Revisiting “target mass corrections” in lepton-nucleus DIS

DIS 2024, Grenoble, France

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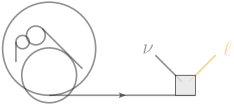

[1] w/ Muzakka, Leger, Olness, Schienbein, et al (nCTEQ), Prog.Part.Nucl.Phys. 136 (2024) 104096 [2301.07715]

thank you for the invitation!

Brief highlights from a "small" ☺ review on **T**arget **M**ass **C**orrections (TMCs) (more in a bit!) in **deep-inelastic scattering** off **nuclear targets**

IFJPAN-IV-2022-18, SMU-HEP-22-12, MS-TP-22-49, ANL-180568, FNAL-PUB-23-142-ND

Target mass corrections in lepton–nucleus DIS: theory and applications to nuclear PDFs

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Abstract

Motivated by the wide range of kinematics covered by current and planned deep-inelastic scattering (DIS) facilities, we revisit the formalism, practical implementation, and numerical impact of target mass corrections (TMCs) for DIS on unpolarized nuclear targets. An important aspect is that we only use nuclear and later partonic degrees of freedom, carefully avoiding a picture of the nucleus in terms of nucleons. After establishing that formulae used for individual nucleon targets (p, n), derived in the Operator Product Expansion (OPE) formalism, are indeed applicable to nuclear targets, we rewrite expressions for nuclear TMCs in terms of re-scaled (or averaged) kinematic variables. As a consequence, we find a representation for nuclear TMCs that is approximately independent of the nuclear target. We go on to construct a single-parameter fit for all nuclear targets that is in good numerical agreement with full computations of TMCs. We discuss in detail qualitative and quantitative differences between nuclear TMCs built in the OPE and the parton model formalisms, as well as give numerical predictions for current and future facilities.

Keywords: DIS, Structure Functions, Target Mass Corrections, OPE, nuclear PDFs
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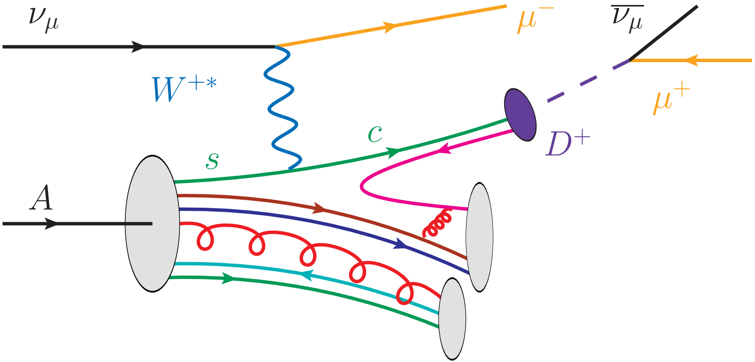
Contents

w/ Muzakka, Leger, Olness, Schienbein, et al (nCTEQ Collaboration), Prog.Part.Nucl.Phys. 136 (2024) 104096 [2301.07715]

the big picture

Deeply inelastic scattering (DIS) is a powerful probe of **hadronic structure**, hadron formation, and **leptonic interactions**

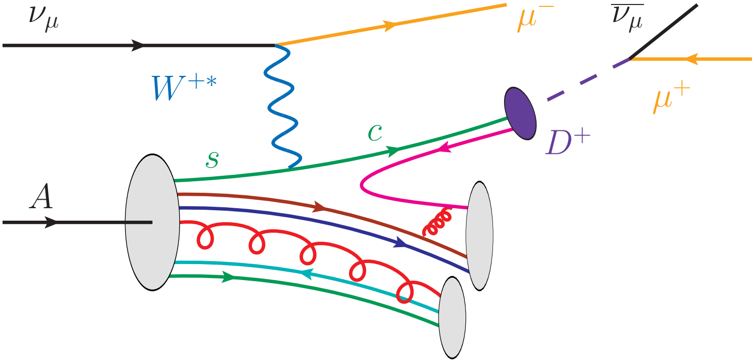
e.g., parity violation, new physics



motivation: ongoing (JLAB) and upcoming (CERN,FNAL,BNL) **precision exp'l** DIS programs require a new level of **theory precision**

