Angular momentum distribution for a quark dressed with a gluon

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Angular momentum of quarks and gluons

Question : can one decompose the nucleon spin into quark and gluon parts, spin and orbital ?

For a long time, it was thought that the gluon angular momentum cannot be separated into orbital and spin part in a gauge invariant way

However, experiments, for example at RHIC, have measured spin asymmetries that are sensitive to the gluon intrinsic spin. Experimental observables are gauge invariant. How to relate the experimentally measured quantity to the spin sum rule of the nucleon ?

Gauge invariant decomposition can be obtained by adding another term called potential angular momentum that can be added either to the quark or gluon part of the orbital angular momentum (OAM) – different decompositions

What is the density of the angular momentum due to the quarks and gluons ? Is the density different for different decompositions ?

In the next few slides, we discuss the key points for different decompositions taking QED as example. The decompositions are similar for QCD.

Jaffe and Manohar, Nucl. Phys. B (1990)
X. Ji, PRL (1997)
Chen et al, PRL (2008), (2009);
Wakamatsu, PRD(2010), PRD(2011)
Leader and Lorce, Phys. Rep, 2014

Different decompositions of angular momentum

In Belinfante decomposition, operator has purely orbital appearance, symmetric. Can be separated into electron and photon AM.

Each part gauge invariant

$$J^{\mu\nu\rho}_{\mathrm{Bel,q}}(x) = x^{\nu} T^{\mu\rho}_{\mathrm{Bel,q}}(x) - x^{\rho} T^{\mu\nu}_{\mathrm{Bel,q}}(x);$$

In Ji's decomposition, the Belinfante AM is rewritten in such a way that the electron AM can be separated into an orbital and spin part. Each part is gauge invariant and measurable, photon AM coincides with Benlinfante.

$$J^{\mu\nu\rho}_{\mathrm{kin,q}}(x) = L^{\mu\nu\rho}_{\mathrm{kin,q}}(x) + S^{\mu\nu\rho}_{\mathrm{q}}(x),$$

No further decomposition of photon AM into orbital and spin

$$L^{\mu\nu\rho}_{\mathrm{kin,q}}(x) + S^{\mu\nu\rho}_{\mathrm{q}}(x) = J^{\mu\nu\rho}_{\mathrm{Bel,q}}(x) - \frac{1}{2}\partial_{\sigma}[x^{\nu} S^{\sigma\mu\rho}_{\mathrm{q}}(x) - x^{\rho} S^{\sigma\mu\nu}_{\mathrm{q}}(x)].$$

X. Ji, PRL (1997)

In Jaffe-Manohar or canonical decomposition, AM is separated into electron and photon spin and OAM parts

Each of them are generators of rotation following Noether's theorem, but not all of them are gauge invariant

$$\begin{aligned} J^{\mu\nu\rho}_{\mathrm{q}}(x) &= L^{\mu\nu\rho}_{\mathrm{q}}(x) + S^{\mu\nu\rho}_{\mathrm{q}}(x) \\ &= \frac{1}{2}\bar{\psi}(x)\gamma^{\mu} x^{[\nu} i \overleftrightarrow{\partial}^{\rho]} \psi(x) + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}(x)\gamma_{\sigma}\gamma_{5}\psi(x). \end{aligned}$$

Jaffe and Manohar, Nucl. Phys. B (1990)

Different decompositions, contd.

Chen et al : photon field is split into two parts, pure and physical

$$\boldsymbol{A} = \boldsymbol{A}_{\text{pure}} + \boldsymbol{A}_{\text{phys}}. \qquad \boldsymbol{\nabla} \times \boldsymbol{A}_{\text{pure}} = \boldsymbol{0}, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{A}_{\text{phys}} = 0.$$

AM decomposed into electron and photon spin and OAM parts : each gauge invariant

Fields involved are non-local; decomposition same as Jaffe-Manohar decomposition in Coulomb gauge

Chen et al, PRL (2008), (2009)

Wakamatsu decomposition : subtracts the potential angular momentum from the electron part and compensates this in the photon part

AM is separated into spin an OAM of electron and photon, each gauge invariant

Makes Coulomb gauge special, physical photon field is non-local

Wakamatsu, PRD(2010), PRD(2011)

Decompositions of angular momentum: contd.

Angular momentum decompositions can be divided into two two families, kinetic and canonical.

Kinetic : potential AM attributed to photon (Belinfante, Ji, Wakamatsu)
Canonical : Potential AM attributed to electron (Jaffe-Manohar, Chen et al)

Covariant generalization of Chen et al decomposition : gauge invariant canonical (gic)

Covariant generalization of Wakamatsu's decomposition : gauge invariant kinetic (gik)

Leader and Lorce, Phys. Rep, 2014

Different decompositions at the density level also differ by superpotential terms, which are terms that become surface terms upon integration, and vanish when the fields vanish at the boundary.

Decomposition of angular momentum in QCD can be made along similar lines.

TABLE I. Properties of all the angular momentum densities derived from EMTs of the kinetic and canonical family.

