DIS2024

XXXI International Workshop on Deep Inelastic Scattering
8-12 April 2024, Grenoble, France

Full next-to-leading order QED contribution to lepton-hadron scattering in joint QED and QCD factorization approach

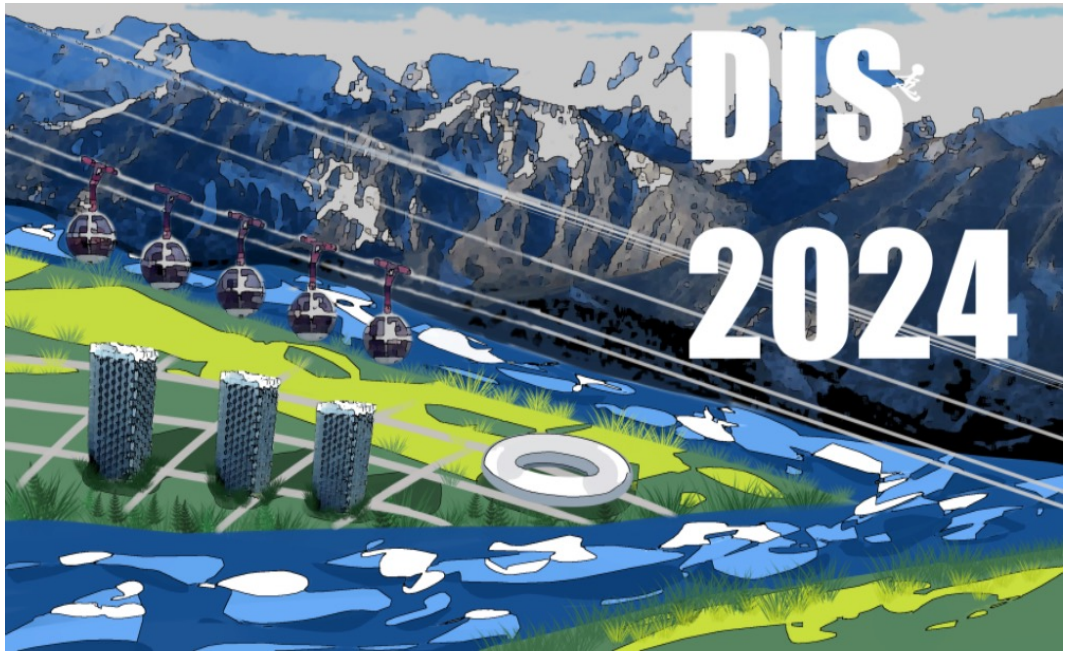
☐ High-energy lepton-hadron collision induces both QED and QCD radiation

☐ Treat QED and QCD radiation equally in terms of factorization approach

☐ Full NLO QED contribution to inclusive DIS

☐ Q^2 is NOT an ideal hard scale at small x_B and/or beyond LO in QED

☐ Summary and Outlook



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Jefferson Lab

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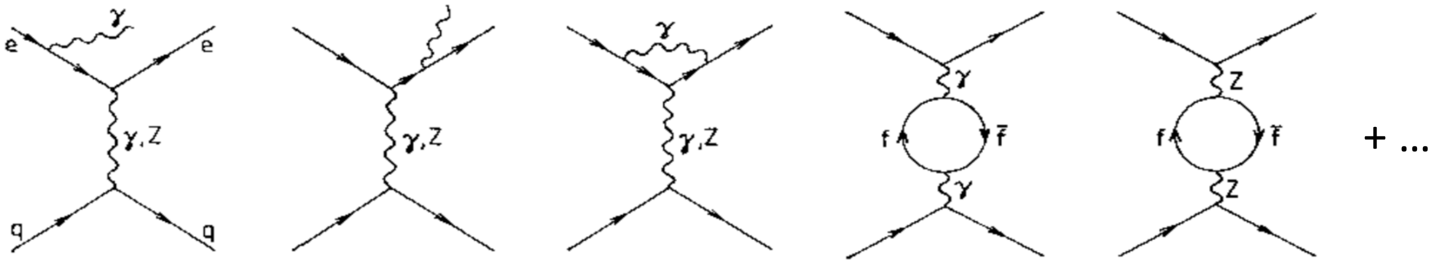
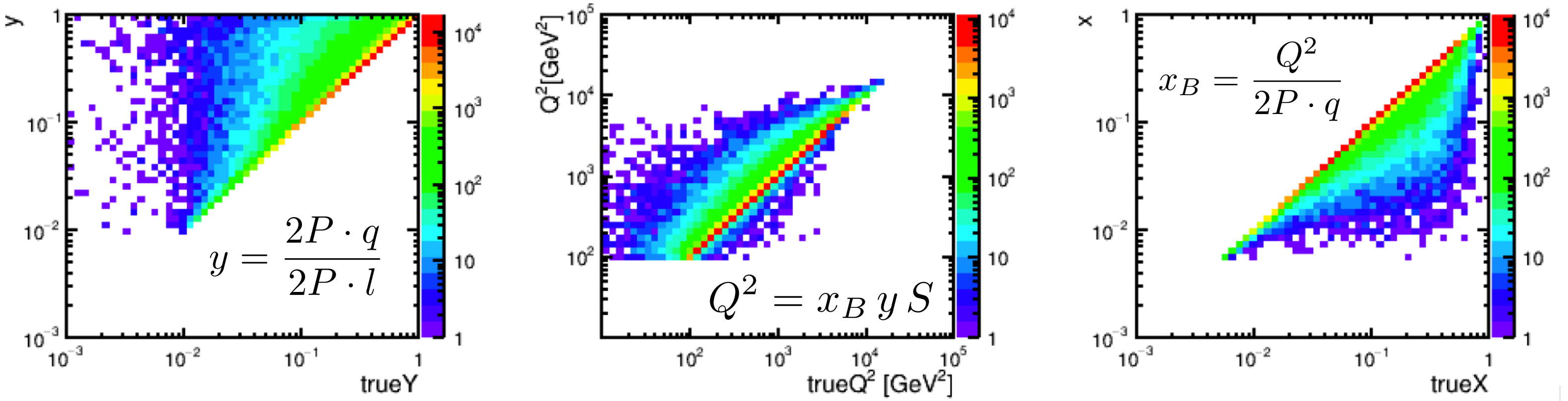
Collision induced QED radiation in high-energy lepton-hadron scattering

- ❑ **“Probe” for the hadron @ EIC is smeared by the induced QED radiation:**

Data sample : Int L = 10 fb^{-1}, Kinematics settings: 0.01<y<0.95, 10^2 GeV^2 <Q^2<10^5 GeV^2



initial **final** **vacuum** **loops**



Instead of a straight line – linear correlation,

the kinematic variables, y, Q^2 , x_B, from the leptons are smeared so much that the proton is not probed by the hard scale that we thought of !!!

Ill-defined “photon-hadron” frame?!

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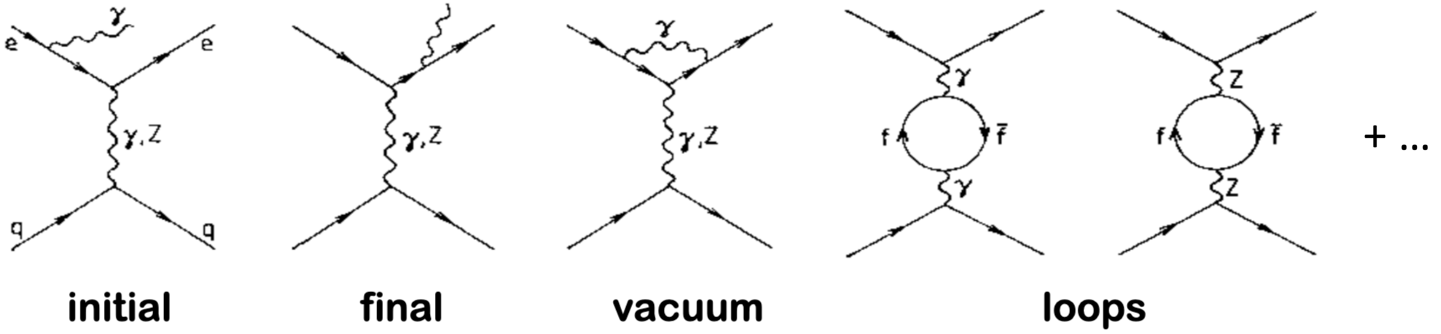

