Andrea Simonelli In collaboration with M. Boglione

Disentangling Soft Effects from TMD Fragmentation Functions







TMD Factorization

• Drell-Yan $pp \rightarrow e^+e^- X$ • Semi-Inclusive DIS $e^-p \rightarrow e^-h X$ • Double-Inclusive $e^+e^- \rightarrow h_1h_2 X$

$$\left(d\sigma = |H|^2 \int \frac{d\vec{b}_T}{(2\pi)^2} e^{-i\vec{q}_T \cdot \vec{b}_T} D_A(z_A, b_T, y_A - y_n) D_B(z_B, b_T, y_n - y_B)\right)$$

Such cross section actually comes from the **re-arranging** of:

$$d\sigma = |H|^2 \int \frac{d\vec{b}_T}{(2\pi)^2} e^{-i\vec{q}_T \cdot \vec{b}_T} \left[D_A^*(z_A, b_T, y_A - y_1) S(b_T, y_1 - y_2) D_B^*(z_B, b_T, y_2 - y_B) \right]$$

"Hidden" and **not directly accessible** soft factor **correlating** the two collinear groups

$$d\sigma = |H|^2 \int \frac{d\vec{b}_T}{(2\pi)^2} e^{-i\vec{q}_T \cdot \vec{b}_T} \left\{ \frac{D_A^{\text{uns.}}(z_A, b_T, y_A - (-\infty))}{S(b_T, y_1 - (-\infty))} \right\} S(b_T, y_1 - y_2) \left\{ \frac{D_B^{\text{uns.}}(z_B, b_T, \infty - y_B)}{S(b_T, \infty - y_2)} \right\}$$

Where:

$$D^{\text{uns}}(z, b_T, y_{\text{had}} - (-\infty)) = \frac{\text{Tr}_c}{N_c} \frac{\text{Tr}_D}{4} \sum_X \frac{1}{z} \int \frac{dx^-}{2\pi} e^{ik^+x^-} \qquad x = \left(0, x^-, \vec{b}_T/2\right)$$
$$\langle 0|\gamma^+ W_-(x/2 \to \infty) |P; X\rangle \langle P; X| W_-^{\dagger}(-x/2 \to \infty) |0\rangle$$

$$d\sigma = |H|^2 \int \frac{d\vec{b}_T}{(2\pi)^2} e^{-i\vec{q}_T \cdot \vec{b}_T} \left\{ \begin{array}{c} D_A^{\text{uns.}}(z_A, b_T, y_A - (-\infty)) \\ S(b_T, y_1 - (-\infty)) \end{array} \right\} S(b_T, y_1 - y_2) \left\{ \begin{array}{c} D_B^{\text{uns.}}(z_B, b_T, \infty - y_B) \\ S(b_T, \infty - y_2) \end{array} \right\}$$

$$TMD Parton Distribution and Fragmentation Functions with QCD Evolution Some related to flysolve 0 as (2011) 11402 \cdot e^{\text{price} 1105 557 (\text{pop pol})} \\ \hline Magic \text{ of } 2\text{-hadron processes} \end{array}$$

$$D(z, b_T, y_{\text{had}} - y_1) = D^{\text{uns}}(z, b_T, y_{\text{had}} - (-\infty)) \sqrt{\frac{S(b_T, \infty - y_1)}{S(b_T, \infty - (-\infty)) S(b_T, y_1 - (-\infty))}}$$

$$\bullet \text{ Light-cone limit } \begin{cases} y_1 \to \infty \\ y_2 \to -\infty \end{cases}$$

• Soft Evolution
$$S(y_A - y_B) \propto S(y_A - y_C) S(y_C - y_B)$$

Soft Factor



$$\mathcal{S}(b_T,\phi_M) = \frac{\mathrm{Tr}}{N} \langle 0|W_{\mathcal{C}}(b_T,\phi_M)|0\rangle = \frac{\mathrm{Tr}}{N} \mathcal{P} Z_S \langle 0|e^{-ig_0} \oint_{\mathcal{C}} dx^{\mu} A^{(0),a}_{\mu}(x) t_a |0\rangle$$

