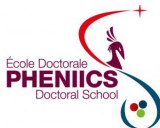


# Sensitivity of DDVCS observables to GPDs

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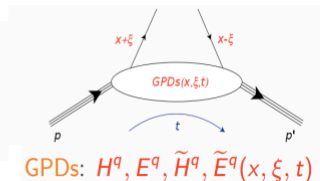
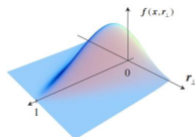
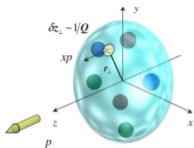
- 1 Introduction**
  - GPDs
  - Exclusive leptonproduction reactions
  - DDVCS experimental observables
  - Motivation
- 2 Sensitivity @ JLab kinematics**
- 3 Sensitivity @ EIC kinematics**
- 4 Conclusions**

# GPDs

- At low energy QCD, we need structure functions to describe the nucleon structure.
- Generalized Parton Distributions (GPDs)** correlate the transverse position and longitudinal momentum of partons in the nucleon.
- They enter the cross-section of exclusive inelastic scattering processes through **Compton Form Factors (CFFs)**.

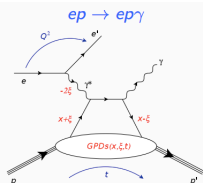
$$\mathcal{F}(\xi, t) \equiv \mathcal{P} \int_{-1}^1 dx F(x, \xi, t) \left( \frac{1}{x - \xi + i\epsilon} \pm \frac{1}{x + \xi + i\epsilon} \right).$$

- For a spin 1/2 particle there are four chiral-even GPDs:  $F = H, E, \tilde{H}, \tilde{E}$ .



# Exclusive leptonproduction reactions

Two golden channels for GPD measurements are **DVCS** and **DDVCS** (See K. Deja, V. Martinez-Fernandez et al. Phys. Rev. D 107.9 (2023), p. 094035.).



Deeply Virtual Compton Scattering (DVCS)

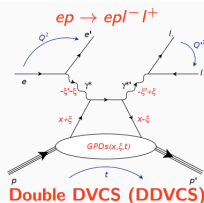
$$\mathcal{H}(\xi, \xi, t) = \sum_q e_q^2 \left\{ \mathcal{P} \int_{-1}^1 dx H^q(x, \xi, t) \left[ \frac{1}{x - \xi} + \frac{1}{x + \xi} \right] - i\pi \left[ H^q(\xi, \xi, t) - H^q(-\xi, \xi, t) \right] \right\}$$

## Pros:

- Easier to access experimentally.
- Direct GPD measurement from  $\Im(\text{CFF})$ .

## Cons:

- GPD measurements only at  $x = \pm \xi$ .



Double DVCS (DDVCS)

$$\mathcal{H}(\xi', \xi, t) = \sum_q e_q^2 \left\{ \mathcal{P} \int_{-1}^1 dx H^q(x, \xi, t) \left[ \frac{1}{x - \xi'} + \frac{1}{x + \xi'} \right] - i\pi \left[ H^q(\xi', \xi, t) - H^q(-\xi', \xi, t) \right] \right\}$$

## Pros:

- GPD measurement at independent  $x = \xi'$  and  $\xi$  values.
- Generalizes the results from DVCS and TCS. **See Victor's talk on Wednesday!**

## Cons:

- Need for a muon detector.
- Smaller cross section.

# DDVCS experimental observables

- We consider a muon pair in the final state, polarized electron/positron beams and a polarized **proton target**.

- At JLab (12 GeV beam energy):
  - ✓ A muon detection is planned (SoLID collaboration) [1].
  - ✓ A positron beam is planned (PEPPo, Ce<sup>+</sup>BAF and JLab positron working group) [2–4].
- At EIC (140 GeV CoM energy):
  - ✓ Electron and positron beams might be expected [5].
  - ✓ Muon detection might be possible.

- Given the notation  $\mathcal{O}_{\text{Beam pol, Target pol}}$ , let us define the following observables:

- Beam Spin Asymmetry (BSA)  $A_{LU} = \frac{d^5\sigma_{+U} - d^5\sigma_{-U}}{d^5\sigma_{UU}}$ .
- Target Spin Asymmetry (TSA)  $A_{UL} = \frac{d^5\sigma_{U+} - d^5\sigma_{U-}}{d^5\sigma_{UU}}$ .
- Double Spin Asymmetry (DSA)  $A_{LL} = \frac{(d^5\sigma_{++} - d^5\sigma_{+-}) - (d^5\sigma_{-+} - d^5\sigma_{--})}{d^5\sigma_{UU}}$ .
- Unpolarized Beam Charge Spin Asymmetry (BCA)  $A_{UU}^C = \frac{d^5\sigma_{UU}^+ - d^5\sigma_{UU}^-}{d^5\sigma_{UU}^+ + d^5\sigma_{UU}^-}$ .

## DDVCS experimental observables

Considering the following asymmetries<sup>a</sup> both for JLab and EIC kinematics<sup>b</sup>.

