Longitudinal spin transfer of semi-inclusive Λ production in deep inelastic scattering

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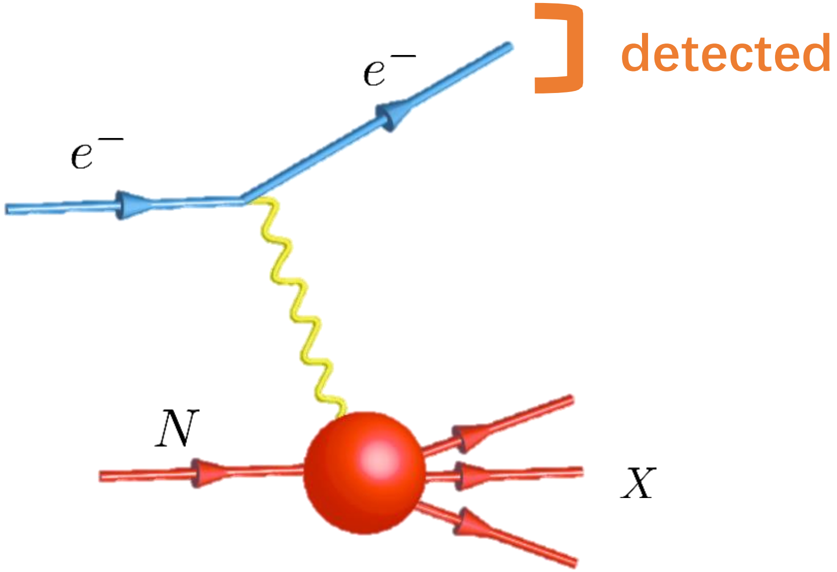
How to “see” the nucleon structure ?

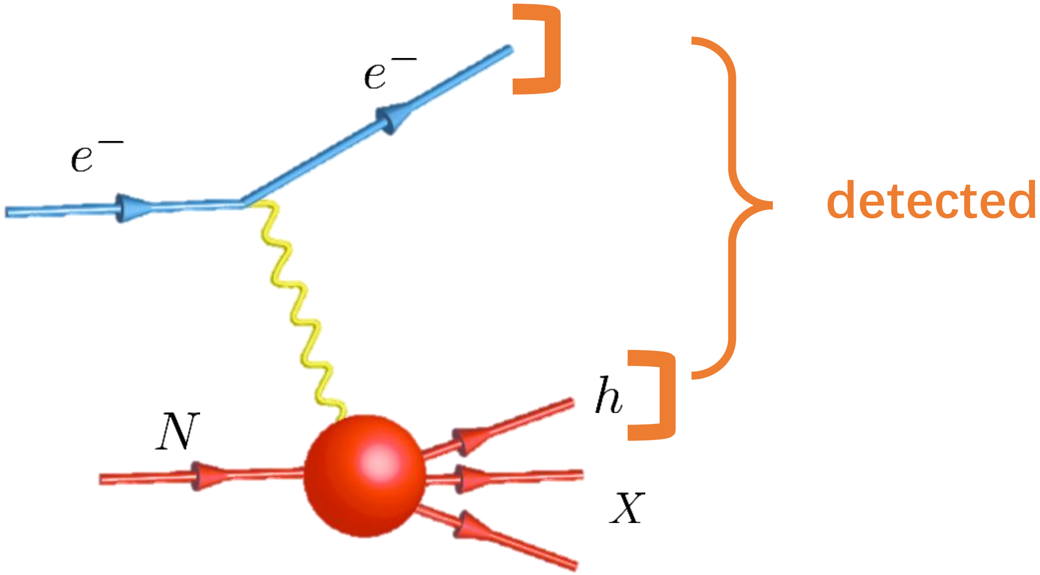
- Deep inelastic scattering (DIS)

$$e^- + N \to e^- + X$$



- Semi-inclusive deep inelastic scattering (SIDIS)

$$e^- + N \to e^- + h + X$$

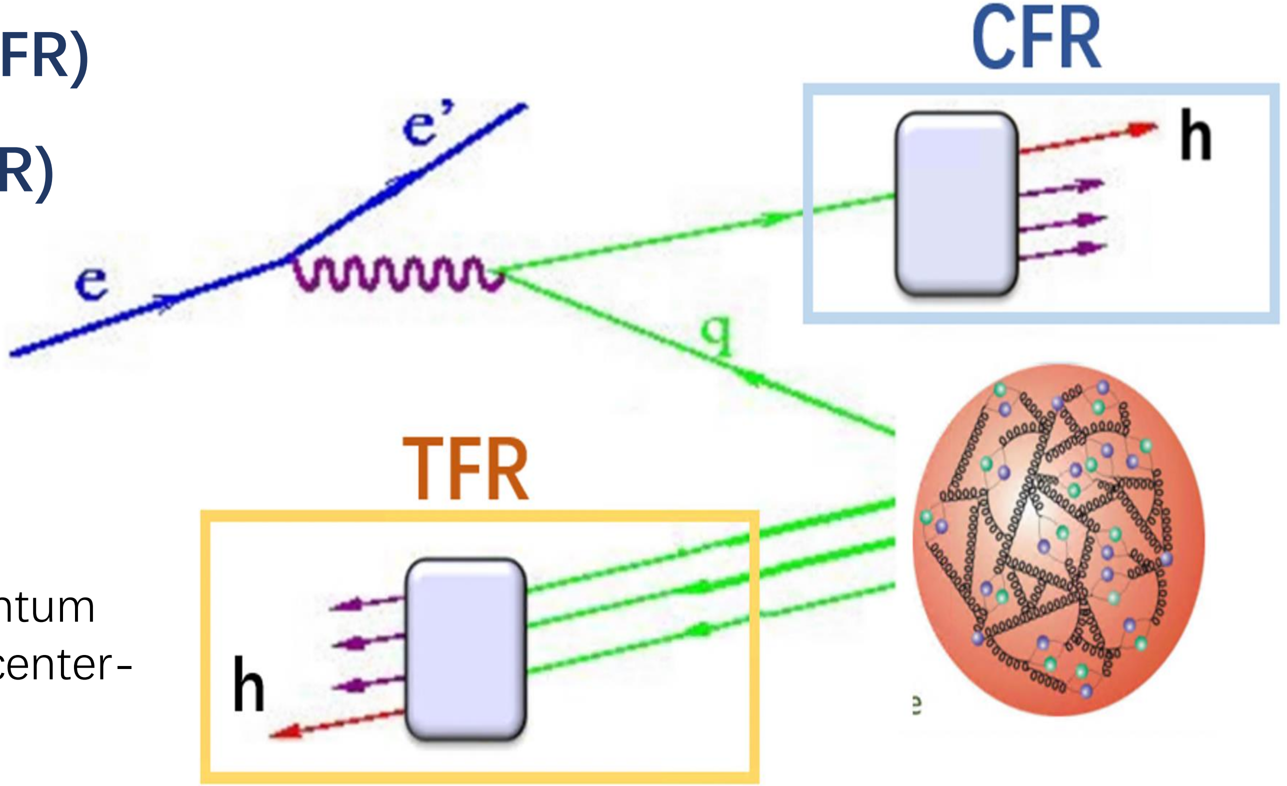


- **The structure of Λ particle & production process** $e^- P \rightarrow e^- \Lambda X$

$$\Lambda: uds; \;\; s = \frac{1}{2}; \qquad \Lambda \rightarrow \mathrm{p}\pi, (BR = (64.1 \pm 0.5)\%); \qquad M = 1.116 GeV$$

- Current Fragmentation Region (CFR)
- Target Fragmentation Region (TFR)

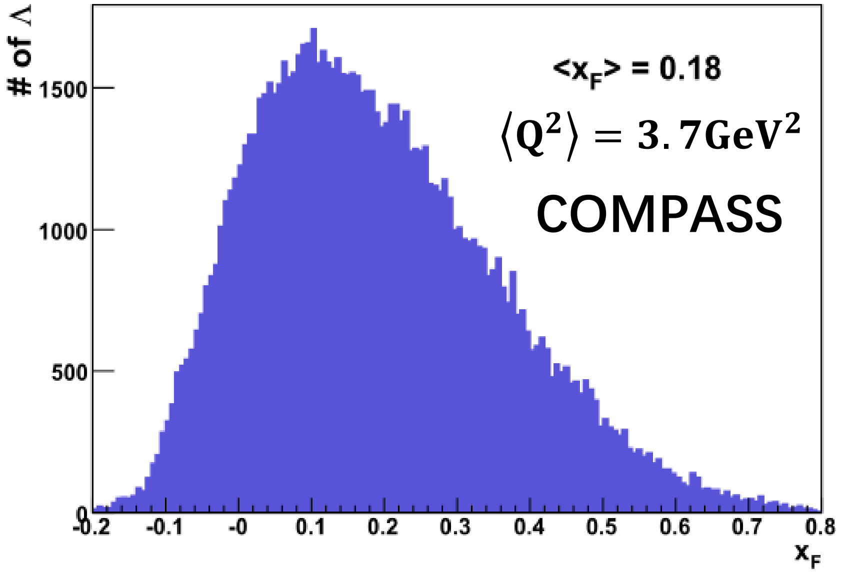
$$x_F = \frac{2P_{hL}}{W}$$



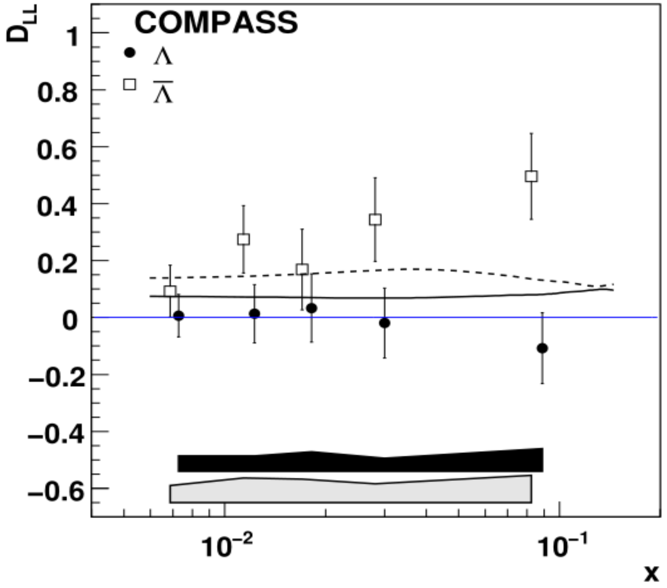
P_{hL} is the projection of the final-state hadron momentum onto the direction of the γ^*-momentum in the $\gamma^* N$ center-of-mass frame, W is invariant mass.

