## Andrea Simonelli

In collaboration with M. Boglione

## Disentangling Soft Effects from TMD Fragmentation Functions



## TMD Factorization

- Drell-Yan

$$
p p \rightarrow e^{+} e^{-} X
$$

- Semi-Inclusive DIS

$$
e^{-} p \rightarrow e^{-} h X
$$

- Double-Inclusive Annihilation

$$
e^{+} e^{-} \rightarrow h_{1} h_{2} X
$$

$$
d \sigma=|H|^{2} \int \frac{d \vec{b}_{T}}{(2 \pi)^{2}} e^{-i \vec{q}_{T} \cdot \vec{b}_{T}} D_{A}\left(z_{A}, b_{T}, y_{A}-y_{n}\right) D_{B}\left(z_{B}, b_{T}, y_{n}-y_{B}\right)
$$

Such cross section actually comes from the re-arranging of:


"Hidden" and not directly accessible soft factor correlating the two collinear groups


Where:

$$
\begin{aligned}
& D^{\text {uns }}\left(z, b_{T}, y_{\text {had }}-(-\infty)\right)=\frac{\operatorname{Tr}_{c}}{N_{c}} \frac{\operatorname{Tr}_{D}}{4} \sum_{X} \frac{1}{z} \int \frac{d x^{-}}{2 \pi} e^{i k^{+} x^{-}} \quad x=\left(0, x^{-}, \vec{b}_{T} / 2\right) \\
& \langle 0| \gamma^{+} W_{-}(x / 2 \rightarrow \infty)|P ; X\rangle\langle P ; X| W_{-}^{\dagger}(-x / 2 \rightarrow \infty)|0\rangle
\end{aligned}
$$

$$
d \sigma=|H|^{2} \int_{-} \frac{d \vec{b}_{T}}{(2 \pi)^{2}} e^{-i \vec{q}_{T} \cdot \vec{b}_{T}}
$$



$$
D\left(z, b_{T}, y_{\mathrm{had}}-y_{1}\right)=D^{\mathrm{uns}}\left(z, b_{T}, y_{\mathrm{had}}-(-\infty)\right) \sqrt{\frac{S\left(b_{T}, \infty-y_{1}\right)}{S\left(b_{T}, \infty-(-\infty)\right) S\left(b_{T}, y_{1}-(-\infty)\right)}}
$$

- Light-cone limit $\left\{\begin{array}{l}y_{1} \rightarrow \infty \\ y_{2} \rightarrow-\infty\end{array}\right.$
- Soft Evolution $S\left(y_{A}-y_{B}\right) \propto S\left(y_{A}-y_{C}\right) S\left(y_{C}-y_{B}\right)$


## Soft Factor



$$
\cosh \phi_{M}=\frac{n \cdot \bar{n}}{\sqrt{n^{2} \bar{n}^{2}}} \equiv \cosh \left(y_{1}-y_{2}\right)
$$

$$
\phi_{M}=y_{1}-y_{2} \quad \text { Minkowskian angle }
$$

$$
\phi_{M} \rightarrow \infty \quad \text { TMD Factorization }
$$

$$
\phi_{M} \rightarrow 0 \quad \text { Bremsstrahlung function }
$$

$$
\phi_{M} \rightarrow i \pi \quad \text { quark-antiquark potential }
$$

$$
\mathcal{S}\left(b_{T}, \phi_{M}\right)=\frac{\operatorname{Tr}}{N}\langle 0| W_{\mathcal{C}}\left(b_{T}, \phi_{M}\right)|0\rangle=\frac{\operatorname{Tr}}{N} \mathcal{P} Z_{S}\langle 0| e^{-i g_{0} \oint_{\mathcal{C}} d x^{\mu} A_{\mu}^{(0), a}(x) t_{a}}|0\rangle
$$

