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High energy $\gamma \gamma$ interactions at the LHeC

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Future Large Hadron-electron Collider (LHeC)

- Future Large Hadron-electron Collider (LHeC):
 - The future collider LHeC, planned at the LHC, is to operate at the center-of-mass energy of 1.2 TeV and is expected to deliver an integrated electron-proton luminosity of about 1 ab⁻¹.
 - Very high ep luminosity & Clean experimental environment & High statistics data event for the rare processes.
 - Negligible event pileup & Excellent particle momentum resolutions and particle identification.
 - High-resolution detectors of the protons and electrons.

Parameter	Unit	LHeC			FCC-eh		
		CDR	$\operatorname{Run}5$	${\rm Run}\; 6$	Dedicated	$E_p {=} 20 \mathrm{TeV}$	$E_p = 50 \text{ TeV}$
E_c	GeV	60	30	50	50	60	60
N_p	10^{11}	1.7	2.2	2.2	2.2	1	1
ϵ_p	μm	3.7	2.5	2.5	2.5	2.2	2.2
I_e	mA	6.4	15	20	50	20	20
N_e	10^{9}	1	2.3	3.1	7.8	3.1	3.1
β^*	cm	10	10	7	7	12	15
Luminosity	$10^{33}{\rm cm}^{-2}{\rm s}^{-1}$	1	5	9	23	8	15



Figure: Summary of luminosity parameter values for the LHeC and FCC-eh; [2007.14491v2 [hep-ex]].

- Another DIS option is studied as part of the possible Future Circular Collider (FCC) at CERN, the FCC-eh [Eur. Phys. J. C 79 (2019) 474], and will reach center-of-mass energies still higher than at the LHeC.
- At Brookhaven, the EIC is under development to perform DIS measurements at lower energies but with higher luminosities than were achieved at HERA [Nuclear Physics A 1026 (2022) 122447].

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Electroweak Physics at the LHeC

- At the proposed electron-proton collider LHeC electroweak interactions can be uniquely studied in a largely unexplored kinematic region.
- Unique measurements of electroweak parameters can be performed with the highest precision [Eur Phys J C 80 (2020) 831].





Figure: Top quark production in CC DIS & W boson production at the LHeC.

Figure: Simultaneous determination of top-quark and the W-boson mass & Z-boson and W-boson.

- Direct W and Z Production at the LHeC:
 - Study of Tripe Gauge Couplings (TGCs), e.g. $WW\gamma$ and WWZ couplings.
- Top Quark Physics at the LHeC:
 - Top quark production is dominated by single top quark production ($\sigma = 1.9$ pb).
 - Photoproduction of top-antitop quark pairs with $\sigma = 0.05$ pb is expected at the LHeC.
 - The top quark Flavour Changing Neutral Currents (FCNCs) interactions, $tq\gamma$, tqZ, tqH at the LHeC \rightarrow New Physics signature [2007.14491v2 [hep-ex]].

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Electroweak Physics at the LHeC

- Higgs Physics at the LHeC:
 - Within the DIS process at the LHeC, the primary mechanism for Higgs boson production involves WW fusion in CC scattering, yielding a cross-section of $\sigma_{CC} = 110$ fb.
 - The next large Higgs production mode in ep is fusion in NC DIS scattering, which has a smaller but still sizable cross-section ($\sigma_{NC} = 20$ fb).



Figure: Higgs boson production in charged (left) and neutral (right) current DIS at the LHeC.



Figure: Coverage of the kinematic plane in DIS at the LHeC.

 Extensive DIS and QCD program at the LHeC → Parton interaction dynamics at small Bjorken x & DIS and diffractive DIS [Phys. Rev. D 107 (2023) 094038] & [Talk by Amanda Cooper-Sarkar].

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Summary of the LHeC: To Move Beyond the HERA Era!

- Cutting-edge laboratory for exploring electroweak & Higgs physics & Beyond Standard Model (BSM) signatures.
- LHeC represents more than just a DIS super-collider \rightarrow Much more than a super-HERA collider.
- The LHeC is designed to move the field of DIS to the energy and intensity frontier of particle physics.
- It extends the accessible kinematic range in lepton-nucleon and lepton-nucleus scattering by several orders of magnitude.
- Due to enhanced luminosity, large energy, and the cleanliness of the hadronic final states, the LHeC
 has a strong Higgs physics program and its own discovery potential for New Physics.
- LHeC could serve as a high-energy electron-ion (eA) collider as well.
- The luminosity of the LHeC exceeds that of HERA by approximately a factor of 1000!

