Approximate N^3LO PDF Benchmarking.

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Aproximate N^{3}LO (and Higher Orders)

Leading source of uncertainties from Missing Higher Orders in perturbation theory. Numerous sources of this, i.e. splitting functions

$$\boldsymbol{P}(x,\alpha_s) = \alpha_s \boldsymbol{P}^{(0)}(x) + \alpha_s^2 \boldsymbol{P}^{(1)}(x) + \alpha_s^3 \boldsymbol{P}^{(2)}(x) + \alpha_s^4 \boldsymbol{P}^{(3)}(x) + \ldots ,$$

but also heavy flavour transition matrix elements and cross-sections

$$F(x,Q^2) = \sum_{\alpha\in\{H,q,g\}} \Big(C_{q,\alpha}^{\mathrm{VF},\, n_f+1} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2) + C_{H,\alpha}^{\mathrm{VF},\, n_f+1} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2) \Big) ,$$

$$\sigma_2^{had}(x_1,x_2,Q^2) = \sum_{\alpha,\beta\in\{H,q,g\}} (\sigma_{\alpha,\beta}^{\mathrm{VF},\, n_f+1} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2) \otimes A_{\beta j}(Q^2/m_h^2) \otimes f_j^{n_f}(Q^2) ,$$

Current knowledge is up to NNLO, with full higher orders unknown. However, already significant progress in calculating at N^{3}LO [1-12].

$\mathrm{N^3LO}$ - What do we know?

Zero-mass structure function $\mathrm{N^3LO}$ coefficient functions are known [1]. Recently, final parts of transition matrix elements $A_{gg,H}$, A_{Hg} [13,14].

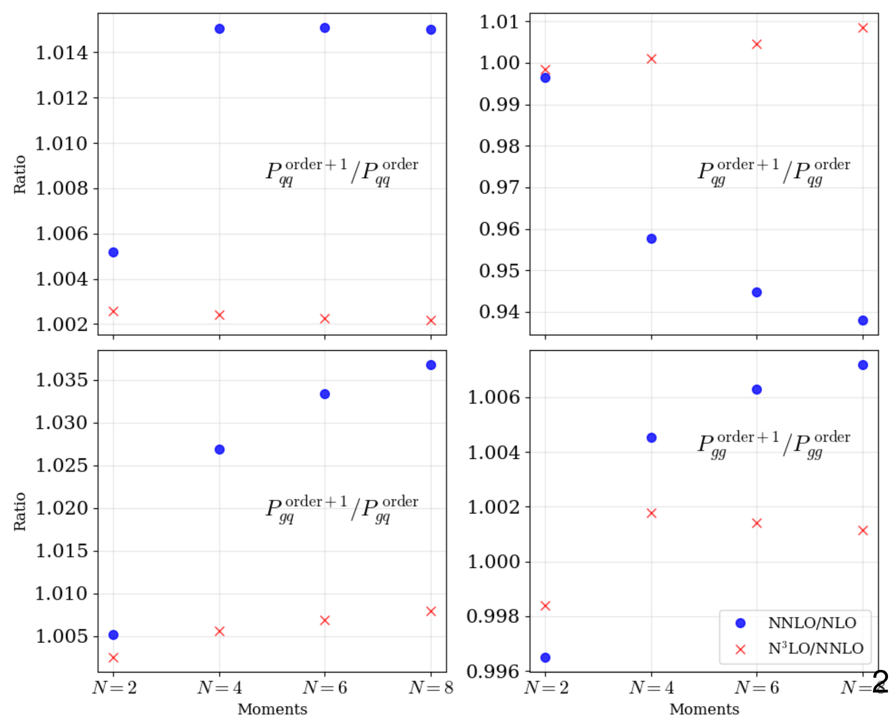
Some knowledge of leading terms in the small x and large x regime. Unknown subleading terms weakly constrained from precedent, approx C_F/C_A relations, smoothness etc. Example case

$$\boldsymbol{P}^{(3)}_{qg}(x) \to \frac{C_A^3}{3\pi^4}\left(\frac{82}{81} + 2\zeta_3\right)\frac{1}{2}\frac{\ln^2 1/x}{x} + \rho_{qg}\,\frac{\ln 1/x}{x},$$

Some numerical constraints (Low-integer Mellin moments), until recently [2-11].

Intuition from lower orders and expectations from perturbation theory.

Very little about many cross-sections (K-factors).



Splitting Functions at aN^{3}LO - MSHT [15]

N_m Mellin moments [1-5] (Moch et al.) can be used as constraints for

$$F(x) = \sum_{i=1}^{N_m} A_i f_i(x) + f_e(x).$$

Choose a set of functions and solve for A_i. (Similar for $A_{\alpha i}(m_h^2)$).

Introduce a degree of freedom a, interpreted as a nuisance parameter allowed to vary in a PDF fit, $f_e(x) \rightarrow f_e(x, a)$. In our treatment it is the coefficient of the most divergent unknown small-x term, e.g. for $\boldsymbol{P}_{qg}^{(3)}(x)$

$$f_1(x) = \frac{1}{x} \quad \text{or} \quad \ln^4 x \quad \text{or} \quad \ln^3 x \quad \text{or} \quad \ln^2 x,$$

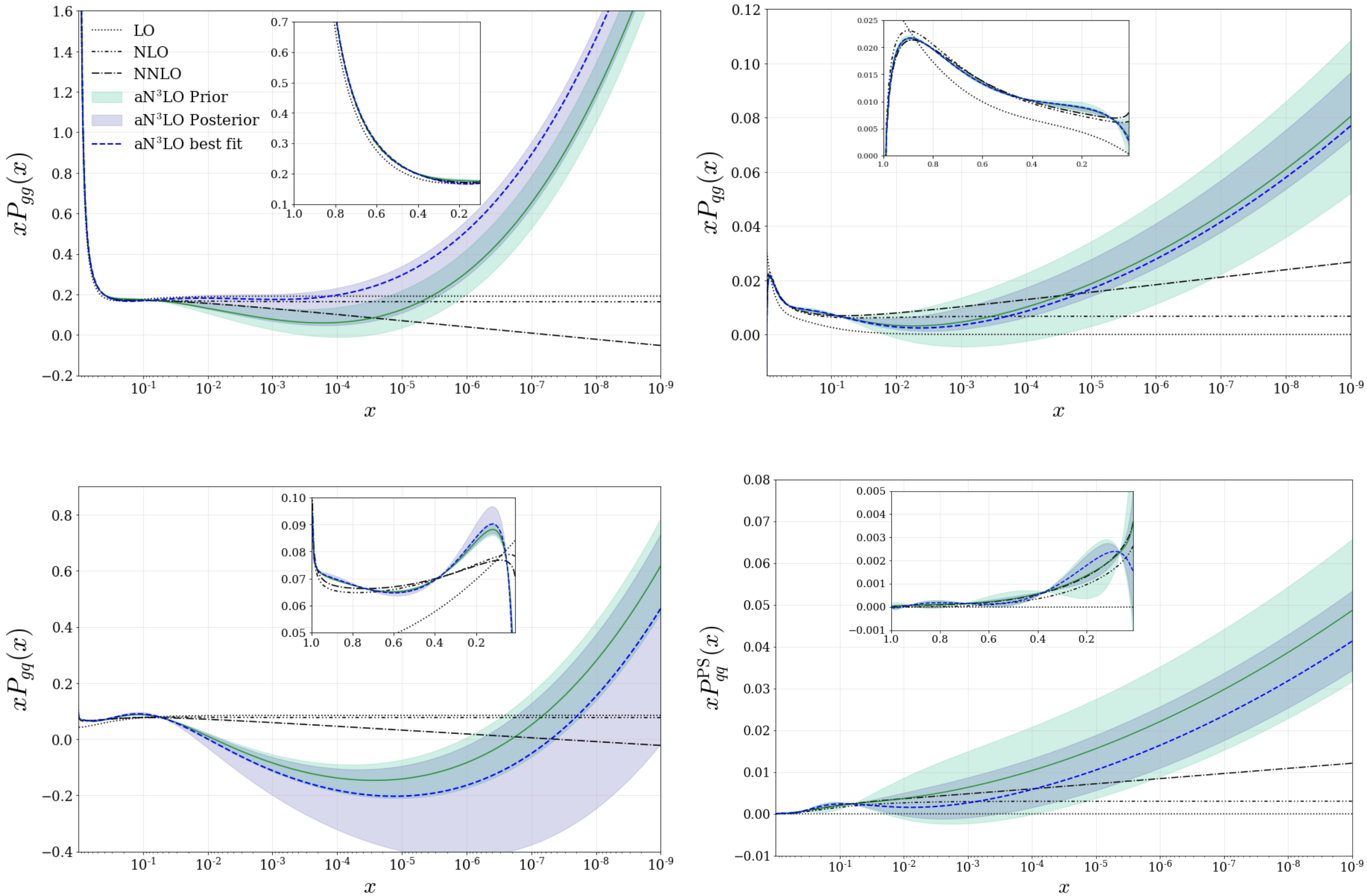
$$f_2(x) = \ln x,$$

$$f_2(x) = 1 \quad \text{or} \quad x \quad \text{or} \quad x^2,$$

$$f_3(x) = \ln^4(1-x) \quad \text{or} \quad \ln^3(1-x) \quad \text{or} \quad \ln^2(1-x) \quad \text{or} \quad \ln(1-x),$$

$$f_e(x, \rho_{qg}) = \frac{C_A^3}{3\pi^4}\left(\frac{82}{81} + 2\zeta_3\right)\frac{1}{2}\frac{\ln^2 1/x}{x} + \rho_{qg}\frac{\ln 1/x}{x}.$$

