Double DVCS including kinematic twist-3 and 4 corrections

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Outline

- Generalized parton distributions.
- DVCS, TCS & DDVCS.
- Energy scale and potential experiments.
- Spin-0 target $\rightarrow$ helicity amplitudes $\rightarrow$ kinematic twist-3 $+$ 4.
- DVCS & TCS limits.
- Phenomenology for the pion.
- Summary and conclusions.

Partonic distribution

GPD

Generalized Parton Distribution ≈ "3D version of a PDF (Parton Distribution Function)." With x the fraction of the hadron's longitudinal momentum carried by a quark:

$$\mathrm{GPD}_f(x,\xi,t)=\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\bar{p}^+z^-} \langle p'|\bar{\mathfrak{q}}_f(-z/2)\gamma^+\mathcal{W}[-z/2,z/2]\mathfrak{q}_f(z/2)|p\rangle\Big|_{z_\perp=z^+=0}$$

$$t=\Delta^2=(p'-p)^2, \quad \xi=-\frac{\Delta n}{2\bar{p}n}, \quad \rho=\xi\frac{qq'}{\Delta q'}, \quad \bar{p}=\frac{p+p'}{2}$$

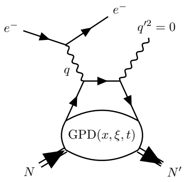
Importance

- Connected to **QCD energy-momentum tensor**. GPDs are a way to study "mechanical" properties and to address the hadron's spin puzzle.
- **Tomography:** distribution of quarks in terms of the longitudinal momentum and in the transverse plane.

$$q(x,\vec{b}_\perp)=\int \frac{d^2\vec{\Delta}_\perp}{4\pi^2}\; e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H^q(x,0,t=-\vec{\Delta}^2_\perp)$$

Deeply virtual Compton scattering (DVCS)

- In the 1990s, Müller, Ji and Radyushkin introduced the Generalized Parton Distributions (GPDs) to study the DVCS process:



Feynman diagram for DVCS

- At LO $\sim O(\alpha_s^0)$ and LT $\sim O(1/Q^0)$:

$$\mathrm{CFF}_{\mathrm{DVCS}} \sim \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x-\xi}\mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx \;\, i\pi\delta(x-\xi)\mathrm{GPD}(x,\xi,t) + \cdots$$

- $\xi = \frac{-n\Delta}{2\bar{p}n}\,, \quad \bar{p} = \frac{p+p'}{2}\,, \quad \Delta = p' - p\,, \quad t = \Delta^2$

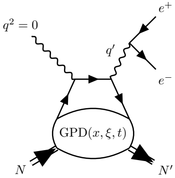
DVCS has been measured in DESY, CERN and JLab.

Timelike Compton scattering (TCS)

- Complementary to DVCS.

  E. R. Berger, M. Diehl and B. Pire, EPJC 23, 675–689 (2002).

  1st measurement of TCS by the CLAS collaboration at JLab: P. Chatagnon et al., PRL **127**, 262501 (2021).



Feynman diagram for TCS

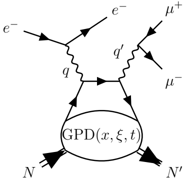
- At LO LT:

$$\mathrm{CFF_{TCS}} \sim \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x+\xi} \mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx \; i\pi\delta(x+\xi)\mathrm{GPD}(x,\xi,t) + \cdots$$

- Like DVCS but $\xi \to -\xi$.

Double deeply virtual Compton scattering (DDVCS)

- **DDVCS vs DVCS/TCS:** extra virtuality $\Rightarrow$ *generalized* Björken variable $\rho \Rightarrow$ GPDs for $x = \rho \neq \xi$.



Double DVCS (DDVCS)

- At LO LT:

$$\mathrm{CFF}_{\mathrm{DDVCS}} \sim \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x-\rho} \mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx \;\; i\pi\delta(x-\rho)\mathrm{GPD}(x,\xi,t) + \cdots$$

$$\xi = -\frac{\Delta n}{2\bar{p}n}, \quad \rho = \xi \frac{qq'}{\Delta q'}, \qquad \rho = \xi \text{ (DVCS)}, \quad \rho = -\xi \text{ (TCS, LT approach)}$$

See talk by Juan Sebastián Alvarado (yesterday).