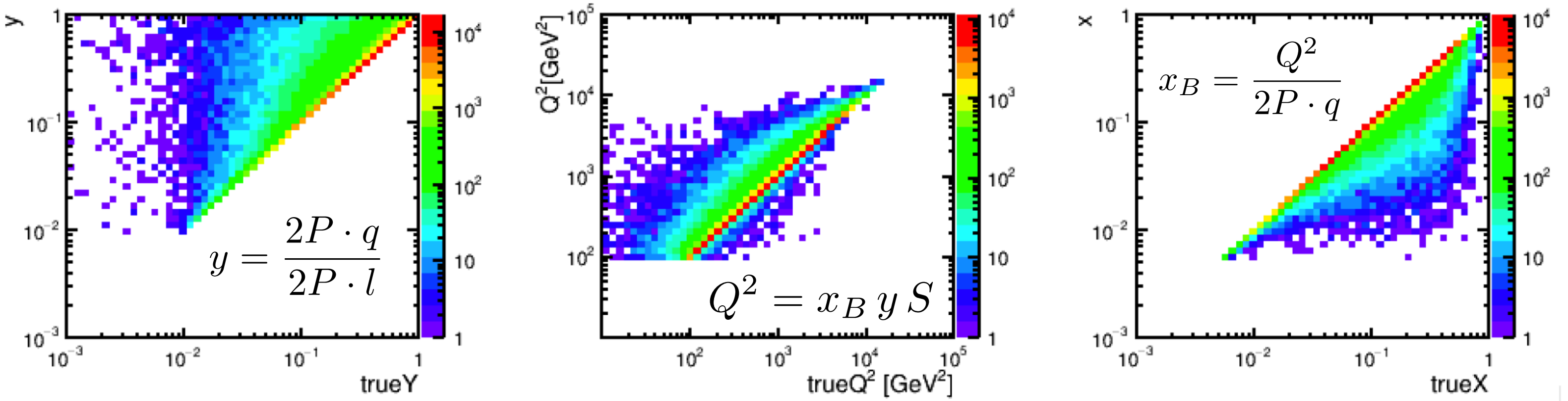

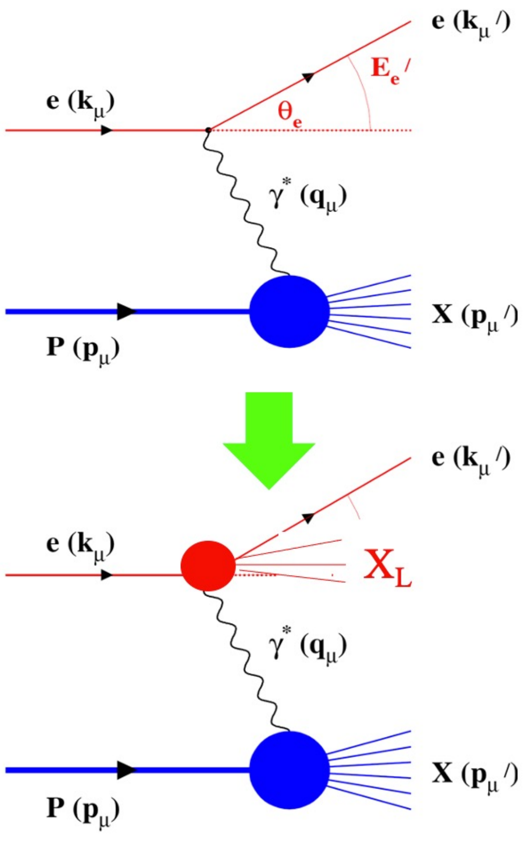

$$q_\mu \;\rightarrow\; \hat{q}_\mu$$

$$Q^2 = -q^2 \;\rightarrow\; \widehat{Q}^2 = -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \;\rightarrow\; \hat{x}_B = \frac{\widehat{Q}^2}{2P\cdot \hat{q}}$$

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Treat QED and QCD radiation equally in terms of factorization approach

❑ Three complementary processes for high-energy lepton-hadron scattering:

Inclusive single high-pT lepton:	$e(\ell) + H(P) \to e(\ell') + X$	Inclusive DIS
Inclusive single high-pT hadron (or jet):	$e(\ell) + H(P) \to h(p)(\mathrm{or\,jet(p)}) + X$	"Photoproduction"
Inclusive high-pT lepton + hadron:	$e(\ell) + H(P) \to e(\ell') + h(p) + X$	Semi-Inclusive DIS

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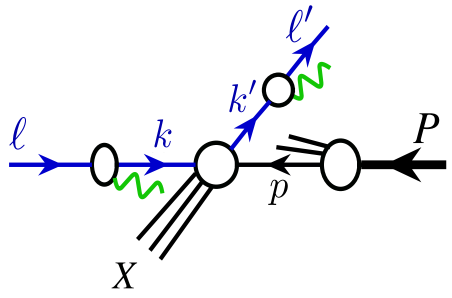
❑ Corresponding factorization formalisms (beyond one-photon exchange approximation):



LFFs **LDFs**

$$E'\frac{d\sigma_{\ell P\to\ell' X}}{d^3\ell'} \approx \frac{1}{2s}\sum_{ija}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$

PDFs

$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi\ell,xP,\ell'/\zeta,\mu^2) + (1/\ell'_T)^{\alpha}$$

IRS hard coefs

- **No DIS “Structure Functions”!**
- **QED & QCD contribution are factorized at the same scale: μ**

$(x_B,Q^2) \;\leftrightarrow\; (y_\ell,\ell'_T)$

$i.j = e,\gamma,\bar{e},...,q,g,...$

$a = q,g,\bar{q},e,\gamma,\bar{e},...$

$$\widehat{H}_{ia\to jX}(\xi\ell,xP,\ell'/\zeta,\mu^2) \approx \widehat{H}^{(m,n)}_{ia\to jX}(\xi\ell,xP,\ell'/\zeta,\mu^2)$$
$$\approx \mathcal{O}(\alpha^m\alpha_s^n)$$

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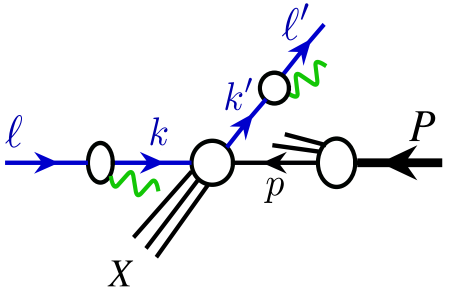

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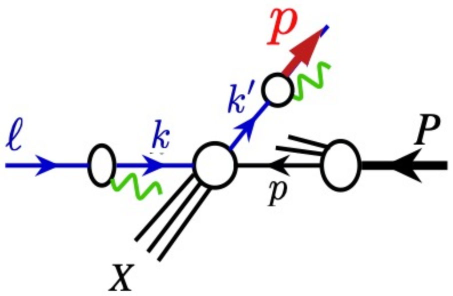

$$E\frac{d\sigma_{\ell P\to pX}}{d^3p} \approx \frac{1}{2s}\sum_{i,a,b}\int_{z_{\min}}^{1}\frac{dz}{z^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\, D_{h/b}(z,\mu^2)\, \boxed{f_{i/e}(\xi,\mu^2)}$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\, f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to bX}(\xi\ell, xP, p/z, \mu^2) + (1/p_T)^{\alpha}$$

Qiu, Watanabe, in preparation

Kang, Meta, Qiu, Zhou, PRD 2011
Hinderer, Schlegel, Vogelsang, PRD 2015, 2016
Abelof, Boughezal, Liu, Petriello, PLB, 2016
Qiu, Wang, Xing, CPL, 2021

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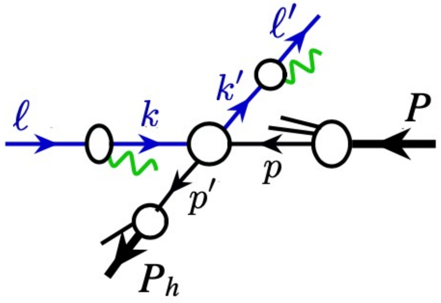

TMD regime: lepton and hadron almost back-to-back

Hybrid Factorization: Collinear factorization for QED + TMD factorization for QCD

$$E_{\ell'} E_{P_h} \frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{\mathrm{d}^3\ell'\,\mathrm{d}^3P_h} \approx \sum_{ij\lambda_k} \int_{\zeta_{\min}}^{1} \frac{\mathrm{d}\zeta}{\zeta^2} D_{e/j}(\zeta) \int_{\xi_{\min}}^{1} \mathrm{d}\xi\, f_{i(\lambda_k)/e(\lambda_\ell)}(\xi) \times \left[E_{k'} E_{P_h} \frac{\mathrm{d}^6\hat{\sigma}_{k(\lambda_k)P(S)\to k' P_h X}}{\mathrm{d}^3k'\,\mathrm{d}^3P_h} \right]_{k=\xi\ell,\, k'=\ell'/\zeta} + \mathcal{O}\left(\frac{m_e^n}{Q^n}\right)$$

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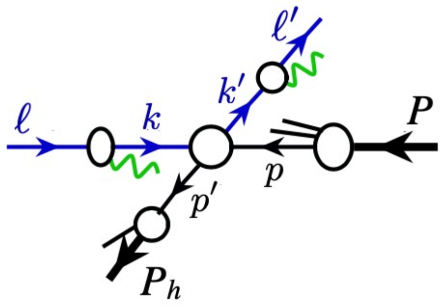

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$$\times\left[E_{k'}E_{P_h}\frac{\mathrm{d}^6\hat\sigma_{k(\lambda_k)P(S)\to k' P_h X}}{\mathrm{d}^3k'\,\mathrm{d}^3P_h}\right]_{k=\xi\ell,k'=\ell'/\zeta}+\mathcal{O}(\frac{m_e^n}{Q^n})$$

