Lattice QCD extraction of the η_c meson's t-dependent parton distribution function

Jose Manuel Morgado Chávez

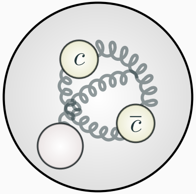
31st International Workshop on Deep Inelastic Scattering
Maison MINATEC. Grenoble, France.
8–12th April 2024.

Email: jose-manuel.morgadochavez@cea.fr

Motivation: η_c mesons

η_c meson

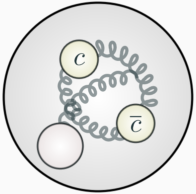
Composition:	$c\bar{c}$
J^{PC}:	0^{-+}
Mass:	2983.9 ± 0.4 MeV
Width:	32.0 ± 0.7 MeV



Motivation: η_c mesons

η_c meson

Composition:	$c\bar{c}$
J^{PC}:	0^{-+}
Mass:	2983.9 ± 0.4 MeV
Width:	32.0 ± 0.7 MeV



η_c-hadron structure

- How does it emerge from the bounding of a pair $c\bar{c}$?
- Comparison with lighter 0^- mesons: Assess quark-mass effect on hadron structure.

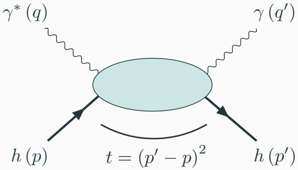
Introduction: t-dependent Parton Distribution Functions (tPDFs)

Hadron structure

How do quarks and gluons combine to make hadrons up?

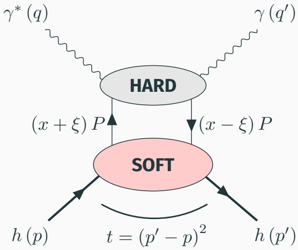
Hadron structure

How do quarks and gluons combine to make hadrons up?



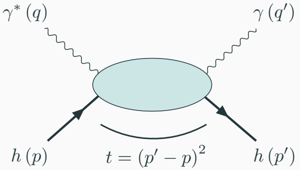
Generalized Bjorken limit
$Q^2 \to \infty$ with $Q^2 >> t$
and ξ fixed.

Factorization
[Phys.Rev.D59(1999)074009]



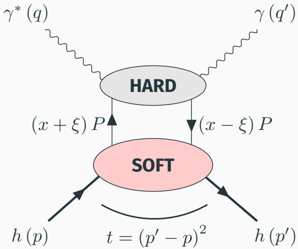
Hadron structure

How do quarks and gluons combine to make hadrons up?



Generalized Bjorken limit
$Q^2 \to \infty$ with $Q^2 >> t$
and ξ fixed.

Factorization
[Phys.Rev.D59(1999)074009]



$$\mathcal{H}\left(\xi, t, Q^2\right) = \sum_{p=q,g} \int_{-1}^{1} \frac{dx}{\xi} \mathcal{K}^p \left(\frac{x}{\xi}, \frac{Q^2}{\mu_F^2}, \alpha_s\left(\mu_F^2\right)\right) H^p\left(x, \xi, t, \mu_F^2\right)$$

Generalized Parton distributions: Off-forward parton distribution functions

Introduction: t-dependent Parton Distribution Functions (tPDFs)

Off-forward parton distribution functions: Non-local, light-like separated, quark or gluon operators, evaluated between hadron states in non-forward kinematics and projected onto the light-front. [Fortsch.Phys.:42(1994)101, Phys.Lett.B:380(1996)417, Phys.Rev.D:55(1997)7114]

Example: Twist-two chiral-even quark GPD of a spinless hadron.

$$H_{q/h}(x,\xi,t,\mu) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix(p+p')\cdot z/2} \langle h(p')|\, \overline{\psi}_q(z/2)\,\gamma^+ \widehat{\mathcal{W}}[z/2,-z/2]\,\psi_q(-z/2)\,|h(p)\rangle\Big|_{\substack{z^+=0\\ \boldsymbol{z}_\perp=\boldsymbol{0}_\perp}}$$

