Matching between TMD and twist-3 factorizations in the transversely polarized hyperon production

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Yuji Koike(Niigata University)

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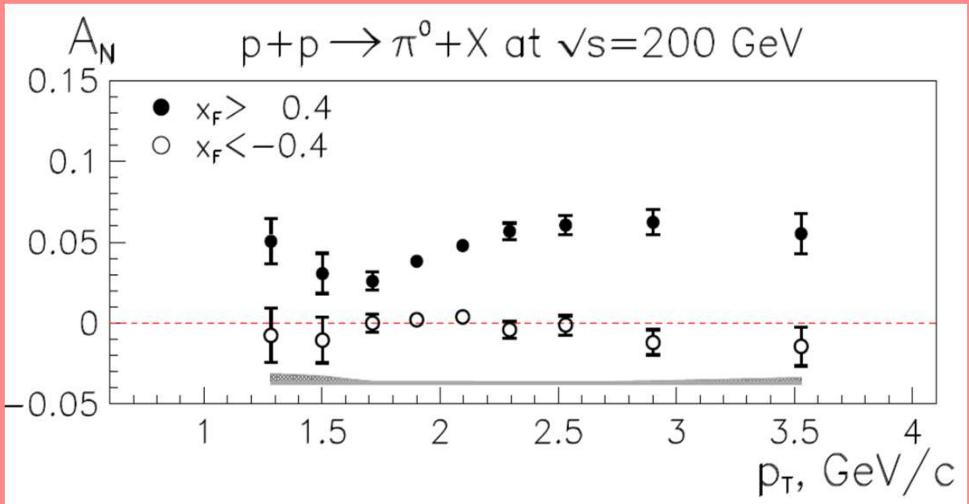
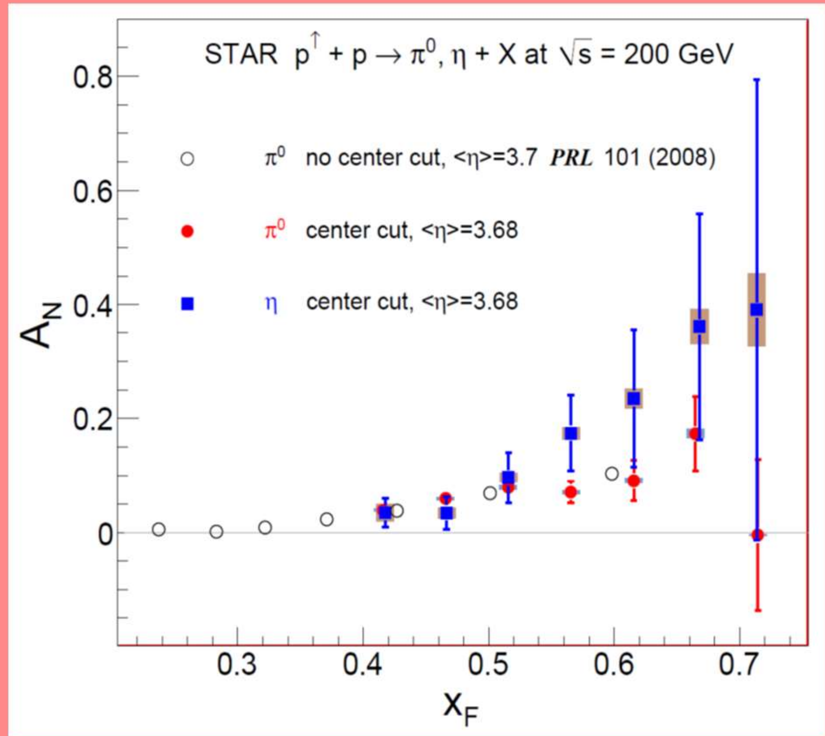
Mysteries in the transverse polarization

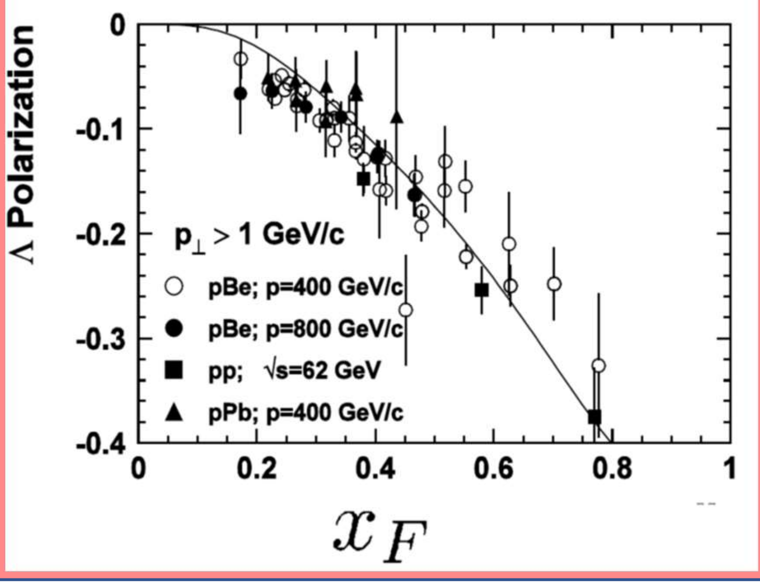
Almost half a century mysteries in high-enegy QCD physics

$p^{\uparrow}p \to \pi X$





$pp \to \Lambda^{\uparrow}X$ (in 80's and 90's)



SSA estimated by pQCD

Kane *et al.* ('78)

$$A_N \sim \alpha_s \frac{m_q}{P_T} \sim \text{negligible!}$$

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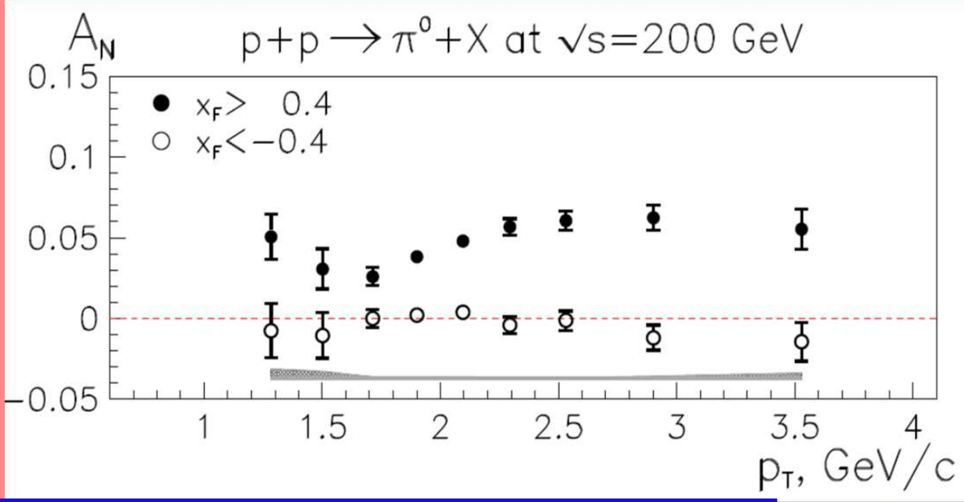

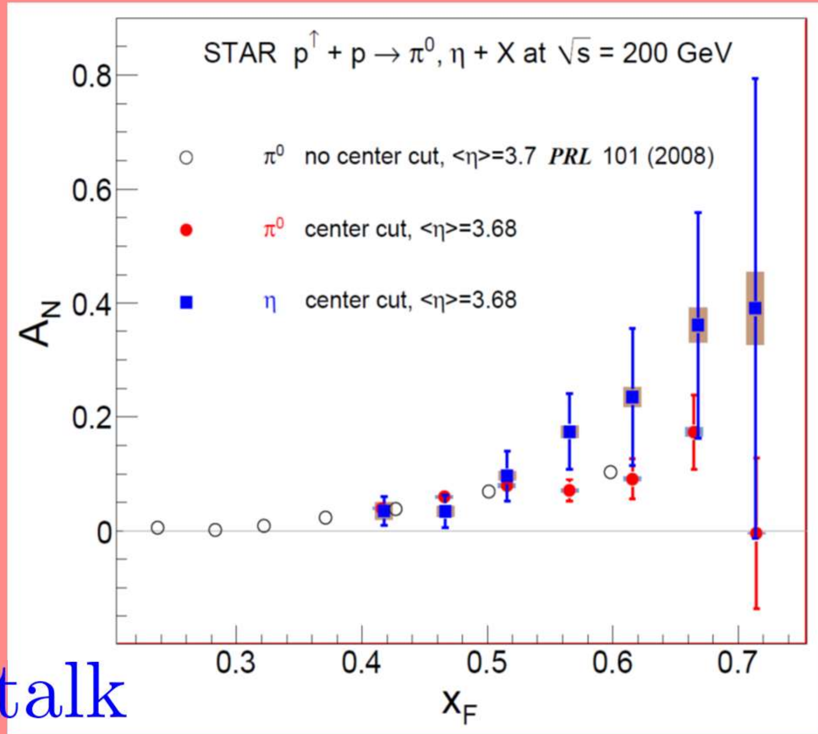

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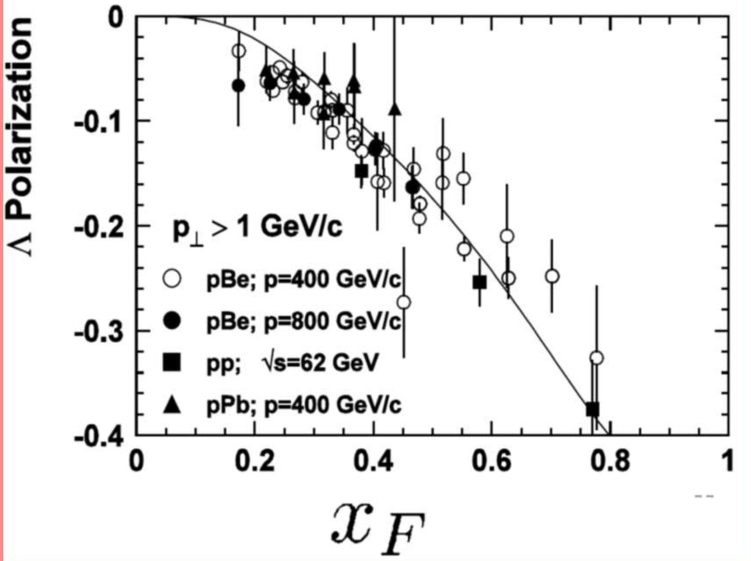

