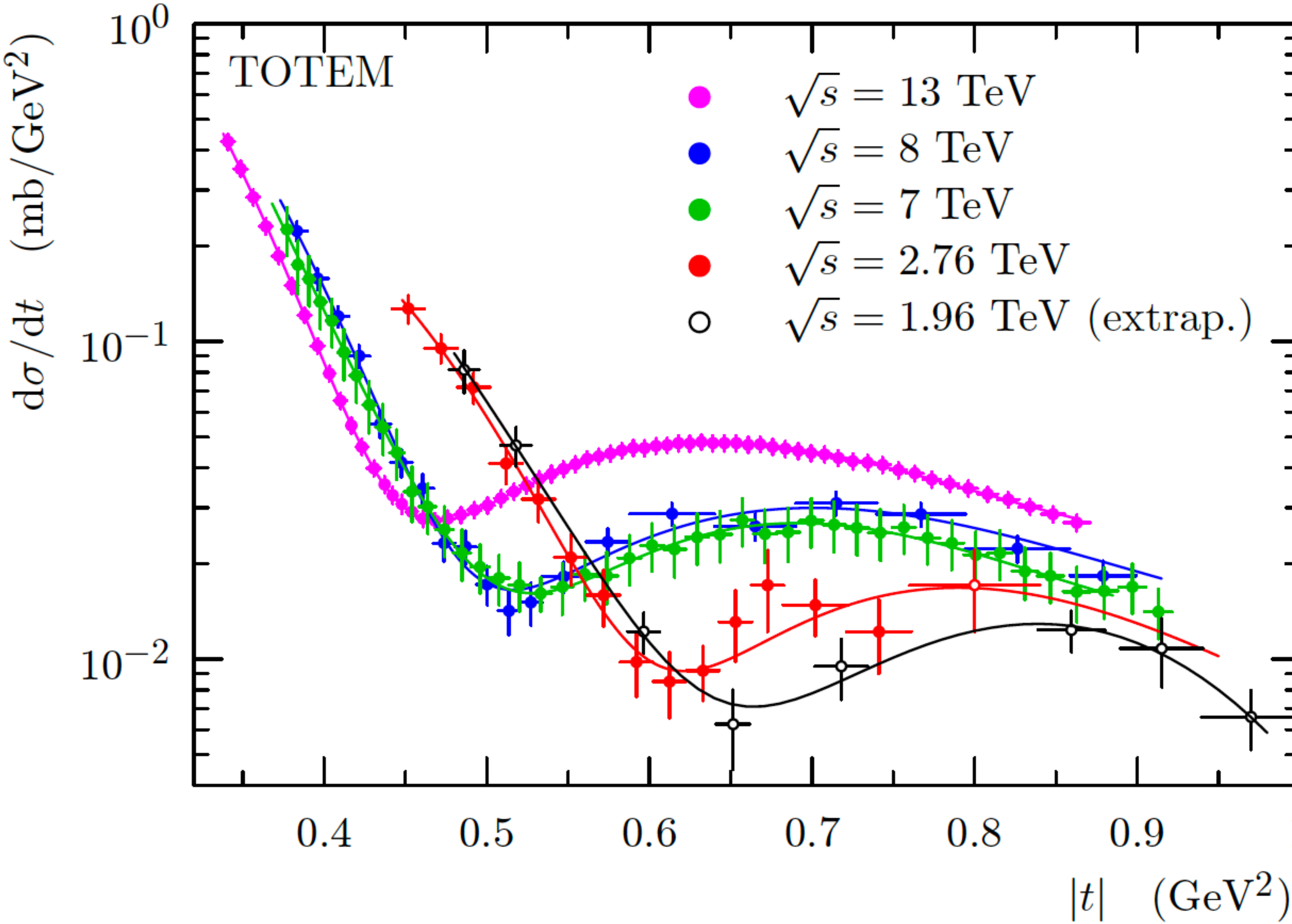
Scaling properties of elastic pp cross-section

Michał Praszałowicz

Cristian Baldenegro, Christophe Royon, Anna Staśto



V.M. Abazov [TOTEM and D0] PRL 102 (2020) 062003 (Royon odderon paper)

Introduction

Impact parameter space (Barone, Predazzi):

$$\sigma_{\rm el} \;=\; \int d^2\boldsymbol{b}\, \left|1 - e^{-\Omega(s,b)+i\chi(s,b)}\right|^2 ,$$

$$\sigma_{\rm tot} \;=\; 2\int d^2\boldsymbol{b}\, {\rm Re}\left[1 - e^{-\Omega(s,b)+i\chi(s,b)}\right] ,$$

$$\sigma_{\rm inel} \;=\; \int d^2\boldsymbol{b}\, \left[1 - \left|e^{-\Omega(s,b)}\right|^2\right] .$$