Off-forward parton distribution functions: Non-local, light-like separated, quark or gluon operators, evaluated between hadron states in non-forward kinematics and projected onto the light-front. [Fortsch.Phys.:42(1994)101, Phys.Lett.B:380(1996)417, Phys.Rev.D:55(1997)7114]

Example: Twist-two chiral-even quark GPD of a spinless hadron.

$$H_{q/h}\left(x,\xi,t,\mu\right)=\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ix\left(p+p'\right)\cdot z/2}\,\langle h\left(p'\right)|\,\overline{\psi}_q\left(z/2\right)\gamma^+\widehat{\mathcal{W}}\left[z/2,-z/2\right]\psi_q\left(-z/2\right)|h\left(p\right)\rangle\Big|_{\substack{z^+=0\\ \boldsymbol{z}_\perp=\boldsymbol{0}_\perp}}$$

t-dependent Parton Distribution Functions

$$q_h\left(x,t,\mu\right)=H_{q/h}\left(x,\xi=0,t,\mu\right)\quad\left[p\cdot z=p'\cdot z\equiv p^+\right]$$

Introduction: t-dependent Parton Distribution Functions (tPDFs)

Off-forward parton distribution functions: Non-local, light-like separated, quark or gluon operators, evaluated between hadron states in non-forward kinematics and projected onto the light-front. [Fortsch.Phys.:42(1994)101, Phys.Lett.B:380(1996)417, Phys.Rev.D:55(1997)7114]

Example: Twist-two chiral-even quark GPD of a spinless hadron.

$$H_{q/h}(x,\xi,t,\mu) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix(p+p')\cdot z/2} \langle h(p')|\, \overline{\psi}_q(z/2)\, \gamma^+ \widehat{\mathcal{W}}[z/2,-z/2]\, \psi_q(-z/2)\, |h(p)\rangle \Big|_{\substack{z^+=0 \\ \boldsymbol{z}_\perp=\boldsymbol{0}_\perp}}$$

t-dependent Parton Distribution Functions

$$q_h(x,t,\mu) = H_{q/h}(x,\xi=0,t,\mu) \quad [p\cdot z = p'\cdot z \equiv p^+]$$

Properties:

- Universality *i.e.* hadron-specific objects.
- Contain parton distribution functions and electromagnetic form factors.
- Non-perturbative description of hadron structure: (3D) Tomography.

Goal of this project: To compute tPDFs of the η_c-meson

Lattice QCD

In Lattice field theory, the expectation value of an observable, $\langle \mathcal{O} \rangle$, is obtained as:

$$\langle \mathcal{O} \rangle \propto \int \mathcal{D}\left[U, \overline{\psi}, \psi\right] \mathcal{O}\left[\overline{\psi}, \psi, U\right] e^{-S_E\left[U, \overline{\psi}, \psi\right]} \simeq \frac{1}{N} \sum_i^N p\left(U_i\right) \langle \mathcal{O} \rangle_F \left[U_i\right],$$

with

- $p\left(U_i\right)$: Boltzmann probability distribution - Obtained, numerically, through Monte Carlo sampling of the Euclidean path integral.
- $\langle \mathcal{O} \rangle_F$: Fermionic expectation value - Evaluated, exactly, through Wick theorem.

Computed expectation values are connected to (Euclidean) correlation functions
[Comm.Math.Phys.:42(1975)281, Comm.Math.Phys.:54(1977)283]

Question: How can we compute tPDFs in Lattice field theory?

The pseudo-distribution formalism (I)

Definition: Ioffe-time tPDF ($\nu \equiv -p \cdot z$)
[Nucl.Phys.B:311(1989)541, Phys.Rev.D:51(1995)6036, Phys.Rev.D:100(2019)116011]

$$q_h\left(\nu, t, \mu\right) \equiv \int dx e^{i\nu x} q_h\left(x, t, \mu\right) = \frac{1}{2p^+} \left\langle h\left(p'\right) \left| \overline{\psi}_q\left(z/2\right) \gamma^+ \widehat{\mathcal{W}}\left[z/2; -z/2\right] \psi_q\left(-z/2\right) \right| h\left(p\right) \right\rangle \Big|_{\substack{z^+=0 \\ \boldsymbol{z}_\perp = \boldsymbol{0}_\perp}}$$

The pseudo-distribution formalism (I)