This talk

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pQCD frameworks for the SSA

Transverse Momentum Dependent(TMD) factorization

· Applicable in small $P_T (Q \gg P_T \geq \Lambda_{QCD})$ region

cf. Sivers function $f_{1T}^{\perp}(x, k_{\perp})$

· Nonperturbative functions depend on the transverse momentum of partons

advantage: TMD functions have definite physical interpretation

disadvantage: limited applicable processes $\times$ $pp \to \pi X$

collinear factorization

· Applicable in large $P_T (P_T \gg \Lambda_{QCD})$ region

· twist-3 multiparton correlation inside hadrons causes the large SSA

advantage: Applicable to many processes such as $pp \to \pi X$

disadvantage: Physical interpretation of the twist-3 functions is unclear

There is overlapping region $\Lambda_{\rm QCD} \ll P_T \ll Q$

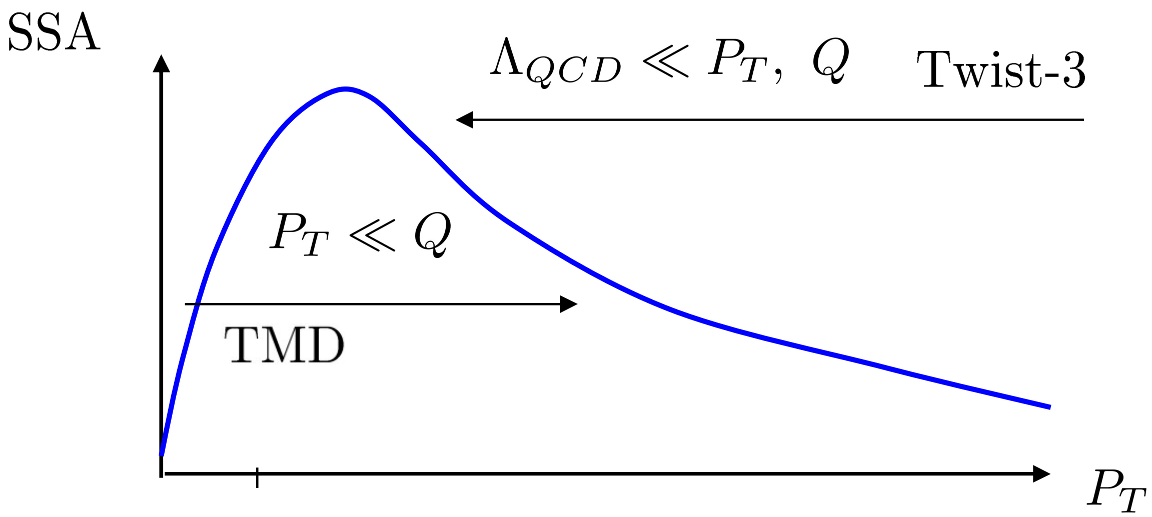
There should not be a quantitative ambiguity in this region
if both are the first principle calculations

Relationship between TMD and twist-3

TMD and the collinear twist-3 gives equivalent results in intermediate P_T region in the pion production

X. Ji, J.-W. Qiu, W.Vogelsang and F. Yuan, PRD73(2006) PLB638(2006)

F. Yuan and J. Zhou, PRL 103 (2009)



QS-function

$$\pi F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

k_T-moment of Sivers

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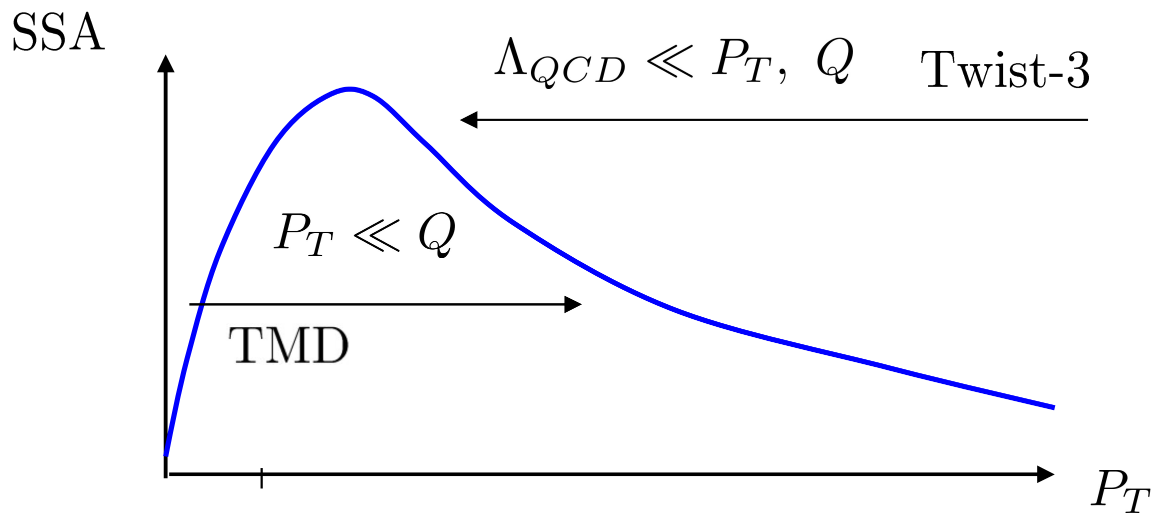

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The Latest parameterizations rely on the relations between TMD and the collinear

L. Gamberg *et al.* [Jefferson Lab Angular Momentum (JAM) collaboration], Phys. Rev. D106 (2022)

$$f(x) = \int d^2\vec{k}_T\, f(x,\vec{k}_T^2)\ \ (f = f_1 \text{ or } h_1)\,, \qquad \pi F_{FT}(x,x) = \int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2} f_{1T}^{\perp}(x,\vec{k}_T^2) \equiv f_{1T}^{\perp(1)}(x)\,,$$

$$H_1^{\perp(1)}(z) = z^2 \int d^2\vec{p}_T\, \frac{\vec{p}_T^{\,2}}{2M_h^2} H_1^{\perp}(z, z^2\vec{p}_T^{\,2})\,.$$

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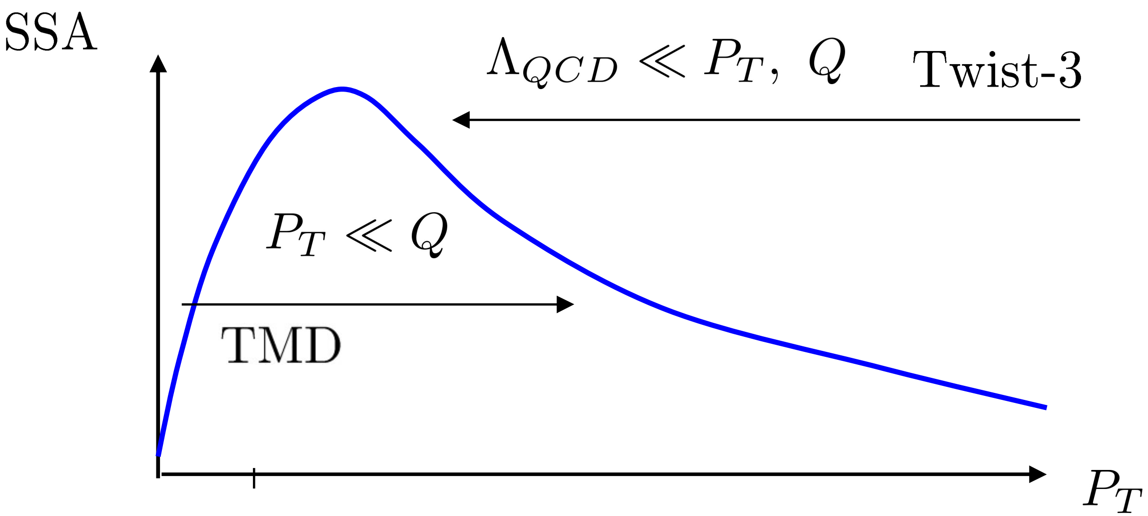

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How about the hyperon polarization ?