Cross-sections

Impact parameter space (Barone, Predazzi):

$$\sigma_{\rm el} = \int d^2\boldsymbol{b} \, \overbrace{\left|1 - e^{-\Omega(s,b)+i\chi(s,b)}\right|}^{-i\,A_{\rm el}}{}^2 ,$$

$$\sigma_{\rm tot} = 2 \int d^2\boldsymbol{b} \, {\rm Re} \left[1 - e^{-\Omega(s,b)+i\chi(s,b)}\right] ,$$

$$\sigma_{\rm inel} = \int d^2\boldsymbol{b} \left[1 - \left|e^{-\Omega(s,b)}\right|^2\right] .$$

A bit of history

Nuclear Physics B59 (1973) 231–236 North-Holland Publishing Company

GEOMETRIC SCALING, MULTIPLICITY DISTRIBUTIONS AND CROSS SECTIONS

J DIAS DE DEUS

The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark

Received 8 March 1973

Abstract From a geometric picture of hadrons as extended objects we arrive at some universal features of high energy collisions In this approach the mean multiplicity, as a function of s and the KNO scaling function are universal and asymptotically the ratio $\sigma_{elastic}/\sigma_{total}$ is expected to be the same for all processes

Geometric scaling

$$\Omega(s,b) = \Omega\left(b/R(s)\right)$$

Opacity is a function of one varible,
and *R*(*s*) grows with energy. Changing variable

$$\boldsymbol{b} \to \boldsymbol{B} = \boldsymbol{b}/R(s)$$

$$\sigma_{\mathrm{inel}} = R^2(s) \underbrace{\int d^2\boldsymbol{B}\, \left[1 - \left|e^{-\Omega(B)}\right|^2\right]}_{\text{constant}}$$

Immediate consequences

$$\sigma_{\rm el} = \int d^2\boldsymbol{b}\,\left|1 - e^{-\Omega(s,b)+i\chi(s,b)}\right|^2,$$

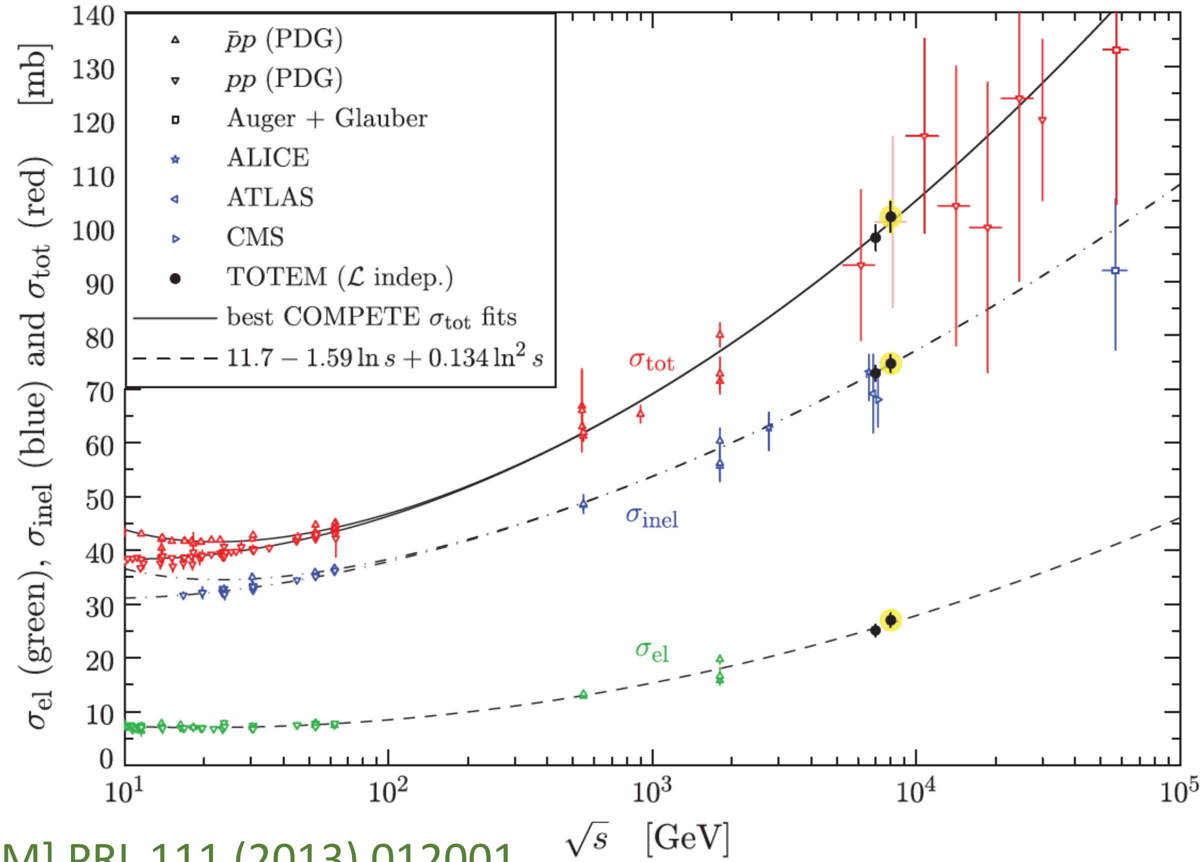
$$\sigma_{\rm tot} = 2\int d^2\boldsymbol{b}\,{\rm Re}\left[1 - e^{-\Omega(s,b)+i\chi(s,b)}\right],$$

