A study of gluon distributions inside the proton

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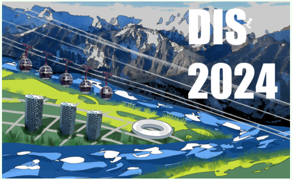


Based on: **PRD.108, 014009 (2023)** and arXiv: **2402.16503**

In collaboration with

D Chakrabarti, B Gurjar, R Kishore, T Maji, C Mondal, A Mukherjee

Deep Inelastic Scattering and Related Subjects (DIS2024)

Apr 9, 2024

Outline of the talk

- Motivation: What is inside the proton? Proton spin puzzle ?
- How much gluon contribute to proton spin and orbital angular momentum ?
- Light front framework
- Gluon light front spectator model
- Gluon Transverse momentum distributions
- Gluon Generalized parton distributions
- Gluon orbital angular momentum .

Light front dynamics

Light-front dynamics describes how a relativistic system changes along a light-front direction.

- In light front,

LF time $x^+ = x^0 + x^3$

$x^- = x^0 - x^3$

$x^\perp = (x^1, x^2)$.

LF energy $p^- = p^0 - p^3$

- No square root in energy dispersion relation $k^2 = k^+ k^- - k_\perp^2 = m^2$.

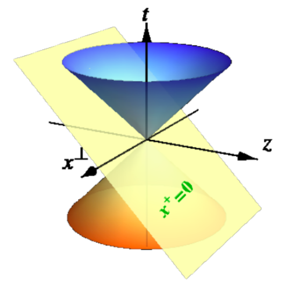


Figure: Leutwyler 1978

$$H_{LF}|\psi\rangle = M^2|\psi\rangle$$

In LF, Solving nonperturbative QCD is equivalent to solving the Hamiltonian eigenvalue problem.

Light front wave functions (LFWFs)

• In LF, the hadron state $|\psi\rangle$ is expanded in multi-particle fock states $|n\rangle$ of free LF Hamiltonian $|\psi\rangle = \sum_n \psi_n |n\rangle$, where

$$|n\rangle = |uud\rangle, |uudg\rangle, |uudq\bar{q}\rangle$$

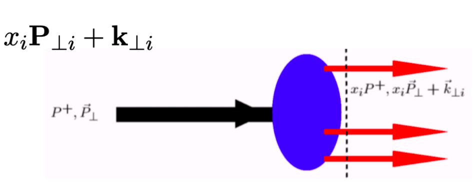
• Fock component ψ_n is known as light front wave function.

• LFWFs $\psi_n(x_i, k_{\perp,,i}, \lambda_i)$ depend only on the relative longitudinal, transverse momentum and spin of the parton.

• Momentum conservation

$\sum_{i=1}^{n} x_i = 1, \sum_{i=1}^{n} k_{\perp,i} = 0$

• Overlap of LFWFs: PDFs, TMDs, GPDs...



• LFWFs are frame independent object and provide intrinsic information of the structure of hadrons

Gluon light front spectator model

- The model is based on the quantum fluctuations of the electron in QED.
- In this simplified model, we describe the proton as a composite state of one active gluon and a spin-$\frac{1}{2}$ spectator.

$$|P;\uparrow(\downarrow)\rangle = \int \frac{\mathrm{d}^2\mathbf{p}_\perp \mathrm{d}x}{16\pi^3\sqrt{x(1-x)}} \times$$

$$\Big[\psi^{\uparrow(\downarrow)}_{+1+\frac{1}{2}}(x,\mathbf{p}_\perp)\left|+1,+\frac{1}{2};xP^+,\mathbf{p}_\perp\right\rangle + \psi^{\uparrow(\downarrow)}_{+1-\frac{1}{2}}(x,\mathbf{p}_\perp)\left|+1,-\frac{1}{2};xP^+,\mathbf{p}_\perp\right\rangle$$

$$+\psi^{\uparrow(\downarrow)}_{-1+\frac{1}{2}}(x,\mathbf{p}_\perp)\left|-1,+\frac{1}{2};xP^+,\mathbf{p}_\perp\right\rangle + \psi^{\uparrow(\downarrow)}_{-1-\frac{1}{2}}(x,\mathbf{p}_\perp)\left|-1,-\frac{1}{2};xP^+,\mathbf{p}_\perp\right\rangle\Big],$$

$$\psi^{\uparrow(\downarrow)}_{\lambda_g\lambda_X} \rightarrow \text{LFWFs}$$

-[PC,D Chakrabarti, B Gurjar, R Kishore, T Maji, C Mondal, A Mukherjee, PRD108, 014009 (2023)]

