

# Role of the twist-3 gluon effect on the single transverse-spin asymmetry in the semi-inclusive $J/\psi$ production

Shinsuke Yoshida

(South China Normal University)



in collaboration with: Longjie Chen(SCNU)

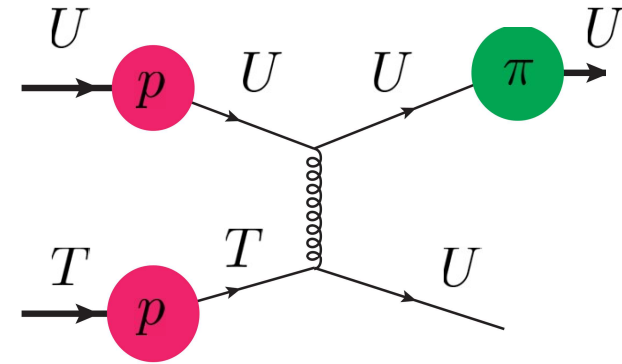
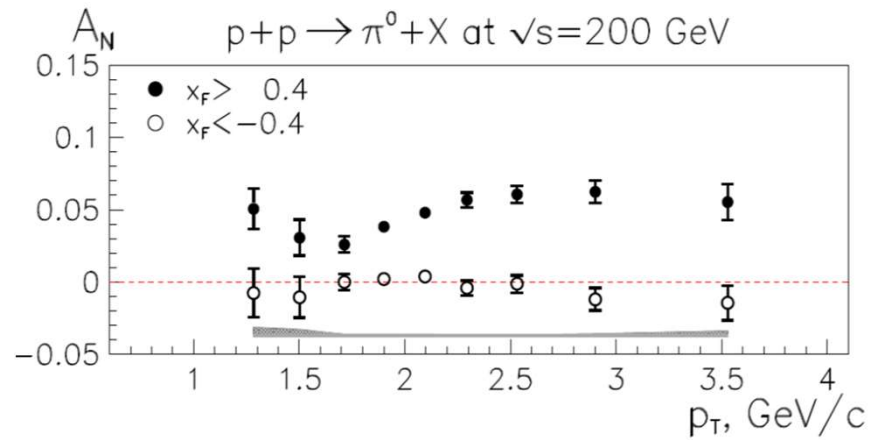
Hongxi Xing(SCNU)

Phys. Rev. D108 (2023)

DIS2024@Grenoble, April 8-12

# New era in the nucleon structure

- Single Transverse-Spin Asymmetry

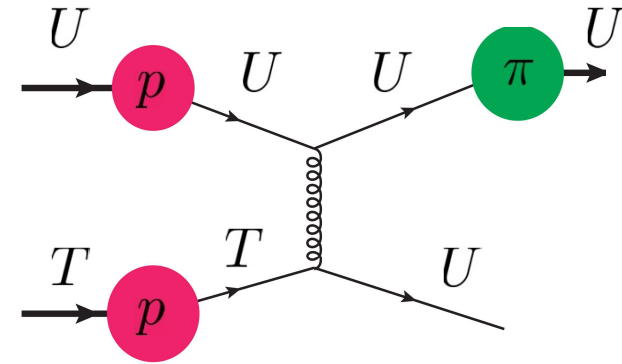
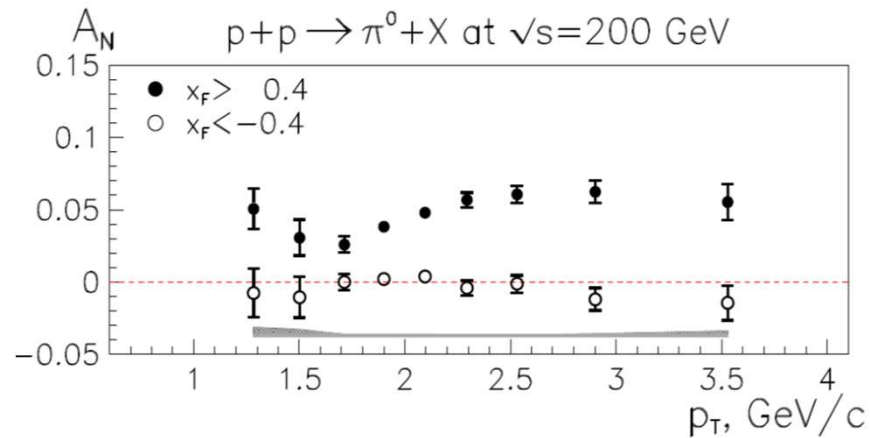


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Conventional picture does not work

