Anomalous dimensions for hard exclusive processes

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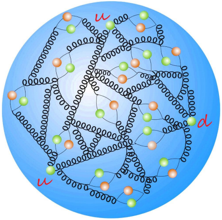
How to gain insight into the structure of hadrons

- Important question: How do hadronic properties emerge from the properties of the constituent partons?
  - For example the proton spin puzzle

    [Aidala et al., 2013],[Leader and Lorcé, 2014],[Deur et al., 2018],[Ji et al., 2021], [Abdulameer et al., 2023]
- Experimentally: Perform high-energy scattering experiments that can resolve the inner hadron structure (e.g. scatter electrons off a proton)
- Hard scale ⇒ Factorization between short-range and long-range physics
- Long-range physics described by non-perturbative parton distributions like PDFs and GPDs



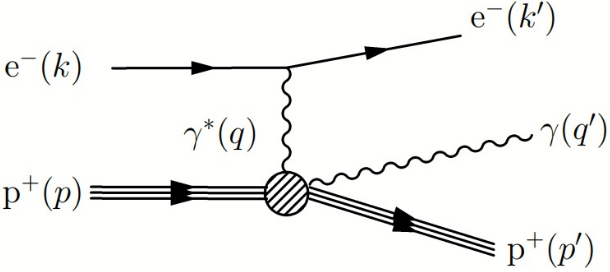
GPDs

- GPDs were independently introduced in '94 by Müller [Müller et al., 1994a] and '96-'97 by Radyushkin [Radyushkin, 1996] and Ji [Ji, 1997]. They generalize other types of non-perturbative QCD quantities like PDFs, form factors and distribution amplitudes.
- GPDs describe (a) transverse distributions of partons and (b) contributions partonic orbital angular momentum to total hadronic spin

  ⇒ Important quantities for describing proton/hadron structure, see e.g. [Pasquini and Boffi, 2008],[Kaiser, 2012],[Bacchetta, 2016]

  → Very precise measurements to come in (near) future! (EIC [Boer et al., 2011],[Abdul Khalek et al., 2021]/EicC [Anderle et al., 2021], LHeC [Abelleira Fernandez et al., 2012], JLab22 upgrade [Accardi et al., 2023], ...)
- Accessible in hard exclusive scattering processes
- Theoretically simplest example: deeply-virtual Compton scattering (DVCS)

Deeply-virtual Compton scattering



- Virtuality: $Q^2 = -q^2$
- Bjorken-x: $x_B = \frac{Q^2}{2p\cdot q}$
- Momentum transfer on hadronic target: $t = (p - p')^2 \equiv \delta^2$
- Skewedness: $\xi = \frac{(p-p')^+}{(p+p')^+}$ [lightcone coordinates: $p^{\pm} = \frac{1}{\sqrt{2}}(p^0 \pm p^3)$]

Deeply-virtual Compton scattering

In the Bjorken limit ($Q^2 \to \infty$ with x_B, t fixed): Factorization of the DVCS amplitude into non-perturbative GPDs and perturbative coefficient functions

- Coefficient functions correspond to partonic amplitudes
- GPDs correspond to hadronic matrix elements of composite QCD operators

The scale dependence of the GPDs is characterized by the evolution equation, which generically takes the following form

[Müller et al., 1994a],[Radyushkin, 1996],[Ji, 1997]

$$\frac{\mathrm{d}\mathcal{G}(x,\xi,t;\mu^2)}{\mathrm{d}\ln\mu^2} = \int_x^1 \frac{\mathrm{d}y}{y}\mathcal{P}\left(\frac{x}{y},\frac{\xi}{y}\right)\mathcal{G}(y,\xi,t;\mu^2)$$

This is a generalization of the well-known DGLAP equation in forward kinematics [Gribov and Lipatov, 1972], [Altarelli and Parisi, 1977], [Dokshitzer, 1977]

$$\frac{\mathrm{d}f(x,\mu^2)}{\mathrm{d}\ln\mu^2} = \int_x^1 \frac{\mathrm{d}y}{y}P(y)f\left(\frac{x}{y},\mu^2\right).$$