Formally, inclusive **DIS** of $\ell \in \{\ell^{\pm}, \nu, \overline{\nu}\}$ off **nucleons** can be described by the **Collinear Factorization Theorem**

Collins, Soper ('87); Collins ('11)



$$d\sigma(\nu\mathcal{A} \to \ell X) = \underbrace{\sum_{k,X_n}}_{inclusive} \underbrace{\Delta_{kk'}}_{\text{shower/RGE}} \otimes \underbrace{f_{k'}}_{\text{PDF}} \otimes \underbrace{d\hat{\sigma}_{\nu k' \to X_n}}_{\text{hard scattering}} + \underbrace{\mathcal{O}\left(\frac{\Lambda_{\text{NP}}^{2+k}}{Q^{2+k}}\right)}_{\text{interesting bit!}}$$

Importance of subleading corrections

$\mathcal{O}\left(\frac{\Lambda_{\rm NP}^{2+k}}{Q^{2+k}}\right)$ corrections increasingly important at **small** Q^2**, large** x!

"target mass corrections" (TM) $\rightarrow$

Georgi, Politzer ('76,'76)

"renormalon" corrections" (HT) $\rightarrow$

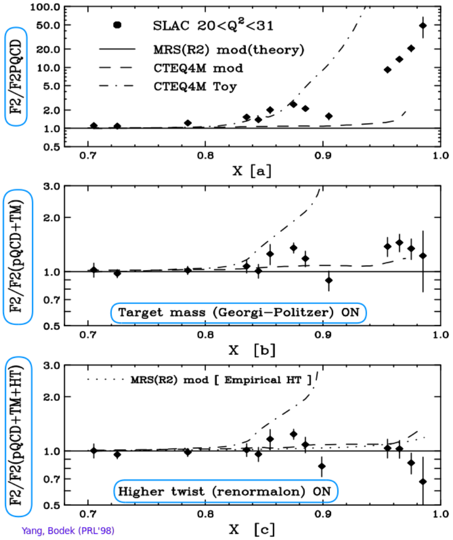
Dasgupta, Webber ('91)

in extreme kinematics, necessary:

– describe DIS data

– extend validity of Fact. Thm.

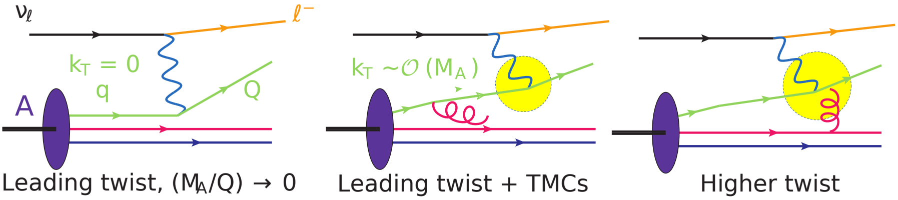
– extract PDFs from structure fns.



Yang, Bodek (PRL'99)

$\mathcal{O}\left(\frac{\Lambda_{\rm NP}^{2+k}}{Q^{2+k}}\right)$ corrections have several origins (kinematical and dynamical)

Georgi, Politzer ('76,'76); Ellis, Furmanski, Petronzio ('82,'82); Dasgupta, Webber ('91); lots more



proton result: kinematical corrections, i.e., **target mass corrections (TMCs)**, can be incorporated in structure functions, $F_i(x, Q^2)$

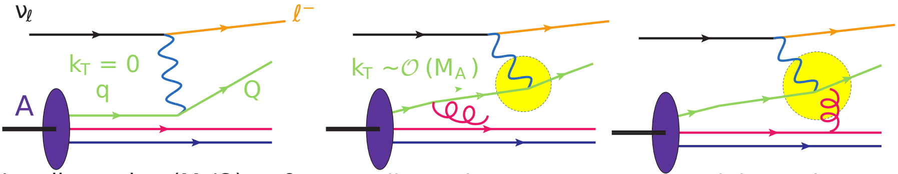
Georgi, Politzer ('76,'76); Ellis, Furmanski, Petronzio ('82,'82); lots more; Kretzer, Reno ('02,'03); Schienbein, et al [0709.1775]

$\Longrightarrow$ **not** obvious such results hold for **arbitrary nuclei**

especially due to questions of original derivation's correctness [Collins ('84)]

do expressions for TMCs for protons hold for nuclei?