- **One-photon approximation:** $i=j=e$

$$\frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell' P_h X}}{\mathrm{d}x_B\mathrm{d}y\,\mathrm{d}\psi\,\mathrm{d}z_h\,\mathrm{d}\phi_h\mathrm{d}P_{hT}^2}=\sum_{ij\lambda_k}\int_{\zeta_{\min}}^{1}\frac{\mathrm{d}\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{\mathrm{d}\xi}{\xi}f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\,D_{e/j}(\zeta)$$
$$\times\overbrace{\frac{\hat x_B}{x_B\xi\zeta}\left[\frac{\alpha^2}{\hat x_B\,\hat y\,\widehat{Q}^2}\frac{\hat y^2}{2(1-\hat\varepsilon)}\left(1+\frac{\hat\gamma^2}{2\hat x_B}\right)\sum_n\hat w_n F_n^h(\hat x_B,\widehat{Q}^2,\hat z_h,\widehat{P}_{hT}^2)\right]}$$

Evaluated in a "virtual photon-hadron" frame

$L(\xi,\zeta):\{\hat q,P,\widehat{P}_h\}\ \Rightarrow\ \{q,P,P_h\}$

Liu, Melnitchouk, Qiu, Sato,
Phys.Rev.D 104 (2021) 094033; JHEP 11 (2021) 157

Inclusive lepton-hadron deep inelastic scattering (DIS)

❑ Recover the concept of structure functions – "one-photon" approximation:

Liu, Melnitchouk, Qiu, Sato,
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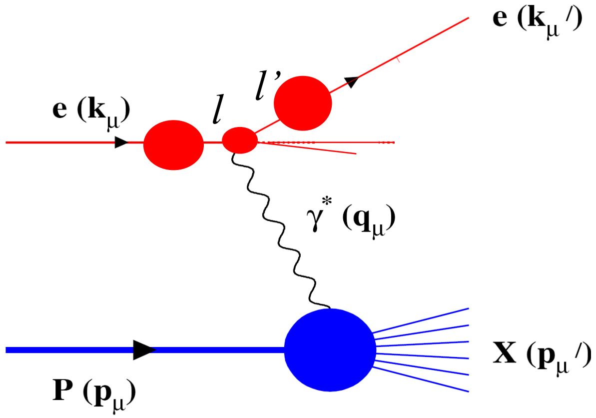
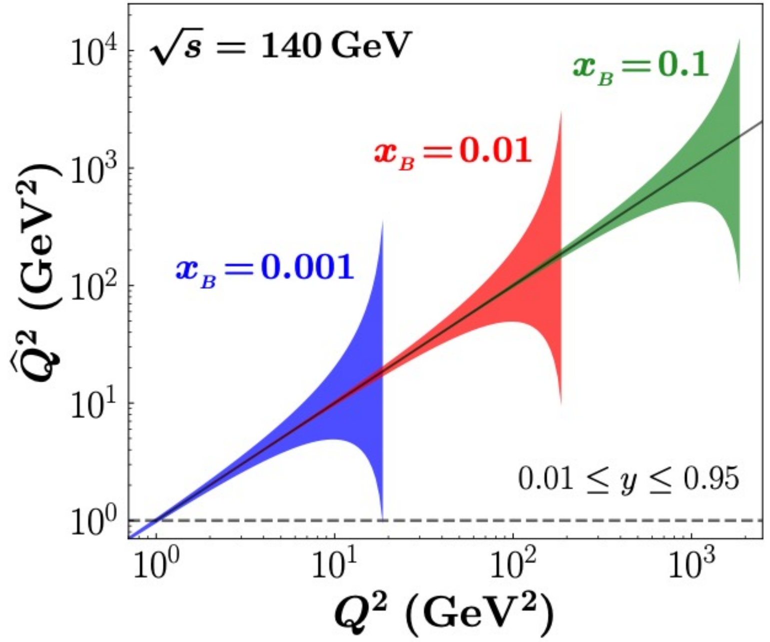


$$i = e, \quad j = e$$

$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B dy} \approx \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^{1} d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\,\frac{\hat{x}_B}{\widehat{Q}^2}\right]$$
$$\times \frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2 F_1(\hat{x}_B,\widehat{Q}^2) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

- QED radiation prevents a well-defined "photon-hadron" frame
- Radiation is CO sensitive as $m_e/Q \to 0$, factorized into LDFs & LFFs
- Hadron is probed by $(x_B, Q^2) \to (\hat{x}_B, \widehat{Q}^2)$

$x_B \to \hat{x}_B \in [x_B, 1]$ $\qquad \widehat{Q}^2_{\min} = Q^2\,\frac{(1-y)}{(1-x_B\,y)}$ $\qquad \widehat{Q}^2_{\max} = Q^2\,\frac{1}{(1-y+x_B\,y)}$



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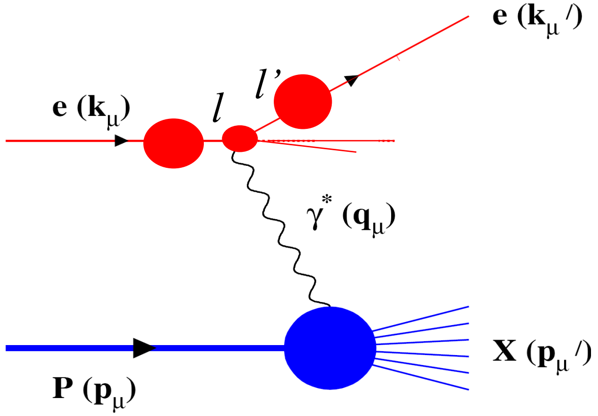

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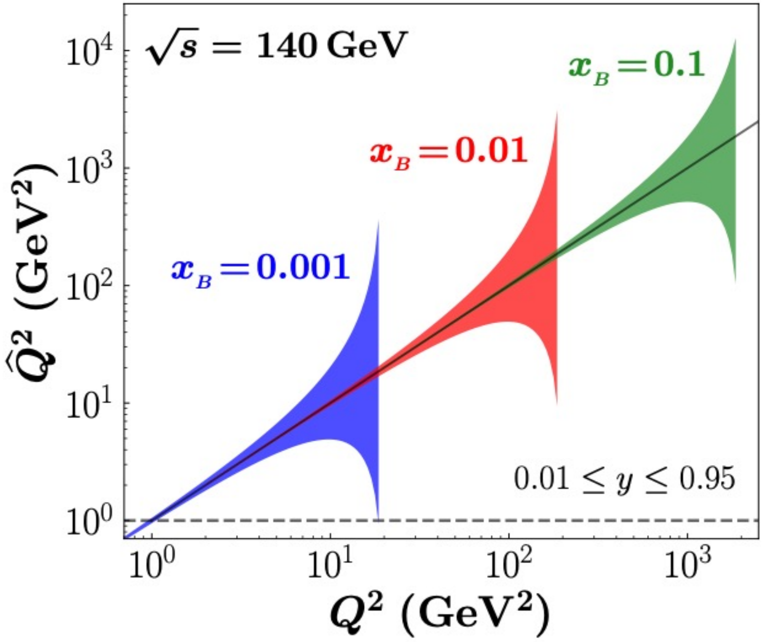

❑ Traditional radiative correction (RC) – Born kinematics:

$$\sigma_{\text{Measured}} \equiv \text{RC} \otimes \sigma_{\text{No QED Radiation}}$$

Necessary requirement: **RC should not depend on the hadronic physics!**

BUT, a simple RC factor at x_B is necessarily sensitive to hadronic information from [x_B, 1] !

Inclusive lepton-hadron deep inelastic scattering (DIS)

☐ Without the “one-photon” approximation:

~ Inclusive single lepton production at high transverse momentum

Liu, Melnitchouk, Qiu, Sato,
Phys.Rev.D 104 (2021) 094033
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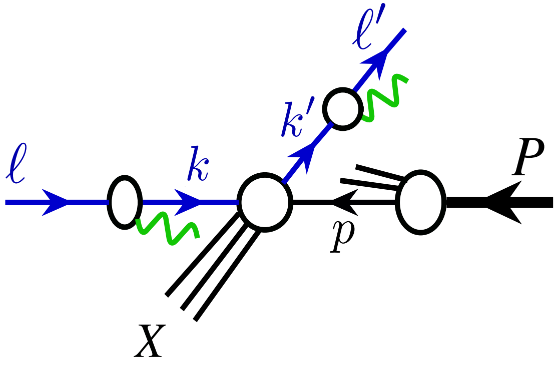
Cammarota, Qiu, Zhang
In preparation

