Single- and double-unresolved limits of polarized matrix elements

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Introduction

- proton structure known to percent level? $\rightarrow$ only unpolarized!
- helicity-dependent PDFs highly unconstrained
- BNL Electron-Ion-Collider promises percent-level accuracy also for polarized PDFs

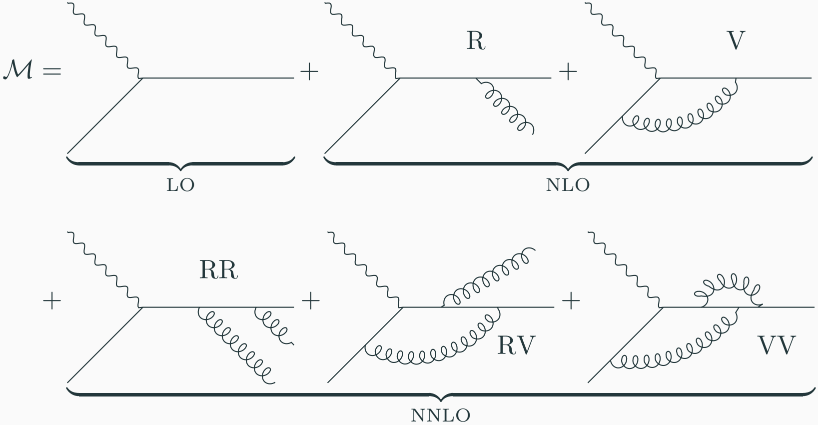
$\rightarrow$ requires inclusive, but also **less inclusive** observables at NNLO

  - polarized SIDIS
  - **jet observables**

- this talk:
  - infrared structure of real radiation in longitudinally polarized matrix elements
  - in Larin γ_5 scheme [Larin, Vermaseren '91; Larin '93]

Anatomy of amplitudes up to NNLO ...

…and where they first appear



$+\mathcal{O}(\mathrm{N^3LO})$

Observables up to NNLO

$$
\begin{aligned}
\sigma = &\int_m |\mathcal{M}_m^{(0)}|^2 J_m \, \mathrm{d}\phi_m \\
&+ \Bigg(\underbrace{\int_{m+1} |\mathcal{M}_{m+1}^{(0)}|^2 J_m \, \mathrm{d}\phi_{m+1}}_{\mathrm{R}} + \underbrace{\int_m (\mathcal{M}_m^{(1)} \mathcal{M}_m^{(0)*} + \mathrm{c.c.}) J_m \, \mathrm{d}\phi_m}_{\mathrm{V}} \Bigg) \\
&+ \Bigg(\underbrace{\int_{m+2} |\mathcal{M}_{m+2}^{(0)}|^2 J_m \, \mathrm{d}\phi_{m+2}}_{\mathrm{RR}} + \underbrace{\int_{m+1} (\mathcal{M}_{m+1}^{(1)} \mathcal{M}_{m+1}^{(0)*} + \mathrm{c.c.}) J_m \, \mathrm{d}\phi_{m+1}}_{\mathrm{RV}} \\
&\qquad + \underbrace{\int_m |\mathcal{M}_m^{(1)}|^2 J_m \, \mathrm{d}\phi_m + \int_m (\mathcal{M}_m^{(2)} \mathcal{M}_m^{(0)*} + \mathrm{c.c.}) J_m \, \mathrm{d}\phi_m}_{\mathrm{VV}} \Bigg) + \mathcal{O}(\mathrm{N^3LO})
\end{aligned}
$$

with $\mathcal{M}_m^{(l)}$ with the amplitude with m final-state partons and l loops, J_m the jet functions selecting m jets out of the final state momenta, and $\mathrm{d}\phi_m$ the m particle phase space

Infrared divergences

$$\sigma = \int_m |\mathcal{M}_m^{(0)}|^2 J_m \, \mathrm{d}\phi_m + \Bigg(\underbrace{\int_{m+1} |\mathcal{M}_{m+1}^{(0)}|^2 J_m \, \mathrm{d}\phi_{m+1}}_{\mathrm{R}} + \underbrace{\int_m (\mathcal{M}_m^{(1)} \mathcal{M}_m^{(0)*} + \mathrm{c.c.}) J_m \, \mathrm{d}\phi_m}_{\mathrm{V}} \Bigg)$$

at NLO:

- V explicit UV ($\to$ renormalization) and IR poles from loop integration
- R implicit infrared (IR) divergences, manifest only after phase space integration
- $\mathrm{V} + \mathrm{R}$ divergence free (modulo mass factorization) [KLN theorem]

