## Probing quark Orbital Angular Momentum in ep collisions

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Based on:
arXiv: 2312.01309

## Wigner function - The "mother function"



Parton Distribution Functions

```
PDFs (x)
```


## Wigner function - The "mother function"



## Wigner function - The "mother function"

Transverse Momentum-dependent Distributions


## Wigner function - The "mother function"

Generalized Transverse Momentum-dependent Distributions
(Meissner, Metz, Schlegel, 2009) GTMDs $\left(x, \vec{k}_{\perp}, \Delta\right)$


## Wigner function - The "mother function"

Wigner functions $\left(x, \vec{k}_{\perp}, \vec{b}_{\perp}\right)$ (Belitsky, Ji, Yuan, 2003)

(Meissner, Metz, Schlegel, 2009) GTMDs $\left(x, \vec{k}_{\perp}, \Delta\right)$


## Spin of proton

## Jaffe-Manohar spin decomposition

An incomplete story:


## Spin of proton

## Jaffe-Manohar spin decomposition

An incomplete story:


Quark helicity $\sim 30 \% \quad$ Gluon helicity $\sim 40 \%$

## Spin of proton



## Wigner functions \& Orbital Angular Momentum

Wigner functions in Quantum Mechanics
(Wigner, 1932)

- Calculate from wave functions:

$$
W(x, k)=\int \frac{d x^{\prime}}{2 \pi} e^{-i k x^{\prime}} \psi\left(x+\frac{x^{\prime}}{2}\right) \psi^{*}\left(x-\frac{x^{\prime}}{2}\right)
$$

- Expectation value of observables:

$$
\langle\mathcal{O}\rangle=\int d x \int d k \mathcal{O}(x, k) W(x, k)
$$

## Wigner functions \& Orbital Angular Momentum

Wigner functions in Quantum Mechanics
(Wigner, 1932)

Wigner functions in parton physics
(Belitsky, Ji, Yuan, 2003)

- Calculate from wave functions:

$$
W(x, k)=\int \frac{d x^{\prime}}{2 \pi} e^{-i k x^{\prime}} \psi\left(x+\frac{x^{\prime}}{2}\right) \psi^{*}\left(x-\frac{x^{\prime}}{2}\right)
$$

- Expectation value of observables:

$$
\langle\mathcal{O}\rangle=\int d x \int d k \mathcal{O}(x, k) W(x, k)
$$

- Calculate from fourier transform of GTMD correlator:

$$
W^{[\Gamma]}\left(x, \vec{k}_{\perp}, \vec{b}_{\perp}\right)
$$

- Application: Orbital Angular Momentum (OAM)

$$
L_{z}^{q, g}=\int d x \int d^{2} k_{\perp} d^{2} b_{\perp}\left(\vec{b}_{\perp} \times \vec{k}_{\perp}\right)_{z} W^{q, g}\left(x, \vec{b}_{\perp}, \vec{k}_{\perp}\right)
$$

Wigner functions \& Orbital Angular Momentum


Wigner functions \& Orbital Angular Momentum

## Big question: Experimental observable?

Expectation value of observables:

- Application: Relation between GTMD $F_{1,4}^{q, g}$ \& OAM

$$
L_{z}^{q, g}=-\int d x \int d^{2} \vec{k}_{\perp} \frac{\vec{k}_{\perp}^{2}}{M^{2}} F_{1,4}^{q, g}\left(x, k_{\perp}, \xi=0, \Delta_{\perp}=0\right)
$$

## Developments

arXiv: 1612.02438 (2016)
Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji, ${ }^{1,2}$ Feng Yuan, ${ }^{3}$ and Yong Zhao ${ }^{1,3}$

## Developments

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Generalized TMDs and the exclusive double Drell-Yan process

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## arXiv: 1802.10550 (2018)

Exclusive double quarkonium production and generalized TM
Shohini Bhattacharya, ${ }^{1}$ Andreas Metz, ${ }^{1}$ Vikash Kumar Ojha, ${ }^{2}$ Jeng-Yuan Tsai, ${ }^{1}$