$$
\begin{align*}
\Gamma & =4 C_{R} \frac{\alpha_{s}}{4 \pi}\left\{\varphi \operatorname{coth} \varphi-1+\frac{\alpha_{s}}{4 \pi}\left[C _ { A } \left[\frac{2}{3} \pi^{2}-\frac{49}{9}+2 \varphi^{2}\right.\right.\right. \\
& +\operatorname{coth} \varphi\left(2 \operatorname{Li}_{2}\left(e^{-2 \varphi}\right)-4 \varphi \log \left(1-e^{-2 \varphi}\right)-\frac{\pi^{2}}{3}-\frac{2}{3} \pi^{2} \varphi+\frac{67}{9} \varphi-2 \varphi^{2}-\frac{2}{3} \varphi^{3}\right) \\
& \left.+\operatorname{coth}^{2} \varphi\left(2 \operatorname{Li}_{3}\left(e^{-2 \varphi}\right)+2 \varphi \operatorname{Li}_{2}\left(e^{-2 \varphi}\right)-2 \zeta_{3}+\frac{\pi^{2}}{3} \varphi+\frac{2}{3} \varphi^{3}\right)\right] \\
- & \left.\left.\frac{20}{9} T_{F} n_{f}(\varphi \operatorname{coth} \varphi-1)\right]+\mathcal{O}\left(\alpha_{s}^{2}\right)\right\} \\
= & 4 C_{R} \frac{\alpha_{s}}{4 \pi}\{\varphi \operatorname{coth} \varphi-1 \\
+ & \frac{\alpha_{s}}{4 \pi}\left[C _ { A } \left[2\left(1+\frac{2}{3} \varphi^{2}\right)-\frac{1}{3}(\varphi \operatorname{coth} \varphi-1)\left(2 \pi^{2}-\frac{67}{3}+2 \varphi^{2}\right)\right.\right. \\
& +\operatorname{coth} \varphi(\varphi \operatorname{coth} \varphi+1)\left(\operatorname{Li}_{2}\left(1-e^{2 \varphi}\right)-\operatorname{Li}_{2}\left(1-e^{-2 \varphi}\right)\right) \\
& \left.-2 \operatorname{coth}{ }^{2} \varphi\left(\operatorname{Li}_{3}\left(1-e^{2 \varphi}\right)+\operatorname{Li}_{3}\left(1-e^{-2 \varphi}\right)\right)\right] \\
- & \left.\left.\frac{20}{9} T_{F} n_{f}(\varphi \operatorname{coth} \varphi-1)\right]+\mathcal{O}\left(\alpha_{s}^{2}\right)\right\} \tag{4.2}
\end{align*}
$$

## Non-Abelian Exponentiation Theorem

See works by E. Gardi, E. Leanen, L. Magnea, C. White etc...

Webs in multiparton scattering using the replica trick
Einan Gardi (Edinburgh U.), Eric Laenen (Amsterdam U. and Utrecht U. and NIKHEF, Amsterdam), Ge Stavenga (Fermilab), Chris D. White (Glasgow U. and Durham U., IPPP and Durham U.) (Aug, 2010) Published in: JHEP 11 (2010) 155 • e-Print: 1008.0098 [hep-ph]
同 bdf
(2) DOI
$\stackrel{r}{\square}$ cite
F ${ }^{\circ}$ claim
层 reference search
$\rightarrow 112$ citá


In the light-cone limit:
$\phi_{M} \rightarrow \infty$

$$
S\left(b_{T} ; \phi_{M}\right)=e^{\phi_{M} K\left(b_{T}\right)+P\left(b_{T}\right)}
$$

Studied a lot in TMD factorization...



Where is it and how can we access it?