yes



Leading twist, $(M_A/Q) \to 0$ Leading twist + TMCs Higher twist

In practice, replace $F_i^{A\ \text{(No TMC)}} \to F_i^{A\ \text{(TMC)}}$ in cross sections:

$$\frac{d^2\sigma^{\rm NC}}{dx\,dy} = x(s - M^2)\,\frac{d^2\sigma^{\rm NC}}{dxdQ^2} = \frac{4\pi\alpha^2}{xyQ^2}\left[\frac{Y_+}{2}\,\sigma^{NC}_{\rm Red.}\right],$$

$$\sigma^{NC}_{\rm Red.} = \left(1 + \frac{2y^2\varepsilon^2}{Y_+}\right)F_2^{\rm NC} \mp \frac{Y_-}{Y_+}\,xF_3^{\rm NC} - \frac{y^2}{Y_+}F_L^{\rm NC},$$

$$F_L = r^2F_2 - 2xF_1,\ r = \sqrt{1+4\varepsilon^2},\ \varepsilon = (xM/Q) \text{ and } Y_\pm = 1 \pm (1-y)^2$$

same holds for charged current scattering

deriving TMCs for DIS with nuclei

light cone dominance

starting point for DIS on p is stipulating kinematic domiain. typically,

$$Q^2 = -q^2 > 0 \gg m^2_{\rm proton} \text{ [proton case]}$$

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naïve application to ^{56}Fe or ^{197}Au would require

$$Q^2 \gg (50\ {\rm GeV})^2 \sim \left(\frac{M_Z}{2}\right)^2 \text{ or } (180\ {\rm GeV})^2 \sim m_t^2 \text{ [incorrect]}$$

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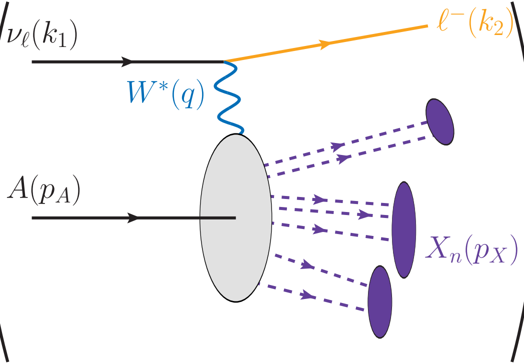
more precise statement

$$Q^2 \gg \Lambda_{\rm non-pert.} \sim \mathcal{O}(1)\ {\rm GeV} \gg \Lambda^2_{\rm QCD} \sim m_q^2 \text{ [general case]}$$

Bjorken scaling still works at moderate energies since $\mathcal{O}(\Lambda^2_{\rm QCD}/Q^2) \ll 1$

Georgi, Politzer ('79); Muta ('98/'10)

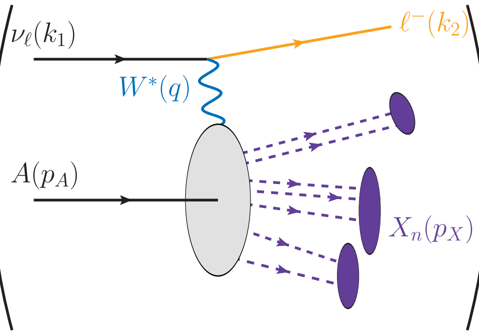
draw diagrams, currents, and build the matrix element



$$-i\mathcal{M}\left(\cdots \right) = J^{\alpha}_{\ell_f \ell_i} \cdot \Delta_{\alpha\mu} \cdot J^{\nu}_{X_n A}$$

n-body phase space integral and summing over n gives us $W_A^{\mu\nu}$

$$\frac{d^3\sigma}{dk_2^3} \sim \int dPS_n \, \Sigma_n \, \Sigma_{\rm spins} \left| \mathcal{M} \left(\cdots \right) \right|^2 \sim L_{\mu\nu} \cdot W_A^{\mu\nu}$$



in the paper, we use exact expressions for $d\sigma$, etc., so $\sim \to =$

Summing over X_n ensures "inclusivity" and closure, $1 = \sum_n |X_n\rangle\langle X_n|$