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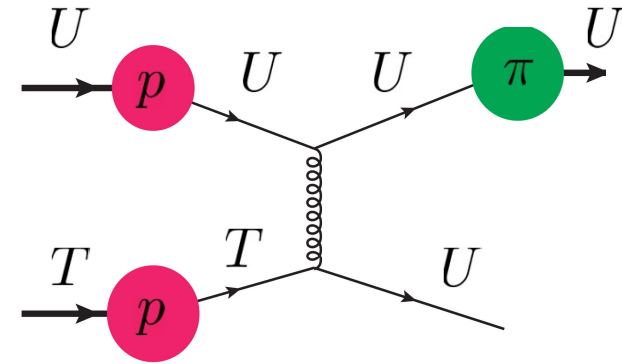
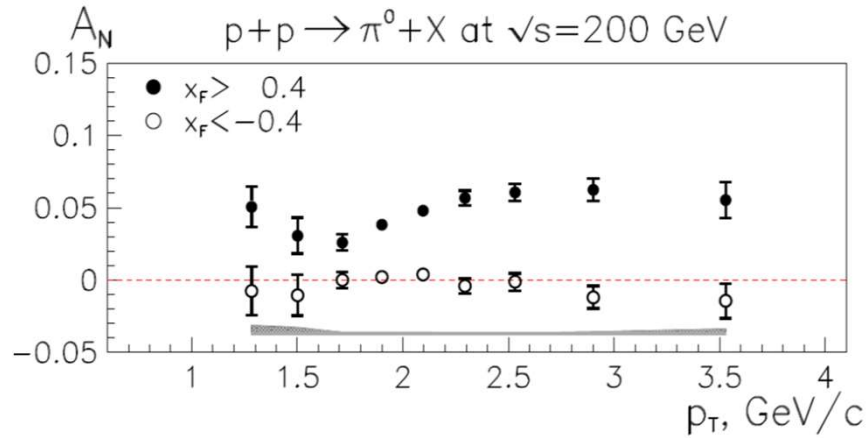
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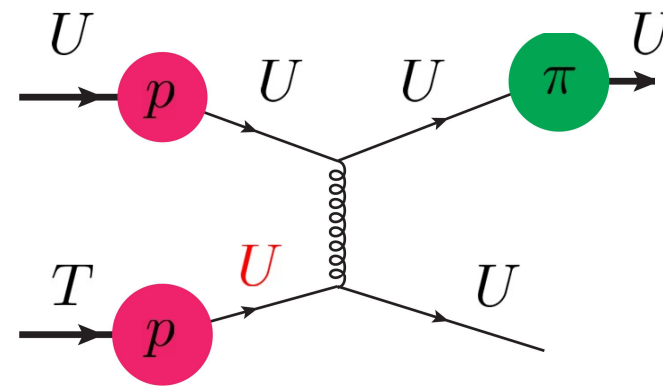
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		quark pol.		
		U	L	T
nucleon pol.	U	$f_1$		$h_1^{\perp}$
	L		$g_{1L}$	$h_{1L}^{\perp}$
	T	$f_{1T}^{\perp}$	$g_{1T}$	$h_1, h_{1T}^{\perp}$

Twist-2 TMDs

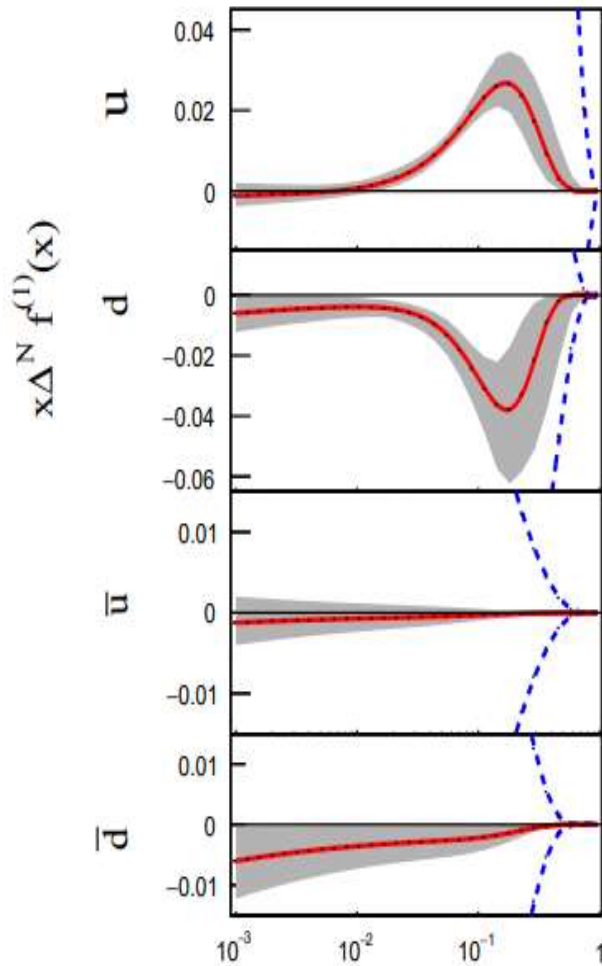


Understanding of the 3D structure and nontrivial polarization effects are tasks of future experiments