Definition: Ioffe-time tPDF $(\nu \equiv -p \cdot z)$

[Nucl.Phys.B:311(1989)541, Phys.Rev.D:51(1995)6036, Phys.Rev.D:100(2019)116011]

$$q_h(\nu, t, \mu) \equiv \int dx e^{i\nu x} q_h(x, t, \mu) = \frac{1}{2p^+} \langle h(p') | \overline{\psi}_q(z/2) \gamma^+ \widehat{\mathcal{W}}[z/2; -z/2] \psi_q(-z/2) | h(p) \rangle \Big|_{\substack{z^+=0 \\ \boldsymbol{z}_\perp = \boldsymbol{0}_\perp}}$$

---

1. Consider a **generic matrix element** with $z \in \mathbb{R}^{3,1}$ (or even $z \in \mathbb{R}^4$)

[Phys.Rev.D:96(2017)034025, Phys.Rev.D:96(2017)094503, Phys.Rev.D:100(2019)116011]

$$\begin{aligned} M_q^\mu(p, p', z) &= \langle h(p') | \overline{\psi}_q(z/2) \gamma^\mu \widehat{\mathcal{W}}[z/2; -z/2] \psi_q(-z/2) | h(p) \rangle \\ &= (p + p')^\mu \mathcal{F}(\nu, t, z^2) - (p' - p)^\mu \mathcal{G}(\nu, t, z^2) + z^\mu \mathcal{Z}(\nu, t, z^2), \quad \nu \equiv -p \cdot z \end{aligned}$$

The pseudo-distribution formalism (I)

Definition: Ioffe-time tPDF $(\nu \equiv -p \cdot z)$
[Nucl.Phys.B:311(1989)541, Phys.Rev.D:51(1995)6036, Phys.Rev.D:100(2019)116011]

$$q_h(\nu, t, \mu) \equiv \int dx e^{i\nu x} q_h(x, t, \mu) = \frac{1}{2p^+} \langle h(p') | \overline{\psi}_q(z/2) \gamma^+ \widehat{\mathcal{W}}[z/2; -z/2] \psi_q(-z/2) | h(p) \rangle \Big|_{\substack{z^+=0 \\ \boldsymbol{z}_\perp = \boldsymbol{0}_\perp}}$$

---

1. Consider a **generic matrix element** with $z \in \mathbb{R}^{3,1}$ (or even $z \in \mathbb{R}^4$)
[Phys.Rev.D:96(2017)034025, Phys.Rev.D:96(2017)094503, Phys.Rev.D:100(2019)116011]

$$\begin{aligned} M_q^\mu(p, p', z) &= \langle h(p') | \overline{\psi}_q(z/2) \gamma^\mu \widehat{\mathcal{W}}[z/2; -z/2] \psi_q(-z/2) | h(p) \rangle \\ &= (p + p')^\mu \mathcal{F}(\nu, t, z^2) - (p' - p)^\mu \mathcal{G}(\nu, t, z^2) + z^\mu \mathcal{Z}(\nu, t, z^2), \quad \nu \equiv -p \cdot z \end{aligned}$$

2. Light-front projection, *i.e.* $z^\mu \to z^\mu \propto (1, 0, 0, -1)$ and $\mu = +$

$$M_q^+(p, p', z)\Big|_{\substack{z^+ \to 0 \\ \boldsymbol{z}_\perp \to \boldsymbol{0}_\perp}} = \langle h(p') | \overline{\psi}_q(z/2) \gamma^+ \widehat{\mathcal{W}}[z/2; -z/2] \psi_q(-z/2) | h(p) \rangle = 2p^+ \mathcal{F}(\nu, t, z^2 \to 0)$$

The pseudo-distribution formalism (I)

Definition: Ioffe-time tPDF $(\nu \equiv -p \cdot z)$
[Nucl.Phys.B:311(1989)541, Phys.Rev.D:51(1995)6036, Phys.Rev.D:100(2019)116011]

$$q_h(\nu, t, \mu) \equiv \int dx e^{i\nu x} q_h(x, t, \mu) = \frac{1}{2p^+} \langle h(p') | \overline{\psi}_q(z/2) \gamma^+ \widehat{\mathcal{W}}[z/2; -z/2] \psi_q(-z/2) | h(p) \rangle \Big|_{\substack{z^+=0 \\ \boldsymbol{z}_\perp = \boldsymbol{0}_\perp}}$$

