GENERALIZED PARTON DISTRIBUTIONS THROUGH UNIVERSAL MOMENT PARAMETERIZATION (GUMP): THE GLUONIC SECTOR WITH DEEPLY VIRTUAL J/ψ PRODUCTION AT NLO

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PAPER IN PREPARATION

CENTER *for* NUCLEAR FEMTOGRAPHY

OUTLINE

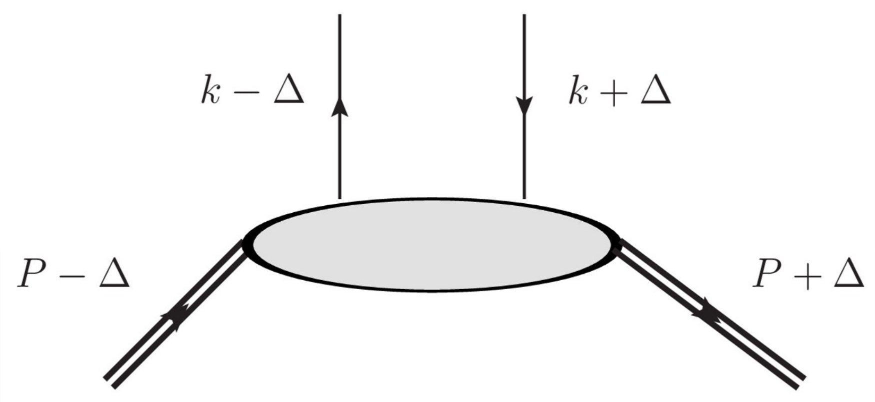
- GPD Review
- GUMP Program
  - Conformal moment parameterization
- Review of Previous Global Analysis: u and d quarks
- Gluons with J/ψ
  - NRQCD treatment
  - NLO corrections
- Gluon GPD fits
  - Comparison to data
  - Small-x vs Moderate-x
- Moving Forward
- Conclusions

GPDS

$$f(x) \to F(x, \xi, t)$$

$$x \sim \text{parton momentum fraction}, \ \xi \sim \text{longitudinal momentum transfer},$$
$$t = \Delta^2 \sim \text{momentum transfer squared}$$

- GPDs generalize the well known PDFs to encode full 3 dimensional information on the quarks and gluons within hadrons



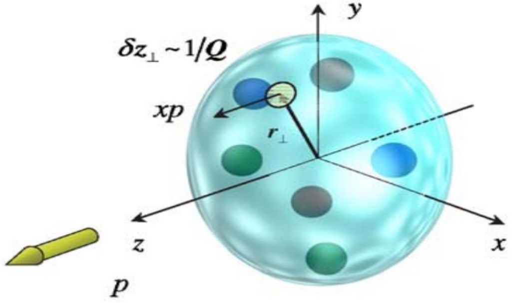
GPDS

- Polarization of the hadron and its parton constituents connects GPDs to the distribution of angular momentum within hadrons (*X. Ji* 1997)
  - Ji sum rule

$$J_i = \frac{1}{2} \int_0^1 \mathrm{d}x\, x \left[H_i(x,\xi) + E_i(x,\xi)\right]$$

- Related via a Fourier transform to the impact parameter distribution of partons (*M. Burkardt* 2003)

$$\rho(x, r_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot r_\perp} H(x, 0, \Delta_\perp^2)$$



- Related to bulk properties of hadron states encoded in form factors

$$\int \mathrm{d}x\, x H_i(x,\xi,t) = A_i(t) + (2\xi)^2 C_i(t), \quad \int \mathrm{d}x\, x E_i(x,\xi,t) = B_i(t) - (2\xi)^2 C_i(t)$$

GUMP PROGRAM: MOMENT PARAMETERIZATION

- Parameterize GPDs by directly parameterizing their conformal moments and resumming with a Mellin-Barnes integral

$$F(x,\xi,t)=\frac{1}{2i}\int_{c-i\infty}^{c+i\infty}\mathrm{d}j\,\frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi,t)$$

(*D. Mueller and A. Schafer* 2006)

- Expansion based on eigenfunctions of evolution – Gegenbauer polynomials

$$(-1)^j p_j(x,\xi)=\xi^{-j-1}\frac{2^j\Gamma(\frac{5}{2}+j)}{\Gamma(\frac{3}{2})\Gamma(j+3)}\left[1-\left(\frac{x}{\xi}\right)^2\right]C_j^{3/2}\left(\frac{x}{\xi}\right)$$

conformal wave function

$$\int_{-1}^{1}\frac{\mathrm{d}x'}{|\xi|}\mathcal{K}\left(\frac{x}{\xi},\frac{x'}{\xi}\right)C_j^{3/2}\left(\frac{x}{\xi}\right)=\gamma_j C_j^{3/2}\left(\frac{x}{\xi}\right)$$

GPD evolution kernel

GUMP PROGRAM: MOMENT PARAMETERIZATION

- Conformal moment parameterization has nice features for fitting GPDs
- Can analytically calculate convolutions in scattering amplitudes – just one Mellin-Barnes integral to compute
- Simple and fast evolution implementation – conformal moments are multiplicatively renormalized at LO
  - Follows from using eigenfunctions of evolution kernel
- Polynomiality condition (*X. Ji* 1998) automatically enforced on conformal moments

Theory constraints can be encoded directly in the moment parameterization!

$$F_{i,n}(\xi,t) = \int_{-1}^{1} \mathrm{d}x\, x^{n-1} F(x,\xi,t) = \sum_{k=0,\ \mathrm{even}}^{n} \xi^k F_{i,n,k}(t)$$

$$\downarrow$$

$$\mathcal{F}_{i,j}(\xi,t) = \sum_{k=0,\ \mathrm{even}}^{j+1} \xi^k \mathcal{F}_{i,j,k}(t)$$

FLEXIBLE MOMENT PARAMETERIZATION

- Our starting point is a relatively simple model for the conformal moments
- Parameterize each GPD moment with five parameters

$$F_{i,j,0} = N_i \underbrace{B(j+1-\alpha_i, 1+\beta_i)}_{\uparrow} \underbrace{\frac{j+1-\alpha_i}{j+1-\alpha_i(t)}}_{\uparrow} \beta(t)$$

Euler Beta Function

Regge trajectory

$$\beta(t) = e^{-b|t|}$$

$$\alpha(t) = \alpha + \alpha' t$$

- Take each moment to be a power series in skewness – polynomiality condition

$$F_{i,j} = F_{i,j,0}(t) + \xi^2 R_{\xi^2} F_{i,j,0}(t) + \xi^4 R_{\xi^4} F_{i,j,0}(t)\ldots$$

NON-ZERO SKEWNESS GLOBAL FIT

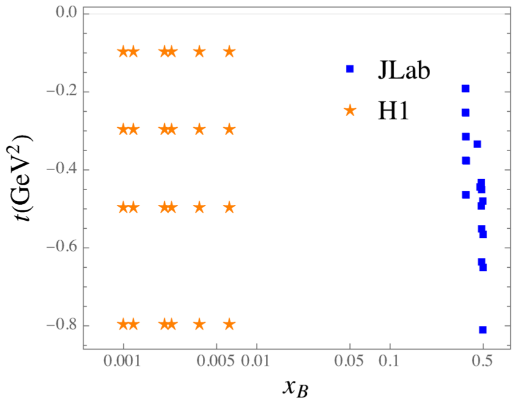
- Even with constraints, lots of parameters!
  - Very high dimensional space to navigate for best fit
  - Very computationally demanding to do error propagation
- We employ a sequential fit, starting with forward (PDF, t-dependent PDF) constraints for each GPD species then apply the off-forward constraints from DVCS data