## arXiv: 1807.08697 (2018)

Probing the Weizsäcker-Williams gluon Wigner distribution in $p p$ collisions Renaud Boussarie, ${ }^{1}$ Yoshitaka Hatta, ${ }^{2}$ Bo-Wen Xiao, ${ }^{3,4}$ and Feng Yuan ${ }^{5}$

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## arXiv: 1912.08182 (2019)

Probing the gluon Sivers function with an unpolarized target:
GTMD distributions and the Odderons
Renaud Boussarie, ${ }^{1}$ Yoshitaka Hatta, ${ }^{1}$ Lech Szymanowski, ${ }^{2}$ and Samuel Wallon ${ }^{3,4}$

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arXiv: 2201.08709/2404.04209 (2022/2024)
Signature of the gluon orbital angular momentum
Shohini Bhattacharya, ${ }^{1, *}$ Renaud Boussarie, ${ }^{2, \dagger}$ and Yoshitaka Hatta ${ }^{1,3, \ddagger}$

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Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou

## arXiv: 2205.00045 (2022)

Angular correlations in exclusive dijet photoproduction in ultra-peripheral PbPb collisions at $\sqrt{s_{N N}}=5.02 \mathrm{TeV}$

## Developments



## Exclusive double Drell-Yan:

## Until now, this has been the sole known process sensitive to quark GTMDs

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Probing quark OAM through double Drell-Yan
Main findings
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Generalized TMDs and the exclusive double Drell-Yan process
Shohini Bhattacharya, ${ }^{1}$ Andreas Metz, ${ }^{1}$ and Jian Zhou ${ }^{2}$


## Probing quark OAM through double Drell-Yan

## Main findings

Example of an observable sensitive to OAM \& spin-orbit correlation :

$$
\begin{aligned}
\frac{1}{2}\left(\tau_{X Y}-\tau_{Y X}\right)=\frac{4}{M_{a}^{2}}\left(\varepsilon_{\perp}^{i j} \Delta q_{\perp}^{i} \Delta_{a \perp}^{j}\right) \operatorname{Re} .\{ & C^{(-)}\left[F_{1,1} \phi_{\pi}\right] C^{(+)}\left[\vec{\beta}_{\perp} \cdot \vec{k}_{a \perp} \boldsymbol{F}_{1,4}^{*} \phi_{\pi}^{*}\right] \\
& \left.-C^{(+)}\left[G_{1,4} \phi_{\pi}\right] C^{(-)}\left[\vec{\beta}_{\perp} \cdot \vec{p}_{a \perp} G_{1,1}^{*} \phi_{\pi}^{*}\right]\right\}
\end{aligned}
$$

Spin-orbit entanglement in the Color Glass Condensate Shohini Bhattacharya, ${ }^{1, *}$ Renaud Boussarie, ${ }^{2, \dagger}$ and Yoshitaka Hatta ${ }^{3,4, \ddagger}$


# Probing quark OAM through double Drell-Yan 

Main findings

Challenges:

- Low count rate (Amplitude $\sim \alpha_{e m}^{2}$ )


## Probing quark OAM through double Drell-Yan

## Main findings

## Challenges:

- Low count rate (Amplitude $\sim \alpha_{e m}^{2}$ )
- Sensitivity to GTMDs only in the ERBL region $-\xi<x<\xi$

$$
\begin{aligned}
& \text { OAM density: } \quad L^{q / g}(x, \boldsymbol{\xi})=-\int d^{2} \vec{k}_{\perp} \frac{\vec{k}_{\perp}^{2}}{M^{2}} F_{1,4}^{q, g}\left(x, k_{\perp}, \boldsymbol{\xi}, \Delta_{\perp}=0\right) \\
& \text { OAM: } \quad L^{q / g}=\int d x L^{q / g}(x, \boldsymbol{\xi}=\mathbf{0})
\end{aligned}
$$

The challenge lies in extrapolating the distribution to the forward limit, where the OAM equation is applicable

## Our work

## arXiv: 2312.01309 (2023)

Probing quark orbital angular momentum at EIC and EicC
Shohini Bhattacharya, ${ }^{1}$ Duxin Zheng, ${ }^{2}$ and Jian Zhou ${ }^{3}$