The distribution part has been done already. J. Zhou, F.Yuan and Z. -T. Liang, PRD78 (2008)

Operator identity in QFT

Two operators are identical

$$\hat{O}_1 = \hat{O}_2$$

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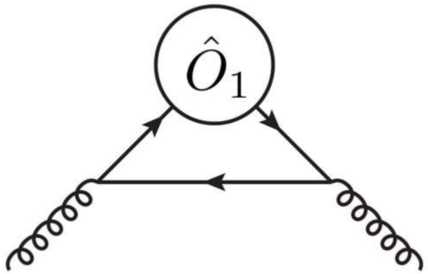

$$= N_f \frac{g^2}{32\pi^2}\left[4\frac{1}{p^2}\frac{1}{(p')^2}\epsilon^{\mu\nu\rho\sigma}p_\rho p'_\sigma\right]$$

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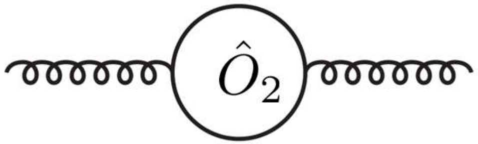

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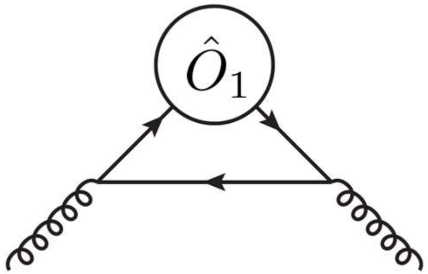

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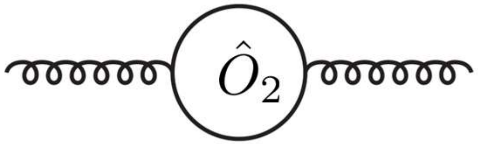

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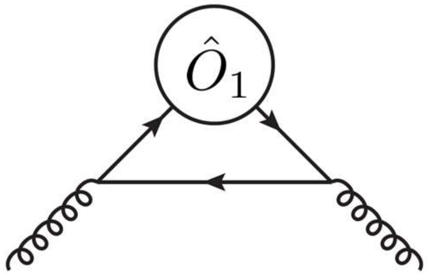

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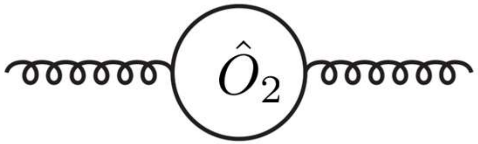

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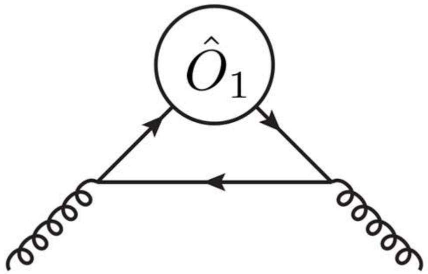

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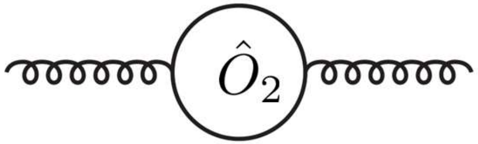

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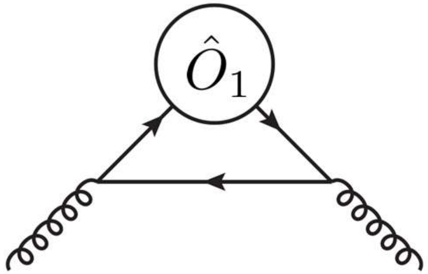

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$$\partial_\mu(\hat{\bar{\psi}}\gamma^5\gamma^\mu\hat{\psi}) = N_f \frac{g^2}{32\pi^2}\hat{F}^{\alpha\beta}\hat{\tilde{F}}_{\alpha\beta}$$

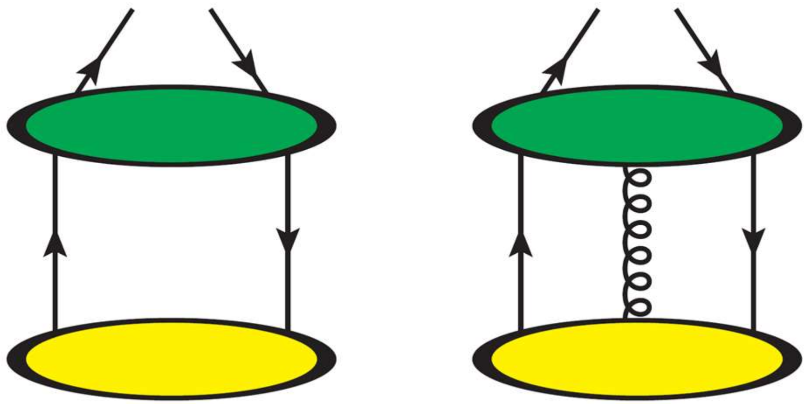
We choose $\hat{O}_1$ as the TMD operator and calculate perturbative corrections to it

$$D_{1T}^{\perp}(z_f, p_\perp) = \frac{M_h}{z_f}\frac{1}{(-4\epsilon^{p_\perp P_h w S_\perp})}\sum_X \int \frac{d\xi^+}{2\pi}\int \frac{d^2\xi_\perp}{(2\pi)^2} e^{-ip^-\xi^+\frac{1}{z}} e^{i\vec{p}_\perp\cdot\xi_\perp}\langle 0|\mathcal{L}_0\psi(0)|P_h X\rangle\langle P_h X|\psi(\xi^+,\xi_\perp)\mathcal{L}_\xi^\dagger|0\rangle$$

D. Boer and P. J. Mulders, PRD57 (1998)

Collinear expansion

Y. Koike, A. Metz, D. Pitonyak, K. Yabe and SY, PRD95 (2017)



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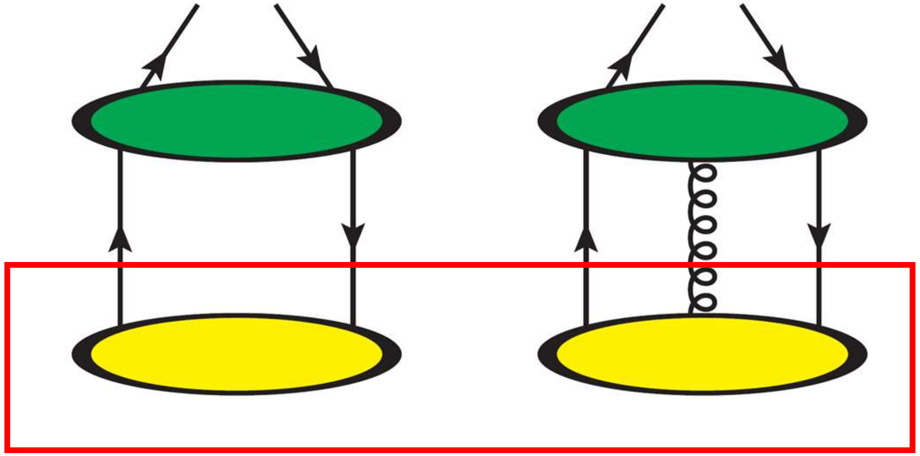
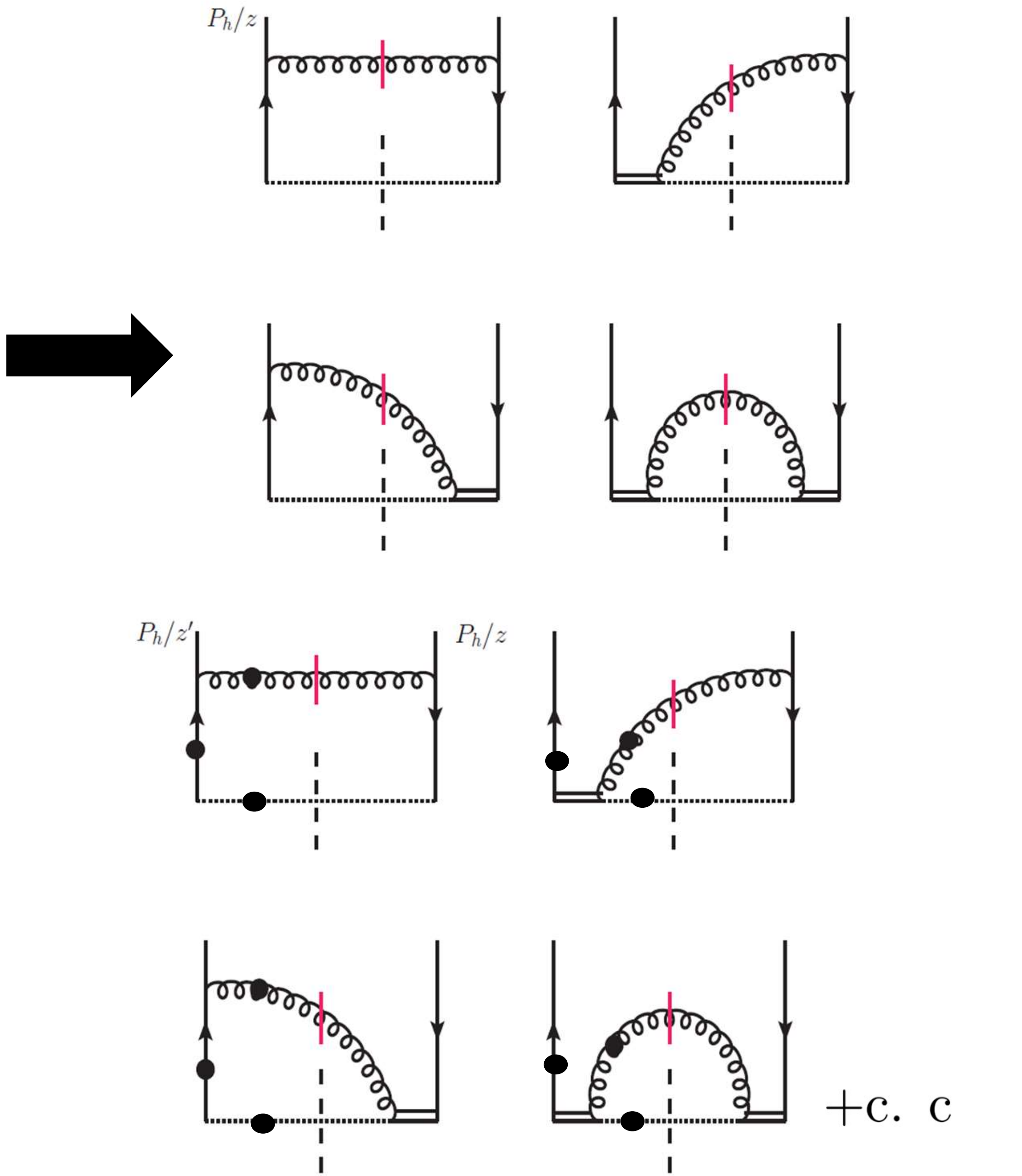