High energy $\gamma\gamma$ interactions at the LHeC

- New electroweak opportunities at the LHeC:
 - Comprehensive survey of studies of high energy photon-photon interactions at the LHeC, for the $\gamma\gamma$ center-of-mass energy of up to 1 TeV.
 - Wide spectrum of γγ processes will be studied at the LHeC, including, in particular, the exclusive production of lepton pairs, Higgs boson, W and Z bosons as well as pairs of charged supersymmetric particles.
 - Very high statistics of these processes are expected to be achieved at the LHeC.



Figure: The exclusive W- and Z-boson pair production via photon-photon fusion at the LHC (left) and future LHeC (right). The additional exchange between protons (yellow band) represents the hadronic re-scattering, absent in e_P collisions.

MC event generators for the photon-photon interactions at colliders

- Ipair [S. P. Baranov, O. Duenger, H. Shooshtari and J. A. M. Vermaseren, Proceedings, Workshop on Physics at HERA, vol. 3, p. 1478, 1991.].
 - A generator for lepton pair production in lepton-lepton, lepton-hadron and hadron-hadron collisions & two-photon Bethe–Heitler (2-γ BH).
- GRAPE [T. Abe, Comput. Phys. Commun. 136 (2001) 126-147, [arXiv:hep-ph/0012029 [hep-ph]]].
 - A generator for dilepton production in ep collisions & Exact matrix elements in the electroweak theory at tree level & dilepton productions via $\gamma\gamma$, γZ and ZZ collisions.
- Cepgen [L. Forthomme, Comput. Phys. Commun. 271 (2022) 108225, arXiv:1807.06059 [hep-ph]].
 - A generic central exclusive processes event generator for the photon-photon physics at the LHC.
 - Integration with Ipair/GRAPE for the electron-proton collision is in progress.
 - The cepgen repository can be found in: https://github.com/cepgen/cepgen.



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Equivalent Photon Approximation (EPA)

• In the EPA [Phys. Rept. 15 (1975) 181-281], cross-sections for electron-proton collisions, specifically those via photon-photon fusion at the LHeC, are calculated by convolving electron and proton equivalent photon fluxes, Φ_e and Φ_p , respectively, with the photon-photon cross-section $\sigma_{\gamma\gamma}$:

$$\begin{split} \sigma_{ep} &= \int \mathrm{d} y_e \mathrm{d} y_p \Phi_e(y_e) \Phi_p(y_p) \sigma_{\gamma \gamma}(W) \\ &= \int \mathrm{d} W \boldsymbol{S}_{\gamma \gamma} \sigma_{\gamma \gamma}(W) \,, \end{split}$$

where the respective photon fractional energies $y_e = E_{\gamma(e)}/E_e$, $y_p = E_{\gamma(p)}/E_p$, the $\gamma\gamma$ center-of-mass energy $W = \sqrt{y_e y_p s}$ and the photon-photon luminosity spectrum $S_{\gamma\gamma}$ is equal to the flux convolution at a given W

$$S_{\gamma\gamma} = \frac{2}{W} \int_{W^2/s}^1 dy_e \Phi_e(y_e) \, y_p \, \Phi_p(y_p) \,,$$

where $y_p = W^2/y_e s$, and hence, one can write

$$S_{\gamma\gamma} = \frac{2W}{s} \int_{W^2/s}^1 \frac{dy_e}{y_e} \Phi_e(y_e) \Phi_p(\frac{W^2}{y_e s})$$

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Photon fluxes for electrons and protons, Φ_e and Φ_p

• It is assumed that $\sigma_{\gamma\gamma}$ is not sensitive to the photon virtualities, so the fluxes could be integrated over the photon Q^2 :

$$\Phi_{\gamma}(y) = \frac{\alpha}{\pi y} \int \frac{\mathrm{d}Q^2}{Q^2} \left[(1-y) \left(1 - \frac{Q_{\min}^2}{Q^2} \right) F_E(Q^2) + \frac{y^2}{2} F_M(Q^2) \right],$$

where α is the fine-structure constant and F_E , F_M are the electric and magnetic form factors, respectively.