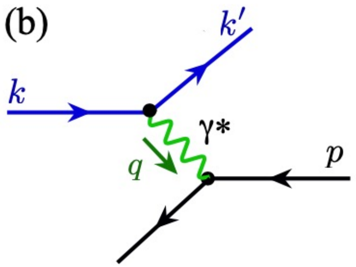


$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2)\;+\;\cdots$$

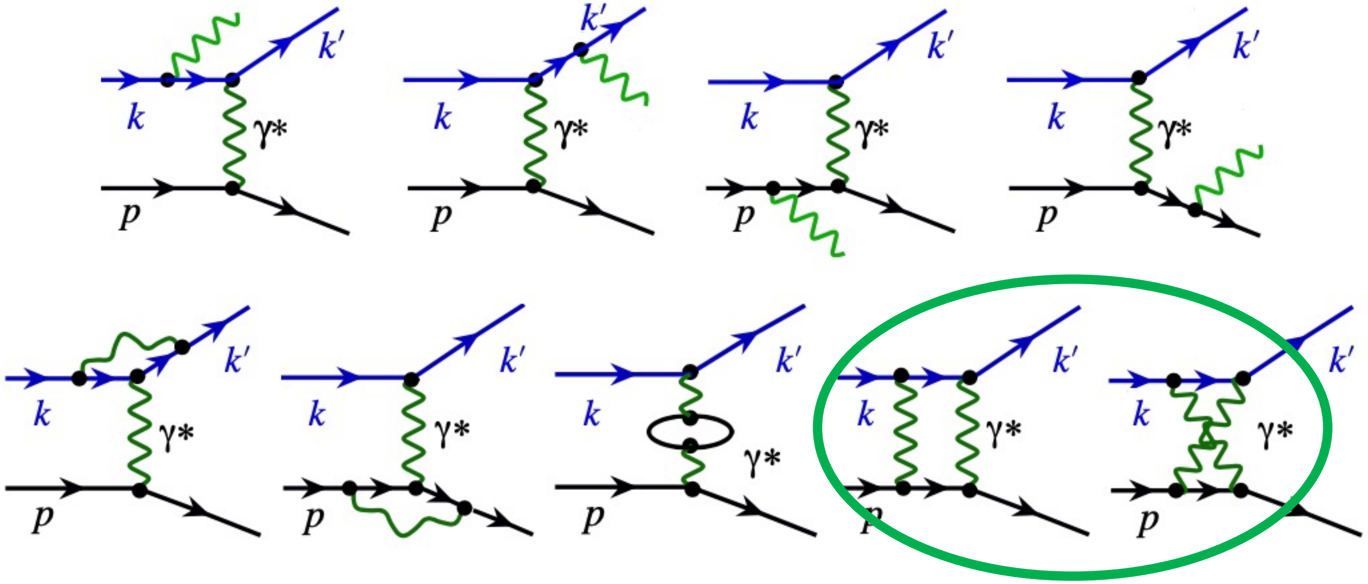
No structure functions, but have PDFs, LDFs, LFFs, …

☐ Calculated hard parts in power of $\alpha^m\alpha_s^n$:

LO



NLO:



More systematic for PVDIS!

Beyond one-photon exchange

Beyond 1-vector boson exchange: NLO QED contribution

- **Project the external particles to leptons or partons:**

Cammarota, Qiu, Zhang
In preparation

NLO: $e(\ell) \to e(k)\,, \quad e(\ell') \to e(k')\,, \quad h(P) \to q(p)\,.$

$$\begin{aligned}
\sigma^{(1)}_{e(k)+q(p)\to e(k')+X} = {} & D^{(0)}_{e/e} \otimes f^{(0)}_{e/e} \otimes f^{(0)}_{q/q} \otimes \hat{H}^{(1)}_{e+q\to e+X} \\
& + D^{(1)}_{e/e} \otimes f^{(0)}_{e/e} \otimes f^{(0)}_{q/q} \otimes \hat{H}^{(0)}_{e+q\to e+X} \\
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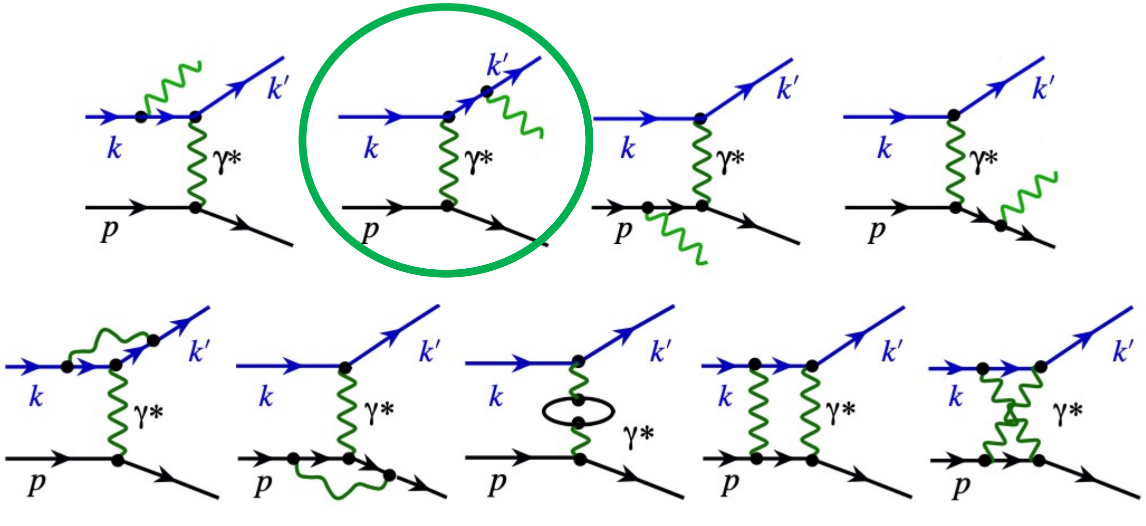
& + D^{(0)}_{e/e} \otimes f^{(0)}_{e/e} \otimes f^{(1)}_{\gamma/q} \otimes \hat{H}^{(0)}_{e+\gamma\to e+X}
\end{aligned}$$



➡ $$\begin{aligned}
\hat{H}^{(1)}_{e+q\to e+X} = {} & \sigma^{(1)}_{e+q\to e+X} - D^{(1)}_{e/e} \otimes \hat{H}^{(0)}_{e+q\to e+X} - f^{(1)}_{e/e} \otimes \hat{H}^{(0)}_{e+q\to e+X} - f^{(1)}_{q/q} \otimes \hat{H}^{(0)}_{e+q\to e+X} \\
& \boxed{- f^{(1)}_{\gamma/q} \otimes \hat{H}^{(0)}_{e+\gamma\to e+X}}
\end{aligned}$$

Completely IR and CO safe! Only depends on factorization scale μ, same in all partonic scattering channels
No need for any “cut-off” parameter(s) in the traditional ”Radiative Correction”

➡ **In joint QCD & QED factorization: Lepton-distributions are not pure QED !**
Hadron’s parton distributions are not pure QCD !

Impact of factorized QED contribution to lepton-hadron scattering

❑ Resummed collinear contribution + Infrared-safe short-distance contribution:

Two-step approach:

(1) Model non-perturbative LDFs, LFFs, PDFs, analytically – everyone can test and verify without numerical complications

(2) QCD + QED evolution of these universal non-perturbative functions

❑ Model distributions:

LDFs: Very different from PDFs, peaked at larger momentum fraction

LFFs:

$$f_{e/e}(x) \approx D_{e/e}(x) = N_e \frac{x^\alpha (1-x)^\beta}{B(1+\alpha, 1+\beta)}$$

with $N_e = 1, \quad \alpha = 5 \gg 0, \quad \beta = 0.5 \sim 0$

PDFs: Valence type

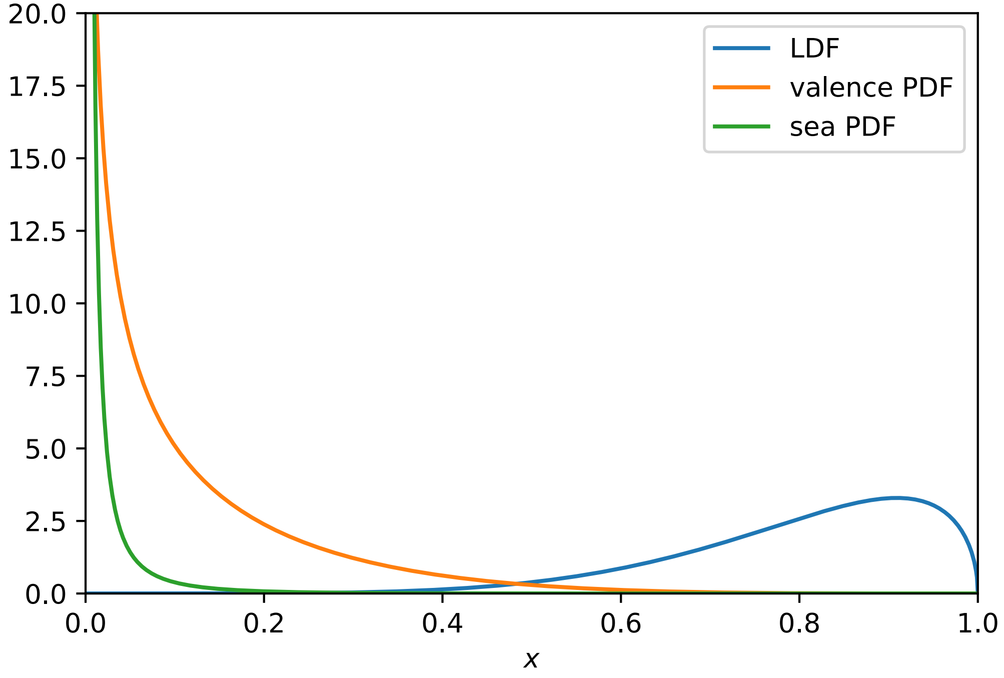
$$f_{q/h}(x) \approx N_q \frac{x^\alpha (1-x)^\beta}{B(1+\alpha, 1+\beta)}$$