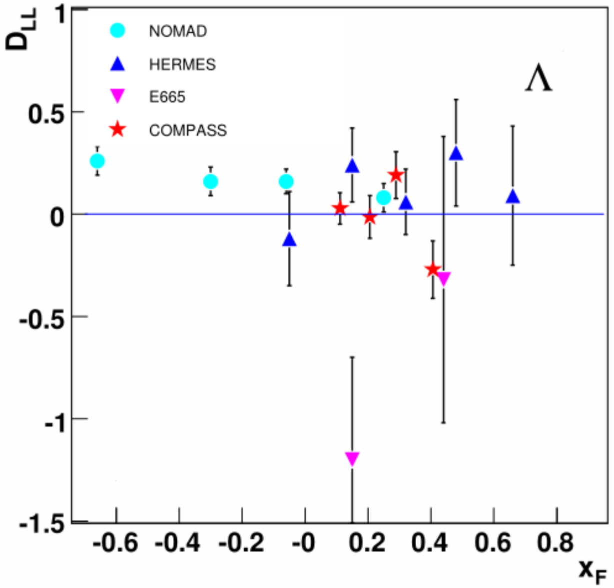
Introduction

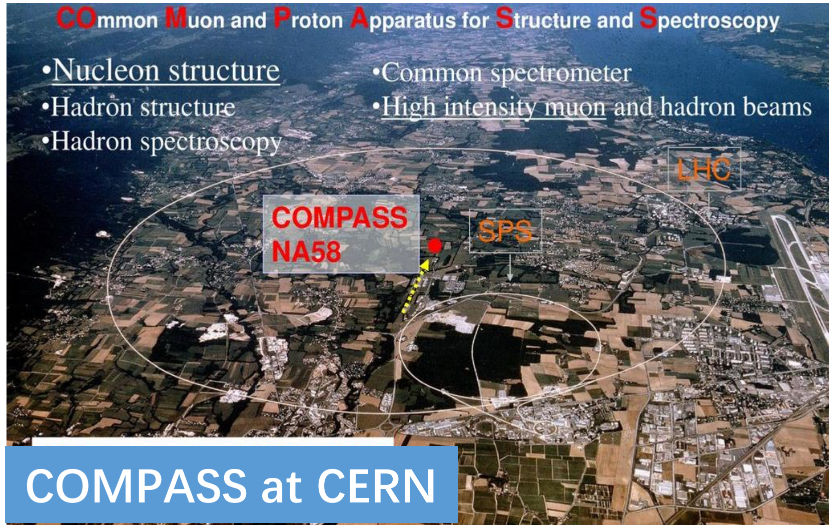
Distribution of Λ in SIDIS:



Λ longitudinal spin transfer: *Eur.Phys.J.C 64 (2009) 171-179*

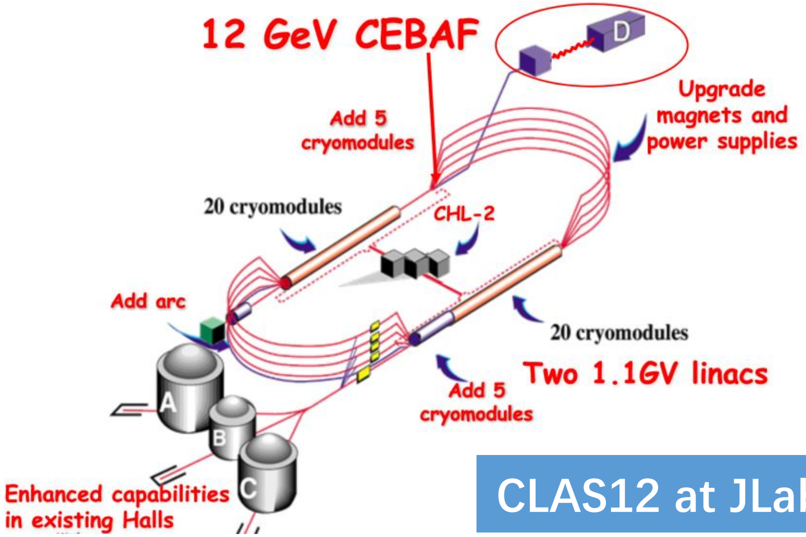






COMPASS at CERN

HERMES at DESY



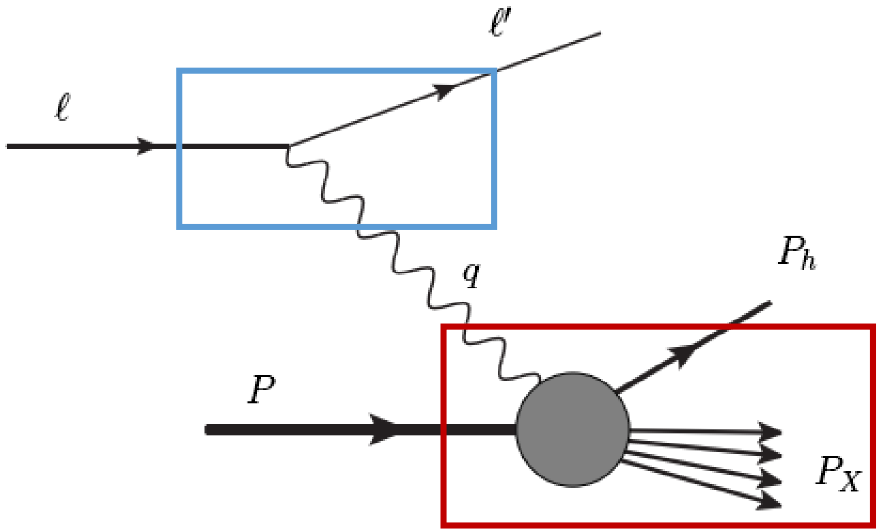
CLAS12 at JLab

Longitudinal spin transfer of Λ Polarization in SIDIS:

$$e^- P \to e^- \Lambda X$$

- The kinematic analysis
- Spectator diquark model calculation
- Numerical estimate

Differential Cross Section

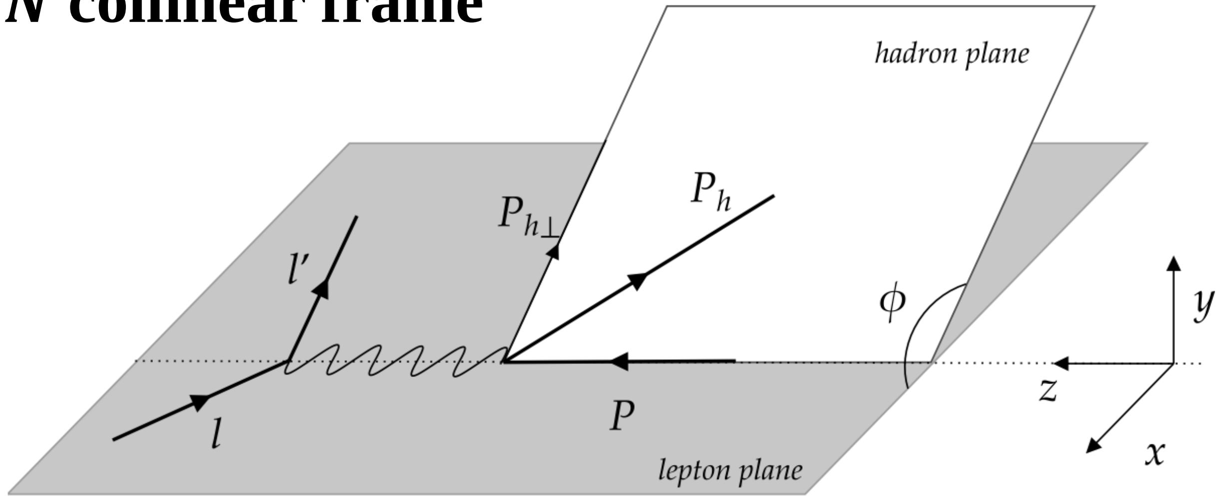


$$\frac{d\sigma^{\mathrm{SIDIS}}}{dx\,dy\,dz_\Lambda d^2\boldsymbol{P}_{\Lambda\perp}} = \frac{\pi\alpha_{\mathrm{em}}^2}{2Q^4}\frac{y}{z_\Lambda}L_{\mu\nu}W^{\mu\nu}$$

$$L_{\mu\nu} = 2\left(l_\mu l'_\nu + l_\nu l'_\mu - g_{\mu\nu} l\cdot l' + i\lambda_e \epsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma\right)$$

$$W^{\mu\nu} = \frac{1}{(2\pi)^4}\sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3\, 2P_X^0}(2\pi)^4\,\delta^4\left(q+P-P_\Lambda-P_X\right) < P,S|J^\mu(0)|P_\Lambda S_\Lambda; P_X >< P_\Lambda S_\Lambda; P_X|J^\nu(0)|P,S >$$

$\gamma^* N$ collinear frame



$$\cos\phi = -\frac{g_\perp^{\mu\nu} l_\mu P_{h\nu}}{|l_\perp|\,|P_{h\perp}|} \qquad \sin\phi = -\frac{\epsilon_\perp^{\mu\nu} l_\mu P_{h\nu}}{|l_\perp||P_{h\perp}|}$$

$$\cos\phi_S = -\frac{g_\perp^{\mu\nu} l_\mu S_\nu}{|l_\perp|\,|S_\perp|} \qquad \sin\phi_S = -\frac{\epsilon_\perp^{\mu\nu} l_\mu S_\nu}{|l_\perp||S_\perp|}$$

$$\cos\phi_{Sh} = -\frac{g_{h\perp}^{\mu\nu} P_{h\mu} S_{h\nu}}{|P_{h\perp}|\,|S_{h\perp}|} \qquad \sin\phi_{Sh} = -\frac{\epsilon_{h\perp}^{\mu\nu} P_{h\mu} S_{h\nu}}{|P_{h\perp}|\,|S_{h\perp}|}$$

Differential Cross Section

$$\frac{d\sigma}{dx\,dy\,dz\,d^2P_{\Lambda\perp}} = \frac{2\pi\alpha_{em}^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\{$$

