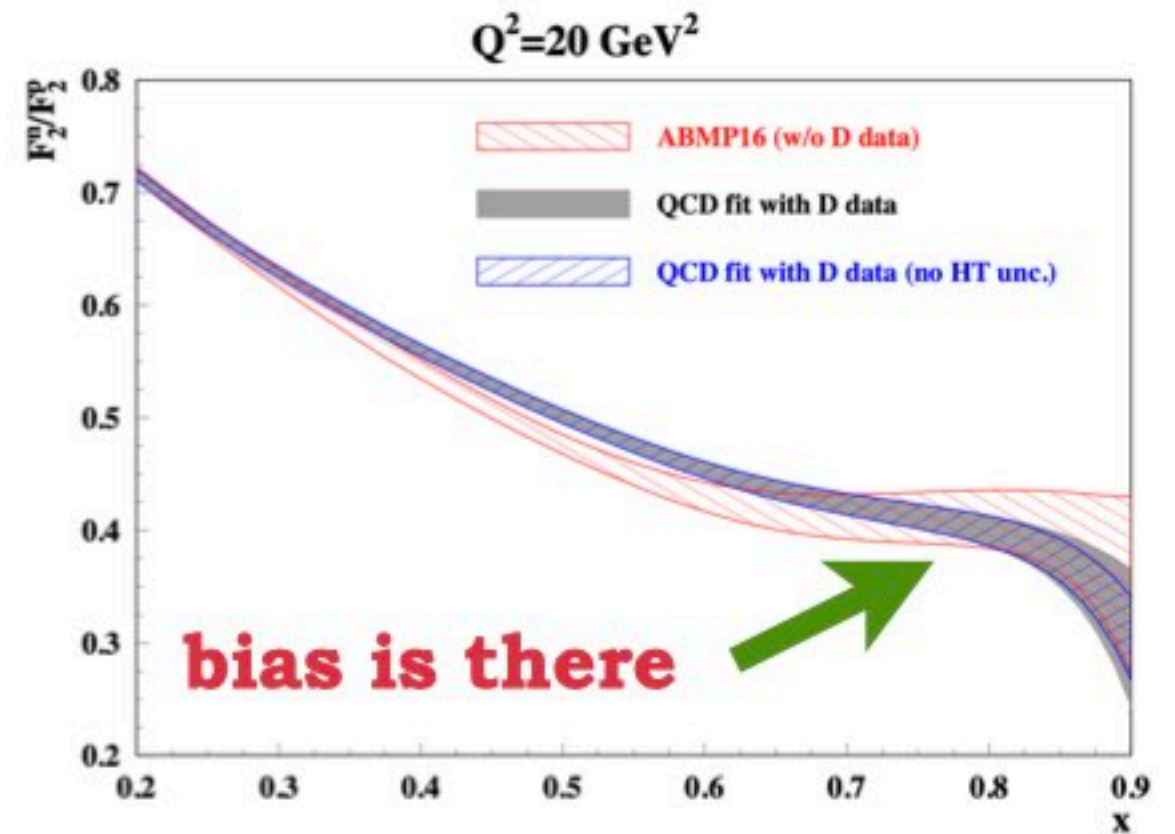
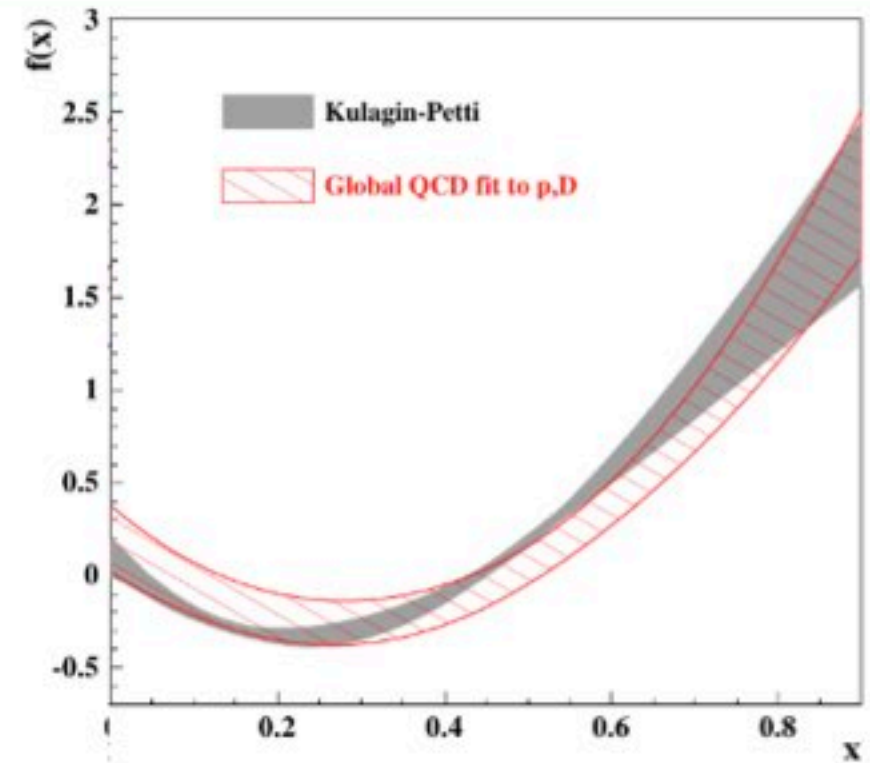
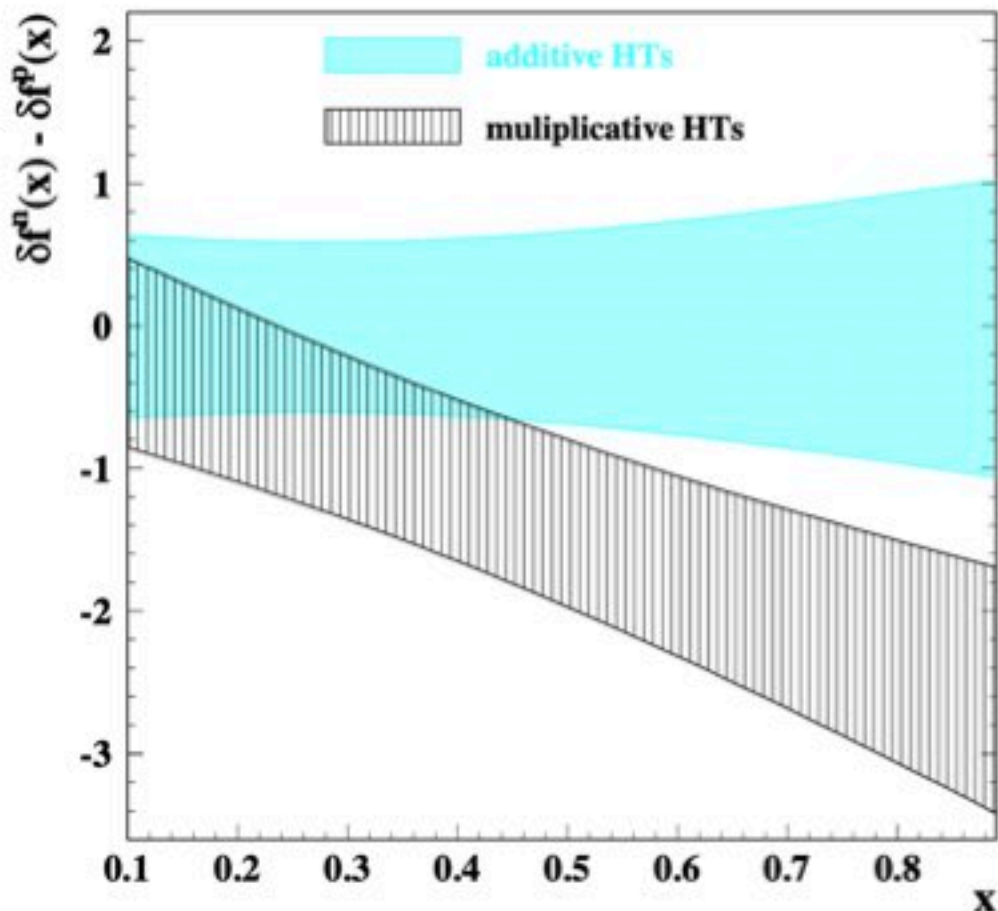