Resulting splitting functions



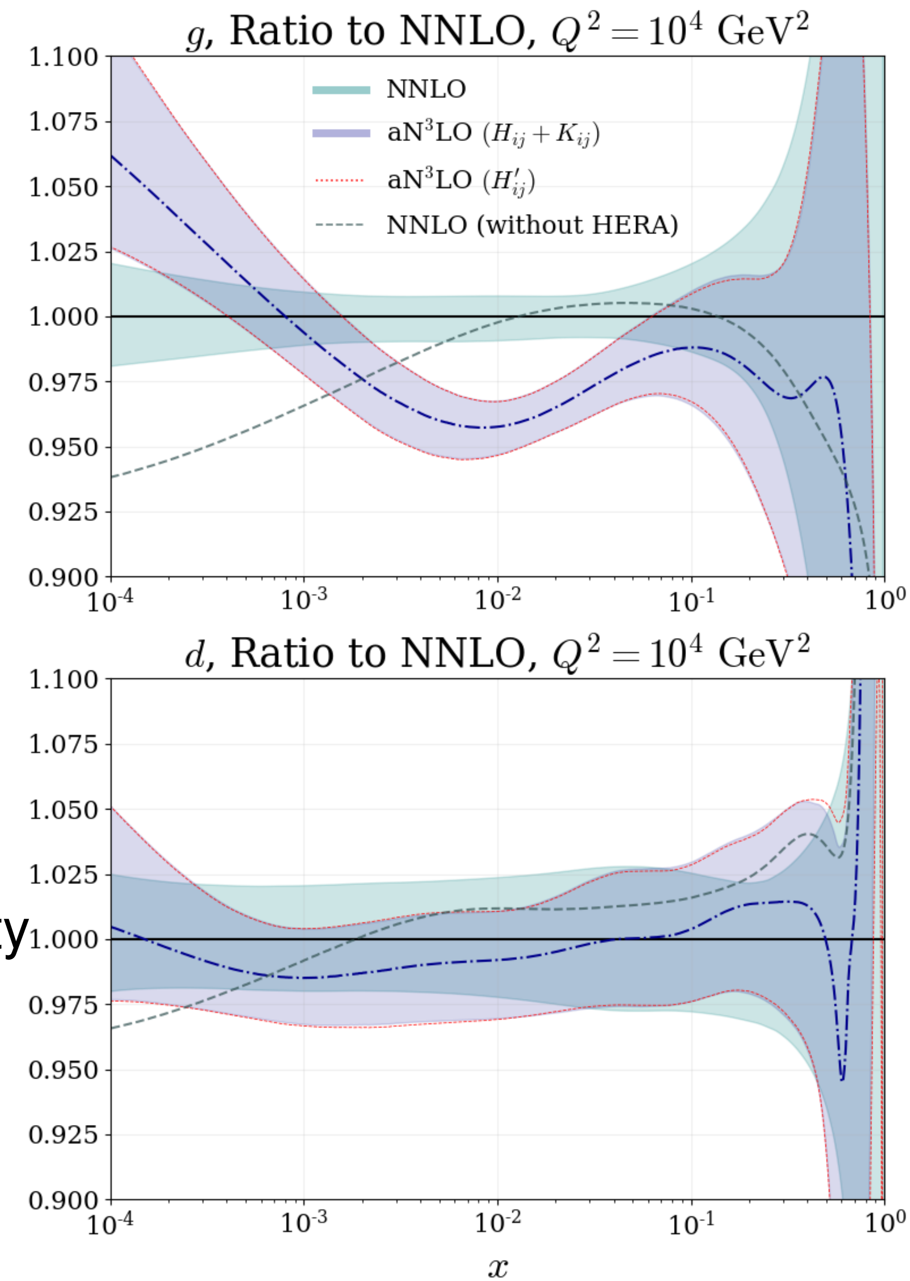
Uncertainty largest at small x. Best fit largely compatible with best estimate.

The PDFs at aN^{3}LO compared to NNLO - detail.

The gluon is enhanced at small-x due to the large logarithms present at higher orders.

Light quarks enhanced slightly at high x.

Correlated and uncorrelated K-factors show consistent uncertainty predictions.



NNPDF study recently completed [16]. Similar in numerous respects.

Approximate N^3LO splitting functions as

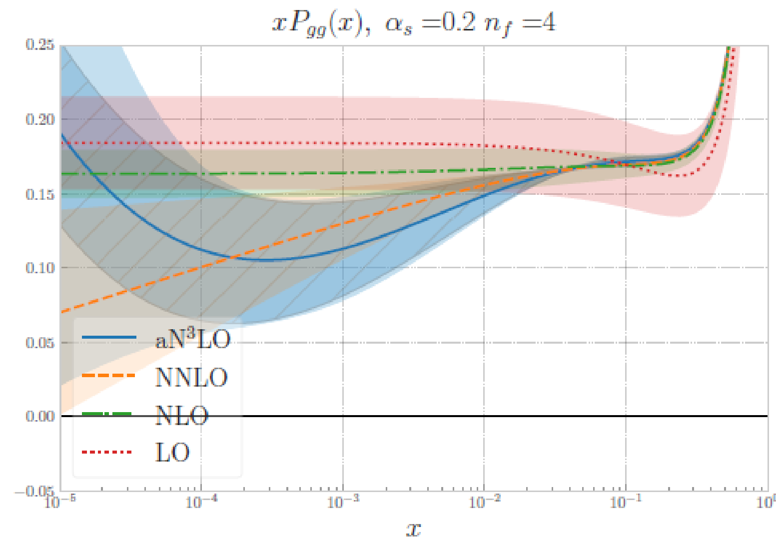
$$\gamma_{ij}^{(3)} = \gamma_{ij,n_f^3}^{(3)} + \gamma_{ij,N\to\infty}^{(3)} + \gamma_{ij,N\to 0}^{(3)} + \tilde{\gamma}_{ij}^{(3)}$$

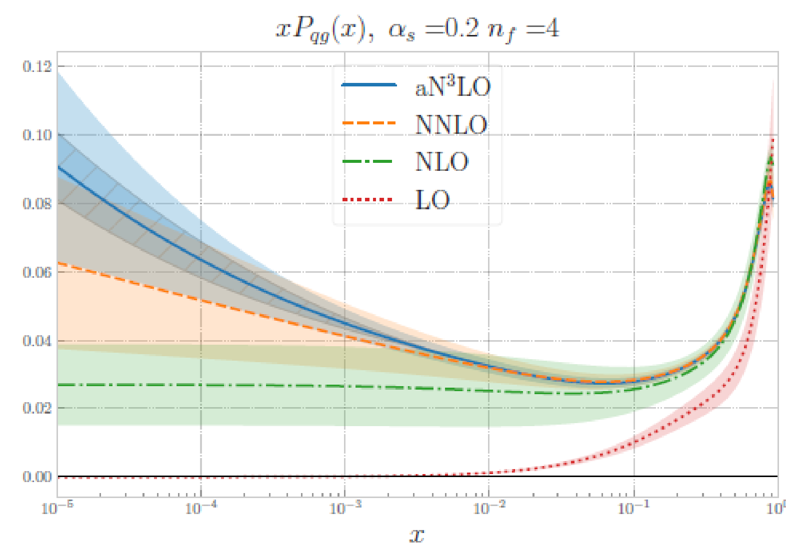
Parametrise $\tilde{\gamma}_{ij}^{(3)} = \sum_l a_{ij}^{(l)} G_l(N)$

– G_1 for the leading unknown large-N term
– G_2 for the leading unknown small-N term
– 3 or 8 G_l for the sub-leading unknown small- and large-N contributions
– vary the functions G_l to generate a variety of approximations and estimate IHOU
– determine the coefficients $a_{ij}^{(l)}$ with known moments and momentum conservation

Adopted basis function for $\tilde{\gamma}_{qq}^{(3)}$

$G_1(N)$	$\mathcal{M}[(1-x)\ln^2(1-x)]$
$G_2(N)$	$-\frac{1}{(N-1)^2} + \frac{1}{N^2}$
$G_3(N)$	$\frac{1}{N^4}$, $\frac{1}{N^3}$, $\mathcal{M}[(1-x)\ln(1-x)]$ $\mathcal{M}[(1-x)^2\ln(1-x)^2]$, $\frac{1}{N-1}-\frac{1}{N}$, $\mathcal{M}[(1-x)\ln(x)]$
$G_4(N)$	$\mathcal{M}[(1-x)(1+2x)]$, $\mathcal{M}[(1-x)x^2]$, $\mathcal{M}[(1-x)x(1+x)]$, $\mathcal{M}[(1-x)]$





Parts unknown at N^{3}LO estimated using existing covariance matrix/scale variation approach.

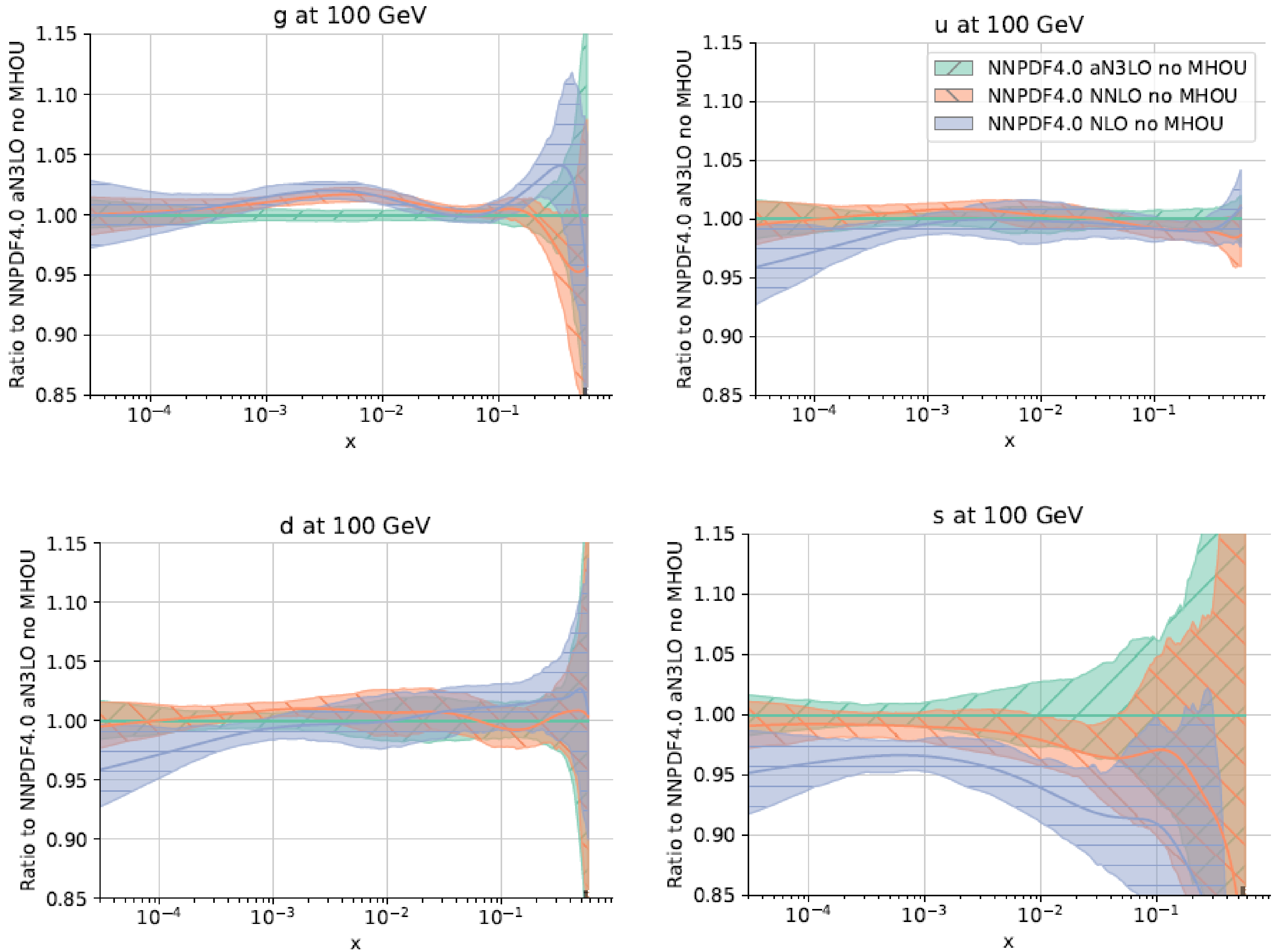
$$\Delta_m(ij,k) = T_m(ij,k) - \bar{T}_m,$$