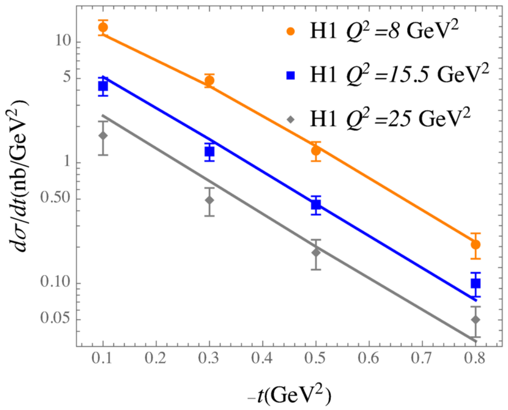
- JAM (2022) PDF global analysis results
- Globally extracted electromagnetic form factors (*Z. Ye et al* 2018)
- Lattice GPDs (*Alexandrou et al* 2020) and form factors (*Alexandrou et al* 2022)

DVCS measurements from JLab (*CLAS* 2019 & 2021, *Hall A* 2018 & 2022) and HERA (*H1* 2010)

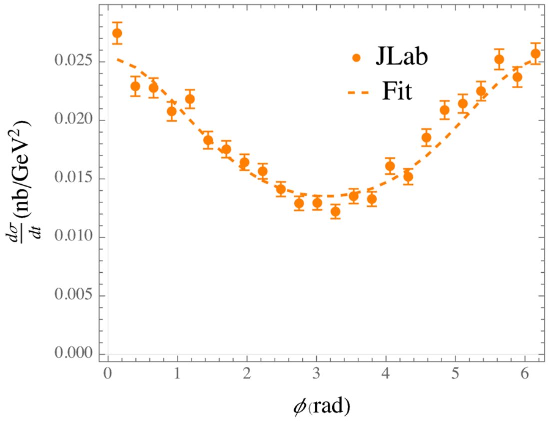
NON-ZERO SKEWNESS GLOBAL FIT

- Total χ^2/dof is approximately 1.4
- Some agreement with both JLAB and H1 data
- Gluon GPDs not well constrained at non-zero skewness
  - Only contribute to DVCS through evolution at LO
- Error propagation is not yet implemented
  - Very computationally expensive with so many parameters!

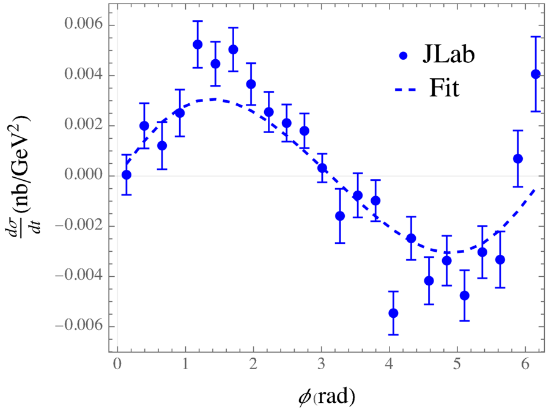




$d\sigma_{UU}$ at $x_B=0.367$, $t=-0.197$ GeV^2

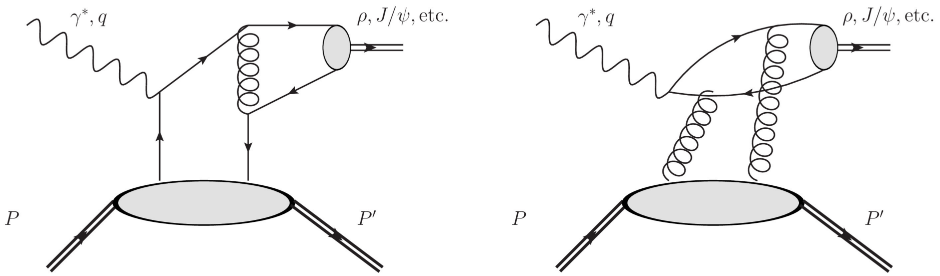


$d\sigma_{LU}$ at $x_B=0.367$, $t=-0.197$ GeV^2



GLUON SENSITIVE PROCESSES

- DVCS at LO is only sensitive to gluon GPDs through scale evolution
- Using Deeply Virtual Meson Production (DVMP) gives a direct probe of gluons at LO



- Light vector mesons have similar sensitivity to quarks and gluons
  - KM framework applied to produce simultaneous fits of DVCS and DVMP for ρ^0 meson production with data from HERA (*arXiv:2310.13837*)
- Add heavy vector meson to obtain better constraints on gluon GPDs – use J/ψ production!

DEEPLY VIRTUAL J/ψ PRODUCTION (DV J/ψ P)

- Charm quark contribution for nucleon target is negligible – direct probe of gluons
- Complementary with GUMP work on quark GPDs, but mostly sensitive to small-x_B region whereas JLab data combined with HERA gives better constraint at moderate x_B
- Caveat: mass of the J/ψ gives significant power corrections to collinear factorization

$$M^2_{J/\psi}/Q^2_{\text{max bin}} \approx 9/20 \to \text{corrections of order } 1/2$$

- Need to take heavy mass corrections into account – non-relativistic (NR) QCD!

NON-RELATIVISTIC MODEL APPROACH

- Including the mass corrections means we have a hard scale even as $Q^2 \to 0$, so we can potentially include photoproduction data in future fits!
- The NRQCD treatment includes both photon polarizations – eliminates largest source of uncertainty in data at the cost of model dependence

$$R = \frac{\mathrm{d}\sigma_L}{\mathrm{d}\sigma_T} = \frac{Q^2}{M_{J/\psi}^2}$$

$$\Rightarrow \mathrm{d}\sigma_{total} = \left(\varepsilon + \frac{M_{J/\psi}^2}{Q^2}\right) \mathrm{d}\sigma_L$$

$\varepsilon \sim$ longitudinal to transverse photon flux ratio

IMPLEMENTING NR J/ψ PRODUCTION IN GUMP

- We can make a hybrid scheme by combining the minimal NRQCD corrections with collinear factorization NLO hard scattering and universal NLO GPD evolution

$$\mathcal{A}_{\mathrm{Hyb.}} \propto \sqrt{\frac{\langle \mathcal{O}_1 \rangle_{J/\psi}}{m_c}} \sum_{i=flavors,g} \int_0^1 \mathrm{d}z \frac{\Phi_{asymptotic}(z)}{z(1-z)} \int_{-1}^{1} \mathrm{d}x \, C_2^i(x,\xi,z) F^i(x,\xi,t)$$