this step sometimes omitted in textbooks, e.g., Halzen & Martin

$$W^A_{\mu\nu} = \frac{1}{4\pi}\int d^4z\ e^{iq\cdot z}\langle A|J^\dagger_{had.\mu}(z)\ J_{had.\nu}(0)|A\rangle$$

$$= -g_{\mu\nu}\ F^A_1\ + \frac{p_{A\mu}p_{A\nu}}{Q^2}\ 2x_A\ F^A_2\ - i\epsilon_{\mu\nu\rho\sigma}\frac{p^\rho_A q^\sigma}{Q^2}\ x_A F^A_3$$

$$+ \frac{q_\mu q_\nu}{Q^2}\ 2F^A_4\ + \frac{p_{A\mu}q_\nu + p_{A\nu}q_\mu}{Q^2}\ 2x_A\ F^A_5\ + \frac{p_{A\mu}q_\nu - p_{A\nu}q_\mu}{Q^2}\ 2x_A\ F^A_6$$

– **point #1:** $F_i(x, Q^2)$ are structure functions and can be measured

and predicted from other experiments since parton model says $F_i = \sum f_{j/p}$

– **point #2:** $W^A_{\mu\nu}$ is defined in the "DIS" limit:

$$x_A = \frac{Q^2}{2p_A\cdot q}\ \text{is fixed and}\ (Q^2/M^2_A) \to \infty$$

Define the time-ordered ME for (virtual) $AV^* \to AV^*$ scattering

$$T^A_{\mu\nu} = \int\ d^4z\ e^{iq\cdot z}\langle A|\ \mathcal{T} J^\dagger_{had.\mu}(z)\ J_{had.\nu}(0)|A\rangle$$

$$= -g_{\mu\nu}\ \Delta T^A_1\ + \frac{p_{A\mu}p_{A\nu}}{M_A^2}\ \Delta T^A_1\ - i\epsilon_{\mu\nu\rho\sigma}\frac{p_A^\rho q^\sigma}{M_A^2}\ \Delta T^A_3$$

$$+ \frac{q_\mu q_\nu}{M_A^2}\ \Delta T^A_4\ + \frac{p_{A\mu}q_\nu + p_{A\nu}q_\mu}{M_A^2}\ \Delta T^A_5\ + \frac{p_{A\mu}q_\nu - p_{A\nu}q_\mu}{M_A^2}\ \Delta T^A_6$$

point #1: related to $W^A_{\mu\nu}$ by Fourier transformations + Cauchy's Thm

see also Collins ('84)!

$$\Delta T^A_i\ =\ \text{(some factor)}\ \times\ \sum_N^\infty \underbrace{F_i^{AN}(Q^2)}_{N^{th}\ \text{Mellin moment}\ =\int_0^1 dy\ \ y^{(N-1)}F_i(y)}\ x_A^{-N}$$

point #2: $T^A_{\mu\nu}$ is defined in the "short-distance" limit:

$$\frac{x_A}{Q}\ \text{is fixed and}\ (Q^2/M_A^2) \to \infty$$

the operator product expansion (in a nutshell)

The OPE is a formalism for decomposing products of operators

Wilson ('69); Brandt, Preparata ('71); Christ, et al ('72)

$$\langle \text{some number of operators } \hat{\mathcal{O}} \rangle = \sum_k \underbrace{\mathcal{C}_k}_{\text{Wilson coeff.}} \times \langle \text{fewer operators } \hat{\mathcal{O}} \rangle$$

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1. Assume $T^A_{\mu\nu}$ has an OPE in the short-distance limit:

$$\lim_{z\to 0} \; T^A_{\mu\nu} \overset{\text{OPE}}{=} (\text{Wilson coeff.}) \times (\text{hadronic ME}) + \ldots$$

2. Take leading term, keeping masses

power counting is ordered by "twist", τ = (dim. of EFT operator) − (# of Lorentz indices); see also Stermen (TASI'95)

3. Organize, simplify...

$$\Delta T^A_i = (\text{some factor}) \times (\textstyle\sum \text{stuff} \times \text{Wilson})\; x_A^{-N}$$