- $A_{LU} \propto \sin(\phi) \Im \left( (F_1 \mathcal{H} - kF_2 \mathcal{E}) + \xi' (F_1 + F_2) \tilde{\mathcal{H}} \right)$
- $A_{UU}^C \propto \cos(\phi) \Re \left( \frac{\xi'}{\xi} (F_1 \mathcal{H} - kF_2 \mathcal{E}) + \xi (F_1 + F_2) \tilde{\mathcal{H}} \right)$
- $A_{UL} \propto \sin(\phi) \Im \left( F_1 \tilde{\mathcal{H}} + \xi' (F_1 + F_2) \left( \mathcal{H} + \frac{\xi}{1+\xi} \mathcal{E} \right) - \xi \left( \frac{\xi}{1+\xi} F_1 + kF_2 \right) \tilde{\mathcal{E}} \right)$
- $A_{LL} \propto A + B \cos(\phi)$ 
  - $A \propto \Re \left( \xi (F_1 + F_2) \left( \mathcal{H} + \frac{\xi}{1+\xi} \mathcal{E} \right) + F_1 \tilde{\mathcal{H}} - \xi \left( \frac{\xi}{1+\xi} F_1 + kF_2 \right) \tilde{\mathcal{E}} \right)$
  - $B \propto \Re \left( \xi (F_1 + F_2) \left( \mathcal{H} + \frac{\xi}{1+\xi} \mathcal{E} \right) + \frac{\xi'}{\xi} F_1 \tilde{\mathcal{H}} - \xi' \left( \frac{\xi}{1+\xi} F_1 + kF_2 \right) \tilde{\mathcal{E}} \right)$

- 
- $A_{LU}$  and  $A_{UU}^C$  are GPD  $\mathcal{H}$  dominated.
  - $A_{LU}$  and  $A_{LL}$  are GPD  $\tilde{\mathcal{H}}$  dominated.
  - Due to the  $\xi'$  dependence, the coefficients in  $A_{LL}$  no longer have the same CFF dependence. Likewise for  $A_{LU}$  and  $A_{UU}^C$ .

<sup>a</sup>Considering the cross-section integrated over muon angles

<sup>b</sup>A. V. Belitsky et al. In: Physical Review D 68.11 (2003), p. 116005

# Motivation

**To model GPDs, the models used for evaluations are:**

- VGG: Orsay's code [6–9].
- GK19: Latest model from PARTONS [10–12]
- KM10, KM15: Models from Gepard<sup>a</sup> [13, 14]
- AFKM12: KM model adaptation for EIC kinematics<sup>b</sup>, also in Gepard.

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**The main goal of this study is to:**

- Quantify the GPD dependence of the DDVCS observables within a reasonable kinematic window.
- Look for kinematic regions where models can be discriminated.
- Study the feasibility of the observable measurements considering JLab and EIC experimental configurations.

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<sup>a</sup>Homemade adapted for DDVCS computations.

<sup>b</sup>E. Aschenauer et al. Journal of High Energy Physics 2013.9 (2013), pp. 1–59

# Sensitivity @ JLab kinematics

It was studied in the LOI12-16-004 (S. Stepanyan et al.) the kinematic reach of DDVCS with CLAS12

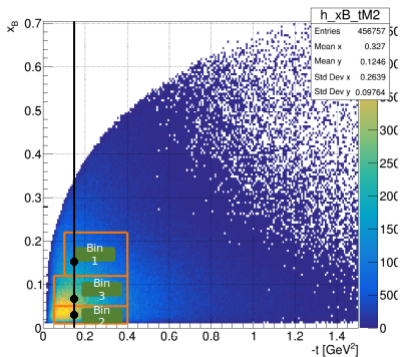


Figure:  $x_B$  vs  $t$ .

We perform a  $(Q^2, Q'^2)$  exploration at:

- $t = -0.15 \text{ GeV}^2$ .
- $x_B = 0.15, 0.07, 0.03$ .

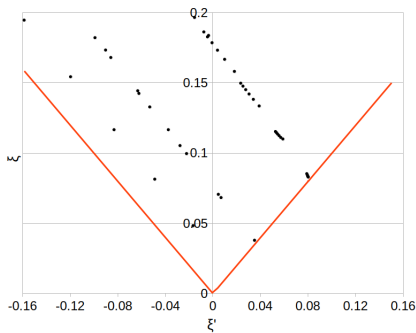


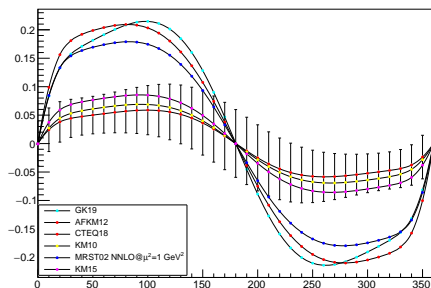
Figure:  $\xi$  vs  $\xi'$

- Assuming  $\mathcal{L} = 10^{37} \text{ cm}^{-2} \text{ s}^{-1}$
- 5% of combined reconstruction and acceptance efficiency
- 100 days of beam time.