GPD scale dependence

Because of the direct relation between GPDs and QCD operators, the scale dependence of the distributions is determined by the scale dependence of the operators, characterized by their anomalous dimension

$$\frac{\mathrm{d}[\mathcal{O}]}{\mathrm{d}\ln\mu^2} = \gamma[\mathcal{O}].$$

These anomalous dimensions can be computed perturbatively in QCD by renormalizing the partonic matrix elements of the operators. In this talk we will focus our attention on the leading-twist flavor-non-singlet quark operators

$$\mathcal{O} = \mathcal{S}\overline{\psi}\lambda^{\alpha}\Gamma D_{\mu_2}\ldots D_{\mu_N}\psi$$

- Wilson operators (e.g. DVCS):

$$\mathcal{O}_{\mu_1\ldots\mu_N} = \mathcal{S}\overline{\psi}\lambda^{\alpha}\gamma_{\mu_1}D_{\mu_2}\ldots D_{\mu_N}\psi$$

- Transversity operators (e.g. transverse meson production):

$$\mathcal{O}^{T}_{\nu\mu_1\ldots\mu_N} = \mathcal{S}\overline{\psi}\lambda^{\alpha}\sigma_{\nu\mu_1}D_{\mu_2}\ldots D_{\mu_N}\psi$$

Operator anomalous dimensions

For exclusive processes like DVCS, one needs to renormalize the non-forward matrix elements of the operators. In this case one has to take into account mixing with total derivative operators

$$\begin{pmatrix} \mathcal{O}_{N+1} \\ \partial \mathcal{O}_N \\ \vdots \\ \partial^N \mathcal{O}_1 \end{pmatrix} = \begin{pmatrix} Z_{N,N} & Z_{N,N-1} & \dots & Z_{N,0} \\ 0 & Z_{N-1,N-1} & \dots & Z_{N-1,0} \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & Z_{0,0} \end{pmatrix} \begin{pmatrix} [\mathcal{O}_{N+1}] \\ [\partial \mathcal{O}_N] \\ \vdots \\ [\partial^N \mathcal{O}_1] \end{pmatrix}$$

Hence we now also have an anomalous dimension matrix (ADM)

$$\hat{\gamma} = -\frac{\mathrm{d} \ln \hat{Z}}{\mathrm{d} \ln \mu^2} = \begin{pmatrix} \gamma_{N,N} & \gamma_{N,N-1} & \cdots & \gamma_{N,0} \\ 0 & \gamma_{N-1,N-1} & \cdots & \gamma_{N-1,0} \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & \gamma_{0,0} \end{pmatrix}$$

Diagonal elements: forward anomalous dimensions

A consistency relation for anomalous dimensions

$$\boxed{\begin{aligned}\gamma^{\mathcal{D}}_{N,k} &= \binom{N}{k}\sum_{j=0}^{N-k}(-1)^j\binom{N-k}{j}\gamma_{j+k,\,j+k} \\ &\quad + \sum_{j=k}^{N}(-1)^k\binom{j}{k}\sum_{l=j+1}^{N}(-1)^l\binom{N}{l}\gamma^{\mathcal{D}}_{l,j}\end{aligned}}$$

- ✓ Can be used to construct the full ADM from the knowledge of the forward anomalous dimensions $\gamma_{N,N}$ + boundary condition to ensure uniqueness of the solution ($\gamma^{\mathcal{D}}_{N,0}$, from Feynman diagrams)

  [Moch and Van Thurenhout, 2021]

For the Feynman diagram computations, one generically needs the Feynman rules of all relevant operators. The generic form of these rules, together with *Mathematica* and *FORM* [Vermaseren, 2000, Kuipers et al., 2013] implementations for their automatic generation, can be found in [Somogyi and Van Thurenhout, 2024]

A consistency relation for anomalous dimensions

Application of this method allowed us to extend the low-N results for $\gamma^{\mathcal{D}}$ in [Shifman and Vysotsky, 1981, Baldracchini et al., 1981, Artru and Mekhfi, 1990, Blümlein, 2001, Gracey, 2009, Kniehl and Veretin, 2020] in the following ways

- Large n_f: 5-loop Wilson, 4-loop transversity anomalous dimensions [Moch and Van Thurenhout, 2021, Van Thurenhout, 2022] (see also [Van Thurenhout and Moch, 2022] for all-order results in this limit)
- Large n_c: 2-loop Wilson anomalous dimensions [Moch and Van Thurenhout, 2021] (subleading color analysis in progress)

Main advantage: The full procedure can be automated using computer algebra methods, e.g.

