Energy-Energy Correlator in DIS

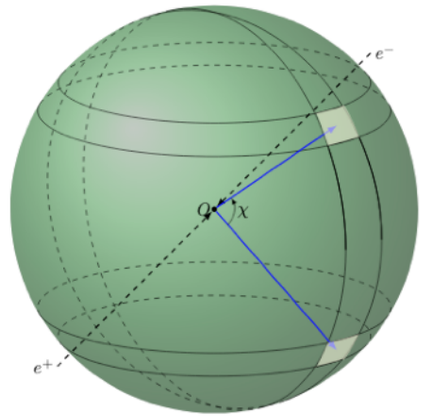
Haitao Li, Shandong University

arXiv:2006.02437, arXiv: 2102.05669, arXiv: 2312.07655
with H. Cao, Y. Makris, Z. Mi, I. Vitev, Y.J. Zhu

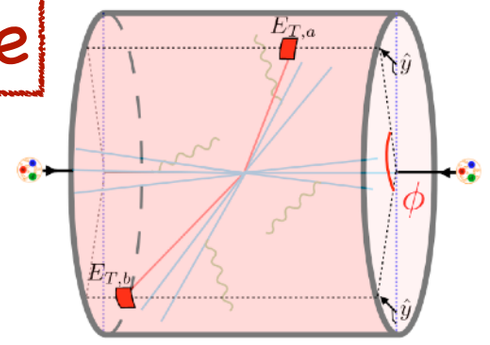
9-April-2024

EEC and TEEC

e^+e^- Collisions



Hadronic initial state



observable

$$\mathrm{EEC} = \sum_{a,b} \int d\sigma_{V\to a+b+X} \frac{2E_aE_b}{Q^2\sigma_{\mathrm{tot}}} \delta(\cos(\theta_{ab}) - \cos(\chi))$$

$$\mathrm{TEEC} = \sum_{a,b} \int d\sigma_{pp\to a+b+X} \frac{2E_{T,a}E_{T,b}}{|\sum_i E_{T,i}|^2} \delta(\cos\phi_{ab} - \cos\phi)$$

- sum over all the jets for each event
- sum over all the particles for each event

Basham et al 1978
Moult, Zhu, 2018

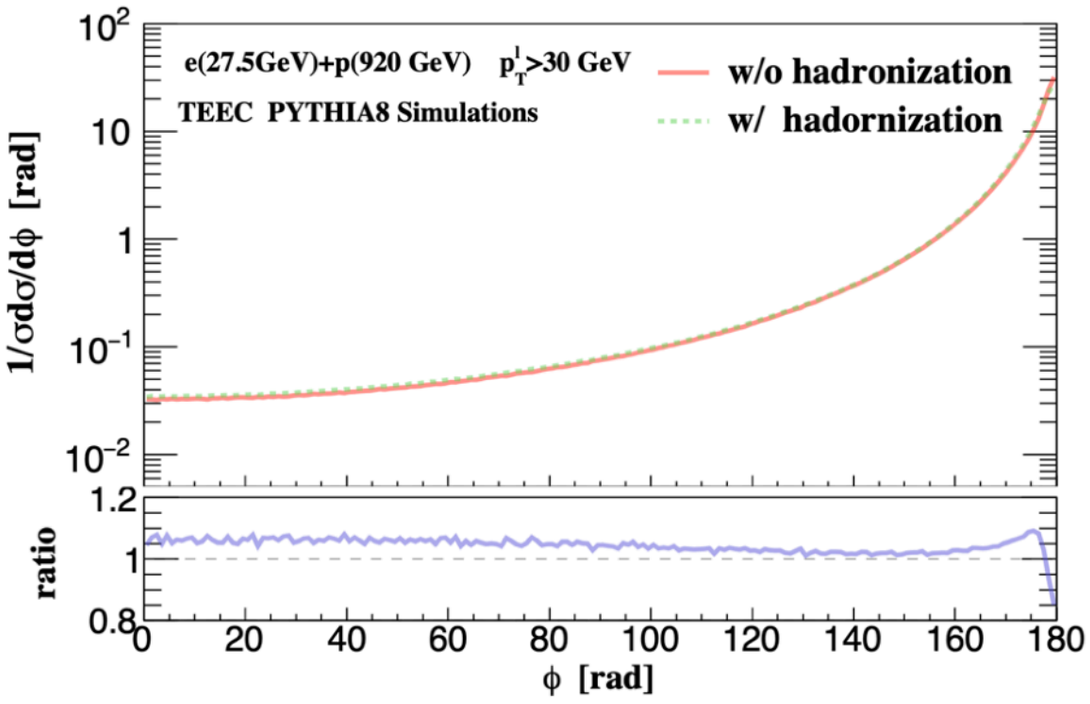
- weighted cross section
- the soft radiation does not contribute directly to the observable at leading power
- soft gluon contributes only via recoil

Ali et al 1984
Gao, HTL, Moult, Zhu, 2019, 2023

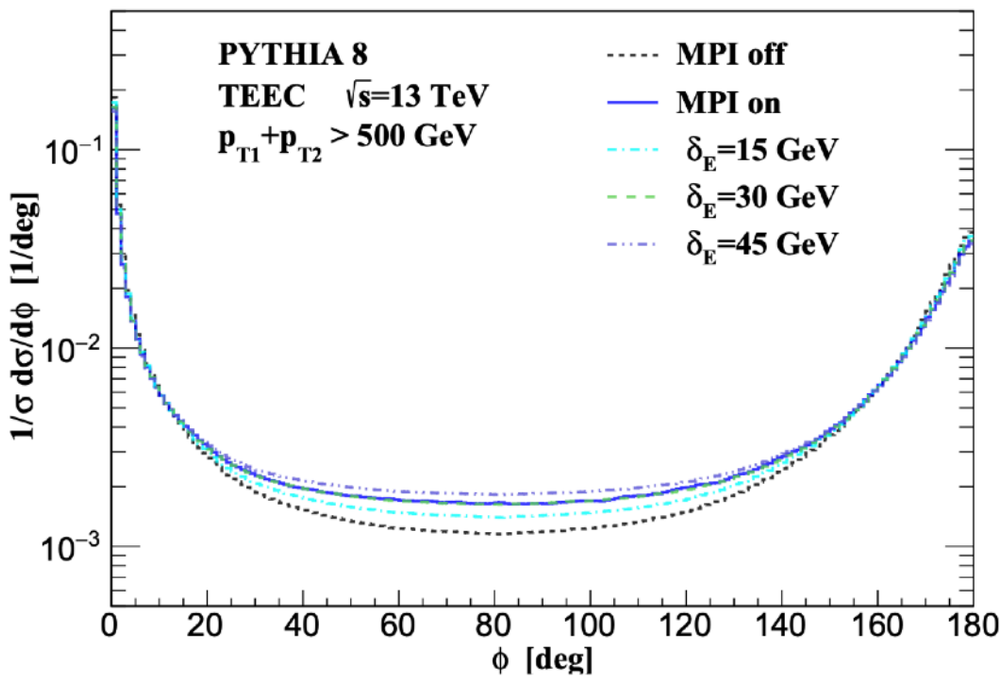
EEC and TEEC

EEC/TEEC is a class of event shape variables

where nonperturbative effect is supposed to be small



HTL, Vitev, Zhu, 2020



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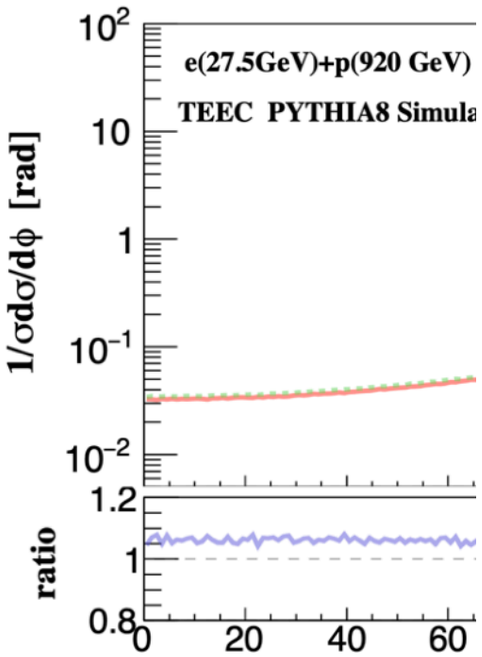