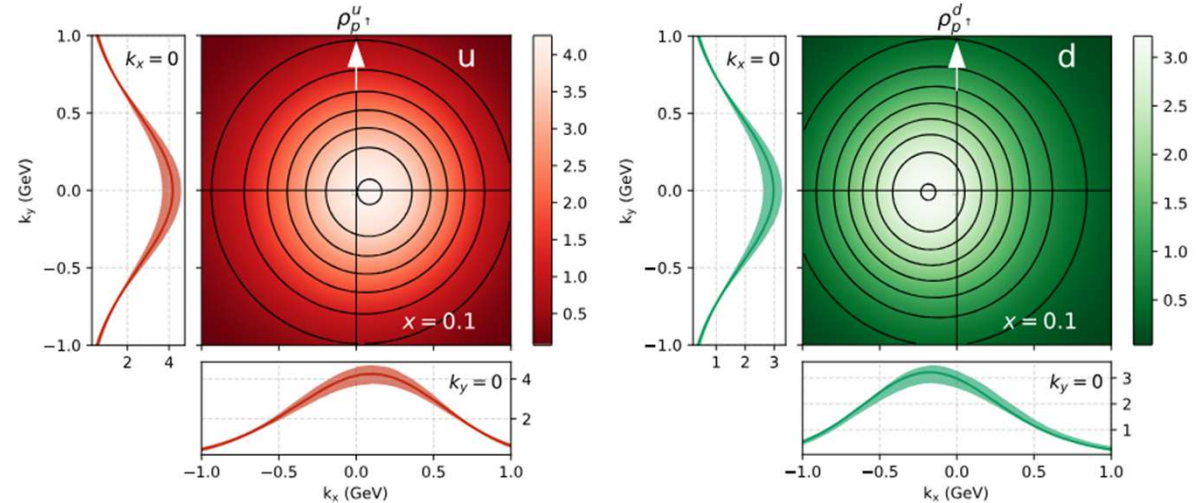
# Quark Sivers effect

well determined in the past couple of decades

TMD Handbook, arXiv:2304.03302



x

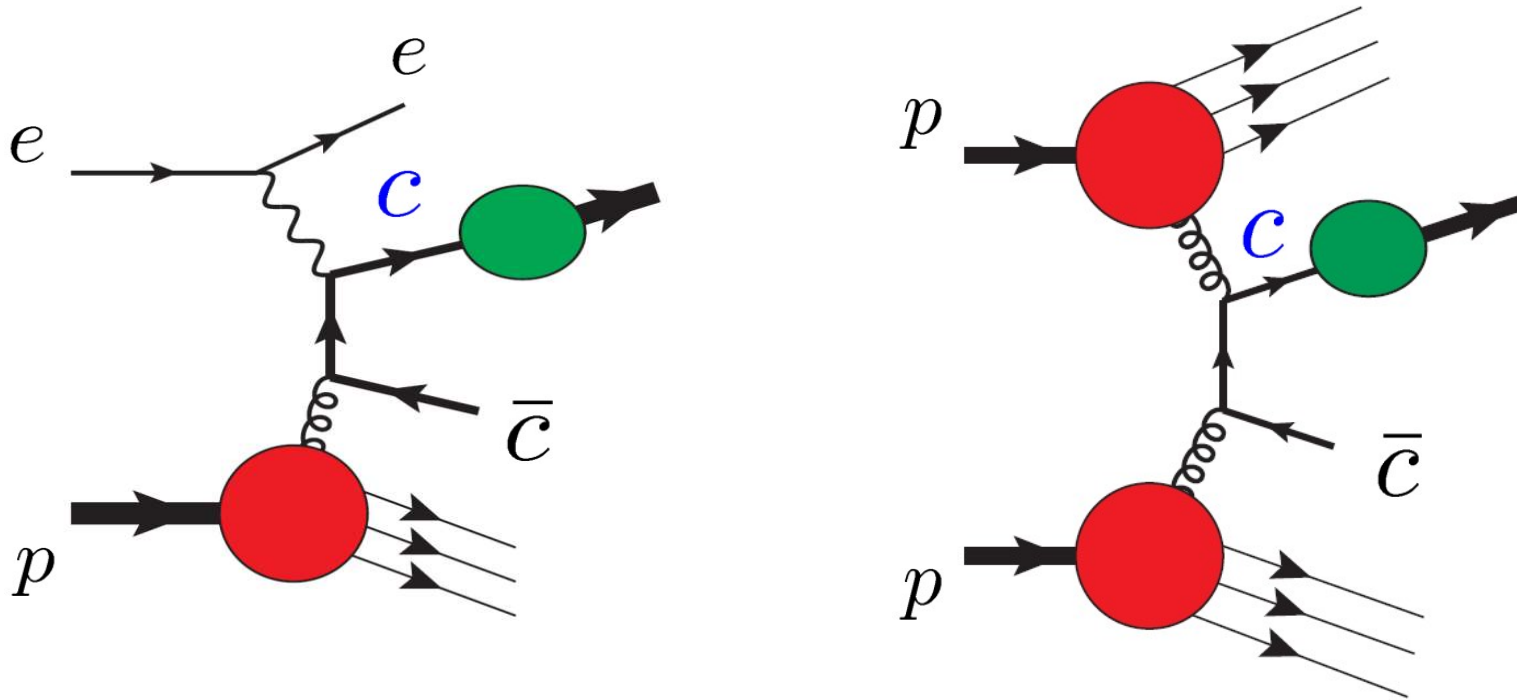


Proceed with precision tests

- Sea distributions
- Role of scale evolution
- Confirmation of the sign change

# Heavy quark production

Heavy quarks are mainly produced by the gluon fusion.



Heavy meson productions like  $D$ ,  $J/\psi$ ,  $\dots$  are ideal observables to investigate gluon distribution functions

# Gluon Sivers effect

A lot of work have been done on the gluon TMD Sivers effect

- *D*-meson production

PRD 70, 074025 (2004) PRD 94, 114022 (2016) PRD 96, 036011 (2017)

PRD 97, 076001 (2018) PRD 99, 036013 (2019)

- *J/ψ* production

PRD 96, 036011 (2017) PRD 99, 036013 (2019) PRD 91, 014005 (2015) EPJC 77, 854 (2017)

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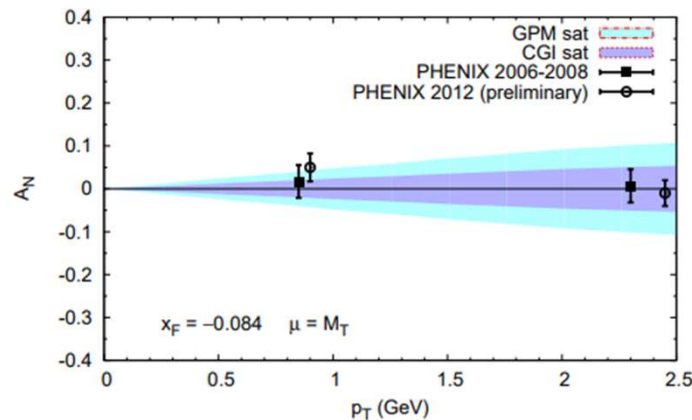
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PRD 101, 054003 (2020) EPJC 79, 1029 (2019) PRD 102, 094011 (2020)

However...



Only a few data points are available

A. Adare et al. [PHENIX Collaboration],  
Phys. Rev. D 82 (2010) 112008

little information on the gluon TMD Sivers function even at the tree-level



# Gluon Sivers in twist-3

Two independent functions

H. Beppu, Y. Koike, K. Tanaka and SY, Phys. Rev. D82 (2010)

$C$ -odd function:  $O(x_1, x_2)$

$C$ -even function:  $N(x_1, x_2)$

Separation of them is always a problem

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The cross section formulas of the SSA have been derived in  $D$ -meson productions.