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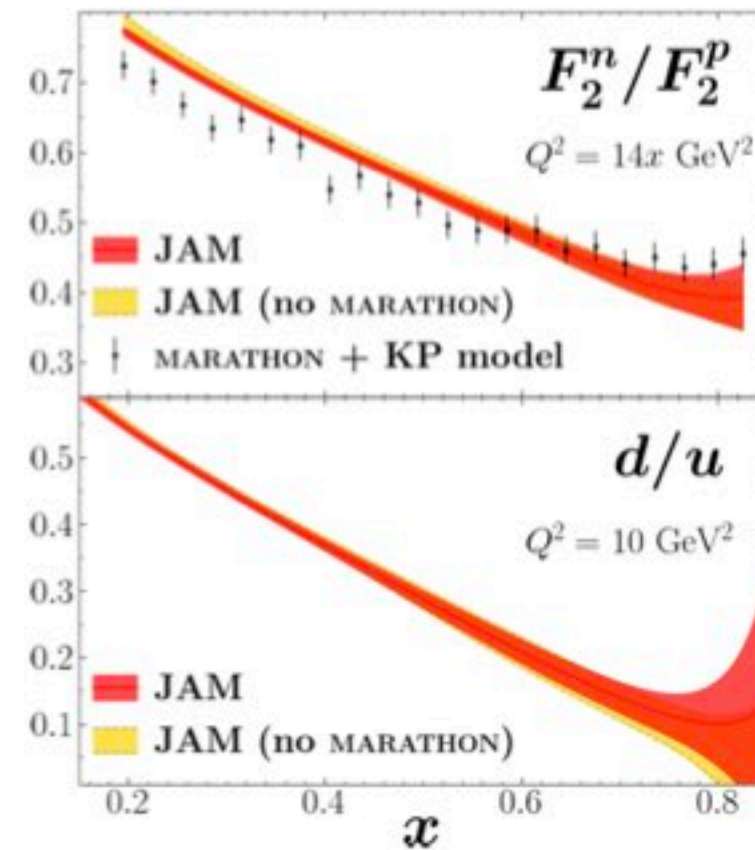
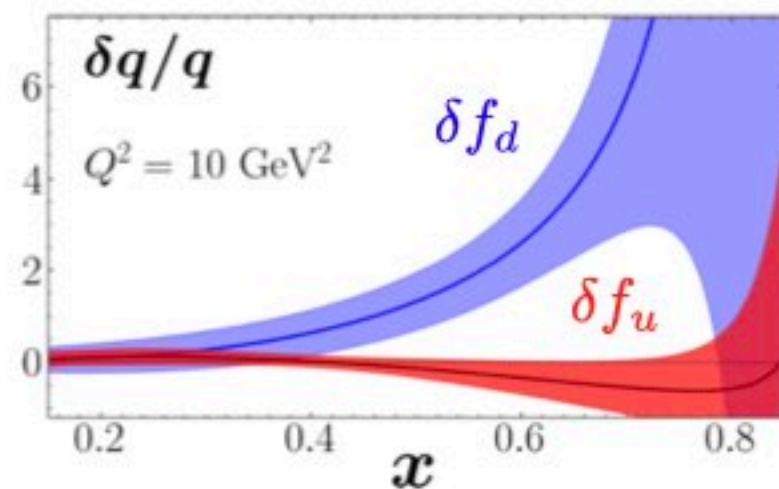
