Breakdown of collinear factorisation in the photoproduction of a $\pi^0\gamma$ pair with large invariant mass

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Based on 2311.09146 with Jakob Schönleber, Lech Szymanowski and Samuel Wallon

$$\gamma(q) + N(p_1) \to \gamma(k) + M(p_M) + N'(p_2)$$

- Calculation at LO and leading twist for $M = \pi^{\pm}$, $\rho^{0,\pm}_{L,T}$:

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  B. Pire, L. Szymanowski, S. Wallon:

  [2212.00655, 2302.12026]

Introduction

Exclusive photon-meson photoproduction

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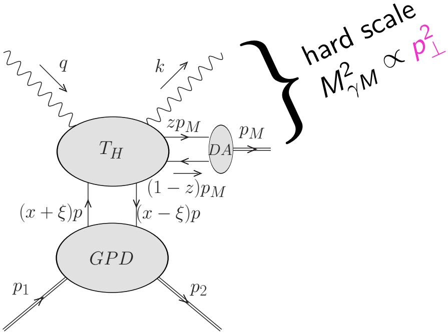
- Sensitive to chiral-odd GPDs at the *leading twist* when $M = \rho_T$.
- Good statistics at various experiments, particularly at *JLab*.
- *Small ξ* limit of quark GPDs can be studied at collider experiments.



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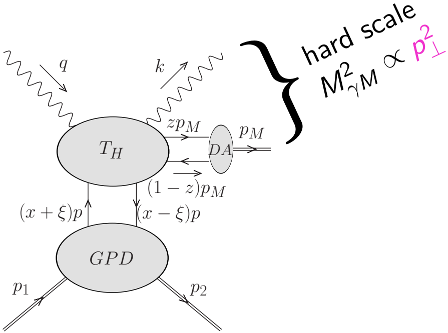

$\Longrightarrow$ See Samuel Wallon's talk on Tuesday

Introduction

Is Collinear factorisation justified?

- Recently, factorisation has been proved for the process $\pi N \to \gamma\gamma N'$ by J. Qiu, Z. Yu [2205.07846].
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- In fact, NLO computation has been performed for $\gamma N \to \gamma\gamma N'$ by O. Grocholski, B. Pire, P. Sznajder, L. Szymanowski, J. Wagner [2110.00048, 2204.00396]
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Issues with exclusive $\pi^0\gamma$ photoproduction...

- Because of the quantum numbers of π^0 ($J^{PC} = 0^{-+}$), the exclusive photoproduction of $\pi^0\gamma$ is also sensitive to *gluon GPD contributions*.

Introduction

Gluon GPD contributions to exclusive $\pi^0\gamma$ photoproduction

- Because of the quantum numbers of π^0 ($J^{PC} = 0^{-+}$), the exclusive photoproduction of $\pi^0\gamma$ is also sensitive to *gluon GPD contributions*.
- A total of 24 diagrams contribute in this case (compared to 20 diagrams from quark GPD contributions), with 6 groups of 4 related by symmetries ($x \to -x$ and $z \to 1 - z$ separately).

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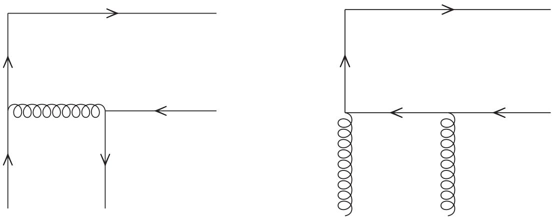
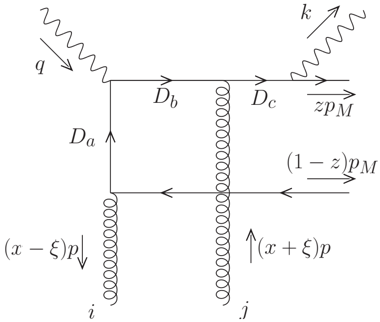
- Diagrams amount to connecting photons to the following two topologies.