Class	EMT	AM densities	Gauge invariant	Follow $SU(2)$ algebra
Kinetic	Belinfante	$\boldsymbol{J}_{\mathrm{Bel,q}}$	✓	✗
		$\boldsymbol{J}_{\mathrm{Bel,g}}$	✓	✗
	Ji	$\boldsymbol{L}_{\mathrm{Ji,q}}$	✓	✗
		$\boldsymbol{S}_{\mathrm{Ji,q}}$	✓	✓
		$\boldsymbol{J}_{\mathrm{Ji,g}}$	✓	✗
	Wakamatsu (gik)	$\boldsymbol{L}_{\mathrm{gic,q}}$	✓	✗
		$\boldsymbol{S}_{\mathrm{gic,q}}$	✓	✓
		$\boldsymbol{L}_{\mathrm{gic,g}}$	✓	✗
		$\boldsymbol{S}_{\mathrm{gic,g}}$	✓	✗
Canonical	Jaffe-Manohar	$\boldsymbol{L}_{\mathrm{JM,q}}$	✗	✓
		$\boldsymbol{S}_{\mathrm{JM,q}}$	✓	✓
		$\boldsymbol{L}_{\mathrm{JM,g}}$	✗	✗
		$\boldsymbol{S}_{\mathrm{JM,g}}$	✗	✗
	Chen *et al.* (gic)	$\boldsymbol{L}_{\mathrm{gic,q}}$	✓	✓
		$\boldsymbol{S}_{\mathrm{gic,q}}$	✓	✓
		$\boldsymbol{L}_{\mathrm{gic,g}}$	✓	✗
		$\boldsymbol{S}_{\mathrm{gic,g}}$	✓	✗

In order to be identified as the generator of rotation, the components of angular momentum have to obey SU(2) algebra. But that is not the case for individual components.

Leader and Lorce, Phys. Rep, (2014), (2019)

Also, individual components are not always gauge invariant

Model calculations of the different components in different decompositions is interesting

Model incorporating gluon ?

Ravi Singh, Sudeep Saha, AM, Nilmani Mathur, PRD 109, 016022(2024)

Angular momentum of a quark dressed with a gluon

Use two-component formalism in light-front Hamiltonian QCD : state expanded in Fock space in terms of multi-parton light-front wave functions (LEWFs)

$$|p,\sigma\rangle = \psi_1(p,\sigma) b_\sigma^\dagger(p)|0\rangle + \sum_{\lambda_1,\lambda_2} \int \frac{dk_1^+ d^2k_1^\perp dk_2^+ d^2k_2^\perp}{(16\pi^3)\sqrt{k_1^+ k_2^+}} \times \sqrt{16\pi^3 p^+}\psi_2(p,\sigma|k_1,\lambda_1;k_2,\lambda_2) \times \delta^{(3)}(p-k_1-k_2) b_{\lambda_1}^\dagger(k_1) a_{\lambda_2}^\dagger(k_2)|0\rangle.$$

Expansion can be truncated upto two-particle sector in a Boost invariant way, boost invariant LFWF :

$$\phi^{\sigma a}_{\lambda_1,\lambda_2}(x_i, \boldsymbol{\kappa}_i^\perp) = \sqrt{P^+}\psi_2(P,\sigma|k_1,\lambda_1;k_2,\lambda_2),$$

$$k_i^+ = x_i p^+, \qquad \boldsymbol{\kappa}_i^\perp = \boldsymbol{\kappa}_i^\perp + x_i \boldsymbol{p}^\perp,$$

$$\phi^{\sigma a}_{\lambda_1,\lambda_2}(x,\boldsymbol{\kappa}^\perp) = \frac{g}{\sqrt{2(2\pi)^3}} \left[\frac{x(1-x)}{\boldsymbol{\kappa}_\perp^2 + m^2(1-x)^2}\right] \frac{T^a}{\sqrt{1-x}} \times \chi^\dagger_{\lambda_1}\left[-\frac{2(\boldsymbol{\kappa}^\perp \cdot \boldsymbol{\epsilon}^{\perp *}_{\lambda_2})}{1-x} - \frac{1}{x}(\tilde{\boldsymbol{\sigma}}^\perp \cdot \boldsymbol{\kappa}^\perp)(\tilde{\boldsymbol{\sigma}}^\perp \cdot \boldsymbol{\epsilon}^{\perp *}_{\lambda_2}) + im(\tilde{\boldsymbol{\sigma}}^\perp \cdot \boldsymbol{\epsilon}^{\perp *}_{\lambda_2})\frac{1-x}{x}\right]\chi_\sigma \psi_1^\sigma,$$

Used two-component formalism of light-front QCD

Eliminated the constrained dof in light front gauge $A^+=0$

Two-particle LFWF can be obtained analytically using light-front eigenvalue equation

Zhang and Harindranath, PRD 48,4881 (1993)

Two-component formalism of light-front QCD

Fermion field is separated into 'good' and 'bad' components $\Psi = \psi^+ + \psi^-$

In light-front gauge, the constrained fields are removed using equations of constraints

Zhang and Harindranath, PRD 48, 4881 (1993)

$$i\partial^+\psi_- = (i\alpha^\perp \cdot \partial^\perp + g\alpha^\perp \cdot A^\perp + \beta m)\psi_+,$$

$$\frac{1}{2}\partial^+ E_a^- = (\partial^i E_a^i + g f^{abc} A_b^i E_c^i) - g\psi_+^\dagger T^a \psi_+, \qquad E_a^{-,i} = -\tfrac{1}{2}\partial^+ A_a^{-,i}$$

Using a suitable gamma matrix representation, we write the four-component fermion field in terms of two-component fields

$$\psi_+ = \begin{bmatrix} \xi \\ 0 \end{bmatrix}, \qquad \psi_- = \begin{bmatrix} 0 \\ \eta \end{bmatrix},$$

Two-component fields are given by :

$$\xi(y) = \sum_\lambda \chi_\lambda \int \frac{[dk]}{\sqrt{2(2\pi)^3}} [b_\lambda(k) e^{-ik\cdot y} + d_{-\lambda}^\dagger(k) e^{ik\cdot y}],$$

$$A^\perp(y) = \sum_\lambda \int \frac{[dk]}{\sqrt{2(2\pi)^3 k^+}} [\epsilon_\lambda^\perp a_\lambda(k) e^{-ik\cdot y} + \epsilon_\lambda^{\perp *} a_\lambda^\dagger(k) e^{ik\cdot y}],$$

$$\eta(y) = \left(\frac{1}{i\partial^+}\right) [\sigma^\perp \cdot (i\partial^\perp + gA^\perp(y)) + im] \xi(y),$$