- Passing to moment space we write

$$\mathcal{A}_{\mathrm{Hyb.}} \propto \sqrt{\frac{\langle \mathcal{O}_1 \rangle_{J/\psi}}{m_c}} \sum_{i=flavors,g} \int_{c-i\infty}^{c+i\infty} \mathrm{d}j \xi^{-j-1} \left[i + \tan\left(\frac{\pi j}{2}\right) \right]$$
$$\times \left[C^{i,LO} E_j^{i,LO}(Q^2) + C^{i,NLO} E_j^{i,LO}(Q^2) + C^{i,LO} E_j^{i,NLO}(Q^2) \right] \mathcal{F}_j^i(\xi,t)$$

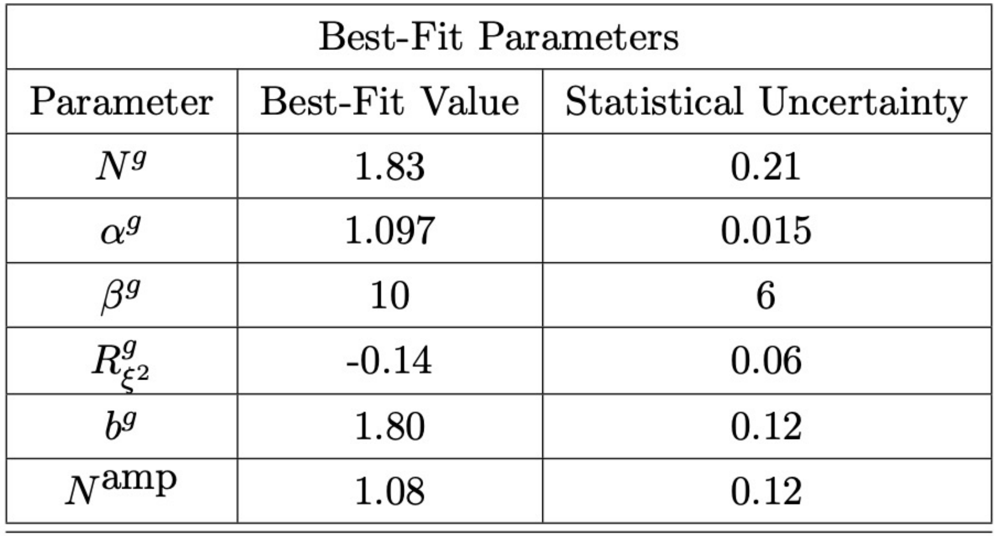
We set the factorization and renormalization scales equal to $Q^2 + M_{J/\psi}^2$

GLUON GPD FIT INPUTS

- We use 17 t-dependent cross section points from H1 (2006) data
  - $< Q^2 >$ in range $7.0 - 22.4\ GeV^2$, x_B in range $9{\times}10^{-4} - 6{\times}10^{-3}$, and $|t|$ in range $0.04\ - 0.64\ GeV^2$
  - The data has negligible sensitivity to the GPD E, so we only fit parameters coming from the GPD H: b^g and R_{ξ^2} as well as the DA normalization parameter N^{DA}
- Given the small values of x_B, we redo the fit of our forward gluon PDF parameters in a simultaneous fit, using 9 points from the JAM22 global analysis with $Q^2 = 4\ GeV^2$ and $x_B = 10^{-4} - 10^{-3}$ to constrain N^g, α^g, β^g
  - Limited number of points constraining forward limit since we have a limited number of off-forward data points

GLUON GPD PRELIMINARY FIT RESULTS

- Minimizing with Minuit2 gives $\chi^2/dof \approx 0.98$ and the following best-fit parameters
- Only statistical uncertainties from Minuit2 right now, full error propagation left for future work



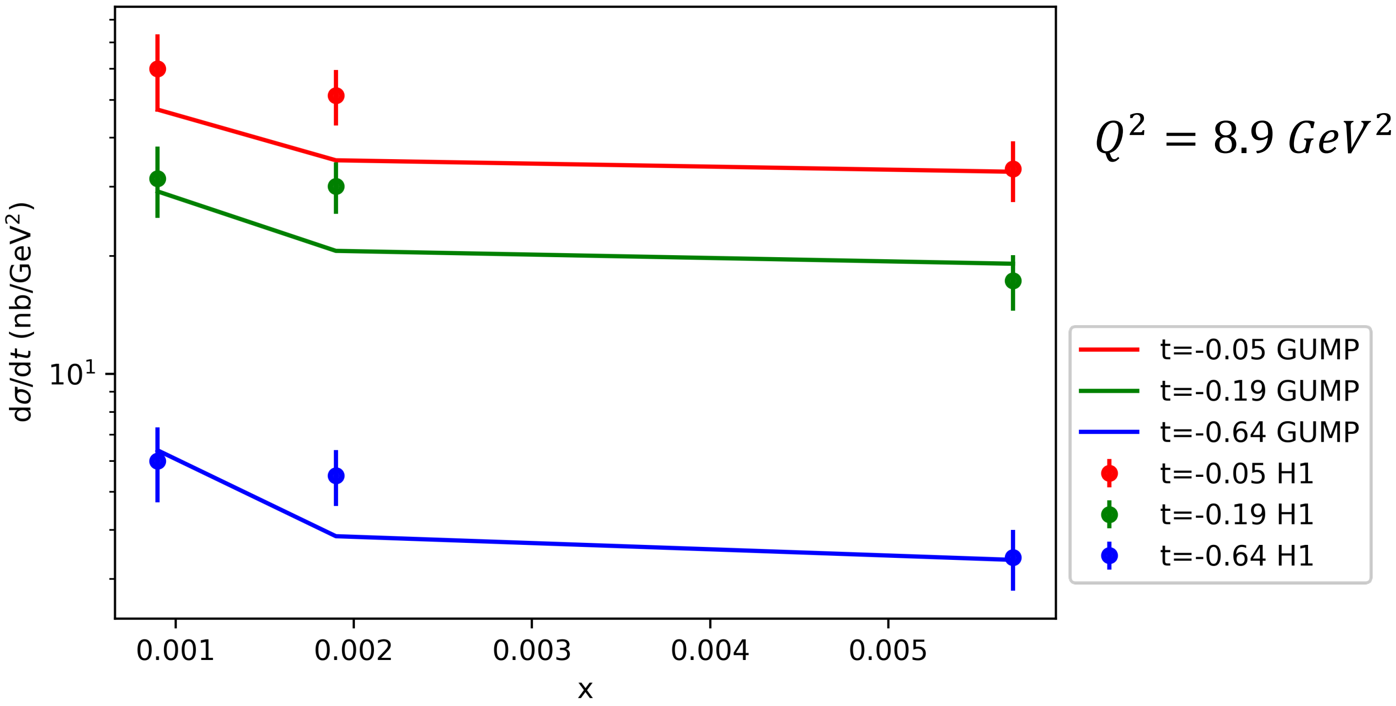
Best-Fit Parameters		
Parameter	Best-Fit Value	Statistical Uncertainty
N^g	1.83	0.21
α^g	1.097	0.015
β^g	10	6
$R^g_{\xi^2}$	-0.14	0.06
b^g	1.80	0.12
N^{amp}	1.08	0.12

Note the large uncertainty in β^g - expected from using small x_B PDFs but also correlation with normalization factors through

$$B(j+1-\alpha, 1+\beta)$$

GLUON GPD PRELIMINARY FIT RESULTS

- For $Q^2 \sim M_{J/\psi}^2$ or larger our hybrid scheme describes the data relatively well
- The x_B-dependence here crucially relies on the large logarithms entering the NLO corrections in our framework