4. Identify $F^{AN}_i(Q^2) = (\text{stuff} \times \text{Wilson})$, then inverse Mellin

Nuclear structure functions with TMCs

$$\tilde{F}_1^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A}\right)\tilde{F}_1^{A,(0)}(\xi_A) + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^2}\right)\tilde{h}_2^A(\xi_A) + \left(\frac{2M_A^4 x_A^3}{Q^4 r_A^3}\right)\tilde{g}_2^A(\xi_A),$$

$$\tilde{F}_2^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A^2}{\xi_A^2 r_A^3}\right)\tilde{F}_2^{A,(0)}(\xi_A) + \left(\frac{6M_A^2 x_A^3}{Q^2 r_A^4}\right)\tilde{h}_2^A(\xi_A) + \left(\frac{12M_A^4 x_A^4}{Q^4 r_A^5}\right)\tilde{g}_2^A(\xi_A),$$

$$\tilde{F}_3^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A^2}\right)\tilde{F}_3^{A,(0)}(\xi_A) + \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_3^A(\xi_A),$$

$$\begin{aligned}\tilde{F}_4^{A,\mathrm{TMC}}(x_A) = {} & \left(\frac{x_A}{\xi_A r_A}\right)\tilde{F}_4^{A,(0)}(\xi_A) - \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^2}\right)\tilde{F}_5^{A,(0)}(\xi_A) + \left(\frac{M_A^4 x_A^3}{Q^4 r_A^3}\right)\tilde{F}_2^{A,(0)}(\xi_A) \\ & + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_5^A(\xi_A) - \left(\frac{2M_A^4 x_A^4}{Q^4 r_A^4}\right)\left(2 - \xi_A^2 M_A^2/Q^2\right)\tilde{h}_2^A(\xi_A) \\ & + \left(\frac{2M_A^4 x_A^3}{Q^4 r_A^5}\right)\left(1 - 2x_A^2 M_A^2/Q^2\right)\tilde{g}_2^A(\xi_A),\end{aligned}$$

$$\begin{aligned}\tilde{F}_5^{A,\mathrm{TMC}}(x_A) = {} & \left(\frac{x_A}{\xi_A r_A^2}\right)\tilde{F}_5^{A,(0)}(\xi_A) - \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3 \xi_A}\right)\tilde{F}_2^{A,(0)}(\xi_A) \\ & + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_5^A(\xi_A) - \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^4}\right)\left(1 - x_A \xi_A M_A^2/Q^2\right)\tilde{h}_2^A(\xi_A) \\ & + \left(\frac{6M_A^4 x_A^3}{Q^4 r_A^5}\right)\tilde{g}_2^A(\xi_A),\end{aligned}$$

$$\tilde{F}_6^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A^2}\right)\tilde{F}_6^{A,(0)}(\xi_A) + \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_6(\xi_A).$$

running numbers

we use NLO PDFs (`nCTEQ15`) to build str. fns. At LO, these are

$$F_1^{\nu A} = (d+s+\bar{u}+\bar{c}), \qquad F_1^{\bar{\nu} A} = (u+c+\bar{d}+\bar{s})$$

$$F_2^{\nu A} = 2x\,(d+s+\bar{u}+\bar{c}), \qquad F_2^{\bar{\nu} A} = 2x\left(u+c+\bar{d}+\bar{s}\right)$$

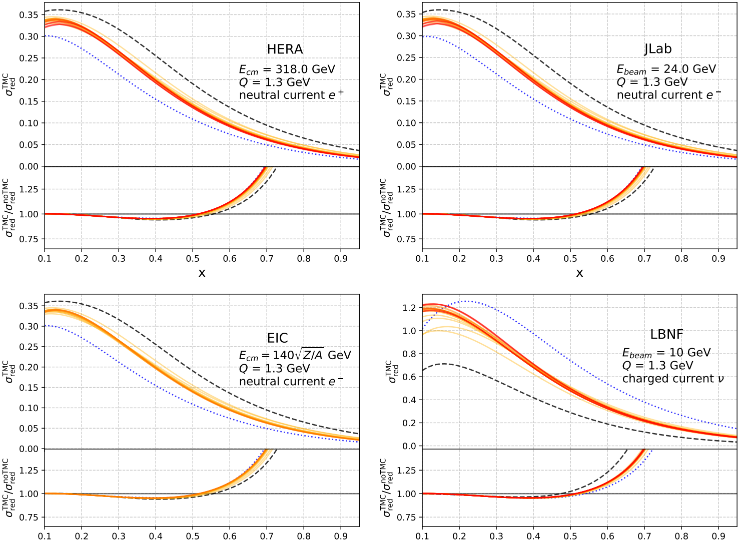
$$F_3^{\nu A} = +2\,(d+s-\bar{u}-\bar{c}), \qquad F_3^{\bar{\nu} A} = -2\left(u+c-\bar{d}-\bar{s}\right)$$

