

# A GMVFN scheme for Z boson associated with a heavy quark production at hadron colliders

Marco Guzzi,  
Kennesaw State University  
with  
P. Nadolsky, L. Reina, D. Wackerlo, K. Xie



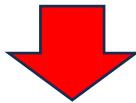
**DIS2024: XXXI International Workshop on Deep-Inelastic Scattering and  
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April the 9<sup>th</sup>, 2024, WG1-WG4



# Main Goals

**Simplify implementation of GMVFN schemes in (N)NLO QCD calculations using the formalism of subtracted PDFs**

Associated production of a Z boson with  $c$ - or  $b$ -quark jets in pp collisions provides direct access to  $c$  and  $b$  PDFs.



- Constrain heavy-flavor PDFs in global QCD analyses;
  - Probe QCD dynamics at small and large  $x$ .
  - Probe nonperturbative  $c, b$  contributions in the proton
- 
- This talk: S-ACOT-MPS scheme to  $\text{pp} \rightarrow \text{Z} + b + X$  in pQCD
  - Previously implemented for inclusive charm [[FPF, 2109.10905, 2203.05090](#)] and bottom [[2203.06207](#)] production. Related S-ACOT-mT scheme (Helenius, Paukkunen JHEP23, 2303.17864) for B-meson production.

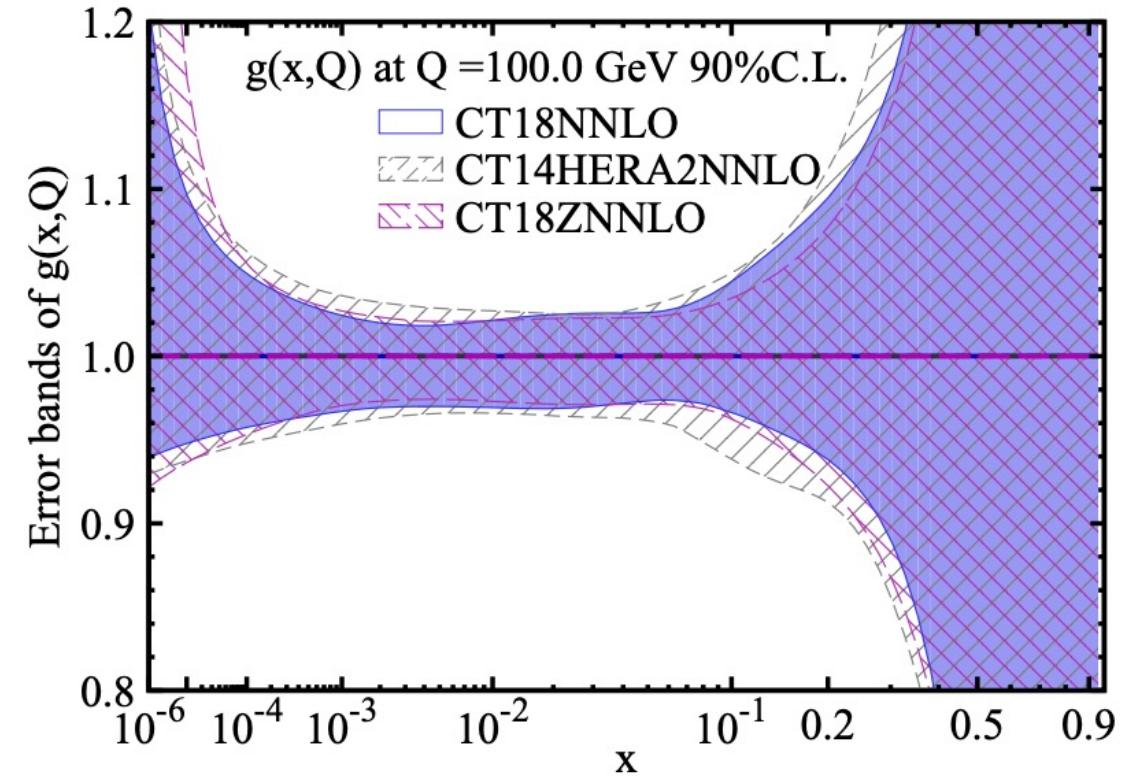
# Motivations

- $Z + c/b$  production at the LHC at small  $p_T$  and large rapidity  $y$  of the heavy quark: sensitive to PDFs at both small and large  $x$

$$x_1 \geq \frac{1}{\sqrt{s}} \left( \sqrt{(p_T^Z)^2 + m_{\ell\ell}^2} \exp(-y_Z) + p_T^{\text{jet}} \exp(-y_{\text{jet}}) \right)$$

$$x_2 \geq \frac{1}{\sqrt{s}} \left( \sqrt{(p_T^Z)^2 + m_{\ell\ell}^2} \exp(y_Z) + p_T^{\text{jet}} \exp(y_{\text{jet}}) \right)$$

- In this kinematic region PDFs are poorly constrained by other experiments in global QCD analyses of PDFs.
- $Z + c/b$  production in the  $3 < |y_Z| < 4$  rapidity range in pp collisions at the LHC 13.6 TeV can probe  $x \approx 10^{-4}$ . When  $p_T \geq 40$  GeV, it can probe  $x \geq 0.3$



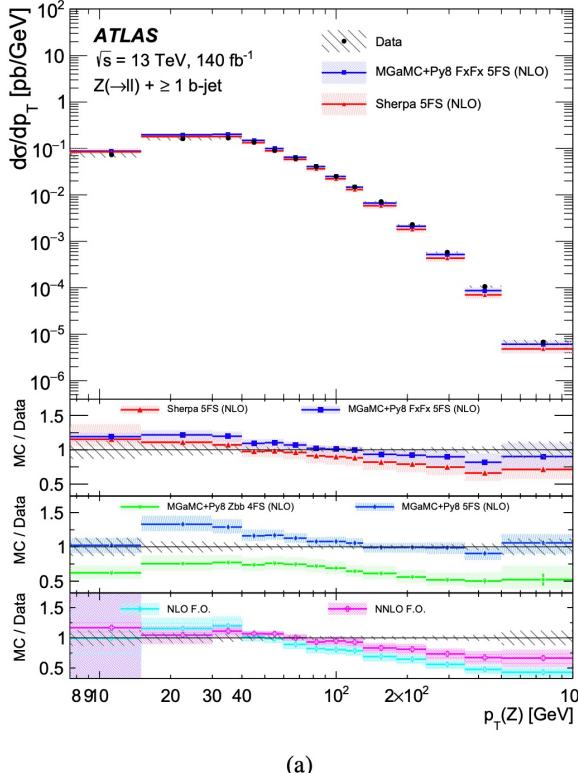
The CT18 gluon PDF *Phys.Rev.D* 103 (2021).  
Small- and large- $x$  regions have wide uncertainty bands.  
(See also: The PDF4LHC21 combination of global PDF fits  
for the LHC Run III, 2203.05506 [hep-ph].)