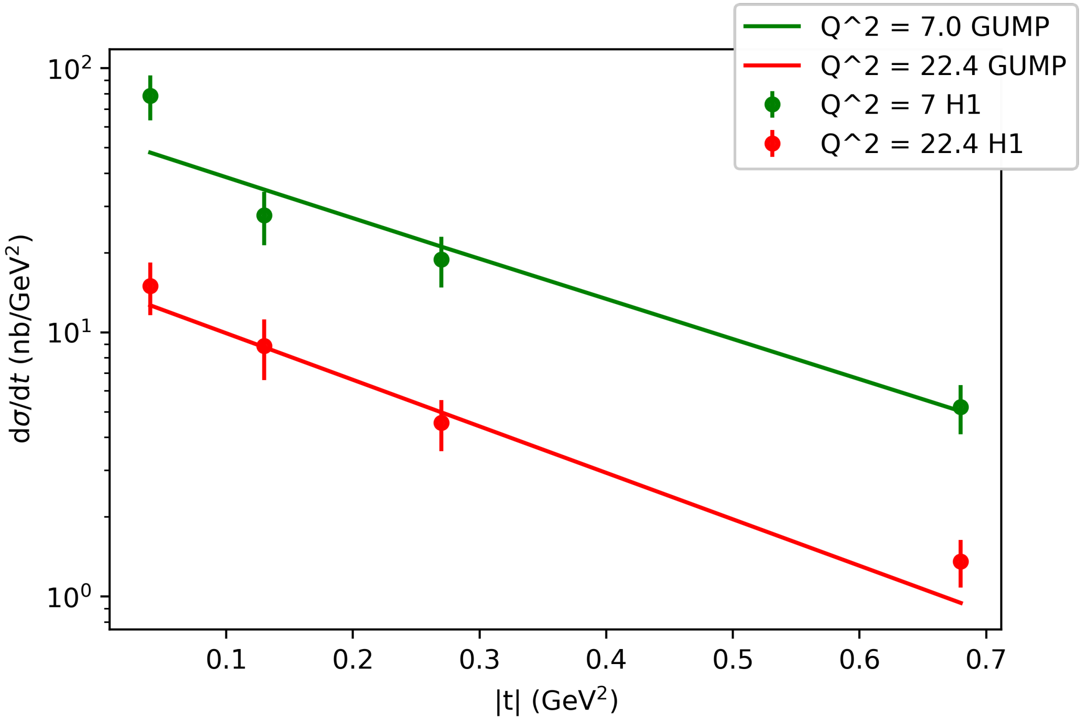


$Q^2 = 8.9\ GeV^2$

GLUON GPD PRELIMINARY FIT RESULTS

- Going to lower Q^2 we start to see discrepancy with the data
  - Lower Q^2 brings in higher twist effects, same issue for DVCS
  - Lower $M_{J/\psi}^2/Q^2$ enhances power corrections which we have dropped in the NLO terms

$$x_B = 1.3\times10^{-3} - 3.2\times10^{-3}$$



FUTURE IMPROVEMENTS/ADDITIONS FOR GLUONS IN GUMP

- Simultaneous fit with DVρ^0P data from HERA
- Further analysis of fit results
  - Uncertainty from renormalization/factorization scale setting
  - Skewness ratio $H(x,x,0)/H(x,0,0)$
- Conversion of NLO mass corrections to moment space
  - Can add photoproduction data to fits
- More sophisticated moment ansatz
  - Inclusion of lattice calculations and moderate x_B experimental data requires more complicated ansatz
- Full DVCS and DVMP global analysis with NLO correctios

FUTURE ADDITIONS TO GUMP

- Full uncertainty propagation
- Add threshold J/ψ production – potentially constrain D-term/DA-terms
- Implement t-integrated cross sections
- Add quark flavors and implement ϕ electroproduction
  - Could examine N_f dependence – so far just u and d quarks
- Add other processes like TCS or recently proposed SDHEP (*Qiu and Yu 2022-2023*)

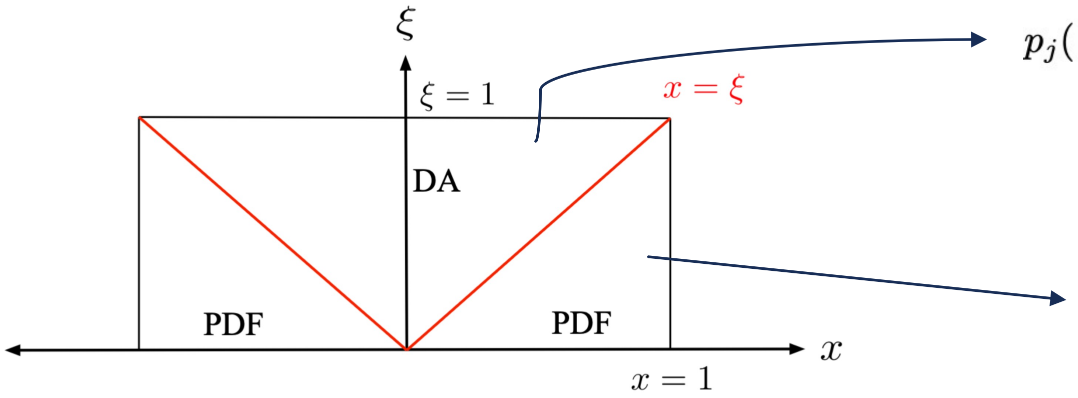
CONCLUSIONS

- Fit DV J/ψ P data from H1 using gluon GPD H parameters in hybrid collinear-NRQCD factorization
- Further analysis of fits and DVρ^0P fits in progress
- Several directions for future improvements available – both for gluon sector and GUMP overall

BACKUP SLIDES

ANALYTIC CONTINUATION OF MOMENTS

- Gegenbauer polynomials are only defined in the DA-like, only give a formal sum for the full GPD
- Analytic continuation to all values of x/ξ and complex values of conformal spin j yields two bases for the DA-like and PDF-like regions and allows for reconstruction of GPD across all (x, ξ)