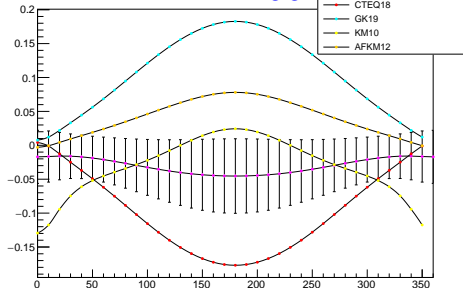


## Sensitivity @ JLab kinematics

$$x_B = 0.15, Q^2 = 2.77 \text{ GeV}^2, Q'^2 = 1.0 \text{ GeV}^2$$

 $A_{LU}$ 


GPD  $H$  dominated

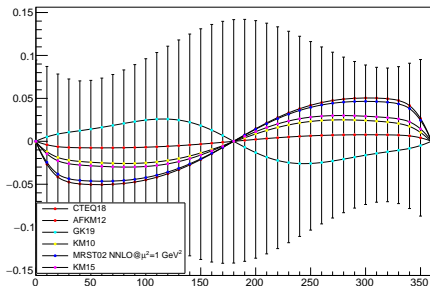
 $A_{UU}^C$ 


- ✓ Measurements are possible under 100 days of beam time.
- ✓  $A_{LU}$  is sensitive to models. VGG & GK19 models vs KM ones.
- ✓  $A_{UU}^C$  is sensitive to models. GK19 vs the others.
- ✓ VGG & KM15 models predict a different sign for  $A_{UU}^C$ .

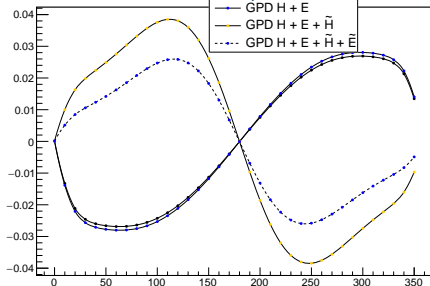
## Sensitivity @ JLab kinematics

$$x_B = 0.15, Q^2 = 2.77 \text{ GeV}^2, Q'^2 = 1.0 \text{ GeV}^2$$

$A_{UL}$



GPD  $\tilde{H}$  dominated sensitive to  $\tilde{E}$ .

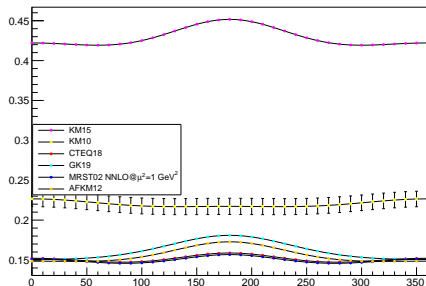


- ✗ Small amplitudes expected.
- ✗ Require more than 100 days of beam time.

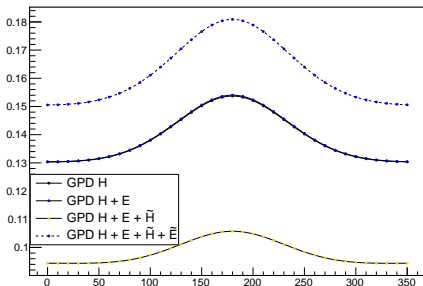
## Sensitivity @ JLab kinematics

$$x_B = 0.07, Q^2 = 0.8 \text{ GeV}^2, Q'^2 = 2.4 \text{ GeV}^2$$

$A_{LL}$



Sensitive to GPD  $H$ ,  $\tilde{H}$  and  $\tilde{E}$

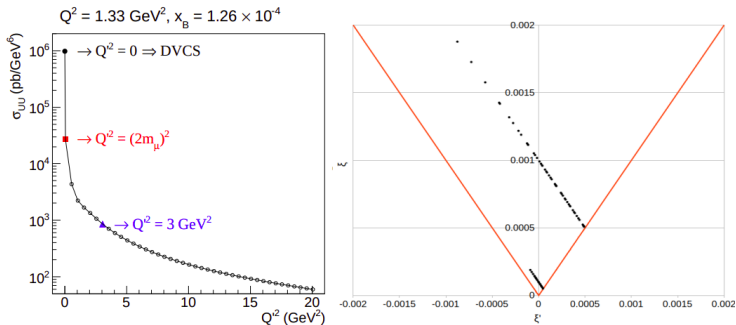


- ✓ Provides big asymmetry values.
- ✓ Measurable under 100 days with very good precision.
- ✗ Only not sensitive to GPD  $E$ .
- ✓ Sensitive to models. KM15 & KM10 vs the other models.

## Sensitivity @ EIC kinematics

The cross-section drops quickly with  $Q'^2$  [15, 16], then we explore on:

- $0.5 < Q^2(\text{GeV}^2) < 3$ , steps of 0.5
  - $4m_\mu^2 < Q^2(\text{GeV}^2) < 3$ , steps of 0.3
  - $t = -0.15 \text{ GeV}^2$ .
  - $x_B = 10^{-3}, 10^{-4}$ .
- Assuming  $\mathcal{L} = 10 \text{ fb}^{-1} \text{ year}^{-1}$
  - 5% of combined reconstruction and acceptance efficiency
  - 1 effective year of beam time.



**Figure:** **Left:** unpolarized cross-section as a function of  $Q^2$ . **Right:**  $\xi$  vs  $\xi'$  explored phase space. Smaller  $x_B$  regions corresponds to lines closer to the vertex.