QCD cusp anomalous dimension: current status Andrey Grozin (Novosibirsk, IYF) (Dec 10, 2022)

e-Print: 2212.05290 [hep-ph]

$$\begin{split} \Gamma &= 4C_R \frac{\alpha_s}{4\pi} \left\{ \varphi \coth \varphi - 1 + \frac{\alpha_s}{4\pi} \left[C_A \left[\frac{2}{3} \pi^2 - \frac{49}{9} + 2\varphi^2 \right] \right. \\ &+ \coth \varphi \left(2 \operatorname{Li}_2(e^{-2\varphi}) - 4\varphi \log(1 - e^{-2\varphi}) - \frac{\pi^2}{3} - \frac{2}{3} \pi^2 \varphi + \frac{67}{9} \varphi - 2\varphi^2 - \frac{2}{3} \varphi^3 \right) \right] \\ &+ \coth^2 \varphi \left(2 \operatorname{Li}_3(e^{-2\varphi}) + 2\varphi \operatorname{Li}_2(e^{-2\varphi}) - 2\zeta_3 + \frac{\pi^2}{3} \varphi + \frac{2}{3} \varphi^3 \right) \right] \\ &- \frac{20}{9} T_F n_f (\varphi \coth \varphi - 1) \right] + \mathcal{O}(\alpha_s^2) \bigg\} \\ &= 4C_R \frac{\alpha_s}{4\pi} \left\{ \varphi \coth \varphi - 1 \right. \\ &+ \frac{\alpha_s}{4\pi} \left[C_A \left[2 \left(1 + \frac{2}{3} \varphi^2 \right) - \frac{1}{3} (\varphi \coth \varphi - 1) \left(2\pi^2 - \frac{67}{3} + 2\varphi^2 \right) \right. \\ &+ \coth \varphi (\varphi \coth \varphi + 1) \left(\operatorname{Li}_2(1 - e^{2\varphi}) - \operatorname{Li}_2(1 - e^{-2\varphi}) \right) \right] \\ &- 2 \coth^2 \varphi \left(\operatorname{Li}_3(1 - e^{2\varphi}) + \operatorname{Li}_3(1 - e^{-2\varphi}) \right) \bigg] \end{split}$$

$$(4.2)$$

Non-Abelian Exponentiation Theorem

See works by E. Gardi, E. Leanen, L. Magnea, C. White etc...

Webs in multiparton scattering using the replica trick

Einan Gardi (Edinburgh U.), Eric Laenen (Amsterdam U. and Utrecht U. and NIKHEF, Amsterdam), Ge Stavenga (Fermilab), Chris D. White (Glasgow U. and Durham U., IPPP and Durham U.) (Aug, 2010) Published in: *JHEP* 11 (2010) 155 • e-Print: 1008.0098 [hep-ph]

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The P-terms **disappear** in the standard TMD factorization...

$$\frac{\mathcal{D}_{A}^{\text{uns.}}(z_{A}, b_{T}; y_{A} - (-\infty))}{\mathcal{S}(b_{T}; y_{1} - (-\infty))} \times \mathcal{S}(b_{T}; y_{1} - y_{2}) \times \frac{\mathcal{D}_{B}^{\text{uns.}}(z_{B}, b_{T}; \infty - y_{B})}{\mathcal{S}(b_{T}; \infty - y_{2})}$$
$$= \frac{\mathcal{D}_{A}^{\text{uns.}}(z_{A}, b_{T}; y_{A} - (-\infty))}{e^{(y_{1} - (-\infty))K + \frac{1}{2}P}} \times e^{(y_{1} - y_{2})K + R} \times \frac{\mathcal{D}_{B}^{\text{uns.}}(z_{B}, b_{T}; \infty - y_{B})}{e^{(\infty - y_{2})K + \frac{1}{2}P}}$$

...as well as in the standard TMD definition:

$$D(y_{\text{had}} - y_1) = D^{\text{uns.}}(y_{\text{had}} - (-\infty))\sqrt{\frac{\mathcal{S}(\infty - y_1)}{\mathcal{S}(\infty - (-\infty))\mathcal{S}(y_1 - (-\infty))}}$$

The standard definition is optimal for standard TMD factorization.

$$= D^{\text{uns.}}(y_{\text{had}} - (-\infty)) \sqrt{\frac{e^{(\infty - y_1)K + \frac{1}{2}P}}{e^{(\infty - (-\infty))K} e^{(y_1 - (-\infty))K + \frac{1}{2}P}}}$$

We can forget about the existence of the P-term **in the standard cases**.



#1

A non-standard case

Single-Inclusive Annihilation (SIA) with thrust $e^+e^- \rightarrow h X$



Data available since 2019

Transverse momentum dependent production cross sections of charged $^{\#11}$ pions, kaons and protons produced in inclusive e^+e^- annihilation at $\sqrt{s}=$ 10.58 GeV

Belle Collaboration • R. Seidl (RIKEN BNL) et al. (Feb 5, 2019)Published in: Phys.Rev.D 99 (2019) 11, 112006 • e-Print: 1902.01552 [hep-ex]

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The transverse momentum of the detected hadron is measured w.r.t. the thrust axis

$$z_h = \frac{E}{Q/2}, \quad T = \frac{\sum_i |\vec{P}_{(\text{c.m.}), i} \cdot \hat{n}|}{\sum_i |\vec{P}_{(\text{c.m.}), i}|}, \quad P_T \text{ w.r.t } \vec{n}$$