The P-terms disappear in the standard TMD factorization...

$$
\begin{aligned}
& \frac{\mathcal{D}_{A}^{\text {uns. }}\left(z_{A}, b_{T} ; y_{A}-(-\infty)\right)}{\mathcal{S}\left(b_{T} ; y_{1}-(-\infty)\right)} \times \mathcal{S}\left(b_{T} ; y_{1}-y_{2}\right) \times \frac{\mathcal{D}_{B}^{\text {uns. }}\left(z_{B}, b_{T} ; \infty-y_{B}\right)}{\mathcal{S}\left(b_{T} ; \infty-y_{2}\right)} \\
& \quad=\frac{\mathcal{D}_{A}^{\text {uns. }}\left(z_{A}, b_{T} ; y_{A}-(-\infty)\right)}{e^{\left(y_{1}-(-\infty)\right) K+\frac{x}{2} P}} \times e^{\left(y_{1}-y_{2}\right) K+R} \times \frac{\mathcal{D}_{B}^{\text {uns. }}\left(z_{B}, b_{T} ; \infty-y_{B}\right)}{e^{\left(\infty-y_{2}\right) K+\frac{y}{2} P}}
\end{aligned}
$$

...as well as in the standard TMD definition:

$$
\begin{aligned}
D\left(y_{\text {had }}-y_{1}\right) & =D^{\text {uns. }}\left(y_{\text {had }}-(-\infty)\right) \sqrt{\frac{\mathcal{S}\left(\infty-y_{1}\right)}{\mathcal{S}(\infty-(-\infty)) \mathcal{S}\left(y_{1}-(-\infty)\right)}} \\
& =D^{\text {uns. }}\left(y_{\text {had }}-(-\infty)\right) \sqrt{\frac{e^{\left(\infty-y_{1}\right) K+\frac{\lambda}{2} R}}{e^{(\infty-(-\infty)) K} e^{\left(y_{1}-(-\infty)\right) K+\frac{1}{2} P}}}
\end{aligned}
$$

The standard definition is optimal for standard TMD factorization.

We can forget about the existence of the P-term

This is a naïve proof! Actual proof requires to in the standard cases. modify also the unsubtracted TMDs

## A non－standard case

Single－Inclusive Annihilation（SIA）with thrust $e^{+} e^{-} \rightarrow h X$


## Data available since 2019

Transverse momentum dependent production cross sections of charged pions，kaons and protons produced in inclusive $e^{+} e^{-}$annihilation at $\sqrt{s}=$ 10.58 GeV

Belle Collaboration • R．Seidl（RIKEN BNL）et al．（Feb 5，2019）
Published in：Phys．Rev．D 99 （2019）11， 112006 • e－Print： 1902.01552 ［hep－ex］
园 pdf © DOI E cite 庿 claim reference search $\rightarrow 34$ citations

The transverse momentum of the detected hadron is measured w．r．t．the thrust axis

$$
z_{h}=\frac{E}{Q / 2}, \quad T=\frac{\sum_{i}\left|\vec{P}_{(\text {c.m. }), i} \cdot \widehat{n}\right|}{\sum_{i}\left|\vec{P}_{(\text {c.m. }), i}\right|}, \quad P_{T} \text { w.r.t } \vec{n}
$$

Complete theoretical treatment and first phenomenology is now available
Full treatment of the thrust distribution in single inclusive $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hX}$ processes
M．Boglione（Turin U．and INFN，Turin），A．Simonelli（Old Dominion U．and Jefferson Lab）（Jun 5，2023） Published in：JHEP 09 （2023） $006 \cdot$ e－Print： 2306.02937 ［hep－ph］


Different kinematics leads to different factorized cross sections

Factorization works in the same way for all the three regions, but it produces different results depending on the underlying kinematics

The hadron is detected very close to the axis of the jet.
A. Simonelli
$\square$ Extremely small $P_{T}$
Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

The hadron is detected in the central region of the jet.
$\square$ Most common scenario
$\square$ Majority of experimental data expected to fall into this case

The hadron is detected near the boundary of the jet.
$\square$ Moderately small $P_{T}$
The hadron transverse momentum affects the topology of the final state directly

|  | soft | soft-collinear | collinear |
| :---: | :---: | :---: | :---: |
| $R_{1}$ | TMD-relevant | TMD-relevant | TMD-relevant $\\|$ |
| $R_{2}$ | TMD-irrelevant | TMD-relevant | TMD-relevant |
| $R_{3}$ | TMD-irrelevant | TMD-irrelevant | TMD-relevant $\\|$ |

$$
\begin{aligned}
& d \sigma_{R_{2}}=|H|^{2} \int \frac{d u}{2 i \pi} e^{u \tau} \int \frac{d \vec{b}_{T}}{(2 \pi)^{2}} e^{i z \vec{P}_{T} \cdot \vec{b}_{T}} \\
& J\left(u, \infty-y_{j}\left\{\begin{array}{l}
\widehat{S}\left(u, y_{1}-y_{2}\right) \\
\widehat{S}\left(u, \infty-y_{2}\right) \\
\end{array} D^{\star}\left(z, b_{T}, y_{\text {had }-y_{1}}\right)\right.\right.
\end{aligned}
$$