## Sensitivity @ EIC kinematics

As  $x_B \rightarrow 0$ ,  $\xi$ ,  $\xi' \rightarrow 0$  as well. Then at LO in  $\xi$  and  $\xi'$  we get:

- $A_{LU} \propto \sin(\phi) \Im (F_1 \mathcal{H} - k F_2 \mathcal{E})$
- $A_{UU}^C \propto \cos(\phi) \Re \left( \frac{\xi'}{\xi} (F_1 \mathcal{H} - k F_2 \mathcal{E}) \right)$
- $A_{UL} \propto \sin(\phi) \Im (F_1 \tilde{\mathcal{H}})$
- $A_{LL} \propto A + B \cos(\phi)$ 
  - $A, B \propto \Re (F_1 \tilde{\mathcal{H}})$

**Consequences of such region kinematic reach are:**

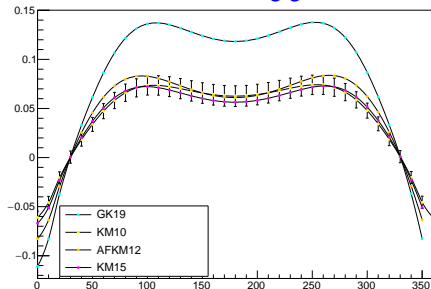
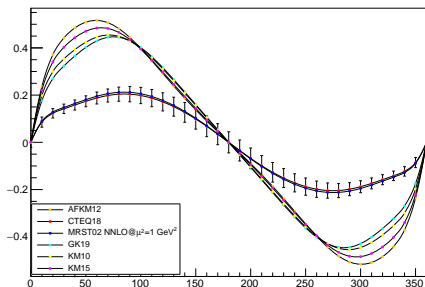
- ✓ Allows for cleaner measurements of GPD  $H$  and  $\tilde{H}$ .
- ✓ Main corrections come from GPD  $E$
- ✗ We lose most of the GPD sensitivity.
- ✗ On the explored region,  $A_{UL}$  and  $A_{LL}$  amplitudes are below 1%.

## Sensitivity @ EIC kinematics

$$x_B = 10^{-4}, Q^2 = 1.5 \text{ GeV}^2, Q'^2 = 0.34 \text{ GeV}^2$$

 $A_{LU}$ 

 GPD  $H$  dominated

 $A_{UU}^C$ 


- ✓ Measurable within 1 year.
- ✓ The increase in the cross-section compensates the smaller luminosity, explaining the smaller error bars.
- ✓  $A_{LU}$  sensitive to models. VGG vs the other models.
- ✓  $A_{UU}^C$  sensitive to models. GK19 model vs the other models.

## Sensitivity @ EIC kinematics

Furthermore, we can consider **transverse target asymmetries** which are mainly **sensitive to GPD  $E$** .

$A_{UT} \sim$

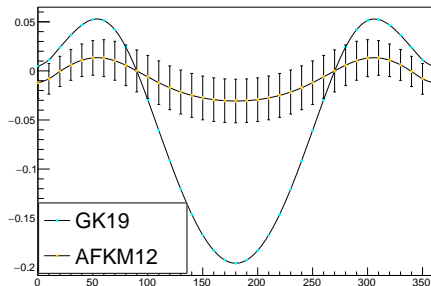
$$\begin{aligned}
 & + \cos(\Phi - \phi) \sin(\phi) \frac{2k}{1 + \xi} \Re \left( \xi(F_1 + F_2)\mathcal{E} - \frac{\xi'}{\xi}(F_2\tilde{\mathcal{H}} - \xi F_1\tilde{\mathcal{E}}) \right) \\
 & + \sin(\Phi - \phi) \cos(\phi) \frac{2k}{1 + \xi} \frac{\xi'}{\xi} \Im(F_2\mathcal{H} + F_1\mathcal{E})
 \end{aligned}$$

Here  $\Phi$  is the azimuthal angle of the target polarization vector and we only keep terms up to linear order in  $\xi$  and  $\xi'$ .

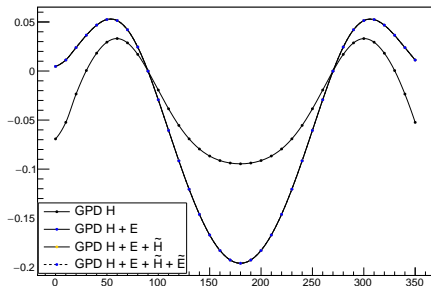
## Sensitivity @ EIC kinematics

$$x_B = 10^{-4}, Q^2 = 1.5 \text{ GeV}^2, Q'^2 = 0.34 \text{ GeV}^2$$

$A_{UT}$



GPD  $E$  dominated



- ✓ Measurable within 1 year.
- ✓ Good opportunity to measure GPD  $E$ .
- ✓ Sensitive to models.



# Conclusions

- The  $\xi'$  dependence of the DDVCS observables allow us to access more information about GPDs.
- At JLab kinematics:
  - Measurements of  $A_{LU}$ ,  $A_{LL}$  and  $A_{UU}^C$  are sensitive to models.
  - Measurements of  $A_{UL}$  are not possible due to a large beam time requirement.
  - Measurements of  $A_{LL}$  can be done with more precision and separate further the KM15 and KM10 models from the others.
  - On  $A_{UU}^C$ , a sign change is predicted by KM15 & VGG models.
  - The above mentioned measurements can be achieved within 100 days of beam time.
- At EIC kinematics:
  - $A_{LU}$  and  $A_{UU}^C$  are sensitive to models. Separating the VGG model and the GK19 model respectively.
  - In the explored kinematics,  $A_{UL}$  and  $A_{LL}$  have negligible amplitudes.
  - $A_{UT}$  has a sizeable amplitude and is very sensitive on GPD  $E$ .
  - The above mentioned measurements can be achieved within 1 effective year of beam time.