Complete theoretical treatment and first phenomenology is now available

Full treatment of the thrust distribution in single inclusive $e^+e^- \rightarrow h X$ processes

M. Boglione (Turin U. and INFN, Turin), A. Simonelli (Old Dominion U. and Jefferson Lab) (Jun 5, 2023) Published in: JHEP 09 (2023) 006 • e-Print: 2306.02937 [hep-ph]



Different kinematics leads to different factorized cross sections

Factorization works in the same way for all the three regions, but it produces different results depending on the underlying kinematics The hadron is detected very close to the **axis** of the jet. A. Simonelli

 \Box Extremely small P_T

Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

The hadron is detected in the **central region** of the jet.

Most common scenario

□ Majority of experimental data expected to fall into this case

The hadron is detected near the **boundary** of the jet.

- □ Moderately small P_T
- The hadron transverse momentum affects the topology of the final state directly

	soft	soft-collinear	collinear			
R_1	TMD-relevant	TMD-relevant	TMD-relevant			
R_2	TMD-irrelevant	TMD-relevant	TMD-relevant			
R_3	TMD-irrelevant	TMD-irrelevant	TMD-relevant			



We are forced to use the definition*. There are disadvantages but also...

- More universal (no soft contamination)
- Inclusion of P-term effects (new physics!)

Accessing the P-term

From comparing **operators**:



This relation is *exact*, provided that:

- 1. Both extractions are performed @ same perturbative accuracy
- 2. Both extractions use the same model for the non-perturbative behavior of the CS-kernel

... in practice, given the extractions available today, none of the assumptions above are satisfied at the same time.

Also, the comparison is cleaner if both extractions use the same prescription for separating out the non-perturbative effects in b_T -space (CSS b*, HSO...)

Simplifications at the numerator: $d\sigma_{R_2} \propto e^{I_R(u,y_1,\mu) - \frac{1}{2}P(b_T,\mu)} \approx e^{\frac{1}{2}g_P(b_T)}$ P-terms for thrust and TMD sector $P(b_T,\mu) = P(a_S(\mu_b^*)) - \int_{\mu_b^*}^{\mu} \frac{d\mu'}{\mu'} \gamma_P(a_S(\mu')) - g_P(b_T)$

The function g_P for the P-term is the counterpart of g_K for the CS-kernel.

Then, effectively:
$$D_{i/h}^{\star}(z, b_T, \mu, y_1) = \frac{1}{z^2} C_{i/j} \otimes d_{j/h}(z, \mu_b)$$
$$e^{\frac{1}{2}K(a_S(\mu_b^{\star}))\log\frac{Qe^{-y_1}}{\mu_b^{\star}} + \int_{\mu_b^{\star}}^{\mu} \frac{d\mu'}{\mu'} \gamma_D\left(a_S(\mu'), \log\frac{Qe^{-y_1}}{\mu'}\right)}$$
$$M_D(z, b_T) e^{\frac{1}{2}g_P(b_T)} e^{-\frac{1}{2}g_K(b_T)\log\frac{Qe^{-y_1}}{M_{had}}}$$
Effective combination extracted from

 $=e^{\frac{1}{2}g_P(b_T)}$

Ideally:

BS23 (NP-model)

Full treatment of the thrust distribution in single inclusive $e^+e^- \rightarrow h X$ #1 processes M. Boglione (Turin U. and INFN, Turin), A. Simonelli (Old Dominion U. and Jefferson Lab) (Jun 5, 2023)

Published in: JHEP 09 (2023) 006 • e-Print: 2306.02937 [hep-ph]

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OR

SV19 (NP-model)

Non-perturbative structure of semi-inclusive deep-inelastic and Drell-Yan

scattering at small transverse momentum

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Ignazio Scimemi (Madrid U.), Alexey Vladimirov (Regensburg U.) (Dec 13, 2019) Published in: *JHEP* 06 (2020) 137 • e-Print: 1912.06532 [hep-ph]

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Unpolarized transverse momentum distributions from a global fit of Drell-Yan ^{#//} and semi-inclusive deep-inelastic scattering data MAP (Multi-dimensional Analyses of Partonic distributions) Collaboration • Alessandro Bacchetta (Pavia U. and INFN, Pavia) et al. (Jun 15, 2022) Published in: JHEP 10 (2022) 127 • e-Print: 2206.07598 [hep-ph]

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The plan is:

Check z-independence of R (insensitive to collinear physics)

□ Infer information on g_P

Beware! This would be an extraction of an extraction!