## Main Observable:

## Longitudinal single-target spin asymmetry

## Exclusive $\pi^{0}$ production in ep collisions

## Known results



$$
\begin{aligned}
\frac{d \sigma_{T}}{d t} & =\frac{1}{2 \kappa}\left(\left|M_{0-,++}\right|^{2}+2\left|M_{0+,++}\right|^{2}\right) \\
\frac{d \sigma_{T T}}{d t} & =-\frac{1}{\kappa}\left|M_{0+,++}\right|^{2} \cos (2 \phi)
\end{aligned}
$$

The coupling of twist-2 helicity-flip chiral odd GPDs with the twist-3 distribution amplitude of the neutral pion yields the leading power contribution to the unpolarized cross-section:

$$
\begin{aligned}
& M_{0-,++}=\frac{e_{0}}{Q} \sqrt{1-\xi^{2}}\left\langle H_{T}\right\rangle \\
& M_{0+,++}=-\frac{e_{0}}{Q} \frac{\sqrt{-t^{\prime}}}{4 m}\left\langle\bar{E}_{T}\right\rangle
\end{aligned}
$$

See for example L. Frankfurt, P. Pobylitsa, M. Polyakov, \& M. Strikman, 1999

# Probing quark OAM through $\pi^{0}$ production in ep collisions 



Scattering amplitude

Probing quark OAM through $\pi^{0}$ production in ep collisions


Probing quark OAM through $\pi^{0}$ production in ep collisions


## Probing quark OAM through $\pi^{0}$ production in ep collisions



## Probing quark OAM through $\pi^{0}$ production in ep collisions

## Scattering amplitude

Scattering amplitude:

$$
A \propto \int d x \int d^{2} k_{\perp} H\left(x, \xi, z, k_{\perp}, \Delta_{\perp}\right) f^{q}\left(x, \xi, k_{\perp}, \Delta_{\perp}\right) \int d z \phi_{\pi}(z)
$$

Collinear twist-expansion of hard part:


$$
H\left(k_{\perp}, \Delta_{\perp}\right)=H\left(k_{\perp}=0, \Delta_{\perp}=0\right)+\left.\frac{\partial H\left(k_{\perp}, \Delta_{\perp}=0\right)}{\partial k_{\perp}^{\mu}}\right|_{k_{\perp}=0} k_{\perp}^{\mu}+\left.\frac{\partial H\left(k_{\perp}=0, \Delta_{\perp}\right)}{\partial \Delta_{\perp}^{\mu}}\right|_{\Delta_{\perp}=0} \Delta_{\perp}^{\mu}+\ldots
$$

Use special-propagator technique to ensure electromagnetic gauge invariance

Probing quark OAM through $\pi^{0}$ production in ep collisions


Probing quark OAM through $\pi^{0}$ production in ep collisions


## Probing quark OAM through $\pi^{0}$ production in ep collisions



## Probing quark OAM through $\pi^{0}$ production in ep collisions

## Angular correlations

Scattering amplitudes depend on different angular correlations:

$$
\begin{aligned}
& \mathcal{M}_{1}=\frac{g_{s}^{2} e f_{\pi}}{2 \sqrt{2}} \frac{\left(N_{c}^{2}-1\right) 2 \xi}{N_{c}^{2} \sqrt{1-\xi^{2}}} \delta_{\lambda \lambda^{\prime}} \frac{\epsilon_{\perp} \times \Delta_{\perp}}{Q^{2}}\left\{\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\right\} \\
& \mathcal{M}_{2}=\frac{g_{s}^{2} e f_{\pi}}{2 \sqrt{2}} \frac{\left(N_{c}^{2}-1\right) 2 \xi}{N_{c}^{2} \sqrt{1-\xi^{2}}} \delta_{\lambda,-\lambda^{\prime}} \frac{M \epsilon_{\perp} \cdot S_{\perp}}{Q^{2}}\left\{\mathcal{F}_{1,2}+\mathcal{G}_{1,2}\right\} \quad S_{\perp}^{\mu}=\left(0^{+}, 0^{-},-i, \lambda\right) \\
& \mathcal{M}_{4}=\frac{i g_{s}^{2} e f_{\pi}}{2 \sqrt{2}} \frac{\left(N_{c}^{2}-1\right) 2 \xi}{N_{c}^{2} \sqrt{1-\xi^{2}}} \lambda \delta_{\lambda \lambda^{\prime}} \frac{\epsilon_{\perp} \cdot \Delta_{\perp}}{Q^{2}}\left\{\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\right\}
\end{aligned}
$$