---

1. Consider a **generic matrix element** with $z \in \mathbb{R}^{3,1}$ (or even $z \in \mathbb{R}^4$)
[Phys.Rev.D:96(2017)034025, Phys.Rev.D:96(2017)094503, Phys.Rev.D:100(2019)116011]

$$\begin{aligned} M_q^\mu(p, p', z) &= \langle h(p') | \overline{\psi}_q(z/2) \gamma^\mu \widehat{\mathcal{W}}[z/2; -z/2] \psi_q(-z/2) | h(p) \rangle \\ &= (p + p')^\mu \mathcal{F}(\nu, t, z^2) - (p' - p)^\mu \mathcal{G}(\nu, t, z^2) + z^\mu \mathcal{Z}(\nu, t, z^2), \quad \nu \equiv -p \cdot z \end{aligned}$$

2. Light-front projection, *i.e.* $z^\mu \to z^\mu \propto (1, 0, 0, -1)$ and $\mu = +$

$$M_q^+(p, p', z)\Big|_{\substack{z^+ \to 0 \\ \boldsymbol{z}_\perp \to \boldsymbol{0}_\perp}} = \langle h(p') | \overline{\psi}_q(z/2) \gamma^+ \widehat{\mathcal{W}}[z/2; -z/2] \psi_q(-z/2) | h(p) \rangle = 2p^+ \mathcal{F}(\nu, t, z^2 \to 0)$$

3. Ioffe-time distributions

$$\widetilde{q}_h(\nu, t, \mu) = \mathcal{F}(\nu, t, z^2 \to 0)$$

The pseudo-distribution formalism (II)

Euclidean setup:

- $p = \left(E, \boldsymbol{p}_\perp, p^3\right)$ and $p' = \left(E, -\boldsymbol{p}_\perp, p^3\right)$
- $z = \left(0, 0, 0, z^3\right)$

1. Compute

$$M^0\left(p, p', z\right) = \langle h\left(p'\right)| \overline{\psi}_q\left(z/2\right)\gamma^0 \widehat{\mathcal{W}}\left[z/2; -z/2\right]\psi_q\left(-z/2\right)|h\left(p\right)\rangle = 2E\mathcal{F}\left(\nu, t, z^2\right)$$

2. Form RGI ratio
[Phys.Lett.B:767(2017)314]

$$\mathfrak{M}\left(p, p', z\right) \equiv \frac{M^0\left(p, p', z\right)}{M^0\left(0, 0, z\right)}\frac{M^0\left(0, 0, 0\right)}{M^0\left(p, p', 0\right)} = \frac{\mathcal{F}\left(\nu, t, z^2\right)}{\mathcal{F}\left(0, 0, z\right)}\frac{\mathcal{F}\left(0, 0, 0\right)}{\mathcal{F}\left(0, t, 0\right)} = \widetilde{q}_h\left(\nu, t, z^2\right) + \text{h.t.}$$

3. Light-front matching
[Phys.Rev.D:98(2018)014019, Phys.Rev.D:98(2018)050004, Phys.Rev.D:97(2018)074508]

$$\widetilde{q}_h\left(\nu, t, z^2\right) = \mathcal{C}\left(\nu, w, t, z\mu\right) \otimes \widetilde{q}_h\left(w, t, \mu\right) = \mathcal{C}\left(\nu, w, t, z\mu\right) \otimes \int dx e^{iwx} q_h\left(x, t, \mu\right)$$

Lattice QCD calculation

Numerical setup

- $N_f = 2$ ensembles (CLS) [Nucl.Phys.B:865(2012)397,PoSLATTICE2013:(2014)475]
  - Wilson gauge action
  - $\mathcal{O}(a)$-improved Wilson fermions

Name	β	a [fm]	$L^3 \times T$	N_f	m_π [MeV]	κ_l	κ_c
D5	5.3	0.0658 (7) (7)	$24^3 \times 48$	u, d	450	0.13625	0.12724
E5	5.3	0.0658 (7) (7)	$32^3 \times 64$	u, d	437	0.13625	0.12724

- One hadron-interpolator and four smearings (source and sink).