Result assuming collinear factorisation

Specific diagram



$$CF \sim \frac{\mathrm{Tr}\left[\not{p}_M \gamma^5 \not{\epsilon}_k \left(\not{k} + z\not{p}_M\right) \gamma^j \left(\not{q} - (x-\xi)\not{p} - \bar{z}\not{p}_M\right) \not{\epsilon}_q \left(-(x-\xi)\not{p} - \bar{z}\not{p}_M\right) \gamma^i\right]}{\left[2z\, kp_M\right]\left[-2\,(x-\xi)\, qp - 2\bar{z}\, qp_M + 2\bar{z}\,(x-\xi)\, pp_M + i\epsilon\right]\left[2\bar{z}\,(x-\xi)\, pp_M + i\epsilon\right]}$$

$$\xrightarrow{x\to\xi,\bar{z}\to 0} \propto \frac{x-\xi}{\left[(x-\xi) + A\bar{z} - i\epsilon\right]\left[\bar{z}\,(x-\xi) + i\epsilon\right]}, \qquad A \equiv \frac{qp_M}{qp} > 0.$$

(Assuming p_M is along minus direction)

Result assuming collinear factorisation

Specific diagram

Need to dress coefficient function CF with gluon GPD $\left(\frac{H_g(x)}{(x-\xi+i\epsilon)(x+\xi-i\epsilon)}\right)$, and DA $(z\bar{z})$. This gives

$$\mathcal{A} \sim \frac{\bar{z}\,(x-\xi)\,H_g(x)}{(x-\xi+i\epsilon)\left[(x-\xi)+A\bar{z}-i\epsilon\right]\left[\bar{z}\,(x-\xi)+i\epsilon\right]}$$

$$\longrightarrow \frac{H_g(x)}{\left[(x-\xi)+A\bar{z}-i\epsilon\right]\left[x-\xi+i\epsilon\right]}$$

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$$\longrightarrow \frac{H_g(x)}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

The integral over z and x diverges if the GPD $H_g(x)$ is non-vanishing at $x=\xi$:

$$\int_{-1}^{1} dx \int_{0}^{1} dz \frac{1}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

$$\supset \int_{-1}^{1} dx \frac{\ln(x-\xi-i\epsilon)}{[x-\xi+i\epsilon]} \quad \Longrightarrow \text{ divergent imaginary part!}$$

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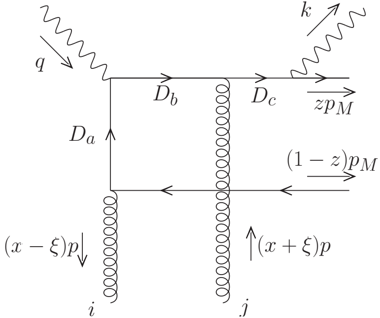

$$\int_{-1}^{1} dx \int_{0}^{1} dz \frac{1}{[(x-\xi)+A\bar{z}-i\epsilon]\,[x-\xi+i\epsilon]}$$

$\Longrightarrow$ The "*pinching*" is caused by propagators D_a and D_b.

What about the sum of diagrams?

$$\sum \mathcal{A} \sim \frac{z\bar{z}\left(x^2-\xi^2\right)\left[-\alpha\left[\left(x^2-\xi^2\right)^2(1-2z\bar{z})+8x^2\xi^2 z\bar{z}\right]-\left(1+\alpha^2\right)z\bar{z}\left(x^4-\xi^4\right)\right]H_g(x)}{z\bar{z}\left[x-\xi+i\epsilon\right]^2\left[\bar{z}\left(x+\xi\right)-\alpha z\left(x-\xi\right)-i\epsilon\right]\left[z\left(x-\xi\right)+\alpha\bar{z}\left(x+\xi\right)-i\epsilon\right]}$$

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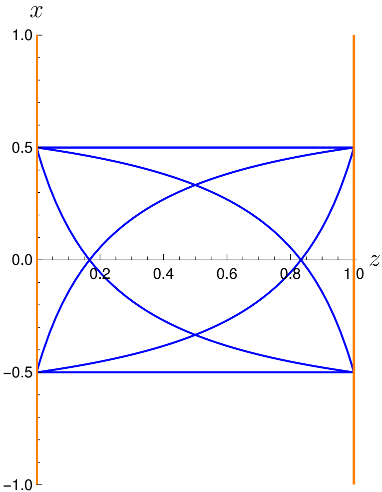
Full amplitude (anti)-symmetric in $x \to -x$ and $z \to \bar{z}$ for (anti)-symmetric GPD. (only symmetric result shown above)

$\Longrightarrow$ *divergence survives*, and actually adds up.