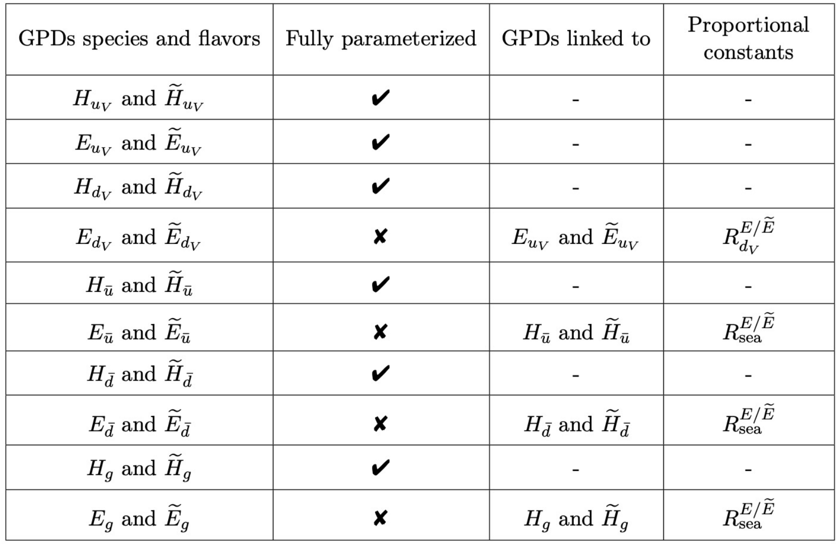


$$p_j(|x| \leq \xi, \xi) = \frac{2^{j+1}\Gamma(5/2+j)\xi^{-j-1}}{\Gamma(1/2)\Gamma(1+j)}(1+x/\xi)\ {}_2F_1\left(-1-j, j+2, 2; \frac{\xi+x}{2\xi}\right)$$

$$p_j(x > \xi, \xi) = \frac{\sin(\pi[j+1])}{\pi}x^{-j-1}{}_2F_1\left(\frac{j+1}{2}, \frac{j+2}{2}, 5/2+j; \frac{\xi^2}{x^2}\right)$$

FIRST STEP TOWARD GLOBAL GPD ANALYSIS

- The number of parameters needed for modelling all the species of GPD grows very quickly
- We impose extra constraints for simplicity



GPDs species and flavors	Fully parameterized	GPDs linked to	Proportional constants
H_{u_V} and $\widetilde{H}_{u_V}$	✔	-	-
E_{u_V} and $\widetilde{E}_{u_V}$	✔	-	-
H_{d_V} and $\widetilde{H}_{d_V}$	✔	-	-
E_{d_V} and $\widetilde{E}_{d_V}$	✘	E_{u_V} and $\widetilde{E}_{u_V}$	$R^{E/\widetilde{E}}_{d_V}$
$H_{\bar{u}}$ and $\widetilde{H}_{\bar{u}}$	✔	-	-
$E_{\bar{u}}$ and $\widetilde{E}_{\bar{u}}$	✘	$H_{\bar{u}}$ and $\widetilde{H}_{\bar{u}}$	$R^{E/\widetilde{E}}_{\text{sea}}$
$H_{\bar{d}}$ and $\widetilde{H}_{\bar{d}}$	✔	-	-
$E_{\bar{d}}$ and $\widetilde{E}_{\bar{d}}$	✘	$H_{\bar{d}}$ and $\widetilde{H}_{\bar{d}}$	$R^{E/\widetilde{E}}_{\text{sea}}$
H_g and $\widetilde{H}_g$	✔	-	-
E_g and $\widetilde{E}_g$	✘	H_g and $\widetilde{H}_g$	$R^{E/\widetilde{E}}_{\text{sea}}$

SEMI-FORWARD INPUTS

- JAM (2022) PDF global analysis results
  - Full global analysis should in principle fit to PDF sensitive data directly, but here we fit to JAM results
  - Limited number of points taken to avoid need for more sophisticated forward limit
- Globally extracted electromagnetic form factors (*Z. Ye et al* 2018)
- Lattice GPDs (*Alexandrou et al* 2020) and form factors (*Alexandrou et al* 2022)
  - x, t -dependent GPDs (semi-forward limit)

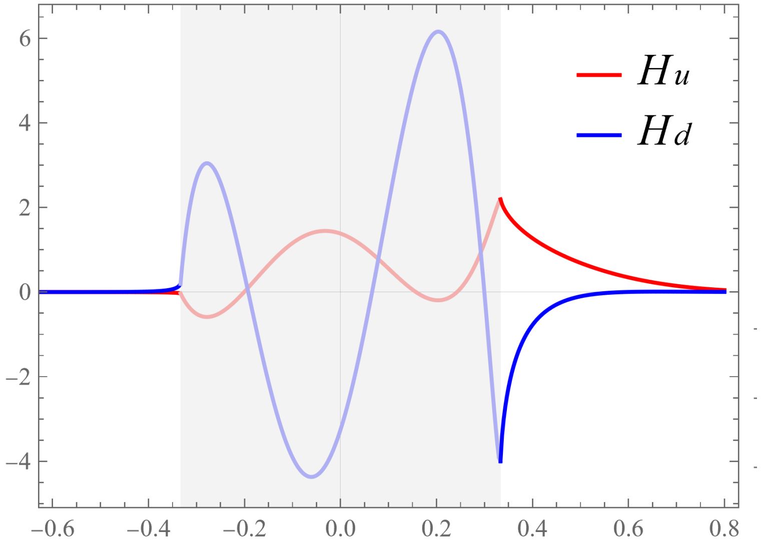
OFF-FORWARD INPUTS

- DVCS measurements from JLab (*CLAS* 2019 & 2021, *Hall A* 2018 & 2022) and HERA (*H1* 2010)
- Only using t-dependent cross sections due to practical limitations
- Far more points from JLab data than from HERA from φ-dependence and both UU and LU polarization channels
- Off-forward lattice GPDs not used in fitting, but can supply crucial constraints for future work!

EXTRACTED GPDS

- GPDs are mostly constrained on the $\xi = x$ line and in the DGLAP region $|\xi| < |x|$
- ERBL region shows large oscillations which are characteristic of the Gegenbauer polynomials used in the moment expansion

GPDs H_q at $\xi = 1/3$ and $-t = 0.69$ GeV^2



NON-RELATIVISTIC MODEL APPROACH

- Encoding the J/ψ formation into NR matrix elements

$$\Gamma[J/\psi \to e^+e^-] = \frac{8\pi\alpha_{EM}^2}{27}\frac{f_{J/\psi}^2}{m_c} \to \frac{8\pi\alpha_{EM}^2}{27}\frac{\langle\mathcal{O}_1\rangle_{J/\psi}}{m_c^2}$$

- Maintain the form of the factorization theorem for the process – still sensitive to leading twist GPDs (*D.Y. Ivanov et al* 2004)

at least to NLO in pQCD!

$$\mathcal{A}_{\text{collinear}} \sim \int_0^1 \mathrm{d}z \int_{-1}^{1} \mathrm{d}x\, \underbrace{C_1(x,\xi,z)}_{\text{hard scattering term}} \overbrace{F^g(x,\xi,t)}^{\text{GPD}} \overbrace{\Phi(z)}^{\text{DA}} \longrightarrow \mathcal{A}_{\text{NR}} \sim \sqrt{\frac{\langle\mathcal{O}_1\rangle_{J/\psi}}{m_c}} \int_{-1}^{1} \mathrm{d}x\, \underbrace{C_2\left(x,\xi,\frac{m_c}{Q}\right)}_{\text{hard scattering term}} \underbrace{F^g(x,\xi,t)}_{\text{GPD}}$$

IMPLEMENTING NR J/ψ PRODUCTION IN GUMP

- Previous studies on J/ψ photoproduction have seen a poor description with LO calculations
  - NLO hard scattering corrections are large and improve the description
- Using the same LO treatment as for our previous global analysis, we see the problem persists for DV J/ψ P
  - Here we will add in both NLO hard scattering corrections and NLO GPD evolution!
- NLO GPD evolution kernel is known in conformal moment space (*Kumerički et al 2008*)
  - Allows for (relatively) fast numerical implementation!
- Finite mass corrections for hard scattering are only known in momentum fraction space (*Flett et al 2021*)
  - Mass corrections make the convolutions for converting to conformal moment space much more complicated
  - Converting these is crucial in order to include photoproduction in our global analysis framework