- Diagram computations using e.g. *FORCER* [Ruijl et al., 2020] in *FORM*
- Evaluation of sums using e.g. the *Mathematica* package *SIGMA* [Schneider, 2007, Schneider, 2013]

⇒ In principle straightforward to go to higher orders in perturbation theory!

Anomalous dimensions from conformal symmetry

- Instead of working with physical 4D QCD, one considers QCD in $D = 4 - 2\varepsilon$ dimensions at the critical point
- The anomalous dimensions $\gamma^{\mathcal{C}}$ can then be reconstructed using consistency relations coming from the conformal algebra
- The physical kernels have the same functional form as the critical ones, up to terms associated to the breaking of conformal symmetry: QCD beta-function and the conformal anomaly (currently known to two-loop accuracy [Müller, 1991, Braun et al., 2016, Braun et al., 2017])
- As generically the L-loop anomalous dimensions depend only on the $(L-1)$-loop conformal anomaly [Müller, 1991], they could be calculated up to three loops using this approach [Braun et al., 2017]

Connecting results

The 2 approaches above follow independent methods and use different bases for the total-derivative operators. They can be connected to each other by constructing a similarity transformation between the 2 bases

[Van Thurenhout, 2024]

$$\boxed{\begin{aligned}\gamma^{\mathcal{D}}_{N,k} &= \frac{(-1)^k (N+1)!}{(k+1)!} \sum_{l=k}^{N} (-1)^l \binom{N}{l} \frac{l!\,(3+2l)}{(N+l+3)!} \sum_{j=k}^{l} \binom{j}{k} \frac{(j+k+2)!}{j!} \gamma^{\mathcal{C}}_{l,j} \\ \gamma^{\mathcal{C}}_{N,k} &= (-1)^k \frac{k!}{N!} (3+2k) \sum_{l=k}^{N} (-1)^l \binom{N}{l} \frac{(N+l+2)!}{(l+1)!} \sum_{j=k}^{l} \binom{j}{k} \frac{(j+1)!}{(j+k+3)!} \gamma^{\mathcal{D}}_{l,j}\end{aligned}}$$

- ✓ Cross-check independent computations
- ✓ Learn about functional form of the ADM

Summary and outlook

- Hard exclusive scattering processes in QCD can be factorized into perturbative coefficient functions and non-perturbative GPDs (hadronic matrix elements of QCD operators)
- The scale dependence of GPDs is characterized by the anomalous dimensions of the operators, which can be determined by renormalizing the corresponding partonic matrix elements
- For exclusive processes, one has to take into account mixing with total-derivative operators during the renormalization procedure
- We have discussed two methods to reconstruct the anomalous dimensions using (a) a consistency relation and (b) conformal symmetry arguments
- More complicated operator mixing in flavor singlet sector: gluon and alien operators

End[1]

Thank you for your attention!

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Appendices and references

1. Computations of evolution kernels and anomalous dimensions
2. All-order results in the large-n_f limit
3. The derivative basis
4. The conformal basis
5. Getting actual predictions for hard exclusive processes
6. References

Computations of evolution kernels and anomalous dimensions

Forward evolution kernels/anomalous dimensions:

[Gross and Wilczek, 1973, Gross and Wilczek, 1974, Floratos et al., 1977, Gonzalez-Arroyo et al., 1979, Floratos et al., 1979, Gonzalez-Arroyo and Lopez, 1980, Gonzalez-Arroyo et al., 1980, Curci et al., 1980, Furmanski and Petronzio, 1980, Shifman and Vysotsky, 1981, Baldracchini et al., 1981, Artru and Mekhfi, 1990, Gracey, 1994, Gracey, 1996, Hayashigaki et al., 1997, Kumano and Miyama, 1997, Blumlein et al., 1997b, Larin et al., 1997, Vogelsang, 1998, Bennett and Gracey, 1998, Blumlein and Vogt, 1998, Blumlein et al., 1998a, Blumlein et al., 1998b, van Neerven and Vogt, 2000, Blümlein, 2001, Gracey, 2003a, Gracey, 2003b, Vogt et al., 2004, Moch et al., 2004, Blumlein, 2004, Gracey, 2006a, Gracey, 2006b, Blumlein et al., 2009, Bierenbaum et al., 2009, Vogt et al., 2010a, Soar et al., 2010, Vogt et al., 2010b, Ablinger et al., 2011, Velizhanin, 2012a, Velizhanin, 2012b, Ablinger et al., 2014a, Ablinger et al., 2014b, Moch et al., 2014, Ruijl et al., 2016, Davies et al., 2017, Moch et al., 2017, Ablinger et al., 2017, Vogt et al., 2018, Moch et al., 2018, Behring et al., 2019, Herzog et al., 2019, Velizhanin, 2020, Blümlein et al., 2021, Blümlein et al., 2022b, Moch et al., 2022, Blümlein et al., 2022a, Falcioni and Herzog, 2022, Blümlein, 2023, Gehrmann et al., 2023c, Gehrmann et al., 2023a, Gehrmann et al., 2023b, Falcioni et al., 2023c, Ji et al., 2023, Falcioni et al., 2023b, Falcioni et al., 2023a, Moch et al., 2023]

Computations of evolution kernels and anomalous dimensions

Non-forward evolution kernels/anomalous dimensions:

[Efremov and Radyushkin, 1980, Makeenko, 1981, Shifman and Vysotsky, 1981, Baldracchini et al., 1981, Geyer, 1982, Gribov et al., 1983, Geyer et al., 1985, Braunschweig et al., 1986, Dittes et al., 1988, Balitsky and Braun, 1989, Artru and Mekhfi, 1990, Müller, 1991, Müller, 1994, Müller et al., 1994b, Ji, 1997, Radyushkin, 1997, Balitsky and Radyushkin, 1997, Blumlein et al., 1997a, Martin and Ryskin, 1998, Belitsky and Müller, 1998, Hoodbhoy and Ji, 1998, Belitsky and Müller, 1999a, Radyushkin, 1999, Blümlein et al., 1999, Belitsky et al., 1999, Belitsky and Müller, 1999b, Belitsky et al., 2000a, Belitsky et al., 2000b, Belitsky and Müller, 2000, Blümlein, 2001, Mikhailov and Vladimirov, 2009a, Mikhailov and Vladimirov, 2009b, Gracey, 2009, Gracey, 2011a, Gracey, 2011b, Braun and Manashov, 2013, Braun and Manashov, 2014, Manashov and Strohmaier, 2015, Braun et al., 2017, Braun et al., 2019a, Braun et al., 2019b, Kniehl and Veretin, 2020, Braun et al., 2021, Moch and Van Thurenhout, 2021, Braun et al., 2022a, Bertone et al., 2022, Van Thurenhout, 2022, Van Thurenhout and Moch, 2022, Ji et al., 2023, Van Thurenhout, 2024, Bertone et al., 2023]

All-order results in the large-n_f limit

In [Gracey, 1994] and [Gracey, 2003b] the all-order expressions for the Wilson and transversity forward anomalous dimensions in the leading-n_f approximation were computed[2].

The calculation relied on exact conformal symmetry at the Wilson-Fisher critical point [Braun et al., 2019c], in which case propagators in the model simply have a power law structure. The anomalous dimensions calculated this way are then functions of the spacetime dimension D and n_f.

In [Van Thurenhout and Moch, 2022] we extended this programme to the computation of the off-diagonal elements of the ADM.

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[2] An independent computation in x-space, based on summation of renormalon-chain insertions, was performed in [Mikhailov, 1998b],[Mikhailov, 1998a],[Mikhailov, 2000].