'Phase Space' for amplitude



$\xi = 0.5$

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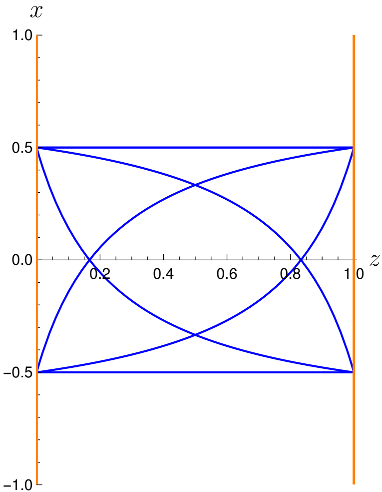

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Result assuming collinear factorisation
Singularity structure of the full amplitude

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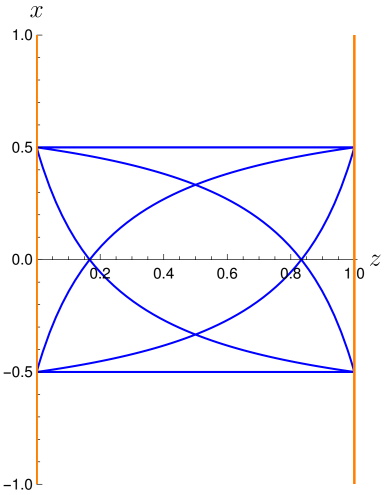

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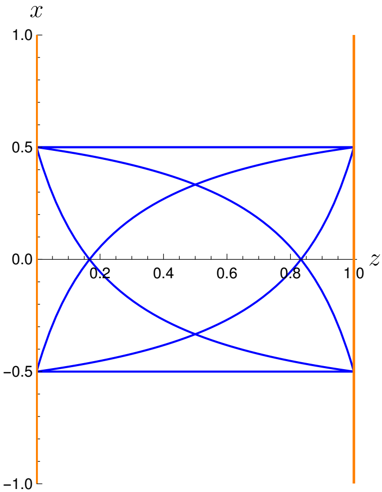

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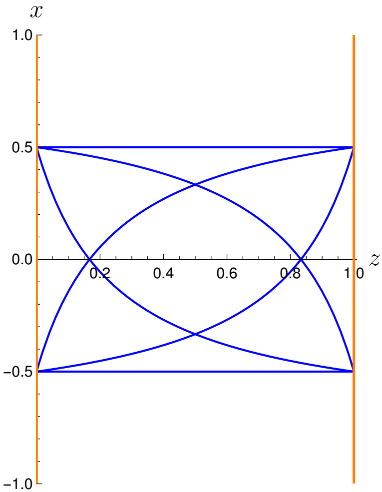

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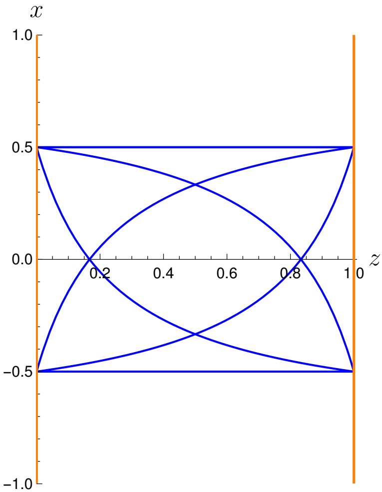

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- *Can this divergence be understood from a theoretical point of view?*
  YES! $\Longrightarrow$ [**S. N.**, J. Schönleber, L. Szymanowski, S. Wallon: 2311.09146]

Reduced diagram analysis

Libby-Sterman power counting

- How to obtain the dominant contribution of an amplitude (in QCD) in a certain specific kinematics (e. g. collinear)?
  $\Longrightarrow$ Libby-Sterman power counting rule [Phys.Rev.D 18 (1978) 3252; Phys.Rev.D 18 (1978) 4737]