IMPLEMENTING NR J/ψ PRODUCTION IN GUMP

- We have implemented NLO GPD evolution for the sea quarks and gluons (valence quarks are insignificant for small x_B HERA kinematics)
  - Huge thanks to Gepard package – full NLO implementation of DVCS and DVMP for light vector mesons available!
- Conversion of NLO finite mass hard scattering terms to moment space is on going
- Collinear factorization NLO hard scattering terms are known in conformal moment space (*Müller et al 2014*)
  - Gives the large logs of $1/x_B$ that are important for HERA data, mass corrections shouldn't be too significant for higher Q^2 data points

IMPLEMENTING NR J/ψ PRODUCTION IN GUMP

- Matching between the NRQCD matrix element and the distribution amplitude in conformal moment space can introduce some ambiguity from expanding a delta function

$$\langle \mathcal{O}_1 \rangle_{J/\psi} \Rightarrow \Phi_{J/\psi}(z) \propto \delta(z-1/2)$$

$$\Rightarrow \delta(z-1/2) = \sum_{k=0,2,4\ldots}^{\infty} 6z(1-z)C_k^{3/2}(2z-1)\Phi_k,$$

$$\Phi_k = \frac{2(2k+3)}{3(k+1)(k+2)}C_k^{3/2}(0)$$

Not clear how to extract size of truncation error!

- For simplicity we keep only the first conformal moment (asymptotic DA), so we introduce an order one normalization factor into the amplitudes to absorb the mismatch

$$\Phi_{J/\psi}(z) = N^{DA}\Phi_{asymptotic}(z)$$

ANALYTIC CONTINUATION OF MOMENTS

- Gegenbauer moments from ERBL region only give a formal sum for the full GPD
- Analytic continuation to all values of x/ξ yields two bases for the ERBL and DLGAP regions

$$p_j(|x| \leq \xi, \xi) = \frac{2^{j+1}\Gamma(5/2+j)\xi^{-j-1}}{\Gamma(1/2)\Gamma(1+j)}(1+x/\xi)\ {}_2F_1\left(-1-j, j+2, 2; \frac{\xi+x}{2\xi}\right)$$

$$p_j(x > \xi, \xi) = \frac{\sin(\pi[j+1])}{\pi} x^{-j-1} {}_2F_1\left(\frac{j+1}{2}, \frac{j+2}{2}, 5/2+j; \frac{\xi^2}{x^2}\right)$$

- These conformal wave functions can then be used to reconstruct the GPD from its conformal moments with a Mellin-Barnes integral

$$F(x.\xi, t) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}j\, \frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi, t)$$

CONFORMAL MOMENT POLYNOMIALITY

- We can expand the Gegenbauer polynomials in a power series, connecting them to the Mellin moment expansion

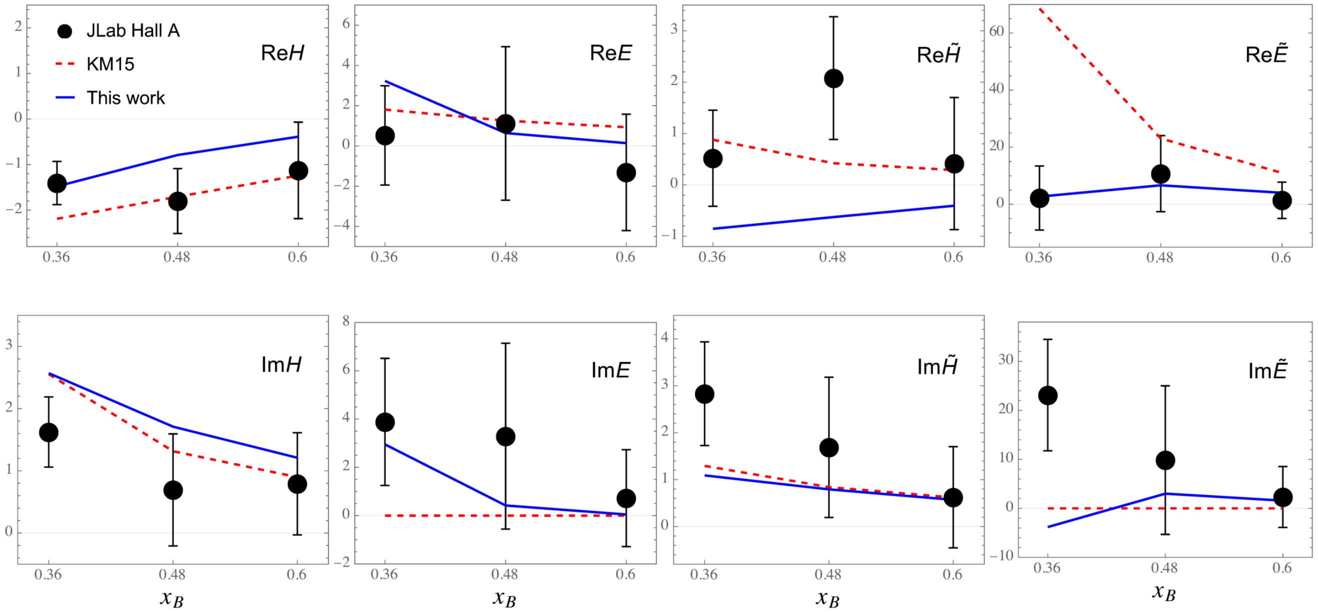
$$C_j^{(\lambda)}(x) = \sum_{k=0}^{j} c_{j,k}^{(\lambda)} x^k$$

- Then using the polynomiality of the Mellin moments we obtain a polynomiality condition on the conformal moments

$$\begin{aligned}\mathcal{F}_j(\xi,t) &\propto \int_{-1}^{1} \mathrm{d}x\ \xi^j C_j^{\frac{3}{2}}\left(\frac{x}{\xi}\right) F(x,\xi,t) \\ &= \int_{-1}^{1} \mathrm{d}x \sum_{k=0}^{j} c_{j,k}^{\frac{3}{2}} \xi^{j-k} x^k F(x,\xi,t) \\ &= \sum_{k=0}^{j} c_{j,k}^{\frac{3}{2}} \xi^{j-k} \int_{-1}^{1} \mathrm{d}x x^k F(x,\xi,t)\end{aligned}$$

NON-ZERO SKEWNESS GLOBAL FIT: CFFS

- CFFs from fit are mostly consistent with local extraction from JLAB Hall A data as well as KM15 extractions
- Some inconsistencies can be expected from degeneracies in CFF contribution to cross sections – need more polarization configurations!