Not same functional form! No magic this
 time...

Red blobs are TMD-relevant
Blue blobs are TMD-irrelevant

We are forced to use the definition*. There are disadvantages but also...

- More universal (no soft contamination)
- Inclusion of P-term effects (new physics!)


## Accessing the P -term

From comparing operators:

$$
\begin{aligned}
& D\left(z, b_{T}, y_{\text {had }}-y_{1}\right)=D^{\star}\left(z, b_{T}, y_{\text {had }}-y_{1}\right)
\end{aligned} e^{-\frac{1}{2} P\left(b_{T}\right)} \begin{gathered}
\text { TMD extraction from beyond } \\
\text { standard process (SIA R2 })
\end{gathered}
$$

Hidden soft effects

Oss: $\frac{d P\left(b_{T}, \mu\right)}{d \log \mu}=-\gamma_{P}\left(a_{S}(\mu)\right)$

$$
\begin{aligned}
& \frac{\partial \log D^{\star}\left(z, b_{T} ; \mu, y_{1}\right)}{\partial y_{1}}=-K\left(b_{T}, \mu\right) \\
& \frac{\partial \log D^{\star}\left(z, b_{T} ; \mu, y_{1}\right)}{\partial \log \mu}=\gamma_{d}\left(a_{S}(\mu)\right)+\frac{1}{2} \gamma_{P}\left(a_{S}(\mu)\right)-\gamma_{K}\left(a_{S}(\mu)\right) \log \frac{Q e^{-y_{1}}}{\mu}
\end{aligned}
$$

$$
R=\frac{\begin{array}{c}
\text { TMD extraction from beyond } \\
\text { standard process }
\end{array}}{\substack{\text { TMD extraction from } \\
\text { standard process }}}=e^{-\frac{1}{2} P\left(b_{T}, \mu\right)}
$$

This relation is exact, provided that:

1. Both extractions are performed @ same perturbative accuracy
2. Both extractions use the same model for the non-perturbative behavior of the CS-kernel
... in practice, given the extractions available today, none of the assumptions above are satisfied at the same time.

Also, the comparison is cleaner if both extractions use the same prescription for separating out the non-perturbative effects in $b_{T}$-space (CSS b*, HSO...)


The function $g_{p}$ for the $P$-term is the counterpart of $g_{\kappa}$ for the CS-kernel.
Then, effectively: $\quad D_{i / h}^{\star}\left(z, b_{T}, \mu, y_{1}\right)=\frac{1}{z^{2}} C_{i / j} \otimes d_{j / h}\left(z, \mu_{b}\right)$

$$
\begin{aligned}
& e^{\frac{1}{2} K\left(a_{S}\left(\mu_{b}^{\star}\right)\right) \log \frac{Q e^{-y_{1}}}{\mu_{b}^{\star}}+\int_{\mu_{b}^{\star}}^{\mu} \frac{d \mu^{\prime}}{\mu^{\prime}} \gamma_{D}\left(a_{S}\left(\mu^{\prime}\right), \log \frac{Q e^{-y_{1}}}{\mu^{\prime}}\right)} \\
& M_{D}\left(z, b_{T}\right) e^{\frac{1}{2} g_{P}\left(b_{T}\right)} e^{-\frac{1}{2} g_{K}\left(b_{T}\right) \log \frac{Q e^{-y_{1}}}{M_{\text {had }}}}
\end{aligned}
$$