•  $D$ -meson production in  $ep$  Z. B. Kang and J. W. Qiu, Phys. Rev. D78 (2008)

H. Beppu, Y. Koike, K. Tanaka and SY, Phys. Rev. D82 (2010)

in  $pp$  Z. B. Kang, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D78 (2008)

Y. Koike and SY, Phys. Rev. D84 (2011)

**No results for  $J/\psi$  production**

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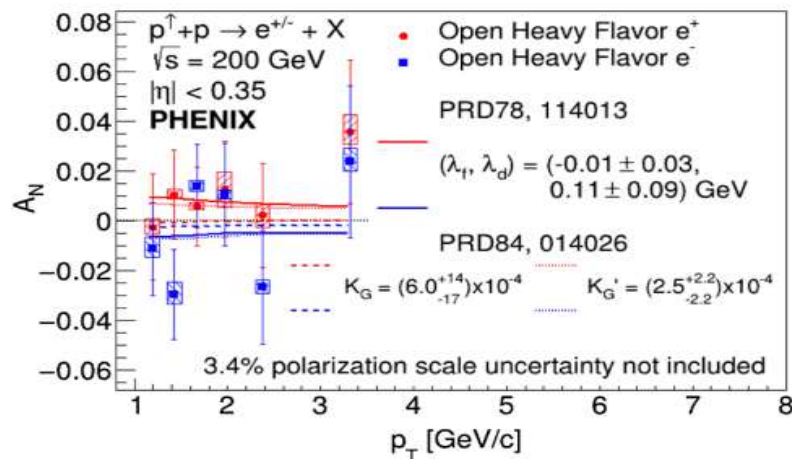
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No results for  $J/\psi$  production



N. J. Abdulameer et al. [PHENIX], Phys. Rev. D107 (2023)

$$O(x, x) = N(x, x) = K_G G(x) \quad K_G = (6.0^{+14}_{-17}) \times 10^{-4}$$

Upper bound of the functions

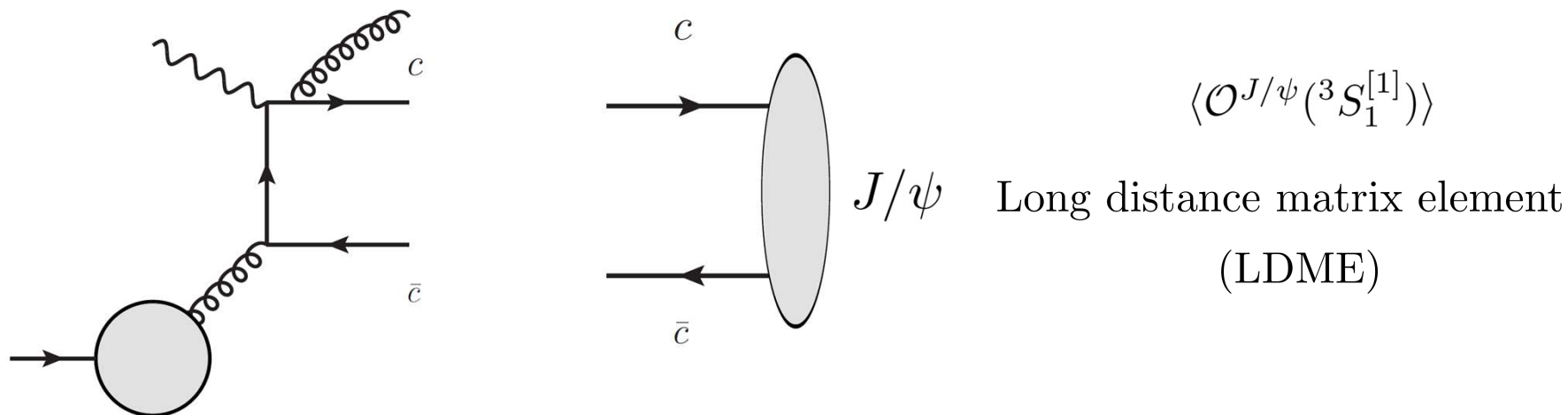
# Collinear twist-3 + NRQCD

We adopt non-relativistic QCD framework for the hadronization into  $J/\psi$

- The charm quark pair is produced through a hard scattering

$$\gamma^* + g \rightarrow \sum_n c\bar{c}[n] + g \quad n: \text{possible Fock state } {}^3S_1^{[1]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}, \dots$$

- Hadronization happens nonperturbatively



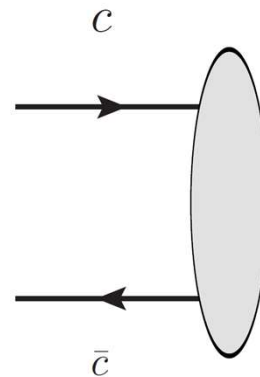
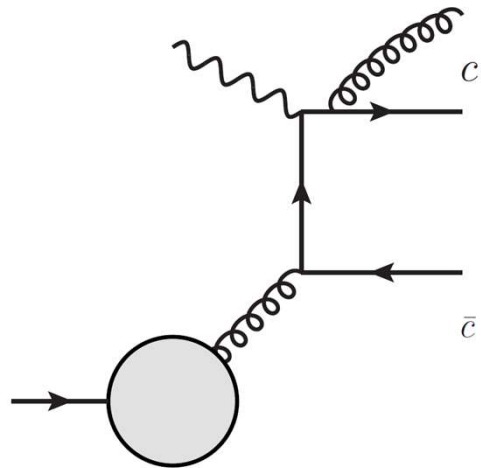
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$J/\psi$

Long distance matrix element  
(LDME)