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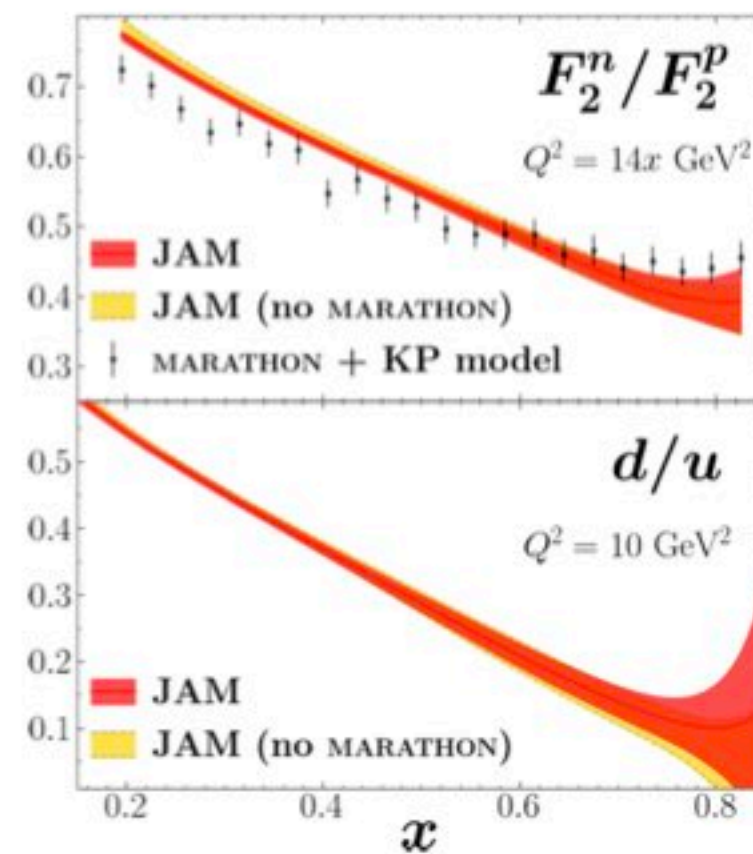
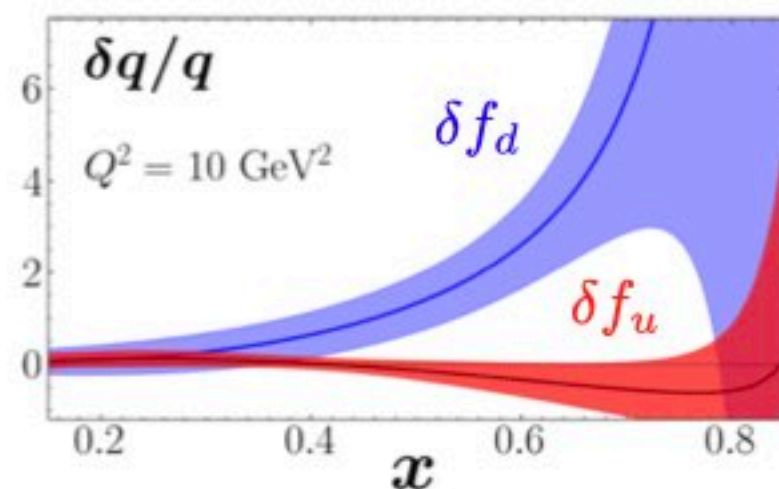