$$\sigma_{\rm inel} = \int d^2\boldsymbol{b}\,\left[1 - \left|e^{-\Omega(s,b)}\right|^2\right].$$

If we neglect χ (indeed ρ parameter is small), then all cross-sections have the same energy dependence.

Geometric scaling at the ISR

	elastic	inelastic	total	$\frac{\text{elastic}}{\text{inelastic}}$	ρ
ISR	$W^{0.1142\pm0.0034}$	$W^{0.1099\pm0.0012}$	$W^{0.1098\pm0.0012}$	$W^{0.0043\pm0.0036}$	$0.02-0.095$



G. Antchev [TOTEM] PRL 111 (2013) 012001

Geometric scaling at the ISR

Nuclear Physics B71 (1974) 481–492

SCALING LAW FOR THE ELASTIC DIFFERENTIAL CROSS SECTION IN pp SCATTERING FROM GEOMETRIC SCALING*

A.J. BURAS and J. DIAS de DEUS

The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen ϕ, Denmark

Received 6 December 1973

Abstract: Plots of $(1/\sigma_{in}^2)d\sigma_{el}/d|t| \equiv \Phi(\tau, s)$ as a function of $\tau \equiv |t|\sigma_{in}$ are shown to scale in the NAL-ISR energy region. Such scaling is shown to be a consequence of geometric scaling for the inelastic overlap function $G_{in}(\beta = \pi b^2/\sigma_{in})$ in the limit $\rho = \mathrm{Re}A/\mathrm{Im}A \to 0$ and in the case of $\sigma_{in} \sim (\ln s)^2$ is equivalent to the scaling proposed by Auberson, Kinoshita and Martin. A possible relation to the KNO multiplicity scaling is indicated.