$$\mathrm{cov}_{mn}^{(ij)} = \frac{1}{\widetilde{N}_{ij}-1}\sum_{k=1}^{\widetilde{N}_{ij}} \Delta_m(ij,k)\Delta_n(ij,k).$$

$$\mathrm{cov}_{mn}^{\mathrm{IHOU}} = \mathrm{cov}_{mn}^{(gg)} + \mathrm{cov}_{mn}^{(gq)} + \mathrm{cov}_{mn}^{(qg)} + \mathrm{cov}_{mn}^{(qq)}.$$

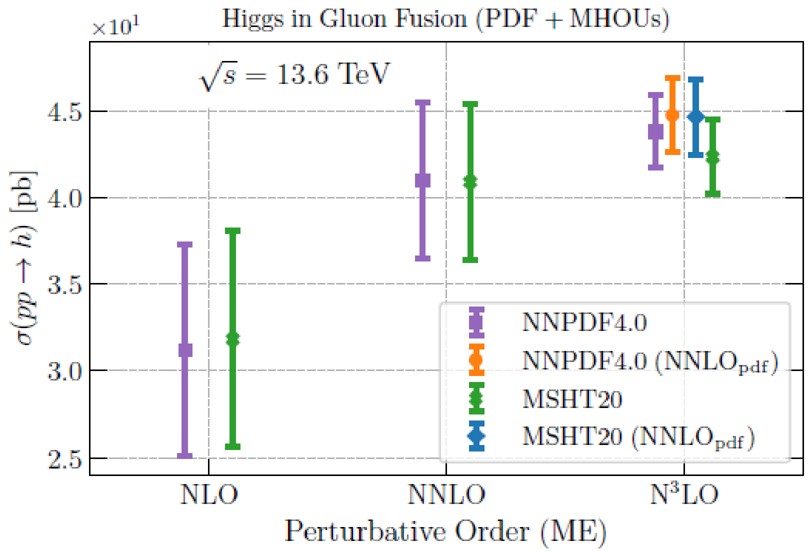
$$(\sigma_{ij}(N))^2 = \frac{1}{\widetilde{N}_{ij}-1}\sum_{k=1}^{\widetilde{N}_{ij}} \left(\gamma_{ij}^{(3),\,(k)}(N) - \gamma_{ij}^{(3)}(N)\right)^2.$$

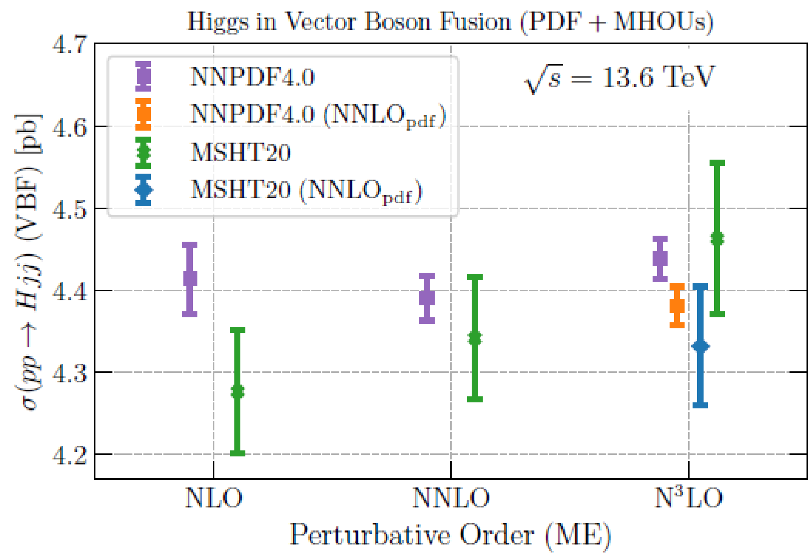
Gives uncertainty on splitting functions, similar approach for other quantities.



PDFs - main change in g

Consequences for Higgs Cross Sections.





Changes in N^{3}LO cross section relative to use of NNLO PDFs obvious.
Smaller for NNPDF than MSHT.

Recent improvements in knowledge of splitting functions.

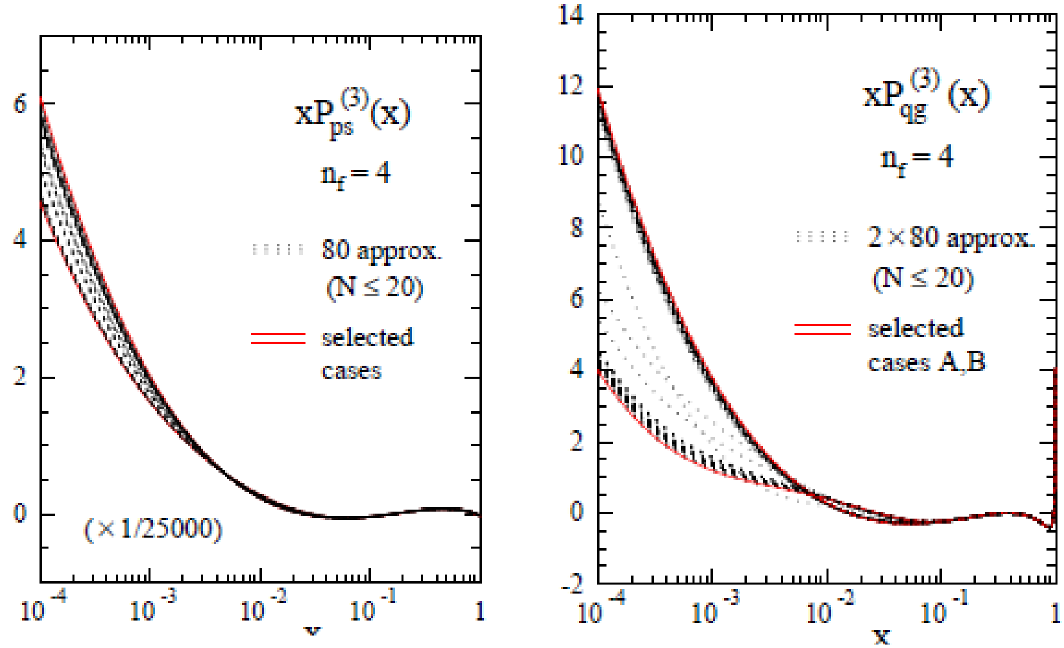
Very recently [17-19] more moments have become availble for splitting functions

Now 5 moments available for P_{gg}, P_{gq}. Allows improved constraint provided by [17] (Moch et al.)

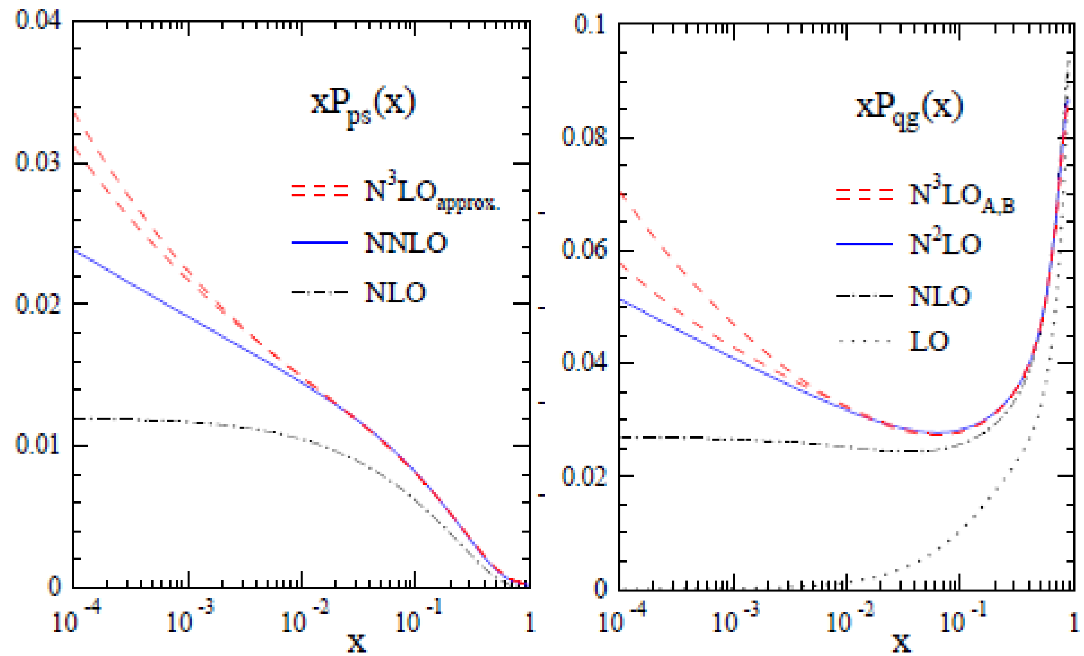
Also now 10 moments for P_{qq}^{PS} and P_{qg}. Allows much improved constraint in [17,18] (Falconi, et al.).