Why DDVCS? / State of art

Why DDVCS?

1. Access to $x \neq \pm\xi \leftarrow$ restriction in DVCS & TCS (at lowest order).
2. Single framework to describe DVCS, TCS & DDVCS:

$$\text{DVCS: } Q'^2 \to 0,\ \rho \to \xi$$

$$\text{TCS: } Q^2 \to 0,\ \rho \to -\xi \ \text{(at LT)}$$

State of art:

- Original papers in DDVCS **(LO LT)**:
  1. Belitsky & Müller, PRL 90, 022001 (2003).
  2. Guidal & Vanderhaeghen, PRL 90, 012001 (2003).
  3. Belitsky & Müller, PRD 68, 116005 (2003).
- Alternative derivation of amplitudes and cross-section, and JLab and EIC's phenomenology **(LO LT)** are discussed in: K. Deja, VMF, B. Pire, P. Sznajder & J. Wagner, PRD 107, 094035 (2023).
- **NLO** is known for DVCS, TCS & DDVCS.

LT and beyond

- LT: Q^2, $Q'^2 \to \infty$, Björken limit.
- Kinematic power corrections = kinematic higher-twist corrections:

$$\sim \left(\frac{|t|}{Q^2}\right)^P, \quad \sim \left(\frac{M^2}{Q^2}\right)^P,$$

$$P = \frac{\tau_{\text{kin}} - 2}{2}$$

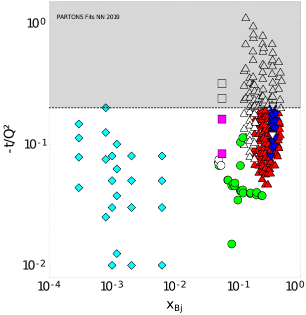
- Similarly for Q'^2.
- LT = kinematic twist-2, $\tau_{\text{kin}} = 2$.

Why higher twists?

1. Nucleon tomography is a Fourier transform in $\Delta_\perp$ that requires data on a sizable range of t:

$$q(x,\vec{b}_\perp)=\int \frac{d^2\vec{\Delta}_\perp}{4\pi^2}\ e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H^q(x,0,t=-\vec{\Delta}^2_\perp)$$

2. Increase the range of useful experimental data:



Data come from the Hall A (▼, ▽), CLAS (▲, △), HERMES (●, ○), COMPASS (■, □) and HERA H1 and ZEUS (♦, ◊) experiments. The gray bands (open markers) indicate phase-space areas (experimental points) being excluded in the analysis of H. Moutarde, P. Sznajder, J. Wagner, EPJC 79, 614 (2019).

Why spin-0 target?

1. It is the simplest case (spin-1/2 and spin-1 targets to be studied later).

2. Study of DVCS on scalar and pseudo-scalar nuclei, e.g. $^4\mathrm{He}$:

   S. Fucini, S. Scopetta, & M. Viviani, PRC 98, 015203 (2018).

3. Study of meson GPDs (π) through the Sullivan process: $\gamma^* p \to \gamma \pi^+ n$

   J. D. Sullivan, PRD 5, 1732 (1972).

   Pion GPDs from J. M. Morgado-Chavez et al., PRD 105, 094012 (2022).

   D. Amrath, M. Diehl, J.-P. Lansberg, EPJC 58, 179–192 (2008).

   J. M. Morgado Chávez, PRL 128, 202501 (2022).

4. Kinematical higher twist contributions recently studied in related process $\gamma^* \gamma \to \pi^+ \pi^-$: access to pion GDAs:

   C. Lorcé, B. Pire, Q.-T. Song, PRD 106, 094030 (2022); B. Pire, Q.-T. Song, PRD 107, 114014 (2023) .

Scale of DDVCS and experiments

- **Scale of DDVCS, not imposed but inferred from the twist expansion:**

$$\mathbb{Q}^2 = Q^2 + Q'^2 + t$$

- **Conclusion:** you may choose

$$\begin{cases} \sigma_{\text{DDVCS}} \text{ grows rapidly with small } Q^2 \Rightarrow Q^2 < 1 \text{ GeV}^2 \\ \text{region in-between resonances} \Rightarrow Q'^2 \in (2.25, 9) \text{ GeV}^2 \end{cases}$$

- Low Q^2 + high Q'^2... as for TCS* $\rightarrow$ potential with CLAS12 (Hall B, JLab) & SoLID† (Hall A, JLab) $\Leftrightarrow$ muon detector or reconstruction trajectories.