# Motivations

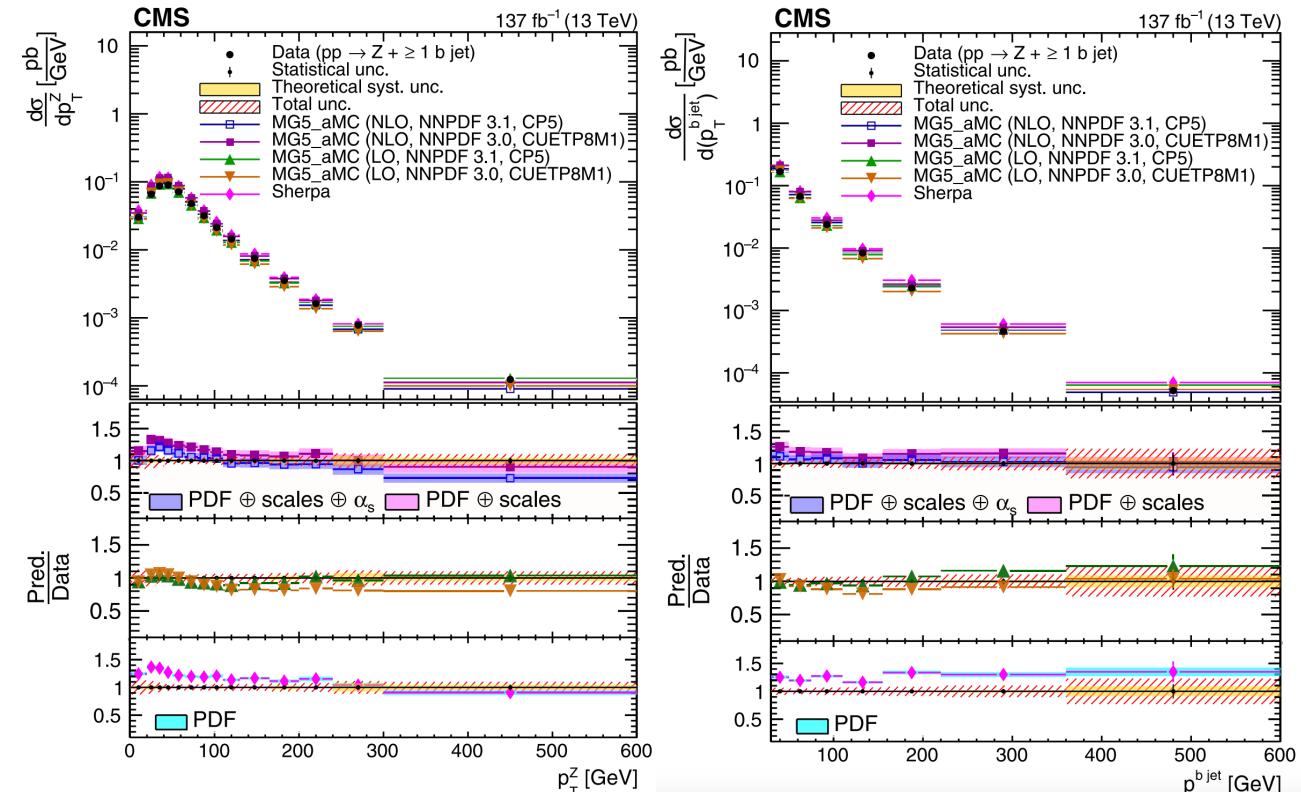
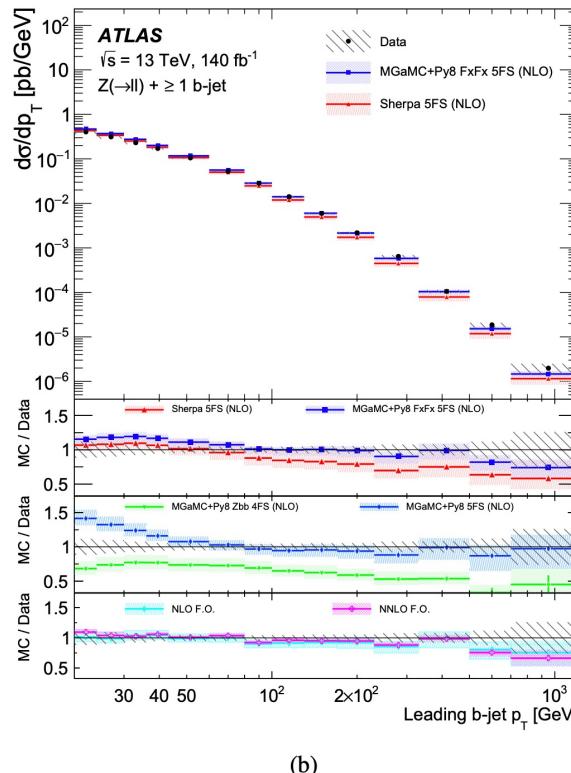
- Modern PDF analyses: extend on wide range of collision energies. Sensitive to mass effects, e.g., phase space suppression, large radiative corrections to collinear  $Q\bar{Q}$  production. Magnitude comparable to NNLO and N3LO corrections.
- Natural to evaluate all fitted cross sections in a GMVFN scheme, which assumes that the number of (nearly) massless quark flavors varies with energy, and at the same time includes dependence on heavy-quark masses in relevant kinematical regions.
- $Z+b$  and  $Z+b\bar{b}$  dominant background for Higgs boson production in association with a  $Z$  boson ( $ZH, H \rightarrow b\bar{b}$ ) in the SM, and in BSM scenarios: SUSY Higgs bosons + b-quark, and new generations of heavy quarks decaying to a  $Z$  boson and a b quark.
- Probing this regime (and beyond, at future facilities) helps us shed light on the **(intrinsic) heavy-flavor content** of the proton, and on **small-x dynamics**.

# Large inflow of new measurements @LHC

Precise measurements  $Z + c/b$ -jets available from the ATLAS, CMS and LHCb collaborations at the LHC

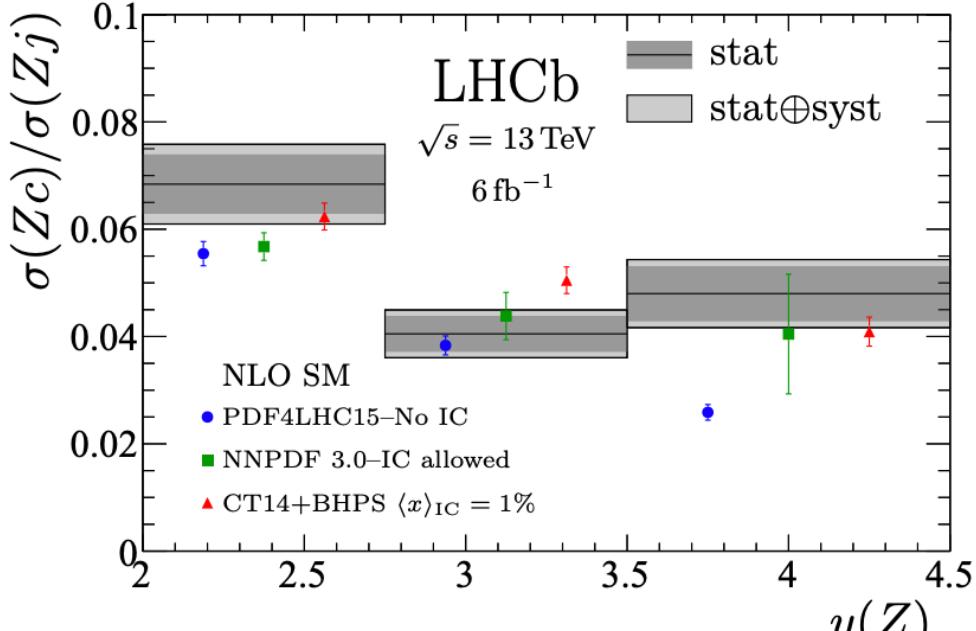


ATLAS13 TeV, Z+b-jet,  $140 \text{ fb}^{-1}$  2403.15093



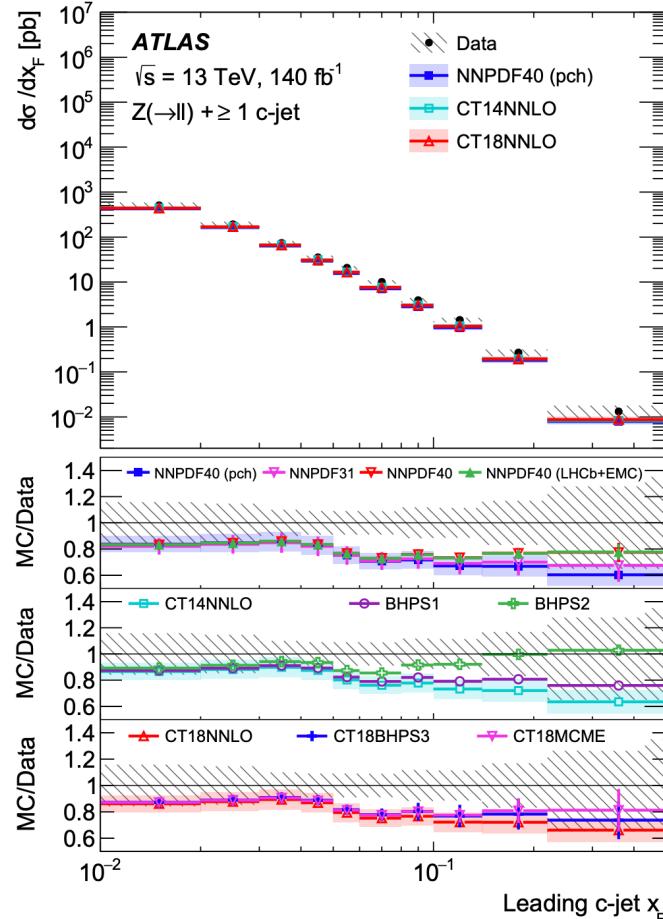
CMS13 TeV, Z+b-jet,  $137 \text{ fb}^{-1}$  2112.09659 PRD 105 (2022)