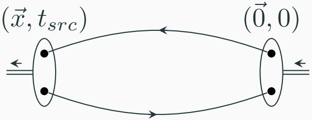
$$\eta_c^s(x) = \psi_c^s(x)\,\gamma_5\psi_c^s(x) \quad , \quad J^{PC} = 0^{-+}$$
$$\psi_q^s(x) = \left(1 + 0.125\Delta_{\mathrm{APE}}\right)^{N_s}\psi_q(x) \quad , \quad N_s \in \{0, 30, 50, 80\}$$

- Twisted boundary conditions and a symmetric frame.

Two-point functions: Spectroscopy (I)

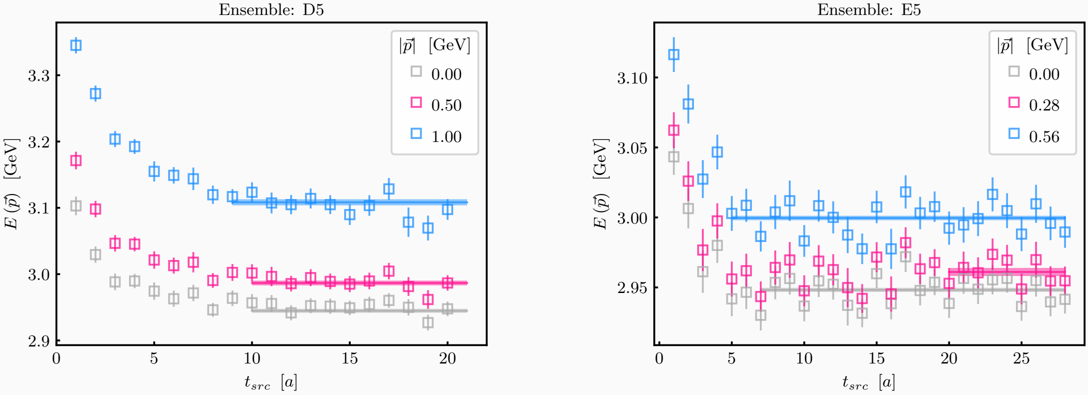
Computation of hadron propagators

$$C_2^{(ss')}(\vec{p}, t_{src}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \eta_c^s(\vec{x}, t_{src}) \overline{\eta}_c^{s'}(\vec{0}, 0) \rangle \propto \mathcal{N}(\vec{p})\, e^{-E(\vec{p})t_{src}}$$



- Consider connected diagrams only.
- Project ground-state ($\eta_c(1s)$) solving GEVP.
- Fit energies: Choose best fit range according to AIC.

Two-point functions: Spectroscopy (II)

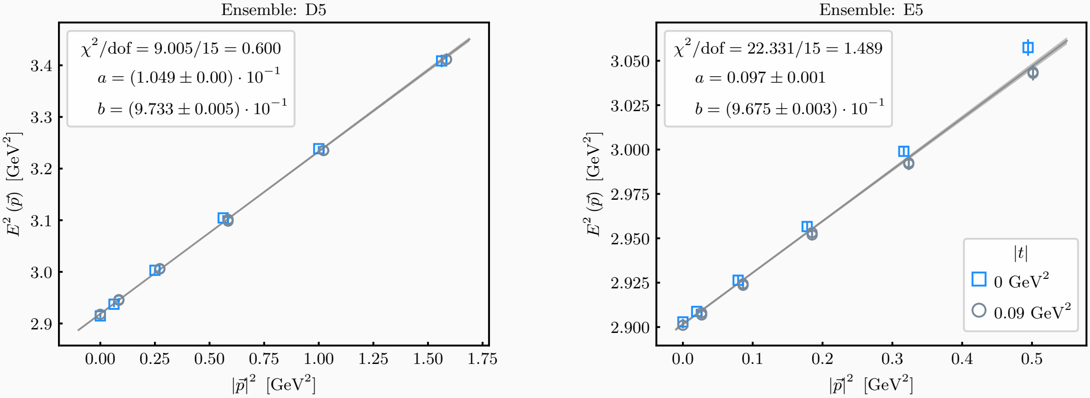


Energy spectrum compatible with expectation within finite-volume and cut-off effects.