- *Elastic* scattering: For the electrons $F_E = F_M = 1$ & the dipole approximation, $F_M = G_M^2$ and $F_E = (4M_p^2 G_E^2 + Q^2 G_M^2)/(4M_p^2 + Q^2).$
- Inelastic scattering: Proton does not survive the interaction and dissociates into a state of the invariant mass $M_N > M_{p_1} F_E = \int dx F_2/x$ and $F_M = \int dx F_2/x^3$, where $F_2(x, Q^2)$ is the proton structure function and $M_N^2 - M_n^2 = Q^2(1/x - 1)$.
- The ALLM parametrization for the proton structure function is used, in which gives a good description of the DIS data for the kinematical region of $10^{-6} < x < 0.85$ and $0 < Q^2 < 5000 \text{ GeV}^2$ [arXiv:hep-ph/9712415 [hep-ph]].
- ALLM parametrization is based on the fit to the experimental data on the measurement of the total $\gamma^* p$ cross-sections.

Luminosity spectrum $S_{\gamma\gamma}$ & Integrated luminosity spectra, $\int dW S_{\gamma\gamma}$

- It represents the fraction of the *ep* luminosity available for $\gamma\gamma$ collisions.
- Large fraction the total $\gamma\gamma$ luminosity \rightarrow To makes the LHeC an excellent place to study the photonphoton interactions \rightarrow An extraordinary $\gamma\gamma$ collider!



Figure: (left) The elastic (inelastic) luminosity spectrum $S_{\gamma\gamma}$ at the LHeC, (right) the integrated luminosity spectra, $\int dW S_{\gamma\gamma}$, as a function of the minimal $\gamma\gamma$ center-of-mass energy, W_0 .

• The production rate of massive objects is limited by the photon luminosity at high invariant mass & However, they could be produced in a very clean experimental environment.

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Exclusive production of electron and muon pairs at LHeC



Figure: The integrated lepton pairs ($\ell = e, \mu$) production cross sections, $\int_{W_0}^{\sqrt{s}} dW S_{\gamma\gamma} \sigma_{\ell+\ell}$ at the LHeC.

- Very large statistics of e^+e^- and $\mu^+\mu^-$ pairs production via two-photon interactions are expected at LHeC, both in the elastic process $ep \rightarrow e\ell^+\ell^-p$ and the inelastic case $ep \rightarrow e\ell^+\ell^-X$, where $\ell = e, \mu$.
- Very clean experimental signature, involving the detection of simple objects such as leptons and photons.
- The $\gamma\gamma \rightarrow \mu^+\mu^-$ and $\gamma\gamma \rightarrow e^+e^-$ pairs serve as excellent calibration tools.
- HERA measurements: Multi-electron production [Eur. Phys. J. C 31 (2003) 17-29] & Production of two isolated muons [Phys. Lett. B 583 (2004) 28-40] both dominated by photon-photon collisions.

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Photon-photon factory of au pairs

- Large statistics of elastic (inelastic) $\tau^+ \tau^-$ pairs produced for W > 10 GeV.
- Signal topologies \rightarrow The hadronic and leptonic decay of the au lepton.
- The most significant background arises from the production of electron or muon pairs, $ep \rightarrow ee^+e^-X$ and $ep \rightarrow e\mu^+\mu^-X$ & Misidentification or mismeasurement of NC DIS ($ep \rightarrow eX$) and photoproduction ($\gamma p \rightarrow X$).
- Another interesting electroweak potential of τ pairs → competitive determinations of the electrical and magnetic dipole moments of the τ lepton which is highly sensitive to the New Physics [Phys. Rev. D 106 (2022) 039902].
- High statistics of τ pair production at LHeC \rightarrow to probe the a_{τ} with high precision.



Figure: Two-photon production of τ pairs ($\gamma\gamma \rightarrow \tau^+\tau^-$) at the LHeC & The integrated τ pair production cross sections, $\int_{W_0}^{\sqrt{s}} dW S_{\gamma\gamma} \sigma_{\tau^+\tau^-}$ at the LHeC & New physics can modify $\tau - \gamma$ couplings affecting the magnetic moment.