All-order results in the large-n_f limit

The general expression from which the anomalous dimensions can be extracted is[3]

$$\gamma\mathcal{O}(z_1,z_2)=\frac{\mu(\mu-1)}{2(\mu-2)(2\mu-1)}\eta\left\{\int_0^1 \mathrm{d}\alpha\,\frac{\overline{\alpha}^{\mu-1}}{\alpha}\left(2[\mathcal{O}(z_1,z_2)]-[\mathcal{O}(z_{12}^{\alpha},z_2)]-[\mathcal{O}(z_1,z_{21}^{\alpha})]\right)\right.$$
$$\left.-(\mu-\delta)^2\int_0^1\mathrm{d}\alpha\int_0^{\overline{\alpha}}\mathrm{d}\beta\,(1-\alpha-\beta)^{\mu-2}[\mathcal{O}(z_{12}^{\alpha},z_{21}^{\beta})]+\frac{\mu-1}{\mu}[O(z_1,z_2)]\right\}$$

with

$$\mu=\frac{D}{2}=2-\varepsilon_*=2+a_s\beta_0\Big|_{n_f}=2-\frac{2}{3}n_f a_s$$
$$\eta=\frac{1}{n_f}\frac{(\mu-2)(2\mu-1)\Gamma(2\mu)}{\Gamma^2(\mu)\Gamma(\mu+1)\Gamma(2-\mu)}$$
$$z_{12}^{\alpha}=z_1\overline{\alpha}+z_2\alpha,\ \ \overline{\alpha}=1-\alpha$$

δ is a parameter controlling the Dirac structure of the considered operators (1 for Wilson and 2 for transversity).

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[3] We thank A. Manashov for useful discussions on this subject.

All-order results in the large-n_f limit

Depending on the case of interest, we now substitute different expressions for the moments of the non-local operators $\mathcal{O}(z_1, z_2)$

- Forward kinematics

$$\mathcal{O}(z_1, z_2) \to z_{12}^{N-1} = (z_1 - z_2)^{N-1}$$

  The forward anomalous dimensions then simply correspond to the prefactor of $(z_1 - z_2)^{N-1}$ and agree with [Gracey, 1994],[Gracey, 2003b]

- Non-forward kinematics: Use that the non-local operators act as generating functions for local ones [Braun et al., 2017]

$$\begin{aligned}[\mathcal{O}(z_1, z_2)] &= \sum_{m,k} \frac{z_1^m z_2^k}{m!\ k!} [\overline{\psi}(x)(\overleftarrow{D} \cdot \Delta)^k (\Delta \cdot \Gamma)(\Delta \cdot \overrightarrow{D})^m \psi(x)] \\ &= \sum_{m,k} \frac{z_1^m z_2^k}{m!\ k!} [\mathcal{O}_{0,k,m}]\end{aligned}$$

All-order results in the large-n_f limit

The latter operators can be written in terms of operators without covariant derivatives acting on ψ as

$$\mathcal{O}_{0,N-k,k} = (-1)^k \sum_{j=0}^{k} (-1)^j \binom{k}{j} \mathcal{O}_{j,N-j,0}.$$

It then follows that

$$[\mathcal{O}(z_1, z_2)] = \sum_{k=0}^{N} \sum_{j=0}^{k} (-1)^{j+k} \binom{k}{j} \frac{z_1^{N-k} z_2^k}{k!\,(N-k)!} [\mathcal{O}_{j,N-j}].$$

All-order results in the large-n_f limit

The resulting integrals can be computed for fixed values of N. Taking the N-th derivative with respect to z_1 and take $z_1, z_2 \to 0$, the expression takes the form

$$\gamma\mathcal{O}(z_1, z_2) = \gamma_{N,N}[\mathcal{O}_{0,N}] + \gamma_{N,N-1}[\mathcal{O}_{1,N-1}] + \gamma_{N,N-2}[\mathcal{O}_{2,N-2}] + \cdots + \gamma_{N,0}[\mathcal{O}_{N,0}],$$

from which the all-order expressions for $\gamma_{N,k}$ with $k = 0, 1, \ldots, N$ can be read off. The results agree with what was computed in

[Moch and Van Thurenhout, 2021], [Van Thurenhout, 2022]