TMD operator part



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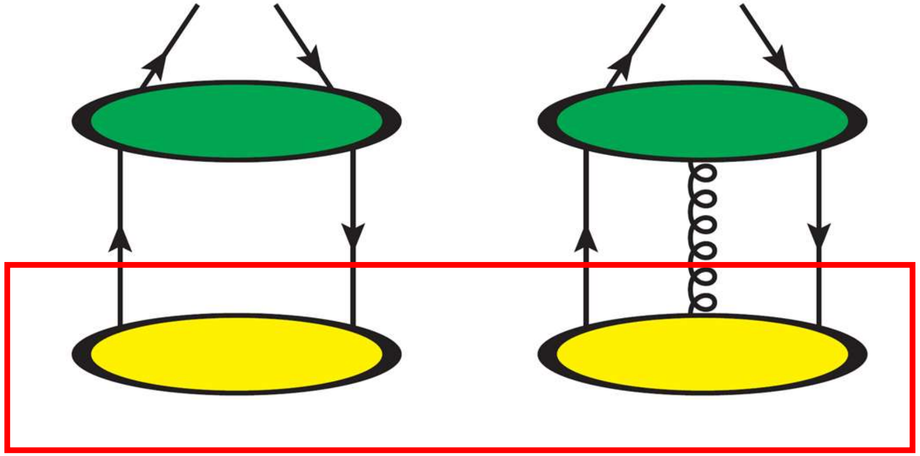

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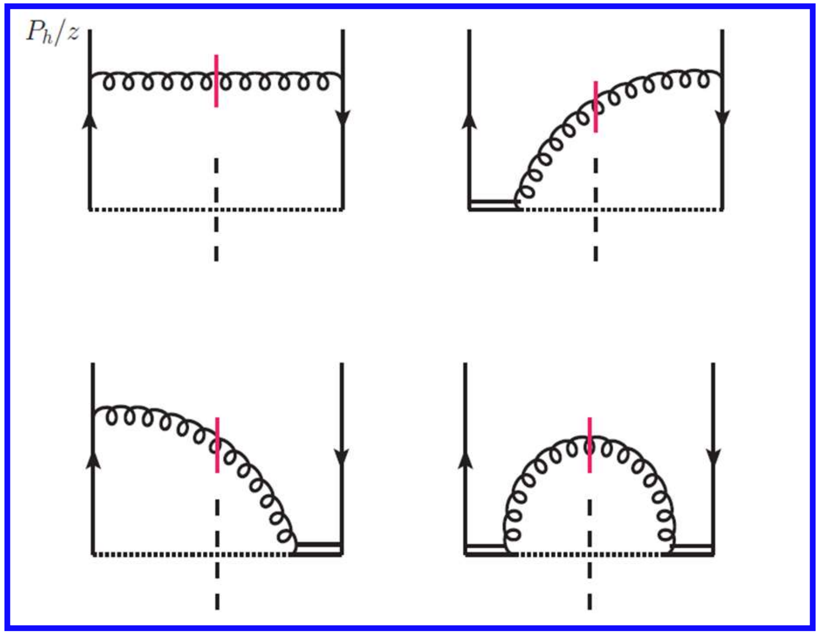
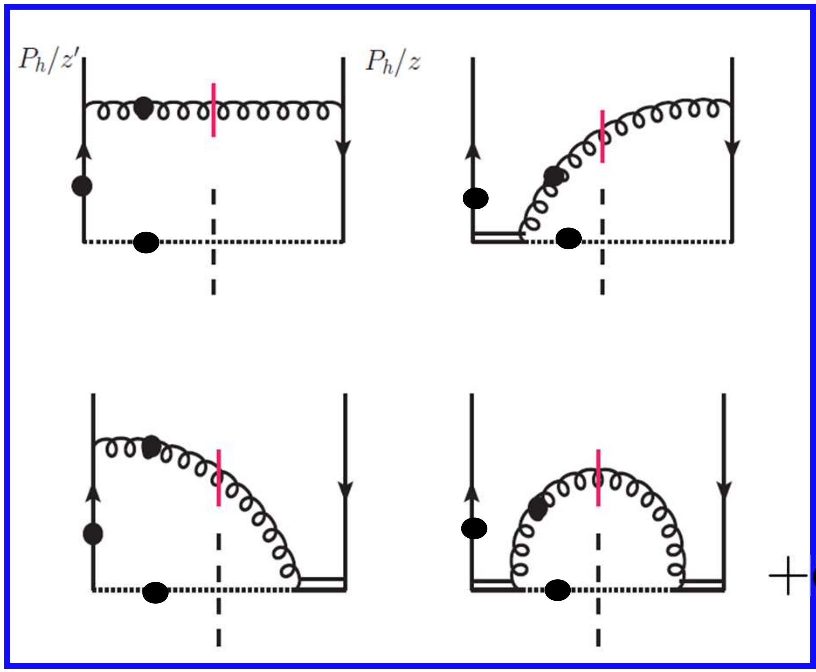


$S(k)$



$S_\alpha(k_1, k_2)$

+c. c

Ward-Takahashi identity

$$(k_2 - k_1)^\alpha S_\alpha(k_1, k_2) = S(k_2) - S(k_1)$$

$$D_{1T}^{\perp}(z_f, z_f^2 p_\perp^2)\Big|_{\text{quark-gluon}}$$

$$= \frac{M_h}{z_f} \frac{1}{(-4\epsilon^{p_\perp P_h w S_\perp})} \boxed{\int \frac{dz}{z^2}\Big[\text{Tr}[\Delta(z) H(\frac{P_h}{z})] - i\text{Tr}[\Omega^\alpha{}_\beta \Delta_\partial^\beta(z) \frac{\partial}{\partial k^\alpha} H(k)\Big|_{k=\frac{P_h}{z}}]}$$

$$\boxed{+2\text{Re}\Big[i\int \frac{dz'}{z'^2} \text{Tr}[\Omega^\alpha{}_\beta \Delta_F^\beta(z', z)\Big(\frac{1}{\frac{1}{z} - \frac{1}{z'}}\Big) H_{L\alpha}(\frac{P_h}{z'}, \frac{P_h}{z})]\Big],}$$

operator part is written in this form

$$D_{1T}^{\perp}(z_f, z_f^2 p_\perp^2)\Big|_{\text{quark-gluon}}$$

intrinsic　　　kinematical

$$= \frac{M_h}{z_f}\frac{1}{(-4\epsilon^{p_\perp P_h w S_\perp})}\int\frac{dz}{z^2}\Big[\mathrm{Tr}[\Delta(z)H(\frac{P_h}{z})] - i\mathrm{Tr}[\Omega^\alpha{}_\beta\Delta_\partial^\beta(z)\frac{\partial}{\partial k^\alpha}H(k)\Big|_{k=\frac{P_h}{z}}]$$

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dynamical

operator part is written in this form

$$\Delta_{ij}(z) = \frac{1}{N}\sum_X\int\frac{d\lambda}{2\pi}e^{-i\frac{\lambda}{z}}\langle 0|[\infty w,0]\psi_i(0)|h(P_h,S_\perp)X\rangle\langle h(P_h,S_\perp)X|\bar\psi_j(\lambda w)[\lambda w,\infty w]|0\rangle$$

$$= \cdots + M_h\epsilon^{\alpha S_\perp w P_h}(\gamma_\alpha)_{ij}\frac{D_T(z)}{z} + \cdots .$$

$$\Delta_{\partial ij}^\alpha(z) = \frac{1}{N}\sum_X\int\frac{d\lambda}{2\pi}e^{-i\frac{\lambda}{z}}\langle 0|[\infty w,0]\psi_i(0)|h(P_h,S_\perp)X\rangle\langle h(P_h,S_\perp)X|\bar\psi_j(\lambda w)[\lambda w,\infty w]|0\rangle\overleftarrow{\partial}^\alpha$$

$$= -iM_h\epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij}\frac{D_{1T}^{\perp(1)}(z)}{z} + \cdots ,$$

$$\Delta_{Fij}^\alpha(z, z_1)$$

$$= \frac{1}{N}\sum_X\int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi}e^{-i\frac{\lambda}{z_1}}e^{-i\mu(\frac{1}{z}-\frac{1}{z_1})}\langle 0|\psi_i(0)|h(P_h,S_\perp)X\rangle\langle h(P_h,S_\perp)X|\bar\psi_j(\lambda w)gF^{\alpha w}(\mu w)|0\rangle$$

$$= M_h\epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij}\frac{\widehat{D}_{FT}^*(z,z_1)}{z} - iM_h S_\perp^\alpha(\gamma_5\not{P}_h)_{ij}\frac{\widehat{G}_{FT}^*(z,z_1)}{z} + \cdots ,$$