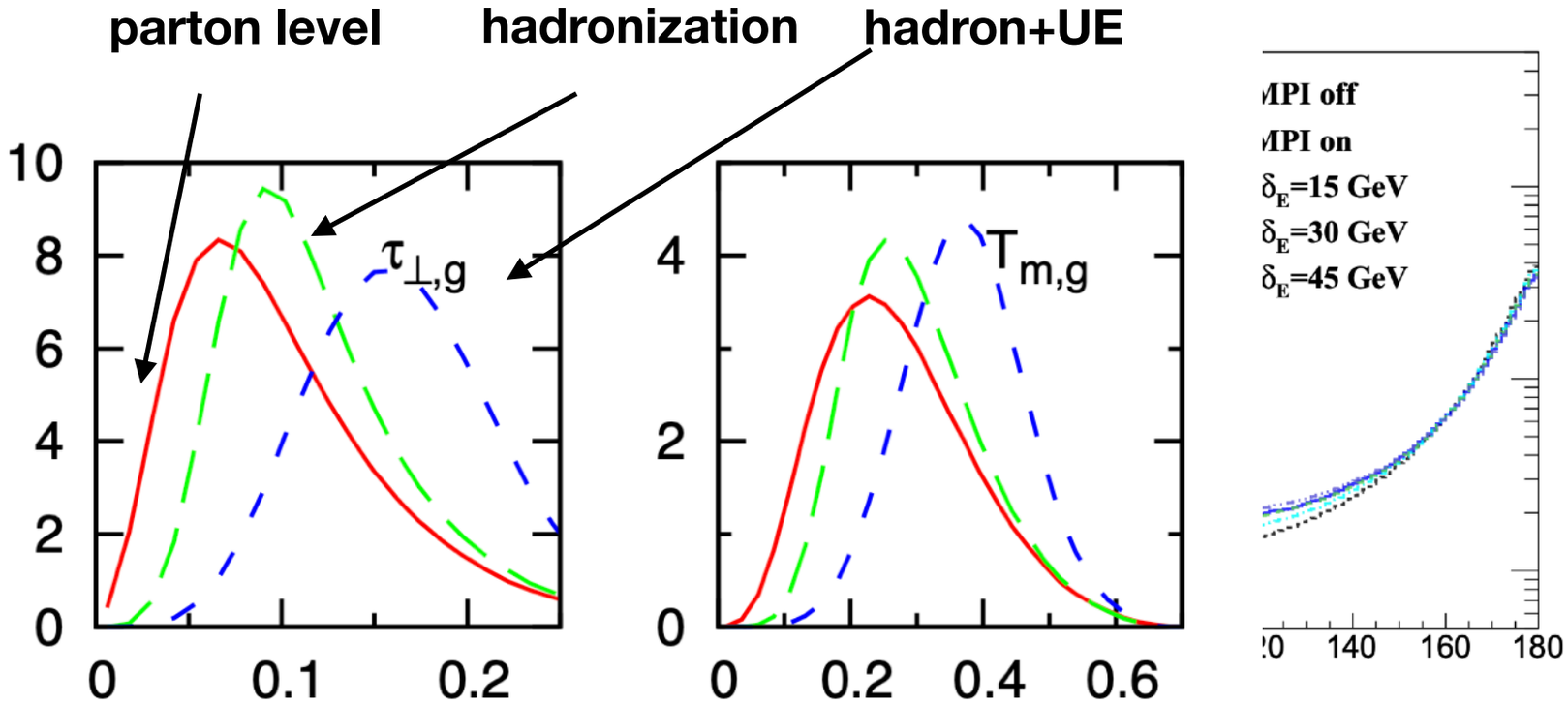

Banfi, Salam and Zanderighi 2010

HTL, V

, Zhu, 2023

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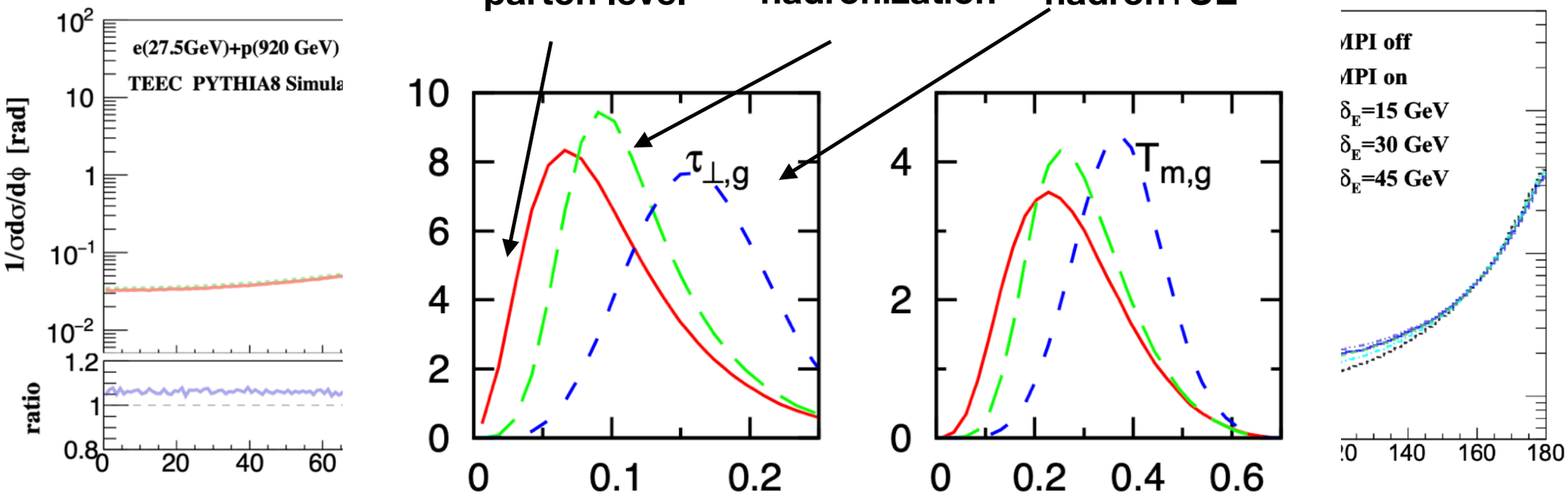

Recent developments (EEC/TEEC in DIS)

Nucleon Energy Correlators for Color Glass Condensate, Liu, Zhu et al, 2022,2023

Collins-type EEC jet in DIS, Kang et al, 2023

Imaging Cold Nuclear Matter with Energy Correlators, Devereaux et al 2023

TEEC in the Color-Glass Condensate at the Electron-Ion Collider, Kang et al 2023

TMDs from Semi-inclusive Energy Correlators, Liu, Zhu, 2024

See Friederike Bock's talk yesterday, Penttala' talk on April 10

TEEC in DIS

In Lab Frame at the EIC

HTL, Vitev, Zhu, 2020

Definition

$$\mathrm{TEEC} = \sum_a \int d\sigma_{lp\to l+a+X} \frac{E_{T,l}E_{T,a}}{E_{T,l}\sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$$

sum over all hadrons

energy weighted

measure azimuthal angle correlations

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measure azimuthal angle correlations

azimuthal angle correction between particles and lepton $\tau = \dfrac{1+\cos\phi}{2}$

For $\tau \to 1$, large angle radiation

For $\tau \to 0$, small angle radiation

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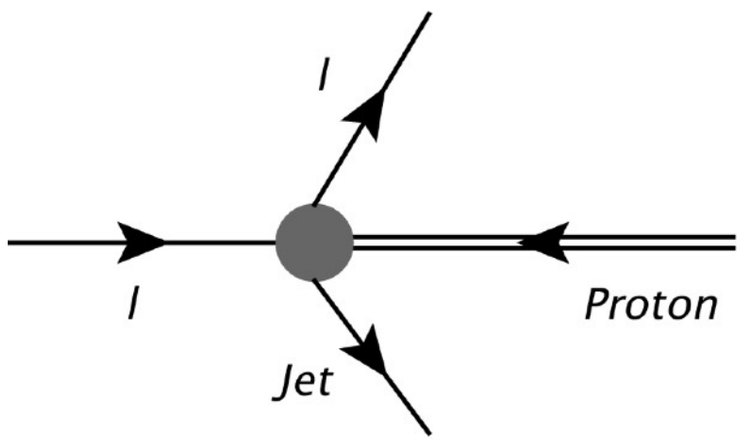
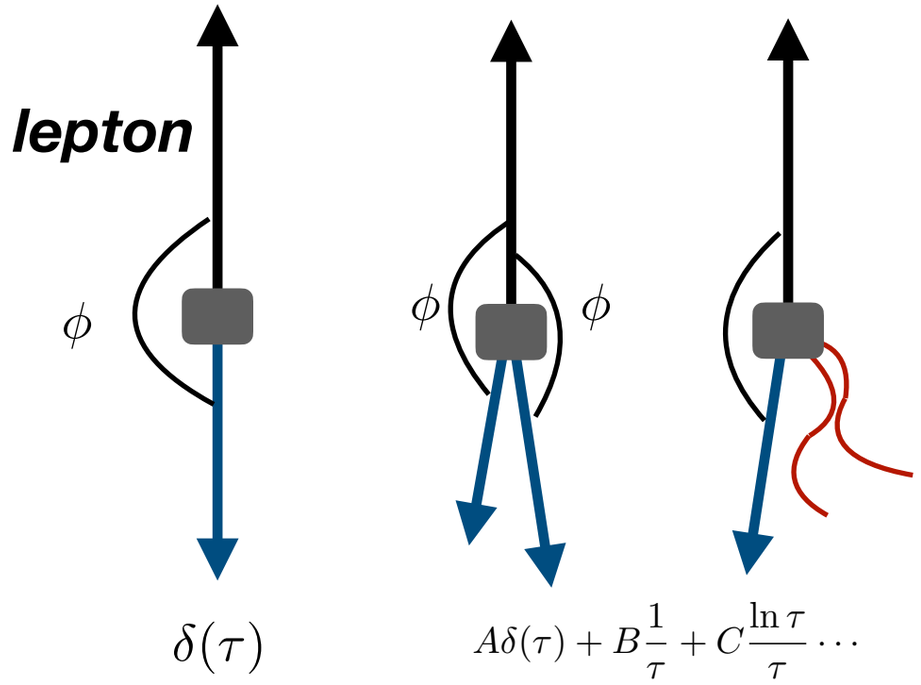