Also now 10 moments for P_{gq} (Falconi - this meeting).

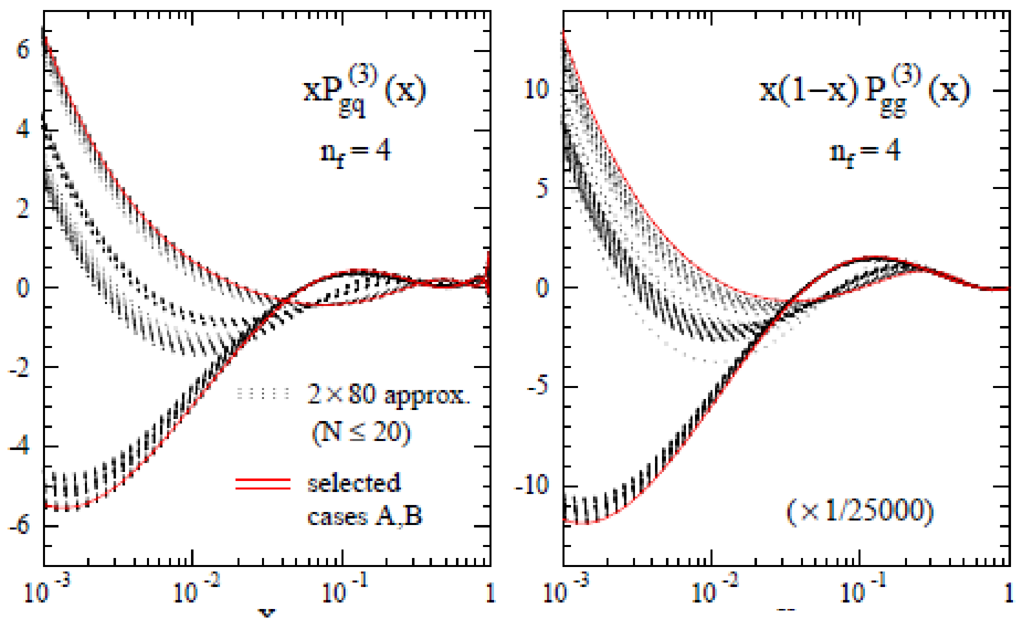
Range of allowed N^3LO splitting functions using constraints.



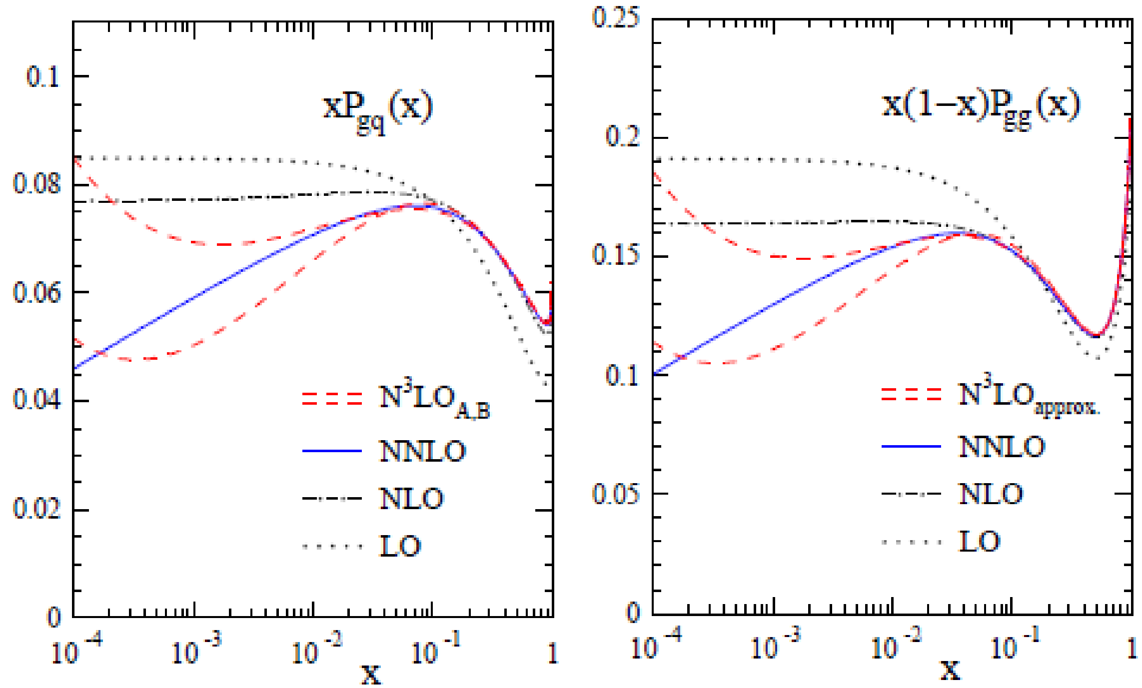
Range of allowed total splitting splitting functions.



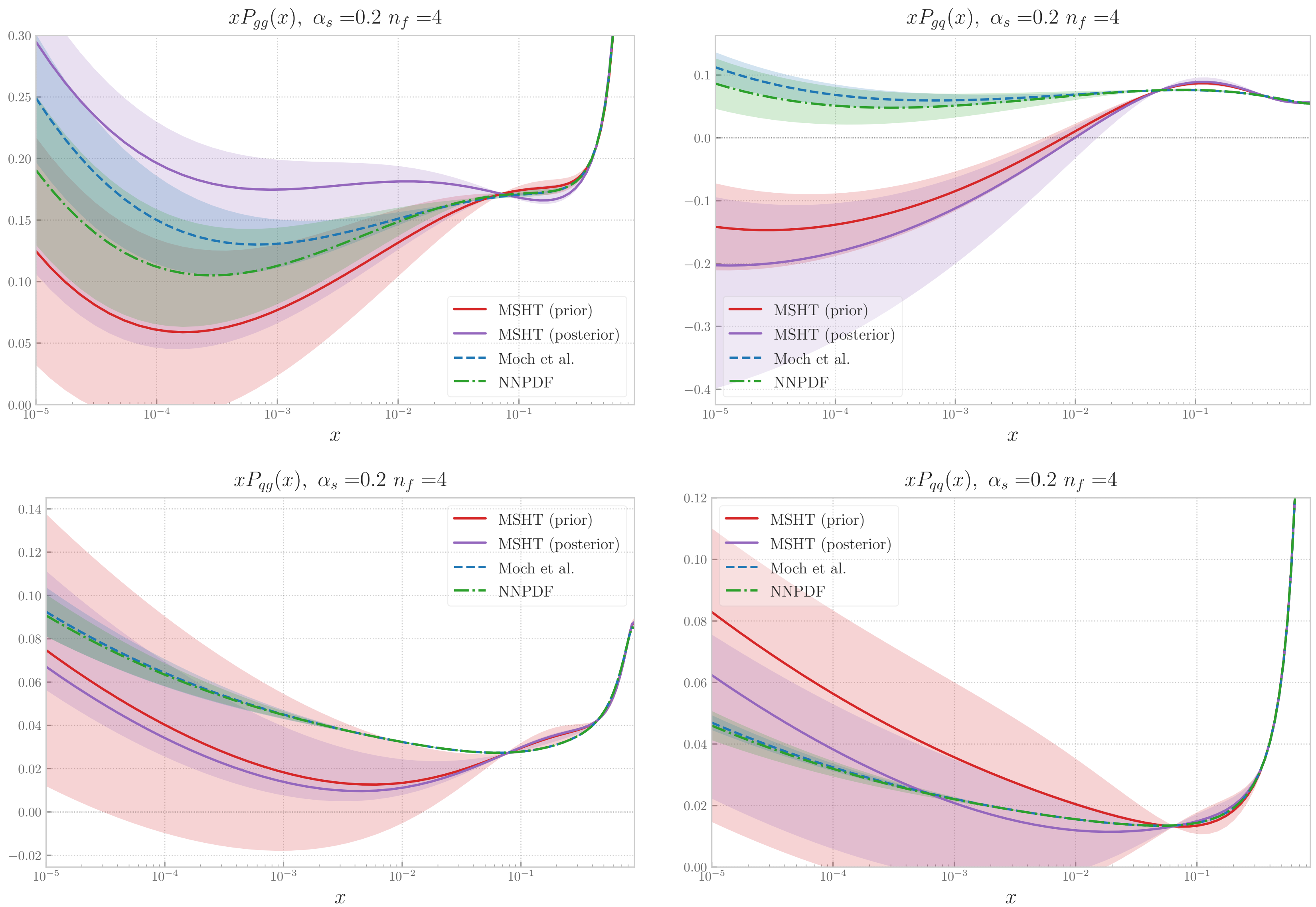
Range of allowed N^3LO splitting functions using constraints.



Range of allowed total splitting splitting functions.



Comparison with MSHT and NNPDF versions



Benchmarking PDFs at N^{3}LO

Given seeming difference in MSHT and NNPDF results, and new results on splitting functions desire for this.

$$
\begin{aligned}
xu_{\rm v}(x,\mu_{\rm f,0}^2) &= 5.107200\, x^{0.8}\,(1-x)^3 \\
xd_{\rm v}(x,\mu_{\rm f,0}^2) &= 3.064320\, x^{0.8}\,(1-x)^4 \\
xg\,(x,\mu_{\rm f,0}^2) &= 1.700000\, x^{-0.1}(1-x)^5 \\
x\bar{d}\,(x,\mu_{\rm f,0}^2) &= .1939875\, x^{-0.1}(1-x)^6 \\
x\bar{u}\,(x,\mu_{\rm f,0}^2) &= (1-x)\, x\bar{d}\,(x,\mu_{\rm f,0}^2) \\
xs\,(x,\mu_{\rm f,0}^2) &= x\bar{s}\,(x,\mu_{\rm f,0}^2) \;=\; 0.2\,x(\bar{u}+\bar{d})(x,\mu_{\rm f,0}^2)
\end{aligned}
$$

Check consistency of PDF evolution, and of effect of N^{3}LO specifically on evolution.