• HERA measurement: [Eur. Phys. J. C 48 (2006) 699-714; JHEP 02 (2011) 11] dominated by the $\gamma\gamma \rightarrow \tau^+ \tau^-$ process & Tau-pair production by DELPHI Collaboration [Eur. Phys. J. C 35 (2004) 159-170].

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Exclusive W boson pair production

- Multiple signal topologies & Fully leptonic decays (involving muon and electron) of the W boson are particularly promising & Requirement of lepton pairs of different flavor with sufficiently high transverse momentum.
- The primary backgrounds at the LHeC in the context of $\gamma\gamma \rightarrow W^+W^-$ pair production: $\gamma\gamma \rightarrow e^+e^-/\mu^+\mu^-$ & $\gamma\gamma \rightarrow \tau^+\tau^-$.
- Considerable cross-section + favorable experimental condition at LHeC \rightarrow Well suited to study the beyond SM phenomena \rightarrow Anomalous Triple γWW and Quartic $\gamma \gamma WW$ Gauge Couplings.



Figure: Leading-order Feynman diagrams contributing to the exclusive photon-induced production of W-boson pairs & The integrated W boson pair cross sections, $\int_{W_0}^{W_0} dW S_{\gamma\gamma} \sigma_{WW}$ at the LHeC.

• Recent experimental efforts by ATLAS [Phys. Lett. B 816 (2021) 136190] and CMS [JHEP 07 (2023) 229] collaborations for the observation of such process at LHC; $pp(\gamma\gamma) \rightarrow p^{(*)}W^+W^-p^{(*)}$.

Unique Z boson laboratory

- The exclusive production of ZZ pairs through two-photon interactions at the LHeC \rightarrow excellent platform for investigating the Anomalous Quartic Gauge Couplings (AQGC), specifically those involving $\gamma\gamma ZZ \rightarrow$ Hint of New Physics beyond the Standard Model [0908.2020 [hep-ph]].
- In its high multiplicity of decay channels, the Z boson provides several final states & The decay channel of interest $\gamma\gamma \rightarrow ZZ \rightarrow \ell^+ \ell^- \nu_\ell \bar{\nu}_\ell$.



Figure: The integrated Z-boson pair production cross sections, $\int_{W_0}^{\sqrt{s}} dW S_{\gamma\gamma} \sigma_{ZZ}$, as a function of the minimal $\gamma\gamma$ center-of-mass energy, W_0 .

• The Standard Model cross-section for the neutral gauge boson production process $ep \rightarrow e\gamma^*\gamma^*p \rightarrow eZZp$ is anticipated to be notably small; not allowed at the tree level.

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Production of the Higgs boson via photon-photon fusion

- The future LHeC and its high-energy upgrade FCC-eh have emerged as potential candidates for a 'Higgs factory' experiment.
- The cross-section for photon-photon production of the Higgs boson at the LHeC is calculated to be 0.145 fb for the case of elastic, and 0.265 fb for the case of inelastic events, with $M_N < 10$ GeV and $Q_{p,\rm max}^2 < 10$ GeV², comparable to that of FCCee/CEPC [Chin. Phys. C 47 (2023) 013001].



Figure: Feynman diagrams of the Higgs photo-production process through W (left) and charged fermion loops (right).

• Various decay channels of the Higgs boson can be taken into account, including the main channels of $h \rightarrow b\bar{b}$, $h \rightarrow ZZ$, and $h \rightarrow W^+W^-$ & Challenging to identify the photo-production of the Higgs boson due to SM backgrounds.

Two-photon exclusive production of supersymmetric pairs

- Exploring non-strongly interacting supersymmetric (SUSY) particle production, like pairs of sleptons $\tilde{\ell}$, via photon-photon ($\gamma\gamma$) collisions at the LHeC.
- Utilizing the high luminosity and clean experimental conditions at the LHeC to study these processes, improving statistical significance and reducing background noise.



Figure: Integrated higgsionos pair production cross sections at the LHeC for a given higgsionos mass of 100 GeV.