Probing quark OAM

## Compton Form Factors:

## Angular correlations

Scattering amplitudes depend on different angular correlations:


$$
\begin{align*}
\mathcal{F}_{1,1}= & \int_{-1}^{1} d x \frac{x^{2} \int d^{2} k_{\perp} F_{1,1}^{u+d}\left(x, \xi, \Delta_{\perp}, k_{\perp}\right)}{(x+\xi-i \epsilon)^{2}(x-\xi+i \epsilon)^{2}} \\
& \times \int_{0}^{1} d z \frac{\phi_{\pi}(z)\left(1+z^{2}-z\right)}{z^{2}(1-z)^{2}},  \tag{8}\\
\mathcal{G}_{1,1}= & \int_{-1}^{1} d x \int_{0}^{1} d z \frac{\phi_{\pi}(z)\left(x^{2}+2 x^{2} z+\xi^{2}\right)}{z^{2}(x+\xi-i \epsilon)^{2}(x-\xi+i \epsilon)^{2}} \\
& \times \int_{\mathcal{F}^{2}} d^{2} k_{\perp} \frac{k_{\perp}^{2}}{M^{2}} G_{1,1}^{u+d}\left(x, \xi, \Delta_{\perp}, k_{\perp}\right), \\
\mathcal{F}_{1,2}= & \int_{-1}^{1} d x x \frac{\xi\left(1-\xi^{2}\right) \int d^{2} k_{\perp} k_{\perp}^{2} F_{1,2}^{u+d}\left(x, \xi, \Delta_{\perp}, k_{\perp}\right)}{M^{2}(x+\xi-i \epsilon)^{2}(x-\xi+i \epsilon)^{2}} \\
& \times \int_{0}^{1} d z \frac{\phi_{\pi}(z)\left(1+z^{2}-z\right)}{z^{2}(1-z)^{2}}, \\
\mathcal{G}_{1,2}= & \int_{-1}^{1} d x \int_{0}^{1} d z \frac{\phi_{\pi}(z)\left(x^{2}+2 x^{2} z+\xi^{2}\right)\left(1-\xi^{2}\right)}{z^{2}(x+\xi-i \epsilon)^{2}(x-\xi+i \epsilon)^{2}} \\
& \times \int_{\mathcal{F}^{2}} d^{2} k_{\perp} \frac{k_{\perp}^{2}}{M^{2}} G_{1,2}^{u+d}\left(x, \xi, \Delta_{\perp}, k_{\perp}\right), \\
\mathcal{F}_{1,4}= & \int_{-1}^{1} d x \frac{x \xi \int d^{2} k_{\perp} k_{\perp}^{2} F_{1,4}^{u+d}\left(x, \xi, \Delta_{\perp}, k_{\perp}\right)}{M^{2}(x+\xi-i \epsilon)^{2}(x-\xi+i \epsilon)^{2}} \\
& \times \int_{0}^{1} d z \frac{\phi_{\pi}(z)\left(1+z^{2}-z\right)}{z^{2}(1-z)^{2}},  \tag{12}\\
\mathcal{G}_{1,4}= & \int_{-1}^{1} d x \int_{0}^{1} d z \frac{x\left(4 \xi^{2} z+\xi^{2}-2 x^{2} z+x^{2}\right)}{z^{2} \xi(x+\xi-i \epsilon)^{2}(x-\xi+i \epsilon)^{2}} \phi_{\pi}(z) \\
& \times \int^{2} d^{2} k_{\perp} G_{1,4}^{u+d}\left(x, \xi, \Delta_{\perp}, k_{\perp}\right) .  \tag{13}\\
& (13)
\end{align*}
$$