# Probing HF content of the proton



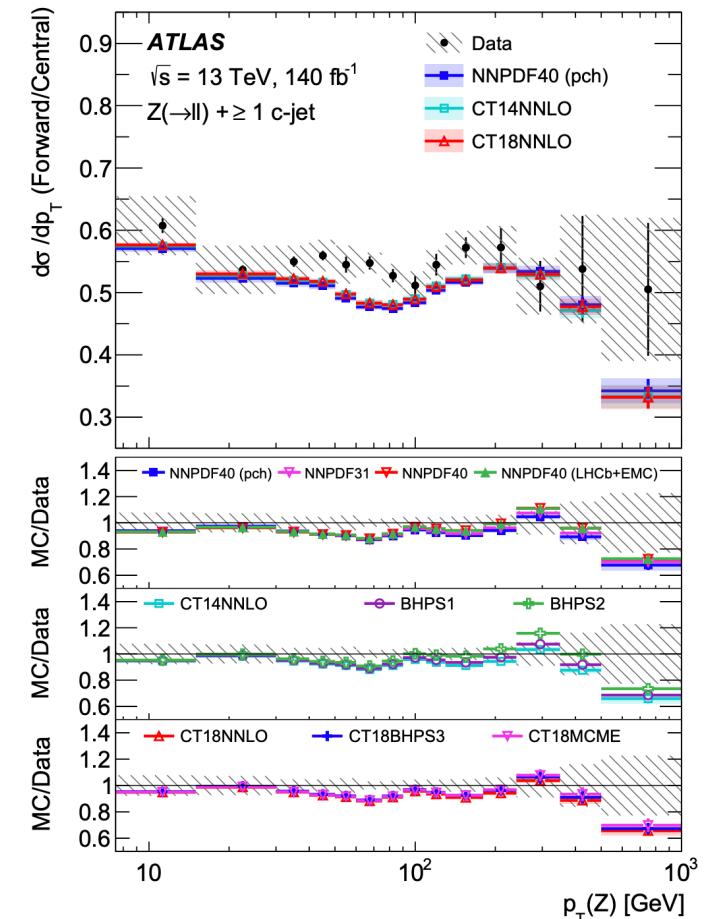
LHCb 13 TeV, arXiv:2109.08084, PRL128 (2022)

Played/will play an important role in recent analyses of IC  
 -NNPDF coll., Nature 2022,  
 -CT18 PLB 2023, 2211.01387  
 -NNPDF coll., 2311.00743



(a)

ATLAS13 TeV, Z+c-jet,  $140 \text{ fb}^{-1}$  arXiv:2403.15093



(b)

# GMVN schemes in a nutshell

Heavy-flavor production dynamics is nontrivial due to the interplay of massless and massive schemes which are different ways of organizing the perturbation series

**Massive Schemes:** final-state HQ with  $p_T \leq m_Q \Rightarrow p_T$ -spectrum can be obtained in the **fixed-flavor number (FFN) scheme**.

- No heavy-quark PDF in the proton. Heavy flavors generated as massive final states.  $m_Q$  is an infrared cut-off.
- Power terms  $(p_T^2/m_Q^2)^p$  are correctly accounted for in the perturbative series.

**Massless schemes:**  $p_T \gg m_Q \gg m_P \Rightarrow$  appearance of log terms  $\alpha_s^m \log^n(p_T^2/m_Q^2)$  that spoil the convergence of the fixed-order expansion. Essentially, a **zero mass (ZM) scheme**.

- Heavy quark is considered essentially massless and enters also the running of  $\alpha_s$ .
- Need to resum these logs with DGLAP: initial-state logs resummed into a heavy-quark PDF, final-state logs resummed into a fragmentation function (FF)

**Interpolating (GMVFN) schemes:** composite schemes that retain key mass dependence and efficiently resum collinear logs, so that they combine the FFN and ZM schemes together. They are crucial for:

- a correct treatment of heavy flavors in DIS and PP,
- accurate predictions of key scattering rates at the LHC,
- global analyses to determine proton PDFs.

# Matching GM schemes in Z/H+b

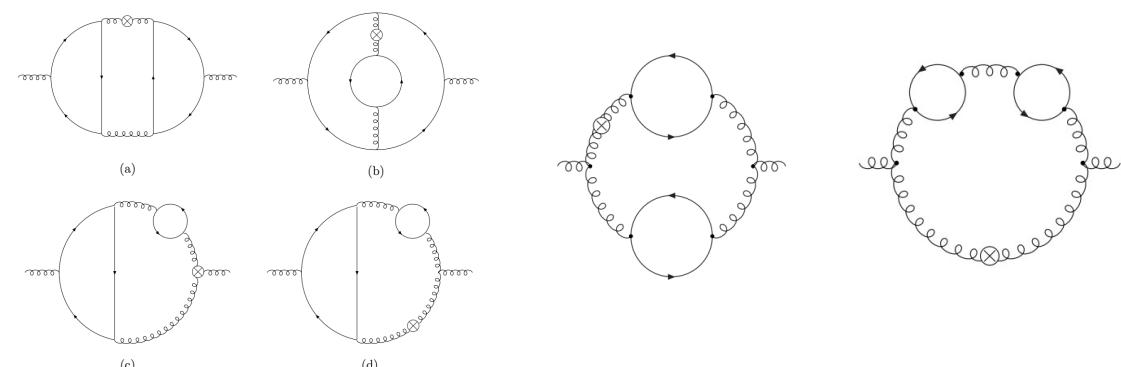
A lot of work has been done in trying to understand the interplay between 4FS and 5FS in **single and double bottom-quark initiated processes** relevant for **Higgs and Z production**.

The list here is of course not exhaustive:

- [Gauld, Gehrmann-De Ridder, Glover, Huss, Majer 2005.03016](#): (FO calculation for Z+ b-jet at  $O(\alpha_s^3)$  in QCD, combines ZM NNLO and FFNS NLO)
- [Forte, Giani, Napoletano EPJC 2019](#): (massive b-scheme)
- [Figueroa, Honeywell, Quackenbush, Reina, Reuschle, Wackerlo, PRD 2018](#): (massive b-scheme, Z + b-jet at  $O(\alpha_s^2 \alpha)$  and  $O(\alpha_s \alpha^2)$  within ACOT and S-ACOT)
- [Forte, Napoletano, Ubiali EPCJ 2018](#): (FONLL method to match 5FS with massless b to 4FS with massive b)
- [Krauss, Napoletano, Schumann, PRD 2017](#): (Z/H + b with SHERPA);
- [Lim, Maltoni, Ridolfi, Ubiali JHEP 2016](#): (b-bbar-initiated processes at the LHC);
- [Bonvini, Papanastasiou, Tackmann, JHEP 2015, JHEP 2016](#): (4 matched calculation b-bar-H);
- [Forte, Napoletano, Ubiali, PLB 2015](#);
- [Maltoni, Ridolfi, Ubiali JHEP 2012](#): (b-initiated processes at the LHC);
- [Campbell, Caola, Cordero, Reina, Wackerlo, PRD 2012](#);
- [Campbell, Ellis, Cordero, Maltoni, Reina, Wackerlo, Willenbrock, PRD 2009](#);
- [Dawson, Jackson, Reina, Wackerlo PRD 2004](#);
- [Maltoni, Sullivan, Willenbrock, PRD 2003](#);
- .....