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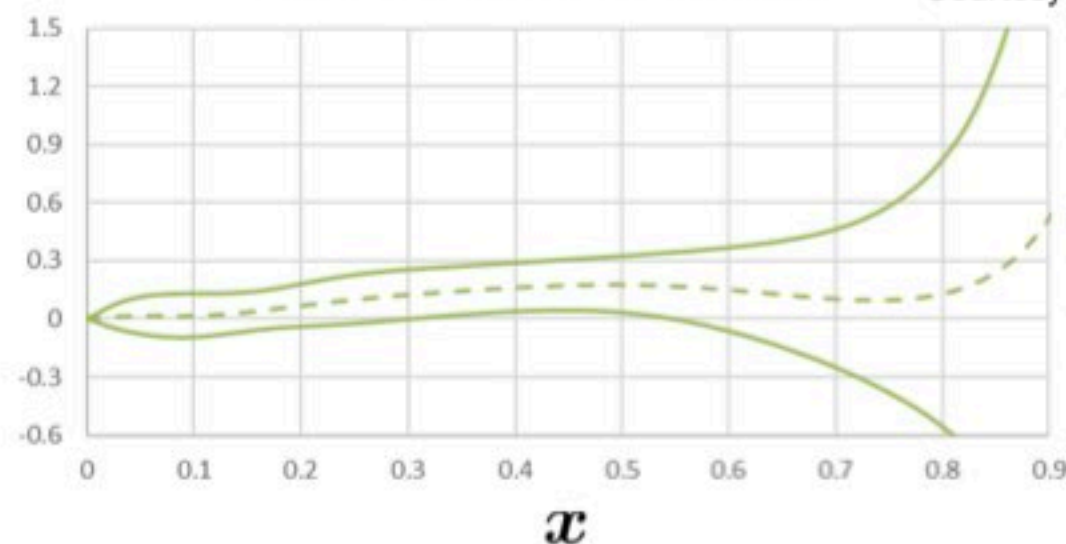
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$$\delta f(x)|_{\text{CJ-like}} = \frac{u\delta f_u + d\delta f_d}{u + d}$$