Problem: analytical integration of exclusive phase spaces too hard
$\to$ numerical integration (MC)

But: numerical integration only in integer dimensions ↯ divergent ↯

Wishful thinking ... Subtraction

We could try to add a zero with the same infrared structure that subtracts the divergences locally in the phase space, but that we are somehow able to integrate analytically:

$$\sigma = \int_m \mathrm{d}\sigma_{\mathrm{LO}} + \left(\int_{m+1} \underbrace{(\mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NLO}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO}})}_{\text{finite}} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO}} + \int_m \mathrm{d}\sigma^{\mathrm{V}}_{\mathrm{NLO}} \right)$$

We are lucky:

- phase spaces factorize, e.g.
  $\mathrm{d}\phi_{m+1}(p_1, \ldots, p_{m+1}; Q) = \mathrm{d}\phi_m(\tilde{p}_1, \ldots, \tilde{p}_m; Q) \otimes [dp_{m+1}(\tilde{p}_1, \ldots, \tilde{p}_m)]$
- matrix elements factorize in singular limits

Established technique for LHC physics:
e.g. FKS [Frixione, Kunszt, Signer '95], Catani-Seymour ['96] @ NLO
polarized extension of Catani-Seymour [Borsa, de Florian, Pedron '20]
e.g. STRIPPER [Czakon '13], Antenna [Gehrmann et al. '05] @ NNLO

Subtraction in polarized processes

What is required?

✓ Phase space factorization
same in polarized and unpolarized processes

- How to build a subtraction term?
  $\rightarrow$ Full understanding of factorization of matrix elements:
  - Do polarized MEs factorize (γ_5) in infrared limits?
  - What do they factorize into?

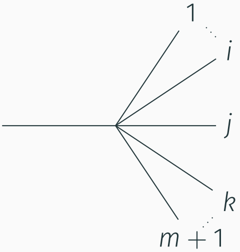
Disclaimer

- from now on consider only color-ordered matrix elements
  $\rightarrow$ singular limits only between color-adjacent partons
- $(\Delta)P_{ij}$ and $(\Delta)P_{ijk}$ (polarized) splitting *amplitudes*:
  only radiative parts, unregulated endpoints, dim. reg. dependent

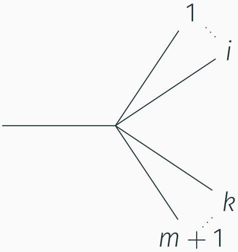
IR limits: the single-unresolved case (1)

Single-soft

parametrize $p_j \to \lambda p_j$ with $\lambda \to 0$



$$\xrightarrow{j\text{ soft}} 4\pi\alpha_s \frac{1}{\lambda^2}\frac{2s_{ik}}{s_{ij}s_{jk}} \times$$



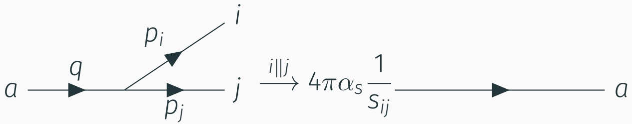
$$|(\Delta)\mathcal{M}^{(0)}_{m+1}(p_1,\ldots,p_i,p_j,p_k,\ldots p_{m+1})|^2$$
$$\xrightarrow{j\text{ soft}} 4\pi\alpha_s \frac{1}{\lambda^2}\frac{2s_{ik}}{s_{ij}s_{jk}}|(\Delta)\mathcal{M}^{(0)}_{m}(p_1,\ldots,p_i,p_k,\ldots p_{m+1})|^2 + \mathcal{O}(\lambda^0)$$

if j is an unpolarized gluon, else 0.

Single-collinear: $i \parallel j$, final-final

$$\begin{cases} p_i^\mu \to z p^\mu + k_\perp^\mu - \frac{k_\perp^2}{2z}\frac{n^\mu}{p\cdot n} \\ p_j^\mu \to (1-z) p^\mu - k_\perp^\mu - \frac{k_\perp^2}{2(1-z)}\frac{n^\mu}{p\cdot n} \end{cases}$$