# Progress on OMEs calculations

- J. Ablinger, A. Behring, J. Blumlein, A. De Freitas, et al., NPB(2024), arXiv:2311.00644.
- J. Ablinger, A. Behring, J. Blumlein, A. De Freitas, et al., JHEP(2022), arXiv:2211.05462.
- J. Ablinger, A. Behring, J. Blumlein, A. De Freitas, et al., NPB(2014), arXiv:1409.1135
- J. Ablinger, J. Blumlein, A. De Freitas, A. Hasselhuhn, et al., NPB(2014), arXiv:1402.0359
- J. Blumlein, J. Ablinger, A. Behring, A. De Freitas, et al., PoS, QCDEV2017(2017), arXiv:1711.07957
- A. Behring, I. Bierenbaum, J. Blumlein, A. De Freitas, et al., EPJC(2014), arXiv:1403.6356.
- J. Ablinger, J. Blumlein, A. De Freitas, A. Hasselhuhn, et al., NPB(2014), arXiv:1405.4259.
- J. Ablinger, A. Behring, J. Blumlein, et al, NPB(2014), arXiv:1406.4654.
- J. Ablinger, J. Blumlein, S. Klein, et al., NPB(2011), arXiv:1008.3347.
- I. Bierenbaum, J. Blumlein, and S. Klein, PLB(2009), arXiv:0901.0669.
- I. Bierenbaum, J. Blumlein, S. Klein, and C. Schneider, NPB(2008), arXiv:0803.0273.
- I. Bierenbaum, J. Blumlein, and S. Klein, NPB(2009), arXiv:0904.3563.



# Main idea behind S-ACOT-MPS (massive phase space)

$$\sigma = \text{FC} + \underbrace{\text{FE} - \text{SB}}_{\text{``Residual FE''}}$$

FC = Flavor creation contributions with full mass dependence

FE = Flavor excitation contribution with approximate mass dependence

(available from public codes)

Mass fully retained in the *PS* in all terms.

Kinematical power corrections under control.

Subtraction well defined at the quark mass threshold

FE and Subtraction  facilitated by introducing residual PDF:

allows us to get (FE-Subtraction ) in one step

$$\delta f_Q(x, \mu^2) = f_Q(x, \mu^2) - \frac{\alpha_s}{2\pi} \log \left( \frac{\mu^2}{m_Q^2} \right) f_Q(x, \mu^2) \otimes P_{Q \leftarrow g}(x)$$

Subtracted and Residual PDFs are provided in the form of LHAPDF grids for phenomenology applications: <https://sacotmps.hepforge.org/downloads?f=PDFs>

at LO

# S-ACOT-MPS Theory framework

The differential cross section for parton a + parton b  $\rightarrow Z + Q + X$  with a, b having zero mass can be written as follows

$$\frac{d\sigma(a b \rightarrow Z, Q, X)}{dQ^2 d\chi} = G_{ab} \left( x_A, x_B, Q; \frac{\mu}{Q}, \frac{m_Q}{\mu}, \alpha_s, N_f, N_f^{fs} \right)$$

The factorization formula can be written as

$$G_{a,b} \left( x_A, x_B, Q; \frac{\mu}{Q}, \frac{m_Q}{\mu}, \alpha_s, N_f, N_f^{fs} \right) = \sum_{c,d=0}^{N_f} \int_{x_A}^1 d\xi_A \int_{x_B}^1 d\xi_B \\ \times f_{c/a}(\xi_A, Q) H_{c,d} \left( \frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q; \frac{\mu}{Q}, \frac{m_Q}{\mu}, \alpha_s, N_f, N_f^{fs} \right) f_{d/b}(\xi_B, Q).$$

Perturbative expansion of terms leads to

$$G_{i,b}(x_A, x_B) = G_{i,b}^{(0)}(x_A, x_B) + a_s G_{i,b}^{(1)}(x_A, x_B) + a_s^2 G_{i,b}^{(2)}(x_A, x_B) + \dots,$$

$$H_{i,a}(\hat{x}_A, \hat{x}_B) = H_{i,a}^{(0)}(\hat{x}_A, \hat{x}_B) + a_s H_{i,a}^{(1)}(\hat{x}_A, \hat{x}_B) + a_s^2 H_{i,a}^{(2)}(\hat{x}_A, \hat{x}_B) + \dots,$$

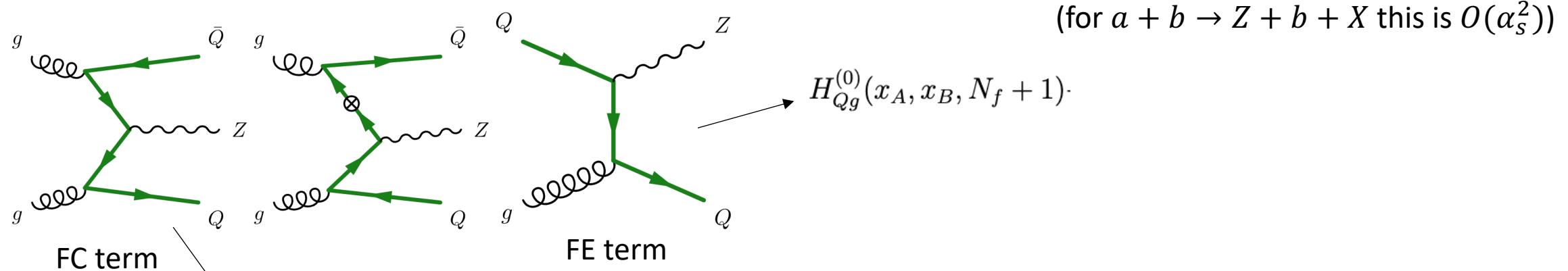
$$f_{a/b}(\xi) = \delta_{ab}\delta(1-\xi) + a_s A_{ab}^{(1)}(\xi) + a_s^2 A_{ab}^{(2)}(\xi) + a_s^3 A_{ab}^{(3)}(\xi) + \dots,$$

$$\hat{x} = x/\xi.$$

$$A_{ab}^{(k)} \quad (k = 0, 1, 2, \dots) \quad \text{OME's}$$

$$A_{hg}^{(1)}(\xi) = 2P_{hg}^{(1)}(\xi) \ln(\mu^2/m_h^2) \quad \text{For } g \rightarrow Q\bar{Q}$$

# S-ACOT-MPS cancellation pattern at the lowest order



$$H_{g,g}^{(1)}(x_A, x_B) = G_{g,g}^{(1)}(x_A, x_B) - A_{Qg}^{(1)}(\xi_A) \otimes H_{Q,g}^{(0)}(\hat{x}_A, x_B) - A_{Qg}^{(1)}(\xi_B) \otimes H_{g,Q}^{(0)}(x_A, \hat{x}_B)$$

$$\sigma_{\text{Sub}}^{(0)} = g(x_A, \mu^2) \tilde{f}_Q^{(1)}(x_B) \hat{\sigma}_{AQ \rightarrow QX}^{(0)} + \{A \leftrightarrow B\}$$

$$\tilde{f}_Q^{(1)}(x, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \log \frac{\mu^2}{m_Q^2} [P_{Qg}^{(1)} \otimes g](x, \mu^2)$$