# References I

- [1] E. Voutier et al. In: *Jefferson Lab Experiment LOI12-15-005* (2015).
- [2] A. Accardi et al. In: *Eur. Phys. J. A* 57 (2021), p. 261.
- [3] E. Voutier et al. In: *IPAC 13-The 4th International Particle Accelerator Conference*. Joint Accelerator Conferences Website. 2013, pp. 2088–2090.
- [4] J. Grames et al. In: *arXiv preprint arXiv:2309.15581* (2023).
- [5] A. Accardi et al. In: *The European Physical Journal A* 52 (2016), pp. 1–100.
- [6] M. Vanderhaeghen et al. In: *Physical review letters* 80.23 (1998), p. 5064.
- [7] M. Vanderhaeghen et al. In: *Physical Review D* 60.9 (1999), p. 094017.

## References II

- [8] K. Goeke et al. In: *Progress in Particle and Nuclear Physics* 47.2 (2001), pp. 401–515.
- [9] M. Guidal et al. In: *Physical Review D* 72.5 (2005), p. 054013.
- [10] S. Goloskokov et al. In: *The European Physical Journal C* 50 (2007), pp. 829–842.
- [11] S. Goloskokov and P. Kroll. In: *The European Physical Journal C* 53 (2008), pp. 367–384.
- [12] B. Berthou et al. In: *Eur. Phys. J. C* 78.6 (2018), p. 478.  
DOI: 10.1140/epjc/s10052-018-5948-0. arXiv: 1512.06174 [hep-ph].
- [13] K. Kumerički and D. Mueller. In: *Nuclear Physics B* 841.1-2 (2010), pp. 1–58.

## References III

- [14] K. Kumerički. *Gepard: Tool for studying the 3d quark and gluon distributions in the nucleon*.  
<https://gepard.phy.hr/credits.html>.
- [15] S. Y. Zhao. “Access to decoupled information of Generalized Parton Distributions via Double Deeply Virtual Compton Scattering”. *HADRON 2019*. 2019. URL:  
[https://indico.ihep.ac.cn/event/9119/contributions/107033/attachments/57434/66243/HADRON\\_Shengying\\_ZHAO.pdf](https://indico.ihep.ac.cn/event/9119/contributions/107033/attachments/57434/66243/HADRON_Shengying_ZHAO.pdf).
- [16] Shengying Zhao. “Access to decoupled information on Generalized Parton Distributions (GPDs) via Double Deeply Virtual Compton Scattering (DDVCS)”. In: *HADRON SPECTROSCOPY AND STRUCTURE: Proceedings of the XVIII International Conference*. World Scientific. 2021, pp. 592–596.

# Thanks

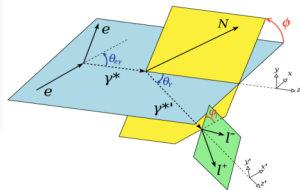
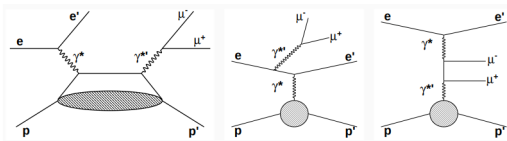
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# Backup

Backup

# DDVCS experimental observables

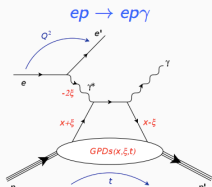
- DDVCS interferes with two kind of BH processes.
- Interference term is accessible through asymmetries.



$$\sigma \propto \mathcal{T}^2 = |\mathcal{T}_{\text{ddvcs}}|^2 + |\mathcal{T}_{\text{BH}_1} + \mathcal{T}_{\text{BH}_2}|^2 + \mathcal{I} \text{ (linear in Compton form factors)}$$

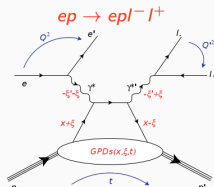
# Exclusive leptonproduction reactions

Two golden channels for GPD measurements are **DVCS** and **DDVCS**.



Deeply Virtual Compton Scattering (DVCS)

$$\mathcal{H}(\xi, \xi, t) = \sum_q e_q^2 \left\{ \mathcal{P} \int_{-1}^1 dx H^q(x, \xi, t) \left[ \frac{1}{x - \xi} + \frac{1}{x + \xi} \right] - i\pi \left[ H^q(\xi, \xi, t) - H^q(-\xi, \xi, t) \right] \right\}$$



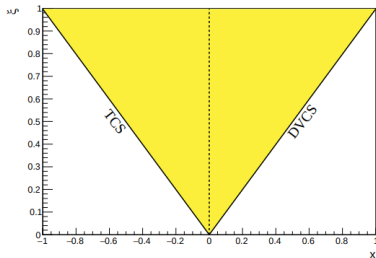
Double DVCS (DDVCS)

$$\mathcal{H}(\xi', \xi, t) = \sum_q e_q^2 \left\{ \mathcal{P} \int_{-1}^1 dx H^q(x, \xi, t) \left[ \frac{1}{x - \xi'} + \frac{1}{x + \xi'} \right] - i\pi \left[ H^q(\xi', \xi, t) - H^q(-\xi', \xi, t) \right] \right\}$$

- ❑ DVCS access GPDs at  $x = \pm\xi$
- ❑ DDVCS access GPDs at independent  $x = \xi'$  and  $\xi$  values ( $|\xi'| < \xi$ ).