In transverse plane



$$\tau \approx \frac{\left| k_{2,y} - k_{s,y} + \frac{k_{4,y}}{\xi_4} \right|^2}{4p_T^2}$$

TEEC in DIS

Definition $$\mathrm{TEEC} = \sum_a \int d\sigma_{lp\to l+a+X} \frac{E_{T,l}E_{T,a}}{E_{T,l}\sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$$

In back-to-back limit, it is similar to 1-dimensional TMD factorization

hadron with small p_T

TMD PDF

$$\frac{d\sigma_h}{d^2p_\perp} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int \frac{db}{2\pi} e^{ib_\perp\cdot p_\perp} f_{f/N}\left(b,\xi,\mu,\nu\right) S\left(b,\frac{n_2\cdot n_4}{2},\mu,\nu\right) \int \frac{dz}{z^2} F_{h/f}\left(z,b/z,E_4,\mu,\nu\right)$$

TMD soft

TMDFF

TEEC in DIS

Definition $$\mathrm{TEEC} = \sum_a \int d\sigma_{lp\to l+a+X} \frac{E_{T,l}E_{T,a}}{E_{T,l}\sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$$

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hadron with small p_T

TMD PDF

sum over all hadrons in the final state

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TMD soft

TMDFF

$$\frac{d\sigma_h}{d\tau} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int dk_y \int \frac{db}{2\pi} e^{-ib_y\cdot k_y} f_{f/N}\left(b,\xi,\mu,\nu\right) S\left(b,\frac{n_2\cdot n_4}{2},\mu,\nu\right) \sum_h \int z dz\ F_{h/f}\left(z,b/z,E_4,\mu,\nu\right) \delta(\tau-\tau(k_y))$$

TEEC in DIS

Definition $$\mathrm{TEEC} = \sum_a \int d\sigma_{lp\to l+a+X} \frac{E_{T,l}E_{T,a}}{E_{T,l}\sum_i E_{T,i}} \delta(\cos\phi_{la} - \cos\phi)$$

In back-to-back limit, it is similar to 1-dimensional TMD factorization

hadron with small p_T — TMD PDF — **sum over all hadrons in the final state**

$$\frac{d\sigma_h}{d^2p_\perp} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int \frac{db}{2\pi} e^{ib_\perp\cdot p_\perp} f_{f/N}\left(b,\xi,\mu,\nu\right) S\left(b,\frac{n_2\cdot n_4}{2},\mu,\nu\right) \int \frac{dz}{z^2} F_{h/f}\left(z,b/z,E_4,\mu,\nu\right)$$

TMD soft — TMDFF

$$\frac{d\sigma_h}{d\tau} = \sum_f \int \frac{d\xi dQ^2}{\xi Q^2} Q_f^2 H(Q,\mu) \int dk_y \int \frac{db}{2\pi} e^{-ib_y\cdot k_y} f_{f/N}\left(b,\xi,\mu,\nu\right) S\left(b,\frac{n_2\cdot n_4}{2},\mu,\nu\right) \sum_h \int z dz\ F_{h/f}\left(z,b/z,E_4,\mu,\nu\right) \delta(\tau-\tau(k_y))$$

Jet function — **the second Mellin-Moment of the TMDFFs**

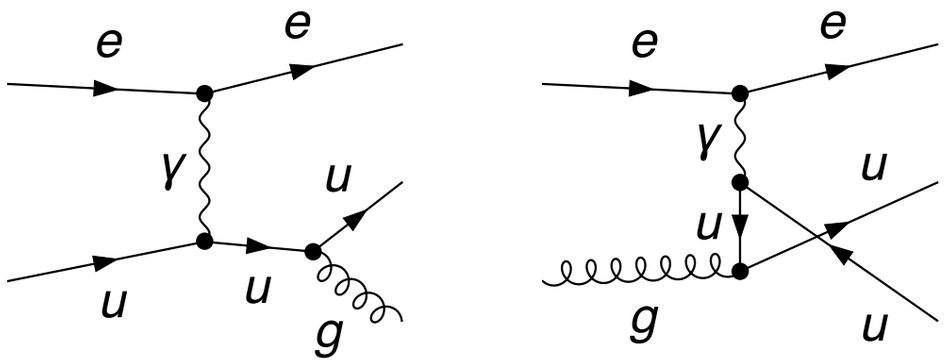
$$\sum_N \int_0^1 dz z F_{N/q}\left(z,b_\perp/z,\nu\right) = \sum_{i,N} \int_0^1 dz z \int_z^1 \frac{d\xi}{\xi}\ d_{N/i}(z/\xi)\ \mathscr{C}_{iq}\left(\xi,b_\perp/\xi,\nu\right) + \mathcal{O}\left(b_T^2\Lambda_{\mathrm{QCD}}^2\right)$$

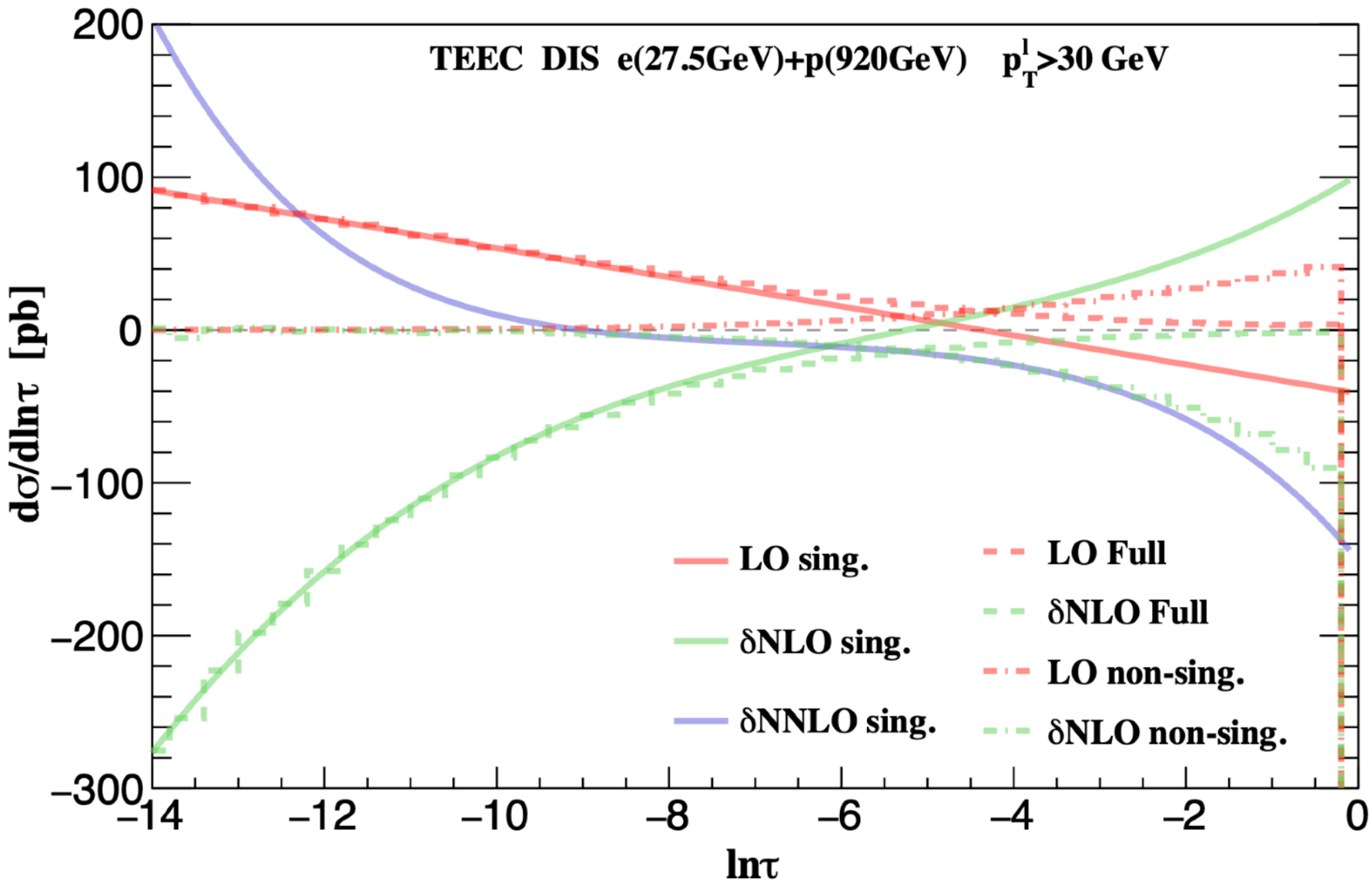
$$= \sum_{i,N} \int_0^1 dx x \mathscr{C}_{iq}\left(x,b_\perp/\xi,\nu\right) \int_0^1 d\xi \xi d_{N/i}(\xi) + \mathcal{O}\left(b_T^2\Lambda_{\mathrm{QCD}}^2\right)$$