Light front wave functions(LFWFs)

The LFWFs for the Fock-state expansion for a proton with $J_z = +1/2$

$$\begin{aligned}
\psi^{\uparrow}_{+1+\frac{1}{2}}(x,\mathbf{p}_\perp) &= -\sqrt{2}\frac{(-p^1_\perp + ip^2_\perp)}{x(1-x)}\varphi(x,\mathbf{p}^2_\perp),\\
\psi^{\uparrow}_{+1-\frac{1}{2}}(x,\mathbf{p}_\perp) &= -\sqrt{2}\left(M - \frac{M_X}{(1-x)}\right)\varphi(x,\mathbf{p}^2_\perp),\\
\psi^{\uparrow}_{-1+\frac{1}{2}}(x,\mathbf{p}_\perp) &= -\sqrt{2}\frac{(p^1_\perp + ip^2_\perp)}{x}\varphi(x,\mathbf{p}^2_\perp),\\
\psi^{\uparrow}_{-1-\frac{1}{2}}(x,\mathbf{p}_\perp) &= 0
\end{aligned}$$

Wave function motivated by modified from soft wall AdS/QCD two particle solution

$$\varphi(x,\mathbf{p}^2_\perp) = N_g\frac{4\pi}{\kappa}\sqrt{\frac{\log[1/(1-x)]}{x}}x^b(1-x)^a\exp\left[-\frac{\log[1/(1-x)]}{2\kappa^2x^2}\mathbf{p}^2_\perp\right]$$

$\kappa = 0.4$ GeV scale parameter

-[S Brodsky, G Teramond, Phys.Rev.D 77 (2008) 056007]

-[D Chakrabarti, C Mondal Eur.Phys.J.C 73 (2013) 2671]

Fitting methodology

- There are four parameters in the LFWFs. : N_g, a, b, M_X
- The spectator mass $M_X > M$ should be.
- Fix the model scale and range of fitting :

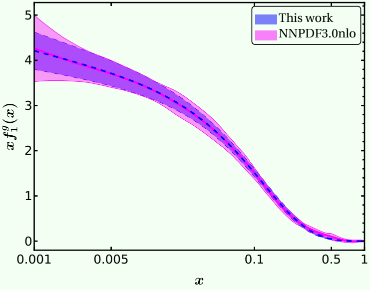
  $0.001 \leq x \leq 1$ at $Q_0 = 2$ GeV

- The unpolarized gluon PDF $f_1^g(x)$ in the model

$$
\begin{aligned}
f_1^g(x) &= \frac{1}{16\pi^3} \int d^2\mathbf{p}_\perp \Big[|\psi^\uparrow_{+1+1/2}(x, \mathbf{p}_\perp^2)|^2 + |\psi^\uparrow_{+1-1/2}(x, \mathbf{p}_\perp^2)|^2 + |\psi^\uparrow_{-1+1/2}(x, \mathbf{p}_\perp^2)|^2 \Big]. \\
&= 2N_g^2 x^{2b+1}(1-x)^{2a-2} \Big[\kappa^2 \frac{(1+(1-x)^2)}{\log[1/(1-x)]} + (M(1-x) - M_X)^2 \Big].
\end{aligned}
$$

Fixing model parameters

- Fixed the model parameters by fitting the model unpolarized gluon PDF with the NNPDF3.0nlo global analysis.



Parameter	Central Value	1σ-Error band	2σ-Error band
a	3.88	$\pm$ 0.1020	$\pm$ 0.2232
b	-0.53	$\pm$ 0.0035	$\pm$ 0.0071

$N_g = 2.088$, $M_X = 0.985$ $\chi^2_{min} = 20.88$ for 300 no of datas.

-R. D. Ball et al. (NNPDF Collaboration), Eur. Phys. J. C 77, 663 (2017).