AMBIGUITY IN ERBL REGION

- We can add terms in the moment expansion which only contribute to the ERBL region

$$(-1)^j p_j(x,\xi) = \xi^{-j-1} \frac{2^j \Gamma(\frac{5}{2}+j)}{\Gamma(\frac{3}{2})\Gamma(j+3)} \left[1 - \left(\frac{x}{\xi}\right)^2\right] C_j^{3/2}\left(\frac{x}{\xi}\right) , \quad |x| < |\xi|$$

- This suggests an interpretation of the GPDs in terms of quark and antiquark pieces as well as a ERBL region distribution amplitude (DA) piece

$$F_q(x,\xi,t) = \underbrace{F_{\hat{q}}(x,\xi,t)}_{\substack{\text{quark} \\ x > -\xi}} \mp \underbrace{F_{\bar{q}}(-x,\xi,t)}_{\substack{\text{antiquark} \\ x < \xi}} + \underbrace{F_{q\bar{q}}}_{\substack{\text{DA} \\ \xi > x > -\xi}}$$

CONNECTION TO D-TERM

- These DA terms don't have a large affect on CFFs, but they do contain information related to the various D-terms in QCD, ex.
  - Gravitational form factor C/D

$$\int_{-1}^{1} \mathrm{d}x \, x H_q(x, \xi, t) = A_q(t) + (2\xi)^2 C_q(t)$$

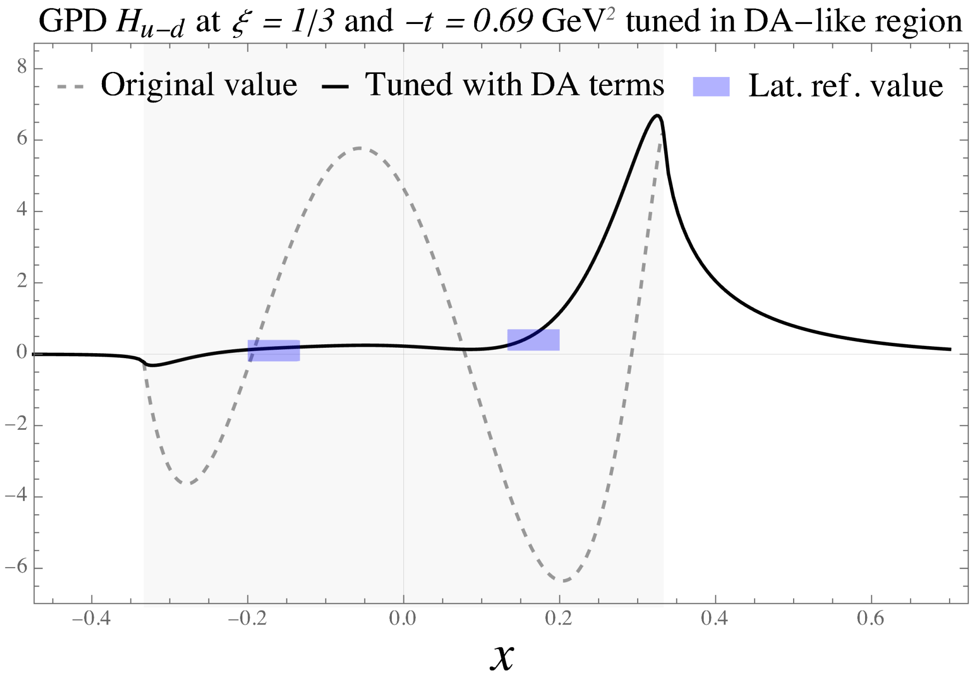
  - Dispersion relation subtraction term

$$F(\xi, t, Q^2) = \frac{1}{\pi} \int_{0}^{1} \mathrm{d}\xi' \left(\frac{1}{\xi - \xi'} \mp \frac{1}{\xi + \xi'} \right) \mathrm{Im} \left[F(\xi' - i0, t, Q^2) \right] + \mathcal{C}(t, Q^2)$$

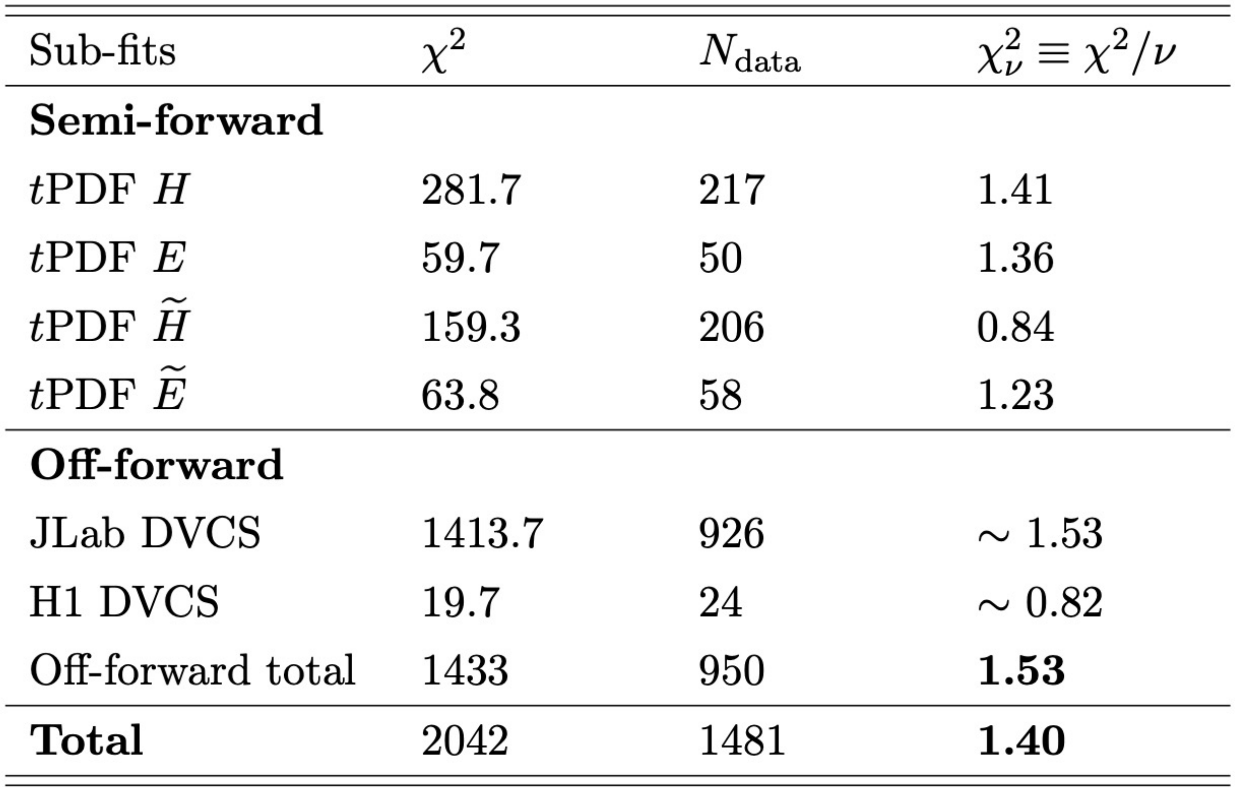
- By constraining the DA terms with further experimental data and lattice calculations, we can access the mechanical properties of hadrons contained in these D-terms!