Geometric scaling at the ISR

$$\tau = \sigma_{\rm inel}(s)\,|t| = R^2(s)|t| \times {\rm const.}$$

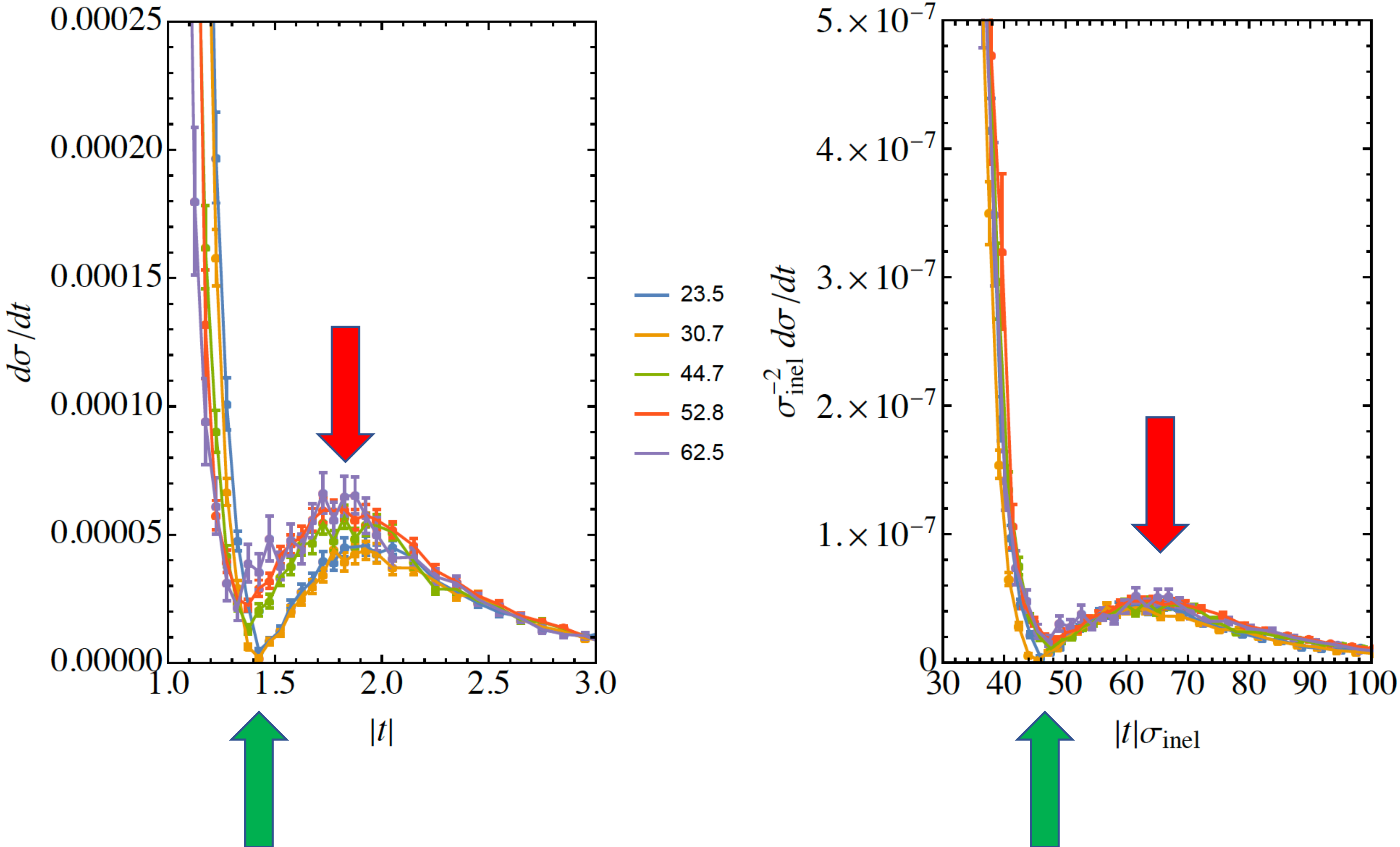
$$\begin{aligned}
\frac{d\sigma_{\rm el}}{d|t|} &\sim \left| \int_0^\infty db^2 A_{\rm el}(b^2,s)\, J_0\left(b\sqrt{|t|}\right)\right|^2 \\
&= \left| \sigma_{\rm inel}(s) \int_0^\infty d\left(b^2/\sigma_{\rm inel}(s)\right)\, A_{\rm el}(b^2/\sigma_{\rm inel}(s))\, J_0\left(\sqrt{\tau}\, b/\sqrt{\sigma_{\rm inel}(s)}\right)\right|^2 \\
&= \sigma^2_{\rm inel}(s) \left| \int_0^\infty dB^2 A_{\rm el}(B^2)\, J_0\left(B\sqrt{\tau}\right)\right|^2 \\
&= \sigma^2_{\rm inel}(s)\, \Phi(\tau).
\end{aligned}$$

Geometric scaling at the ISR

$$\tau = \sigma_{\rm inel}(s)\,|t| \; = R^2(s)|t| \times \text{const.}$$

$$\frac{1}{\sigma^2_{\rm inel}(s)}\frac{d\sigma_{\rm el}}{d|t|}(s,t) = \Phi(\tau)$$