Following outline of previous benchmarking up to NNLO in arXiv:hep-ph/0511119.

Evolve specific PDF inputs at $Q_0^2 = 2{\rm GeV}^2$ up to higher scales using FFNS ($n_f = 4$) and VFNS.

Ongoing study to be written up for Les Houches proceedings.

Check output of various PDF flavours at $Q^2 = 10^4\ \mathrm{GeV}^2$.

first check consistency between groups and previous results at NNLO.

Table 15: As Table 14, but for the variable-N_f evolution using the flavour matching conditions of Ref. [156, 158, 159]. The corresponding values for the strong coupling $\alpha_s(\mu_r^2 = 10^4\ \mathrm{GeV}^2)$ are given by 0.115818, 0.115605 and 0.115410 for $\mu_r^2/\mu_f^2 = 0.5$, 1 and 2, respectively. For brevity the small, but non-vanishing valence distributions s_v, c_v and b_v are not displayed.

NNLO, $N_f = 3 \ldots 5$, $\mu_f^2 = 10^4\ \mathrm{GeV}^2$								
x	xu_v	xd_v	xL_-	$2xL_+$	xs_+	xc_+	xb_+	xg
$\mu_r^2 = \mu_f^2$								
10^{-7}	1.5978^{-4}	1.0699^{-5}	6.0090^{-6}	1.3916^{+2}	6.8509^{+1}	6.6929^{+1}	5.7438^{+1}	9.9694^{+3}
10^{-6}	7.1787^{-4}	4.5929^{-4}	2.6569^{-5}	7.1710^{+1}	3.5003^{+1}	3.3849^{+1}	2.8332^{+1}	4.8817^{+2}
10^{-5}	3.1907^{-3}	1.9532^{-3}	1.1116^{-4}	3.4732^{+1}	1.6690^{+1}	1.5875^{+1}	1.2896^{+1}	2.2012^{+2}
10^{-4}	1.4023^{-2}	8.2749^{-3}	4.3744^{-4}	1.5617^{+1}	7.2747^{+0}	6.7244^{+0}	5.2597^{+0}	8.8804^{+1}
10^{-3}	6.0019^{-2}	3.4519^{-2}	1.6296^{-3}	6.4173^{+0}	2.7954^{+0}	2.4494^{+0}	1.8139^{+0}	3.0404^{+1}
10^{-2}	2.3244^{-1}	1.3000^{-1}	5.6100^{-3}	2.2778^{+0}	8.5749^{-1}	6.6746^{-1}	4.5073^{-1}	7.7912^{+0}
0.1	5.4993^{-1}	2.7035^{-1}	9.9596^{-3}	3.8526^{-1}	1.1230^{-1}	6.4466^{-2}	3.7280^{-2}	8.5266^{-1}
0.3	3.4622^{-1}	1.2833^{-1}	2.9572^{-3}	3.4600^{-2}	8.8410^{-3}	4.0134^{-3}	2.1047^{-3}	7.8898^{-2}
0.5	1.1868^{-1}	3.0811^{-2}	3.6760^{-4}	2.3198^{-3}	5.6309^{-4}	2.3752^{-4}	1.2004^{-4}	7.6398^{-3}
0.7	1.9486^{-2}	2.9901^{-3}	1.2957^{-5}	5.2352^{-5}	1.2504^{-5}	5.6038^{-6}	2.8888^{-6}	3.7080^{-4}
0.9	3.3522^{-4}	1.6933^{-5}	8.209^{-9}	2.574^{-8}	6.856^{-9}	4.337^{-9}	2.679^{-9}	1.1721^{-6}
$\mu_r^2 = 2\mu_f^2$								
10^{-7}	1.3950^{-4}	9.0954^{-5}	5.2113^{-6}	1.3549^{+2}	6.6672^{+1}	6.5348^{+1}	5.6851^{+1}	1.0084^{+3}
10^{-6}	6.4865^{-4}	4.0691^{-4}	2.3344^{-5}	6.9214^{+1}	3.3753^{+1}	3.2772^{+1}	2.7818^{+1}	4.8816^{+2}
10^{-5}	2.9777^{-3}	1.8020^{-3}	9.9329^{-5}	3.3385^{+1}	1.6015^{+1}	1.5306^{+1}	1.2601^{+1}	2.1838^{+2}
10^{-4}	1.3452^{-2}	7.9078^{-3}	4.0036^{-4}	1.5035^{+1}	6.9818^{+0}	6.4880^{+0}	5.1327^{+0}	8.7550^{+1}
10^{-3}	5.8746^{-2}	3.3815^{-2}	1.5411^{-3}	6.2321^{+0}	2.7012^{+0}	2.3747^{+0}	1.7742^{+0}	3.0060^{+1}
10^{-2}	2.3063^{-1}	1.2923^{-1}	5.4954^{-3}	2.2490^{+0}	8.4141^{-1}	6.5083^{-1}	4.4354^{-1}	7.7495^{+0}
0.1	5.5279^{-1}	2.7222^{-1}	1.0021^{-2}	3.8897^{-1}	1.1312^{-1}	6.2917^{-2}	3.7048^{-2}	8.5897^{-1}
0.3	3.5141^{-1}	1.3051^{-1}	3.0134^{-3}	3.5398^{-2}	9.0559^{-3}	3.8727^{-3}	2.0993^{-3}	8.0226^{-2}
0.5	1.2140^{-1}	3.1590^{-2}	3.7799^{-4}	2.3919^{-3}	5.8148^{-4}	2.2376^{-4}	1.1918^{-4}	7.8098^{-3}
0.7	2.0120^{-2}	3.0955^{-3}	1.3462^{-5}	5.4194^{-5}	1.2896^{-5}	5.0329^{-6}	2.8153^{-6}	3.8099^{-4}
0.9	3.5230^{-4}	1.7849^{-5}	8.687^{-9}	2.568^{-8}	6.513^{-9}	3.390^{-9}	2.407^{-9}	1.2188^{-6}
$\mu_r^2 = 1/2\mu_f^2$								
10^{-7}	1.8906^{-4}	1.3200^{-4}	6.9268^{-6}	1.3739^{+2}	6.7627^{+1}	6.5548^{+1}	5.5295^{+1}	9.4403^{+2}
10^{-6}	8.1001^{-4}	5.3574^{-4}	3.0345^{-5}	7.2374^{+1}	3.5337^{+1}	3.3846^{+1}	2.7870^{+1}	4.7444^{+2}
10^{-5}	3.4428^{-3}	2.1524^{-3}	1.2531^{-4}	3.5529^{+1}	1.7091^{+1}	1.6065^{+1}	1.2883^{+1}	2.1802^{+2}
10^{-4}	1.4580^{-2}	8.6744^{-3}	4.8276^{-4}	1.6042^{+1}	7.4886^{+0}	6.8276^{+0}	5.3044^{+0}	8.9013^{+1}
10^{-3}	6.0912^{-2}	3.5030^{-2}	1.7393^{-3}	6.5544^{+0}	2.8656^{+0}	2.4802^{+0}	1.8362^{+0}	3.0617^{+1}
10^{-2}	2.3327^{-1}	1.3022^{-1}	5.7588^{-3}	2.2949^{+0}	8.6723^{-1}	6.7688^{-1}	4.5597^{-1}	7.8243^{+0}
0.1	5.4798^{-1}	2.6905^{-1}	9.9470^{-3}	3.8192^{-1}	1.1124^{-1}	6.7091^{-2}	3.7698^{-2}	8.4908^{-1}
0.3	3.4291^{-1}	1.2693^{-1}	2.9239^{-3}	3.4069^{-2}	8.6867^{-3}	4.3924^{-3}	2.1435^{-3}	7.8109^{-2}
0.5	1.1694^{-1}	3.0310^{-2}	3.6112^{-4}	2.2828^{-3}	5.5537^{-4}	2.7744^{-4}	1.2416^{-4}	7.5371^{-3}
0.7	1.9076^{-2}	2.9217^{-3}	1.2635^{-5}	5.2061^{-5}	1.2677^{-5}	7.2083^{-6}	3.0908^{-6}	3.6441^{-4}
0.9	3.2404^{-4}	1.6333^{-5}	7.900^{-9}	2.850^{-8}	8.407^{-9}	6.795^{-9}	3.205^{-9}	1.1411^{-6}

VFNS

MSHT

q	100	100	100	100	100	100	100	100	100	100	100
x	1E-07	1E-06	1E-05	1E-04	0.001	0.0099999998	0.1000000015	0.3000000119	0.5	0.6999999881	0.8999999762
xuv	0.0001705248	0.0007283112	0.0032008776	0.0140315385	0.060021575	0.2324358591	0.549938192	0.3462376847	0.118684053	0.0194850682	0.0003352296
xdv	0.0001131562	0.0004652558	0.0019591481	0.0082798121	0.0345202167	0.1299957679	0.2703601075	0.1283337156	0.0308105987	0.0029899376	1.6982463E-05
xL-	6.3407967E-06	2.6838085E-05	0.000111361	0.0004375606	0.0016295458	0.0056097858	0.0099601319	0.0029571574	0.0003675655	1.2966913E-05	8.3946228E-09
2xL+	139.22938531	71.723535696	34.733495032	15.616175163	6.4163010768	2.2774283946	0.3851792319	0.0345886861	0.0023190149	5.2370537E-05	2.5672906E-08
xs+	68.542346121	35.010180373	16.691032972	7.2740522141	2.7949296951	0.8572896433	0.1122608987	0.00883556	0.000562623	1.2491318E-05	6.6688885E-09
xc+	66.954641827	33.851618557	15.87230924	6.722279948	2.4481510177	0.6669270751	0.0643587162	0.0040017131	0.0002370764	5.5280208E-06	2.8139933E-09
xb+	57.472577322	28.34001755	12.897157111	5.2593976479	1.8135182473	0.4505455055	0.0372484074	0.0021013203	0.0001199588	2.8618412E-06	2.1064032E-09
xg	997.62937958	488.35333788	220.16054651	88.809881995	30.403337232	7.7909407531	0.8526027179	0.0788775885	0.007636451	0.0003706604	1.1764031E-06