(after azimuthal avg. $\int \mathrm{d}\varphi\, k_T^\mu(\varphi) k_T^\nu(\varphi) \propto -g^{\mu\nu} + p^\mu p^\nu$ for gluonic a)



$$|(\Delta)\mathcal{M}_{m+1}^{(0)}(p_1,\dots,p_i,p_j,\dots p_{m+1})|^2$$
$$\xrightarrow{i\parallel j} 4\pi\alpha_s \frac{1}{s_{ij}} \mathcal{P}_{ia}(z)\, |(\Delta)\mathcal{M}_m^{(0)}(p_1,\dots,p,\dots,p_{m+1})|^2 + \mathcal{O}(k_T^0)$$

with $\mathcal{P}_{ia}(z) = \begin{cases} P_{ia}(z) & i,j \text{ unpolarized} \\ \Delta P_{ia}(z) & i \text{ polarized} \end{cases}$

Unpolarized and Polarized single-collinear splitting amplitudes

Unpolarized and polarized splitting amplitudes differ:

$$P_{qq}(z) = C_F \left[\frac{1+z^2}{1-z} - (1-z)\varepsilon\right], \qquad \Delta P_{qq}(z) = C_F \left[\frac{1+z^2}{1-z} + 3(1-z)\varepsilon\right]$$

$$P_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z} - z\varepsilon\right], \qquad \Delta P_{gq}(z) = C_F \left[2 - z + 2\varepsilon(1-z)\right]$$

$$P_{qg}(z) = 2n_f T_R \left[1 - 2\frac{z(1-z)}{1-\varepsilon}\right], \qquad \Delta P_{qg}(z) = 2n_f T_R \left[1 - 2\frac{1-z}{1-\varepsilon}\right]$$

$$P_{gg}(z) = C_A \left[2z(1-z) + \frac{2}{1-z} + \frac{2}{z} - 4\right], \qquad \Delta P_{gg}(z) = C_A \left[\frac{1}{1-z} - 1 + 2\frac{1-z}{1-\varepsilon}\right]$$

$\rightarrow$ different infrared structure!

To achieve proper factorization of matrix element need to retain *entire d*-dimensional structure.

Unconnected
singularities factorize individually like single-unresolved case

Double soft Like single-soft described by eikonal factors, initiated by gluon

- color-ordered double-soft eikonal factors S_{gg}, $S_{\gamma\gamma}$, and $S_{q\bar{q}}$
- $S_{\gamma\gamma}$[1] and S_{gg} contain iterated soft limits
- S_{gg} and $S_{q\bar{q}}$ contain genuine double-soft limit

in the case of unpolarised soft partons i, j; otherwise zero

Soft & collinear
Overlap between subsequent triple-collinear & soft, and soft & single-collinear, depending on position of soft parton in cluster

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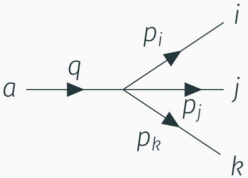
[1] γ refers to an abelian gluon in the cluster

IR limits: the double-unresolved case (2)

Triple-collinear: $i \parallel j \parallel k$ final-final

$$\begin{cases} p_i^\mu = z_i p^\mu + k_{T,i}^\mu + \frac{k_T^2}{2z_i}\frac{n^\mu}{p\cdot n} \\ p_j^\mu = z_j p^\mu + k_{T,j}^\mu + \frac{k_T^2}{2z_j}\frac{n^\mu}{p\cdot n} \\ p_k^\mu = z_k p^\mu + k_{T,k}^\mu + \frac{k_T^2}{2z_k}\frac{n^\mu}{p\cdot n} \end{cases}$$

with $k_{T,i}^\mu + k_{T,j}^\mu + k_{T,k}^\mu = 0, \quad z_i + z_j + z_k = 1$



$$\xrightarrow{i\|j} (4\pi\alpha_s)^2 \frac{1}{s_{ijk}^2} \mathcal{P}_{ijk}(z_i, z_j, z_k, s_{ij}, s_{ik}, s_{jk}) \times \longrightarrow a$$