OAM Distribution

Orbital angular momentum distribution in the front form is given by

$$\langle L^z\rangle(\boldsymbol{b}^\perp) = -i\epsilon^{3jk}\int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\left[\frac{\partial\langle T^{+k}\rangle_{\rm LF}}{\partial\Delta_\perp^j}\right],$$

$$\langle T^{+k}\rangle_{\rm LF} = \frac{\langle p',s|T^{+k}(0)|p,s\rangle}{2\sqrt{p'^+p^+}}$$

Instead of a proton, we use a dressed quark state

We use light front gauge and Drell-Yan frame, where the average transverse momentum of the system is zero

$$T^{+k}_{\rm kin,q}(0) = \sum_{\lambda,\lambda'}\int \frac{dk'^+d^2k'^\perp dk^+d^2k^\perp}{(16\pi^3)^2\sqrt{k'^+k^+}} b^\dagger_{\lambda'}(k')b_\lambda(k)\chi^\dagger_{\lambda'}[k'^k+k^k]\chi_\lambda$$
$$+2g\sum_{\lambda',\lambda,\lambda_3}\int \frac{dk'^+d^2k'^\perp dk^+d^2k^\perp dk_3^+d^2k_3^\perp}{(16\pi^3)^3k_3^+\sqrt{k'^+k^+}}[\epsilon^k_{\lambda_3}b^\dagger_{\lambda'}(k')b_\lambda(k)a_{\lambda_3}(k_3)+\epsilon^{k*}_{\lambda_3}b^\dagger_{\lambda'}(k')b_\lambda(k)a^\dagger_{\lambda_3}(k_3)].$$

Contribution to the matrix element comes from both diagonal and off-diagonal overlaps

$$p^\mu = \left(P^+, -\frac{\boldsymbol{\Delta}^\perp}{2}, \frac{1}{P^+}\left(m^2+\frac{\boldsymbol{\Delta}^{\perp 2}}{4}\right)\right),$$

$$p'^\mu = \left(P^+, \frac{\boldsymbol{\Delta}^\perp}{2}, \frac{1}{P^+}\left(m^2+\frac{\boldsymbol{\Delta}^{\perp 2}}{4}\right)\right),$$

$$\Delta^\mu = (p'-p)^\mu = (0, \boldsymbol{\Delta}^\perp, 0),$$

Orbital angular momentum density

OAM density in impact parameter space

$$\langle L^z \rangle(\boldsymbol{b}^\perp) = -i\epsilon^{3jk} \int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}^\perp \cdot \boldsymbol{b}^\perp} \left[\frac{\partial \langle T^{+k} \rangle_{\text{LF}}}{\partial \Delta^j_\perp} \right],$$

Contribution coming from the quark part of the EMT for a dressed quark state

For the kinetic OAM, we obtain

$$\langle L^z_{\text{kin,q}} \rangle(\boldsymbol{b}^\perp) = \frac{g^2 C_F}{72\pi^2} \int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{b}^\perp \cdot \boldsymbol{\Delta}^\perp} \left[-7 + \frac{6}{\omega}\left(1 + \frac{2m^2}{\Delta^2}\right) \log\left(\frac{1+\omega}{-1+\omega}\right) - 6 \log\left(\frac{\Lambda^2}{m^2}\right) \right],$$

Λ : cutoff on transverse momentum

$$\omega = \sqrt{1 + \frac{4m^2}{\Delta^2}}$$

Off-diagonal term corresponds to the potential angular momentum and it vanishes

Also observed in scalar diquark model and QED

D.A. Amor-Quiroz, M. Burkardt, W. Focillon, and C. Lorcé, Eur. Phys. J. C 81, 589 (2021). X. Ji, A. Schäfer, F. Yuan, J.-H. Zhang, and Y. Zhao, Phys. Rev. D 93, 054013 (2016).

OAM density : details

We have calculated the matrix element of the EMT in terms of the LFWF of a dressed quark

$$\frac{\langle 2,\uparrow|T^{+k}_{\text{kin,q}}(0)|2,\uparrow\rangle}{2p^+} = \frac{1}{2}\sum_{\lambda_1',\lambda_1,\lambda_2}\int dx d^2\boldsymbol{\kappa}^\perp \phi^{*\uparrow}_{\lambda_1',\lambda_2}(x,\boldsymbol{\kappa}'^\perp)\chi^\dagger_{\lambda_1'}(2\boldsymbol{\kappa}^k + (1-x)\boldsymbol{\Delta}^k)\chi_{\lambda_1}\phi^\uparrow_{\lambda_1,\lambda_2}(x,\boldsymbol{\kappa}^\perp),$$

$$\boldsymbol{\kappa}'^\perp = \boldsymbol{\kappa}^\perp + (1-x)\boldsymbol{\Delta}^\perp.$$