TEEC in DIS

The leading order process is

calculated using NLOJET++

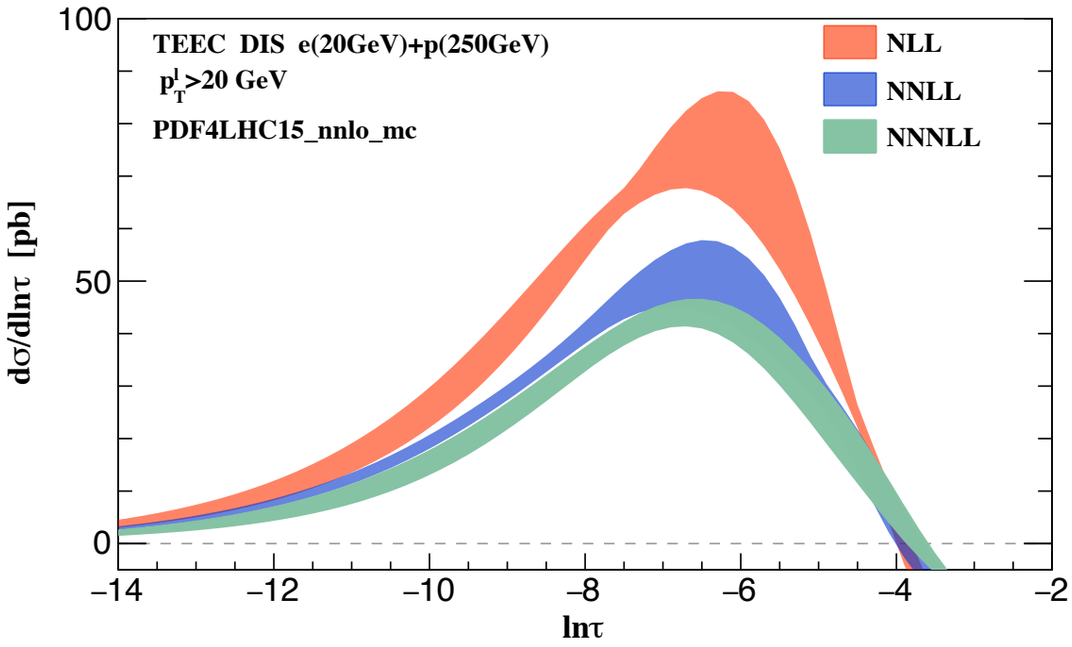




TEEC in DIS

resummation accuracy

Accuracy	H, J, S, B	$\gamma_{\rm cusp}$	γ	β
LL	Tree	1 loop	-	1 loop
NLL	Tree	2 loop	1 loop	2 loop
NNLL	1 loop	3 loop	2 loop	3 loop
$\mathrm{N^3LL}$	2 loop	4 loop	3 loop	4 loop



- Convergence in back-to-back limit after resummation
- Huge difference from NLL to NNLL and good perturbative convergence from NNLL to NNNLL
- Reduction of scale uncertainties order by order from NLL to NNNLL

EEC in DIS

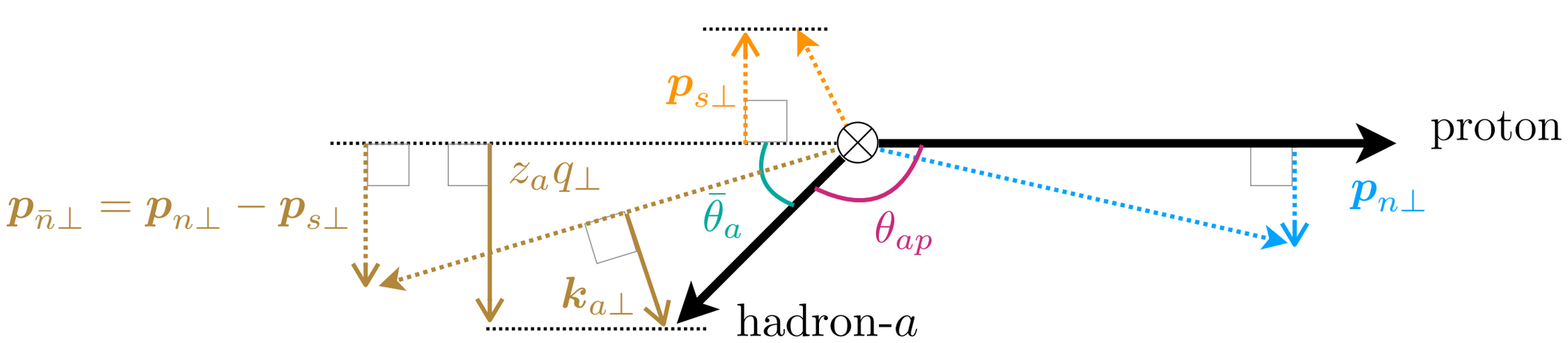
In Breit Frame at the EIC

HTL, Makris, Vitev 2021

From Lab frame to Breit Frame

- **boost the system to proton rest frame**
- **rotate the system: virtual photon has zero $\vec{q}_T$**
- **boost along z direction: virtual photon has zero energy**

$$\gamma^* + \text{proton} \to \text{jet/hadron} + X$$



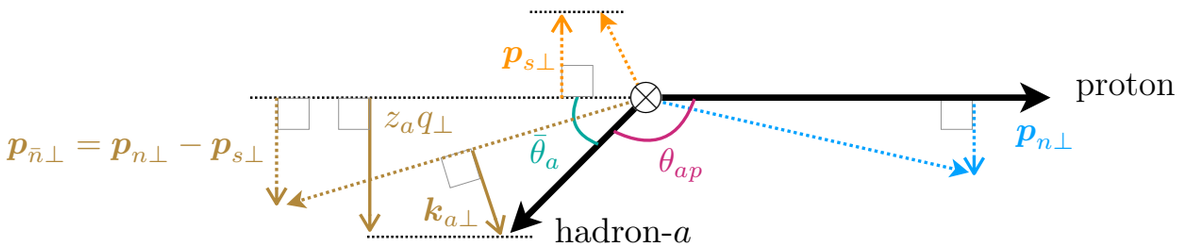
We proposed a new definition of EEC in DIS:

correlation between initial proton and final state hadron

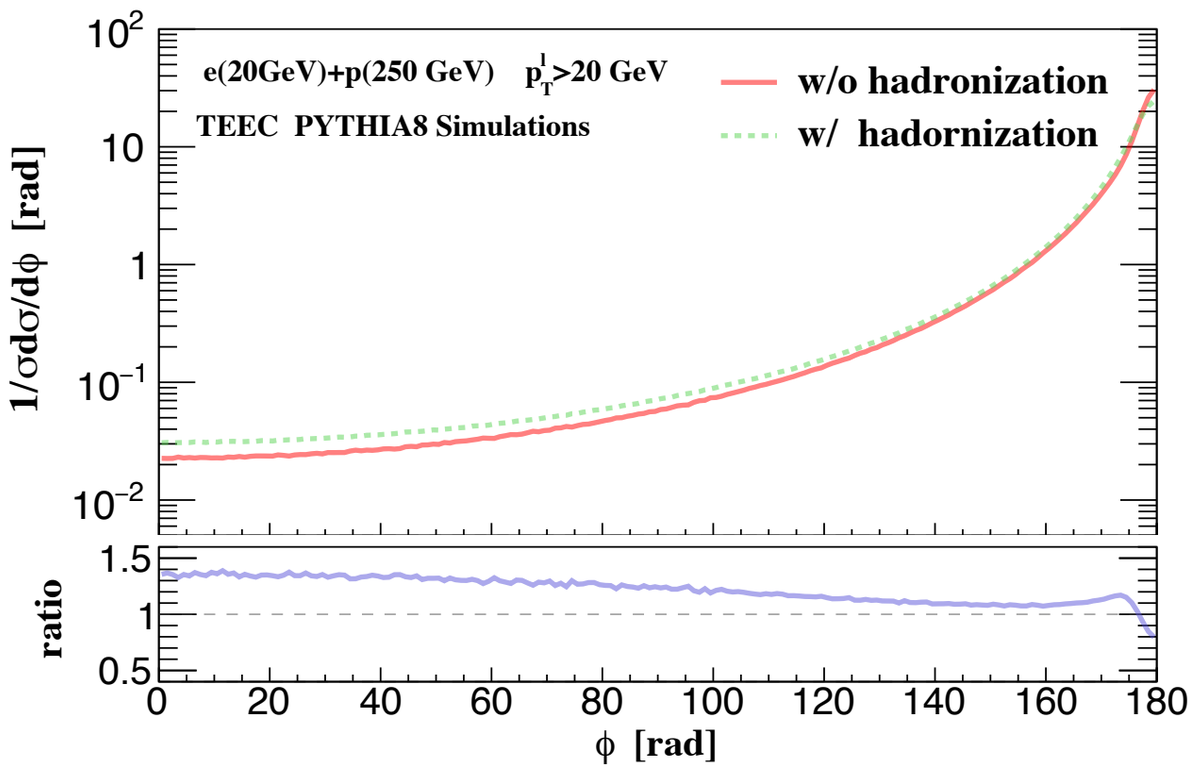
$$\text{EEC} = \sum_a \int d\sigma_{lp\to l+a+X} \left(\frac{p\cdot p_a}{\sum_i p\cdot p_i}\right) \delta(\cos\chi - \cos\theta_{ap})$$