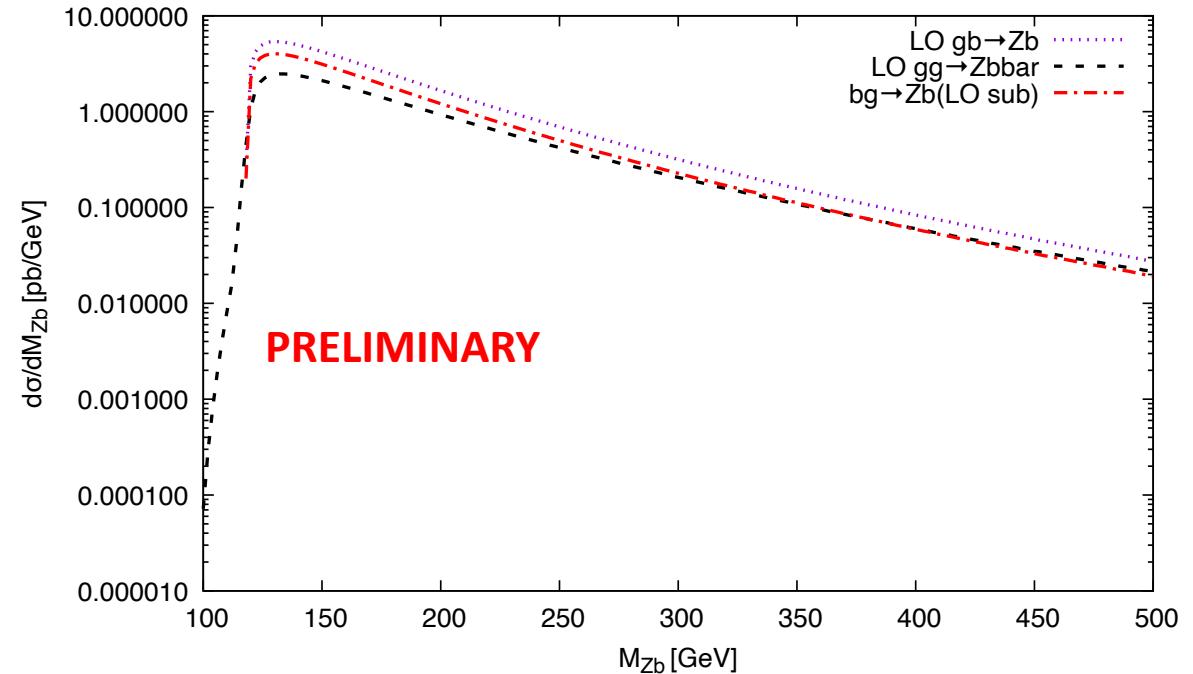
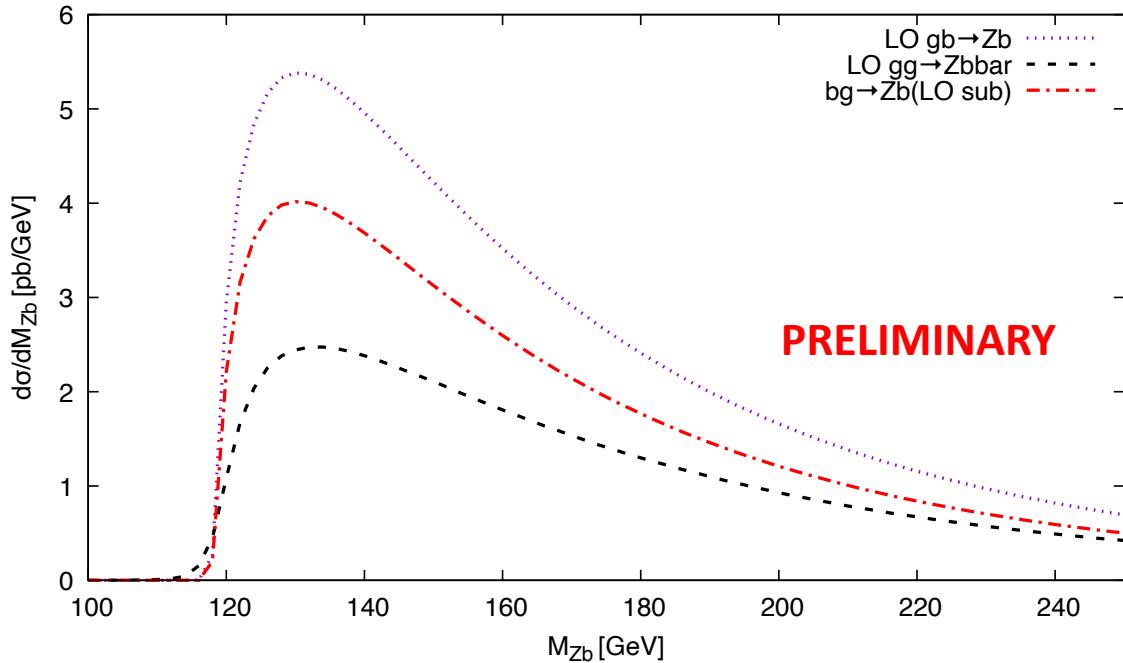
$$\text{Residual PDF } \delta f_Q(x, \mu^2) = f_Q(x, \mu^2) - \tilde{f}_Q(x, \mu^2)$$

Subtracted PDF at LO

Subtracted and Residual PDFs are provided in the form of LHAPDF grids for phenomenology applications:  
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# MZb distribution at the lowest order

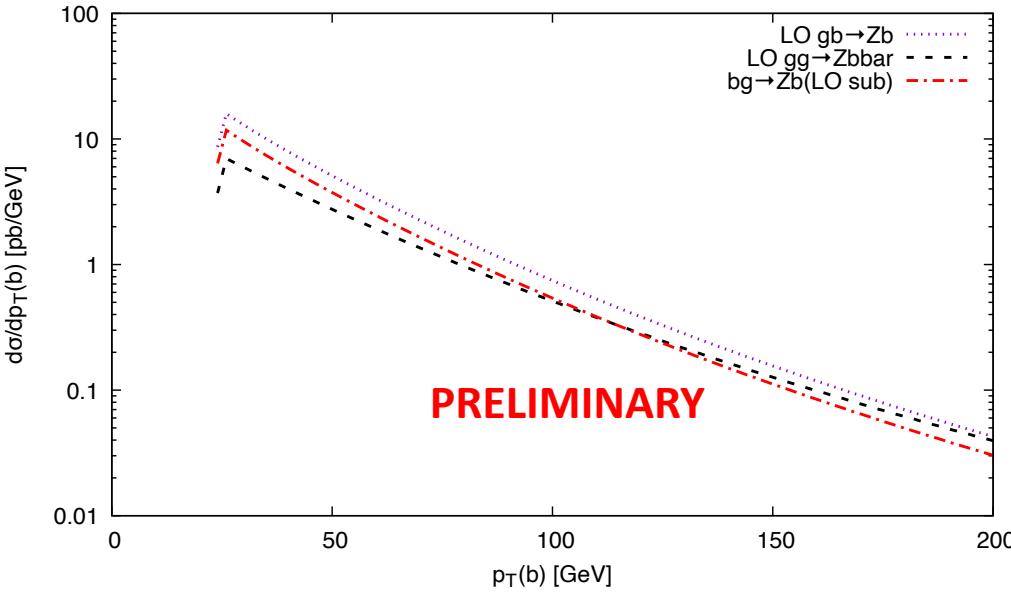
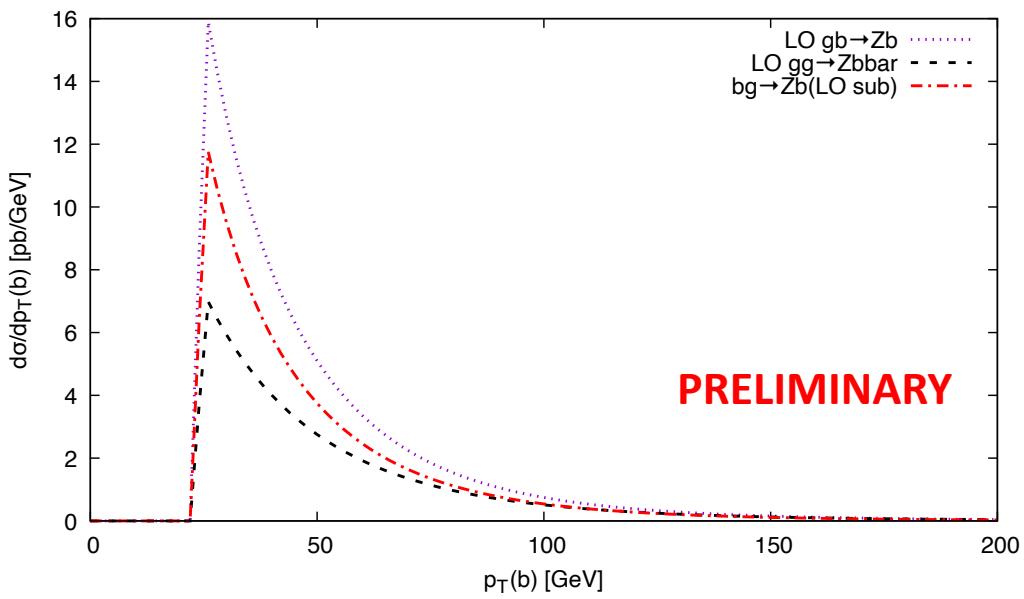
$$\mu = M_Z$$