Effective combination extracted from

## Ideally： <br> BS23（NP－model）

Full treatment of the thrust distribution in single inclusive $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hX}$
processes
M．Boglione（Turin U．and INFN，Turin），A．Simonelli（Old Dominion U．and Jefferson Lab）（Jun 5，2023）
Published in：JHEP 09 （2023） 006 • e－Print： 2306.02937 ［hep－ph］
$R=\xrightarrow{\text { and }}$

SV19（NP－model）
Non－perturbative structure of semi－inclusive deep－inelastic and Drell－Yan scattering at small transverse momentum
Ignazio Scimemi（Madrid U．），Alexey Vladimirov（Regensburg U．）（Dec 13，2019）
Published in：JHEP 06 （2020） 137 • e－Print： 1912.06532 ［hep－ph］
（2）pdf © DOI E cite 庿 claim reference search $\bigoplus 138$ citation

MAP22（NP－model）
Unpolarized transverse momentum distributions from a global fit of Drell－Yan
and semi－inclusive deep－inelastic scattering data
MAP（Multi－dimensional Analyses of Partonic distributions）Collaboration • Alessandro Bacchetta（Pavia
and INFN，Pavia）et al．（Jun 15， 2022
⓪ pdf © DOI 巨 cite 局 claim 层 reference search Э 50 citation：

The plan is：
$\square$ Check z－independence of R （insensitive to collinear physics）
$\square$ Infer information on $g_{p}$

## Ratio w．r．t SV19

## Full treatment of the thrust distribution in single inclusive $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hX}$

processes
M．Boglione（Turin U．and INFN，Turin），A．Simonelli（Old Dominion U．and Jefferson Lab）（Jun 5，2023）
Published in：JHEP 09 （2023） 006 • e－Print： 2306.02937 ［hep－ph］
（ ${ }^{2}$ pdf
（2）DOI
を cite
四 claim
层 reference search
$\ni 1$ citatior
$M_{D}\left(z, b_{T}\right)=\frac{2}{\Gamma(p(z)-1)}\left(\frac{b_{T} m(z)}{2}\right)^{p(z)-1} K_{p(z)-1}\left(b_{T} m(z)\right)$


## 4 free parameters

Published in：JHEP 06 （2020） 137 • e－Print： 1912.06532 ［hep－ph］
K pdf © DOI E cite 层 claim reference search $\rightleftharpoons 138$ citation
$D_{N P}(x, b)=\exp \left(-\frac{\eta_{1} z+\eta_{2}(1-z)}{\sqrt{1+\eta_{3}(\boldsymbol{b} / z)^{2}}} \frac{\boldsymbol{b}^{2}}{z^{2}}\right)\left(1+\eta_{4} \frac{\boldsymbol{b}^{2}}{z^{2}}\right)$


Test the ansatz:

- Extractions @ same perturbative accuracy
- Extractions use the same $g_{k}$
$\times$
$\times$


## BS23 SV19

| NLL | N 2 LL |
| :--- | :--- |
| $\sim$ constant | $\sim$ linear $\longrightarrow$z-independence <br> affected |
| $\mathrm{b}_{\mathrm{T}}$-dependence |  |
| affected |  |

Still:


Test the ansatz:

- Extractions @ same perturbative accuracy
- Extractions use the same $g_{k}$
$\times$
$\times$


## BS23 SV19

| NLL | N 2 LL |
| :--- | :--- |
| $\sim$ constant | $\sim$ linear $\longrightarrow$z-independence <br> affected |
| $\mathrm{b}_{\mathrm{T}}$-dependence |  |
| affected |  |

Still:


Test the ansatz:

- Extractions @ same perturbative accuracy
- Extractions use the same $g_{k}$

$\times$


## BS23 SV19

| NLL | N 2 LL |
| :--- | :--- |
| $\sim$ constant | $\sim$ linear $\longrightarrow$z-independence <br> affected |
| $\mathrm{b}_{\mathrm{T}}$-dependence |  |
| affected |  |

Still:


Test the ansatz:

- Extractions @ same perturbative accuracy
- Extractions use the same $g_{k}$

$\times$


## BS23 SV19

| NLL | N 2 LL |
| :--- | :--- |
| $\sim$ constant | $\sim$ linear $\longrightarrow$z-independence <br> affected |
| $\mathrm{b}_{\mathrm{T}}$-dependence |  |
| affected |  |

Still:


Test the ansatz:

- Extractions @ same perturbative accuracy
- Extractions use the same $g_{k}$

$\times$


## BS23 SV19

| NLL | N 2 LL |
| :--- | :--- |
| $\sim$ constant | $\sim$ linear $\longrightarrow$z-independence <br> affected |
| $\mathrm{b}_{\mathrm{T}}$-dependence |  |
| affected |  |

Still:


Test the ansatz:

- Extractions @ same perturbative accuracy
- Extractions use the same $g_{k}$

$\times$
BS23 SV19

| NLL N 2 LL | $\longrightarrow$ |
| :--- | :--- | | z-independence |
| :--- |
| affected |
| $\mathrm{b}_{\mathrm{T}}$-dependence |
| affected |

Still:


| $T$ | $z$ |  |  |  |  |  |  |  |  |  |  |  | $P_{T} / z \max$ | N |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $$ | $$ | $\circ$ 0 1 1 0 0 0 | $\begin{aligned} & 10 \\ & \hline 1 \\ & 1 \\ & 1 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} 8 \\ 0 \\ 0 \\ 1 \\ 10 \\ \vdots \\ 0 \end{gathered}$ | $\begin{aligned} & 10 \\ & 0 \\ & 0 \\ & 1 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 8 \\ & 0 \\ & 0 \\ & 1 \\ & 1 \\ & 10 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 10 \\ & 0 \\ & 0 \\ & 1 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 8 \\ & 0 \\ & 0 \\ & 1 \\ & 10 \\ & 0 \\ & 0 \end{aligned}$ | 10 1 0 1 1 0 0 | $\circ$ 0 0 1 10 1 0 |  |  |
| 0.80-0.85 |  |  |  |  |  |  |  |  |  |  |  |  | $0.16 Q$ | 57 |
| 0.85-0.90 |  |  |  |  |  |  |  |  |  |  |  |  | $0.15 Q$ | 60 |
| $0.90-0.95$ |  |  |  |  |  |  |  |  |  |  |  |  | $0.14 Q$ | 61 |
| 0.95-1.00 |  |  |  |  |  |  |  |  |  |  |  |  | $0.13 Q$ | 52 |

SV19

| Experiment | Reaction | ref. | Kinematics | $N_{\mathrm{pt}}$ <br> after cuts |
| :---: | :---: | :---: | :---: | :---: |
| HERMES | $p \rightarrow \pi^{+}$ | [66] | $\begin{aligned} 0.023 & <\mathrm{x}<0.6(6 \mathrm{bins}) \\ 0.2 & <\mathrm{z}<0.8(6 \text { bins }) \\ 1.0 & <\mathrm{Q}<\sqrt{20} \mathrm{GeV} \end{aligned}$ | 24 |
|  | $p \rightarrow \pi^{-}$ |  |  | 24 |
|  | $\overline{p \rightarrow K^{\prime}}$ |  |  | 24 |
|  | $p \rightarrow K^{-}$ |  |  | 24 |
|  | $D \rightarrow \pi^{+}$ |  |  | 24 |
|  | $D \rightarrow \pi^{-}$ |  | $\begin{aligned} W^{2} & >10 \mathrm{GeV}^{2} \\ 0.1 & <\mathrm{y}<0.85 \end{aligned}$ | 24 |
|  | $D \rightarrow K^{+}$ |  |  | 24 |
|  | $D \rightarrow K^{-}$ |  |  | 24 |
| COMPASS | $d \rightarrow h^{+}$ | [67] | $\begin{gathered} 0.003<\mathrm{x}<0.4(8 \text { bins }) \\ 0.2<\mathrm{z}<0.8(4 \text { bins }) \\ 1.0<\mathrm{Q} \simeq 9 \mathrm{GeV}(5 \text { bins }) \end{gathered}$ | 195 |
|  | $d \rightarrow h^{-}$ |  |  | 195 |
| Total |  |  |  | 582 |