Systematics: - Fit range: Model averaging (AIC) [Phys.Rev.D:103(2021)114502]

- Excited states: GEVP [Nucl.Phys.B:259(1985)58, JHEP:04(2009)094]

Two-point functions: Spectroscopy (III)

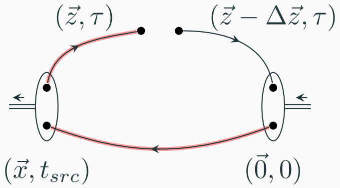


Consistency check: Expected energy-momentum dispersion relations fulfilled.

Three-point functions: tPDF (I)

Computation of hadron three-point functions

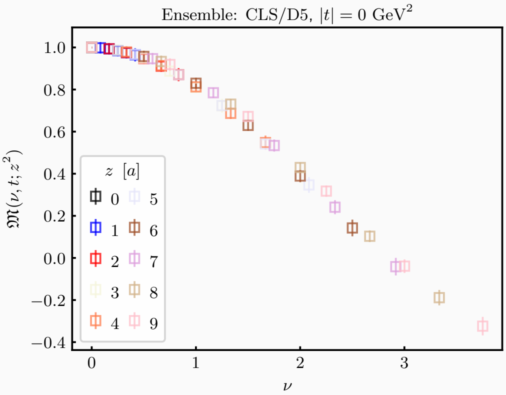
$$C_3^{(ss')}(\vec{p}, t_{src}) = \sum_{\vec{x},\vec{z}} e^{-i\vec{p}\cdot\vec{x}-i\vec{q}\cdot\vec{z}} \langle \eta_c^s(\vec{x}, t_{src}) \overline{\psi}_c(\vec{z}, \tau)\, \gamma^0 \widehat{\mathcal{W}}[\vec{z}, \tau; \vec{z}-\Delta\vec{z}, \tau] \psi_c(\vec{z}-\Delta\vec{z}, \tau)\, \overline{\eta}_c^s(\vec{0}, 0)\rangle$$

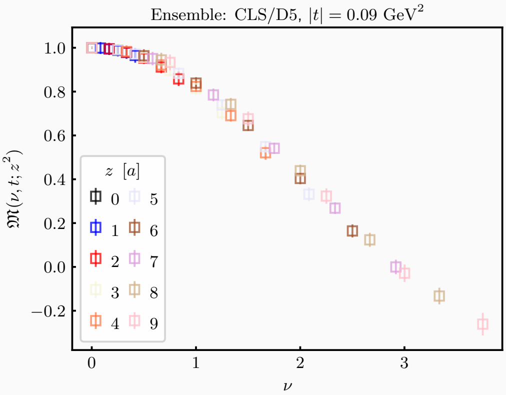


- Consider connected diagrams only: Sequential propagator technique.
- Project ground state ($\eta_c(1s)$) according to GEVP.
- Compute ratios to isolate matrix elements: [PoSLATTICE2005:(2006)360]

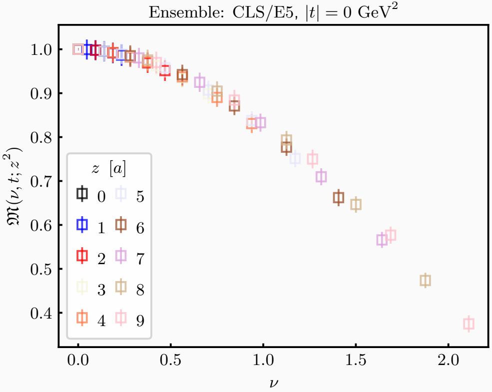
$$R(\tau) = \frac{C_3^{(\mathrm{P})}(\vec{p}, \vec{p}', t_{src}, \tau)}{\sqrt{C_2^{(\mathrm{P})}(\vec{p}', t_{src})\, C_2^{(\mathrm{P})}(\vec{p}, t_{src})}} \sqrt{\frac{C_2^{(\mathrm{P})}(\vec{p}, t_{src}-\tau)\, C_2^{(\mathrm{P})}(\vec{p}', \tau)}{C_2^{(\mathrm{P})}(\vec{p}', t_{src}-\tau)\, C_2^{(\mathrm{P})}(\vec{p}, \tau)}} = \frac{M^0(p, p', z)}{4\sqrt{E(\vec{p})\, E(\vec{p}')}}$$