## Probing quark OAM through $\pi^{0}$ production in ep collisions



Probing quark OAM through $\pi^{0}$ production in ep collisions

## Cross section

$$
\begin{aligned}
& \frac{d \sigma}{d t d Q^{2} d x_{B} d \phi}=\frac{\left(N_{c}^{2}-1\right)^{2} \alpha_{e m}^{2} \alpha_{s}^{2} f_{\pi}^{2} \xi^{3} \Delta_{\perp}^{2}}{2 N_{c}^{4}\left(1-\xi^{2}\right) Q^{10}(1+\xi)}\left[1+(1-y)^{2}\right] \\
& \times\left\{\left[\left|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\right|^{2}+\left|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\right|^{2}+2 \frac{M^{2}}{\Delta_{\perp}^{2}}\left|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}\right|^{2}\right]+\cos (2 \phi) a\left[-\left|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\right|^{2}+\left|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\right|^{2}\right]\right. \\
& \left.+\lambda \sin (2 \phi) 2 a \operatorname{Re}\left[\left(i \mathcal{F}_{1,4}+i \mathcal{G}_{1,4}\right)\left(\mathcal{F}_{1,1}^{*}+\mathcal{G}_{1,1}^{*}\right)\right]\right\} \\
& \quad \quad \mid \\
& \quad a=\frac{2(1-y)}{1+(1-y)^{2}}
\end{aligned}
$$

## Probing quark OAM through $\pi^{0}$ production in ep collisions

## Cross section

$$
\frac{d \sigma}{d t d Q^{2} d x_{B} d \phi}=\frac{\left(N_{c}^{2}-1\right)^{2} \alpha_{e m}^{2} \alpha_{s}^{2} f_{\pi}^{2} \xi^{3} \Delta_{\perp}^{2}}{2 N_{c}^{4}\left(1-\xi^{2}\right) Q^{10}(1+\xi)}\left[1+(1-y)^{2}\right]
$$



Distinguished experimental signature of quark OAM
$\phi=\phi_{l_{\perp}}-\phi_{\Delta_{\perp}}$
$e$

Leptonic plane

Hadronic plane
 P

$\pi^{0}$

## Probing quark OAM through $\pi^{0}$ production in ep collisions

## Cross section

$$
\frac{d \sigma}{d t d Q^{2} d x_{B} d \phi}=\frac{\left(N_{c}^{2}-1\right)^{2} \alpha_{e m}^{2} \alpha_{s}^{2} f_{\pi}^{2} \xi^{3} \Delta_{\perp}^{2}}{2 N_{c}^{4}\left(1-\xi^{2}\right) Q^{10}(1+\xi)}\left[1+(1-y)^{2}\right]
$$

$$
\times\left\{\left[\left|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\right|^{2}+\left|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\right|^{2}+2 \frac{M^{2}}{\Delta_{\perp}^{2}}\left|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}\right|^{2}\right]+\cos (2 \phi) a\left[-\left|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\right|^{2}+\left|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\right|^{2}\right]\right.
$$



- Probe quark Sivers function through an unpolarized target

$$
\left.\operatorname{Im}\left[\boldsymbol{F}_{1,2}\right]\right|_{\Delta=0}=-f_{1 T}^{\perp}
$$

(Similar to the gluon GTMD $F_{1,2}$, as discussed in Boussarie, Hatta, Szymanowski, Wallon, 2019)

## Probing quark OAM through $\pi^{0}$ production in ep collisions

## Cross section

$$
\frac{d \sigma}{d t d Q^{2} d x_{B} d \phi}=\frac{\left(N_{c}^{2}-1\right)^{2} \alpha_{e m}^{2} \alpha_{s}^{2} f_{\pi}^{2} \xi^{3} \Delta_{\perp}^{2}}{2 N_{c}^{4}\left(1-\xi^{2}\right) Q^{10}(1+\xi)}\left[1+(1-y)^{2}\right]
$$

$$
\times\left\{\left[\left|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\right|^{2}+\left|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\right|^{2}+2 \frac{M^{2}}{\Delta_{\perp}^{2}}\left|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}\right|^{2}\right]+\cos (2 \phi) a\left[-\left|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\right|^{2}+\left|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\right|^{2}\right]\right.
$$

## Surprise!

- Probe quark Sivers function through an unpolarized target

$$
\left.\operatorname{Im}\left[\boldsymbol{F}_{1,2}\right]\right|_{\Delta=0}=-f_{1 T}^{\perp}
$$