Result in TMD side

$$D_{1T}^{\perp}(z_h, P_{h\perp}^2)\Big|_{\rm quark-gluon} = \frac{\alpha_s}{2\pi^2}(2M_h^2)\frac{z_f^2}{[P_{h\perp}^2]^2}\Big[A + C_F D_{1T}^{\perp(1)}(z_h)\Big(\ln\frac{\hat{\zeta}^2}{z_f^2 q_T^2}\Big)\Big]$$

$$\begin{aligned}
A \;=\; & \int\frac{dz}{z}\Bigg[\frac{D_T(z)}{z}\Big(-C_F(1+2\hat{z}) - \frac{1}{2N_c}\frac{1+\hat{z}^2}{\hat{z}}\Big) + \frac{d}{d(1/z)}\Big(\frac{D_{1T}^{\perp(1)}}{z}\Big)\Big(-\frac{1}{2N_c}\frac{1+\hat{z}^2}{\hat{z}}\Big) + D_{1T}^{\perp(1)}\Big(C_F\frac{\hat{z}(1+\hat{z})}{(1-\hat{z})_+}\Big) \\
& + \int d\Big(\frac{1}{z'}\Big)\frac{1/z}{1/z - 1/z'}\Big\{\Im\widehat{D}_{FT}(z,z')\Big[\frac{1}{1/z'}\Big(\frac{1}{2N_c}\frac{2-\hat{z}}{\hat{z}}\Big) + \frac{1}{1/z' - 1/z_f}[C_F + \frac{1}{2N_c}](1+\hat{z})\Big] \\
& + \Im\widehat{G}_{FT}(z,z')\Big[\frac{1}{1/z'}\Big(\frac{1}{2N_c}\Big) - \frac{1}{1/z' - 1/z_f}[C_F + \frac{1}{2N_c}](1-\hat{z})\Big]\Big\}\Bigg]
\end{aligned}$$

Nonperturbative part is written only in terms of the collinear twist-3 functions

$D_T,\ D_{1T}^{\perp(1)},\ \widehat{D}_{FT},\ \widehat{G}_{FT}$

Result in TMD side

$$D_{1T}^{\perp}(z_h, P_{h\perp}^2)\Big|_{\text{quark-gluon}} = \frac{\alpha_s}{2\pi^2}(2M_h^2)\frac{z_f^2}{[P_{h\perp}^2]^2}\left[A + C_F D_{1T}^{\perp(1)}(z_h)\left(\ln\frac{\hat{\zeta}^2}{z_f^2 q_T^2}\right)\right]$$

$$\begin{aligned} A \; = \; & \int \frac{dz}{z}\Bigg[\frac{D_T(z)}{z}\left(-C_F(1+2\hat{z}) - \frac{1}{2N_c}\frac{1+\hat{z}^2}{\hat{z}}\right) + \frac{d}{d(1/z)}\left(\frac{D_{1T}^{\perp(1)}}{z}\right)\left(-\frac{1}{2N_c}\frac{1+\hat{z}^2}{\hat{z}}\right) + D_{1T}^{\perp(1)}\left(C_F\frac{\hat{z}(1+\hat{z})}{(1-\hat{z})_+}\right) \\ & + \int d\left(\frac{1}{z'}\right)\frac{1/z}{1/z - 1/z'}\Big\{\Im\widehat{D}_{FT}(z,z')\Big[\frac{1}{1/z'}\left(\frac{1}{2N_c}\frac{2-\hat{z}}{\hat{z}}\right) + \frac{1}{1/z' - 1/z_f}[C_F + \frac{1}{2N_c}](1+\hat{z})\Big] \\ & + \Im\widehat{G}_{FT}(z,z')\Big[\frac{1}{1/z'}\left(\frac{1}{2N_c}\right) - \frac{1}{1/z' - 1/z_f}[C_F + \frac{1}{2N_c}](1-\hat{z})\Big]\Big\}\Bigg] \end{aligned}$$

Nonperturbative part is written only in terms of the collinear twist-3 functions

$D_T,\ D_{1T}^{\perp(1)},\ \widehat{D}_{FT},\ \widehat{G}_{FT}$

- TMD cross section in $ep \to \Lambda^{\uparrow} X$

$$\frac{d^6\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = -\sigma_0 \sin\Phi_s z_f^2 \int d^2k_\perp d^2p_\perp d^2\lambda_\perp \delta^2(k_\perp + p_\perp + \lambda_\perp - q_T)\frac{p_\perp^2}{q_T M_h} f(x_{bj}, k_\perp^2) D_{1T}^{\perp}(z_f, z_f^2 p_\perp^2) S^{-1}(\lambda_\perp^2)$$

$$\vec{p}_\perp, \vec{\lambda}_\perp \ll \vec{k}_\perp \simeq \vec{q}_T, \quad \vec{k}_\perp, \vec{\lambda}_\perp \ll \vec{p}_\perp \simeq \vec{q}_T, \quad \vec{k}_\perp, \vec{p}_\perp \ll \vec{\lambda}_\perp \simeq \vec{q}_T$$

$$\sim -2\sigma_0 \sin\Phi_s \frac{1}{q_T} M_h\left(f(x_{bj}, k_\perp^2) D_{1T}^{\perp(1)}(z_f) + \frac{P_{h\perp}^2}{2M_h^2} f(x_{bj}) D_{1T}^{\perp}(z_f, P_{h\perp}^2) + f(x_{bj}) D_{1T}^{\perp(1)}(z_f) S^{-1}(q_T^2)\right),$$

Result in TMD side

$$D_{1T}^{\perp}(z_h, P_{h\perp}^2)\Big|_{\text{quark-gluon}} = \frac{\alpha_s}{2\pi^2}(2M_h^2)\frac{z_f^2}{[P_{h\perp}^2]^2}\left[A + C_F D_{1T}^{\perp(1)}(z_h)\left(\ln\frac{\hat{\zeta}^2}{z_f^2 q_T^2}\right)\right]$$

$$A = \int \frac{dz}{z}\left[\frac{D_T(z)}{z}\left(-C_F(1+2\hat{z}) - \frac{1}{2N_c}\frac{1+\hat{z}^2}{\hat{z}}\right) + \frac{d}{d(1/z)}\left(\frac{D_{1T}^{\perp(1)}}{z}\right)\left(-\frac{1}{2N_c}\frac{1+\hat{z}^2}{\hat{z}}\right) + D_{1T}^{\perp(1)}\left(C_F\frac{\hat{z}(1+\hat{z})}{(1-\hat{z})_+}\right)\right.$$
$$+\int d\left(\frac{1}{z'}\right)\frac{1/z}{1/z - 1/z'}\left\{\Im\widehat{D}_{FT}(z,z')\left[\frac{1}{1/z'}\left(\frac{1}{2N_c}\frac{2-\hat{z}}{\hat{z}}\right) + \frac{1}{1/z' - 1/z_f}[C_F + \frac{1}{2N_c}](1+\hat{z})\right]\right.$$
$$\left.\left.+\Im\widehat{G}_{FT}(z,z')\left[\frac{1}{1/z'}\left(\frac{1}{2N_c}\right) - \frac{1}{1/z' - 1/z_f}[C_F + \frac{1}{2N_c}](1-\hat{z})\right]\right\}\right]$$