% Diff cf Table 15

6.724777805	1.454466004	0.318978085	0.060889348	0.004290249	-0.00178148	0.001489646	0.005107935	0.003415103	-0.00478208	0.002872995
5.763357279	1.298917918	0.304533463	0.059361691	0.003524803	-0.00325544	0.003738674	0.002895372	-0.00130244	-0.00543217	0.29210766
5.521662538	1.012778192	0.180796357	0.027580135	-0.00332867	-0.00381901	0.005341067	-0.00143928	-0.00937704	0.076507725	2.261210857
0.049860097	0.018875605	0.004304481	-0.00528166	-0.0155661	-0.01631422	-0.02096457	-0.03269922	-0.03384259	0.035408441	-0.26066171
0.048674074	0.020513593	0.006189168	-0.00890464	-0.01682424	-0.02336549	-0.03481861	-0.0615312	-0.08292746	-0.10142276	-2.72916493
0.038311983	0.007735995	-0.01694967	-0.03152775	-0.05099136	-0.07984372	-0.16641915	-0.29119796	-0.18676312	-1.35228188	-35.1165933
0.060199384	0.028298568	0.008972637	-0.00574847	-0.02104596	-0.04093238	-0.0847442	-0.16058089	-0.06762597	-0.9332165	-21.3735278
0.069149556	0.037556155	0.018420186	0.00662357	-0.00217987	-0.00332743	-0.00671805	-0.02587071	-0.04383567	-0.03765187	0.367129326

NNPDF

	0	1	2	3	4	5	6	7	8	9	10
u_v	0.0001597824	0.0007178658	0.0031907024	0.0140232687	0.0600191996	0.2324397383	0.5499296887	0.346220217	0.1186795105	0.0194859521	0.0003331513
d_v	0.0001069924	0.0004592943	0.001953159	0.0082748689	0.0345191769	0.1299988962	0.2703522541	0.1283272126	0.0308119336	0.0029916486	1.7424361E-05
L_m	6.0090155E-06	2.6568708E-05	0.0001111556	0.0004374367	0.00162957	0.0056099937	0.009959485	0.0029571098	0.000367636	1.294161E-05	1.0526658E-08
L_p	139.13928063	71.682267454	34.722989246	15.611573097	6.4149915243	2.277054108	0.3852489055	0.0346046497	0.0023218853	5.2561757E-05	5.649527E-08
s_p	68.497319784	34.989734427	16.685881942	7.2718502374	2.7943465111	0.8571489068	0.1122996567	0.0088431966	0.0005641707	1.2683117E-05	2.0212418E-08
c_p	66.916979839	33.836111497	15.870039212	6.7216684468	2.4483610211	0.6671721948	0.0644633379	0.0040153303	0.0002386119	5.8264856E-06	1.4645617E-08
b_p	57.425685927	28.320322847	12.89214786	5.2575166766	1.8131378129	0.4505094514	0.0372798125	0.0021060685	0.0001207829	3.0407997E-06	7.7238412E-09
g	996.66751451	487.94381715	220.04499947	88.76429755	30.390653078	7.7876349099	0.8526568834	0.0788978737	0.007639902	0.0003705933	1.2675973E-06

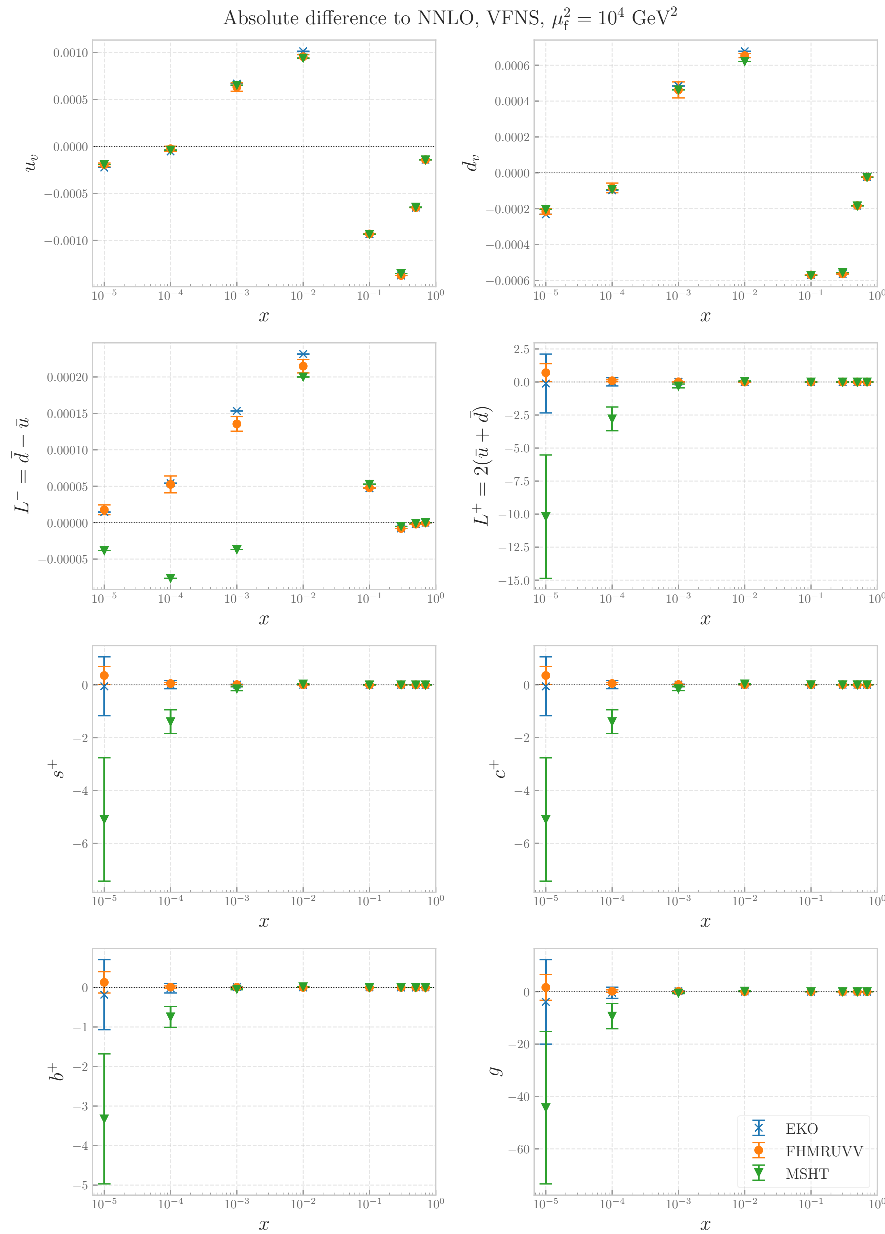
0.001485973	-0.00059008	7.43775E-05	0.001915836	0.00033249	-0.0001126	-5.6614E-05	6.26777E-05	-0.00041246	-0.00024593	-0.61711715
0.002261055	0.000929842	-0.00210089	-0.00037529	0.000512543	-0.00084905	0.000833789	-0.00217203	0.003029981	0.051792487	2.9017955
0.000257795	-0.00109868	-0.00400207	-0.00075643	-0.00184175	-0.00011245	-0.00115487	-0.00305163	0.0097949	-0.11877484	28.23313583
-0.01488888	-0.03867319	-0.02594367	-0.03474997	-0.03597269	-0.03274616	-0.00287974	0.01343856	0.089890278	0.400666515	119.4843415
-0.01704917	-0.03789839	-0.0246738	-0.03917361	-0.03768652	-0.03977809	-0.00030573	0.024845748	0.191931593	1.43247983	194.8135676
-0.01795957	-0.03807647	-0.03124906	-0.04062152	-0.04241769	-0.04311946	-0.00412943	0.048096474	0.45972368	3.973832494	237.6900296
-0.0214389	-0.04121542	-0.02987081	-0.04151042	-0.04201924	-0.04893142	-0.00050294	0.065022367	0.618848018	5.261690384	188.310608
-0.02733219	-0.0463328	-0.03407256	-0.04470795	-0.04389857	-0.04575791	-0.00036552	-0.00016012	0.001334512	-0.05573542	8.14754079

MSHT very similar except at very low x, mainly in extrapolation region of grids.

NNPDF very similar except at very high x.

Both excellent agreement except this.

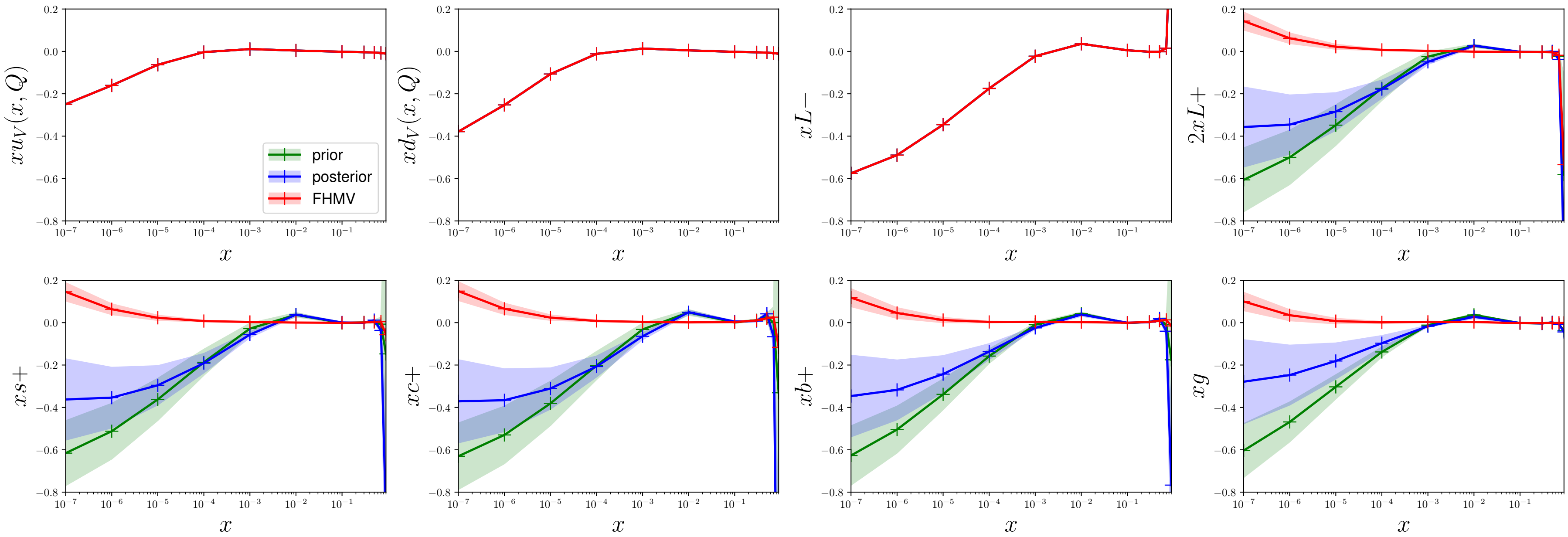
MSHT and NNPDF evolution at N^{3}LO compared to NNLO (plot by G, Magni) for various splitting function choices. **Prelimary**



Some difference in own versions, particularly when based on less information than most up-to-date versions, at very small x.

initially some different choices in non-singlet. Differences disappear when common choices made (later).