- Probe quark worm-gear function through an unpolarized target

$$
\left.\operatorname{Re}\left[G_{1,2}\right]\right|_{\Delta=0}=g_{1 T}
$$

## Probing quark OAM through $\pi^{0}$ production in ep collisions

## Cross section

$$
\frac{d \sigma}{d t d Q^{2} d x_{B} d \phi}=\frac{\left(N_{c}^{2}-1\right)^{2} \alpha_{e m}^{2} \alpha_{s}^{2} f_{\pi}^{2} \xi^{3} \Delta_{\perp}^{2}}{2 N_{c}^{4}\left(1-\xi^{2}\right) Q^{10}(1+\xi)}\left[1+(1-y)^{2}\right]
$$

$$
\times\left\{\left[\left|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\right|^{2}+\left|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\right|^{2}+2 \frac{M^{2}}{\Delta_{\perp}^{2}}\left|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}\right|^{2}\right]+\cos (2 \phi) a\left[-\left|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\right|^{2}+\left|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\right|^{2}\right]\right.
$$

## $+\lambda \sin (2 \phi) 2 a \operatorname{Re}\left\lceil\left(i F_{1,4}+i G_{1,4}\right)\left(F_{1.1}^{*}+G_{1}^{*} \uparrow\right.\right.$



Helicity flip terms persist even when $\Delta_{\perp} \rightarrow 0$

## Probing quark OAM through $\pi^{0}$ production in ep collisions

## Cross section

$$
\begin{aligned}
& \frac{d \sigma}{d t d Q^{2} d x_{B} d \phi}=\frac{\left(N_{c}^{2}-1\right)^{2} \alpha_{e m}^{2} \alpha_{s}^{2} f_{\xi}^{2} \xi^{3} \Delta_{\perp}^{2}}{2 N_{c}^{4}\left(1-\xi^{2}\right) Q^{10}(1+\xi)}\left[1+(1-y)^{2}\right] \\
& \times\left\{\left[\left|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\right|^{2}+\left|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\right|^{2}+2 \frac{M^{2}}{\Delta_{\perp}^{2}}\left|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}\right|^{2}\right]+\cos (2 \phi) a\left[-\left|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\right|^{2}+\left|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\right|^{2}\right]\right. \\
& \left.\quad+\lambda \sin (2 \phi) 2 a \operatorname{Re}\left[\left(i \mathcal{F}_{1,4}+i \mathcal{G}_{1,4}\right)\left(\mathcal{F}_{1,1}^{*}+\mathcal{G}_{1,1}^{*}\right)\right]\right\}
\end{aligned}
$$

Since both unpolarized and polarized cross sections contribute at twist-3, the magnitudes of the asymmetries are not power-suppressed

## Probing quark OAM through $\pi^{0}$ production in ep collisions

## Model input for numerical estimations

## Ingredients for non-perturbative functions:

- Model $\left(H^{q}, \tilde{H}^{q}\right)$ according to the Double distribution approach (see Radyushkin, 9805342)

Example:

$$
H^{q}(x, \boldsymbol{\xi}, \boldsymbol{t})=\int_{-1}^{1} d \beta \int_{-1+|\beta|}^{1-|\beta|} d \alpha \delta(\beta+\boldsymbol{\xi} \alpha-x) \times \frac{3}{4}|\beta|^{-0.9 t} \frac{\left[(1-|\beta|)^{2}-\alpha^{2}\right]}{(1-|\beta|)^{3}} q(|\beta|)
$$

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1. "OAM density": (Hatta, Yoshida, 1207.5332)

$$
L_{c a n}^{q}(\boldsymbol{x})=x \int_{x}^{1} \frac{d x^{\prime}}{x^{\prime}} q\left(x^{\prime}\right)-x \int_{x}^{1} \frac{d x^{\prime}}{x^{\prime 2}} \Delta q\left(x^{\prime}\right)+\text { genuine twist-three }
$$

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$$

2. Use the Double distribution approach to construct $x L^{q}(x, \boldsymbol{\xi}) e^{\boldsymbol{t} / \Lambda}$ from $x L^{q}(x)$