CONSTRAINING DA TERMS

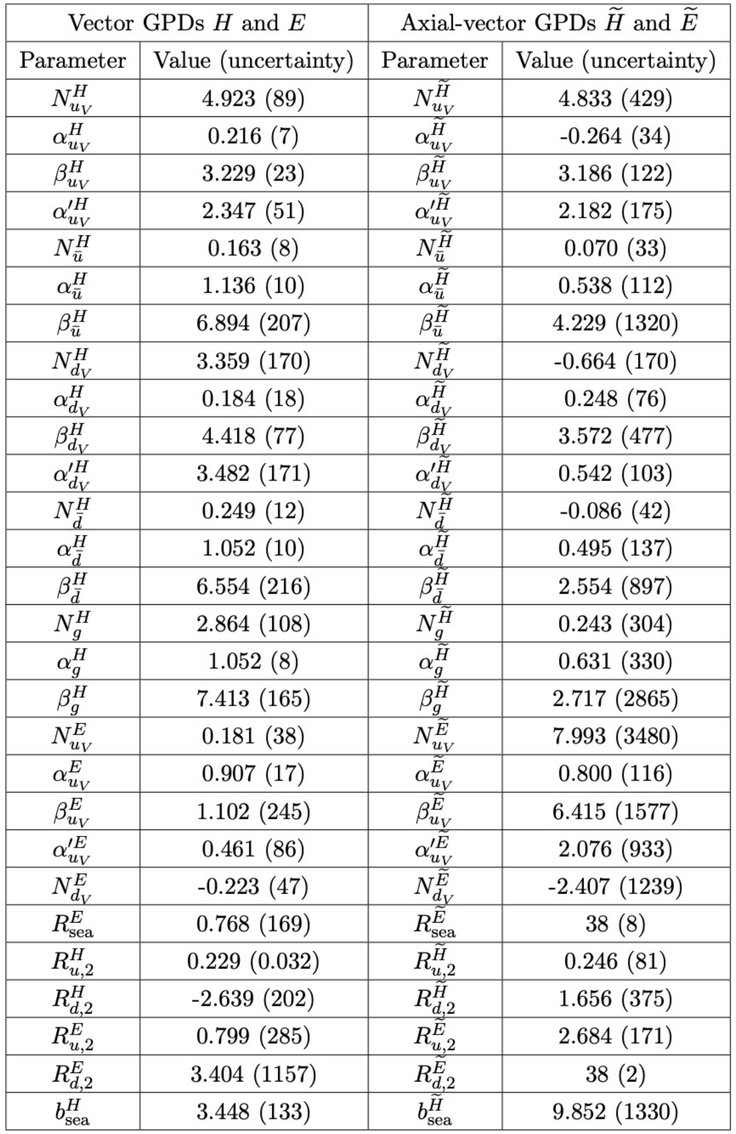
- Adding in lattice GPD calculations can give us constrains directly in the ERBL region
- Adding just a few terms to the moment expansion can remove the unphysical oscillations



BEST FIT χ^2 BREAKDOWN



Sub-fits	χ^2	N_{data}	$\chi^2_\nu \equiv \chi^2/\nu$
Semi-forward			
tPDF H	281.7	217	1.41
tPDF E	59.7	50	1.36
tPDF $\widetilde{H}$	159.3	206	0.84
tPDF $\widetilde{E}$	63.8	58	1.23
Off-forward			
JLab DVCS	1413.7	926	~ 1.53
H1 DVCS	19.7	24	~ 0.82
Off-forward total	1433	950	**1.53**
Total	2042	1481	**1.40**



Vector GPDs H and E		Axial-vector GPDs $\widetilde{H}$ and $\widetilde{E}$	
Parameter	Value (uncertainty)	Parameter	Value (uncertainty)
$N_{u_V}^{H}$	4.923 (89)	$N_{u_V}^{\widetilde{H}}$	4.833 (429)
$\alpha_{u_V}^{H}$	0.216 (7)	$\alpha_{u_V}^{\widetilde{H}}$	-0.264 (34)
$\beta_{u_V}^{H}$	3.229 (23)	$\beta_{u_V}^{\widetilde{H}}$	3.186 (122)
$\alpha_{u_V}'^{H}$	2.347 (51)	$\alpha_{u_V}'^{\widetilde{H}}$	2.182 (175)
$N_{\bar{u}}^{H}$	0.163 (8)	$N_{\bar{u}}^{\widetilde{H}}$	0.070 (33)
$\alpha_{\bar{u}}^{H}$	1.136 (10)	$\alpha_{\bar{u}}^{\widetilde{H}}$	0.538 (112)
$\beta_{\bar{u}}^{H}$	6.894 (207)	$\beta_{\bar{u}}^{\widetilde{H}}$	4.229 (1320)
$N_{d_V}^{H}$	3.359 (170)	$N_{d_V}^{\widetilde{H}}$	-0.664 (170)
$\alpha_{d_V}^{H}$	0.184 (18)	$\alpha_{d_V}^{\widetilde{H}}$	0.248 (76)
$\beta_{d_V}^{H}$	4.418 (77)	$\beta_{d_V}^{\widetilde{H}}$	3.572 (477)
$\alpha_{d_V}'^{H}$	3.482 (171)	$\alpha_{d_V}'^{\widetilde{H}}$	0.542 (103)
$N_{\bar{d}}^{H}$	0.249 (12)	$N_{\bar{d}}^{\widetilde{H}}$	-0.086 (42)
$\alpha_{\bar{d}}^{H}$	1.052 (10)	$\alpha_{\bar{d}}^{\widetilde{H}}$	0.495 (137)
$\beta_{\bar{d}}^{H}$	6.554 (216)	$\beta_{\bar{d}}^{\widetilde{H}}$	2.554 (897)
N_{g}^{H}	2.864 (108)	$N_{g}^{\widetilde{H}}$	0.243 (304)
α_{g}^{H}	1.052 (8)	$\alpha_{g}^{\widetilde{H}}$	0.631 (330)
β_{g}^{H}	7.413 (165)	$\beta_{g}^{\widetilde{H}}$	2.717 (2865)
$N_{u_V}^{E}$	0.181 (38)	$N_{u_V}^{\widetilde{E}}$	7.993 (3480)
$\alpha_{u_V}^{E}$	0.907 (17)	$\alpha_{u_V}^{\widetilde{E}}$	0.800 (116)
$\beta_{u_V}^{E}$	1.102 (245)	$\beta_{u_V}^{\widetilde{E}}$	6.415 (1577)
$\alpha_{u_V}'^{E}$	0.461 (86)	$\alpha_{u_V}'^{\widetilde{E}}$	2.076 (933)
$N_{d_V}^{E}$	-0.223 (47)	$N_{d_V}^{\widetilde{E}}$	-2.407 (1239)
R_{sea}^{E}	0.768 (169)	$R_{\text{sea}}^{\widetilde{E}}$	38 (8)
$R_{u,2}^{H}$	0.229 (0.032)	$R_{u,2}^{\widetilde{H}}$	0.246 (81)
$R_{d,2}^{H}$	-2.639 (202)	$R_{d,2}^{\widetilde{H}}$	1.656 (375)
$R_{u,2}^{E}$	0.799 (285)	$R_{u,2}^{\widetilde{E}}$	2.684 (171)
$R_{d,2}^{E}$	3.404 (1157)	$R_{d,2}^{\widetilde{E}}$	38 (2)
b_{sea}^{H}	3.448 (133)	$b_{\text{sea}}^{\widetilde{H}}$	9.852 (1330)