Diagonal overlap

$$= g^2 C_F \int \frac{dx d^2\boldsymbol{\kappa}^\perp}{16\pi^3}\frac{(2\kappa^k + (1-x)\Delta^k)}{(1-x)D_1 D_2}[m^2(1-x)^4 + (1+x^2)\boldsymbol{\kappa}^{\perp 2} + (1-x)(1+x^2)\boldsymbol{\kappa}^\perp\cdot\boldsymbol{\Delta}^\perp$$
$$+ i(1-x)(1-x^2)(\boldsymbol{\kappa}^{(1)}\Delta^{(2)} - \boldsymbol{\kappa}^{(2)}\Delta^{(1)})],$$

$$D_1 = \boldsymbol{\kappa}^{\perp 2} + m^2(1-x)^2,$$

$$D_2 = (\boldsymbol{\kappa}^\perp + (1-x)\boldsymbol{\Delta}^\perp)^2 + m^2(1-x)^2.$$

$$\frac{1}{2p^+}[\langle 1,\uparrow|T^{+k}_{\text{kin,q}}(0)|2,\uparrow\rangle + \langle 2,\uparrow|T^{+k}_{\text{kin,q}}(0)|1,\uparrow\rangle]$$

Non-diagonal overlap : gives zero

$$= \frac{g}{\sqrt{16\pi^3}}\sum_{\lambda_1,\lambda_2}\int\frac{dx d^2\boldsymbol{\kappa}^\perp}{\sqrt{1-x}}[\psi_1^*(P,\sigma')\chi^\dagger_{\sigma'}\epsilon^k_{\lambda_2}\chi_{\lambda_1}\phi^\sigma_{\lambda_1,\lambda_2}(x,\boldsymbol{\kappa}^\perp) + \phi^{*\sigma'}_{\lambda_1,\lambda_2}(x,\boldsymbol{\kappa}^\perp)\chi^\dagger_{\lambda_1}\epsilon^{k*}_{\lambda_2}\chi_\sigma\psi_1(P,\sigma)],$$

Spin density

$$\langle S^z\rangle(\boldsymbol{b}^\perp) = \frac{1}{2}\epsilon^{3jk}\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\langle S^{+jk}\rangle_{\mathrm{LF}}.$$

Kinetic spin angular momentum is calculated using the overlap

$$\frac{\langle 2,\uparrow|S_q^{+jk}(0)|2,\uparrow\rangle}{2p^+}$$

$$= \frac{1}{4}\epsilon^{+jk-}\sum_{\lambda_1,\lambda_1',\lambda_2}\int dx d^2\boldsymbol{\kappa}^\perp \phi^{*\uparrow}_{\lambda_1',\lambda_2}(x,\boldsymbol{\kappa}'^\perp)(\chi^\dagger_{\lambda_1'}\sigma^{(3)}\chi_{\lambda_1})\phi^{\uparrow}_{\lambda_1,\lambda_2}(x,\boldsymbol{\kappa}^\perp)$$

$$= \frac{g^2C_F}{4}\epsilon^{+jk-}\int\frac{dxd^2\boldsymbol{\kappa}^\perp}{8\pi^3}\frac{1}{(1-x)D_1D_2}$$
$$\times[\boldsymbol{\kappa}^{\perp 2}(1+x^2)+\boldsymbol{\kappa}^\perp\cdot\boldsymbol{\Delta}^\perp(1-x)(1+x^2)+i(1-x)(1-x^2)(\boldsymbol{\kappa}^{(1)}\Delta^{(2)}-\boldsymbol{\kappa}^{(2)}\Delta^{(1)})-m^2(1-x)^4].$$

$$\langle S^z_{\mathrm{kin,q}}\rangle(\boldsymbol{b}^\perp) = -\frac{g^2C_F}{32\pi^2}\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\int\frac{dx}{1-x}$$
$$\times\left[\omega(1+x^2)\log\left(\frac{1+\omega}{-1+\omega}\right)+\left(\frac{1-\omega^2}{\omega}\right)x\log\left(\frac{1+\omega}{-1+\omega}\right)-(1+x^2)\log\left(\frac{\Lambda^2}{m^2(1-x)^2}\right)\right].$$

Belinfante AM

$$\langle J^z_{\rm Bel}\rangle(\boldsymbol{b}^\perp) = -i\epsilon^{3jk}\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\left[\frac{\partial\langle T^{+k}_{\rm Bel}\rangle_{\rm LF}}{\partial\Delta^j_\perp}\right].$$

Connection between Belinfante and kinetic AM is through the superpotential term

$$\langle M^z\rangle(\boldsymbol{b}^\perp) = \frac{1}{2}\epsilon^{3jk}\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\Delta^l_\perp\frac{\partial\langle S^{l+k}\rangle_{\rm LF}}{\partial\Delta^j_\perp}.$$

$$\begin{aligned}\langle J^z_{\rm Bel,q}\rangle(\boldsymbol{b}^\perp) = g^2C_F\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{b}^\perp\cdot\boldsymbol{\Delta}^\perp}\int\frac{dx}{16\pi^2}\frac{1}{(1-x)\Delta^4\omega^3}\\ \times\Big[(8m^4(1-2x)(1-x(1-x))+6m^2(1-(2-x)x(1+2x))\Delta^2+(1-(2-x)x(1+2x))\Delta^4)\log\left(\frac{1+\omega}{-1+\omega}\right)\\ -\omega\Delta^2\left(4m^2(1-(1-x)x)+(1+x^2)\Delta^2+(1-(2-x)x(1+2x))(4m^2+\Delta^2)\log\left(\frac{\Lambda^2}{m^2(1-x)^2}\right)\right)\Big].\end{aligned}$$

$$\begin{aligned}\langle M^z_{\rm q}\rangle(\boldsymbol{b}^\perp) = \frac{g^2C_F}{32\pi^2}\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\int\frac{dx}{(1-x)}\frac{1}{\omega^3\Delta^4}\\ \times[\omega\Delta^2((4m^2+\Delta^2)(1+x^2)-4m^2x)\\ -2m^2((4m^2+\Delta^2)(1+x^2)-4m^2x-2x\Delta^2)].\end{aligned}$$