Unpolarized P and Λ — 5

$$A(y)F_{UUU}^{T} + B(y)F_{UUU}^{L} + C(y)\cos\phi F_{UUU}^{\cos\phi} + B(y)\cos 2\phi F_{UUU}^{\cos 2\phi} + \lambda_e E(y)\sin\phi F_{LUU}^{\sin\phi}$$

Polarized P, Unpolarized Λ — 13

$$+\lambda C(y)\sin\phi F_{ULU}^{\sin\phi} + \lambda B(y)\sin 2\phi F_{ULU}^{\sin 2\phi} + \mathbf{S}_\perp A(y)\sin(\phi-\phi_S)\,F_{UTU}^{T\,\sin(\phi-\phi_S)}$$
$$+\mathbf{S}_\perp B(y)\sin(\phi-\phi_S)\,F_{UTU}^{L\,\sin(\phi-\phi_S)} + \mathbf{S}_\perp C(y)\sin(2\phi-\phi_S)\,F_{UTU}^{\sin(2\phi-\phi_S)}$$
$$+\mathbf{S}_\perp C(y)\sin\phi_S F_{UTU}^{\phi_S} + \mathbf{S}_\perp B(y)\sin(3\phi-\phi_S)\,F_{UTU}^{\sin(3\phi-\phi_S)} + \mathbf{S}_\perp B(y)\sin(\phi+\phi_S)\,F_{UTU}^{\sin(\phi+\phi_S)}$$
$$+\lambda_e\lambda D(y)F_{LLU}^{T} + \lambda_e\lambda E(y)\cos\phi F_{LLU}^{\cos\phi} + \lambda_e\mathbf{S}_\perp D(y)\cos(\phi-\phi_S)\,F_{LTU}^{T\,\cos(\phi-\phi_S)}$$
$$+\lambda_e\mathbf{S}_\perp E(y)\cos\phi_S F_{LTU}^{\cos\phi_S} + \lambda_e\mathbf{S}_\perp E(y)\cos(2\phi-\phi_S)\,F_{LTU}^{\cos(2\phi-\phi_S)}$$

Unpolarized P, Polarized Λ — 13

$$+\lambda_h C(y)\sin\phi F_{UUL}^{\sin\phi} + \lambda_h B(y)\sin 2\phi F_{UUL}^{\sin 2\phi} + \mathbf{S}_{h\perp}\sin\phi_{Sh}B(y)F_{UUT}^{L} + \mathbf{S}_{h\perp}\sin\phi_{Sh}A(y)F_{UUT}^{T}$$
$$+\mathbf{S}_{h\perp}B(y)\sin(2\phi+\phi_{Sh})F_{UUT}^{\sin(2\phi+\phi_{Sh})} + \mathbf{S}_{h\perp}B(y)\sin(2\phi-\phi_{Sh})F_{UUT}^{\sin(2\phi-\phi_{Sh})}$$
$$+\mathbf{S}_{h\perp}C(y)\sin(\phi+\phi_{Sh})F_{UUT}^{\sin(\phi+\phi_{Sh})} + \mathbf{S}_{h\perp}C(y)\sin(\phi-\phi_{Sh})F_{UUT}^{\sin(\phi-\phi_{Sh})}$$
$$+\lambda_e\lambda_h D(y)F_{LUL}^{T} + \lambda_e\mathbf{S}_{h\perp}\cos\phi_{Sh}D(y)F_{LUT}^{T} + \lambda_e\lambda_h E(y)\cos\phi F_{LUL}^{\cos\phi}$$
$$+\lambda_e\mathbf{S}_{h\perp}E(y)\cos(\phi-\phi_{Sh})F_{LUT}^{\cos(\phi-\phi_{Sh})} + \lambda_e\mathbf{S}_{h\perp}E(y)\cos(\phi+\phi_{Sh})F_{LUT}^{\cos(\phi+\phi_{Sh})}$$

F_{ZXY}

Z: lepton; X: nucleon; Y: Λ

U: unpolarized; L: longitudinal;

T: transvers

$$A(y) = \frac{y^2}{4}(2+\gamma^2) - y + 1$$

$$B(y) = 1 - y - \frac{1}{4}\gamma^2 y^2$$

$\cdots\cdots$

Differential Cross Section

$$
\begin{aligned}
&+ \lambda\lambda_h A(y) F_{ULL}^{T} + \lambda\lambda_h B(y) F_{ULL}^{L} + \lambda\lambda_h C(y)\cos\phi F_{ULL}^{T\cos\phi} + \lambda\lambda_h B(y)\cos 2\phi F_{ULL}^{T\cos 2\phi} + \lambda \mathbf{S}_{h\perp}\cos\phi_{Sh} B(y) F_{ULT}^{L} \\
&+ \lambda \mathbf{S}_{h\perp}\cos\phi_{Sh} A(y) F_{ULT}^{T} + \lambda \mathbf{S}_{h\perp} C(y)\cos(\phi-\phi_{Sh}) F_{ULT}^{\cos(\phi-\phi_{Sh})} + \lambda \mathbf{S}_{h\perp} B(y)\cos(2\phi-\phi_{Sh}) F_{ULT}^{\cos(2\phi-\phi_{Sh})} \\
&+ \lambda \mathbf{S}_{h\perp} C(y)\cos(\phi+\phi_{Sh}) F_{ULT}^{\cos(\phi+\phi_{Sh})} + \lambda \mathbf{S}_{h\perp} B(y)\cos(2\phi+\phi_{Sh}) F_{ULT}^{\cos(2\phi+\phi_{Sh})} \\
&+ \mathbf{S}_{\perp}\lambda_h C(y)\cos\phi_S F_{UTL}^{\phi_S} + \mathbf{S}_{\perp}\lambda_h A(y)\cos(\phi-\phi_S) F_{UTL}^{T\cos(\phi-\phi_S)} + \mathbf{S}_{\perp}\lambda_h B(y)\cos(\phi-\phi_S) F_{UTL}^{L\cos(\phi-\phi_S)} \\
&+ \mathbf{S}_{\perp}\lambda_h C(y)\cos(2\phi-\phi_S) F_{UTL}^{\cos(2\phi-\phi_S)} + \mathbf{S}_{\perp}\lambda_h B(y)\cos(\phi+\phi_S) F_{UTL}^{\cos(\phi+\phi_S)} + \mathbf{S}_{\perp}\lambda_h B(y)\cos(3\phi-\phi_S) F_{UTL}^{\cos(3\phi-\phi_S)} \\
&+ \lambda_e \mathbf{S}_{\perp}\lambda_h E(y)\sin\phi_S F_{LTL}^{\sin\phi_S} + \lambda_e \mathbf{S}_{\perp}\lambda_h E(y)\sin(2\phi-\phi_S) F_{LTL}^{\sin(2\phi-\phi_S)} + \lambda_e \mathbf{S}_{\perp}\lambda_h D(y)\sin(\phi-\phi_S) F_{LTL}^{T\sin(\phi-\phi_S)} \\
&+ \lambda_e\lambda\lambda_h E(y)\sin\phi F_{LLL}^{\sin\phi} + \lambda_e\lambda \mathbf{S}_{h\perp} E(y)\left[\sin(\phi+\phi_{Sh}) F_{LLT}^{\sin(\phi+\phi_{Sh})} + \sin(\phi-\phi_{Sh}) F_{LLT}^{\sin(\phi-\phi_{Sh})}\right] + \lambda_e\lambda \mathbf{S}_{h\perp}\sin\phi_{Sh} D(y) F_{LLT}^{T} \\
&+ \lambda_e \mathbf{S}_{\perp}\mathbf{S}_{h\perp} D(y)\left[\sin(\phi-\phi_S+\phi_{Sh}) F_{LTT}^{T\sin(\phi-\phi_S+\phi_{Sh})} + \sin(\phi-\phi_S-\phi_{Sh}) F_{LTT}^{T\sin(\phi-\phi_S-\phi_{Sh})}\right] \\
&+ \lambda_e \mathbf{S}_{\perp}\mathbf{S}_{h\perp} E(y)\left[\sin(\phi_S+\phi_{Sh}) F_{LTT}^{\sin(\phi_S+\phi_{Sh})} + \sin(\phi_S-\phi_{Sh}) F_{LTT}^{\sin(\phi_S-\phi_{Sh})}\right. \\
&\qquad \left. + \sin(2\phi-\phi_S-\phi_{Sh}) F_{LTT}^{\sin(2\phi-\phi_S-\phi_{Sh})} + \sin(2\phi-\phi_S+\phi_{Sh}) F_{LTT}^{\sin(2\phi-\phi_S+\phi_{Sh})}\right] \\
&+ \mathbf{S}_{\perp}\mathbf{S}_{h\perp}\cos(\phi-\phi_S-\phi_{Sh}) A(y) F_{UTT}^{T\cos(\phi-\phi_S-\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\sin(\phi-\phi_S-\phi_{Sh}) F_{UTT}^{L\sin(\phi-\phi_S-\phi_{Sh})} \\
&+ \mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\cos(3\phi-\phi_S-\phi_{Sh}) F_{UTT}^{\cos(3\phi-\phi_S-\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\cos(\phi+\phi_S-\phi_{Sh}) F_{UTT}^{\cos(\phi+\phi_S-\phi_{Sh})} \\
&+ \mathbf{S}_{\perp}\mathbf{S}_{h\perp} C(y)\cos(2\phi-\phi_S-\phi_{Sh}) F_{UTT}^{\cos(2\phi-\phi_S-\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} C(y)\cos(\phi_S-\phi_{Sh}) F_{UTT}^{\cos(\phi_S-\phi_{Sh})} \\
&+ \mathbf{S}_{\perp}\mathbf{S}_{h\perp}\cos(\phi-\phi_S+\phi_{Sh}) A(y) F_{UTT}^{T\cos(\phi-\phi_S+\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\sin(\phi-\phi_S+\phi_{Sh}) F_{UTT}^{L\sin(\phi-\phi_S+\phi_{Sh})} \\
&+ \mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\cos(3\phi-\phi_S+\phi_{Sh}) F_{UTT}^{\cos(3\phi-\phi_S+\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\cos(\phi+\phi_S+\phi_{Sh}) F_{UTT}^{\cos(\phi+\phi_S+\phi_{Sh})} \\
&+ \mathbf{S}_{\perp}\mathbf{S}_{h\perp} C(y)\cos(2\phi-\phi_S+\phi_{Sh}) F_{UTT}^{\cos(2\phi-\phi_S+\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} C(y)\cos(\phi_S+\phi_{Sh}) F_{UTT}^{\cos(\phi_S+\phi_{Sh})}
\end{aligned}
$$