# Probing quark OAM through $\pi^{0}$ production in ep collisions 

## Model input for numerical estimations

Ingredients for non-perturbative functions:

- Pion distribution amplitude:

$$
\text { Asymptotic form } \quad \phi_{\pi}(z)=6 z(1-z)
$$

## Probing quark OAM through $\pi^{0}$ production in ep collisions

## Model input for numerical estimations

End-point singularity \& discontinuity:

$$
\mathcal{F}_{1,4}=\int_{-1}^{1} d x \frac{x \xi \int d^{2} k_{\perp} k_{\perp}^{2} F_{1,4}^{u+d}\left(x, \xi, \Delta_{\perp}, k_{\perp}\right)}{M^{2}(x+\xi-i \epsilon)^{2}(x-\xi+i \epsilon)^{2}} \times \int_{0}^{\mathbf{1}} d \boldsymbol{d} \frac{\phi_{\pi}(z)\left(1+z^{2}-z\right)}{z^{2}(1-z)^{2}}
$$

Model-dependent method:

$$
\int_{\left\langle p_{\perp}^{2}\right\rangle / Q^{2}}^{1-\left\langle p_{\perp}^{2}\right\rangle / Q^{2}} d z
$$

S. V. Goloskokov and P. Kroll, 2005

## Probing quark OAM through $\pi^{0}$ production in ep collisions

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$$

Model-dependent method:

S. V. Goloskokov and P. Kroll, 2005

$$
\frac{1}{(x-\xi+i \epsilon)^{2}} \rightarrow \frac{1}{\left(x-\xi-\left\langle\boldsymbol{p}_{\perp}^{2}\right\rangle / Q^{2}+i \epsilon\right)^{2}}
$$

Probing quark OAM through $\pi^{0}$ production in ep collisions

## Numerical results

Kinematics:

|  | $Q^{2}\left(\mathrm{GeV}^{2}\right)$ | $\sqrt{s}_{\text {ep }}(\mathrm{GeV})$ |
| :---: | :---: | :---: |
| EIC | 10 | 100 |
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## Probing quark OAM through $\pi^{0}$ production in ep collisions

## Numerical results

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- We focus on large skewness $(\xi)$ region to suppress gluon contribution
- We focus on large momentum transfer $(t)$ region to suppress contribution from Primakoff process

$\propto \frac{1}{t}$


## Probing quark OAM through $\pi^{0}$ production in ep collisions

## Remark:

Accessing the gluon GTMD $\boldsymbol{F}_{1,4}$ in exclusive $\pi^{0}$ production in $e p$ collisions

Shohini Bhattacharya, ${ }^{1}$ Duxin Zheng, ${ }^{2}$ and Jian Zhou ${ }^{3}$


$$
\frac{d \Delta \sigma}{d t d Q^{2} d x_{B} d \phi}=-\sin (2 \phi) \frac{x_{e m}^{3} \alpha_{s} f_{\pi}^{2}(1-y) \xi x_{B} \mathcal{F}(t)}{3 Q^{8} N_{c}}\left[\int_{0}^{1} d z \frac{\phi_{\pi}(z)}{z(1-z)}\right]^{2} \operatorname{Im}\left[\int_{-1}^{1} d x \frac{F_{1,4}^{(1)}\left(x, \xi, \Delta_{\perp}\right) / M^{2}}{(x+\xi-i \epsilon)^{2}(x-\xi+i \epsilon)^{2}}\right]
$$

The same azimuthal asymmetry, precisely mirroring what we observe in this study, emerges from the interference between the Primakoff process and the contribution from the gluon GTMD

# Probing quark OAM through $\pi^{0}$ production in ep collisions 



Probing quark OAM through $\pi^{0}$ production in ep collisions


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- Generalized TMDs/Wigner functions are the holy grail of spin physics


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- Probe quark OAM via exclusive $\pi^{0}$ production in ep collisions
- Circumvent challenges associated with double Drell-Yan process


- Longitudinal single-target spin asymmetry is not power suppressed
- Asymmetry is substantial \& thus exclusive $\pi^{0}$ production in ep collisions maybe a promising route to constrain quark OAM