Nonperturbative part is written only in terms of the collinear twist-3 functions

$$D_T,\ D_{1T}^{\perp(1)},\ \widehat{D}_{FT},\ \widehat{G}_{FT}$$

○ TMD cross section in $ep \to \Lambda^\uparrow X$

$$\frac{d^6\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = -\sigma_0 \sin\Phi_s z_f^2 \int d^2k_\perp d^2p_\perp d^2\lambda_\perp \delta^2(k_\perp + p_\perp + \lambda_\perp - q_T)\frac{p_\perp^2}{q_T M_h} f(x_{bj}, k_\perp^2) D_{1T}^{\perp}(z_f, z_f^2 p_\perp^2) S^{-1}(\lambda_\perp^2)$$

$$\vec{p}_\perp, \vec{\lambda}_\perp \ll \vec{k}_\perp \simeq \vec{q}_T, \quad \vec{k}_\perp, \vec{\lambda}_\perp \ll \vec{p}_\perp \simeq \vec{q}_T, \quad \vec{k}_\perp, \vec{p}_\perp \ll \vec{\lambda}_\perp \simeq \vec{q}_T$$

$$\sim -2\sigma_0 \sin\Phi_s \frac{1}{q_T} M_h\left(f(x_{bj}, k_\perp^2) D_{1T}^{\perp(1)}(z_f) + \frac{P_{h\perp}^2}{2M_h^2} f(x_{bj}) D_{1T}^{\perp}(z_f, P_{h\perp}^2) + f(x_{bj}) D_{1T}^{\perp(1)}(z_f) S^{-1}(q_T^2)\right),$$

$$= -2\sigma_0 \sin\Phi_s\left(\frac{1}{q_T^3}\right)\frac{\alpha_s}{2\pi^2} M_h\left\{f(x_{bj})\Big[A\Big] + C_F D_{1T}^{\perp(1)}(z_f)\int\frac{dx}{x}\left(f(x)\frac{1+\hat{x}^2}{(1-\hat{x})_+} + \frac{1}{2}G(x)[\hat{x}^2 + (1-\hat{x})^2]\right)\right.$$
$$\left. + 2C_F f(x_{bj}) D_{1T}^{\perp(1)}(z_f)\ln\frac{Q^2}{q_T^2}\right\}$$

only in terms of the collinear functions

Result in TMD side

$$D_{1T}^{\perp}(z_h, P_{h\perp}^2)\Big|_{\rm quark-gluon} = \frac{\alpha_s}{2\pi^2}(2M_h^2)\frac{z_f^2}{[P_{h\perp}^2]^2}\left[A + C_F D_{1T}^{\perp(1)}(z_h)\left(\ln\frac{\hat{\zeta}^2}{z_f^2 q_T^2}\right)\right]$$

$$A = \int\frac{dz}{z}\left[\frac{D_T(z)}{z}\left(-C_F(1+2\hat{z}) - \frac{1}{2N_c}\frac{1+\hat{z}^2}{\hat{z}}\right) + \frac{d}{d(1/z)}\left(\frac{D_{1T}^{\perp(1)}}{z}\right)\left(-\frac{1}{2N_c}\frac{1+\hat{z}^2}{\hat{z}}\right) + D_{1T}^{\perp(1)}\left(C_F\frac{\hat{z}(1+\hat{z})}{(1-\hat{z})_+}\right)\right.$$
$$+\int d\left(\frac{1}{z'}\right)\frac{1/z}{1/z-1/z'}\left\{\Im\widehat{D}_{FT}(z,z')\left[\frac{1}{1/z'}\left(\frac{1}{2N_c}\frac{2-\hat{z}}{\hat{z}}\right) + \frac{1}{1/z'-1/z_f}[C_F + \frac{1}{2N_c}](1+\hat{z})\right]\right.$$
$$\left.\left. +\Im\widehat{G}_{FT}(z,z')\left[\frac{1}{1/z'}\left(\frac{1}{2N_c}\right) - \frac{1}{1/z'-1/z_f}[C_F + \frac{1}{2N_c}](1-\hat{z})\right]\right\}\right]$$

Nonperturbative part is written only in terms of the collinear twist-3 functions

$D_T,\ D_{1T}^{\perp(1)},\ \widehat{D}_{FT},\ \widehat{G}_{FT}$

- TMD cross section in $ep \to \Lambda^\uparrow X$

$$\frac{d^6\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = -\sigma_0 \sin\Phi_s z_f^2 \int d^2k_\perp d^2p_\perp d^2\lambda_\perp \delta^2(k_\perp + p_\perp + \lambda_\perp - q_T)\frac{p_\perp^2}{q_T M_h} f(x_{bj}, k_\perp^2) D_{1T}^{\perp}(z_f, z_f^2 p_\perp^2) S^{-1}(\lambda_\perp^2)$$

$$\vec{p}_\perp, \vec{\lambda}_\perp \ll \vec{k}_\perp \simeq \vec{q}_T, \quad \vec{k}_\perp, \vec{\lambda}_\perp \ll \vec{p}_\perp \simeq \vec{q}_T, \quad \vec{k}_\perp, \vec{p}_\perp \ll \vec{\lambda}_\perp \simeq \vec{q}_T$$

$$\sim -2\sigma_0 \sin\Phi_s \frac{1}{q_T} M_h\left(f(x_{bj}, k_\perp^2) D_{1T}^{\perp(1)}(z_f) + \frac{P_{h\perp}^2}{2M_h^2} f(x_{bj}) D_{1T}^{\perp}(z_f, P_{h\perp}^2) + f(x_{bj}) D_{1T}^{\perp(1)}(z_f) S^{-1}(q_T^2)\right),$$

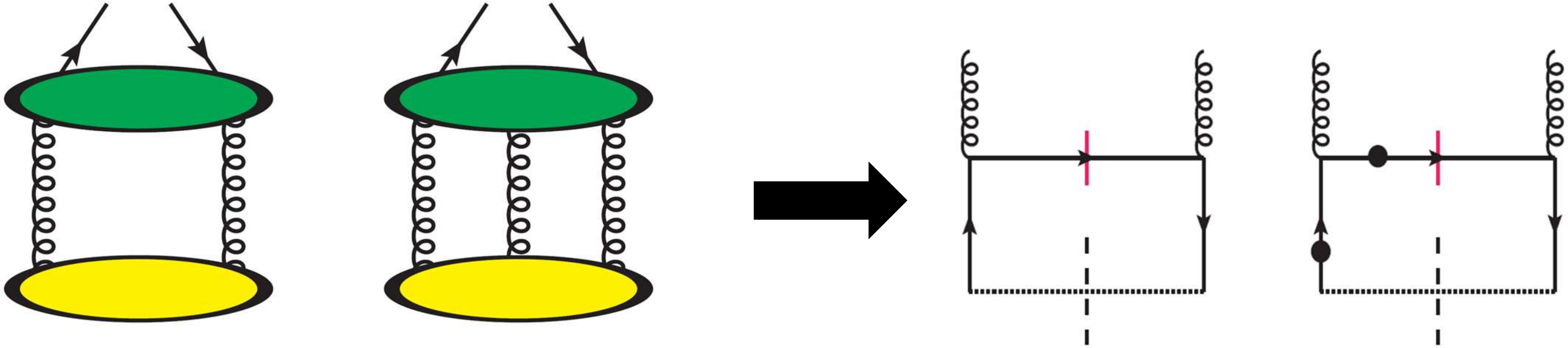
$$= -2\sigma_0 \sin\Phi_s\left(\frac{1}{q_T^3}\right)\frac{\alpha_s}{2\pi^2} M_h\left\{f(x_{bj})\left[A\right] + C_F D_{1T}^{\perp(1)}(z_f)\int\frac{dx}{x}\left(f(x)\frac{1+\hat{x}^2}{(1-\hat{x})_+} + \frac{1}{2}G(x)[\hat{x}^2 + (1-\hat{x})^2]\right)\right.$$
$$\left. +2C_F f(x_{bj}) D_{1T}^{\perp(1)}(z_f)\ln\frac{Q^2}{q_T^2}\right\}$$

only in terms of the collinear functions

What is the relation with the collinear twist-3 result ?

· gluon mixing

R. Ikarashi, Y. Koike, K. Yabe and SY, PRD106 (2022)



$$D_{1T}^{\perp}(z_f, p_\perp'^2)\Big|_{3-\text{gluon}} = -\frac{\alpha_s}{2\pi^2}(M_h^2)\frac{z_f^2}{[p_\perp^2]^2}\Big[B\Big]$$

$$\begin{aligned}
B \;=\; & \int dz\, z\Big[\hat{G}_T^{(1)}(z)\sigma_1 + \Delta\hat{H}_T^{(1)}(z)\sigma_2 \\
& + \int d(\frac{1}{z'})\frac{1}{1/z-1/z'}\Im\Big(\hat{N}_1(\frac{1}{z'},\frac{1}{z})\sigma_{N1} + \hat{N}_2(\frac{1}{z'},\frac{1}{z})\sigma_{N2} + \hat{N}_2(\frac{1}{z}-\frac{1}{z'},\frac{1}{z})\sigma_{N3}\Big) \\
& + \int d(\frac{1}{z'})\frac{1/z}{(1/z-1/z')^2}\Im\Big(\hat{N}_1(\frac{1}{z'},\frac{1}{z})\sigma_{DN1} + \hat{N}_2(\frac{1}{z'},\frac{1}{z})\sigma_{DN1} + \hat{N}_2(\frac{1}{z}-\frac{1}{z'},\frac{1}{z})\sigma_{DN3}\Big) \\
& + \int d(\frac{1}{z'})\frac{1}{1/z-1/z'}\Im\Big(\hat{O}_1(\frac{1}{z'},\frac{1}{z})\sigma_{O1} + \hat{O}_2(\frac{1}{z'},\frac{1}{z})\sigma_{O2} + \hat{O}_2(\frac{1}{z}-\frac{1}{z'},\frac{1}{z})\sigma_{O2}\Big) \\
& + \int d(\frac{1}{z'})\frac{1/z}{(1/z-1/z')^2}\Im\Big(\hat{O}_1(\frac{1}{z'},\frac{1}{z})\sigma_{DN1} + \hat{O}_2(\frac{1}{z'},\frac{1}{z})\sigma_{DN1} + \hat{O}_2(\frac{1}{z}-\frac{1}{z'},\frac{1}{z})(-\sigma_{DN3})\Big) \\
& + \frac{1}{C_F}\int d(\frac{1}{z'})\Big\{\Im\hat{D}_{FT}(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z})\Big(-\frac{1}{2}\sigma_{N1} + \frac{1/z}{1/z-1/z'}\sigma_{DF2} + \frac{1/z}{1/z'-1/z_f}\sigma_{DF4}\Big) \\
& + \Im\hat{G}_{FT}(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z})\Big(\frac{1/z}{1/z-1/z'}\sigma_{GF2} + \frac{1/z}{1/z'-1/z_f}\sigma_{GF4}\Big)\Big\}\Big]
\end{aligned}$$