Isoscalar offshell function (JAM)

Courtesy of C. Cocuzza



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**Experimental data differential on the off-shell proton virtuality  $p^2$**  would allow us to pin down the off-shell correction in a more clean way



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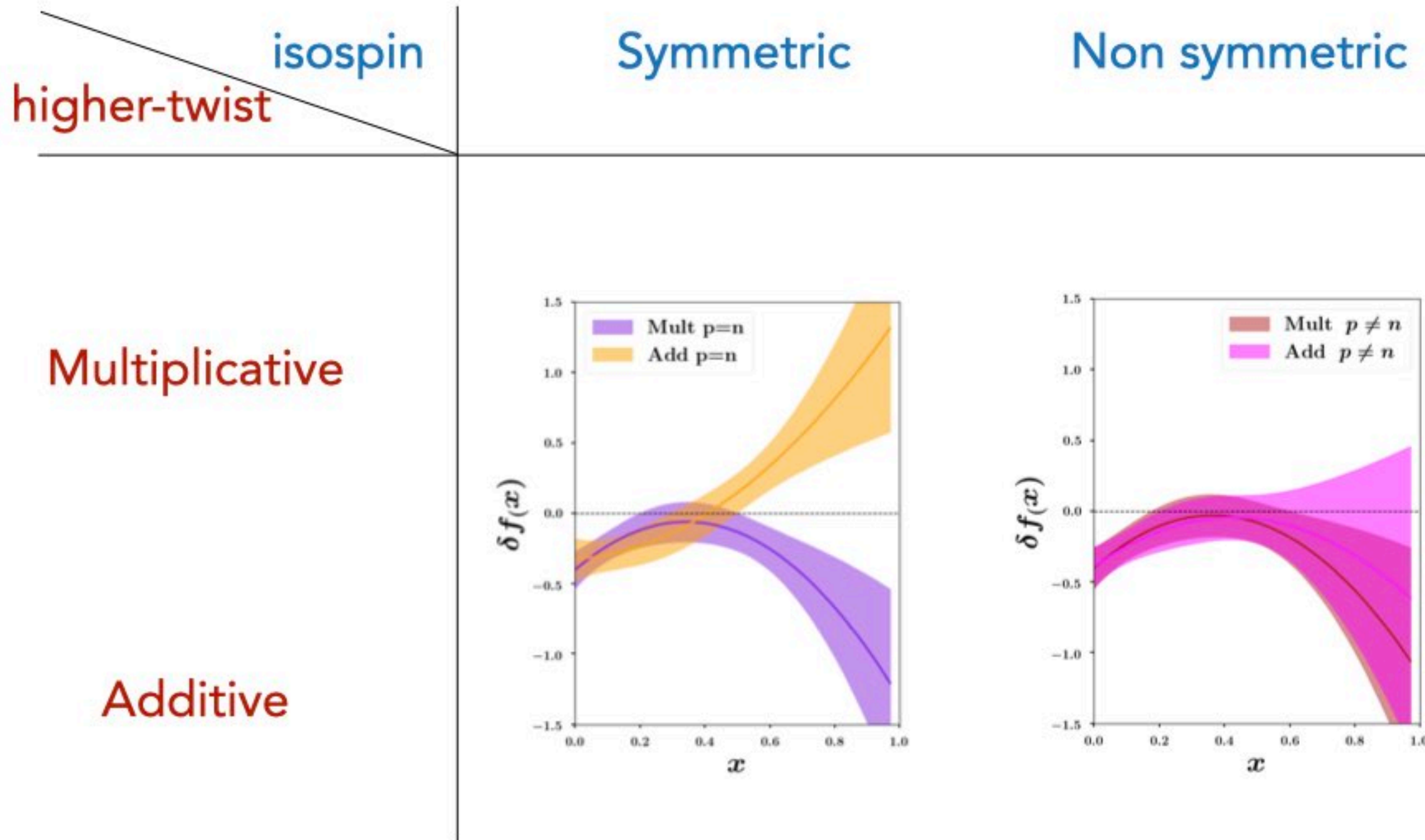
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JAM  $A=3$  fit not in agreement with AKP. Average result compatible with CJ