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Polarized P
Polarized Λ

Current Fragmentation Region (CFR)

Spin asymmetry： $A = \frac{d\sigma(\uparrow) - d\sigma(\downarrow)}{d\sigma(\uparrow) + d\sigma(\downarrow)}$ ⇒ $A_{XY}^{\omega(\phi_h,\phi_S)} \equiv \frac{F_{XY}^{\omega(\phi_h,\phi_S)}}{F_{UU}}$

QCD factorization： $d\sigma \sim d\hat{\sigma} \otimes PDF \otimes FF$

Unpolarized P&Λ

$$\frac{d\sigma(\ell H \to \ell' h X)}{dx dz dy d^2 \boldsymbol{P}_{\Lambda\perp}} = \frac{2\pi\alpha^2}{yQ^2}\{A(y)\mathcal{I}[f_1 D_1] + B(y) cos2\phi \mathcal{I}\left[-\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{MM_h}h_1^\perp H_1^\perp\right]\}$$

Polarized P Unpolarized Λ

$$\frac{d\sigma(\ell \vec{H} \to \ell' h X)}{dx dz dy d^2 \boldsymbol{P}_{\Lambda\perp}} = \frac{2\pi\alpha^2}{yQ^2}\{\lambda\lambda_e C(y)\mathcal{I}[g_1 D_1] + \lambda B(y)\sin(2\phi)\mathcal{I}\left[-\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{MM_h}h_{1L}^\perp H_1^\perp\right] + \cdots$$

Unpolarized P Polarized Λ

$$\frac{d\sigma(\ell H \to \ell' \vec{h} X)}{dx dz dy d^2 \boldsymbol{P}_{\Lambda\perp}} = \frac{2\pi\alpha^2}{yQ^2}\{\lambda_e\lambda_h C(y)\mathcal{I}[f_1 G_1] + \lambda_h B(y)\sin 2\phi\mathcal{I}\left[\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{MM_h}h_1^\perp H_{1L}^\perp\right] + \cdots$$

Polarized P Polarized Λ

$$\frac{d\sigma(\ell \vec{H} \to \ell' \vec{h} X)}{dx dz dy d^2 \boldsymbol{P}_{\Lambda\perp}} = \frac{2\pi\alpha^2}{yQ^2}\{\lambda\lambda_h A(y)\mathcal{I}[g_1 G_1] + \lambda\lambda_h B(y)\cos 2\phi\mathcal{I}\left[-\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{MM_h}h_{1L}^\perp H_{1L}^\perp\right] + \lambda|\boldsymbol{S}_{h\perp}|A(y)\cos\phi_{S_h}\mathcal{I}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}g_1 G_{1T}\right] + \lambda_h|\boldsymbol{S}_T|B(y)\cos(3\phi-\phi_S)\mathcal{I}\left[-\frac{4\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)^2 - 2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\boldsymbol{p}_T\cdot\boldsymbol{k}_T - \hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\boldsymbol{p}_T^2}{2M_h M^2}h_{1T}^\perp H_{1L}^\perp\right] + \cdots$$

Structure Function &PDFs⊗FFs

$$F_{UUU}^{T} = \mathcal{I}\left[f_1 D_1\right]$$

$$F_{UUU}^{\cos 2\phi} = \mathcal{I}\left[-\left(2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)\frac{h_1^\perp H_1^\perp}{M M_h}\right]$$

$$F_{ULU}^{\sin 2\phi} = \mathcal{I}\left[-\left(2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)\frac{h_{1L}^\perp H_1^\perp}{M M_h}\right]$$

$$F_{UTU}^{T\sin(\phi-\phi_S)} = \mathcal{I}\left[-\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\frac{f_{1T}^\perp D_1}{M}\right]$$

$$F_{UTU}^{T\sin(\phi+\phi_S)} = \mathcal{I}\left[-\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\frac{h_1 H_1^\perp}{M_h}\right]$$

$$F_{UTU}^{T\sin(3\phi-\phi_S)} = \mathcal{I}\left[-\left(4\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)^2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - 2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\boldsymbol{p}_T\cdot\boldsymbol{k}_T - \boldsymbol{p}_T^2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\frac{h_{1T}^\perp H_1^\perp}{2M^2 M_h}\right]$$

⋮

$$F_{LLU}^{T} = \mathcal{I}\left[g_1 D_1\right]$$

$$F_{LTU}^{T\cos(\phi-\phi_S)} = \mathcal{I}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\frac{g_{1T}^\perp D_1}{M}\right]$$

$$F_{LUL}^{T} = \mathcal{I}\left[f_1 G_{1L}\right]$$

$$F_{LUT}^{T\cos\phi_{S_h}} = \mathcal{I}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\frac{f_1 G_{1T}}{M_h}\right]$$

$$F_{LLT}^{T\sin\phi_{S_h}} = \mathcal{I}\left[-\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\frac{g_{1L} D_{1T}^\perp}{M_h}\right]$$

$$F_{LTL}^{T\sin(\phi-\phi_S)} = \mathcal{I}\left[-\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\frac{f_{1T}^\perp G_{1L}}{M}\right]$$

$$F_{LTT}^{T\sin(\phi-\phi_S-\phi_{S_h})} = \mathcal{I}\left[-\left(2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)\frac{f_{1T}^\perp G_{1T} - g_{1T} D_{1T}^\perp}{2M M_h}\right]$$

⋮

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Numerical Estimate

◆ Longitudinal Spin Transfer D_{LL} in current fragmentation region:

$$P_L(x,z,Q^2) = \frac{d\sigma_{\Downarrow} - d\sigma_{\Uparrow}}{d\sigma_{\Downarrow} + d\sigma_{\Uparrow}} = \frac{F_{UL}}{F_{UU}} = D(y)D_{LL'}$$

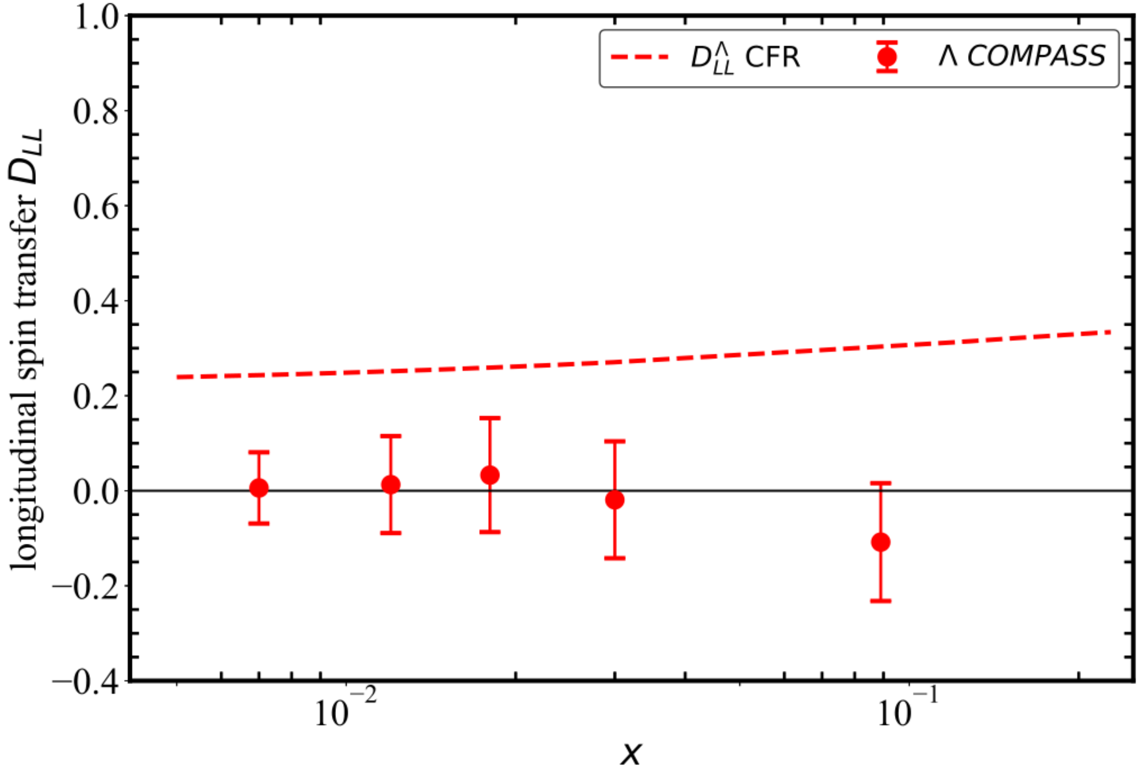
$$D_{LL'} = \frac{\sum_a e_a^2 f_a(x,Q^2)G_a(z,Q^2)}{\sum_a e_a^2 f_a(x,Q^2)D_a(z,Q^2)}\ \omega_q(z_\Lambda, P_{h\perp})$$

$$f_{1q}(x,k_\perp) = f_{1q}(x)\frac{1}{\pi\Delta_p^2}e^{-k_\perp^2/\Delta_p^2}, \quad \Delta_p^2 = 0.61\ \mathrm{GeV}^2$$

$$D_{1q}^{\Lambda}(z,p_T) = D_{1q}^{\Lambda}(z)\frac{1}{\pi\Delta_\Lambda^2}e^{-p_T^2/\Delta_\Lambda^2}, \quad \Delta_\Lambda^2 = 0.118\ \mathrm{GeV}^2$$