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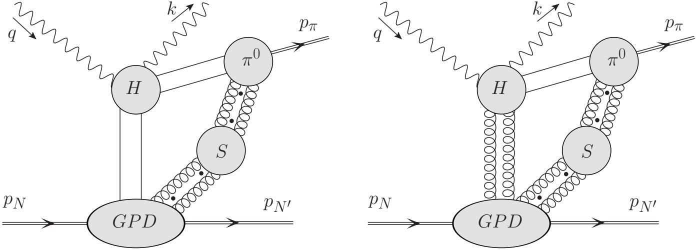
- Basic idea is to identify regions of loop momenta of partons (also number of partons), which gives the dominant contribution to the full amplitude.
- Collect all contributions to the *smallest* α:

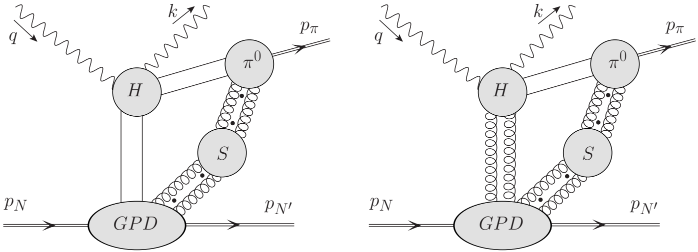
$$\mathcal{A} = Q^{\beta} \sum_{\alpha} f_{\alpha} \lambda^{\alpha} , \qquad \lambda = \frac{\Lambda_{\mathrm{QCD}},\ m_{\pi},\ m_N}{Q} \ll 1$$

Reduced diagram analysis
Classic Collinear pinch



In both of the above cases, the power counting is [**S. N.**, J. Schönleber, L. Szymanowski, S. Wallon: 2311.09146]:

$$\mathcal{A} \sim Q^{-1}\lambda^{\alpha}\,, \qquad \lambda = \frac{\Lambda_{\rm QCD},\, m_\pi,\, m_N}{Q} \ll 1\,, \qquad \alpha = 1$$



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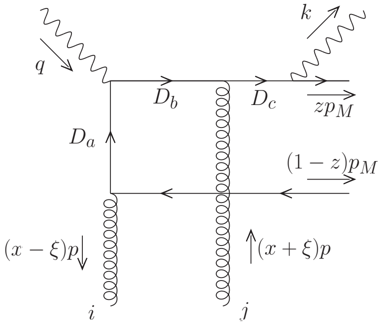
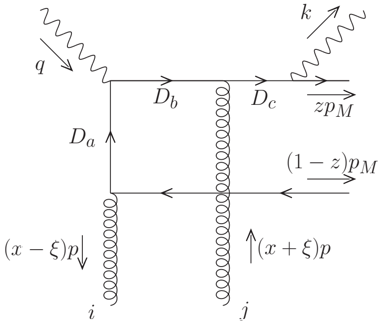
Collinear factorisation at *all orders* and *leading power* provided:

- the above (classic) collinear pinch diagrams are the *only ones contributing to the leading power of $\alpha = 1$*
- the *soft factor S 'cancels'*



Divergence obtained when $(x-\xi)\,p$ and $(1-z)\,p_M$ lines become soft:

$\Longrightarrow$ D_a becomes soft and D_b becomes collinear with respect to q.



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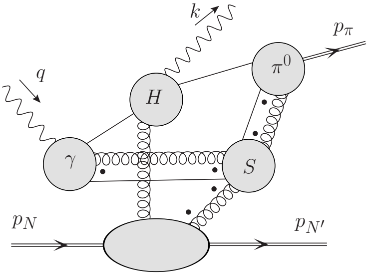
Is there a *leading* pinch diagram that corresponds to this region?
Yes!

Reduced diagram analysis
Other leading pinch surfaces?