$$\xi' = \frac{Q^2 - Q'^2 + t/2}{2Q^2/x_B - Q^2 - Q'^2 + t}$$

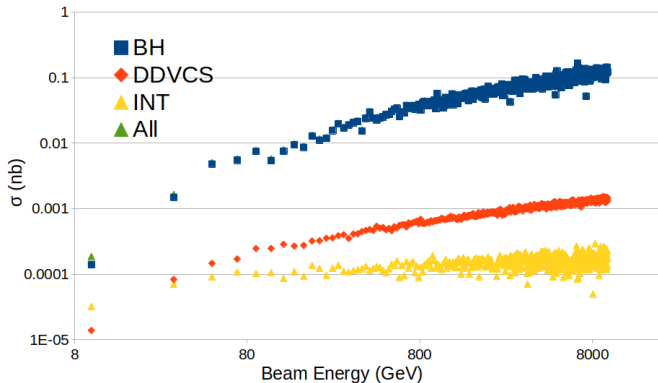
$$\xi = \frac{Q^2 + Q'^2}{2Q^2/x_B - Q^2 - Q'^2 + t}$$





## Sensitivity @ EIC kinematics

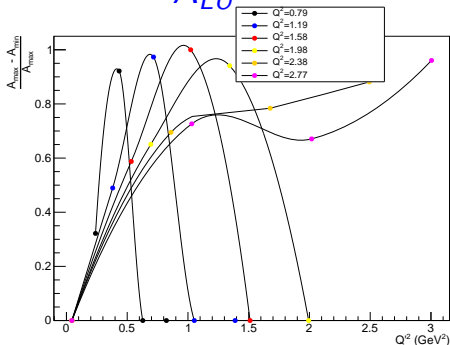
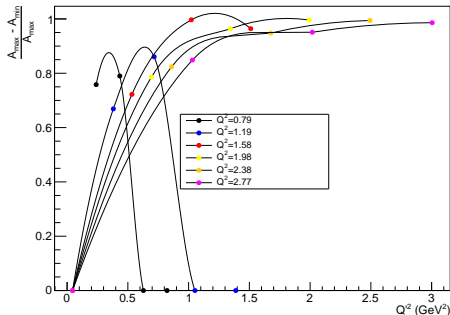
- As  $W = 140$  GeV, the equivalent beam energy is  $E \approx \frac{W^2}{2M} \approx 9800$  GeV.
- At EIC kinematics, the DDVCS cross section increases importantly.
- The cross section increase compensates the smaller luminosity.



**Figure:** Integrated cross section from JLab to EIC kinematics. Obtained with PARTONS [12].

## General scan JLab

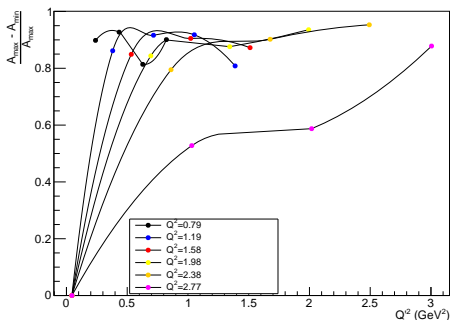
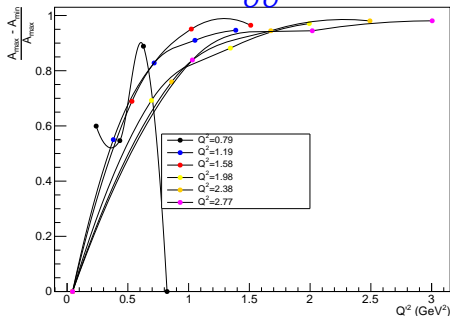
$$x_B = 0.15$$

 $A_{LU}$ 

 $A_{UL}$ 


$\frac{A_{max} - A_{min}}{A_{max}}$  over models on the general scan. Only points with amplitudes greater than 1% are considered.

## General scan JLab

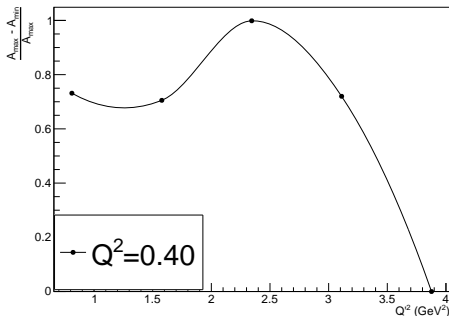
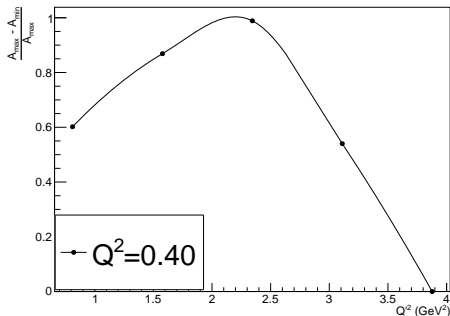
$$x_B = 0.15$$

 $A_{LL}$ 

 $A_{UU}^C$ 


$\frac{A_{max} - A_{min}}{A_{max}}$  over models on the general scan. Only points with amplitudes greater than 1% are considered.