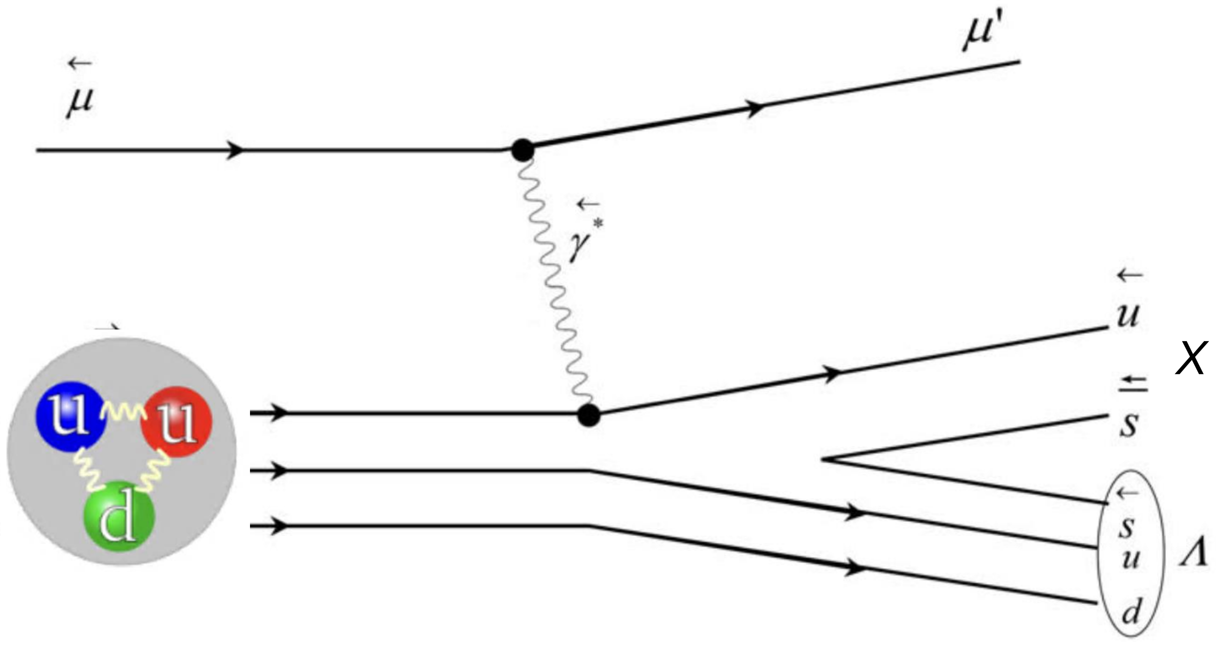
$f_{q/p}(x_B, Q)$	**CT18NLO PDF** *Progress in the CTEQ-TEA NNLO global QCD analysis*

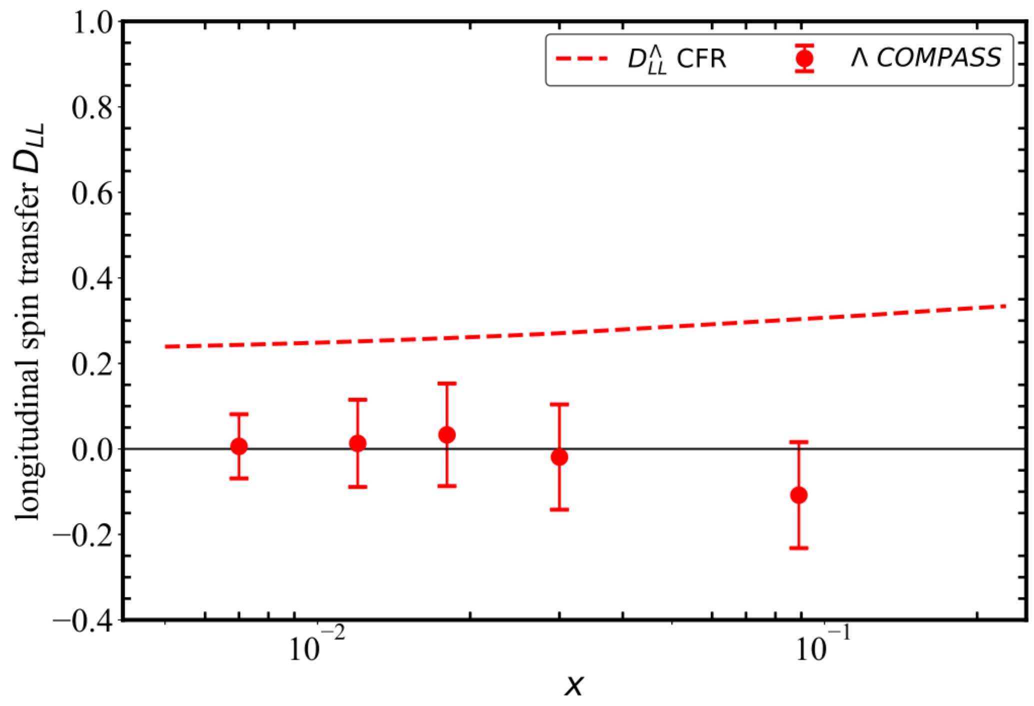
$\frac{G_a(z,Q^2)}{D_a(z,Q^2)}$	**DSV FFs** *Phys.Rev.D 57 (1998) 5811-5824*



COMPASS data from *Eur. Phys. J. C64, 171-179 (2009)*

Target Fragmentation Region (TFR)

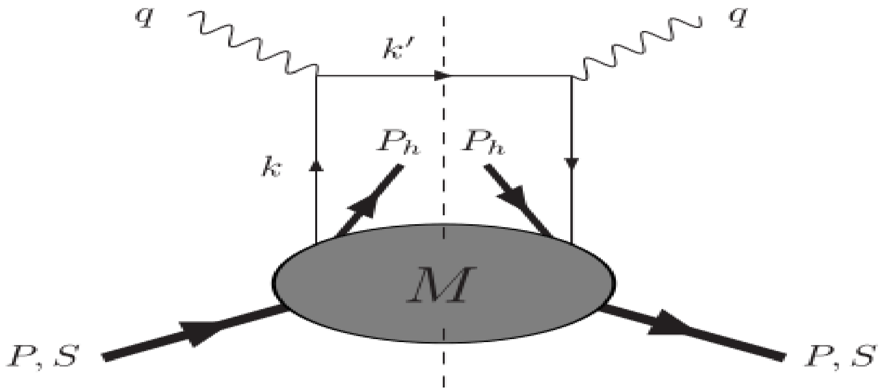




$$\psi = \eta_{\text{flavor}}\chi_{\text{spin}}\xi_{\text{color}}\phi_{\text{space}}$$

$$\Lambda^{\uparrow} = \frac{1}{\sqrt{3}}(ud)_{0,0}s^{\uparrow} + \frac{1}{\sqrt{12}}(us)_{0,0}d^{\uparrow} - \frac{1}{\sqrt{12}}(ds)_{0,0}u^{\uparrow}$$
$$+\frac{1}{2}\left(\sqrt{\frac{2}{3}}(us)_{1,1}d^{\downarrow} - \sqrt{\frac{1}{3}}(us)_{1,0}d^{\uparrow}\right) - \frac{1}{2}\left(\sqrt{\frac{2}{3}}(ds)_{1,1}u^{\downarrow} - \sqrt{\frac{1}{3}}(ds)_{1,0}u^{\uparrow}\right)$$

◆ The fracture matrix $\mathcal{M}$



$$W^{\mu\nu} = \sum_a e_a^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} 2\pi\,\delta[(k+q)^2]\,\mathrm{Tr}\,[\mathcal{M}\gamma^\mu(\not{k}+\not{q})\gamma^\nu]$$

$$\mathcal{M}_{ij}(k;P,S;P_\Lambda,S_\Lambda) = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2E_X} \int \frac{d^4\xi}{(2\pi)^4} e^{ik\xi} \langle P,S|\overline{\Psi}_j(0)|P_\Lambda,S_\Lambda;X\rangle \langle P_\Lambda,S_\Lambda;X\,|\Psi_i(\xi)|\,P,S\rangle$$

decompose it on a basis of Dirac structures： $\mathcal{M} = \frac{1}{2}(\mathcal{S}I + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^\mu\gamma_5 + i\mathcal{P}\gamma_5 + i\mathcal{T}_{\mu\nu}\sigma^{\mu\nu}\gamma_5)$

$\mathcal{M}_{ij}(k;P,S;P_\Lambda,S_\Lambda)$ ⇒

Dirac Struture：$1, \gamma^\mu, \gamma^\mu\gamma_5, \gamma_5, \sigma^{\mu\nu}\gamma_5$

Five Vectors：$k^\mu, P^\mu, P_\Lambda^\mu, S^\mu, S_\Lambda^\mu.$

⇒ the most general decomposition of $\mathcal{M}$

Use $\gamma^+, \gamma^+\gamma_5, i\sigma^{i+}\gamma_5$ to pick out the leading-twist terms

Differential Cross Section

$$\frac{d\sigma^{(\mathrm{TFR})}}{dx\,dy\,d\zeta\,d^2\mathbf{P}_{h\perp}} = \frac{4\alpha^2}{yQ^2}\sum_a e_a^2 \Big\{ (\frac{y^2}{2} - y + 1)\Big[M_{UU} + \lambda\lambda_h M_{LL} - S_\perp \frac{P_{h\perp}}{m_h}\sin(\phi-\phi_S)\, M_{TU}^h$$

$$- S_{h\perp}\frac{P_{h\perp}}{m_h}\sin(\phi-\phi_{S_h})\, M_{UT}^h + S_\perp S_{h\perp}\frac{P_{h\perp}^2}{m_h^2}\left(\cos(2\phi-\phi_S-\phi_{S_h}) + \cos(\phi_S-\phi_{S_h})\right) M_{TT}^h$$

$$+ S_\perp S_{h\perp}\cos(\phi_S-\phi_{S_h})\, M_{TT} + \lambda S_{h\perp}\frac{P_{h\perp}}{m_h}\cos(\phi-\phi_{S_h})\, M_{LT}^h + S_\perp \lambda_h \frac{P_{h\perp}}{m_h}\cos(\phi-\phi_S)\, M_{TL}^h \Big]$$

$$+ \lambda_e y(1-\frac{y}{2})\Big[\lambda \Delta M_{LU} + \lambda_h \Delta M_{UL} + S_\perp \frac{P_{h\perp}}{m_h}\cos(\phi-\phi_S)\,\Delta M_{TU}^h + S_{h\perp}\frac{P_{h\perp}}{m_h} cos(\phi-\phi_{S_h})\,\Delta M_{UT}^h$$

$$- S_\perp S_{h\perp}\sin(\phi_S-\phi_{S_h})\Delta M_{TT} - S_\perp S_{h\perp}\frac{P_{h\perp}^2}{m_h^2}\left(\sin(2\phi_h-\phi_S-\phi_{S_h}) + \sin(\phi_S-\phi_{S_h})\right)\Delta M_{TT}^h$$

$$- \lambda S_{h\perp}\frac{P_{h\perp}}{m_h}\sin(\phi-\phi_{S_h})\,\Delta M_{LT}^h - S_\perp \lambda_h \frac{P_{h\perp}}{m_h}\sin(\phi-\phi_S)\,\Delta M_{TL}^h \Big] \Big\}$$