$$F_2^{l^\pm A} = x\frac{1}{9}\left[4(u+\bar{u}) + (d+\bar{d}) + 4(c+\bar{c}) + (s+\bar{s})\right]$$

for many targets

Symbol	A	Z	Symbol	A	Z	Symbol	A	Z	Symbol	A	Z
H	1	1	Be	9	4	Ca	40	20	Xe	131	54
D	2	1	C	12	6	Fe	56	26	W	184	74
^{3}He	3	2	N	14	7	Cu $_{\rm iso}$	64	32	Au	197	79
He	4	2	Ne	20	10	Kr $_{\rm iso}$	84	42	Au $_{\rm iso}$	197	98.5
Li	6	3	Al	27	13	Ag $_{\rm iso}$	108	54	Pb $_{\rm iso}$	207	103.5
Li	7	3	Ar	40	18	Sn $_{\rm iso}$	119	59.5	Pb	208	82

Plotted: (upper) reduced cross sections with nTMCs; (lower) ratio to w/o



something interesting

the operator product expansion (in a nutshell)

The OPE is a formalism for decomposing products of operators

Wilson ('69); Brandt, Preparata ('71); Christ, et al ('72)

$$\langle \text{some number of operators } \hat{\mathcal{O}} \rangle = \sum_k \underbrace{\mathcal{C}_k}_{\text{Wilson coeff.}} \times \langle \text{fewer operators } \hat{\mathcal{O}} \rangle$$

As an intermediate step, we set $(M_A^2/Q^2) \to 0$:

$$\tilde{F}_i^{AN}\Big|_{\text{No TMC}} = C_i^N A_{\tau=2}^N + \mathcal{O}(\tau > 2) \quad \text{for} \quad i = 1, 3-6 \,,$$

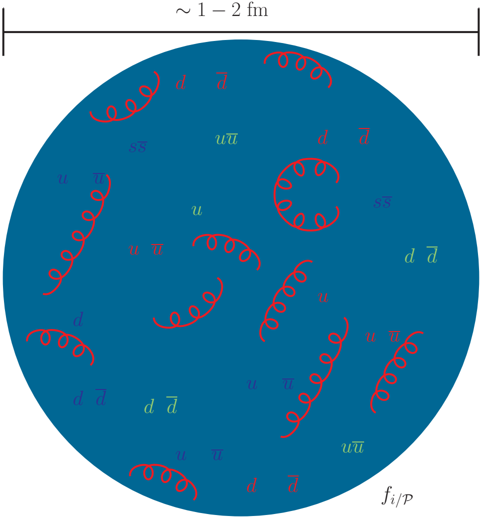
$$\tilde{F}_2^{A(N-1)}\Big|_{\text{No TMC}} = C_2^N A_{\tau=2}^N + \mathcal{O}(\tau > 2)$$

structure fns. = (short-dist. phys.) × (hadronic matrix element)

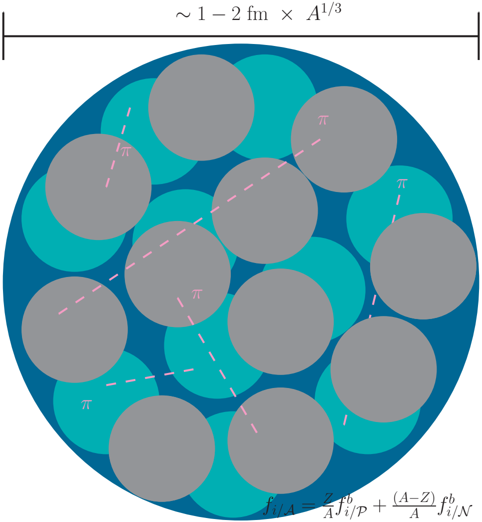
what does this mean?