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle$$

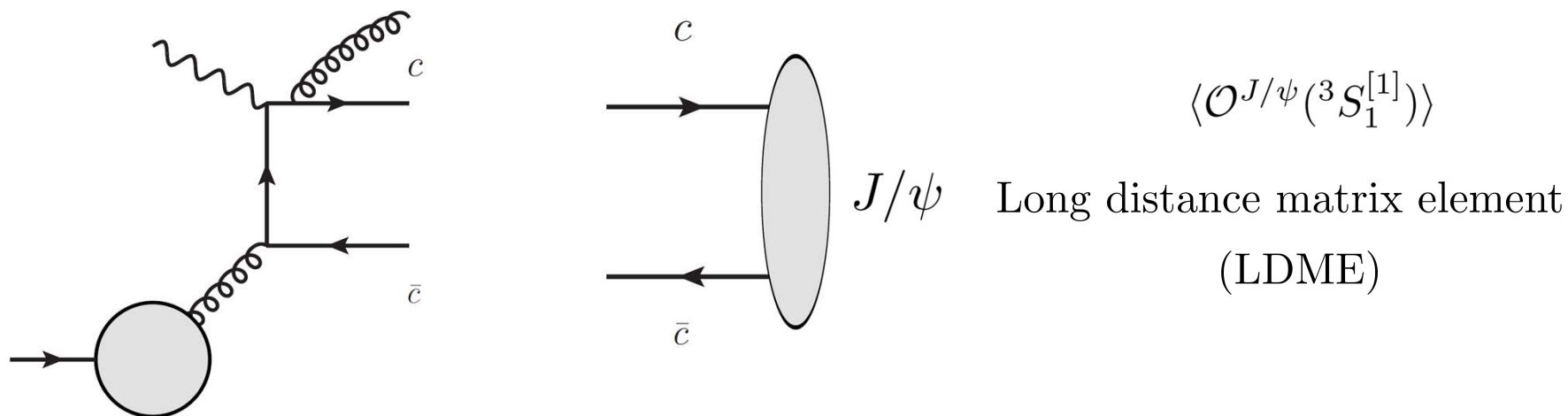
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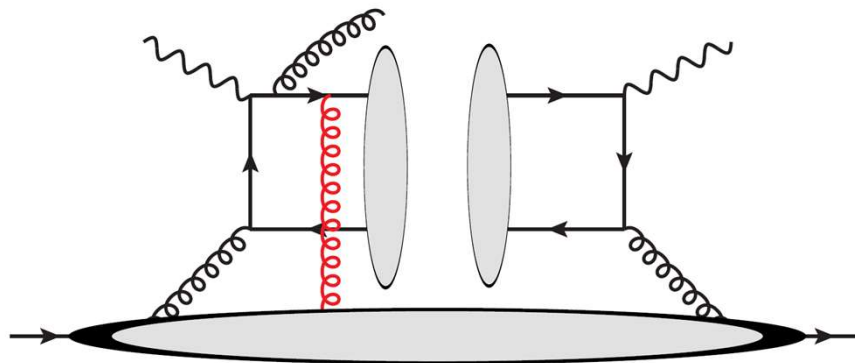
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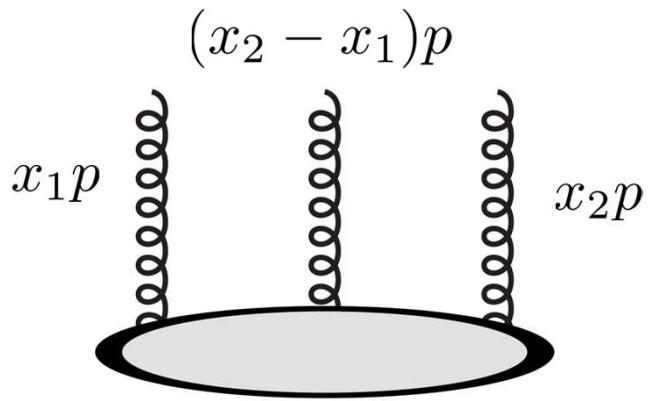
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- Hadronization happens nonperturbatively



- Twist-3 contribution requires one more gluon line





H. Beppu, Y. Koike, K. Tanaka and SY, Phys. Rev. D82 (2010)

$$\begin{aligned}
 & \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle PS_{\perp} | \left\{ \begin{array}{l} d_{bca} \\ i f_{bca} \end{array} \right\} (-i) F_b^{\beta n}(0) [0, \mu n] i F_c^{\gamma n}(\mu n) [\mu n, \lambda n] i F_a^{\alpha n}(\lambda n) | PS_{\perp} \rangle \\
 & = 2i M_N [g^{\alpha\beta} \epsilon^{\gamma p n} S \left\{ \begin{array}{l} O(x_1, x_2) \\ N(x_1, x_2) \end{array} \right\} + g^{\beta\gamma} \epsilon^{\alpha p n} S \left\{ \begin{array}{l} O(x_2, x_2 - x_1) \\ N(x_2, x_2 - x_1) \end{array} \right\} + g^{\gamma\alpha} \epsilon^{\beta p n} S \left\{ \begin{array}{l} O(x_1, x_1 - x_2) \\ N(x_1, x_1 - x_2) \end{array} \right\}
 \end{aligned}$$