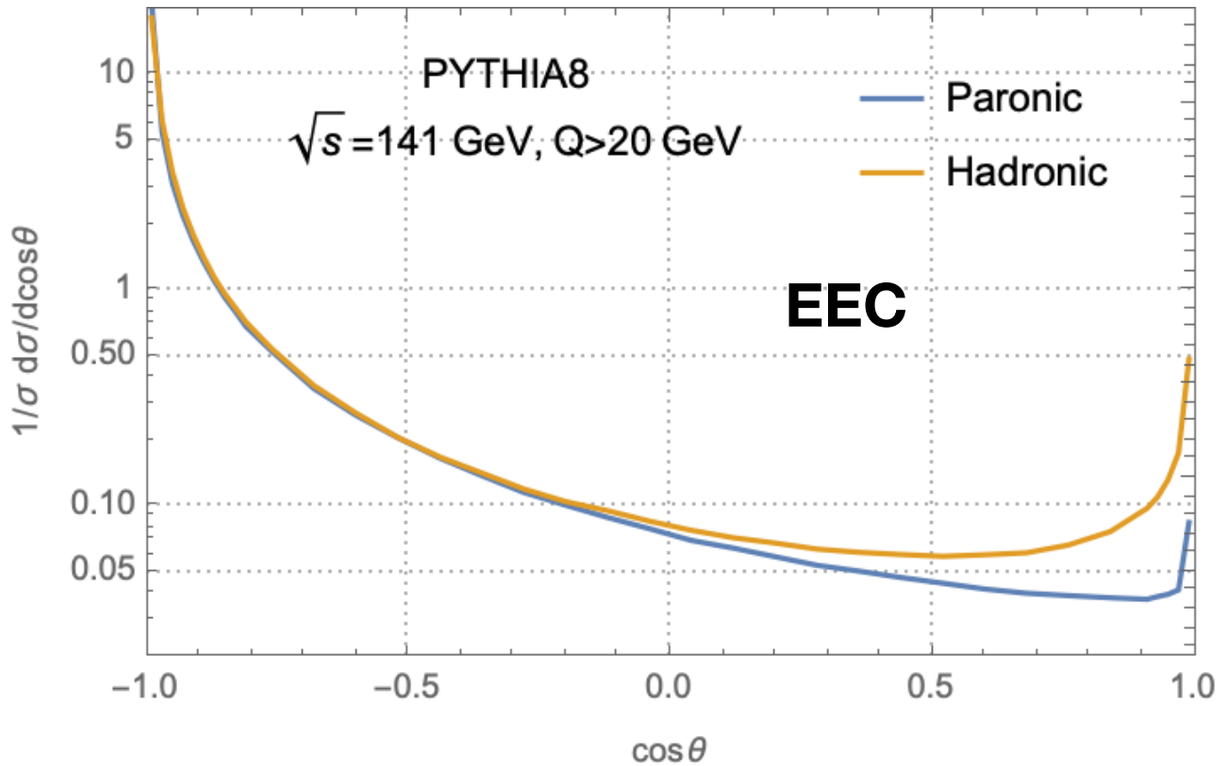
TEEC/EEC in DIS

$$\mathrm{EEC} = \sum_a \int d\sigma_{lp\to l+a+X} \left(\frac{p\cdot p_a}{\sum_i p\cdot p_i}\right) \delta(\cos\chi - \cos\theta_{ap})$$



- weight function is Lorentz Invariant
- radiation close to the beam direction is suppressed
- soft radiation/hadronization effect is suppressed





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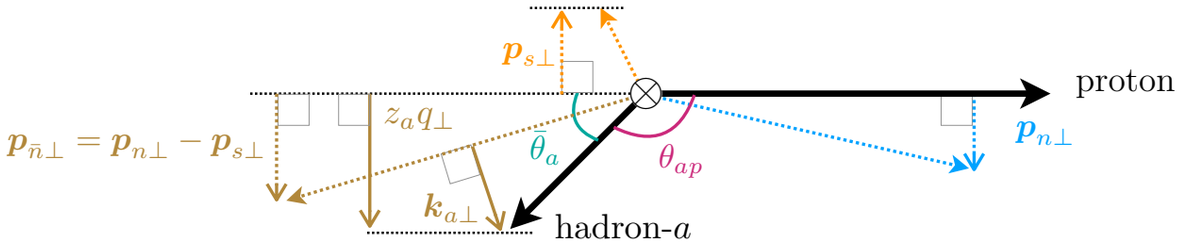

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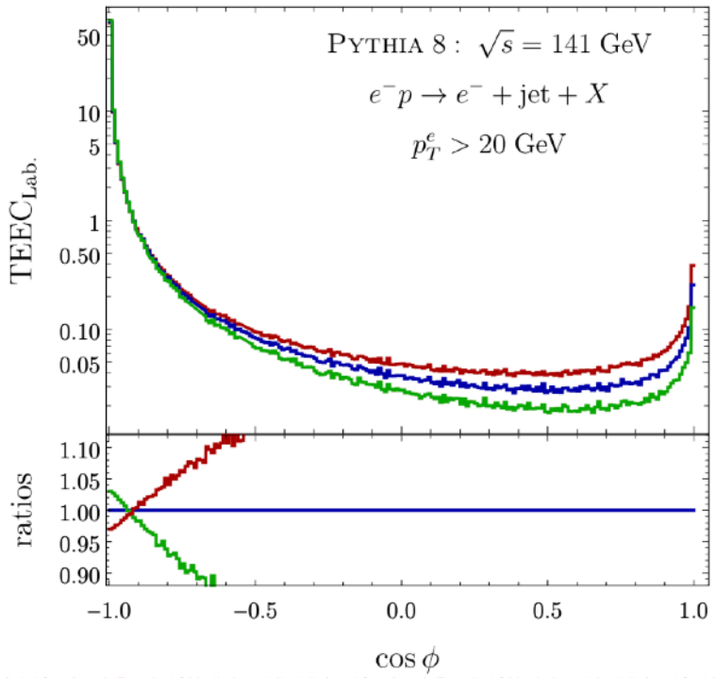

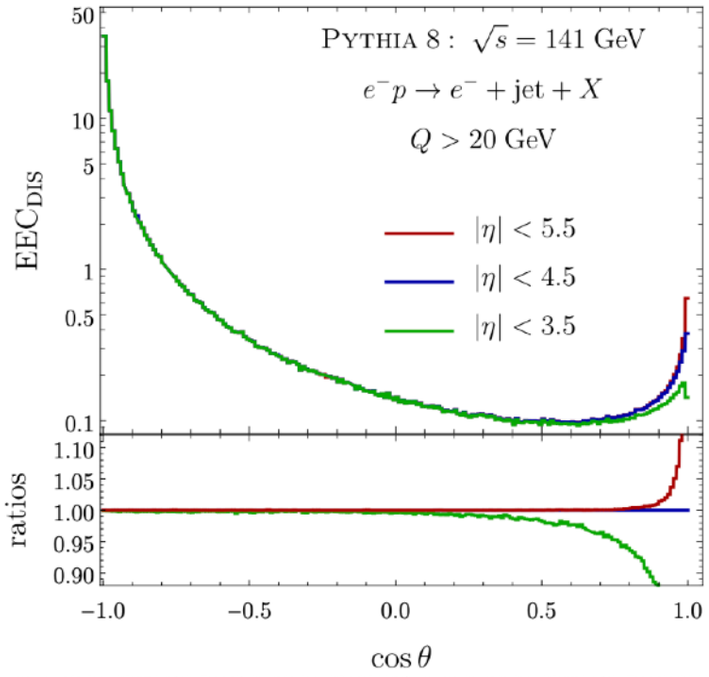

normalized to the cross section with cut $|\eta| < 5.5$

In Breit frame, rapidity cut only changes the cross section tail region

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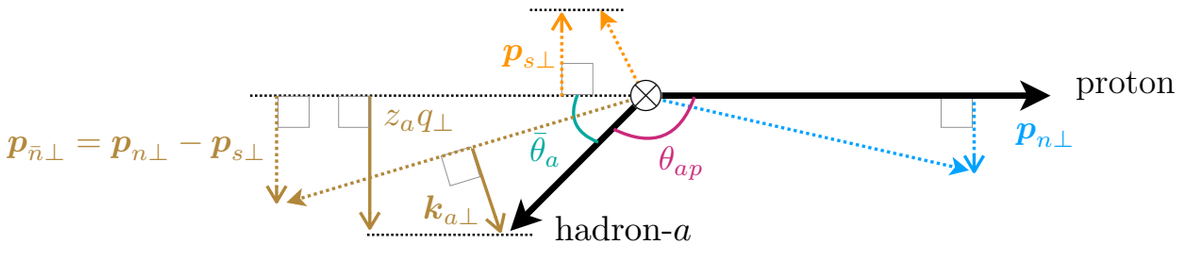