* Chatagnon et al., PRL 127, 262501.

† Arrington et al., J. Phys. G 50 (2023) 11, 110501.

Conformal twist expansion

- **Breakthrough:** OPE based on conformal field theory calculation by Braun, Ji & Manashov, JHEP 2021, 51 (2021). OPE to all twists!!
- Use of conformal symmetry to constrain the coefficients of the expansion around the light-cone.
- Proper kinematic power expansion. No higher-twist GPDs.
- Natchman-like corrections are more involved as

$$\langle p'|\partial^\mu \mathcal{O}|p\rangle = i\Delta^\mu \langle p'|\mathcal{O}|p\rangle \,.$$

- This formalism satisfies QED gauge and translation invariance.

Starting point: OPE + CFT (Braun-Ji-Manashov)

$$T^{\mu\nu} = i \int d^4z\, e^{iq'z} \langle p' | \mathcal{T}\{ j^\nu(z) j^\mu(0) \} | p \rangle =$$

$$\frac{1}{i\pi^2} i \int d^4z\, e^{iq'z} \Bigg\{ \frac{1}{(-z^2+i0)^2} \left[g^{\nu\mu} \mathscr{O}(1,0) - z^\nu \partial^\mu \int_0^1 du\; \mathscr{O}(\bar{u},0) - z^\mu (\partial^\nu - i\Delta^\nu) \int_0^1 dv\; \mathscr{O}(1,v) \right]$$

$$- \frac{1}{-z^2+i0} \left[\frac{i}{2} (\Delta^\mu \partial^\nu - (\nu \leftrightarrow \mu)) \int_0^1 du \int_0^{\bar{u}} dv\; \mathscr{O}(\bar{u},v) - \frac{t}{4} z^\nu \partial^\mu \int_0^1 du\; u \int_0^{\bar{u}} dv\; \mathscr{O}(\bar{u},v) \right]$$

$$+ \cdots$$

Operators $\mathscr{O}$ above are understood as matrix elements, that is:

$$\langle p' | \mathscr{O}(\lambda_1,\lambda_2) | p \rangle = \frac{2i}{\lambda_{12}} \iint_{\mathbb{D}} d\beta d\alpha \left[e^{-i\ell_{\lambda_1,\lambda_2} z} \right]_{\mathrm{LT}} \Phi^{(+)}(\beta,\alpha,t)\,,$$

where

$$\ell_{\lambda_1,\lambda_2} = -\lambda_1 \Delta - \lambda_{12} \left[\beta \bar{p} - \tfrac{1}{2}(\alpha+1)\Delta \right]$$

and $\Phi^{(+)}$ is given by the usual DDs h_f, g_f as

$$\Phi^{(+)}(\beta,\alpha,t) = \sum_f \left(\frac{e_f}{e} \right)^2 \Phi_f^{(+)}(\beta,\alpha,t)\,, \quad \Phi_f^{(+)}(\beta,\alpha,t) = \partial_\beta h_f + \partial_\alpha g_f$$

More details on: Braun, Ji & Manashov, JHEP 03 (2021) 051; and JHEP 01 (2023) 078.

Scalar and pseudo-scalar target

- Spin-0 target $\Rightarrow$ vector component of $T^{\mu\nu}$ is enough.
- Parameterization of $T^{\mu\nu} \to$ **helicity amplitudes, $\mathcal{A}^{AB}$.**
- Spin-0 $\Rightarrow \mathcal{A}^{AB} \equiv \mathcal{H}^{AB}/2$ **(5 CFFs in total).**