for proton, $F_i^N = C_i^N \times A^N$ + power corrections

$\Longrightarrow$ **"PDFs = QCD $\times$ hadronic matrix element"**



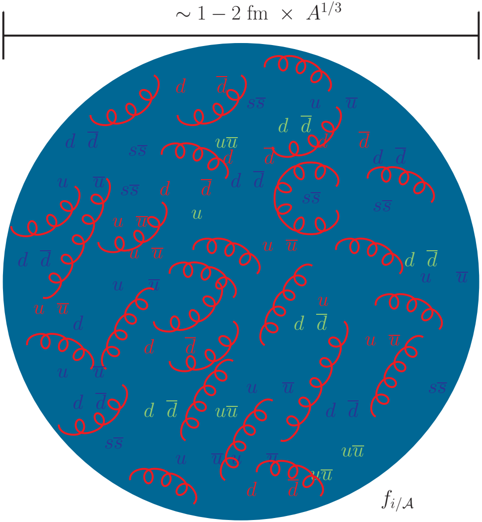
for A, it is common to parameterize PDF as combination of "bound" $\mathcal{P}$ and $\mathcal{N}$ PDFs



$$f_{i/\mathcal{A}} = \frac{Z}{A} f^b_{i/\mathcal{P}} + \frac{(A-Z)}{A} f^b_{i/\mathcal{N}}$$

for A, $F_i^{AN} = C_i^N \times A^N$ + power corrections

$\Longrightarrow$ "PDFs = QCD $\times$ had. ME" ("nucleon" picture not necessary)



Rescaling

Moreover, TMCs have particular kinematical dependence:

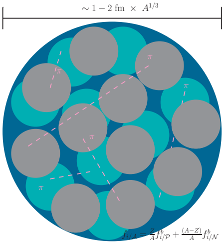
$$\frac{x_A}{\xi_A} \quad \text{or} \quad \left(\frac{x_A^2 M_A^2}{Q^2}\right)$$

Define "average (nucleon) kinematics": $M_N \equiv M_A/A$ **and** $x_N \equiv A x_A$

$$\frac{x_A}{\xi_A} = \frac{x_N}{\xi_N} \quad \text{or} \quad \left(\frac{x_A^2 M_A^2}{Q^2}\right) = \left(\frac{x_N^2 M_N^2}{Q^2}\right)$$

Consequence: TMCs for ***A*-independent, "averaged" nucleon** str. fns. matches intuitive picture of nuclei →

– same expressions as for A but replace "$_A$" with "$_N$"



Summary and conclusion

The nCTEQ collaboration has revisited the theory and phenomenology TMCs in DIS off nuclear targets nCTEQ Collaboration [2301.07715]

- **extended** formalism for protons to nuclei
- **pedagogical appendix** that fills in gaps in literature/texts
- **lots of phenomenology, numbers, and plots** (... so many plots - JLAB, EIC, LBNF)
- hope this work **guides future discussions**
- lots not covered (ACOT, uncertainties, $x_N > 1$, fit results), so see the paper!

Thank you!

backup

more numbers

running the numbers

we use NLO PDFs (`nCTEQ15`) to build str. fns. At LO, these are

$$F_1^{\nu A} = \quad (d + s + \bar{u} + \bar{c}), \qquad F_1^{\bar{\nu} A} = \quad (u + c + \bar{d} + \bar{s})$$
$$F_2^{\nu A} = 2x\left(d + s + \bar{u} + \bar{c}\right), \qquad F_2^{\bar{\nu} A} = 2x\left(u + c + \bar{d} + \bar{s}\right)$$
$$F_3^{\nu A} = +2\left(d + s - \bar{u} - \bar{c}\right), \qquad F_3^{\bar{\nu} A} = -2\left(u + c - \bar{d} - \bar{s}\right)$$