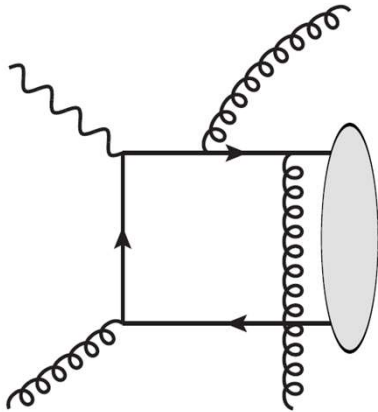
Two independent correlation functions



# Pole contributions

Sivers effect arises from pole contributions in the twist-3 calculation

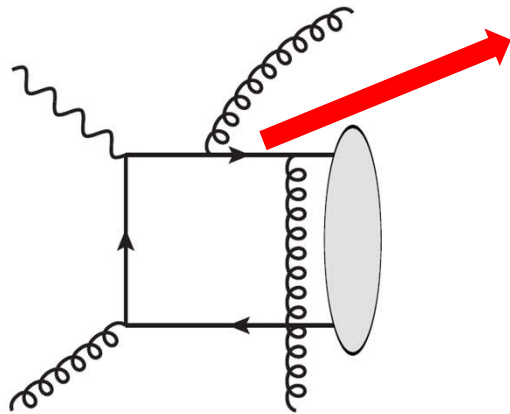
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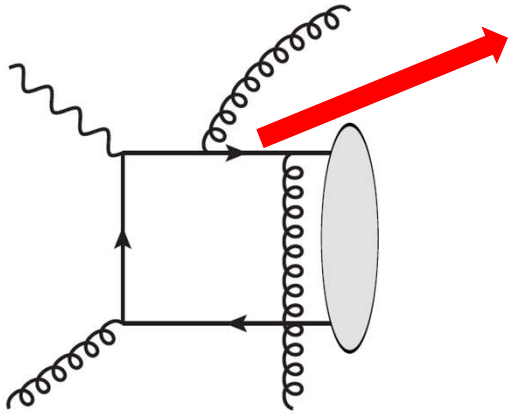
$$\frac{1}{\left(\frac{P_{J/\psi}}{2} - (x_2 - x_1)p\right)^2 - \frac{m_{J/\psi}^2}{4} + i\epsilon}$$

$$P \frac{1}{\left(\frac{P_{J/\psi}}{2} - (x_2 - x_1)p\right)^2 - \frac{m_{J/\psi}^2}{4}} - i\pi\delta\left(\left(\frac{P_{J/\psi}}{2} - (x_2 - x_1)p\right)^2 - \frac{m_{J/\psi}^2}{4}\right)$$

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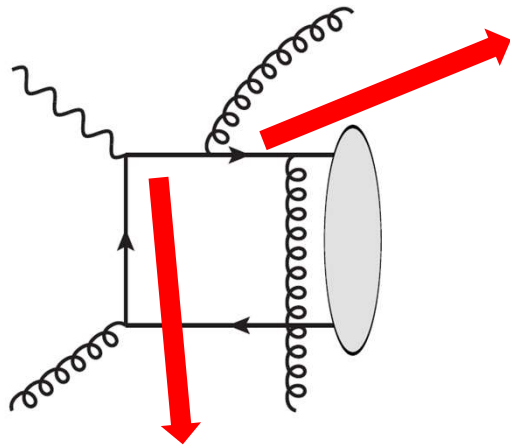
Soft-gluon-pole (SGP)  $-i\pi \frac{1}{P_{J/\psi} \cdot p} \delta(x_2 - x_1)$

$$O(x, x), O(x, 0), N(x, x), N(x, 0)$$

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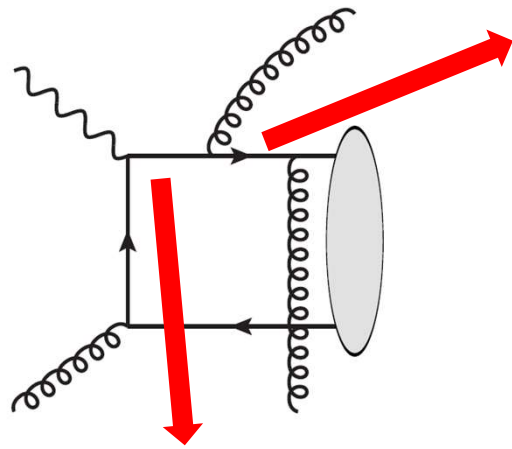
$$O(x, x), O(x, 0), N(x, x), N(x, 0)$$

$$\frac{1}{\left(x_1 p + q - \frac{P_{J/\psi}}{2}\right)^2 - \frac{m_{J/\psi}^2}{4} + i\epsilon}$$

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There are two types of poles in  $J/\psi$  production



$$\frac{1}{\left(\frac{P_{J/\psi}}{2} - (x_2 - x_1)p\right)^2 - \frac{m_{J/\psi}^2}{4} + i\epsilon}$$

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$$\frac{1}{\left(x_1 p + q - \frac{P_{J/\psi}}{2}\right)^2 - \frac{m_{J/\psi}^2}{4} + i\epsilon}$$

Hard-pole(HP)  $-i\pi \frac{1}{2p \cdot \left(q - \frac{P_{J/\psi}}{2}\right)} \delta\left(x_1 - Ax\right) \quad A \neq 0$

$O(x, Ax), O(x, (1 - A)x), O(Ax, -(1 - A)x)$

$N(x, Ax), N(x, (1 - A)x), N(Ax, -(1 - A)x)$

# Cross section formula

$$\text{SIDIS } e^-(\ell) + p^\uparrow(p) \rightarrow e^-(\ell') + J/\psi(P_{J/\psi}) + X$$

$$S_{ep} = (p + \ell)^2 \quad x_B = \frac{Q^2}{2p \cdot q} \quad Q^2 = -(\ell - \ell')^2 \quad z_f = \frac{p \cdot P_{J/\psi}}{p \cdot q}$$

$$\begin{aligned} \frac{d^6 \Delta\sigma}{dx_B dQ^2 dz_f dP_T^2 d\phi d\chi} &= \frac{\alpha_{em}^2 \alpha_s^2 e_c^2 (2\pi M_N)}{4\pi S_{ep} x_B^2 Q^2} \left( \mathcal{N} \langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle \right) \sum_{i=1, \dots, 4, 8, 9} \mathcal{A}_i(\phi - \chi) \mathcal{S}_i(\Phi_S - \chi) \\ &\times \int \frac{dx}{x^2} \delta \left[ \frac{P_T^2}{Q^2} - \left( 1 - \frac{1}{\hat{x}} + \frac{m_{J/\psi}^2}{z_f Q^2} \right) \left( 1 - \frac{1}{z_f} \right) \right] \left[ N(x, x) \sigma_i^{N1} + N(x, 0) \sigma_i^{N2} + N(x, Ax) \sigma_i^{N3} \right. \\ &\left. + N(x, (1-A)x) \sigma_i^{N4} + N(Ax, -(1-A)x) \sigma_i^{N5} \right] \end{aligned}$$