Geometric scaling at the ISR

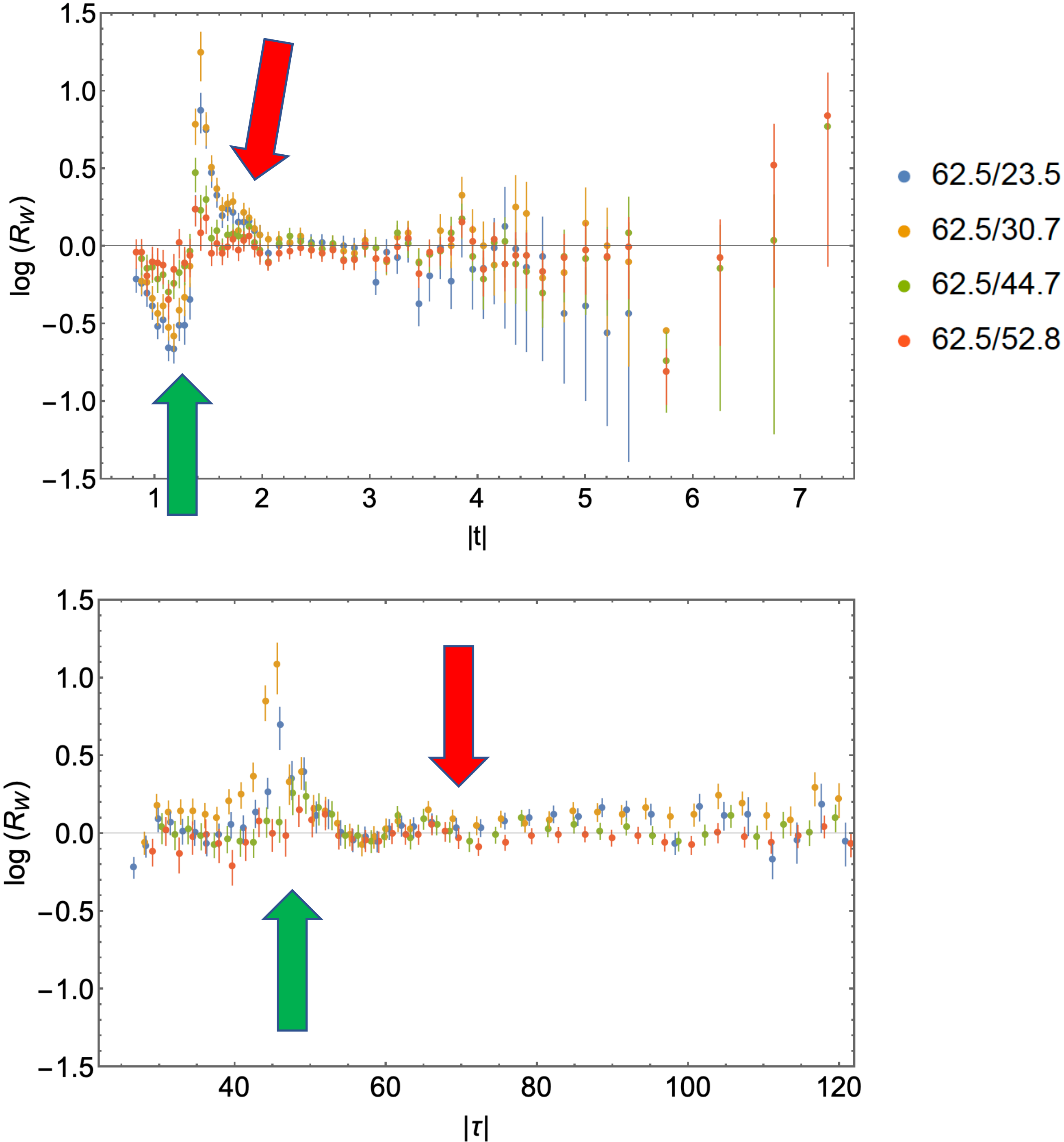


Ratio method

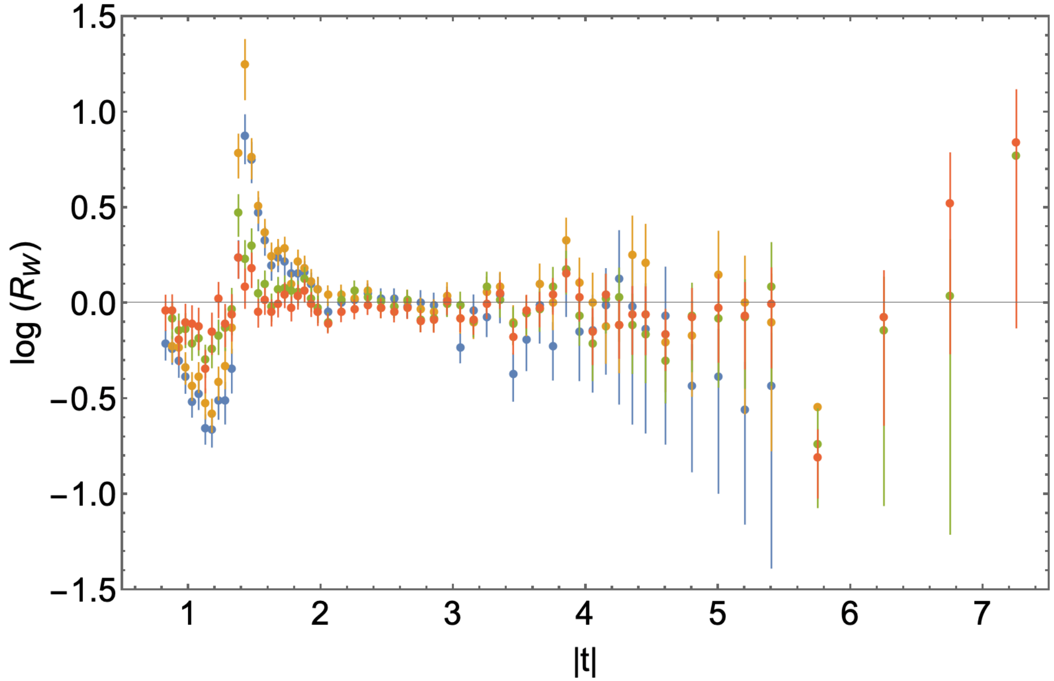
$$\frac{d\tilde{\sigma}_{\rm el}}{d|t|}(W,\tau_i) = \frac{1}{\sigma^2_{\rm inel}(W)}\,\frac{d\sigma_{\rm el}}{d|t|}(W,\tau_i)$$

$$R_W(\tau_i) = \frac{d\tilde{\sigma}_{\rm