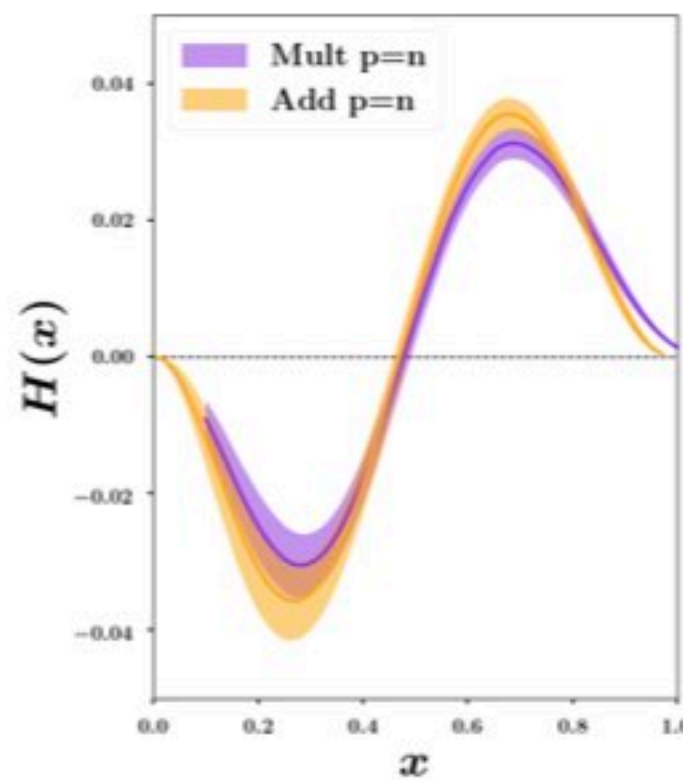
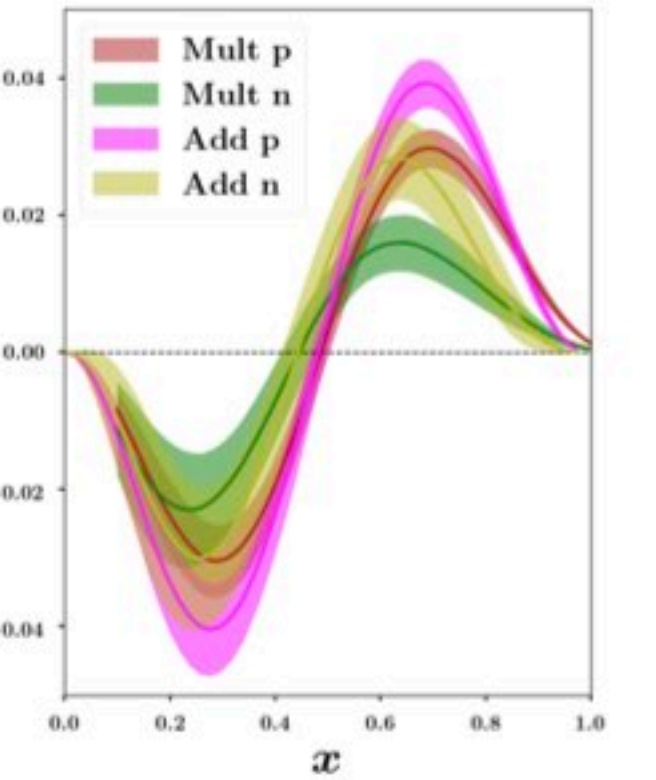