MSHT evolution at N^{3}LO compared to NNLO for various splitting function choices. **Prelimary**



Some differences at small-x in particular, including between MSHT prior and posterior.

MSHT and NNPDF evolution at N^{3}LO when both using Moch et al. splitting functions and common non-singlet choices.

VFNS

MSHT FHMRUVV NS all updated

q	100	100	100	100	100	100	100	100	100	100	100
x	1E-07	1E-06	1E-05	0.0001	0.001	0.01	0.1	0.3	0.5	0.7	0.9
xuv	0.00011639	0.00060389	0.0030183	0.014024	0.060636	0.23325	0.54876	0.34472	0.11798	0.019335	0.00033148
xdv	5.563E-05	0.00033257	0.0017563	0.0082158	0.034976	0.13055	0.26967	0.12772	0.030615	0.0029654	1.6782E-05
xL-	8.3396E-06	3.2692E-05	0.00012872	0.00048879	0.0017596	0.0058026	0.0099483	0.0029024	0.00034653	9.2473E-06	-6.3382E-08
2xL+	159.15	76.153	35.49	15.724	6.4308	2.2752	0.38453	0.034706	0.0023789	6.21E-05	1.8337E-07
xs+	78.505	37.227	17.071	7.329	2.8038	0.85774	0.11219	0.0088422	0.00056438	1.2548E-05	6.2892E-09
xc+	76.906	36.062	16.248	6.7755	2.4567	0.66785	0.064546	0.0040495	0.0002437	5.9902E-06	8.4763E-09
xb+	64.245	29.625	13.056	5.2766	1.8204	0.45172	0.037236	0.0021095	0.00012144	2.9754E-06	3.533E-09
xg	1097.3	505.44	221.75	88.931	30.517	7.816	0.85051	0.078671	0.0076252	0.00037046	1.1753E-06

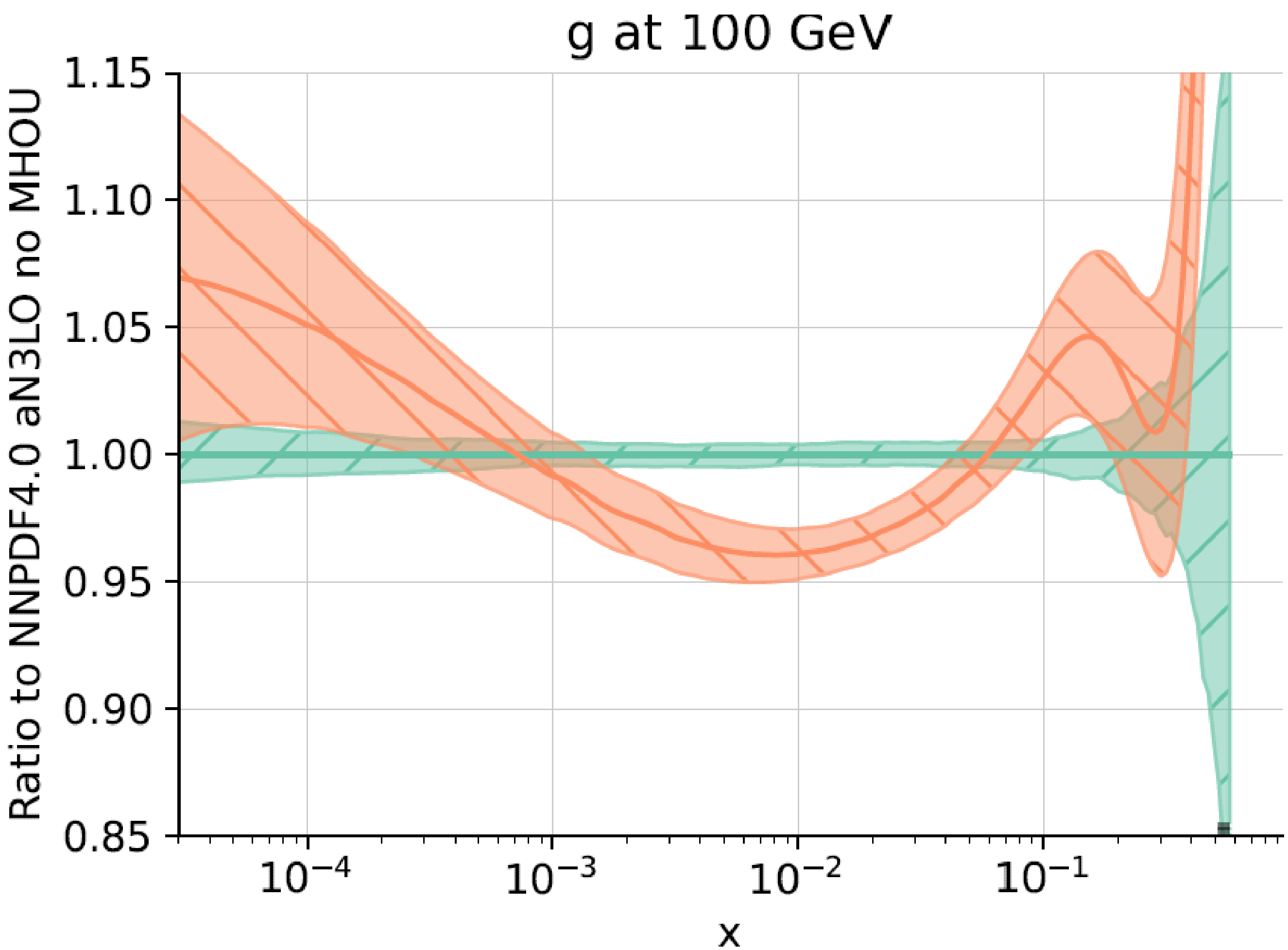
NNPDF FHMRUVV

q	100	100	100	100	100	100	100	100	100	100	100
x	1E-07	1E-06	1E-05	0.0001	0.001	0.01	0.1	0.3	0.5	0.7	0.9
xuv	0.00010347	0.0005885	0.0029963	0.013998	0.060651	0.23339	0.549	0.34485	0.11803	0.019344	0.00032955
xdv	4.7297E-05	0.00032158	0.001738	0.0081908	0.034981	0.13065	0.26978	0.12776	0.030629	0.0029683	1.7232E-05
xL-	7.987E-06	3.2458E-05	0.00012878	0.00048992	0.0017651	0.0058248	0.010007	0.0029491	0.00036563	1.2839E-05	1.035E-08
2xL+	158.46	75.933	35.427	15.705	6.4269	2.2744	0.38425	0.034522	0.0023176	5.2471E-05	5.57E-08
xs+	78.161	37.117	17.039	7.3201	2.802	0.85749	0.11225	0.0088517	0.00056611	1.2749E-05	1.9893E-08
xc+	76.573	35.959	16.221	6.7688	2.4559	0.66796	0.064658	0.0040601	0.00024371	5.9801E-06	1.4418E-08
xb+	63.897	29.511	13.021	5.2666	1.8183	0.45145	0.037274	0.002115	0.00012203	3.0847E-06	7.7074E-09
xg	1094.1	504.76	221.69	88.942	30.527	7.8173	0.85068	0.078667	0.0076246	0.0003702	1.2669E-06

% Diff cf NNPDF

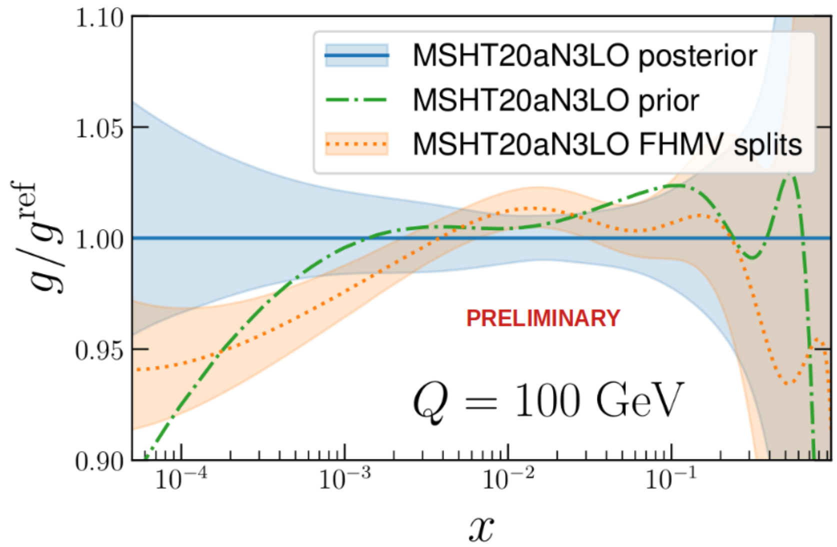
12.48671112	2.615123195	0.734238895	0.18574082	-0.02473166	-0.05998543	-0.04371585	-0.03769755	-0.04236211	-0.04652605	0.585647095
17.6184536	3.417501088	1.052934407	0.305220491	-0.01429347	-0.07654038	-0.04077396	-0.0313087	-0.04570832	-0.09769902	-2.61142061
4.414673845	0.720931666	-0.04659109	-0.2306499	-0.31159708	-0.38112897	-0.58658939	-1.58353396	-5.22386019	-27.9749202	-712.386473
0.435441121	0.289729103	0.177830468	0.120980579	0.060682444	0.035174112	0.072869226	0.532993453	2.644977563	18.35108917	229.2100539
0.440117194	0.296360158	0.187804449	0.121583038	0.064239829	0.029154859	-0.05345212	-0.10732402	-0.30559432	-1.57659424	-68.384859
0.434879135	0.286437331	0.166450897	0.098983572	0.032574616	-0.01646805	-0.17321909	-0.26107731	-0.00410324	0.168893497	-41.2102927
0.544626508	0.386296635	0.268796559	0.189875821	0.115492493	0.059807288	-0.10194774	-0.26004728	-0.48348767	-3.54329432	-54.1609362
0.292477836	0.134717489	0.02706482	-0.01236761	-0.03275789	-0.01662978	-0.01998401	0.005084724	0.007869265	0.070232307	-7.23024706