Collinear twist-3 side

Y. Koike, K. Takada, S. Usui, K. Yabe and SY, PRD105 (2022)

$$\frac{d^6\Delta\sigma^{\rm tw3-frag}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2\alpha_s(-M_h)}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2}\sum_{k=1}^{9}\mathcal{A}_k(\phi-\chi)\mathcal{S}_k\int\frac{dx}{x}f_1(x)\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right) \times\left[\frac{D_T(z)}{z}\hat{\sigma}_T^k-\left\{\frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\right\}\hat{\sigma}_{\perp D}^k - D_{1T}^{\perp(1)}(z)\hat{\sigma}_{\perp}^k\right.$$

$$+\int\frac{dz'}{z'^2}\mathrm{P}\left(\frac{1}{1/z-1/z'}\right)\left\{\Im\widehat{D}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{DF1}^k+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)\,z_f}\right)^{-1}\hat{\sigma}_{DF2}^k\right]\right.$$

$$\left.\left.+\Im\widehat{G}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{GF1}^k+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)\,z_f}\right)^{-1}\hat{\sigma}_{GF2}^k\right]\right\}\right].$$

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$$+\int\frac{dz'}{z'^2}{\rm P}\left(\frac{1}{1/z-1/z'}\right)\left\{\Im\widehat{D}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{DF1}^k+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)\,z_f}\right)^{-1}\hat{\sigma}_{DF2}^k\right]\right.$$

$$\left.\left.+\Im\widehat{G}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{GF1}^k+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)\,z_f}\right)^{-1}\hat{\sigma}_{GF2}^k\right]\right\}\right].$$

84 hard cross sections

$q\gamma \to q$

$q\gamma \to g$

Collinear twist-3 side

Y. Koike, K. Takada, S. Usui, K. Yabe and SY, PRD105 (2022)

$$\frac{d^6\Delta\sigma^{\rm tw3-frag}}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi} = \frac{\alpha_{em}^2\alpha_s(-M_h)}{16\pi^2x_{bj}^2S_{ep}^2Q^2}\sum_{k=1}^{9}\mathcal{A}_k(\phi-\chi)\mathcal{S}_k\int\frac{dx}{x}f_1(x)\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$

$$\times\left[\frac{D_T(z)}{z}\hat{\sigma}_T^k-\left\{\frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\right\}\hat{\sigma}_{\perp D}^k-D_{1T}^{\perp(1)}(z)\hat{\sigma}_\perp^k\right.$$
$$+\int\frac{dz'}{z'^2}\mathrm{P}\left(\frac{1}{1/z-1/z'}\right)\left\{\Im\widehat{D}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{DF1}^k+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)\,z_f}\right)^{-1}\hat{\sigma}_{DF2}^k\right]\right.$$
$$\left.\left.+\Im\widehat{G}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{GF1}^k+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)\,z_f}\right)^{-1}\hat{\sigma}_{GF2}^k\right]\right\}\right].$$

84 hard cross sections

$q\gamma\to q$

$q\gamma\to g$

$$\hat{\sigma}_1^1=4\frac{Q^2}{q_T^3}(1-\hat{z})\frac{1-9\hat{x}+14\hat{x}^2-6\hat{x}^3+2\hat{z}^2(1-6\hat{x}+6\hat{x}^2)+\hat{z}(-4+23\hat{x}-30\hat{x}^2+12\hat{x}^3)}{\hat{x}\hat{z}(1-\hat{x}-\hat{z})}$$

Collinear twist-3 side

Y. Koike, K. Takada, S. Usui, K. Yabe and SY, PRD105 (2022)

$$\frac{d^6\Delta\sigma^{\rm tw3-frag}}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi} = \frac{\alpha_{em}^2\alpha_s(-M_h)}{16\pi^2x_{bj}^2S_{ep}^2Q^2}\sum_{k=1}^{9}\mathcal{A}_k(\phi-\chi)\mathcal{S}_k\int\frac{dx}{x}f_1(x)\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$

$$\times\left[\frac{D_T(z)}{z}\hat{\sigma}_T^k-\left\{\frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\right\}\hat{\sigma}_{\perp D}^k-D_{1T}^{\perp(1)}(z)\hat{\sigma}_\perp^k\right.$$
$$+\int\frac{dz'}{z'^2}\mathrm{P}\left(\frac{1}{1/z-1/z'}\right)\left\{\Im\widehat{D}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{DF1}^k+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)\,z_f}\right)^{-1}\hat{\sigma}_{DF2}^k\right]\right.$$
$$\left.\left.+\Im\widehat{G}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{GF1}^k+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)\,z_f}\right)^{-1}\hat{\sigma}_{GF2}^k\right]\right\}\right].$$

84 hard cross sections

$q\gamma \to q$

$q\gamma \to g$

$q_T^2 \ll Q^2$

X. Ji, J.-W. Qiu, W.Vogelsang and F. Yuan, PRD73(2006) PLB638(2006)

$$\hat{x}\hat{z}\Big[\frac{1}{(1-\hat{z})_+}\delta(1-\hat{x}) \quad +\frac{1}{(1-\hat{x})_+}\delta(1-\hat{z}) \quad +\delta(1-\hat{x})\delta(1-\hat{z})\ln\frac{Q^2}{q_T^2}\Big]$$

$$\hat{\sigma}_1^1 = 4\frac{Q^2}{q_T^3}(1-\hat{z})\frac{1-9\hat{x}+14\hat{x}^2-6\hat{x}^3+2\hat{z}^2(1-6\hat{x}+6\hat{x}^2)+\hat{z}(-4+23\hat{x}-30\hat{x}^2+12\hat{x}^3)}{\hat{x}\hat{z}(1-\hat{x}-\hat{z})}$$

Collinear twist-3 side

Y. Koike, K. Takada, S. Usui, K. Yabe and SY, PRD105 (2022)

$$\frac{d^6\Delta\sigma^{\rm tw3-frag}}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi} = \frac{\alpha_{em}^2\alpha_s(-M_h)}{16\pi^2x_{bj}^2S_{ep}^2Q^2}\sum_{k=1}^{9}\mathcal{A}_k(\phi-\chi)\mathcal{S}_k\int\frac{dx}{x}f_1(x)\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$

$$\times\Bigg[\frac{D_T(z)}{z}\hat{\sigma}_T^k-\left\{\frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\right\}\hat{\sigma}_{\perp D}^k-D_{1T}^{\perp(1)}(z)\hat{\sigma}_\perp^k$$
$$+\int\frac{dz'}{z'^2}\mathrm{P}\left(\frac{1}{1/z-1/z'}\right)\Bigg\{\Im\widehat{D}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{DF1}^k+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)\,z_f}\right)^{-1}\hat{\sigma}_{DF2}^k\right]$$
$$+\Im\widehat{G}_{FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}_{GF1}^k+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)\,z_f}\right)^{-1}\hat{\sigma}_{GF2}^k\right]\Bigg\}\Bigg].$$

84 hard cross sections

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X. Ji, J.-W. Qiu, W.Vogelsang and F. Yuan, PRD73(2006)
PLB638(2006)

$q_T^2 \ll Q^2$

$$\hat{x}\hat{z}\left[\frac{1}{(1-\hat{z})_+}\delta(1-\hat{x}) \quad +\frac{1}{(1-\hat{x})_+}\delta(1-\hat{z}) \quad +\delta(1-\hat{x})\delta(1-\hat{z})\ln\frac{Q^2}{q_T^2}\right]$$

$$\hat{\sigma}_1^1 = -4C_F\frac{Q^2}{q_T^3}(1+2\hat{z}) \qquad 0 \qquad 0$$

$$\hat{\sigma}_1^1 = 4\frac{Q^2}{q_T^3}(1-\hat{z})\frac{1-9\hat{x}+14\hat{x}^2-6\hat{x}^3+2\hat{z}^2(1-6\hat{x}+6\hat{x}^2)+\hat{z}(-4+23\hat{x}-30\hat{x}^2+12\hat{x}^3)}{\hat{x}\hat{z}(1-\hat{x}-\hat{z})}$$

$$\frac{d^6\Delta\sigma^{\rm tw3-frag}}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi}$$

$$= -2\sigma_0 \sin\Phi_s\left(\frac{1}{q_T^3}\right)\frac{\alpha_s}{2\pi^2}M_h\Big\{f(x_{bj})\Big[A\Big] + C_F D_{1T}^{\perp(1)}(z_f)\int\frac{dx}{x}\Big(f(x)\frac{1+\hat{x}^2}{(1-\hat{x})_+} + \frac{1}{2}G(x)[\hat{x}^2+(1-\hat{x})^2]\Big)$$

$$+2C_F f(x_{bj})D_{1T}^{\perp(1)}(z_f)\ln\frac{Q^2}{q_T^2}\Big\}$$

$$A = \int\frac{dz}{z}\Big[\frac{D_T(z)}{z}\Big(-C_F(1+2\hat{z}) - \frac{1}{2N_c}\frac{1+\hat{z}^2}{\hat{z}}\Big) + \frac{d}{d(1/z)}\Big(\frac{D_{1T}^{\perp(1)}}{z}\Big)\Big(-\frac{1}{2N_c}\frac{1+\hat{z}^2}{\hat{z}}\Big) + D_{1T}^{\perp(1)}\Big(C_F\frac{\hat{z}(1+\hat{z})}{(1-\hat{z})_+}\Big)$$

$$+\int d(\frac{1}{z'})\frac{1/z}{1/z-1/z'}\Big\{\Im\widehat{D}_{FT}(z,z')\Big[\frac{1}{1/z'}\Big(\frac{1}{2N_c}\frac{2-\hat{z}}{\hat{z}}\Big) + \frac{1}{1/z'-1/z_f}[C_F+\frac{1}{2N_c}](1+\hat{z})\Big]$$

$$+\Im\widehat{G}_{FT}(z,z')\Big[\frac{1}{1/z'}\Big(\frac{1}{2N_c}\Big) - \frac{1}{1/z'-1/z_f}[C_F+\frac{1}{2N_c}](1-\hat{z})\Big]\Big\}\Big]$$

Consistent with the TMD result !