## General scan JLab

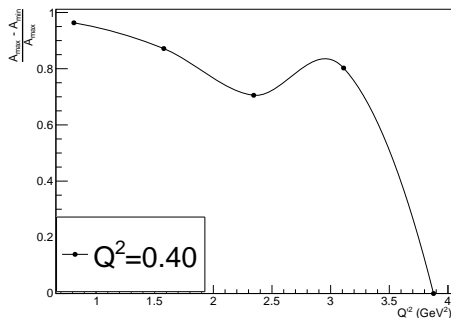
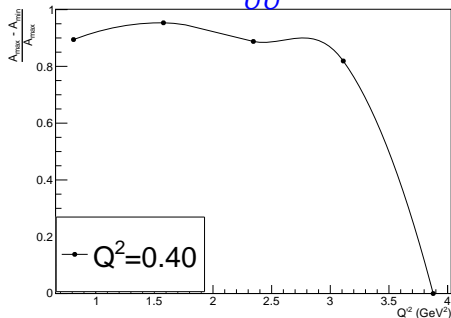
$$x_B = 0.03$$

 $A_{LU}$ 

 $A_{UL}$ 


$\frac{A_{max} - A_{min}}{A_{max}}$  over models on the general scan. Only points with amplitudes greater than 1% are considered.

## General scan JLab

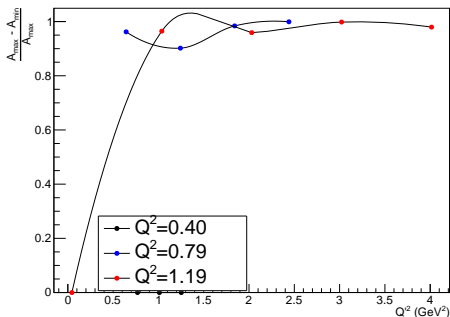
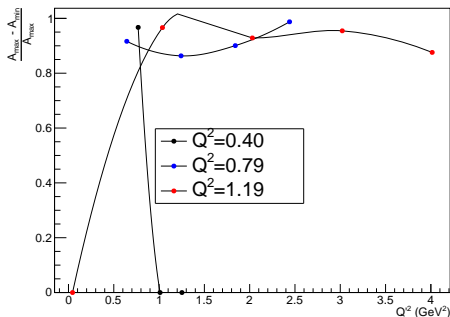
$$x_B = 0.03$$

 $A_{LL}$ 

 $A_{UU}^C$ 


$\frac{A_{max} - A_{min}}{A_{max}}$  over models on the general scan. Only points with amplitudes greater than 1% are considered.

## General scan JLab

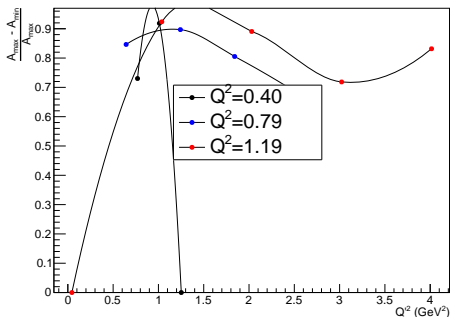
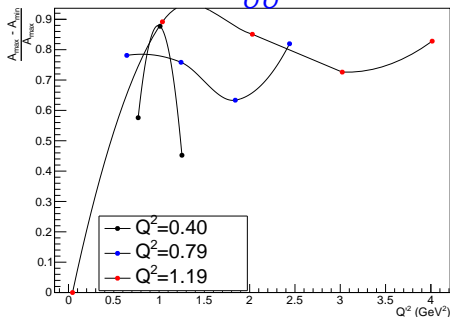
$$x_B = 0.07$$

 $A_{LU}$ 

 $A_{UL}$ 


$\frac{A_{max} - A_{min}}{A_{max}}$  over models on the general scan. Only points with amplitudes greater than 1% are considered.

## General scan JLab

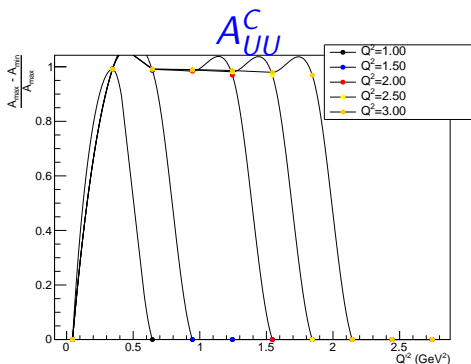
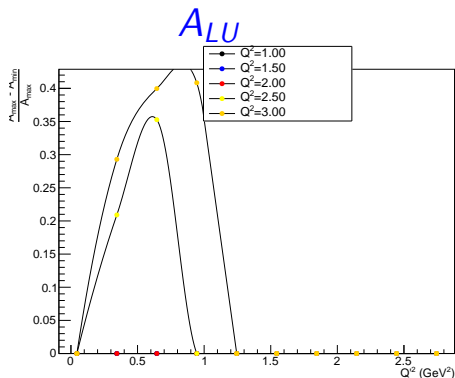
$$x_B = 0.07$$

 $A_{LL}$ 

 $A_{UU}^C$ 


$\frac{A_{max} - A_{min}}{A_{max}}$  over models on the general scan. Only points with amplitudes greater than 1% are considered.

# General scan EIC

$$x_B = 10^{-3}$$

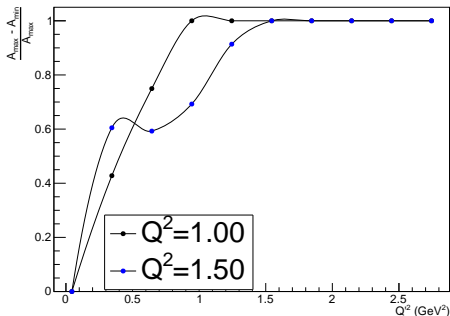
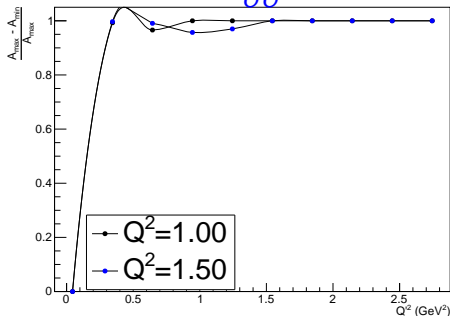


$\frac{A_{max} - A_{min}}{A_{max}}$  over models on the general scan. Only points with amplitudes greater than 1% are considered.