Average longitudinal momentum

$$\langle x \rangle_g = \int_{0.001}^{1} dx x f_1^g(x) = 0.416^{+0.048}_{-0.041},$$

	This work	Bacchetta	Ma-Lu	Pion model	Lattice
$\langle x \rangle_g$	0.416	0.424	0.411	0.409	0.427

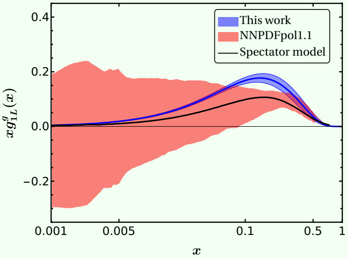
-[Bacchetta, Celiberto, Radici, Taels, EPJC 80 (2020)]

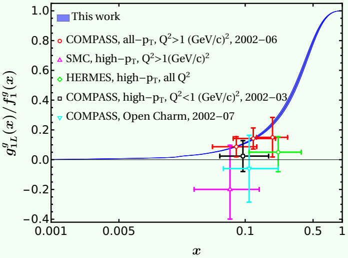
-[Lattice - C. Alexandrou et al. Phys. Rev. D 101, 094513, 2020]

-[Zhun Lu et al. Phys.Rev.D 94 (2016) 9, 094022]

-[Kaur, Dahiya DAE Symp.Nucl.Phys. 64 (2019) 641-642]

Helicity prediction and comparison





Gluon helicity	Central Value	our predictions
$\Delta G = \int_{0.05}^{0.3} dx g_{IL}^{g}(x)$	0.20 [PHENIX-2008]	$0.28^{+0.047}_{-0.037}$
$\Delta \mathrm{G} = \int_{0.05}^{0.2} dx g_{IL}^{g}(x)$	0.23(6) [NNPDF-2014]	$0.22^{+0.033}_{-0.024}$
$\Delta \mathrm{G} = \int_{0.05}^{1} dx g_{IL}^{g}(x)$	0.19(6) [RHIC-2014]	$0.326^{+0.066}_{-0.050}$

Gluon Transverse momentum distributions (TMDs)

The TMD correlator is defined as matrix elements of non-local products of gluon fields between proton states.

$$\Gamma^{ij}(x, \mathbf{k}_\perp, \mathbf{S}) = \frac{1}{xP^+} \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ik\cdot\xi} \left\langle P; S \right| F_a^{+j}(0)\ \mathcal{W}_{+\infty,ab}(0;\xi)\, F_b^{+i}(\xi) \left| P; S \right\rangle \Big|_{\xi^+=0}$$

-[S. Meissner et al. PhysRevD.76.034002(2007)]

-[Mulders, Rodrigues, PRD 63 (2001)]

- There are eight gluon TMDs
- T-even $\rightarrow f_1^g, g_{1L}^g, g_{1T}^g, h_1^{\perp g}$.
- T-odd $\rightarrow f_{1T}^g, h_{1T}^g, h_{1T}^{\perp,g}, h_{1L}^{\perp g}$.

GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}$
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

- TMDs in a spectator model $\rightarrow$ overlap of LFWFs.

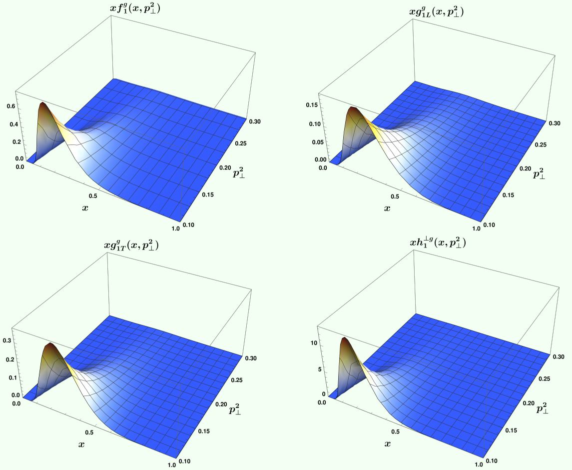
T-even TMDs

$f_1^g(x, \mathbf{p}_\perp^2) > 0$

$f_1^g(x, \mathbf{p}_\perp^2) \geq |g_{1L}^g(x, \mathbf{p}_\perp^2)|$

$f_1^g(x, \mathbf{p}_\perp^2) \geq \frac{|\mathbf{p}_\perp|}{M} |g_{1T}^g(x, \mathbf{p}_\perp^2)|$