$$
\begin{aligned}
T^{\mu\nu} = \mathcal{A}^{00} i \frac{(qq')^2}{R^2 Q^2} & \left[q^\mu q^\nu \sqrt{\frac{R+qq'}{R-qq'}} + q^{(\mu} q'^{\nu)} \frac{2Q}{Q'} - q^\mu q'^\nu \frac{R^2 Q}{2(qq')^2 Q'} + q'^\mu q'^\nu \frac{Q^3}{(qq')Q'} \right] \\
& + \mathcal{A}^{+0} \frac{i\sqrt{2}}{R|\bar{p}_\perp|} \left[Q' q^\mu - \frac{qq'}{Q'} q'^\mu \right] \bar{p}_\perp^\nu \\
& - \mathcal{A}^{0+} \frac{\sqrt{2}}{R|\bar{p}_\perp|} \bar{p}_\perp^\mu \left[\frac{qq'}{Q} q^\nu + Q q'^\nu \right] \\
& + \mathcal{A}^{+-} \frac{1}{|\bar{p}_\perp|^2} \left[\bar{p}_\perp^\mu \bar{p}_\perp^\nu - \widetilde{\bar{p}}_\perp^\mu \widetilde{\bar{p}}_\perp^\nu \right] \\
& - \mathcal{A}^{++} g_\perp^{\mu\nu} ,
\end{aligned}
\qquad R = \sqrt{(qq')^2 + Q^2 Q'^2}
$$

- Read out projectors $\to \mathcal{A}^{AB} = \boldsymbol{\Pi}_{\mu\nu}^{(AB)} T^{\mu\nu}$.

$\mathcal{A}^{+-}$

$\mathcal{A}^{+-}$ is of special interest as it enters the amplitude at:

- LO, kinematic twist-4,
- NLO, kinematic twist-2 through transversity GPDs.[$]
  For the spin-0 target: 1 (chiral-odd) transversity GPD + 1 gluon transversity GPD.

[$] Belitsky, Müller, PLB 486, 369–377, (2000).

$\mathcal{A}^{+-} = O(\text{twist-}4)$

$$\mathcal{A}^{+-}_{1/\mathbb{Q}^2} = \frac{1}{2|\bar{\boldsymbol{p}}_\perp|^2}\left(\bar{\boldsymbol{p}}_{\perp,\mu}\bar{\boldsymbol{p}}_{\perp,\nu} - \widetilde{\bar{\boldsymbol{p}}}_{\perp,\mu}\widetilde{\bar{\boldsymbol{p}}}_{\perp,\nu}\right) T^{\mu\nu}$$

$$= \frac{4\bar{p}_\perp^2}{\mathbb{Q}^2} D_\xi^2 \int_{-1}^{1} \frac{dx}{2\xi} Y\left(\frac{x}{\xi},\frac{\rho}{\xi}\right) H^{(+)}(x,\xi,t)\,, \quad D_\xi = \xi^2\partial_\xi$$

$$Y\left(\frac{x}{\xi},\frac{\rho}{\xi}\right) = -\frac{\xi+\rho}{\xi+x}\log\frac{\rho-x-i0}{\xi+\rho} - \frac{\xi-\rho}{\xi-x}\log\frac{x-\rho+i0}{\xi-\rho} + 2\log\frac{\rho-x-i0}{2\xi}\,.$$

Conclusion: $\mathcal{A}^{+-}$ factorizes at twist-4 in DDVCS and TCS.

- **DVCS# limit:** $Q'^2 \to 0\,, \rho \to \xi$

$$Y_{DVCS}\left(\frac{x}{\xi}\right) = \frac{2x}{\xi+x}\log\frac{\xi-x-i0}{2\xi}$$

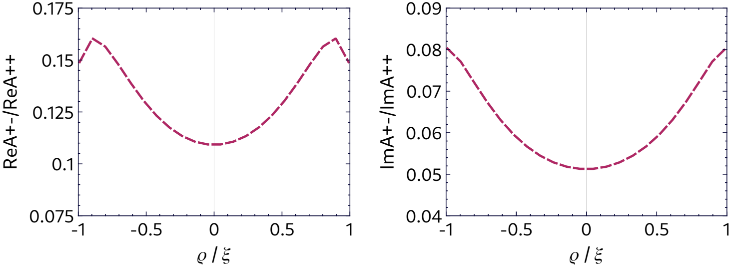
- **TCS limit:** $Q^2 \to 0\,, \rho \to -\xi(1-2t/Q'^2)$

$$Y_{TCS}\left(\frac{x}{\xi}\right) = \frac{2x}{x-\xi}\log\frac{\xi+x+i0}{2\xi}$$

#Results for the spin-0 target in DVCS were already published by Braun, Manashov & Pirnay in PRD 86, 014003 (2012); and with an alternative method by Braun, Ji & Manashov in JHEP 01 (2023) 078.