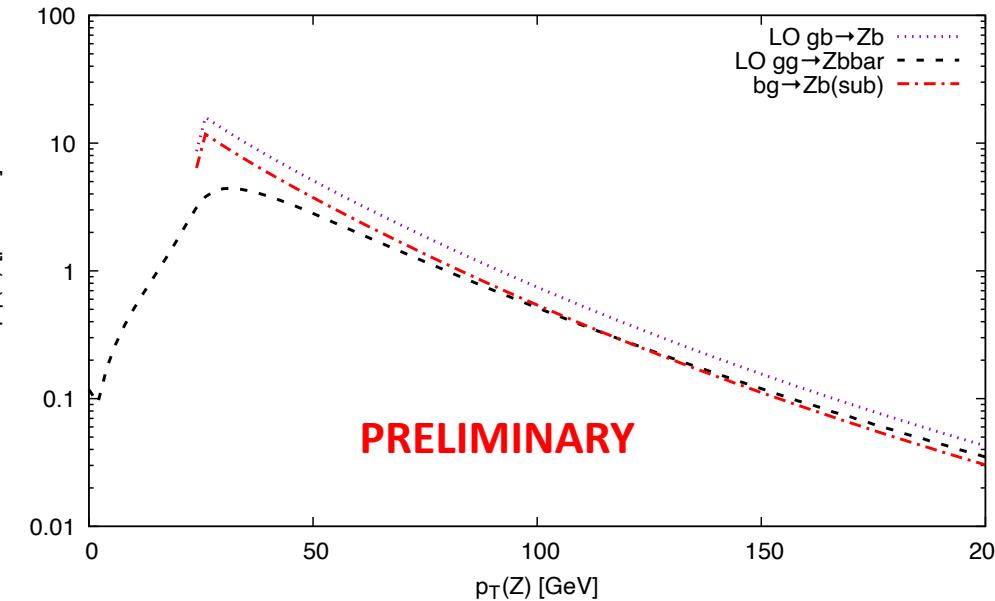
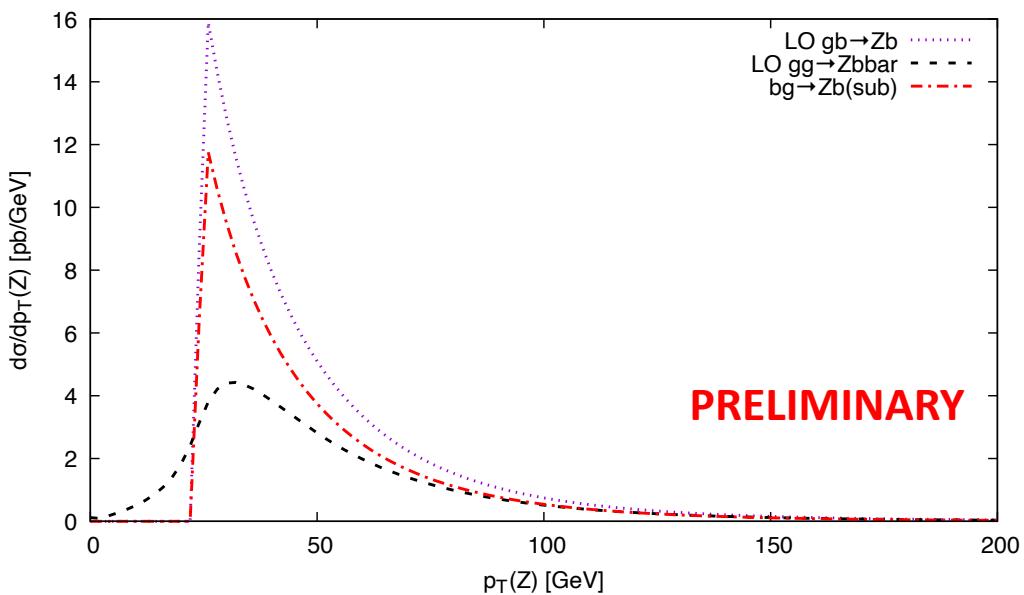
Cancellation between the various terms is clearly visible.

**For the theory calculation:** The combined b-bbar jet can be declared either as a b-jet (an experimental-driven definition) or unflavored jet, such as flavored-kT algorithm (a theoretical infrared-safe definition, adopted in the recent W+c (Czakon, Mitov, et al., 2011.01011) and W+b+bbar (Hartanto Poncelet, et al. 2205.01687), and Z+c (Gauld, Gehrmann-DeRidder et al., 2005.03016) calculations at NNLO in QCD.

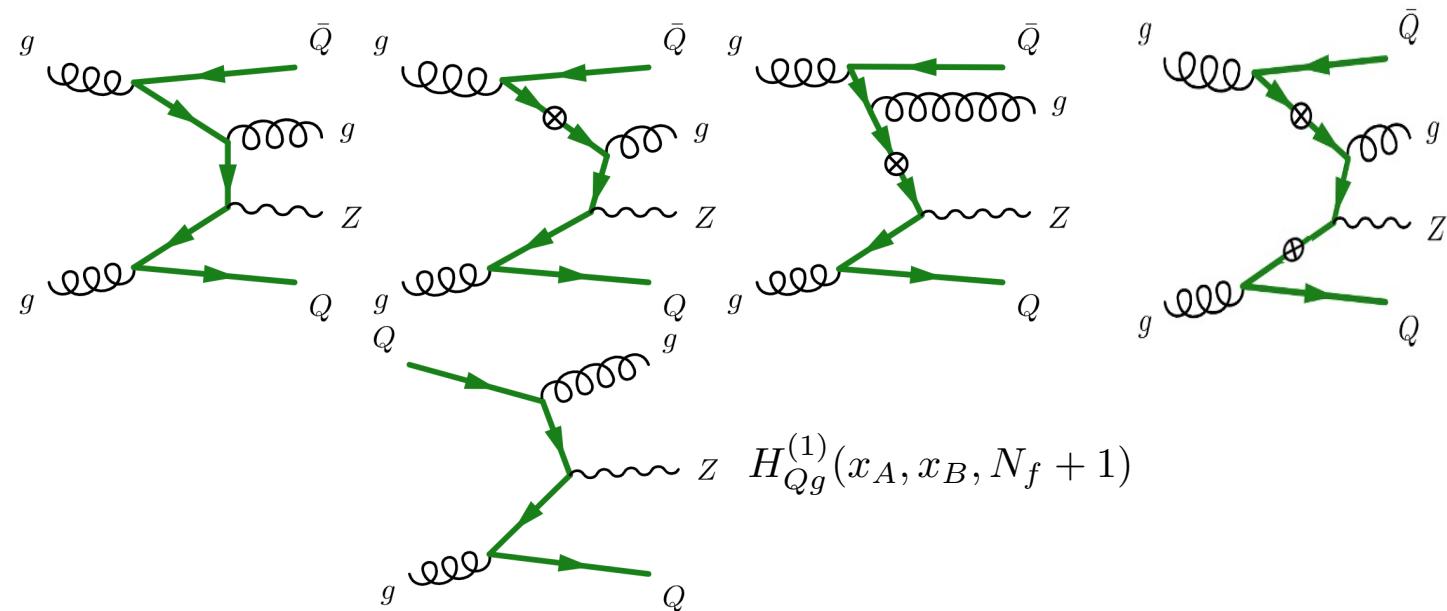
# $p_T$ of b and $p_T$ of Z at lowest order $\mu = M_Z$