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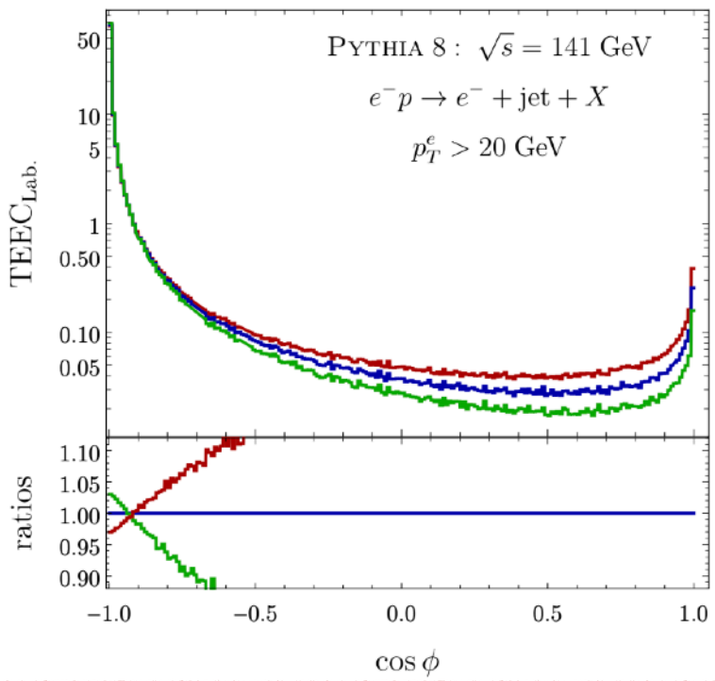
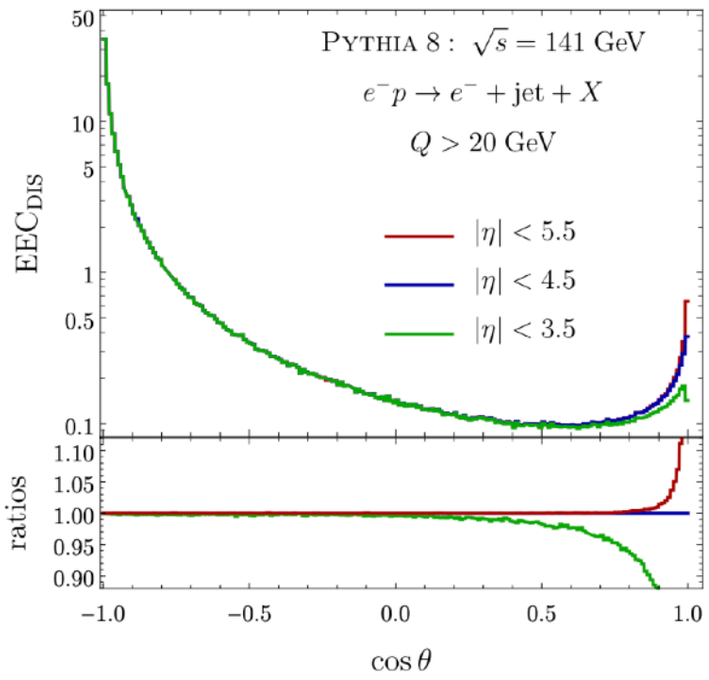




normalized to the cross section with cut $|\eta| < 5.5$

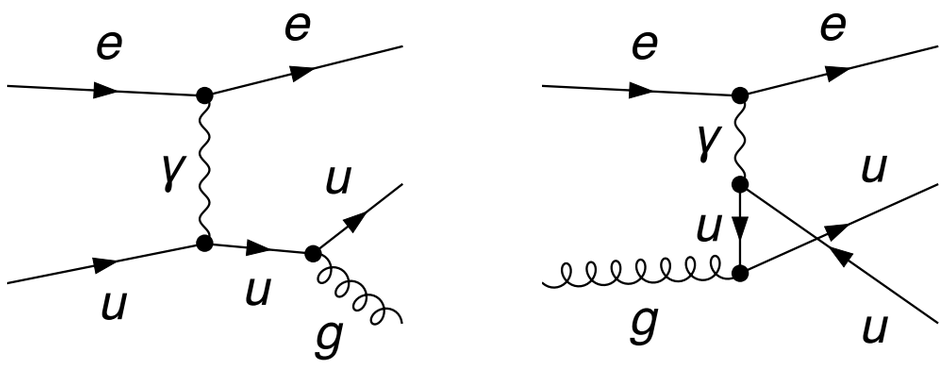
In Breit frame, rapidity cut only changes the cross section tail region

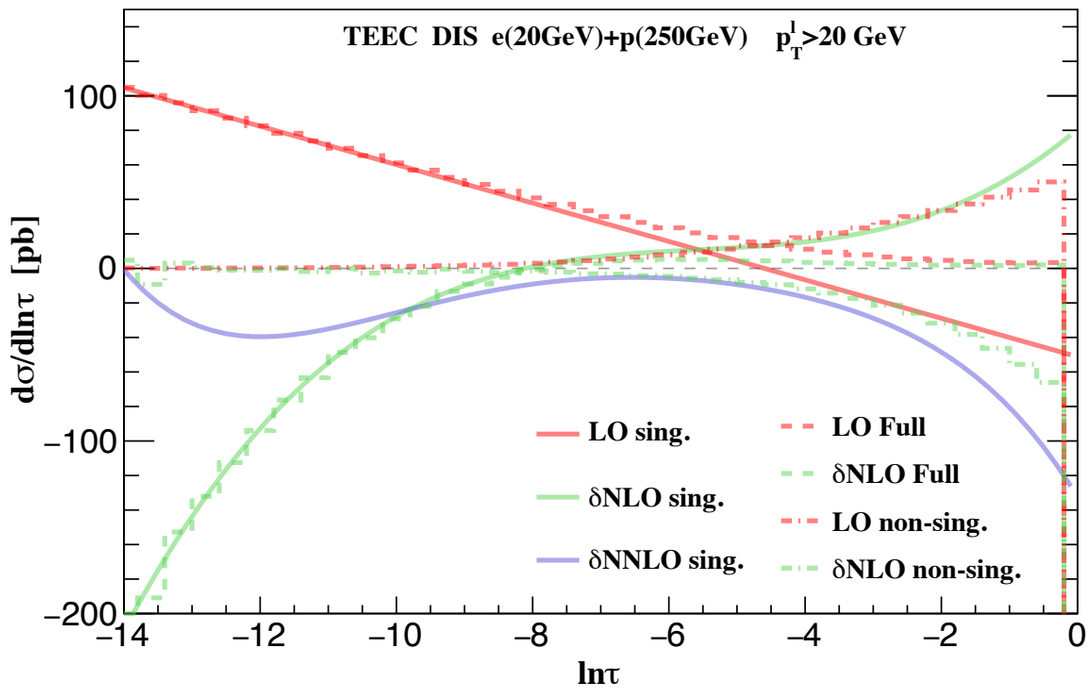
In back-to-back limit, TMD factorization can be used which has a better connection to the usual TMD physics