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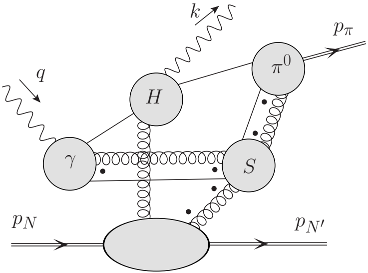

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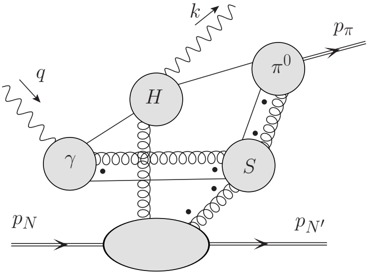
$\Longrightarrow$ power counting is the same as the collinear region!



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Note: Corresponding reduced diagram for quark GPD case is power suppressed.

What exactly does the pinch surface correspond to?

- Use Sudakov basis $(+, -, \perp)$:

$$\text{Collinear} \quad k \sim Q\left(1, \lambda^2, \lambda\right) \quad (\text{or} \quad k \sim Q\left(\lambda^2, 1, \lambda\right))$$

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- Need to distinguish between *ultrasoft*, *soft* and *Glauber* gluons:

$$\begin{array}{ll} \text{Ultrasoft} & k \sim Q\left(\lambda^2, \lambda^2, \lambda^2\right) \\ \text{Soft} & k \sim Q\left(\lambda, \lambda, \lambda\right) \\ \text{Glauber} & k \sim Q\left(\lambda^2, \lambda^2, \lambda\right) \quad (\text{or similar with} \quad |k_\perp^2| \gg k^+ k^-) \end{array}$$

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- Glauber gluons *cannot* be eliminated/suppressed by the use of Ward identities.

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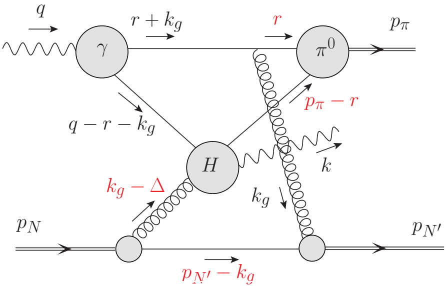
&\text{Soft} && k \sim Q\left(\lambda, \lambda, \lambda\right) \\
&\text{Glauber} && k \sim Q\left(\lambda^2, \lambda^2, \lambda\right) \quad (\text{or similar with} \quad |k_\perp^2| \gg k^+ k^-)
\end{aligned}$$

- Libby-Sterman power counting formula strictly applies for *ultrasoft gluons* only.
- However, these are typically eliminated by the use of *Ward identities*.
- Glauber gluons *cannot* be eliminated/suppressed by the use of Ward identities.
- Key Question: Is there a Glauber pinch that contributes at leading power?

Glauber pinch



(Notation: $(+,-,\perp)$)

$$p_N,\ p_{N'},\ \Delta \sim Q\left(1,\lambda^2,\lambda\right), \quad \Delta^+ < 0.$$
$$p_\pi \sim Q\left(\lambda^2,1,\lambda\right)$$
$$q,\ k \sim Q\left(1,1,1\right), \quad q^2,\ k^2 \sim \lambda^2 Q^2$$
$$[\text{Loop}]\ \text{Glauber}\ k_g \sim Q\left(\lambda,\lambda^2,\lambda\right)$$
$$[\text{Loop}]\ \text{Soft}\ r \sim Q\left(\lambda,\lambda,\lambda\right)$$

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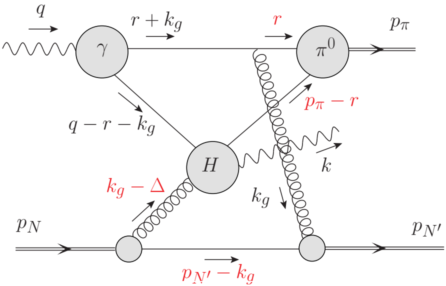

Glauber gluon, since $k_g^+ k_g^- << |k_g^\perp|^2$

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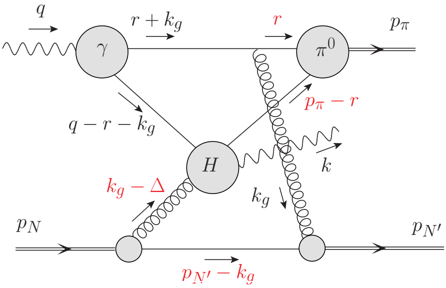