- C-odd function is canceled  $O(x_1, x_2)$

- Derivative terms  $\frac{d}{dx} N(x, x)$ ,  $\frac{d}{dx} N(x, 0)$  are canceled,

but nonderivative functions  $N(x, x)$ ,  $N(x, 0)$  survive

Feng Yuan, Phys. Rev. D78 (2008)

- Hard-pole contribution exist in  $J/\psi$  production

A. Schafer and J. Zhou, Phys. Rev. D88 (2013)

The cross section has five types azimuthal dependences

$$\frac{d^6 \Delta\sigma}{dx_B dQ^2 dz_f dP_T^2 d\phi d\chi} = \sin(\phi_h - \phi_S)(\mathcal{F}_1 + \mathcal{F}_2 \cos \phi_h + \mathcal{F}_3 \cos 2\phi_h) + \cos(\phi_h - \phi_S)(\mathcal{F}_4 \sin \phi_h + \mathcal{F}_5 \sin 2\phi_h)$$

We perform numerical calculations for five normalized structure functions

$$\frac{\mathcal{F}_1}{\sigma_1^U}, \quad \frac{\mathcal{F}_2}{2\sigma_1^U}, \quad \frac{\mathcal{F}_3}{2\sigma_1^U}, \quad \frac{\mathcal{F}_4}{2\sigma_1^U}, \quad \frac{\mathcal{F}_5}{2\sigma_1^U} \quad \sigma_1^U : \text{unpolarized cross section}$$

$$\frac{\mathcal{F}_1}{\sigma_1^U} = \frac{2\pi M_N}{[\frac{4}{y^2}(1-y+\frac{y^2}{2})\hat{\sigma}_1 - 2\hat{\sigma}_2]\bar{x}G(\bar{x})} \left[ \frac{4}{y^2}(1-y+\frac{y^2}{2}) \left( \sum_{i=1}^5 N^i(\bar{x})\sigma_1^{Ni} \right) - 2 \left( \sum_{i=1}^5 N^i(\bar{x})\sigma_2^{Ni} \right) \right] \quad y = \frac{Q^2}{x_B S_{ep}}$$

LDME is canceled in the ratio

$$N^{1,2,3,4,5}(x) = \{N(x, x), N(x, 0), N(x, Ax), N(x, (1-A)x), N(Ax, -(1-A)x)\}$$

We use two models

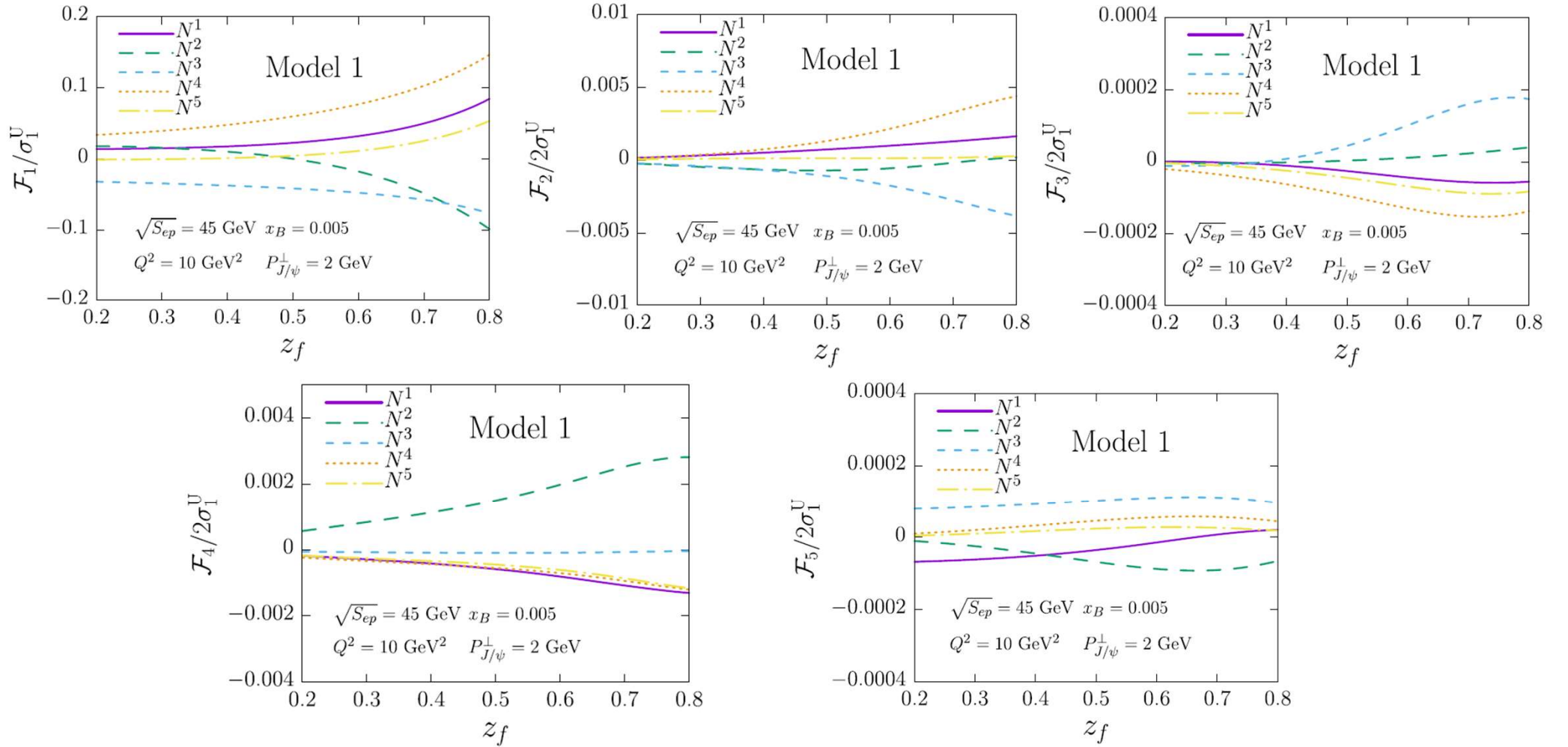
$$\left\{ \begin{array}{l} N(x, x) = 0.002xG(x) \\ N(x, x) = 0.0005x^{\frac{1}{2}}G(x) \end{array} \right. \quad \text{Upper bound of the experimental data}$$