All-order results in the large-n_f limit

Non-trivial example:

$$\begin{aligned}
\gamma_{3,2} &= -4(a_s n_f - 3)[36 + a_s n_f(2a_s n_f - 15)]\mathcal{F}(a_s, n_f),\\
\gamma_{3,1} &= 9[18 + a_s n_f(2a_s n_f - 11)]\mathcal{F}(a_s, n_f),\\
\gamma_{3,0} &= -24(a_s n_f - 3)\mathcal{F}(a_s, n_f)
\end{aligned}$$

for the Wilson operators and

$$\begin{aligned}
\gamma^T_{3,2} &= (3 - a_s n_f)[135 + 8a_s n_f(a_s n_f - 6)]\mathcal{F}(a_s, n_f),\\
\gamma^T_{3,1} &= 9[15 + a_s n_f(2a_s n_f - 7)]\mathcal{F}(a_s, n_f),\\
\gamma^T_{3,0} &= \frac{-3}{2a_s n_f - 3}[45 + 4a_s n_f(4a_s n_f - 9)]\mathcal{F}(a_s, n_f)
\end{aligned}$$

for the transversity ones with

$$\mathcal{F}(a_s, n_f) = -\frac{2^{3-4a_s n_f/3}}{9\pi^{3/2} n_f}\frac{\Gamma(5/2 - 2a_s n_f/3)\sin(2\pi a_s n_f/3)}{\Gamma(6 - 2a_s n_f/3)}.$$

The derivative basis

In this basis the operators are written as

$$\mathcal{O}^{\mathcal{D}}_{k,N-k} = (\Delta \cdot \partial)^k \{\overline{\psi}\lambda^\alpha(\Delta \cdot \Gamma)(\Delta \cdot D)^{N-k}\psi\}$$

with $\Delta^2 = 0$.

- This choice of operator basis is used for hadronic studies on the lattice, see e.g. [Göckeler et al., 2005] and [Gracey, 2009]
- In this basis, the Wilson anomalous dimensions for low-N operators were computed up to $O(a_s^3)$ (see [Gracey, 2009] for analytical results and [Kniehl and Veretin, 2020] for a numerical extension of these). For the transversity operators, the $O(a_s)$ anomalous dimensions are known [Shifman and Vysotsky, 1981], [Baldracchini et al., 1981], [Artru and Mekhfi, 1990], [Blümlein, 2001]
- We have extended these results by deriving a consistency relation for the anomalous dimensions [Moch and Van Thurenhout, 2021]

The conformal basis

In this basis the operators are written in terms of Gegenbauer polynomials

$$\mathcal{O}^{\mathcal{C}}_{N,k} = (\Delta\cdot\partial)^k \overline{\psi'}(\Delta\cdot\Gamma) C_N^{3/2}\left(\frac{\overleftarrow{D}\cdot\Delta - \Delta\cdot\overrightarrow{D}}{\overleftarrow{\partial}\cdot\Delta + \Delta\cdot\overrightarrow{\partial}}\right)\psi$$

with [Olver et al., 2010]

$$C_N^{\nu}(z) = \frac{\Gamma(\nu+1/2)}{\Gamma(2\nu)}\sum_{j=0}^{N}(-1)^j\binom{N}{j}\frac{(N+j+2)!}{(j+1)!}\left(\frac{1}{2}-\frac{z}{2}\right)^j.$$

- This choice of operator basis is natural within conformal schemes [Efremov and Radyushkin, 1980], [Belitsky and Müller, 1999a], [Braun et al., 2017].

Getting actual predictions for hard exclusive processes

To obtain predictions for physical observables in hard exclusive processes, like cross-sections and spin/charge asymmetries, one needs to combine the coefficient functions (state of the art: NNLO for DVCS [Braun et al., 2022b]) with a GPD model. The GPD evolution kernels (operator anomalous dimensions) are needed to evolve the GPDs from some reference scale to the scale of interest.

→ Several numeric codes for this purpose exist, e.g.

- PARTONS (numeric code for GPD phenomenology) [Berthou et al., 2018]
  → `https://partons.cea.fr/partons/doc/html/index.html`
- Vinnikov code (LO GPD evolution) [Vinnikov, 2006]
- GPD evolution for DVCS @ NLO [Freund and McDermott, 2002]
- Gepard [Kumericki et al., 2008]
  → `https://gepard.phy.hr/index.html`
- Twist-2 GPD evolution in momentum space

  [Bertone et al., 2022, Bertone et al., 2023]

  → available through APFEL++ [Bertone et al., 2014, Bertone, 2018] and PARTONS

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