with $N_u = 2, \quad \alpha = -0.5, \quad \beta = 3.5$

Sea type

$$f_{s/h}(x) \approx N_s \frac{x^\alpha (1-x)^\beta}{B(\alpha, 1+\beta)}$$

with $N_s = 0.5, \quad \alpha = -1.5, \quad \beta = 5$



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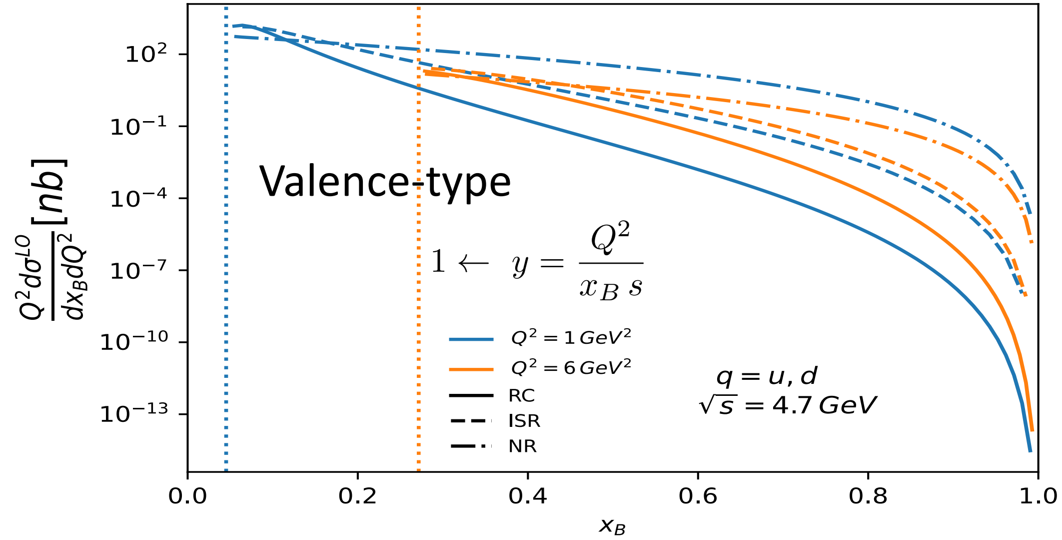
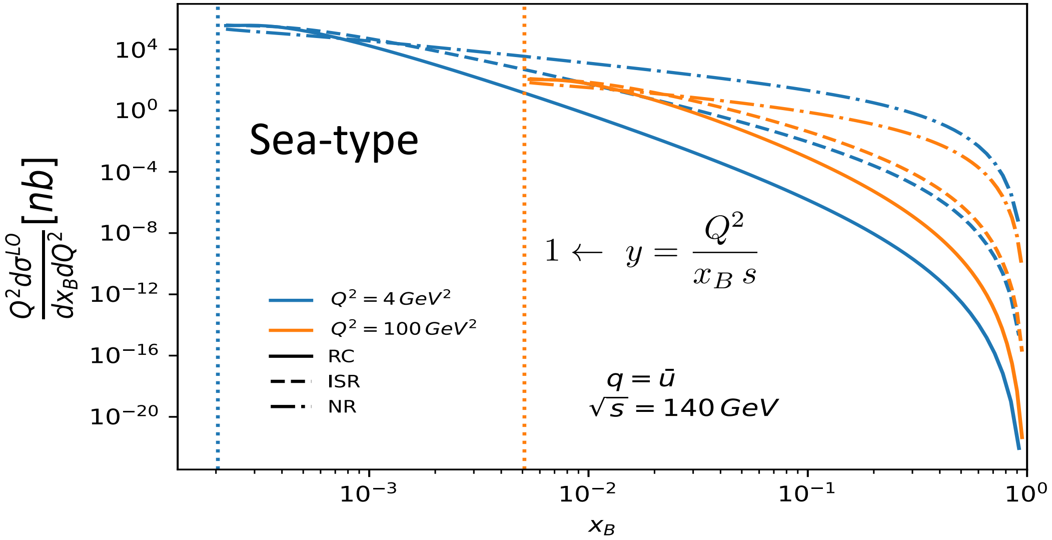
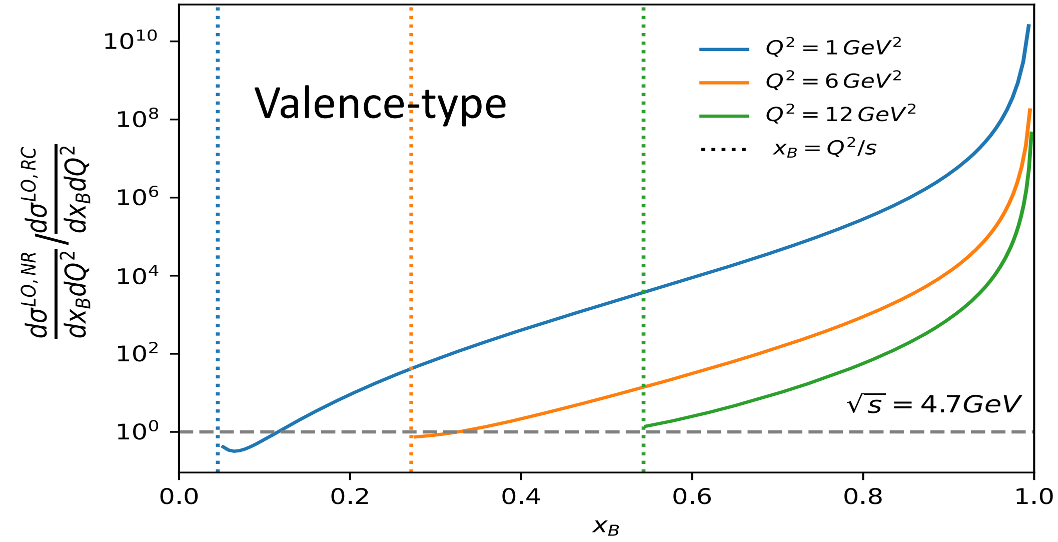
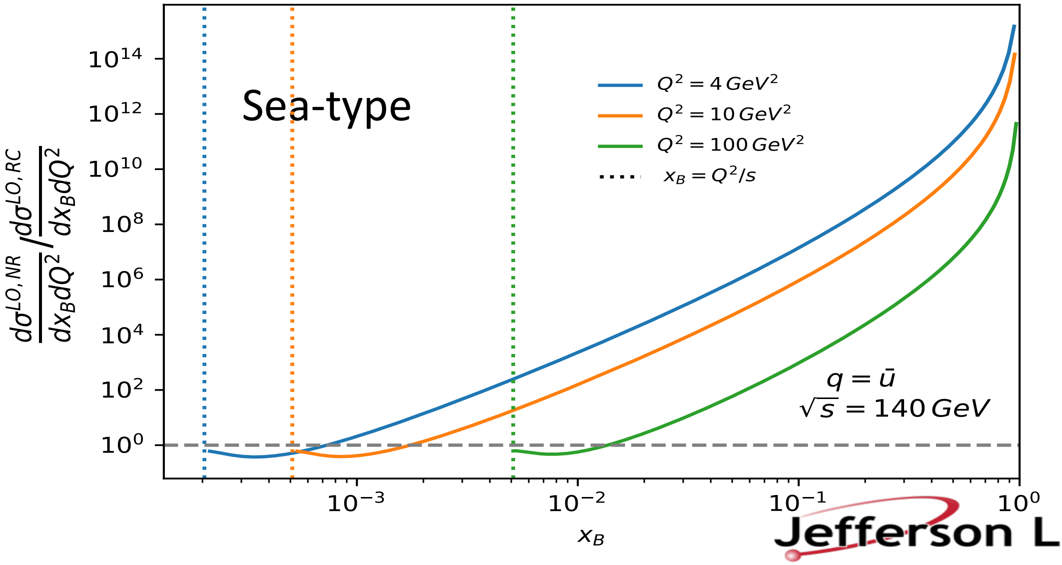