Z. B. Kang, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D78 (2008)

Y. Koike and SY, Phys. Rev. D84 (2011)



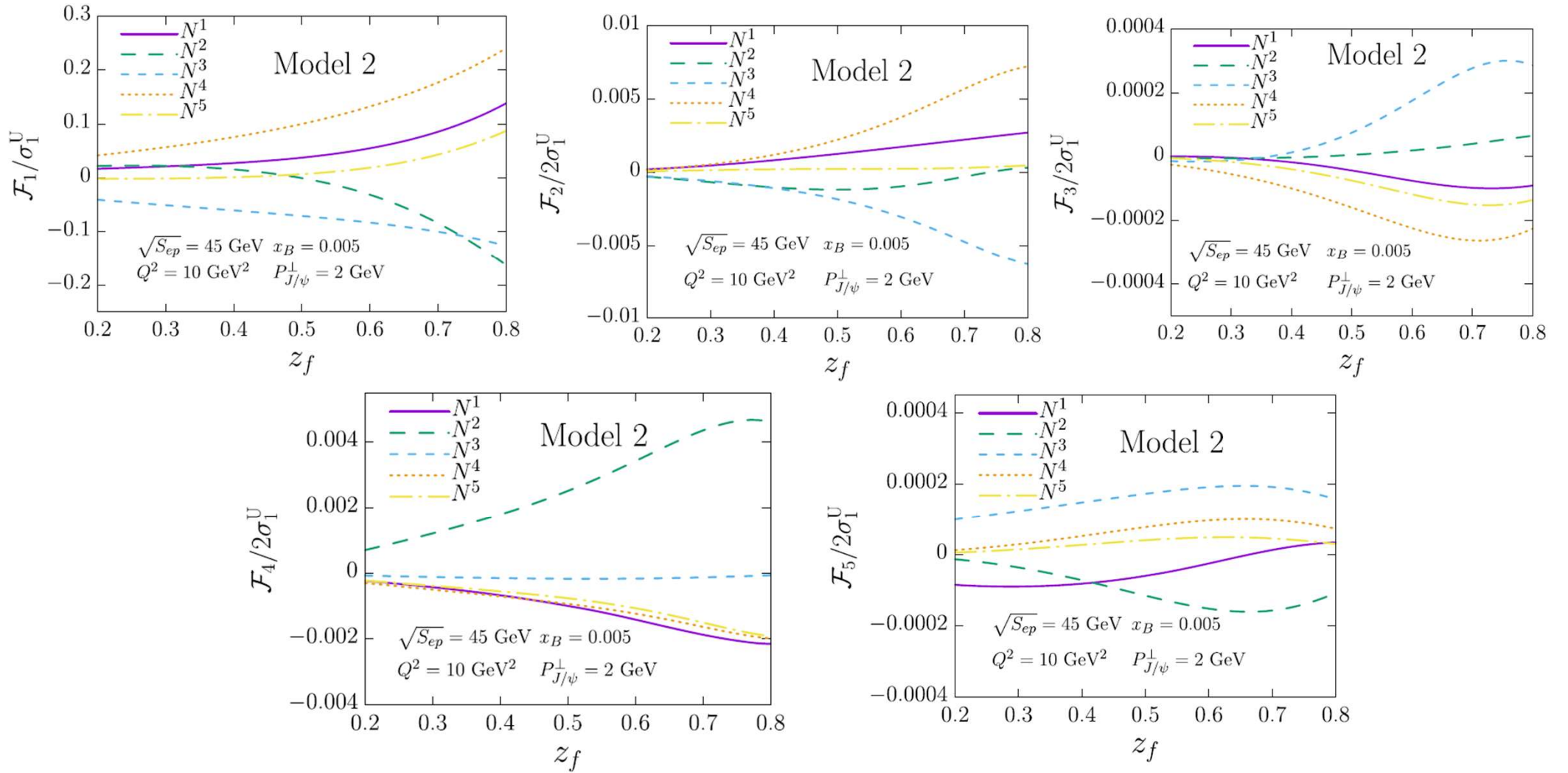
# Numerical simulation(model1)



$$N^{1,2,3,4,5} = \{N(x, x), N(x, 0), N(x, Ax), N(x, (1 - A)x), N(Ax, -(1 - A)x)\} = 0.002xG(x)$$

EIC energy:  $\sqrt{S_{ep}} = 45 \text{ GeV}$   $x_B = 0.005$   $Q^2 = 10 \text{ GeV}^2$   $P_{J/\psi}^\perp = 2 \text{ GeV}$

# Numerical simulation(model2)



$$N^{1,2,3,4,5} = \{N(x, x), N(x, 0), N(x, Ax), N(x, (1 - A)x), N(Ax, -(1 - A)x)\} = 0.0005x^{\frac{1}{2}}G(x)$$

# Summary

- We calculated the twist-3 gluon contribution to the SSA in the  $J/\psi$  production
- The  $C$ -odd function  $O(x_1, x_2)$  is canceled and the LDME is also canceled in the ratio if the color singlet contribution is dominant

—————> Ideal observable to measure  $N(x_1, x_2)$

Relation with TMD (analogy with  $f_{1T}^{\perp(1)}(x) = \pi F_{FT}(x, x)$ )

$$G_T^{(1)}(x) = -4\pi(N(x, x) - N(x, 0)) \qquad \Delta H_T^{(1)}(x) = 8\pi N(x, 0)$$

- The SSA could be sizable in the EIC energy
- Future work
- Calculation for  $pp$  collision
  - Calculation for color-octet channels
  - Matching between TMD and the collinear twist-3