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Glauber gluon, since $k_g^+ k_g^- << |k_g^\perp|^2$

r^+ pinch:

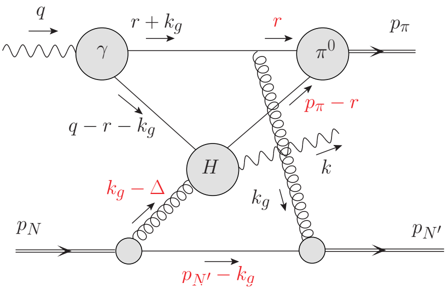
$$r^2 + i0 = r^+ r^- - |r_\perp|^2 + i0\,,$$

$$\implies r^+ = \mathcal{O}(\lambda) - \mathrm{sgn}(r^-)\, i0\,.$$

$$(p_\pi - r)^2 + i0 = -2p_\pi^- r^+ + \mathcal{O}(\lambda^2) + i0\,,$$

$$\implies r^+ = \mathcal{O}(\lambda^2) + i0\,.$$

Glauber pinch



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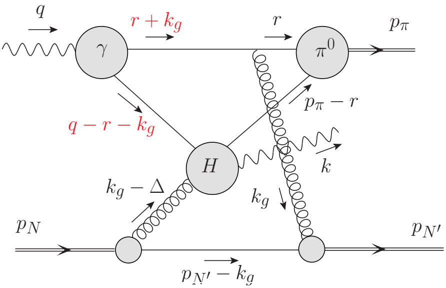
k_g^- pinch:

$$(k_g - \Delta)^2 + i0 = -2\Delta^+ k_g^- + \mathcal{O}(\lambda^2) + i0$$
$$\implies k_g^- = \mathcal{O}(\lambda^2) - i0$$

$$(p_{N'} - k_g)^2 + i0 = -2p_{N'}^+ k_g^- + \mathcal{O}(\lambda^2) + i0$$
$$\implies k_g^- = \mathcal{O}(\lambda^2) + i0\,.$$

Glauber pinch



r^- pinch:

$$(q - r - k_g)^2 + i0$$
$$= -2q^+ r^- - 2q^- k_g^+ + \mathcal{O}(\lambda) + i0$$
$$\implies r^- = \mathcal{O}(\lambda) + i0$$
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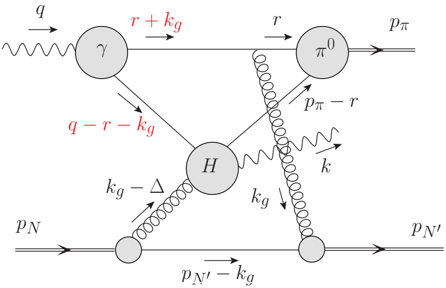
Glauber pinch



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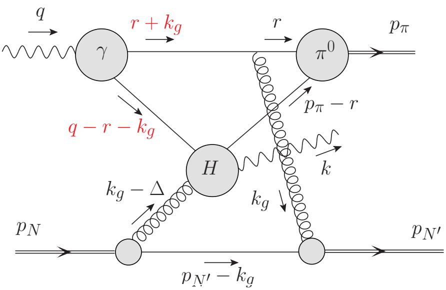
Pinch when $k_g^+ > 0 \implies$ DGLAP region