∘ twist-3 gluon fragmentation effect

R. Ikarashi, Y. Koike, K. Yabe and SY, PRD105 (2022)

$g\gamma \to q$

$$\frac{d^6\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi}$$
$$= \frac{\alpha_{em}^2 \alpha_s M_h}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2} \sum_k \mathscr{A}_k(\phi-\chi)\mathcal{S}_k \int_{x_{min}}^1 \frac{dx}{x} \int_{z_{min}}^1 \frac{dz}{z} z^2 f_1(x) \delta\left(\frac{q_T^2}{Q^2} - \left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)$$
$$\times \Big\{ \widehat{G}_T^{(1)}(z)\hat{\sigma}_G^k + \Delta\widehat{H}_T^{(1)}(z)\hat{\sigma}_H^k$$
$$+ \int d\left(\frac{1}{z'}\right) \left[\frac{1}{1/z-1/z'} \Im\left(\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N1}^k + \widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N2}^k + \widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N3}^k\right)\right.$$
$$+ \frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2 \Im\left(\widehat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN1}^k + \widehat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN2}^k + \widehat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN3}^k\right)$$
$$+ \frac{1}{1/z-1/z'} \Im\left(\widehat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{O1}^k + \widehat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{O2}^k + \widehat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{O3}^k\right)$$
$$\left.+ \frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2 \Im\left(\widehat{O}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DO1}^k + \widehat{O}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DO2}^k + \widehat{O}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DO3}^k\right)\right]$$
$$+ \int d\left(\frac{1}{z'}\right) \frac{2}{C_F} \left[\Im\widetilde{D}_{FT}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left(\hat{\sigma}_{DF1}^k + \frac{1}{z}\frac{1}{1/z-1/z'}\hat{\sigma}_{DF2}^k + \frac{z'}{z}\hat{\sigma}_{DF3}^k\right.\right.$$
$$\left.+ \frac{1}{1-(1-q_T^2/Q^2)z_f/z'}\hat{\sigma}_{DF4}^k + \frac{1}{1-(1-q_T^2/Q^2)z_f(1/z-1/z')}\hat{\sigma}_{DF5}^k\right)$$
$$+ \Im\widetilde{G}_{FT}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\left(\frac{1}{z}\frac{1}{1/z-1/z'}\hat{\sigma}_{GF2}^k + \frac{z'}{z}\hat{\sigma}_{GF3}^k\right.$$
$$\left.\left.+ \frac{1}{1-(1-q_T^2/Q^2)z_f/z'}\hat{\sigma}_{GF4}^k + \frac{1}{1-(1-q_T^2/Q^2)z_f(1/z-1/z')}\hat{\sigma}_{GF5}^k\right)\right]\Big\}$$

➡

$$d\sigma = \sigma_0 \sin\Phi_s \frac{\alpha_s}{2\pi^2} f(x_{bj})\left[B\right]$$

Consistent with the gluon-mixing term in TMD !

Summary

- The twist-3 cross section for the SSA in $\Lambda^{\uparrow}$ production was completed very recent at the lowest order

- We calculated the one-loop corrections to the TMD operator and found the relation with the collinear twist-3 functions

- Using this relation, we confirmed that the TMD and the collinear twist-3 give the consistent results in $\Lambda_{QCD} \ll P_T \ll Q$

- The consistency will play a role in future phenomenological studies as that in the pion production has done

Back up

pQCD frameworks for the SSA

Two possible ways are known as extensional pQCD frameworks

Conventional parton model $\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p|\bar{\psi}_j(0)\psi(\lambda n)|p\rangle = i(\gamma^5 \sigma^{\alpha\beta} P_\beta)_{ij} \times S_{\perp\alpha} \times h_1(x)$

Chiral-odd

The parent proton is transversely polarized

Transverse Momentum Dependent(TMD) factorization

$$\int \frac{d\lambda}{2\pi} \int \frac{d^2\xi_\perp}{2\pi} e^{i\lambda x} e^{-i\vec{\xi}_\perp \cdot \vec{k}_T} \langle p|\bar{\psi}_j(0)\psi(\lambda n, x_\perp)|p\rangle = \frac{1}{2} f(x, k_T)(\not{p})_{ij} + \frac{1}{2M_N}(\gamma_\alpha)_{ji} \epsilon^{\alpha\nu\rho\sigma} P_\nu k_{T\rho} S_{\perp\sigma} f_{1T}^\perp(x, k_T) \cdots$$

Chiral-even!

· Applicable in small $P_T (Q \gg P_T \geq \Lambda_{QCD})$ region $\quad \frac{m_q}{P_T} \to \frac{\Lambda_{\rm QCD}}{P_T}$

Twist-3 mechanism in the collinear factorization

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x} e^{i\mu(x_2 - x_1)} \langle p|\bar{\psi}_j(0) g F^{\alpha+}(\mu n)\psi(\lambda n)|p\rangle = -\frac{M_N}{2} \epsilon^{\alpha\beta-+} (\not{p})_{ij} S_{\perp\beta} F_{FT}(x_1, x_2) + \cdots$$

Chiral-even!

· Applicable in large $P_T (P_T \gg \Lambda_{QCD})$ region

There is overlapping region $\Lambda_{\rm QCD} \ll P_T \ll Q$

There should not be a quantitative ambiguity in this region
if both are the first principle calculations

$$\frac{d^6\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = -\sigma_0 \sin\Phi_s z_f^2 \int d^2k_\perp d^2p_\perp d^2\lambda_\perp \delta^2(k_\perp + p_\perp + \lambda_\perp - q_T)\frac{p_\perp^2}{q_T M_h} f(x_{bj}, k_\perp^2) D_{1T}^\perp(z_f, z_f^2 p_\perp^2) S^{-1}(\lambda_\perp^2)$$

$$\vec{p}_\perp, \vec{\lambda}_\perp \ll \vec{k}_\perp \simeq \vec{q}_T, \quad \vec{k}_\perp, \vec{\lambda}_\perp \ll \vec{p}_\perp \simeq \vec{q}_T, \quad \vec{k}_\perp, \vec{p}_\perp \ll \vec{\lambda}_\perp \simeq \vec{q}_T$$

$$\sim -2\sigma_0 \sin\Phi_s \frac{1}{q_T} M_h \Big(f(x_{bj}, k_\perp^2) D_{1T}^{\perp(1)}(z_f) + \frac{P_{h\perp}^2}{2M_h^2} f(x_{bj}) D_{1T}^\perp(z_f, P_{h\perp}^2) + f(x_{bj}) D_{1T}^{\perp(1)}(z_f) S^{-1}(q_T^2) \Big),$$

X. Ji, J.-W. Qiu, W.Vogelsang and F. Yuan, PRD73(2006)
PLB638(2006)

$$f(x_{bj}, q_T^2) = \frac{\alpha_s}{2\pi^2}\frac{1}{q_T^2} C_F \int \frac{dx}{x} f(x) \Big[\frac{1+\hat{x}^2}{(1-\hat{x})_+} + \delta(1-\hat{x}) \Big(\ln \frac{x_{bj}^2 \zeta^2}{q_T^2} - 1 \Big) \Big]$$

$$S^{-1}(q_T^2) = -\frac{\alpha_s}{2\pi^2}\frac{1}{q_T^2} C_F (\ln \rho^2 - 1), \quad \ln \frac{x_{bj}^2 \zeta^2 \hat{\zeta}^2}{z_f^2 \rho^2} = \ln Q^4$$

Wilson line

F. Yuan and J. Zhou, PRL 103 (2009)

$$D_{1T}^{\perp}(z_f, p_\perp) = \frac{M_h}{z_f} \frac{1}{(-4\epsilon^{p_\perp P_h w S_\perp})} \sum_X \int \frac{d\xi^+}{2\pi} \int \frac{d^2\xi_\perp}{(2\pi)^2} e^{-ip^-\xi^+\frac{1}{z}} e^{i\vec{p}_\perp\cdot\xi_\perp} \langle 0|\mathcal{L}_0 \psi(0)|P_h\, X\rangle \langle P_h\, X|\psi(\xi^+, \xi_\perp)\mathcal{L}_\xi^\dagger|0\rangle$$

$$\mathcal{L}_\xi = \exp\Big(-ig\int_0^\infty d\lambda\, v\cdot A(\lambda v + \xi)\Big)$$