$f_1^g(x, \mathbf{p}_\perp^2) \geq \frac{|\mathbf{p}_\perp|^2}{2M^2} |h_1^{\perp g}(x, \mathbf{p}_\perp^2)|$

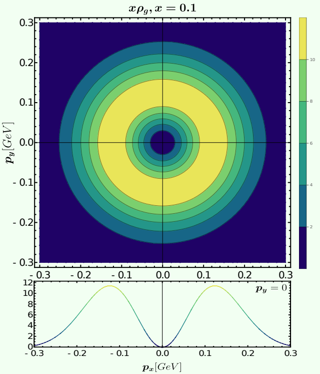


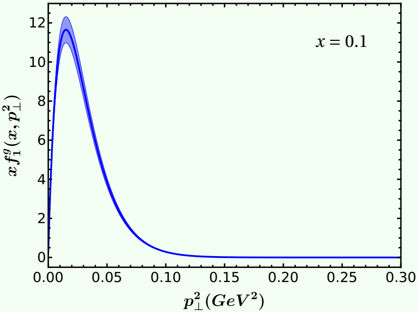
$$[f_1^g(x, \mathbf{p}_\perp^2)]^2 = [g_{1L}^g(x, \mathbf{p}_\perp^2)]^2 + \left[\frac{|\mathbf{p}_\perp|}{M} g_{1T}^g(x, \mathbf{p}_\perp^2)\right]^2 + \left[\frac{\mathbf{p}_\perp^2}{2M^2} h_1^{\perp g}(x, \mathbf{p}_\perp^2)\right]^2$$

-[PC,D Chakrabarti, B Gurjar, R Kishore, T Maji, C Mondal, A Mukherjee, PRD108, 014009 (2023)]

Gluon densities

Density of an unpolarized gluon inside an unpolarized proton



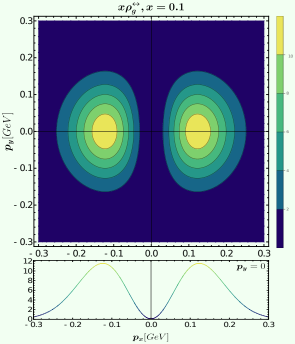


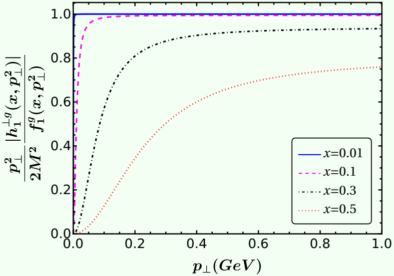
-Comparable with Bacchetta et al EPJC80

Boer-Mulders density

The longitudinally polarized gluon density

$$x\rho_g^{\leftrightarrow}(x, p_x, p_y) = \frac{1}{2}\left[x f_1^g(x, \mathbf{p}_\perp^2) + \frac{p_x^2 - p_y^2}{2M^2} x h_1^{\perp g}(x, \mathbf{p}_\perp^2) \right]$$





Spherical symmetry gets distorted due to the second term and shows dipolar structure in momentum space.

-Comparable with Bacchetta et al EPJC80

Generalized Parton distributions (GPDs)

- GPDs appear in exclusive processes such as DVCS, DDVCS, DVMS etc.
- GPDs are off forward matrix elements of bilocal operators.
- GPDs encode spatial as well as spin structure of the nucleon.