Phenomenology for pion target

PRELIMINARY RESULTS



Real (left) and imaginary (right) parts of $\mathcal{A}^{+-}$ relative to $\mathcal{A}^{++}\big|_{\mathrm{LT}}$ as a function of ρ/ξ for $\xi = 0.25$, $\mathbb{Q}^2 = 1.9\ \mathrm{GeV}^2$ and $t = -0.8\ \mathrm{GeV}^2$.

- **Conclusion:** $\mathcal{A}^{+-}$ is of the order $\sim 10\%$ of $\mathcal{A}^{++}|_{\mathrm{LT}}$ at the order of $-t = 1\ \mathrm{GeV}^2$.

π-GPD model: J. M. Morgado-Chávez et al., PRD 105, 094012 (2022).

Summary and conclusions

- By means of the conformal twist expansion by Braun, Ji & Manashov we compute the higher-twist corrections of DDVCS, DVCS & TCS (these last two by taking the limits of small outgoing and incoming virtuality on the DDVCS results, respectively).

Our results:

1. $\mathcal{A}^{+-}$ factorizes at twist-4 in DDVCS and TCS.
2. $\mathcal{A}^{+-}$ is of the order $\sim 10\%$ of $\mathcal{A}^{++}|_{\mathrm{LT}}$ at the order of $-t = 1$ GeV^2.
3. Other higher-twist amplitudes are in an advanced stage of calculation: $\mathcal{A}^{++}, \mathcal{A}^{0+}, \mathcal{A}^{+0}, \mathcal{A}^{00}$.

Merci beaucoup!

Complementary slides

Kinematic vs geometric twist

- **Kinematic** twist $\rightarrow$ power expansion of observables $\sim \frac{|t|}{\mathbb{Q}^2}, \frac{M^2}{\mathbb{Q}^2} \Rightarrow$ **frame dependent.**

- **Geometric** twist $\rightarrow$ expansion of operators $\mathscr{O}(z)$ in components belonging to irreps of the Lorentz group $\Rightarrow$ **Lorentz invariant.**

- $\mathscr{O}(z)$ **such that** $z^2 \neq 0 \Rightarrow$ **relaxation of the Björken limit in** $\int d^4z\ e^{-irz}\langle p'|\mathscr{O}(z)|p\rangle$ **(in observables).**

Energy scale of two-photon processes

- Compton tensor:

$(\ell = \ell(\Delta, \bar{p}) \, , \Delta^2 = t \, , \bar{p} = (p + p')/2 \, , p^2 = p'^{\,2} = M^2)$

$$T^{\mu\nu}_{s_2 s_1} = i \int d^4 z \; e^{iq' z} \langle p', s_2 | \mathcal{T} \left\{ j^\nu(z) j^\mu(0) \right\} | p, s_1 \rangle$$

$$\sim i \int d^4 z \; e^{iq' z} \frac{f^{\mu\nu}(z, \partial)}{(-z^2 + i0)^J} \underbrace{\left[e^{-i\ell z} \right]_{\mathrm{LT}}}_{\text{geom. LT}} \longrightarrow$$

geom. LT to connect to the usual **GPDs**

$$\sim i \int d^4 z \int_0^1 dw \; e^{i(q' - w\ell)z} \frac{\widetilde{f}^{\mu\nu}(z, w)}{(-z^2 + i0)^K}$$

$$\sim \sum_{n,m} f^{\mu\nu}_{n,m}(\Delta, \bar{p}, q') \times I_{n,m} \longrightarrow \quad I_{n,m} = \int_0^1 dw \; \frac{w^n}{(\ell^2 w^2 - 2q'\ell w + Q'^2 + i0)^m}$$

$$\sim \underbrace{O(1)}_{\text{kin. LT}} + \left(\text{powers of } \frac{|t|}{-2q'\Delta} \, , \frac{M^2}{-2q'\Delta} \right) .$$

- Scale is **in general:** $-2q'\Delta = Q^2 + Q'^{\,2} + t \equiv \mathbb{Q}^2$.

More details on conformal operator-product expansion: Braun, Ji & Manashov, JHEP 03 (2021) 051; and JHEP 01 (2023) 078.