# General scan EIC

$$x_B = 10^{-4}$$

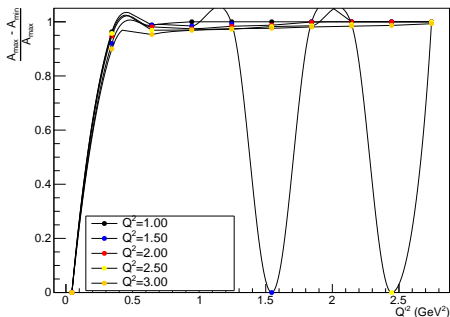
 $A_{LU}$ 

 $A_{UU}^C$ 


$\frac{A_{max} - A_{min}}{A_{max}}$  over models on the general scan. Only points with amplitudes greater than 1% are considered.

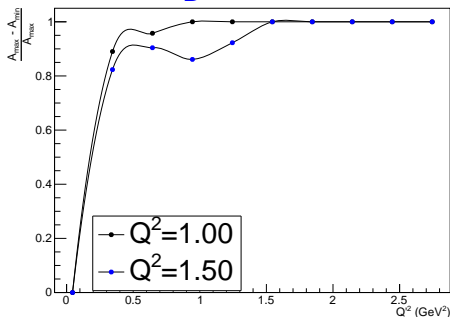
# General scan EIC

## $A_{UT}$

$x_B = 10^{-3}$



$x_B = 10^{-4}$



$\frac{A_{max} - A_{min}}{A_{max}}$  over models on the general scan. Only points with amplitudes greater than 1% are considered.

## Backup: Gepard modifications

- For CFF  $\mathcal{H}$ , the dispersion relation is based on the imaginary part.

- For DVCS:  $H(x, x, t) = \frac{nr}{1+x} \left(\frac{2x}{1+x}\right)^{-\alpha(t)} \left(\frac{1-x}{1+x}\right)^b \frac{1}{\left(1 - \frac{1-x}{1+x} \frac{t}{M^2}\right)^p}$ .

- For DDVCS, an inverse engineering was done to find a DD that reproduces the DVCS case, then it was generalized

$$H(x, \xi, t) =$$

$$\frac{nr\vartheta}{\vartheta+x} \frac{1+\vartheta}{2} \left(\frac{(1+\vartheta)x}{\vartheta+x}\right)^{-\alpha(t)} \left(\frac{\vartheta^2(1+\vartheta)}{2} \frac{1-x}{\vartheta+x}\right)^b \frac{1}{\left(1 - (1-\vartheta^2) \frac{t}{M^2} - \frac{\vartheta^2(1+\vartheta)}{2} \frac{(1-x)}{\vartheta+x} \frac{t}{M^2}\right)^p}$$

being  $\vartheta = x/\xi$ . Notice that for  $\vartheta = 1$  reproduces the DVCS case.

## Backup: Gepard modifications

- Likewise for CFF  $\tilde{\mathcal{H}} = \pi \left( 2\frac{4}{9} + \frac{1}{9} \right)$ 

$$\frac{\tilde{n}\vartheta}{\vartheta+x} \frac{1+\vartheta}{2} \left( \frac{(1+\vartheta)x}{\vartheta+x} \right)^{-\alpha(t)} \left( \frac{\vartheta^2(1+\vartheta)}{2} \frac{1-x}{\vartheta+x} \right)^{3/2} \frac{1}{1-(1-\vartheta^2)\frac{t}{M^2} - \frac{\vartheta^2(1+\vartheta)(1-x)}{\vartheta+x} \frac{t}{2M^2}}$$
- For CFF  $\mathcal{E}$  there is no implementation for a dispersion relation.
- for CFF  $\tilde{\mathcal{E}}$  we can perform exact computations:
  - $\int_{-1}^1 dx \left[ \frac{1}{x-\xi+i\epsilon} - \frac{1}{x+\xi-i\epsilon} \right] \tilde{E}(x, \xi, t) = -\frac{1}{2\xi} h_A(t)$
  - $\int_{-1}^1 dx \left[ \frac{1}{x-\xi'+i\epsilon} - \frac{1}{x+\xi'-i\epsilon} \right] \tilde{E}(x, \xi, t) =$   
 $-\frac{1}{8\xi^3} h_A(t) \left( 4\xi\xi' + 2(\xi^2 - \xi'^2) \log \left( \frac{\xi+\xi'}{\xi-\xi'} \right) \right)$

## Backup: Remarks

- ❑ KM  $\text{Im}(H)$  and  $\text{Im}(H_t)$  parametrization are not odd functions as required, so I force that condition by mirroring the  $x > 0$  region.
- ❑ AFKM12 only has GPD  $H$  and  $E$
- ❑ For regular KM models  $\text{Im}(E) = \text{Im}(E_t) = 0$
- ❑ In VGG, GPD  $E$  DD is chosen with a factorized  $t$  ansatz (option 4)