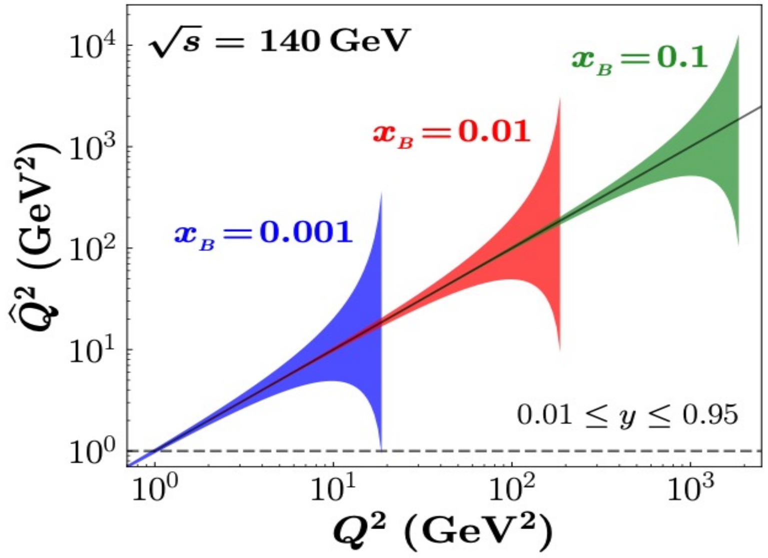


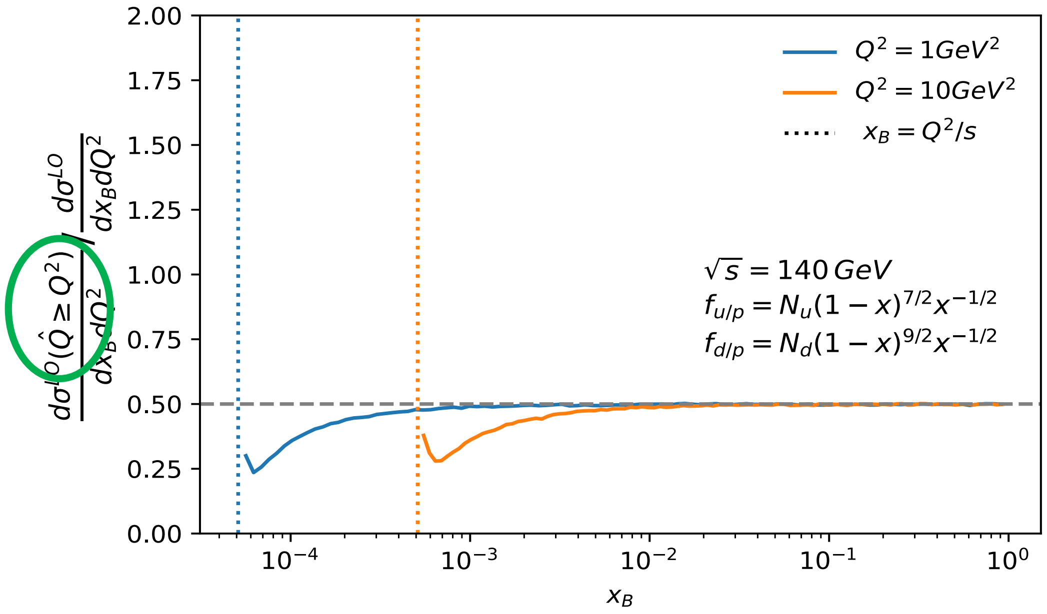
Impact of factorized QED contribution to lepton-hadron scattering

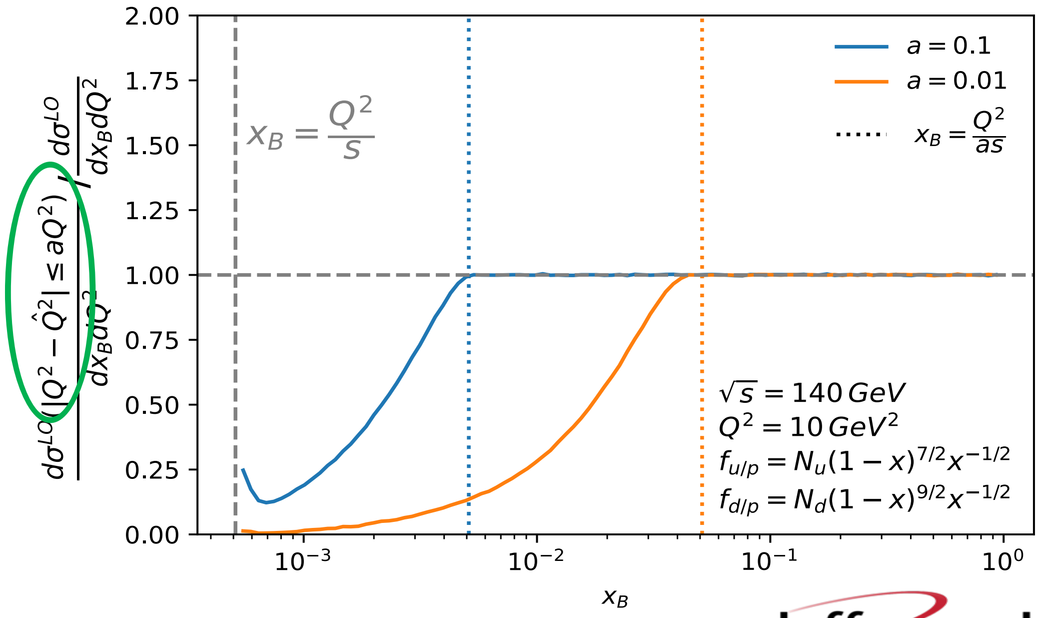
☐ If we require $\hat{Q}^2 \geq Q^2$:

$$\hat{Q}^2_{\min} = Q^2 \, \frac{(1-y)}{(1-x_B\, y)} \qquad \hat{Q}^2_{\max} = Q^2 \, \frac{1}{(1-y+x_B\, y)}$$

- More than ½ total cross sections are from events with $\hat{Q}^2 \leq Q^2$
- Very significant events are NOT from the region where $\hat{Q}^2 \sim Q^2$