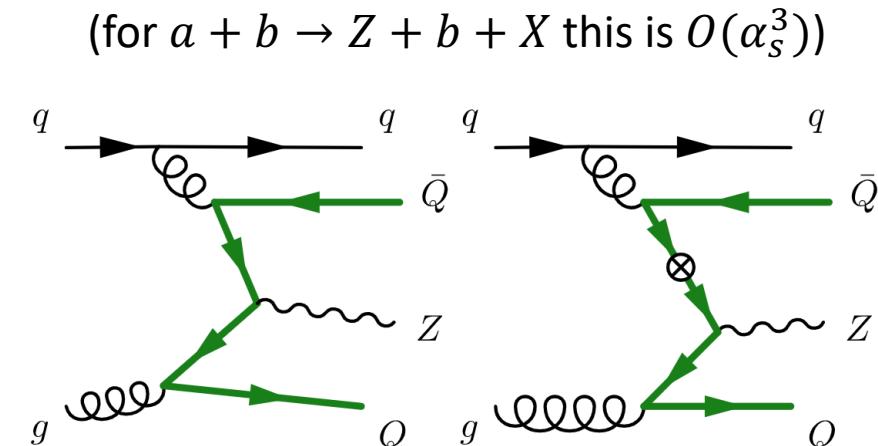
Matching of terms  
affected by PS  
integration and cuts



# S-ACOT-MPS cancellation pattern at NLO



$$\begin{aligned}
 H_{a,b}^{(2)}(x_A, x_B) &= G_{a,b}^{(2)}(x_A, x_B) - \sum_{c=0}^{N_f} A_{ca}^{(1)}(\xi_A) \otimes H_{c,b}^{(1)}(\hat{x}_A, x_B) - \sum_{d=0}^{N_f} A_{db}^{(1)}(\xi_B) \otimes H_{a,d}^{(1)}(x_A, \hat{x}_B) \\
 &\quad - \sum_{c=0}^{N_f} A_{ca}^{(2)}(\xi_A) \otimes H_{c,b}^{(0)}(\hat{x}_A, x_B) - \sum_{d=0}^{N_f} A_{db}^{(2)}(\xi_B) \otimes H_{a,d}^{(0)}(x_A, \hat{x}_B) \\
 &\quad - \sum_{c,d=0}^{N_f} A_{ca}^{(1)}(\xi_A) \otimes H_{c,d}^{(0)}(\hat{x}_A, \hat{x}_B) \otimes A_{db}^{(1)}(\xi_B),
 \end{aligned}$$



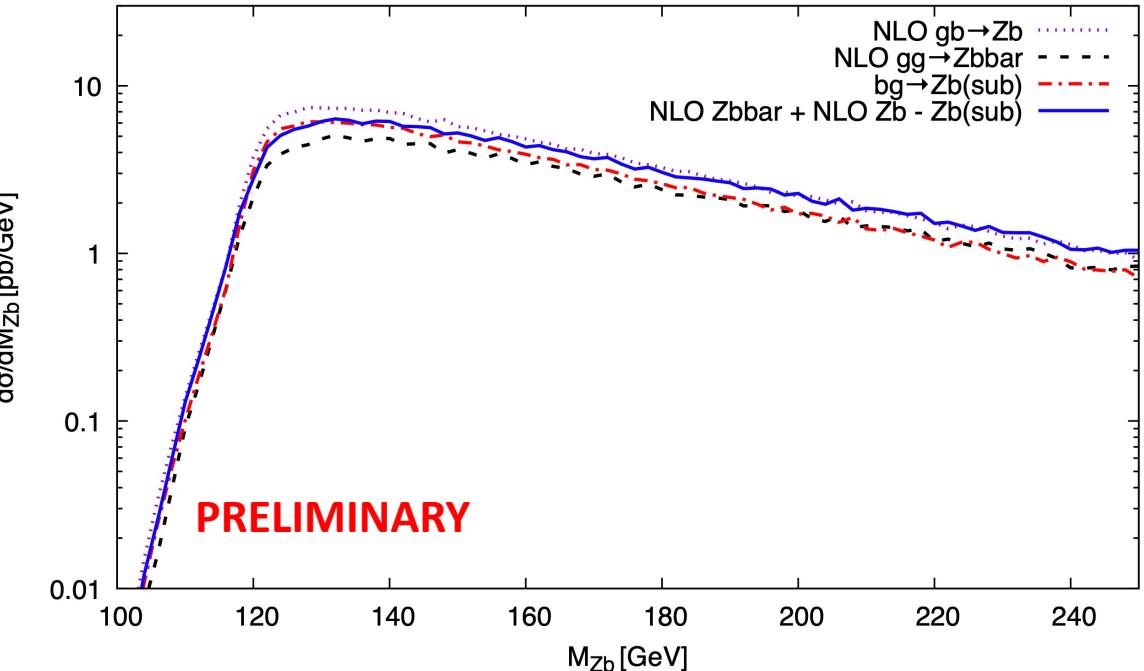
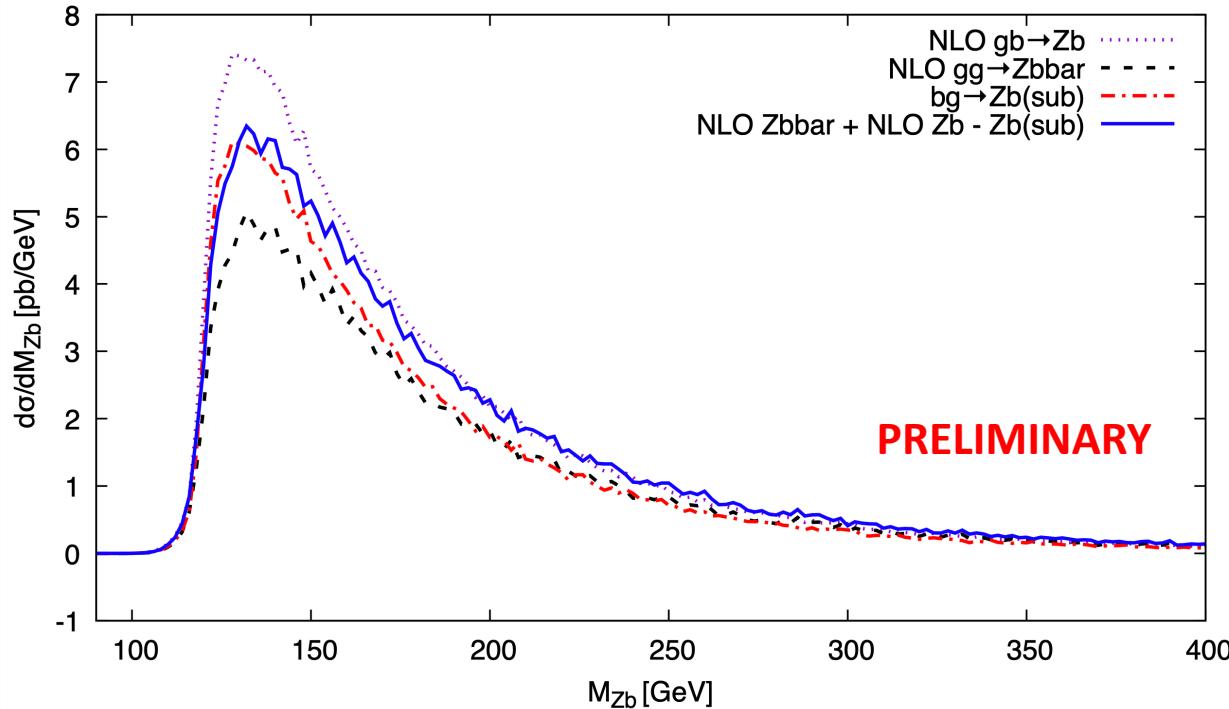
Subtracted PDF at NLO

$$\tilde{f}_Q^{(2)}(x, \mu^2) = \left( \frac{\alpha_s(\mu^2)}{2\pi} \right) \log \left( \frac{\mu^2}{m_Q^2} \right)^2 \left[ P_{Qg}^{(2)} \otimes g + P_{Qq}^{(2)} \otimes q + P_{Q\bar{q}}^{(2)} \otimes \bar{q} \right]$$