Backup

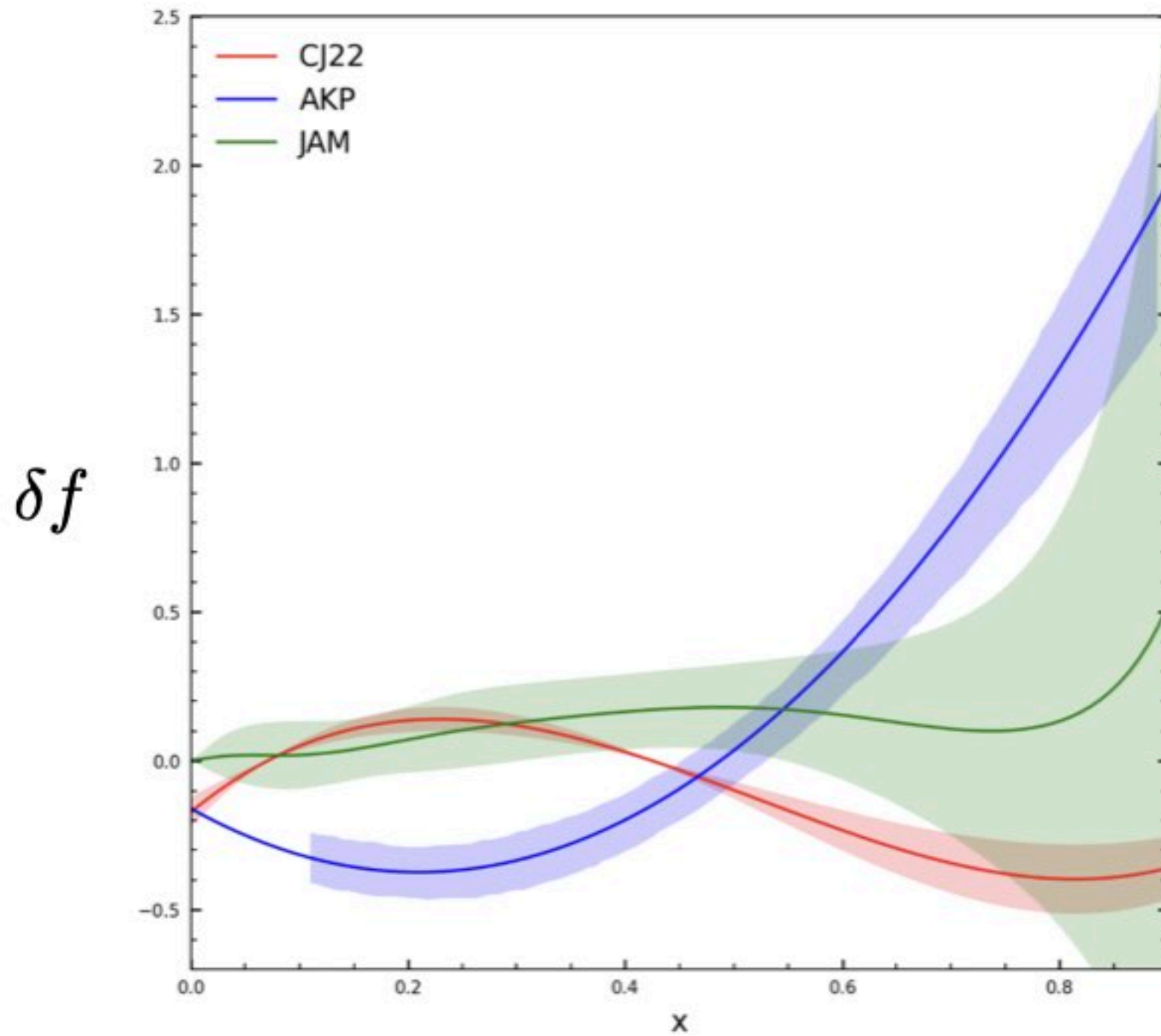
# Off-shell table



# Higher-Twist table

isospin	Symmetric	Non symmetric
higher-twist	$\tilde{H} = F_{2,N}(x, Q^2) H_{\text{Mult}}(x)$	$\delta\tilde{H} = F_{2,N}(x, Q^2) \delta H_{\text{Mult}}(x)$
Multiplicative		
Additive		

# AKP vs CJ





# Some implementation differences

Theoretical choices  $\longrightarrow$

Corrections (increasing-x)



	KP	AKP	CJ15	AKP-like
shadowing	yes	yes (which one?)	MST $x < 0.1$	(same)
smearing	Paris	AV18	AV18 $x > 0.1$	(same)
pi-cloud	yes	yes	----	----
TMC	GP O(Q4)?	GP O(Q4)??	GP approx.	(same)
HT	H (p=n ??)	H (p=n)	C (p=n)	H & C, p=n & p!=n
HT(x)	??	5 pt. spline	parametrized	parametrized
off-shell	O(p2-M2)	O(p2-M2)	O(p2-M2)	(same)
df(x)	factorized	polyn. 2nd/3rd	factorized + sum rule	polyn. 2nd/3rd
pi thresh.	yes	yes	----	----