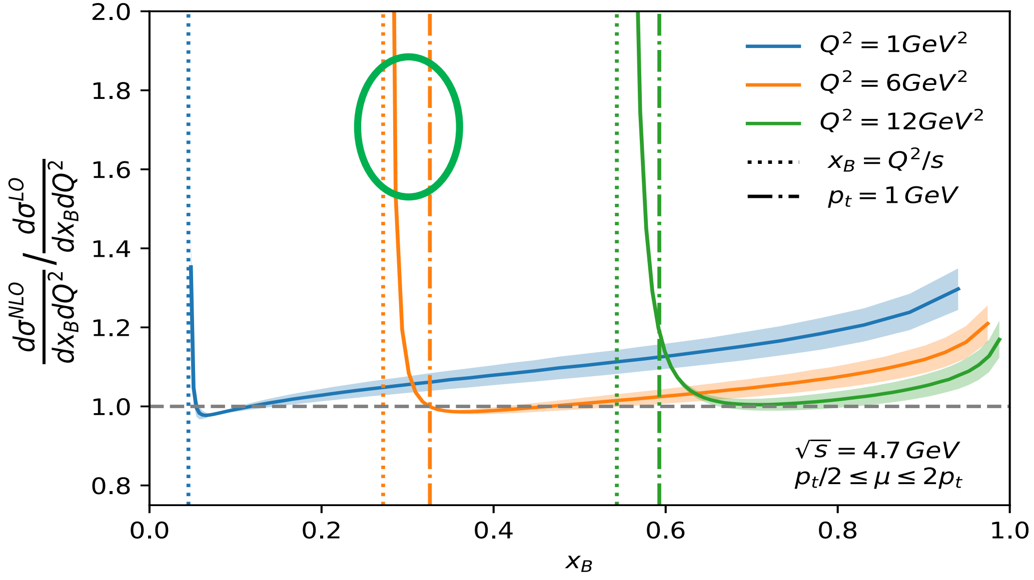


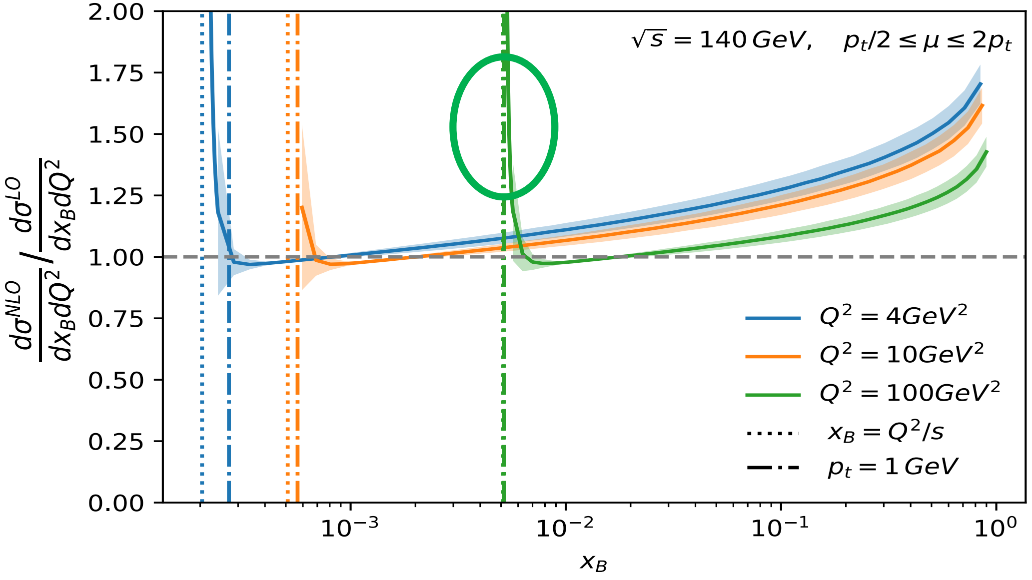




Impact of NLO hard QED contribution to lepton-hadron scattering

❑ **Infrared-safe short-distance contribution:**

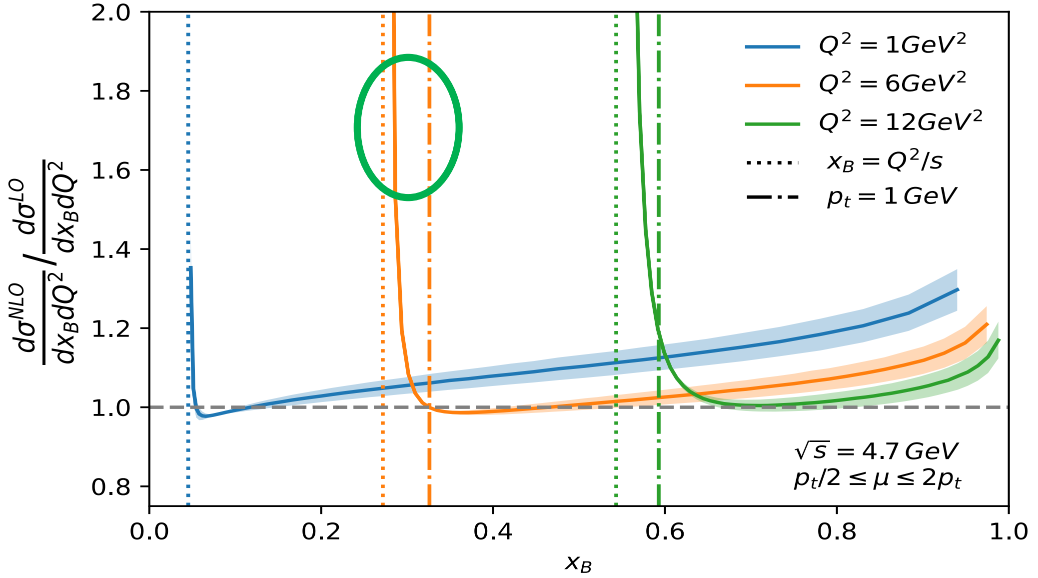


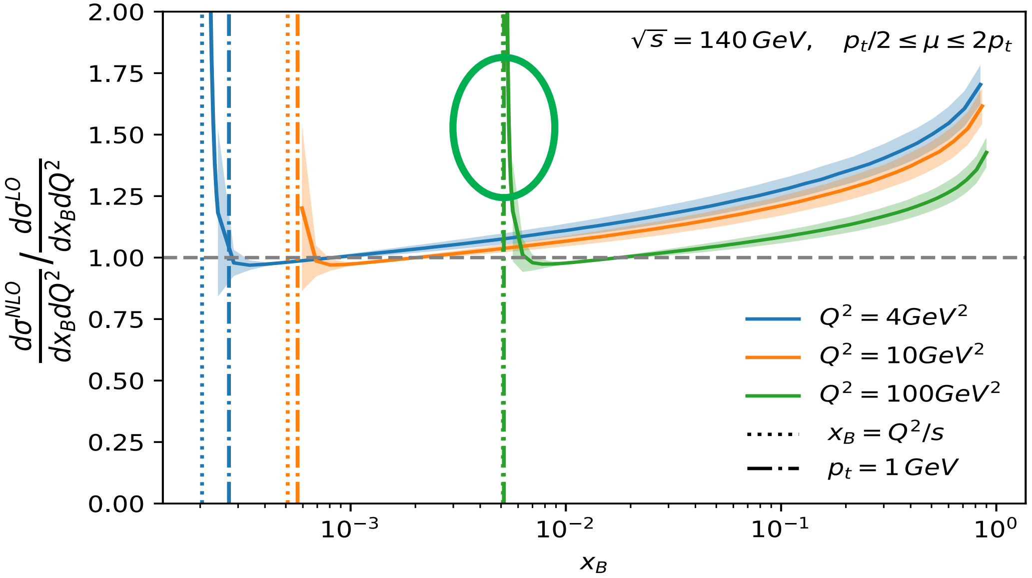


Jefferson Lab

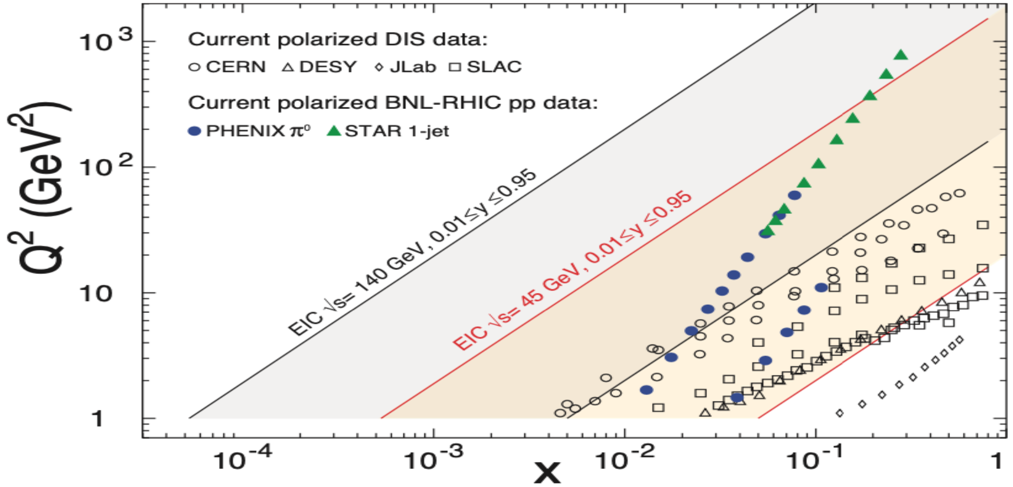
Impact of NLO hard QED contribution to lepton-hadron scattering

- **Infrared-safe short-distance contribution:**





- **Q^2 is NOT an ideal hard scale at small x_B and/or beyond LO in QED:**

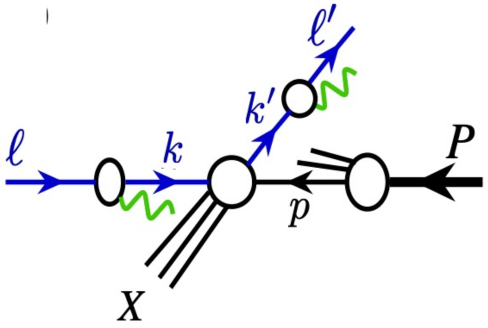


$$\ell_T'^2 = Q^2(1-y) = Q^2\left(1 - \frac{Q^2}{x_B\, s}\right)$$

For EIC: $y \le 0.95$

$$\ell_T'^2 = Q^2(1-y) \ge \frac{Q^2}{20} \text{ !!!}$$

What happens if requiring $\ell_T'^2 \ge 1\,\mathrm{GeV}^2$



Step 2: QCD + QED evolution of these universal non-perturbative functions

❑ Example – Modified DGLAP equation for LDFs:

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In preparation

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix} = \left(\begin{array}{ccc|ccc} P_{ee}^{(1,0)} & P_{e\bar{e}}^{(2,0)} & P_{e\gamma}^{(1,0)} & P_{eq}^{(2,0)} & P_{e\bar{q}}^{(2,0)} & P_{eg}^{(2,1)} \\ P_{\bar{e}e}^{(2,0)} & P_{\bar{e}\bar{e}}^{(1,0)} & P_{\bar{e}\gamma}^{(1,0)} & P_{\bar{e}q}^{(2,0)} & P_{\bar{e}\bar{q}}^{(2,0)} & P_{\bar{e}g}^{(2,1)} \\ P_{\gamma e}^{(1,0)} & P_{\gamma\bar{e}}^{(1,0)} & P_{\gamma\gamma}^{(1,0)} & P_{\gamma q}^{(1,0)} & P_{\gamma\bar{q}}^{(1,0)} & P_{\gamma g}^{(1,1)} \\ \hline P_{qe}^{(2,0)} & P_{q\bar{e}}^{(2,0)} & P_{q\gamma}^{(1,0)} & P_{qq}^{(0,1)} & P_{q\bar{q}}^{(0,2)} & P_{qg}^{(0,1)} \\ P_{\bar{q}e}^{(2,0)} & P_{\bar{q}\bar{e}}^{(2,0)} & P_{\bar{q}\gamma}^{(1,0)} & P_{\bar{q}q}^{(0,2)} & P_{\bar{q}\bar{q}}^{(0,1)} & P_{\bar{q}g}^{(0,1)} \\ P_{ge}^{(2,1)} & P_{g\bar{e}}^{(2,1)} & P_{g\gamma}^{(1,1)} & P_{gq}^{(0,1)} & P_{g\bar{q}}^{(0,1)} & P_{gg}^{(0,1)} \end{array}\right) \otimes \begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix}$$