# MZb distribution at NLO

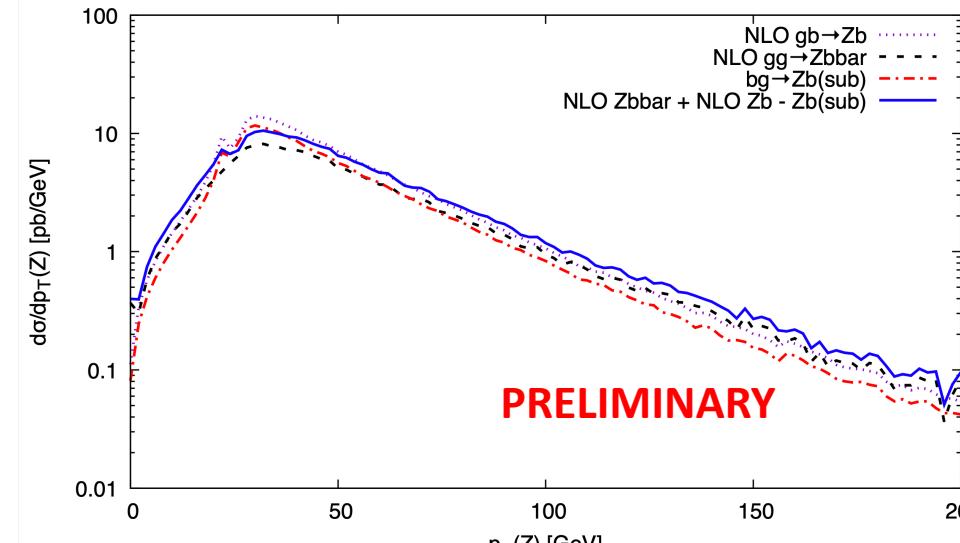
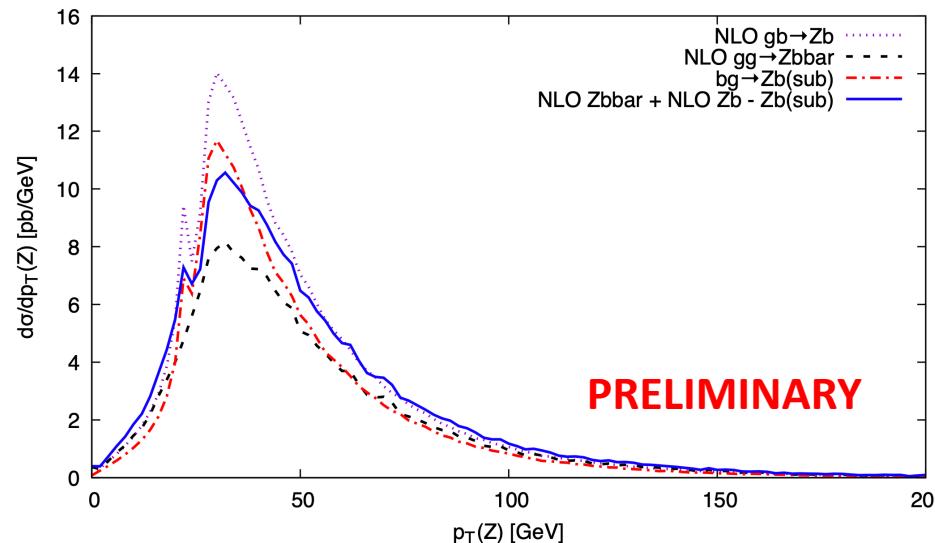
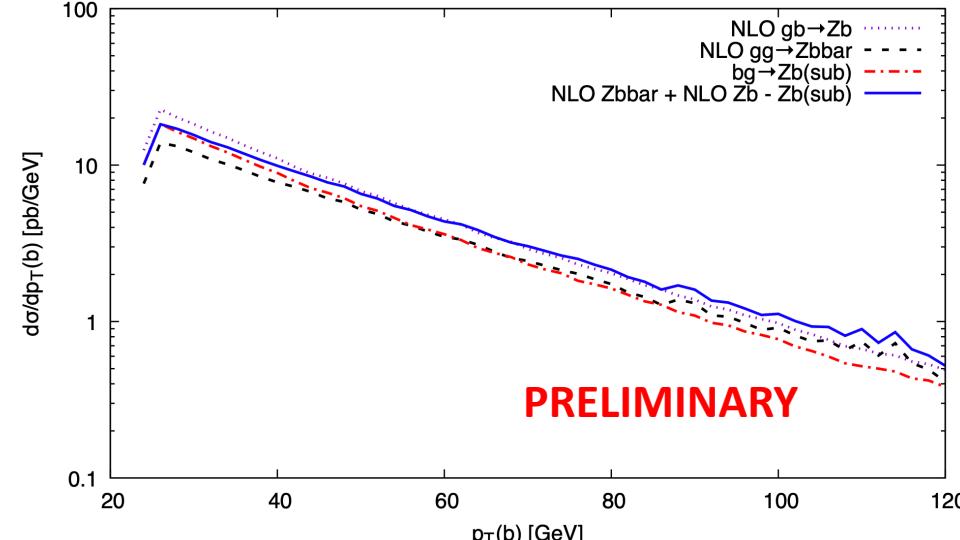
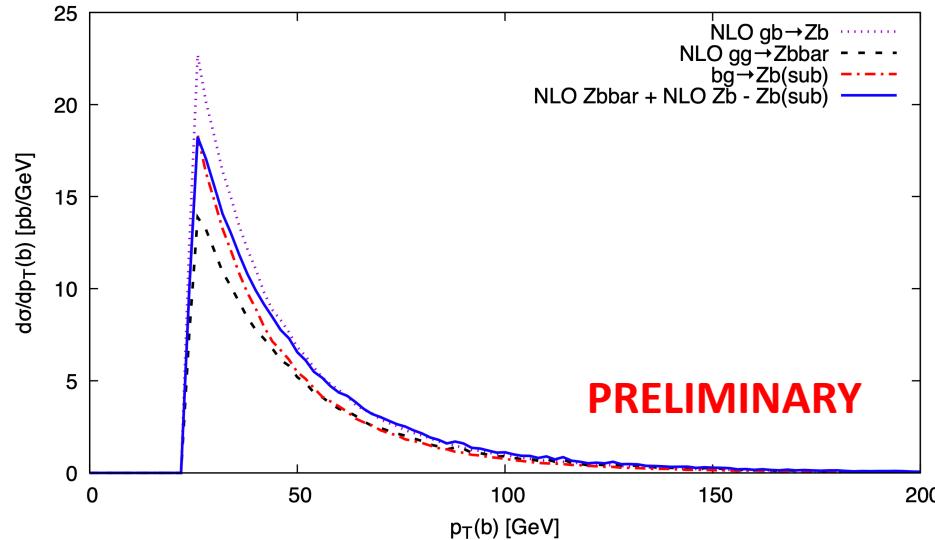
$$\mu = M_Z$$

Theory predictions for Z+at least one b jet obtained with an in-house code +  
NLOX for virtual (Figeroa, et al. CPC(2022) arXiv:2101.01305; Honeywell, et al. CPC(2020) arXiv: 1812.11925 )



# $p_T$ of b and $p_T$ of Z at NLO      $\mu = M_Z$

Theory predictions for Z+at least one b jet obtained with an in-house code +  
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Again, matching of terms affected by PS integration and cuts

MC errors will further be reduced

# Concluding remarks

- We applied the S-ACOT-MPS at NLO to Z+Q production in pp collisions at the LHC
- S-ACOT-MPS developed at NLO: used to describe Z+Q production differentially
- Technically possible to generate predictions within the S-ACOT-MPS scheme at NNLO with K-factors (NNLO/NLO) at hand.
- Direct access to c/b-PDF: Important to constrain heavy-flavor PDFs.
- Residual PDFs are provided in the form of LHAPDF grids to allow users for multiple pheno applications
- Work toward simplifying implementation of GMVFN schemes in (N)NLO QCD calculations using the formalism of subtracted PDFs