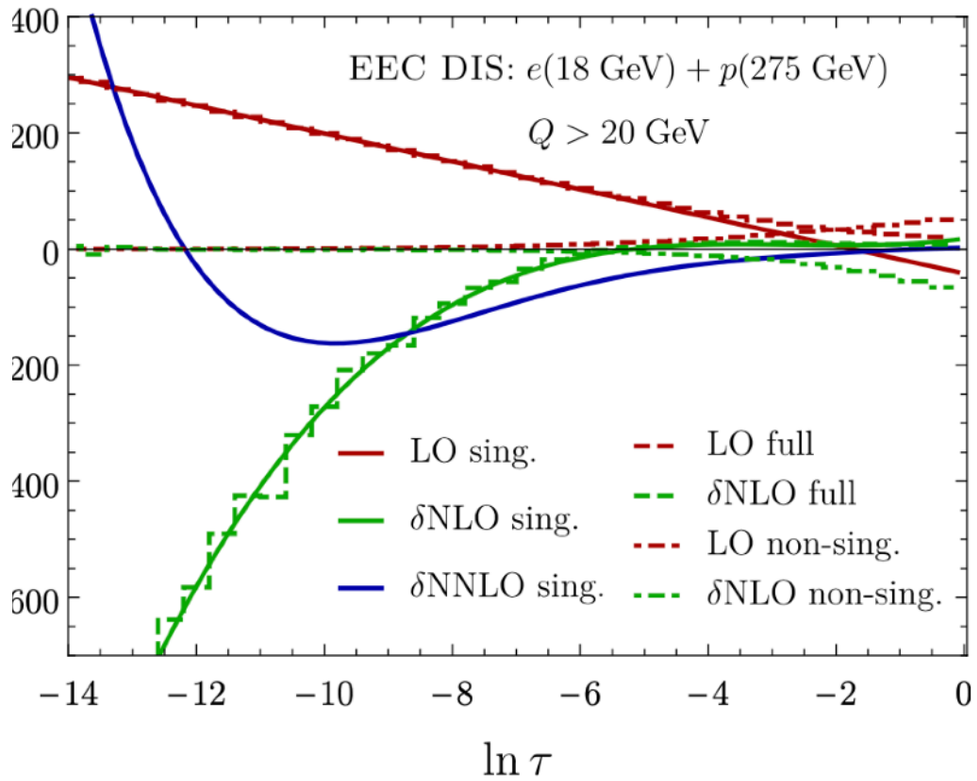
Fixed order results

The leading order process is

calculated using NLOJET++

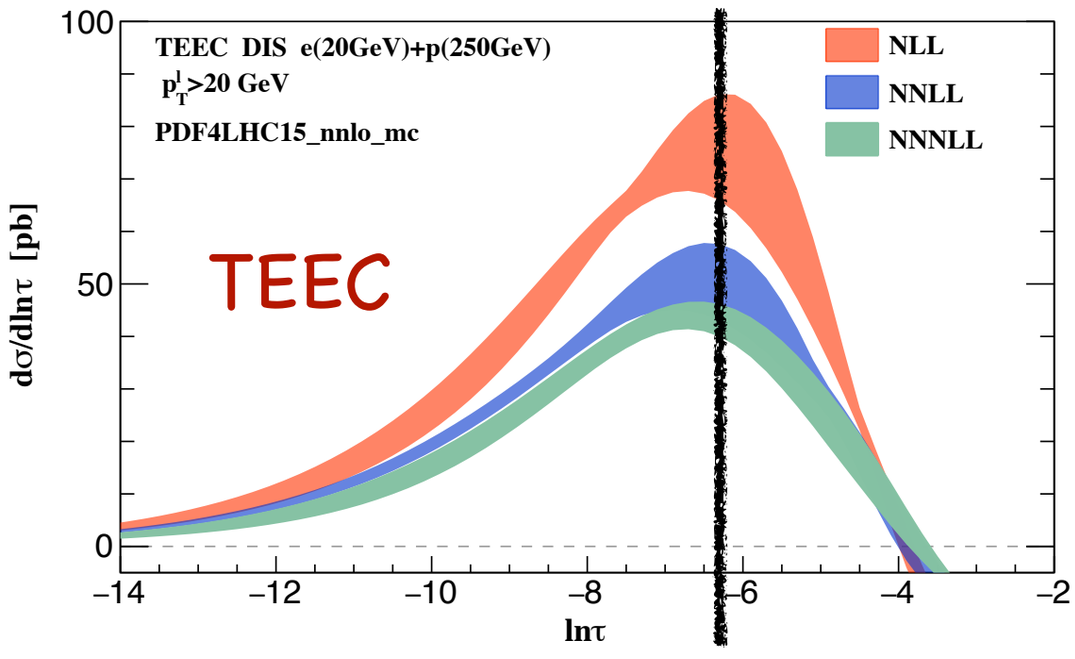


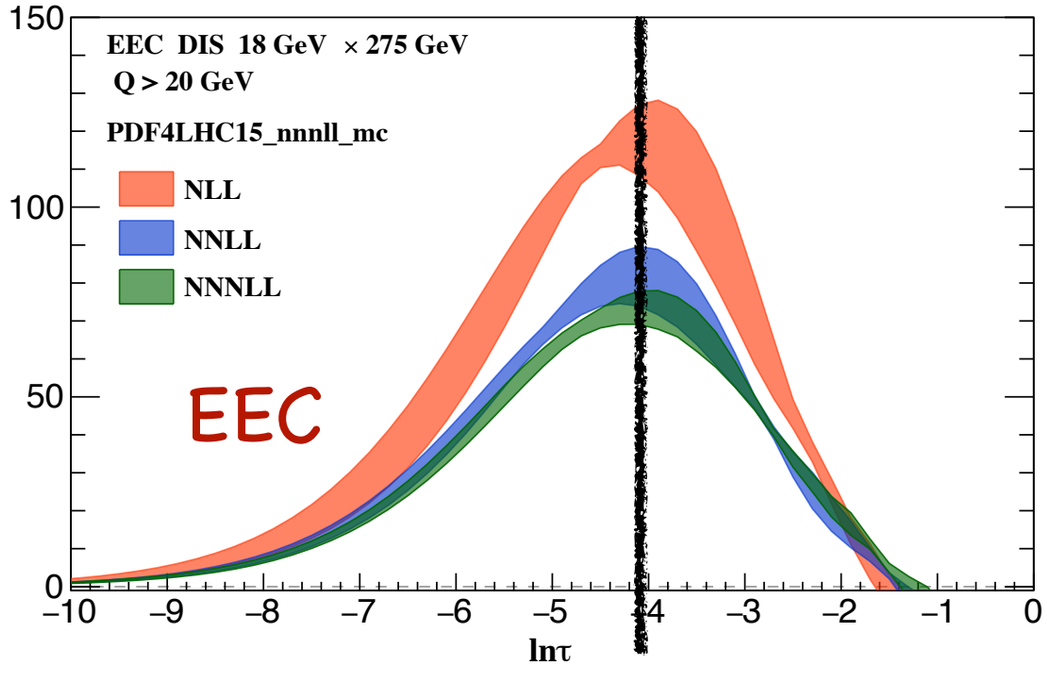




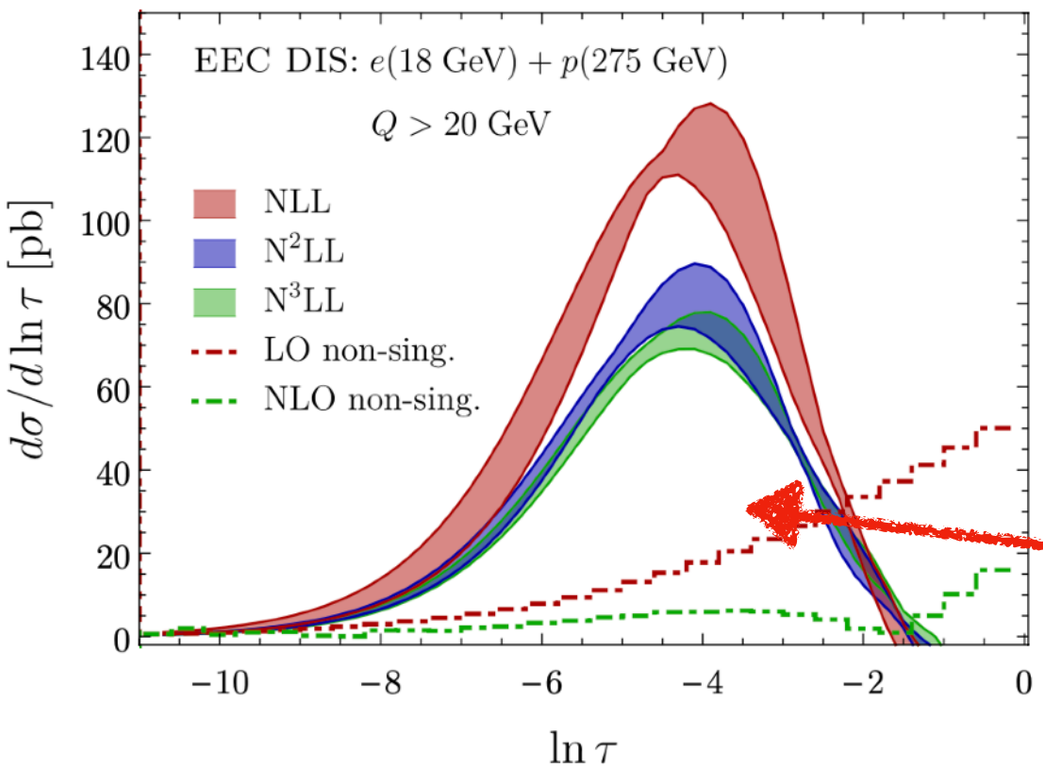
- Reproduced the singular behaviors
- Full control of the distributions in the back-back limit at LO and NLO.
- We obtained singular distribution up to NNLO (three loop anomalous dimensions)

Resummation





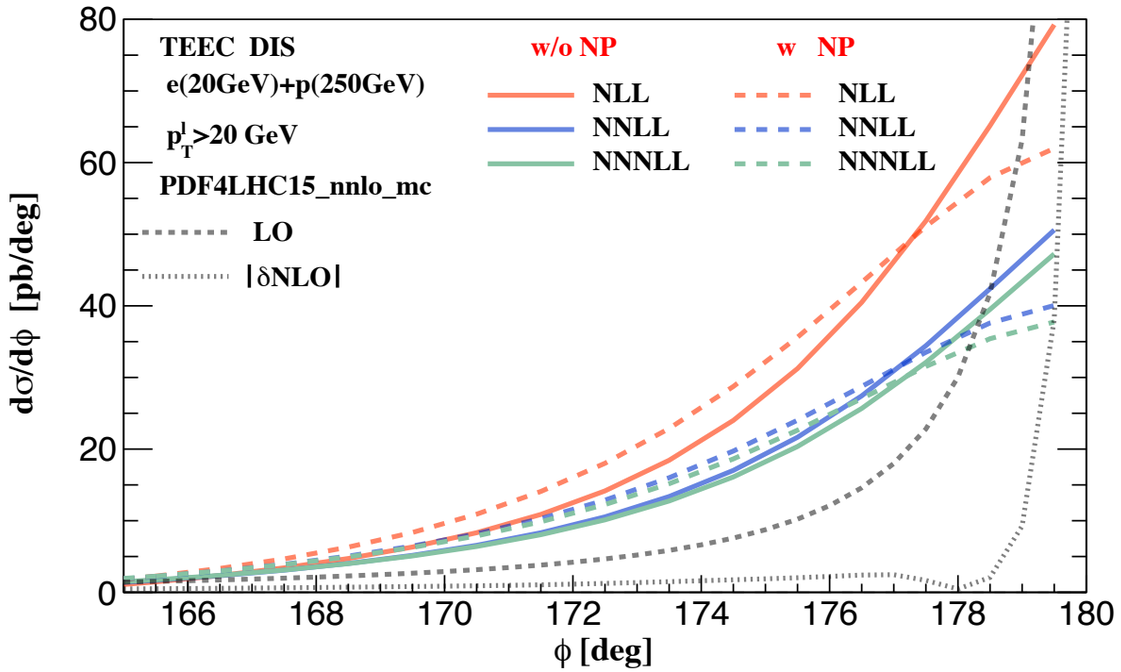
for EEC peak at larger τ, means small NP effects



- Convergence in back-to-back limit after resummation
- Huge difference from NLL to NNLL and good perturbative convergence from NNLL to NNNLL
- Reduction of scale uncertainties order by order from NLL to NNNLL