Figure: Integrated sleptons pair production cross sections at the LHeC for a given value of $m_{\tilde{\ell}}=100~{\rm GeV}.$

 $\label{eq:Figure: Exclusive pair production} figure: Exclusive pair production of scalar leptons (sleptons) <math display="inline">\tilde{\ell}$ at the LHeC with specific decay to ℓ and the lightest neutralino dark matter $\tilde{\chi}_1^0.$

 The production rates are expected to be low for larger values of masses & Experimental search for the supersymmetric particles could impose limits on their masses [Phys. Rept. 364 (2002) 359-450; Phys. Rev. Lett. 123 (2019) 141801].

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- LHeC as a combination of very high luminosities available & advanced high-resolution detectors & very clean experimental environment.
- Favorable LHeC experimental conditions in which include very low event pileup & powerful exclusivity selections & covering very wide rapidity ranges.
- LHeC would play as an optimal condition for further investigations into $\gamma\gamma$ processes.
- LHeC reveals excellent prospects for studying the exclusive production of lepton pairs, pairs of W and Z bosons, Higgs boson, as well as pairs of charged supersymmetric particles via $\gamma\gamma$ processes.
- LHeC will significantly enhance and complement the scientific endeavors of the HL-LHC, particularly in the realms of QCD and Electroweak physics.



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Processes (elastic)	EPA	GRAPE	GRAPE (ISR=1)
$\tau^+ \tau^- (Q_{e,max}^2/Q_{p,max}^2 = 1/1 \text{ GeV}^2)$	41.3	41.2	41.7
$\tau^+ \tau^- (Q_{e,max}^2/Q_{p,max}^2 = 10/10 \text{ GeV}^2)$	46.2	46.3	46.8
$\tau^+ \tau^- (Q_{e,max}^2/Q_{p,max}^2 = 100/100 \text{ GeV}^2)$	51.2	49.5	50.3

Table: The $\tau^+\tau^-$ production cross sections in pb at LHeC calculated at W=10 GeV using the EPA approach in comparison with GRAPE MC event generator.

Processes (elastic)	EPA	GRAPE	GRAPE (ISR=1)
$\mu^{+}\mu^{-} (Q_{e,\max}^2/Q_{p,\max}^2 = 1/1 \text{ GeV}^2)$	107.47	109.11	115.05
$\mu^+\mu^- (Q_{e,\max}^2/Q_{p,\max}^2 = 10/10 \text{ GeV}^2)$	120.90	121.37	127.61
$\mu^+\mu^- (Q_{e,\max}^2/Q_{p,\max}^2 = 50/50 \text{ GeV}^2)$	129.81	127.14	133.87

Table: The $\mu^+\mu^-$ production cross sections in pb at LHeC calculated at W=10 GeV using the EPA approach, and GRAPE MC event generator.



Figure: The integrated lepton pairs ($\ell = e, \mu$) production cross sections, $\int_{W_0}^{\sqrt{s}} dW S_{\gamma\gamma} \sigma_{\ell^+\ell^-}$ at the LHeC.

- Very large statistics of e^+e^- and $\mu^+\mu^-$ pairs production via two-photon interactions are expected at LHeC.
- Very clean experimental signature, involving the detection of simple objects such as leptons and photons.
- $\gamma\gamma \rightarrow \mu^+\mu^-$ and $\gamma\gamma \rightarrow e^+e^-$ pairs serve as excellent calibration tools.
- HERA measurements: Multi-electron production [Eur. Phys. J. C 31 (2003) 17-29] & Production of two isolated muons [Phys. Lett. B 583 (2004) 28-40] both dominated by photonphoton collisions.

EPA/GRAPE Comparison

Processes (elastic)	EPA	GRAPE
$(Q_{e,max}^2/Q_{p,max}^2 = 1/1 \text{ GeV}^2)$	107.47	109.11
$(Q_{e,\max}^2/Q_{p,\max}^2 = 10/10 \text{ GeV}^2)$	120.90	121.37
$(Q_{\rm e,max}^2/Q_{\rm p,max}^2$ = 50/50 GeV ²)	129.81	127.14

Table: The $\mu^+\mu^-$ production cross sections at LHeC at W=10 GeV calculated using the EPA approach in comparison with GRAPE MC event generator.

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