Canonical, gic and gik distributions

$$\langle L^z_{\text{kin,q}}\rangle(\boldsymbol{b}^\perp) = \langle L^z_{\text{q}}\rangle(\boldsymbol{b}^\perp), \quad \langle S^z_{\text{kin,q}}\rangle(\boldsymbol{b}^\perp) = \langle S^z_{\text{q}}\rangle(\boldsymbol{b}^\perp).$$

This is because the nondiagonal matrix element in the kinetic term, that corresponds to quark-gluon interaction gives zero contribution. So effectively we have the same operator structure contributing in both kinetic and canonical, in light-front gauge

$$T^{+k}_{\text{gic,q}} = \frac{1}{2}\bar{\psi}(x)\gamma^+ i\overleftrightarrow{D}^k_{\text{pure}}\psi(x),$$

$$\overleftrightarrow{D}^\mu_{\text{pure}} = \overleftrightarrow{\partial}^\mu - 2igA^\mu_{\text{pure}} \qquad A^\mu_{\text{pure}} = A^\mu - A^\mu_{\text{phys}}.$$

$$T^{+k}_{\text{gic,q}} = \frac{1}{2}\bar{\psi}(x)\gamma^+ i\overleftrightarrow{\partial}^k\psi(x) = T^{+k}_{\text{q}}.$$

In light front gauge, the gic decomposition gives the same distributions as canonical

$$\langle L^z_{\text{gic,q}}\rangle(\boldsymbol{b}^\perp) = \langle L^z_{\text{q}}\rangle(\boldsymbol{b}^\perp), \quad \langle S^z_{\text{gic,q}}\rangle(\boldsymbol{b}^\perp) = \langle S^z_{\text{q}}\rangle(\boldsymbol{b}^\perp).$$

Finally the gik decomposition gives the same distribution as kinetic for a dressed quark

$$T^{\mu\nu}_{\text{gik,q}} = T^{\mu\nu}_{\text{kin,q}}.$$

$$\langle L^z_{\text{gik,q}}\rangle(\boldsymbol{b}^\perp) = \langle L^z_{\text{kin,q}}\rangle(\boldsymbol{b}^\perp), \quad \langle S^z_{\text{gik,q}}\rangle(\boldsymbol{b}^\perp) = \langle S^z_{\text{kin,q}}\rangle(\boldsymbol{b}^\perp).$$

OAM distributions

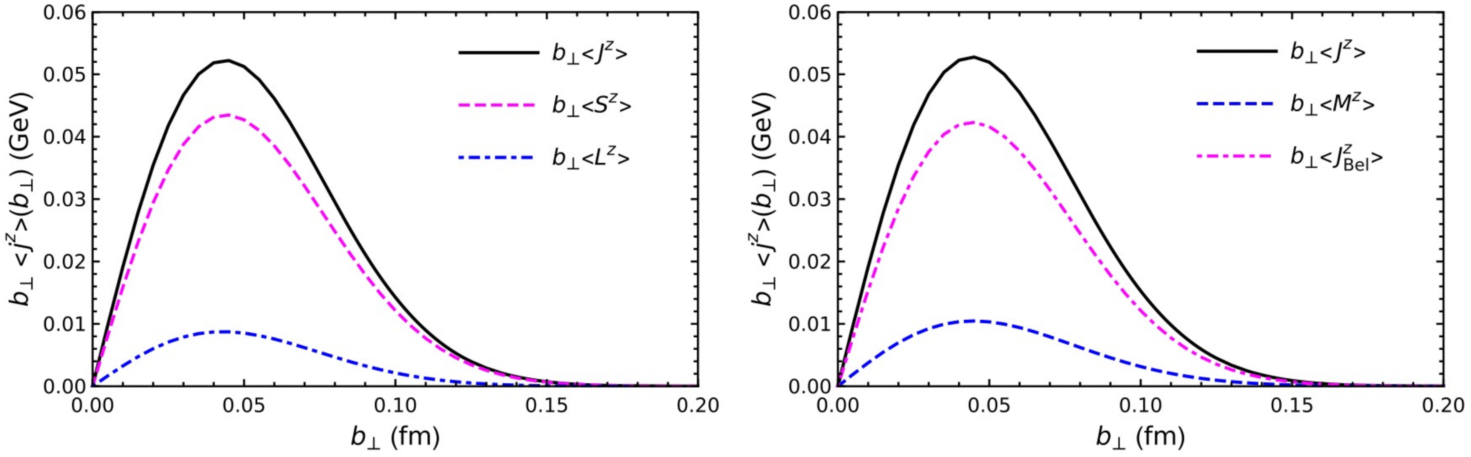

FIG. 1. Longitudinal angular momentum distribution of quarks as a function of impact parameter $b_\perp$. Left: Sum of the kinetic orbital AM $b_\perp\langle L^z\rangle$ (dot-dashed line) and spin AM $b_\perp\langle S^z\rangle$ (dashed line) given by kinetic total AM $b_\perp\langle J^z\rangle$ (solid line). Right: Kinetic total AM $b_\perp\langle J^z\rangle$ (solid line) is given by the sum of Belinfante total AM $b_\perp\langle J^z_{\mathrm{Bel}}\rangle$ (dot-dashed line) and the correction term corresponding to the total divergence $b_\perp\langle M^z\rangle$ (dashed line). Here, $m = 0.3$ GeV, $g = 1$, $C_f = 1$, and $\Lambda = 1.7$ GeV. We chose the Gaussian width $\sigma = 0.1$ GeV.