M_{XY}^h — X: nucleon; Y: Λ hyperon

ΔM: longitudinally polarized quark

Fracture Functions

$$F_{UUU}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} M_{UU}(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{ULL}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} M_{LL}(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{UTT}^{\cos(\phi_S-\phi_{S_h})} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\left[M_{TT}(x,\zeta,\mathbf{P}_{h\perp}^2) + \frac{P_{h\perp}^2}{m_h^2} M_{TT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)\right]$$

$$F_{UTT}^{T\cos(2\phi-\phi_S-\phi_{S_h})} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}^2}{m_h^2} M_{TT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{UTU}^{T\sin(\phi-\phi_S)} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{TU}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{UUT}^{T\sin(\phi-\phi_{S_h})} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{UT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{ULT}^{\cos(\phi-\phi_S)} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{LT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{UTL}^{\cos(\phi-\phi_{S_h})} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{TL}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LLU}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} \Delta M_{LU}(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LUL}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} \Delta M_{UL}(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LTU}^{\cos(\phi-\phi_S)} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{TU}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LUT}^{\cos(\phi-\phi_{S_h})} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{UT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LLT}^{T\sin(\phi_\Lambda-\phi_{S_\Lambda})} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{LT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LTL}^{T\sin(\phi_\Lambda-\phi_S)} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{TL}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LTT}^{\sin(\phi_S-\phi_{S_h})} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\left[\Delta M_{TT}(x,\zeta,\mathbf{P}_{h\perp}^2) + \frac{P_{h\perp}^2}{m_h^2} \Delta M_{TT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)\right]$$

$$F_{LTT}^{\sin(2\phi-\phi_S-\phi_{S_h})} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}^2}{m_h^2} \Delta M_{TT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

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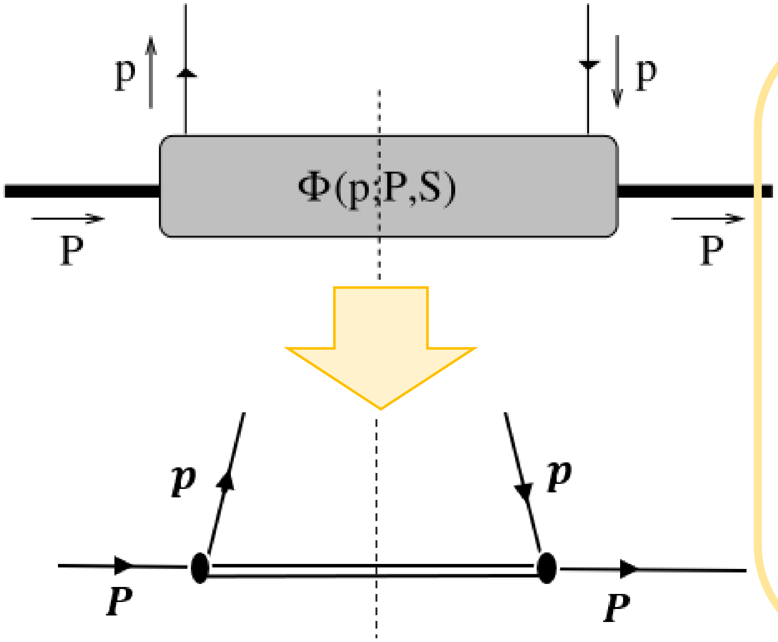
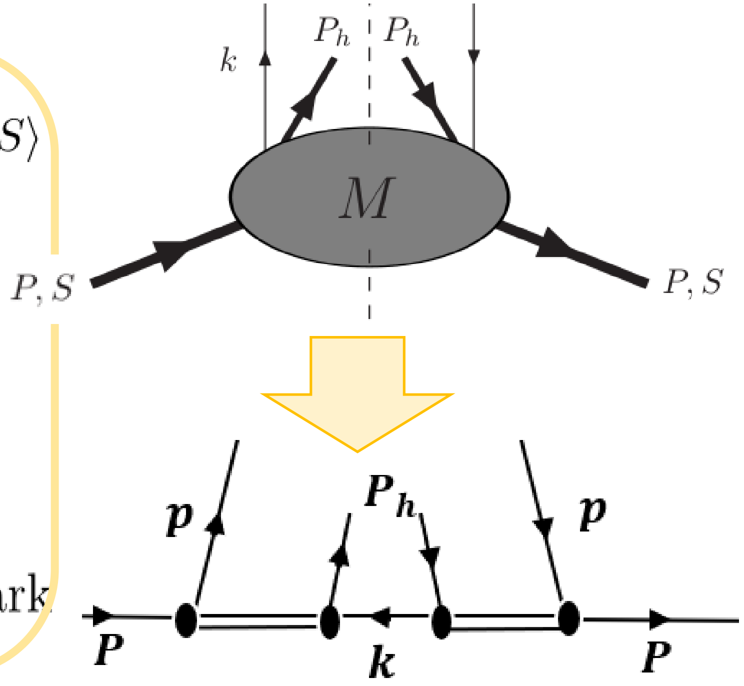
Spectator Diquark Model

R. P. Feynman, "Photon Hadron Interactions," New York 1972-01-01.



$$\Phi_{ij}^{R} = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2P_X^0} \langle P,S|\bar{\psi}_j(0)|X\rangle \langle X|\psi_i(0)|P,S\rangle$$
$$* (2\pi)^4 \delta(P-p-P_X)$$

$$\langle P,S|\bar{\psi}(0)|X\rangle = \begin{cases} \bar{U}(P,S)\Upsilon_s \frac{i}{\not{p}-m_q}, & \text{scalar diquark} \\ \bar{U}(P,S)\Upsilon_a^{\mu} \frac{i}{\not{p}-m_q}\epsilon_\mu, & \text{axial-vector diquark} \end{cases}$$



Parton distribution function

Target Fragment

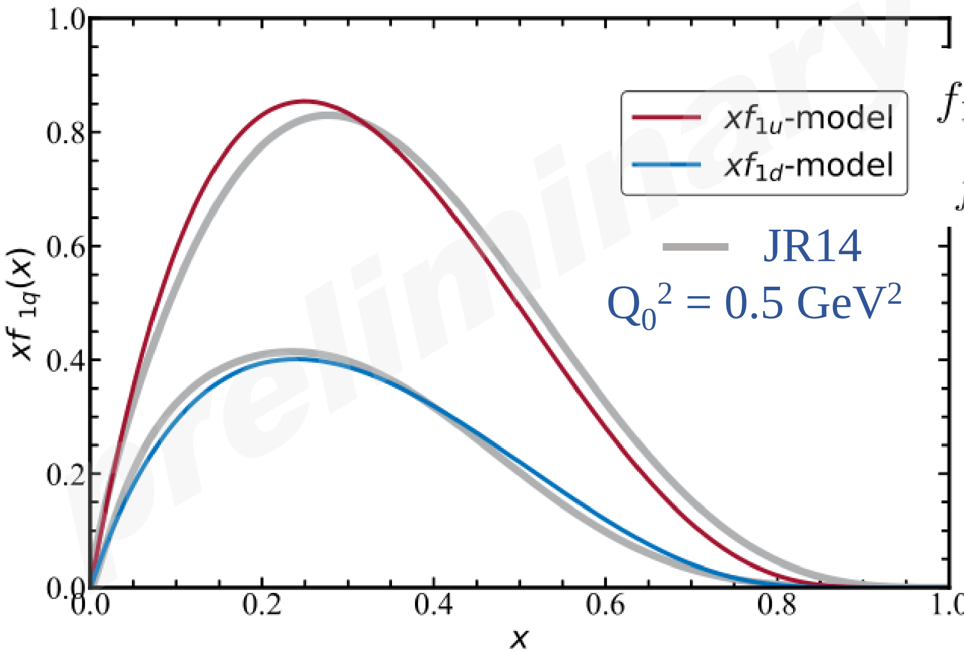
$$M_{UU}^{(s)}(x,\zeta,\mathbf{P}_{h\perp}) = \frac{|g(p,p')|^2 \, |g(k,p')|^2}{8(2\pi)^6 \zeta(1-x-\zeta)}$$
$$* \frac{Tr\left[(\not{p}+m)(\not{P}+M)(\not{p}+m)\gamma^+\right] Tr\left[(\not{k}-m_q)(\not{P}_h+M_h)\right]}{2(p^2-m^2)^2(p'^2-M_R^2)^2 P^+}$$

Model Calculation

$$f_1^{q(s)}(x, \boldsymbol{p}_T^2) = \frac{g_s^2}{(2\pi)^3}\frac{[(m+xM)^2+\boldsymbol{p}_T^2](1-x)}{2[\boldsymbol{p}_T^2+L_s^2(m^2)]^2}$$

$$f_1^{q(a)}(x, \boldsymbol{p}_T^2) = \frac{g_a^2}{(2\pi)^3}\frac{1}{4[\boldsymbol{p}_T^2+L_a^2(m^2)]^2M_a^2(1-x)}[\boldsymbol{p}_T^4 + xM_a^2(2\boldsymbol{p}_T^2+xM_a^2) + (1-x)^2[\boldsymbol{p}_T^2(M^2+m^2+2M_a^2) + 2m^2M_a^2 + 6xmMM_a^2 + 2x^2M^2M_a^2 + m^2M^2(1-x)^2]],$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M^2$$