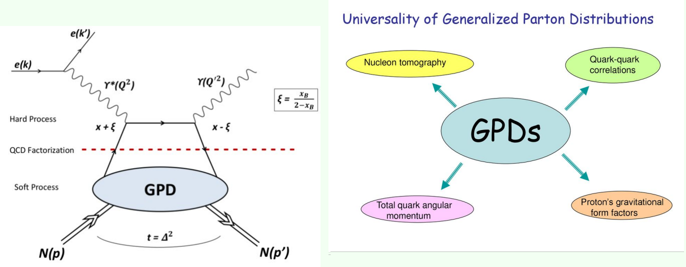


Figure: V.Burkert talk

- In forward limit GPDS $\rightarrow$ PDFs

Chiral even GPDs at non-zero skewness

$$A_{\lambda'\mu',\lambda\mu} = \frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p', s'|\, \epsilon^i(\mu') F^{+i}(-\tfrac{z}{2})\, F^{+j}(\tfrac{z}{2})\epsilon^{*j}(\mu)\,|p, s\rangle\Big|_{z^+=0,\, \mathbf{z}_T=0}$$

-[M Diehl, Phys.Rept. 388 (2003)]

- [Boffi, Pasquini Riv.Nuovo Cim. 30 (2007) 9, 387-448]

where μ (μ') gluon helicity of the initial (final) state

$$H^g = \frac{1}{\sqrt{1-\xi^2}} T_1^g - \frac{2M\xi^2}{\sqrt{t_0 - t}(1-\xi^2)} T_3^g,$$

$$\widetilde{H}^g = \frac{1}{\sqrt{1-\xi^2}} T_2^g + \frac{2M\xi}{\sqrt{t_0 - t}(1-\xi^2)} T_4^g,$$

$$E^g = -\frac{2M}{\epsilon\sqrt{t_0 - t}} T_3^g, \qquad \widetilde{E}^g = \frac{2M}{\epsilon\xi\sqrt{t_0 - t}} T_4^g,$$

$$T_1^g = A_{++,++} + A_{-+,-+}, \qquad T_2^g = A_{++,++} - A_{-+,-+},$$

$$T_3^g = A_{++,-+} + A_{+-,--}. \qquad T_4^g = A_{++,-+} - A_{+-,--},$$

Chiral odd GPDs at non-zero skewness

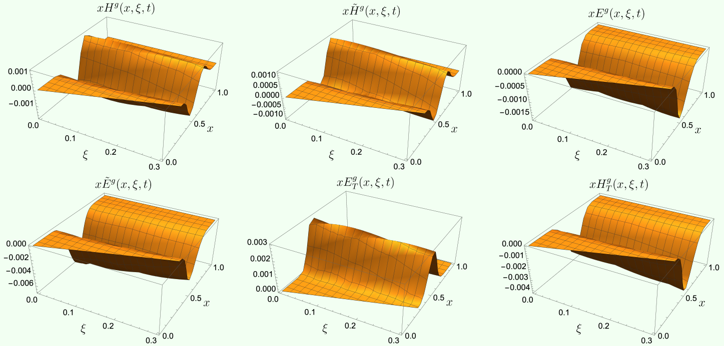
$$
\begin{aligned}
H_T^g &= \frac{2M}{\epsilon\sqrt{t_0 - t}(1-\xi^2)}\widetilde{T}_1^g - \frac{4M^2\xi}{(t_0-t)(1-\xi^2)\sqrt{1-\xi^2}}\widetilde{T}_3^g, \\
E_T^g &= \frac{4M^2}{(t_0-t)(1-\xi^2)\sqrt{1-\xi^2}}\left(\xi\widetilde{T}_3^g + \widetilde{T}_4^g\right) \\
\tilde{E}_T^g &= \frac{4M^2}{(t_0-t)(1-\xi^2)\sqrt{1-\xi^2}}\left(\widetilde{T}_3^g + \xi\widetilde{T}_4^g\right) \\
\tilde{H}_T^g &= 0
\end{aligned}
$$

$$
\widetilde{T}_1^g = A_{++,--} + A_{-+,+-}, \qquad \widetilde{T}_2^g = A_{++,--} - A_{-+,+-},
$$

$$
\widetilde{T}_3^g = A_{++,+-} + A_{+-,++}, \qquad \widetilde{T}_4^g = A_{++,+-} - A_{+-,++}
$$

The matrix elements T_i^g and $\widetilde{T}_i^g$ can be written in terms of overlap of light-front wavefunctions.