For the densities, we use a Gaussian wave packet state

$$\frac{1}{16\pi^3}\int \frac{d^2\boldsymbol{p}^\perp dp^+}{p^+}\phi(p)|p^+,\boldsymbol{p}^\perp,\lambda\rangle$$

$$\phi(p) = p^+\delta(p^+ - p_0^+)\phi(\boldsymbol{p}^\perp).$$

$$\phi(\boldsymbol{p}^\perp) = e^{-\frac{\boldsymbol{p}^{\perp 2}}{2\sigma^2}},$$

Spin distribution dominates over OAM distribution, similar to other calculations for proton

Superpotential term is positive throughout, in contrast to some other model calculations, where it has a positive core but negative near the periphery

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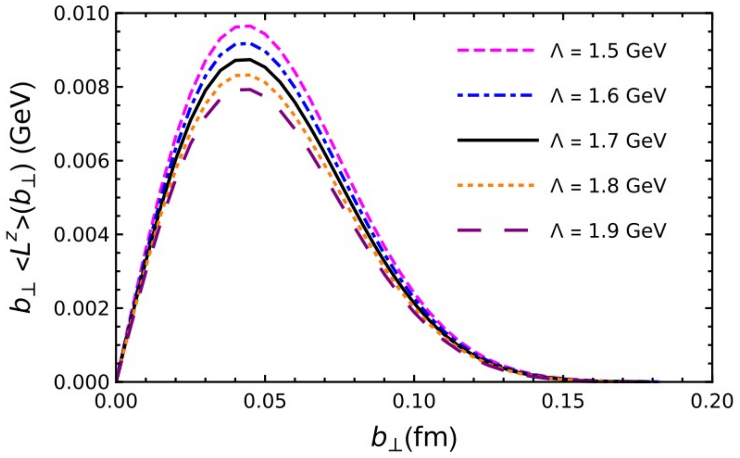
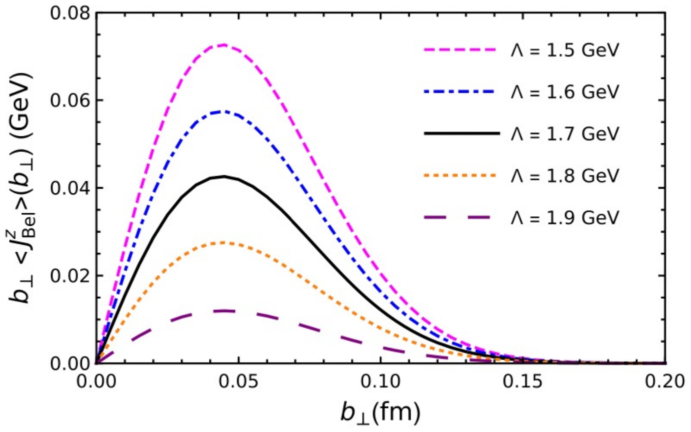
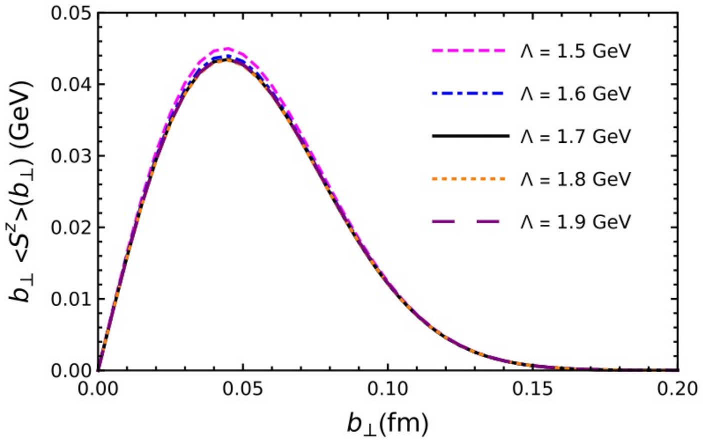
Scale dependence







Considered only quark part of EMT ; Contribution from gluon part needs to be calculated

Contribution from quark part depends on renormalization scale, in our approach cutoff on transverse momentum