Unpolarized PDF $f_1(x)$ vs x for u quark (red line) and d quark (blue line). The gray band from the parametrizations of JR14, and the curves represent the best fit obtained with our spectator model. *Eur. Phys. J. C75 (2015) 3, 132*

$$f_{1u} = \frac{3}{2}f_1^s + \frac{1}{2}f_1^a$$

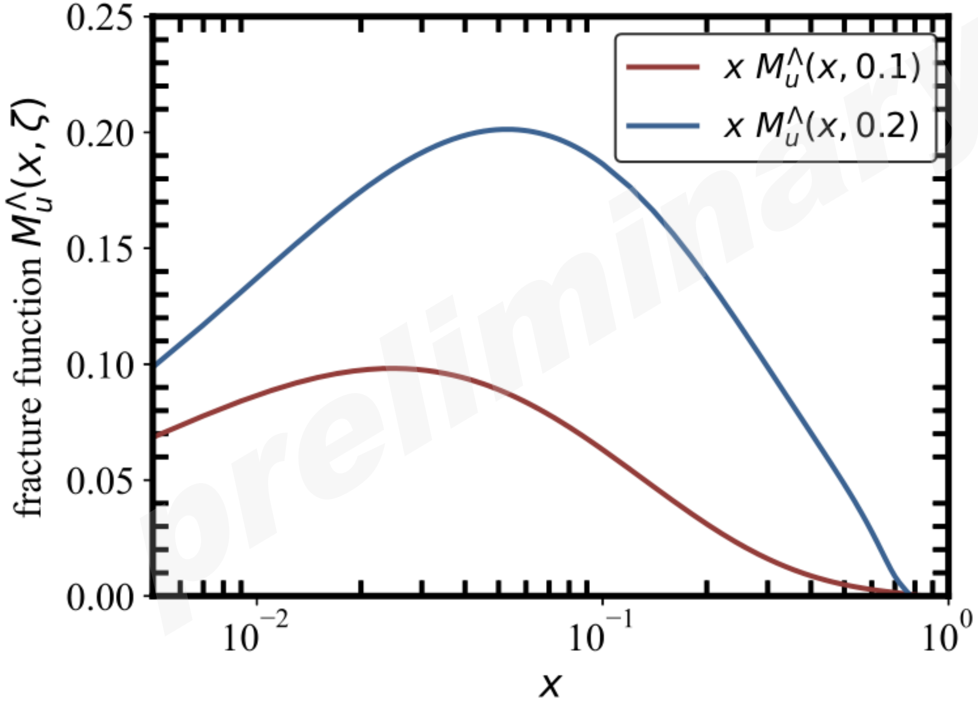
$$f_{1d} = f_1^a$$

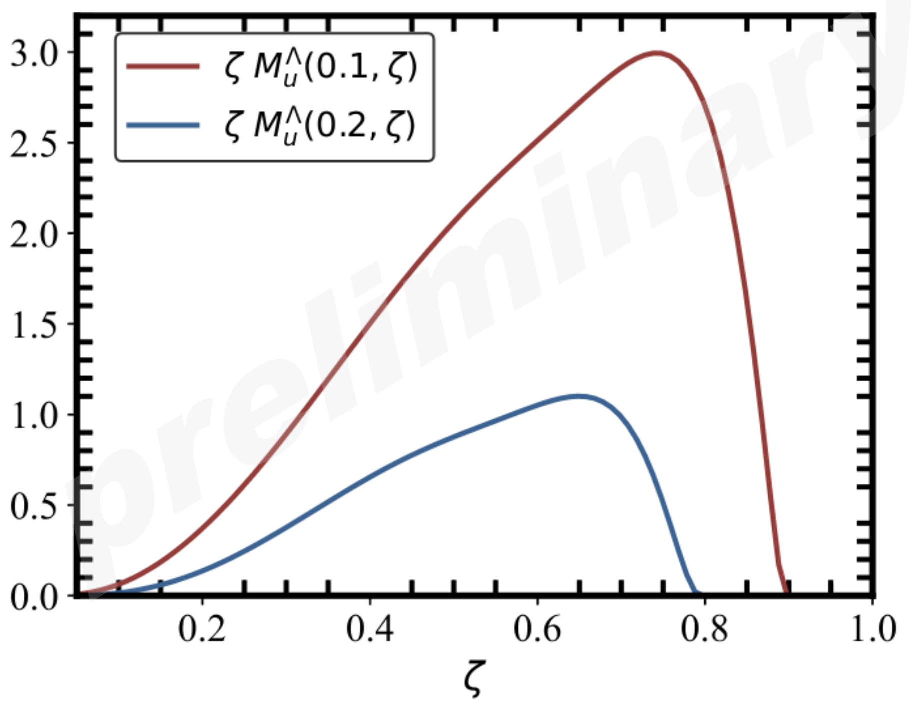
$$m = 0.3 \text{ GeV},$$
$$M_s = 1.2 \text{ GeV}, \Lambda_s = 2.3 \text{ GeV},$$
$$M_a = 1.3 \text{ GeV}, \Lambda_a = 1.6 \text{ GeV},$$
$$g_s = 14.98, \quad g_a = 15.33.$$

Model Calculation

$$m = 0.3\ \text{GeV},\ g_s = 14.98,$$
$$M_s = 1.2\ \text{GeV},\ \Lambda_s = 2.3\ \text{GeV},$$

$$\hat{M} = \frac{g_{1s}^2 g_{2s}^2 x\left[(m + xM)^2 + \boldsymbol{p}_T^2\right]}{2(2\pi)^6\zeta^2(1-\zeta-x)^2(p^2-m^2)^2} \times \frac{\left[(1-x-\zeta)M_h - \zeta m_{\bar{q}}\right]^2 + \left[(1-x)\boldsymbol{P}_{h\perp} + \zeta \boldsymbol{p}_T\right]^2}{x(1-x)M^2 - xM_s^2 - (1-x)p^2 + \boldsymbol{p}_T^2}.$$





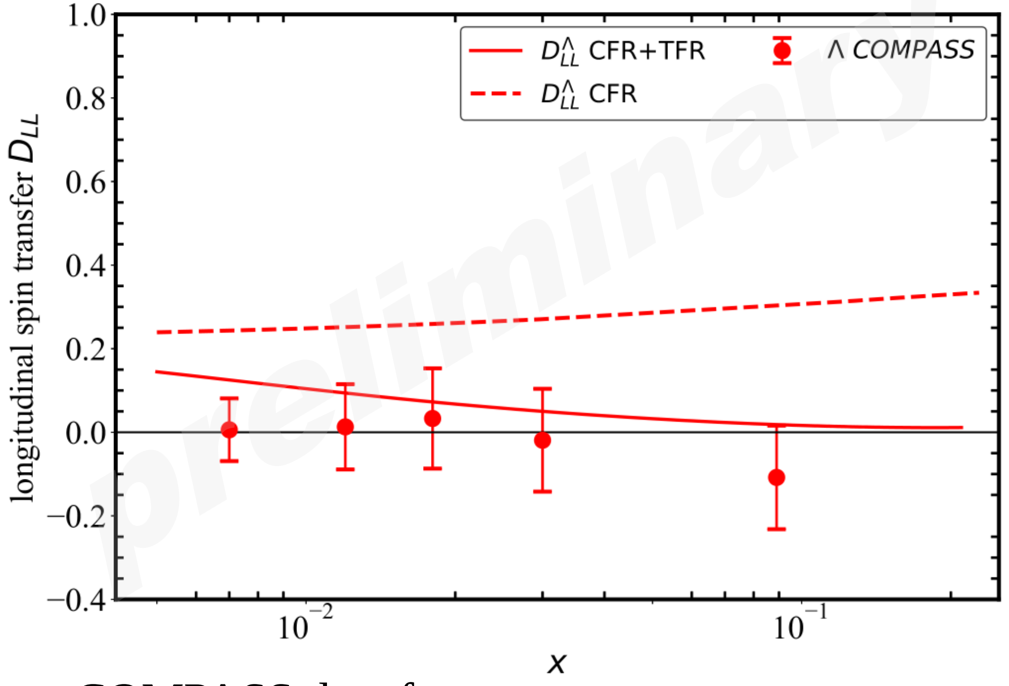
The results for unploarized fracture function $xM_u^\Lambda(x)$ and $\zeta M_u^\Lambda(\zeta)$ on x and ζ dependences using the spectator diquark model.

Longitudinal Spin Transfer $D_{LL}(x)$

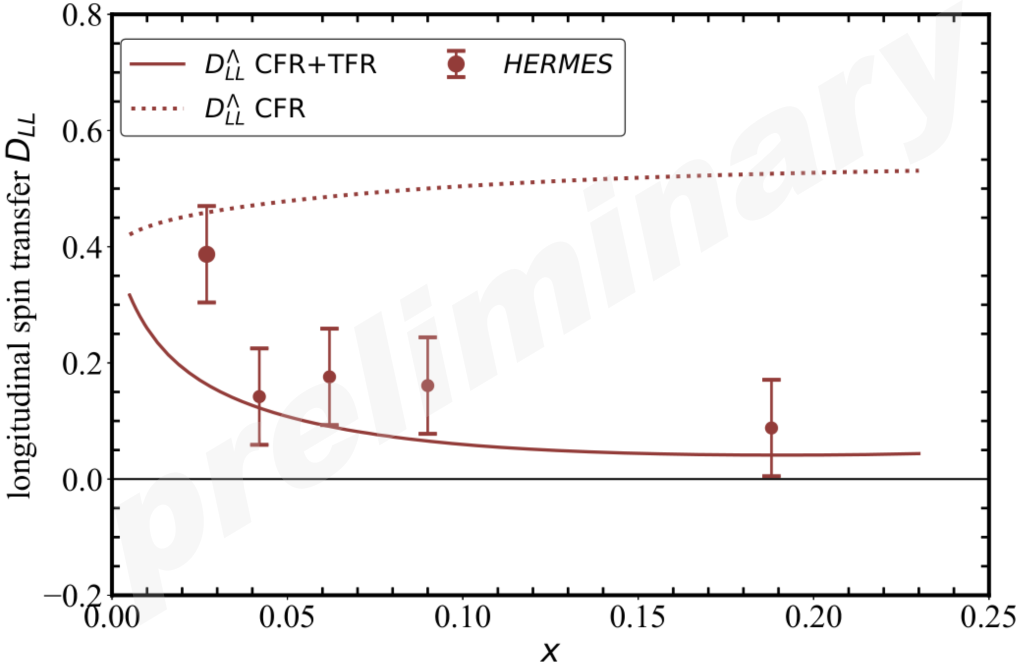
$$D_{LL}^{\Lambda}(x,z,Q^2) = \frac{\sum_q e_q^2 z^2 f_{1q}(x_B,Q^2) G_{1Lq}^{\Lambda}(z_\Lambda,Q^2)}{\sum_q e_q^2 \left[z^2 f_{1q}(x_B,Q^2) D_{1q}^{\Lambda}(z_\Lambda,Q^2) + \frac{\zeta}{z} M_q^{\Lambda}(x_B,\zeta,Q^2)\right]}.$$

$$\zeta = \frac{2x_B(M_h^2 + \boldsymbol{P}_{h\perp}^2)}{z_h Q^2 + \sqrt{z_h^2 Q^4 - 4x_B^2 M^2 (M_h^2 + \boldsymbol{P}_{h\perp}^2)}}$$