Triple-collinear splitting amplitudes

- contain eikonal factors, iterated splitting, and “true” triple-collinear splitting
- unpolarized case: 7 independent splitting amplitudes
  P_{ggg}, P_{qgg}, $P_{q\gamma\gamma}$, $P_{gq\bar{q}}$, $P_{\gamma q\bar{q}}$, $P_{q\bar{q}'q'}$, $P_{q\bar{q}q}$ [Campbell, Glover ’97]
- polarized case: 7 unpolarized + 16 polarized = 23
  $\Delta P_{\Delta q\, g_1 g_2}$, $\Delta P_{\Delta q\, \gamma\gamma}$, $\Delta P_{\Delta g\, gq}$, $\Delta P_{g\, \Delta g\, q}$, $\Delta P_{\Delta\gamma\, \gamma q}$, $\Delta P_{\Delta q\, q'\bar{q}'}$,
  $\Delta P_{\Delta q'\, \bar{q}' q}$, $\Delta P_{\Delta q\, \bar{q} q}$, $\Delta P_{\Delta \bar{q}\, qq}$, $\Delta P^{\text{unconn.}}_{\Delta g\, gg}$, $\Delta P^{\text{conn.}}_{\Delta g\, gg}$,$\Delta P^{\text{conn.}}_{\Delta q\, \bar{q} g}$, $\Delta P^{\text{unconn.}}_{\Delta q\, \bar{q} g}$,
  $\Delta P_{\Delta q\, \bar{q}\gamma}$, $\Delta P_{\Delta g\, q\bar{q}}$, $\Delta P_{\Delta\gamma\, q\bar{q}}$
- complicated structure in terms of momentum fractions
- physically understood and strongly constrained structure in terms of Mandelstam variables [Braun-White, Glover ’22]

Example of a triple-collinear splitting function

$$
\begin{aligned}
&\Delta P_{\Delta q\, gg} = \\
&-\frac{2s_{13}^2}{s_{12}s_{23}}\left(\frac{z_2^2-2z_2+2}{z_3}+\frac{z_3^2-2z_3+2}{z_2+z_3}+2z_2+z_3-2\right)\\
&-\frac{4s_{12}^2z_3^2}{s_{23}^2(z_2+z_3)^2}+\frac{8s_{12}s_{13}}{s_{23}^2}\left(\frac{z_3}{z_2+z_3}-\frac{z_3^2}{(z_2+z_3)^2}\right)\\
&-\frac{2s_{13}}{s_{12}}\left(\frac{2z_2^2-3z_2+2}{z_3}+\frac{2}{z_2z_3}-\frac{2}{z_2(z_2+z_3)}+\frac{z_3^2-z_3}{z_2+z_3}+4z_2+2z_3-4\right)\\
&-\frac{2s_{23}}{s_{12}}\left(\frac{z_2^2-z_2}{z_3}+\frac{2}{z_2z_3}-\frac{2}{z_2(z_2+z_3)}+\frac{z_3-2}{z_2+z_3}+2z_2+z_3-1\right)\\
&-\frac{2s_{12}}{s_{23}}\left(\frac{2z_2^2-4z_2+4}{z_3}+\frac{4z_3^2}{(z_2+z_3)^2}+\frac{-z_3^2-2z_3-6}{z_2+z_3}+z_2+2z_3+2\right)\\
&-\frac{4s_{13}^2}{s_{23}^2}\left(\frac{z_3^2}{(z_2+z_3)^2}-\frac{2z_3}{z_2+z_3}+1\right)-\frac{2s_{13}}{s_{23}}\left(\frac{3z_2^2-6z_2+6}{z_3}+\frac{4z_3^2}{(z_2+z_3)^2}+\frac{-8z_3-4}{z_2+z_3}+3z_2+3z_3\right)\\
&+\left(-\frac{6z_2^2-10z_2+8}{z_3}-\frac{4}{z_2z_3}+\frac{4}{z_2(z_2+z_3)}-\frac{4z_3^2}{(z_2+z_3)^2}-\frac{-2z_3^2-2z_3-16}{z_2+z_3}-6z_2-6z_3-2\right)\\
&+\mathcal{O}(\varepsilon)
\end{aligned}
$$

… and this is just one out of the 16 color-ordered azimuthally averaged polarized structures

How to obtain splitting amplitudes?

- unpolarized: proof of factorization with constructive method [Catani, Grazzini '99]
- polarized:
  same proof not possible for g_1
  instead extraction with above parametrizations from DIS coefficient functions à la [Glover, Campbell '97]
    - g_1 coefficient function in Photon-DIS [e.g. Zijlstra, van Neerven '93]
    - g_1 coefficient functions in Graviton-DIS [Moch et al. '14]

$\rightarrow$ only with averaged "azimuthal correlation",
suffer from coefficient function features in soft limit ($1/z_{\text{pol}}$)

- Check: factorization still true with one extra radiated particle
  $\rightarrow$ non-trivial factorization

Conclusion

- derived all universal objects appearing in single- and double-unresolved limits up to tree-level NNLO
- simple factorization
- next step: 1-loop single-collinear (NNLO RV)
- goals:
    - complete picture of IR structure up to NNLO
    - subtraction for polarized exclusive observables
    - similar event generator machinery for EIC as for LHC
- a lot (of work) lies ahead …

Thanks for your attention!