## The two extractions overlapping core is

$0.3 \lesssim z \lesssim 0.7$


## z-independence

Keep in mind that:

- Very different functional forms at numerator and denominator
- Extractions from completely different data sets (BELLE vs SIDIS/DY)

Spread of error bands can be used to constraint the "extraction" of $g_{P}$


Asymptotically sub-linear

$$
\begin{aligned}
& g_{P}=-\alpha \log \left(1+\beta b_{T}^{2}\right) \\
& \alpha=2.51 \pm 0.21 \\
& \beta=0.43 \pm 0.07 \mathrm{GeV}^{2}
\end{aligned}
$$

Asymptotically linear
$g_{P}=-\frac{b_{T}^{2}}{\alpha \sqrt{1+\frac{b_{T}^{2}}{\beta^{2}}}}$
$\alpha=0.48 \pm 0.15 \mathrm{GeV}^{-2}$
$\beta=0.65 \pm 0.21 \mathrm{GeV}^{-1}$


## Ratio w．r．t MAP22

Full treatment of the thrust distribution in single inclusive $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{hX}$
processes
M．Boglione（Turin U．and INFN，Turin），A．Simonelli（Old Dominion U．and Jefferson Lab）（Jun 5，2023）
Published in：JHEP 09 （2023） 006 • e－Print： 2306.02937 ［hep－ph］
＊ Adf
（2）DOI
■ cite
届 claim
层 reference search
$\ni 1$ citatior
$M_{D}\left(z, b_{T}\right)=\frac{2}{\Gamma(p(z)-1)}\left(\frac{b_{T} m(z)}{2}\right)^{p(z)-1} K_{p(z)-1}\left(b_{T} m(z)\right)$

## 2 free parameters

Unpolarized transverse momentum distributions from a global fit of Drell－Yan \＃ and semi－inclusive deep－inelastic scattering data
MAP（Multi－dimensional Analyses of Partonic distributions）Collaboration • Alessandro Bacchetta（Pavia U．and INFN，Pavia）et al．（Jun 15，2022）
Published in：JHEP 10 （2022） 127 • e－Print： 2206.07598 ［hep－ph］
K pdf
（2）DOI
E cite
Eoc claim
艮 reference search
$\rightleftharpoons 50$ citations
$D_{1 N P}\left(z, \boldsymbol{b}_{T}^{2} ; \zeta, Q_{0}\right)=\frac{g_{3}(z) e^{-g_{3}(z) \frac{b_{T}^{2}}{4 z^{2}}}+\frac{\lambda_{F}}{z^{2}} g_{3 B}^{2}(z)\left[1-g_{3 B}(z) \frac{b_{T}^{2}}{4 z^{2}}\right] e^{-g_{3 B}(z) \frac{b_{T}^{2}}{4 z^{2}}}}{g_{3}(z)+\frac{\lambda_{F}}{z^{2}} g_{3 B}^{2}(z)}$

## 9 free parameters

$$
g_{3}(z)+\frac{\lambda_{F}}{z^{2}} g_{3 B}^{2}(z)
$$



Test the ansatz:

- Extractions @ same perturbative accuracy
- Extractions use the same $g_{k}$

However:

## BS23 MAP22



z-independence

Not satisfied, or, at least, not constrained.

## Conclusions

$\square$ Hidden soft effects, that cancel out in standard cross sections

$$
S\left(b_{T} ; \phi_{M}\right)=e^{\phi_{M} K\left(b_{T}\right)}+P\left(b_{T}\right)
$$

$\square$ Need to go beyond standard processes to have sensitivity on P-term

$$
d \sigma_{R_{2}} \propto e^{I_{R}\left(u, y_{1}, \mu\right)-\frac{1}{2} P\left(b_{T}, \mu\right)} \approx e^{\frac{1}{2} g_{P}\left(b_{T}\right)}=
$$



Lattice applications