Glauber pinch



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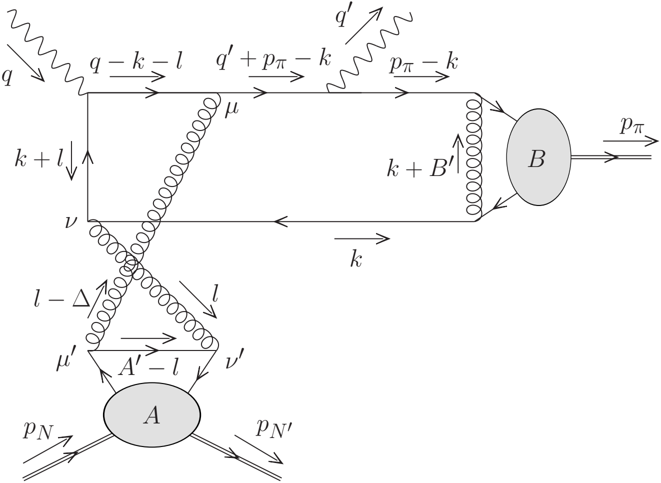
Recall:

$$k_g \sim Q\,(\lambda, \lambda^2, \lambda)$$
$$r \sim Q\,(\lambda, \lambda, \lambda)$$

Glauber pinch is leading



Explicit 2-loop analysis shows that the Glauber pinch demonstrated previously is leading, i.e. it scales as λ^α, with $\alpha = 1$.

Glauber pinch

Exclusive double diffractive processes

Very similar to the exclusive double diffractive process, where the Glauber gluon is pinched between the two pairs of incoming and outgoing collinear hadrons.

$$p(p_{N_1}) + p(p_{N_2}) \longrightarrow p(p_{N_1'}) + p(p_{N_2'}) + \gamma(k_1) + \gamma(k_2)$$

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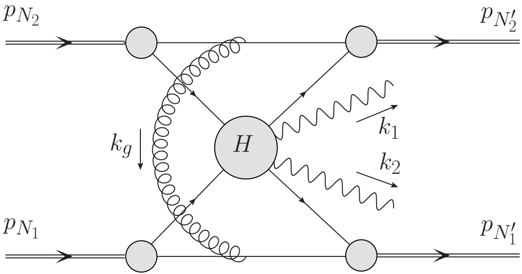

Here, the Glauber pinch corresponds to $k_g \sim (\lambda^2, \lambda^2, \lambda)$

Instead, in our case, the Glauber gluon (which corresponds to one of the active partons) is pinched between *a pair of collinear hadrons*, and *a soft line joining the outgoing pion and the incoming photon*.

Conclusions

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- The existence of the Glauber pinch can be also demonstrated using the *generalised* Landau conditions and the Coleman-Norton picture [ongoing]
- Compute $\gamma N \to \gamma\pi^0 N$ in high-energy (k_T) factorisation [ongoing]

BACKUP SLIDES

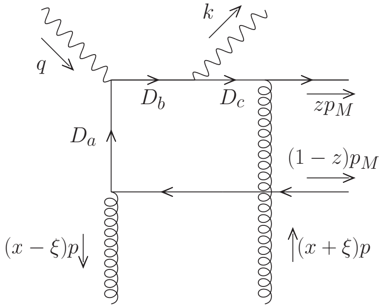
Very useful *Sudakov decomposition* of a generic 4-vector v in lightcone directions n_+ and n_-:

$$v^\mu = v^+ n_+^\mu + v^- n_-^\mu + v_\perp^\mu$$

with

$$\begin{aligned} n_+^2 = n_-^2 &= 0 \\ n_+ \cdot n_- &= 1 \\ v^\pm &= \frac{v^0 \pm v^3}{\sqrt{2}} \\ v^2 &= 2v^+ v^- + v_\perp^2 \end{aligned}$$

In other words, n_+^μ (n_-^μ) defines a *lightlike* 4-vector with spatial components purely in the positive (negative) z-direction



$$D_a = ((x-\xi)p + \bar{z}p_M)^2 + i\epsilon$$
$$= s\bar{\alpha}\bar{z}\left[x - \xi + i\epsilon\right] \ ,$$

$$D_b = (k + zp_M - (x+\xi)p)^2 + i\epsilon$$
$$= -s\left[z\left(x - \xi - i\epsilon\right) + \alpha\bar{z}\left(x + \xi - i\epsilon\right)\right] \ ,$$

$$D_c = (zp_M - (x+\xi)p)^2 + i\epsilon$$
$$= -s\bar{\alpha}z\left[x + \xi - i\epsilon\right]$$

$\Longrightarrow$ *pinching of poles in the propagators (D_a and D_b) in the limit of $z \to 1$*