Ratio w.r.t SV19



4 free parameters

Non-perturbative structure of semi-inclusive deep-inelastic and Drell-Yan scattering at small transverse momentum

Ignazio Scimemi (Madrid U.), Alexey Vladimirov (Regensburg U.) (Dec 13, 2019) Published in: *JHEP* 06 (2020) 137 • e-Print: 1912.06532 [hep-ph]

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$$D_{NP}(x,b) = \exp\left(-\frac{\eta_1 z + \eta_2 (1-z)}{\sqrt{1+\eta_3 (b/z)^2}} \frac{b^2}{z^2}\right) \left(1+\eta_4 \frac{b^2}{z^2}\right)$$















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BS23 (BELLE data $e^+e^- o \pi^\pmX$ $$ with thrust)										thr	rust		SV19							
T						;	z						$P_T/z \max$	Ν	ſ	Experiment	Reaction	ref	Kinematics	$N_{ m pt}$
	25	30	35	40	45	50	55	60	65	70	75	80				Experiment	reaction	101.	minimatics	after cuts
	0.	0.	- 0.	0.	0.	0	0.	0.	0.	0.	0.	- 0.			ſ		$p \rightarrow \pi^+$			24
	20 -	25 -	30 -	35 -	40 -	45 -	20-	55 -	- 09	65 -	- 02	- 21					$p \rightarrow \pi^{-}$		0.023 < x < 0.6 (6 bins)	24
	0.	0	0.	0.	°.	Ö	0	0.	0	0.	0.	0.					$p \rightarrow K^+$		$0.2 {<} z {<} 0.8 ~(6 hins)$	24
0.80 - 0.85													0.16Q	57		HERMES	$p \rightarrow K^-$	[66]	$1.0 {<} \mathrm{Q} {<} \sqrt{20} \mathrm{GeV}$	24
0.85 - 0.90													0.15Q	60		IIEItiviES	$D \to \pi^+$	lool		24
0.90 - 0.95													0.14Q	61			$D \rightarrow \pi^{-}$		$W^2 > 10 \text{GeV}^2$	24
0.95 - 1.00													0.13Q	52			$D \to K^+$		0.1 < y < 0.85	24
																	$D \to K^-$			24
															[COMPASS	$d ightarrow h^+$	[67]	0.003 < x < 0.4 (8 bins)	195
																COMITADD	$d \rightarrow h^-$	[07]	0.2 < z < 0.8 (4 bins)	195

Total

0.2 < z < 0.8 (4 bins)

 $1.0 < Q \simeq 9 \text{GeV} (5 \text{ bins})$

_ _ +

The two extractions overlapping core is

 $0.3 \lesssim z \lesssim 0.7$





Keep in mind that:

- Very different functional forms at numerator and denominator
- Extractions from completely different data sets (BELLE vs SIDIS/DY)

6

5

Spread of error bands can be used to constraint the "extraction" of g_P



Ratio w.r.t MAP22



9 free parameters

Unpolarized transverse momentum distributions from a global fit of Drell-Yan ^{#2} and semi-inclusive deep-inelastic scattering data MAP (Multi-dimensional Analyses of Partonic distributions) Collaboration • Alessandro Bacchetta (Pavia U. and INFN, Pavia) et al. (Jun 15, 2022)

Published in: JHEP 10 (2022) 127 • e-Print: 2206.07598 [hep-ph]

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$$D_{1\,NP}(z,\boldsymbol{b}_{T}^{2};\zeta,Q_{0}) = \frac{g_{3}(z)\,e^{-g_{3}(z)\frac{\boldsymbol{b}_{T}^{2}}{4z^{2}} + \frac{\lambda_{F}}{z^{2}}\,g_{3B}^{2}(z)\left[1 - g_{3B}(z)\frac{\boldsymbol{b}_{T}^{2}}{4z^{2}}\right]e^{-g_{3B}(z)\frac{\boldsymbol{b}_{T}^{2}}{4z^{2}}}}{g_{3}(z) + \frac{\lambda_{F}}{z^{2}}\,g_{3B}^{2}(z)}$$



Test the ansatz:

- Extractions @ same perturbative accuracy
- \circ Extractions use the same g_{κ}

However:







Not satisfied, or, at least, not constrained.

Thank

You!

Conclusions

□ Hidden soft effects, that cancel out in standard cross sections

 $S(b_T;\phi_M) = e^{\phi_M K(b_T) + P(b_T)}$

□ Need to go **beyond** standard processes to have sensitivity on P-term

$$d\sigma_{R_2} \propto e^{I_R(u,y_1,\mu) - \frac{1}{2}P(b_T,\mu)} \approx e^{\frac{1}{2}g_P(b_T,\mu)}$$

 $R = \begin{array}{c} \text{TMD extraction from beyond} \\ \text{standard process} \\ \hline \\ \text{TMD extraction from} \\ \text{standard process} \end{array} =$





Lattice applications