D_{LL}^{Λ} in CFR (dashed curves) and CFR+TFR (solid curves):



COMPASS data from *Eur. Phys. J. C64, 171-179 (2009)*
at $\bar{Q}^2 = 3.7\ GeV^2$, $\bar{z} = 0.27$

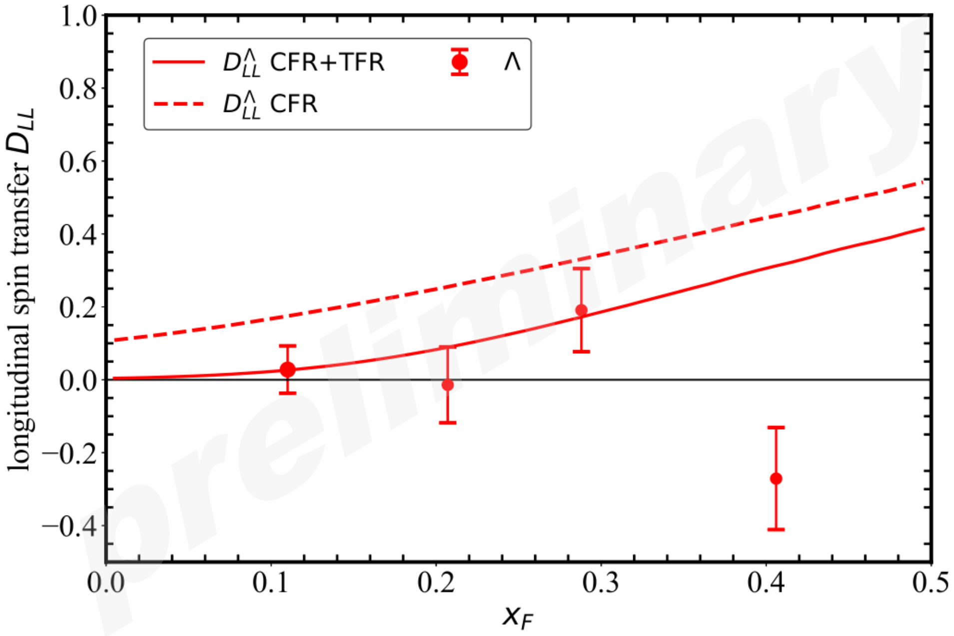


HERMES data from *J. Phys. Conf. Ser, 295,02114 (2011)*
at $\bar{Q}^2 = 2.4\ GeV^2$, $\bar{z} = 0.45$

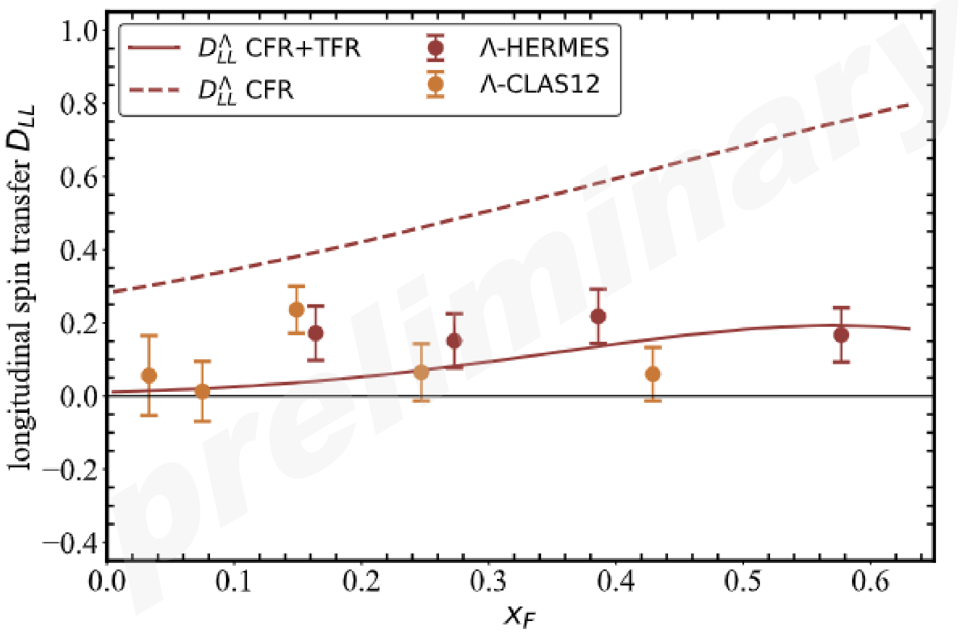
Numerical Estimate

$$x_F = \frac{z_\Lambda}{\frac{x_B M^2}{Q^2} + (1 - x_B)} \left[\left(1 + \frac{Q^2}{2x_B M^2}\right) \sqrt{1 - \frac{4x_B^2 M^2 (M_h^2 + \mathbf{P}_{h\perp}^2)}{z_\Lambda^2 Q^4}} - \sqrt{\frac{Q^4}{4x_B^2 M^4} + \frac{Q^2}{M^2}} \right]$$

D_{LL}^{Λ} in CFR (dashed curves) and CFR+TFR (solid curves):



The comparison of x_F-dependent results and COMPASS data for Λ and $\bar{\Lambda}$ at $\bar{Q}^2 = 3.7\ GeV^2$. The curves are the theoretical calculation while the dots with error bars represent the experimental data.



The comparison of x_F-dependent results and data from HERMES and CLAS12 at $\bar{Q}^2 = 2.4\ GeV^2$.

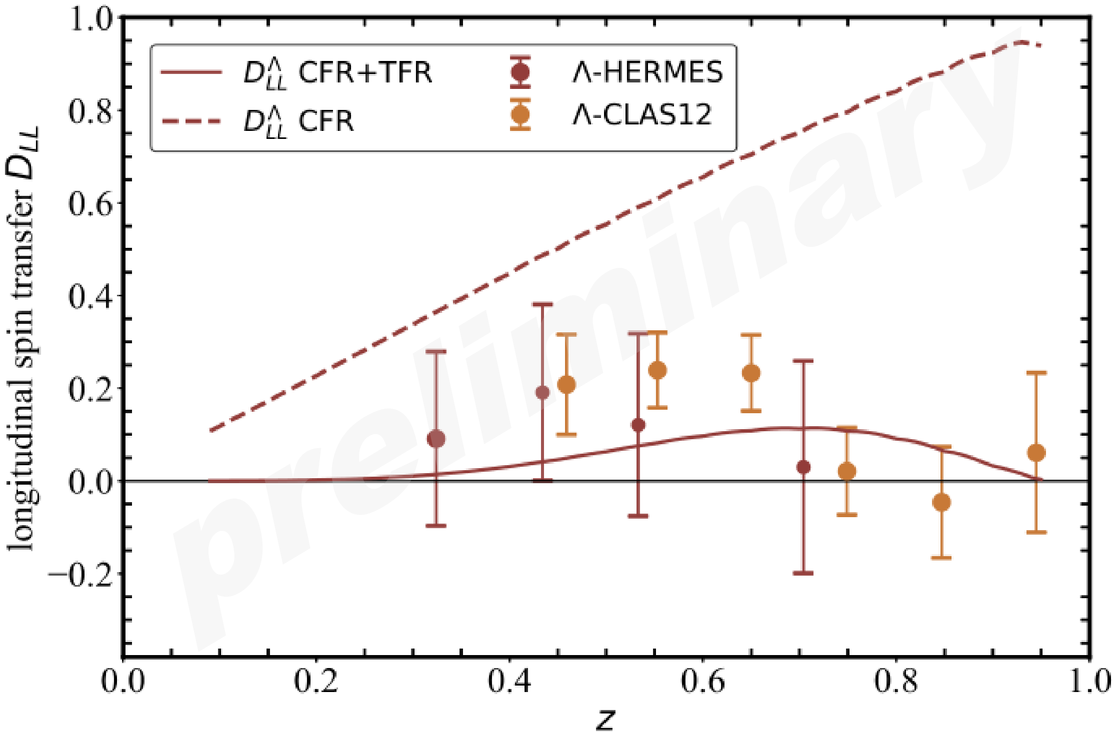
Numerical Estimate

◆ Longitudinal Spin Transfer $D_{LL}(z)$ $\sigma = \sigma^{CFR} + \sigma^{TFR}$

$$D_{LL}^{\Lambda}(x,z,Q^2) = \frac{\sum_q e_q^2 z^2 f_{1q}(x_B,Q^2) G_{1Lq}^{\Lambda}(z_\Lambda,Q^2)}{\sum_q e_q^2 \left[z^2 f_{1q}(x_B,Q^2) D_{1q}^{\Lambda}(z_\Lambda,Q^2) + \frac{\zeta}{z} M_q^{\Lambda}(x_B,\zeta,Q^2)\right]}.$$

D_{LL}^{Λ} in CFR (dashed curves) and CFR+TFR (solid curves):

$$\zeta = \frac{2x_B(M_h^2 + \boldsymbol{P}_{h\perp}^2)}{z_h Q^2 + \sqrt{z_h^2 Q^4 - 4x_B^2 M^2 (M_h^2 + \boldsymbol{P}_{h\perp}^2)}}$$



HERMES data from *Phys.Rev.D 74 (2006) 072004*
CLAS12 data from *JPS Conf. Proc. 37, 020304(2022)*
at $\bar{Q}^2 = 2.4\ GeV^2$, $\bar{x} = 0.088$

Summary

- We derived the general form of cross section for spin-1/2 hadrons, and obtained expressions of structure functions at the leading twist in CFR and TFR.
- We studied the contribution from TFR to the Λ production in SIDIS and perform the estimation to quantitatively demonstrate the effect by diquark model.
- We estimated the spin transfer D_{LL}, while considering the TFR, our calculation results can explain the COMPASS, HERMES and CLAS12 data reasonably.

Thank you!

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