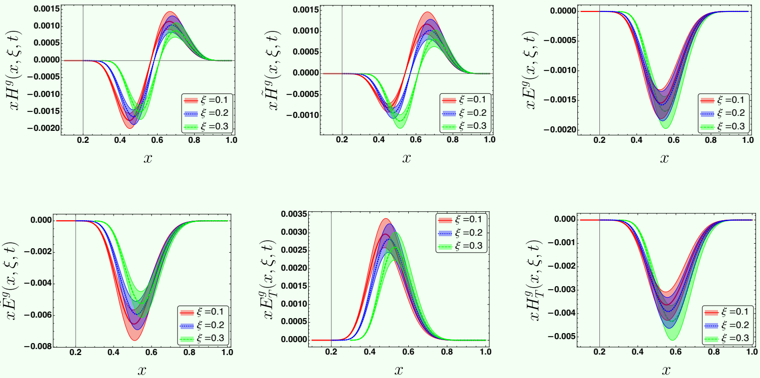
Gluon GPDs at non -zero skewness



at transverse momentum transfer $-|t| = 3$ GeV^2

-[PC,D Chakrabarti, B Gurjar, T Maji, C Mondal, A Mukherjee arXiv: 2402.16503]

2D plots in DGLAP region



at transverse momentum transfer $-|t| = 3$ GeV^2

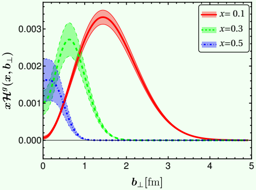
-[PC,D Chakrabarti, B Gurjar, T Maji, C Mondal, A Mukherjee arXiv: 2402.16503]

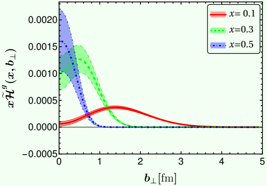
The behaviour is comparable with Tan, Lu PRD 108 (2023) 5, 054038

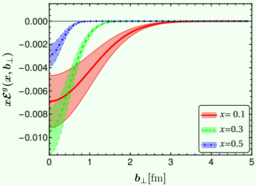
Gluon Impact parameter distribution(IPDs)

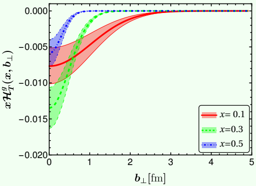
- IPDs reveal the gluon distributions in transverse-coordinate space and longitudinal momentum for different gluon and target polarizations.

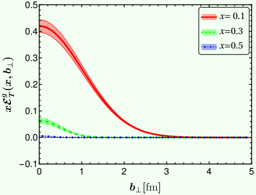
$$\mathcal{F}(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp . b_\perp} F^g(x, \xi = 0, t = -\Delta_\perp^2)$$











Gluon Orbital angular momentum

According to Ji's sum rule, the total angular momentum J_z^g of the gluon

$$J_z^g = \frac{1}{2}\int dxx\,[H^g(x,0,0) + E^g(x,0,0)]$$

-[Xiang-Dong Ji, Phys Rev Lett. 78, 610613 (1996)]

$$J_z^g = 0.058, \qquad J_z^g|_{\text{BLFQ}} = 0.066, \qquad J_z^g|_{\text{Lattice}} = 0.187(46)(10)$$

Kinetic OAM of gluon

$$L_z^g = \int dx \left\{ \frac{1}{2}x\left[H^g(x,0,0) + E^g(x,0,0)\right] - \widetilde{H}^g(x,0,0) \right\}$$

Our result $L_z^g = -0.42$

while $L_z^g|_{\text{Ma-Lu}} = -0.123$ [Tan-Lu, PRD 108, 054038]

- Spin-orbit correlation

$$C_z^g(x) = \int dx\, d^2k_\perp\, d^2b_\perp\, \left(\vec{b}_\perp \times \vec{k}_\perp\right)_z \rho_{UL}^q(\vec{b}_\perp, \vec{k}_\perp, x) \qquad C_z^g = -7.7480$$

$C_z^g < 0$ means gluon spin and OAM antialigned.

Conclusion and future work

• We have explored the light front dynamics and its implication to understand hadron dynamics in terms of LFWFs.

• We have discussed light-front spectator model for the gluon with the light-front wave functions modeled from the soft-wall holographic AdS/QCD prediction for two-body bound states.

• In a gluon spectator model, we have shown the results for the gluon average momentum, helicity, TMDs, densities, GPDs and OAM contribution of the gluon.

Thanks for listening....