This is because of the quark-gluon interaction present

Belinfante AM depends strongly on the cutoff

For other plots we have used Λ =1.7 GeV

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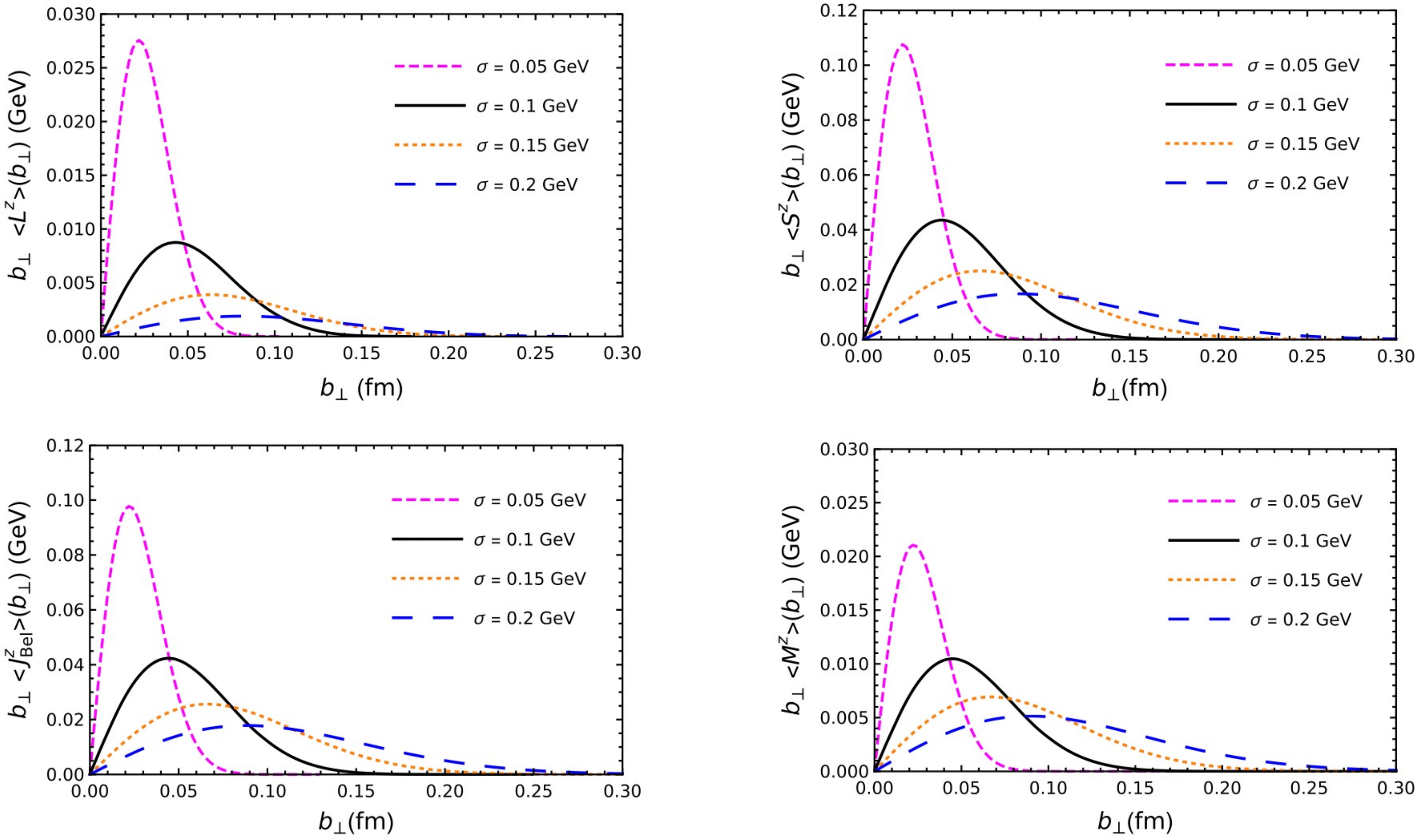
Dependence on Gaussian Width



Distributions spread and are broader for larger width of the Gaussian wave packet

Peak shifts towards larger impact parameter

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Axial vector form factor

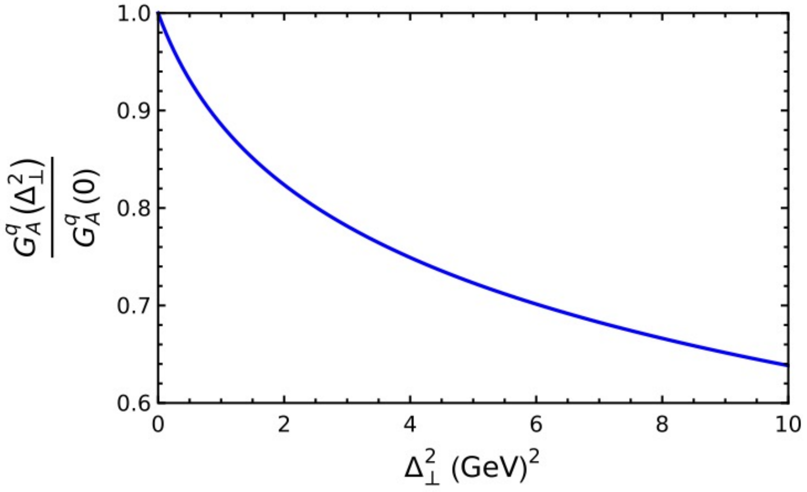

FIG. 4. Axial vector form factor $\frac{G_A^q(\Delta_\perp^2)}{G_A^q(0)}$ as a function of $\Delta_\perp^2$. Here $m = 0.3$ GeV, $g = 1$, $\Lambda = 1.7$ GeV, and $C_f = 1$.

Qualitative behavior similar to other models of the proton and lattice calculation

$$\langle p', s'|S^{\mu\alpha\beta}(0)|p, s\rangle = \frac{1}{2}\epsilon^{\mu\alpha\beta\lambda}\bar{u}(p', s')\left[\gamma_\lambda\gamma_5 G_A^q(t) + \frac{\Delta_\lambda\gamma_5}{2M}G_P^q(t)\right]u(p, s),$$

Matrix element of spin density operator is parametrized in terms of axial vector and pseudoscalar form factors

Antisymmetric part of EMT is related to the divergence of the spin operator

$$\bar{\psi}(x)[\gamma^\alpha i\overleftrightarrow{D}^\beta - \gamma^\beta i\overleftrightarrow{D}^\alpha]\psi(x) = -\epsilon^{\mu\alpha\beta\lambda}\partial_\mu[\bar{\psi}(x)\gamma_\lambda\gamma_5\psi(x)].$$

$$D_q(t) = -G_A^q(t). \qquad \frac{\langle p', s|S_q^{+jk}(0)|p, s\rangle}{2p^+} = \frac{1}{2}\epsilon^{+jk-}s^z G_A^q(t),$$

$$D_q(t) = -\frac{g^2 C_F}{16\pi^2}\int\frac{dx}{1-x}\left[\omega(1+x^2)\log\left(\frac{1+\omega}{-1+\omega}\right) + \left(\frac{1-\omega^2}{\omega}\right)x\log\left(\frac{1+\omega}{-1+\omega}\right) - (1+x^2)\log\left(\frac{\Lambda^2}{m^2(1-x)^2}\right)\right],$$