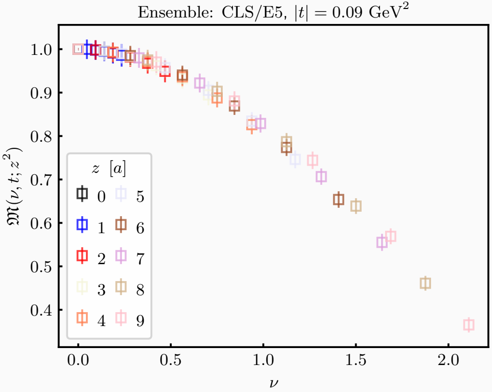
Three-point functions: tPDF (II)





Three-point functions: tPDF (III)





Conclusions and future steps

Summary

- Study of η_c-meson's structure through GPDs within lattice QCD.
- tPDFs give a comprehensive picture about hadron structure.
- Ongoing effort for the extraction of tPDFs

Future steps

- Extend kinematics: t-values
- Tame excited state contamination
- Handle lattice artifacts: Include new ensembles
  - Finite volume
  - Discretization
- Matching to the light-cone: Take mass effects into account
- Reconstruction of light-cone distribution functions.

Thank you!

Back-up slides

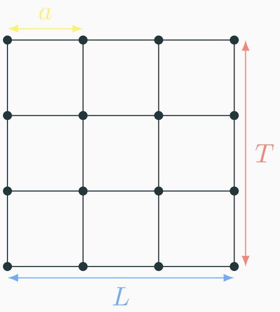
(Continuum) Quantum field theory

$$\langle\Omega|\,\mathcal{O}\,|\Omega\rangle \propto \int \mathcal{D}\left[A_\mu,\overline{\psi},\psi\right](x)\,\mathcal{O}\left[A_\mu,\overline{\psi},\psi\right](x)\,e^{iS\left[A_\mu,\overline{\psi},\psi\right](x)}$$

Extremely hard to assess beyond perturbation theory

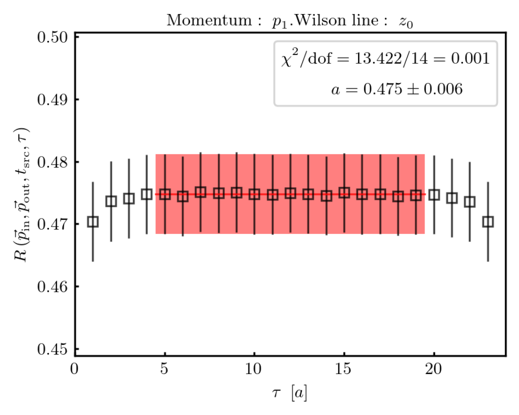
(Lattice) Quantum field theory

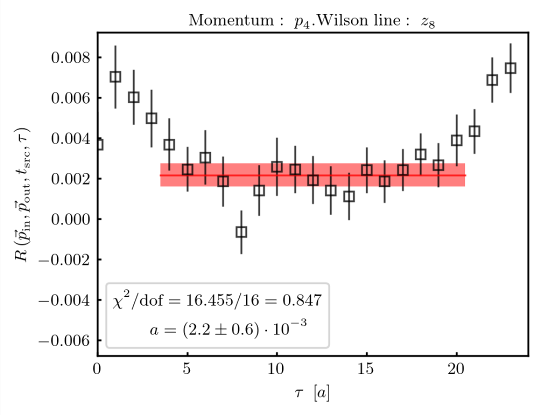
- Analytic continuation: $t \to -it_E \Rightarrow e^{iS} \to e^{-S_E}$
- Spacetime discretization:
  - a (lattice spacing): UV cut-off.
  - $L^3 \times T$ (finite box): Finite number of degrees of freedom.



Amenable for numerical evaluation of the path integral:
Non-perturbative calculations!

Three-point functions





Three-point functions