Evolution kernels in both QCD and QED:

$$P_{ij}(\xi,\mu^2) = \sum_{n,m=0}^{\infty}\left(\frac{\alpha_{em}(\mu^2)}{2\pi}\right)^n \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^m \hat{P}_{ij}^{(n,m)}(\xi) = \sum_{n,m=0}^{\infty} P_{ij}^{(n,m)}(\xi,\mu^2)$$

with $P_{ij}^{(0,0)} = 0\,,\quad N_F\,,\quad N_l$

- **Factorization scale:**

  $$\mu^2 \sim m_c^2$$

- **Input LDFs at μ^2:**
  - **Perturbatively generated by solving QED evolution from lepton mass threshold**
  - **With perturbatively calculated fixed-order MSbar LDFs**
  - **Test the size of non-perturbative hadronic contribution**
  - **…**

Evolution of lepton distribution functions (LDFs)

❑ Lepton distribution functions (LDFs):

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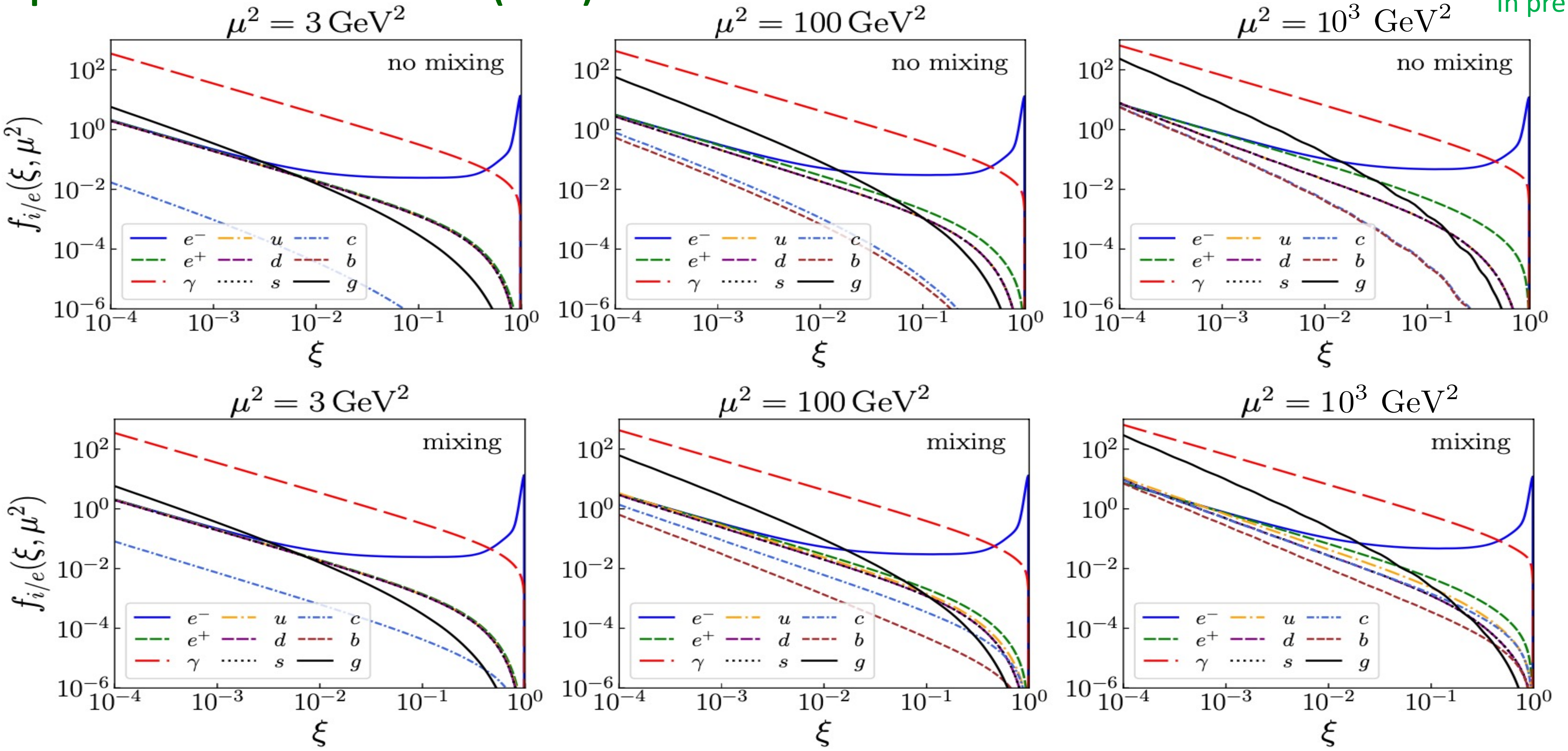
In preparation



With LDFs, we can calculate single hadron production, including J/ψ or jet production at the EIC

Evolution of lepton distribution functions (LDFs)

☐ Example – Photon distribution of an electron:

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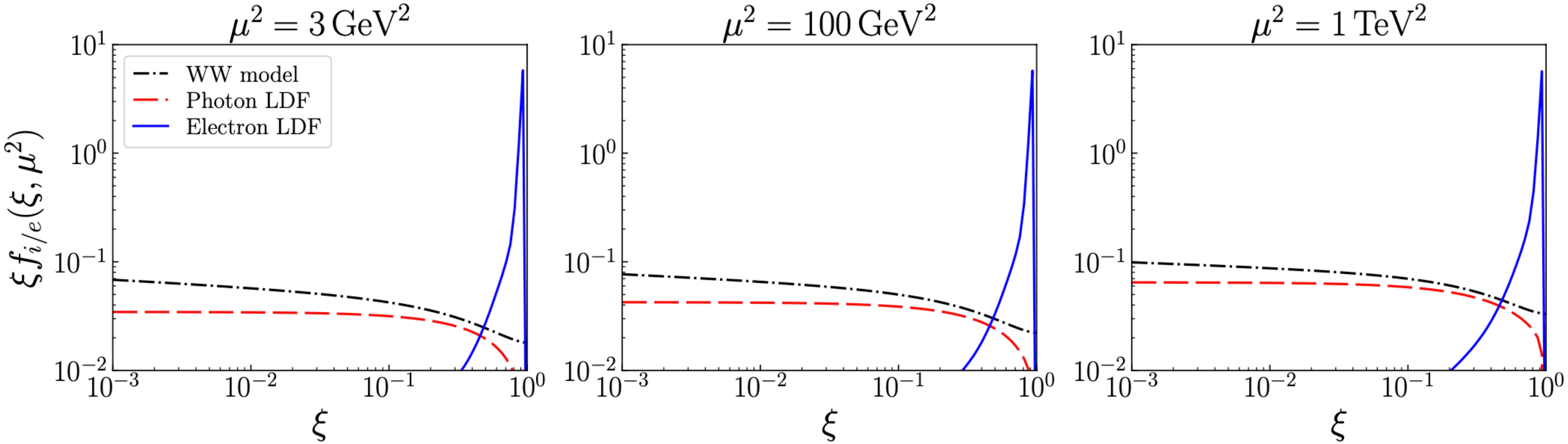
In preparation

- **Weizsa ̈cker-William photon distribution:**

$$f_{\gamma/e}^{\mathrm{WW}}(\xi,\mu^2) = \frac{\alpha_{em}(\mu^2)}{2\pi} P_{\gamma e}(\xi) \left[\ln\left(\frac{\mu^2}{\xi^2 m_e^2}\right) - 1 \right]$$



- **LDFs are not purely perturbative in QED – need global analysis!**
  - **Precision measurements for BSM physics at the EIC needs reliable lepton distributions**
  - **Joint global analysis of lepton and hadron distribution functions should be carried out.**
  - **Impact on searching BSM at ILC or CEPC, FCC, …**

Summary and Outlook – Thank you!

- ❑ Collision induced QED radiation is an integrated part of lepton-hadron scattering, just like QCD radiation
  - ○ Radiative correction approach is difficult for a consistent treatment beyond the inclusive DIS
  - ○ No well-defined photon-hadron frame, if we cannot recover all QED radiation
  - ○ Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC

- ❑ Factorization approach to include both QCD and QED radiative contributions provides a consistent and controllable approximation to high-energy lepton-hadron scattering processes
  - ○ QED radiation is a part of production cross sections, treated in the same way as QCD radiation from quarks and gluons (Have not be able to extend this to full EW+QCD factorization!)
  - ○ No artificial and/or process dependent scale(s) introduced for treating QED radiation, other than the standard factorization scale, universal lepton distribution and fragmentation functions
  - ○ All perturbatively calculable hard parts are IR safe for both QCD and QED
  - ○ All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)

- ❑ Beyond LO in QED, NO F_1, F_2, g_1, and g_2 structure functions (results of the 1-photon approximation) for lepton-hadron scattering processes, but, the universal lepton and hadron distribution functions!