Non-singular terms start to contribute which is less important for EEC

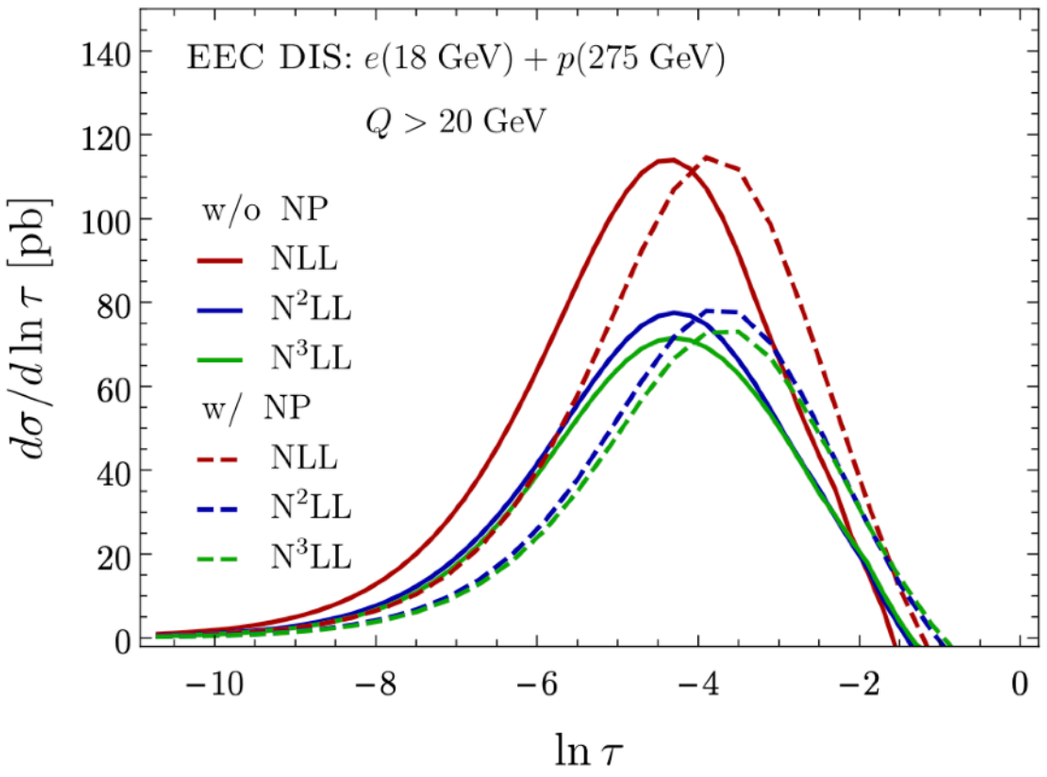
Non-perturbative effects from TMD



- ☑ corrections to rapidity evolution
- ☑ corrections to the TMD matrix element

Non-perturbative form factors, which extracted from the semi-inclusive hadron production in DIS.

$$S_{\rm NP} = \exp\left[-0.106\ b^2 - 0.84 \ln Q/Q_0 \ln b/b^*\right]$$



from TMD FFs

$$D_{i/a}^{\rm NP}(y,b) = \exp\left(-0.042\frac{b^2}{y^2}\right)$$

$$j_i(b) = \exp\left(-0.59b - 0.03b^2\right)$$

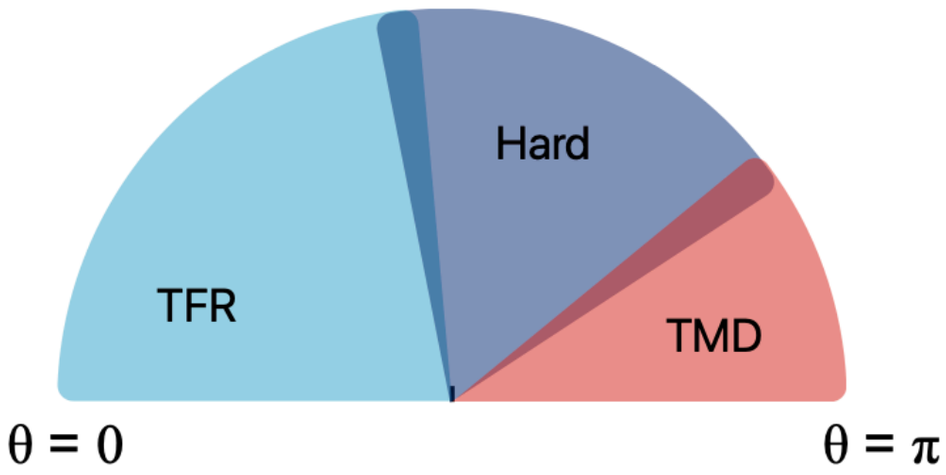
NP shifts the cross section

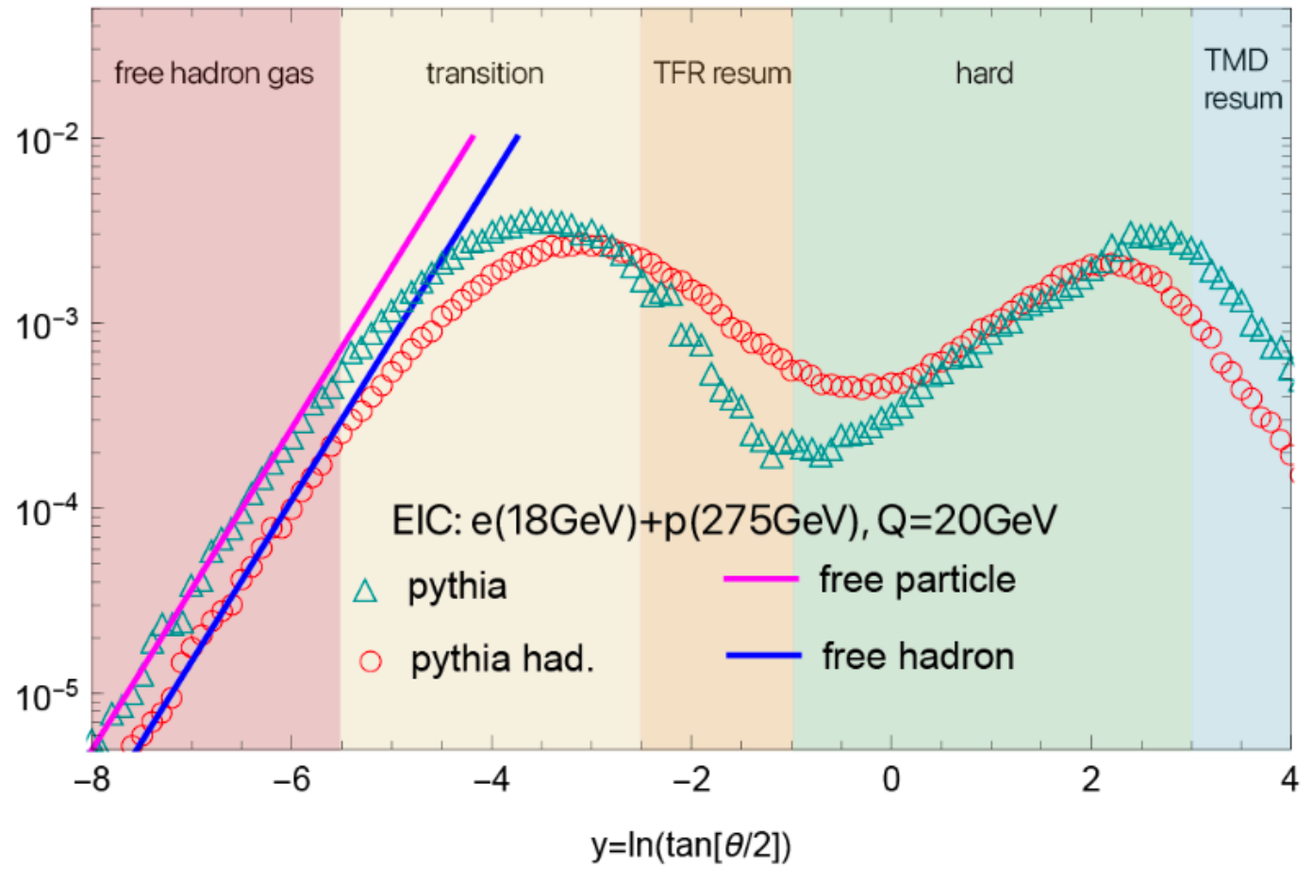
Sizable NP effects in back-to-back limit

Bjorken x weighted EEC

$$\mathrm{EEC} = \frac{1}{\sigma}\sum_a \int d\sigma(\ell + h \to \ell + a + X) x_B^{N-1} \frac{P\cdot p_a}{P\cdot q}\delta\left(\cos\theta_{ap} - \cos\theta\right)$$

NEEC, Liu, Zhu, arXiv:2209.02080; Cao, Liu, Zhu, arXiv:2303.01530.





TFR: the correlation of the energy flows from the initial nucleon.

Hard: measures the perturbative behavior of QCD

TMD: measures perturbative and nonperturbative TMD physics

Cao, HTL, Mi, arXiv:2312.07655

Conclusion

Motivation

- ❑ Event shapes serve as a QCD laboratory, a tool for QCD study
- ❑ EEC/TEEC can been studied for various processes

Observables

- ❑ TEEC and EEC in DIS

Application

- ❑ investigate QCD in low and high energy limits
- ❑ test and study TMD factorization
- ❑ extract TMD PDFs and TMD FFs

Conclusion

Motivation

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OI

Thank you!

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