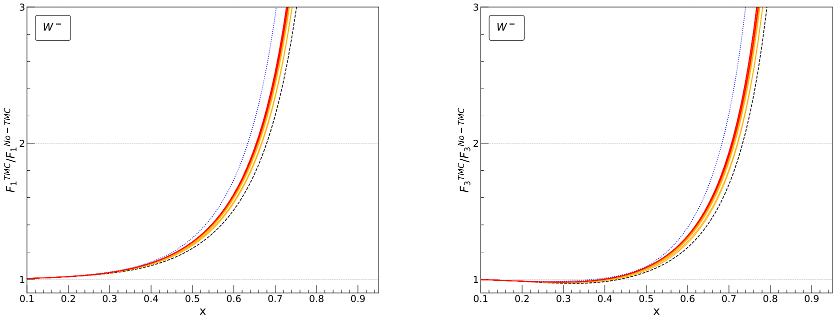
$$F_2^{l^\pm A} = x\frac{1}{9}\left[4(u+\bar{u}) + (d+\bar{d}) + 4(c+\bar{c}) + (s+\bar{s})\right]$$

for many targets

Symbol	A	Z	Symbol	A	Z	Symbol	A	Z	Symbol	A	Z
H	1	1	Be	9	4	Ca	40	20	Xe	131	54
D	2	1	C	12	6	Fe	56	26	W	184	74
^{3}He	3	2	N	14	7	Cu $_{\rm iso}$	64	32	Au	197	79
He	4	2	Ne	20	10	Kr $_{\rm iso}$	84	42	Au $_{\rm iso}$	197	98.5
Li	6	3	Al	27	13	Ag $_{\rm iso}$	108	54	Pb $_{\rm iso}$	207	103.5
Li	7	3	Ar	40	18	Sn $_{\rm iso}$	119	59.5	Pb	208	82

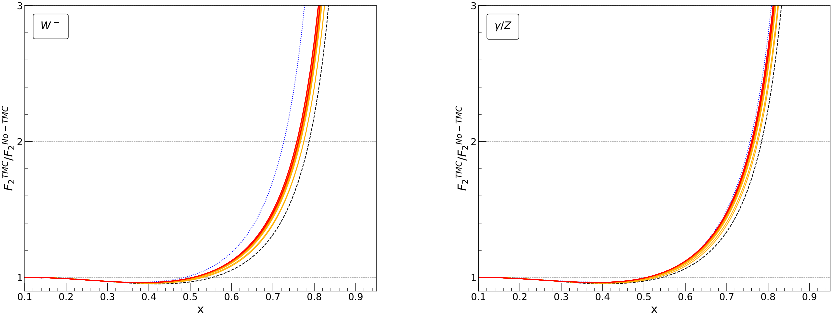
ratio of F_i^{TMC} / $F_i^{\mathrm{no\ TMC}}$

Plotted: ratio for (L) $F_1^{W^-}$ and (R) $F_3^{W^-}$ at $Q = 1.5$ GeV



Can you spot the ^{1}H and ^{2}D curves?

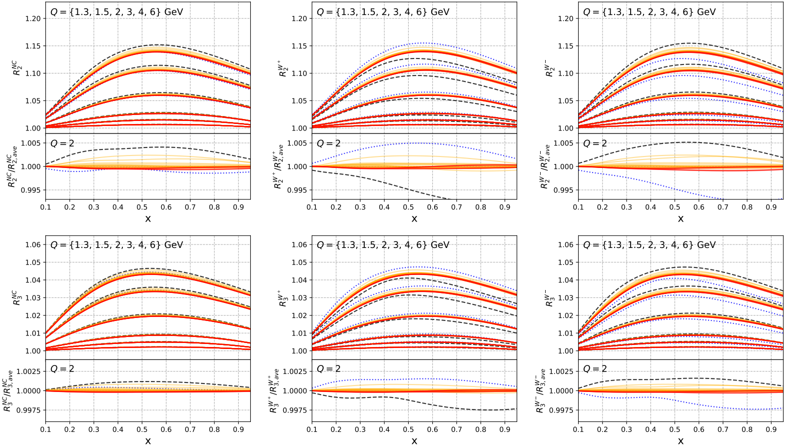
Plotted: ratio for (L) $F_2^{W^-}$ and (R) $F_2^{\gamma/Z}$ at $Q = 1.5$ GeV



Can you spot the ^{1}H and ^{2}D curves?

ratio of F_i^{TMC} / $F_i^{\text{leading TMC}}$

Plotted: ratio for (L) $F_i^{Z/\gamma}$, (C) $F_i^{W^+}$, (R) $F_i^{W^-}$ for $i = 2$ (upper) and $i = 3$ (lower)



remarkable uniformity! (good enough to fit! ☺)

reduced cross sections

Plotted: (upper) reduced cross sections with nTMCs; (lower) ratio to w/o