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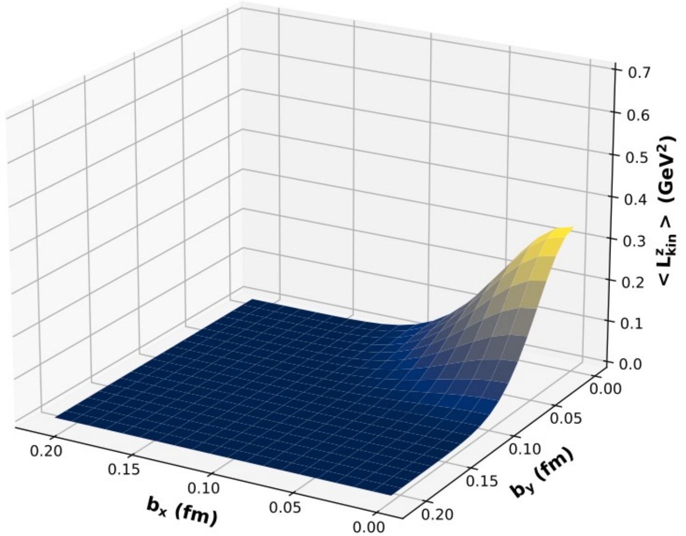
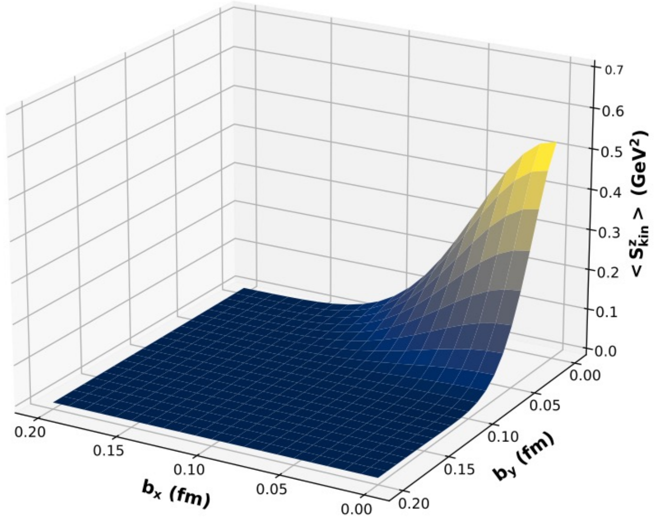
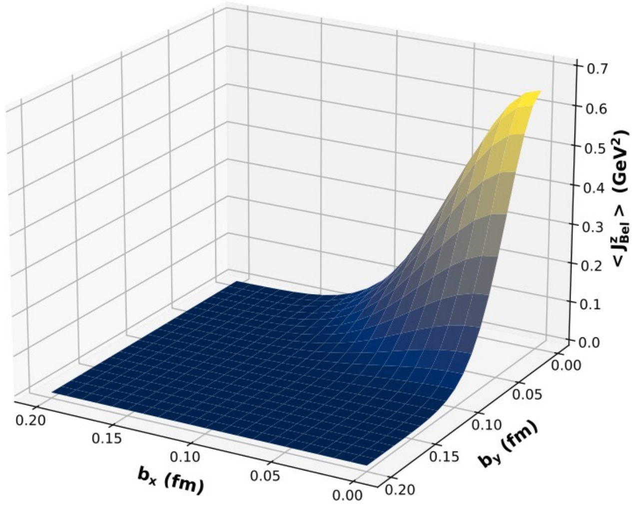
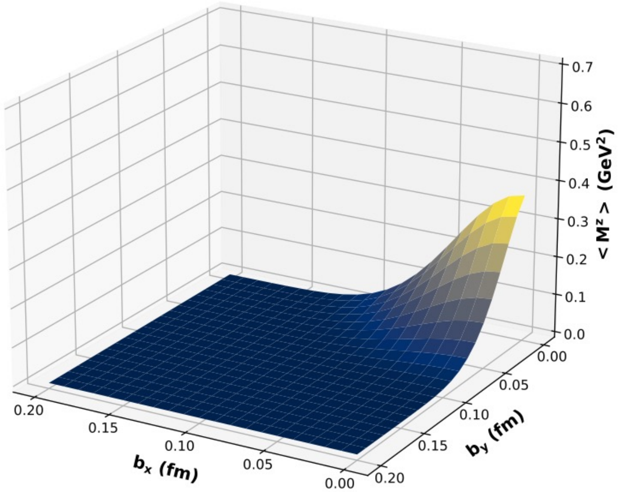




3D plots in impact parameter space

OAM and spin distribution





Belinfante total AM and correction term

Results depend on scale. Contribution from gluon part of the EMT needs to be included to obtain the spin sum rule, with spin of the state independent of the scale

Summary and conclusion

Presented a calculation of angular momentum in different decompositions for a spin ½ relativistic composite state, namely a quark dressed with a gluon

Used two-component formalism in light-front Hamiltonian QCD in light cone gauge, constrained fields are eliminated using equations of constraint

Result in gauge invariant kinetic (gik) decomposition agrees with kinetic decomposition and result for gauge invariant canonical (gic) agrees with canonical decomposition. Also, kinetic and canonical decomposition give the same results.

As we have calculated the contribution from the quark part of the EMT only, result depends on the renormalization scale, which is the cutoff on transverse momentum integral in this approach.

Potential term is zero, as already seen in QED

So far, presented only contribution coming from the quark part of AM, contribution from the gluon part is being calculated. Important to verify the spin sum rule.