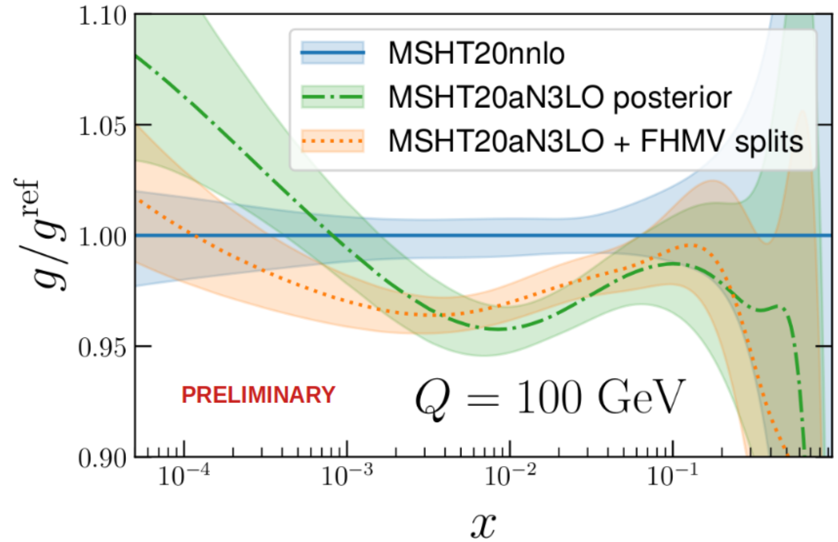
Comparison of MSHT and NNPDF fit PDFs.



As with benchmark evolved PDFS the main difference is in the gluon at $x \sim 0.01$ but discrepency a little larger.

Effect of MSHT fits with improved splitting functions.



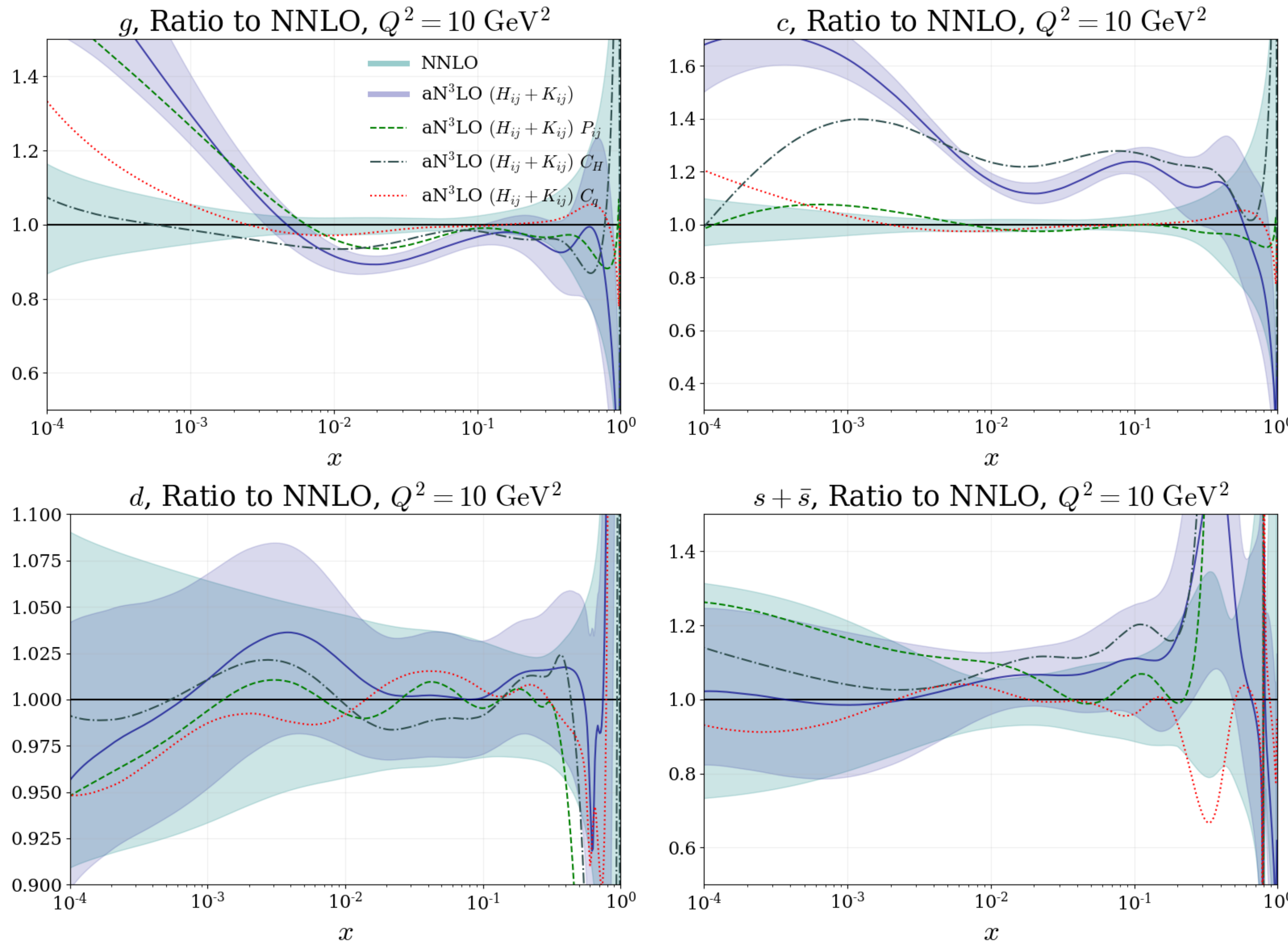


Note - no uncertainties used for improved splitting functions - only central value. Now almost exclusively at small x.

Use of (central value of) improved N^{3}LO splitting functions changes N^{3}LO gluon a little compared to published MSHT PDFs, raising 1.5% near $x = 0.01$.

Main features of N^{3}LO comparison to NNLO remain the same.

Effect of each individual N^{3}LO change.



Not only splitting functions responsible for change in PDFs.

Conclusions

Approximate N^3LO PDFs are available and we encourage their use.

Designed so that theoretical uncertainties represent the missing parts of N^3LO, i.e. assume this is the dominant source of missing higher order corrections. Approaches to this differ.

Better precision, control of uncertainties, and better fit quality.

MSHT and now also NNPDF PDFs available as LHAPDF grids.

Some apparent differences between MSHT and NNPDF versions.

Benchmarking largely complete underway. Shows evolution consistent when same splitting functions used. Differences in evolution from the applied splitting functions – recent updates have led to significant improvements.

Indications from fits with more similar splitting functions and further analyses (e.g. cuts) reveal converenge and/or understanding of differences.

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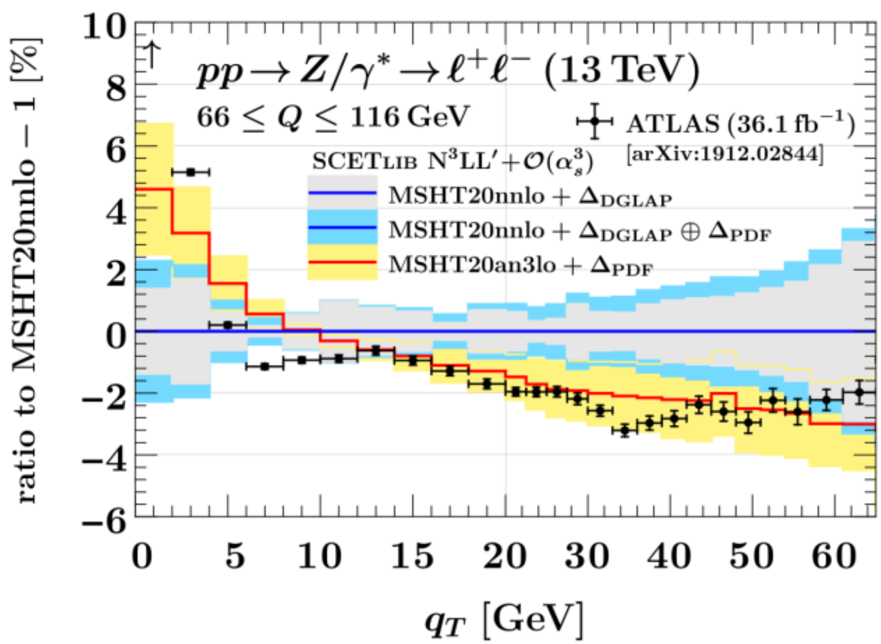
Back-up

Application of aN^{3}LO PDFs.

aN3LO PDFs for Zp_T at low q_T:

- MSHT20aN3LO PDFs already starting to be used by theory community - e.g. resummed (+ fixed order) predictions for Zp_T spectrum at low transverse momenta:



- aN3LO PDFs fit the measured ATLAS data better, likely due to indirect effects of gluon shape change.... need to look into this more!

Figure Credit: SCETlib - Georgios Billis, shown by Johannes Michel at LHC EW WG Sep 2022.