el}/d|t|(W_{\rm ref},\tau_i)}{d\tilde{\sigma}_{\rm el}/d|t|(W,\tau_i)}$$

$$W_{\rm ref} = 62.5\ {\rm GeV}$$

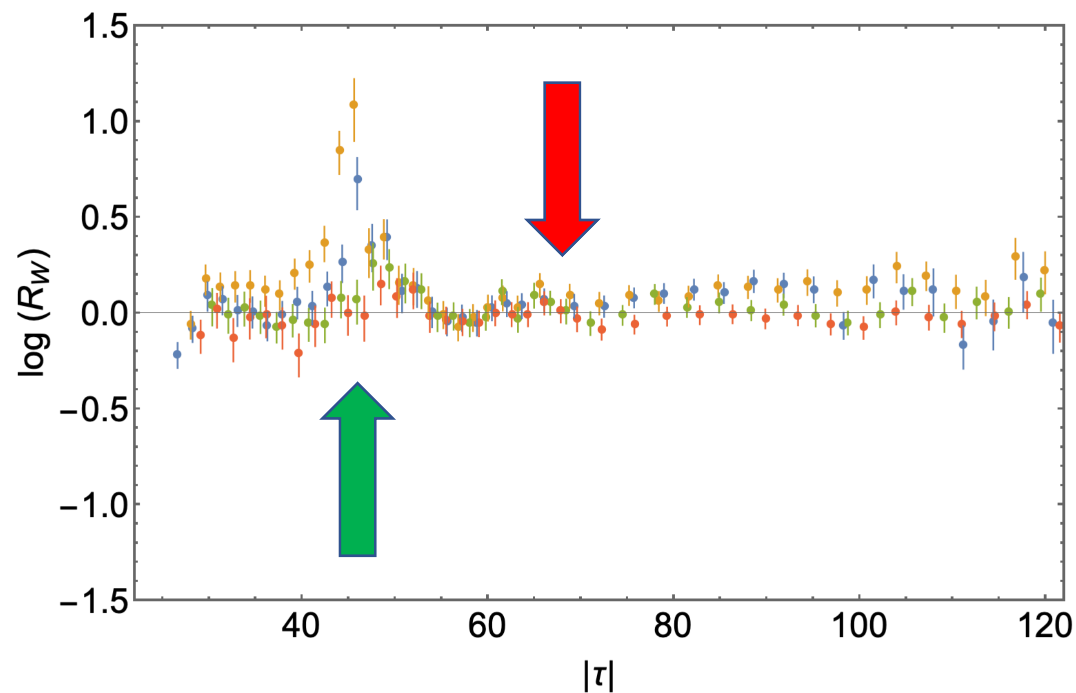


log (R_W)
|t|
|τ|
62.5/23.5
62.5/30.7
62.5/44.7
62.5/52.8



- 62.5/23.5
- 62.5/30.7
- 62.5/44.7
- 62.5/52.8

J. Dias de Deus, P. Kroll, APP B9 (78) 157

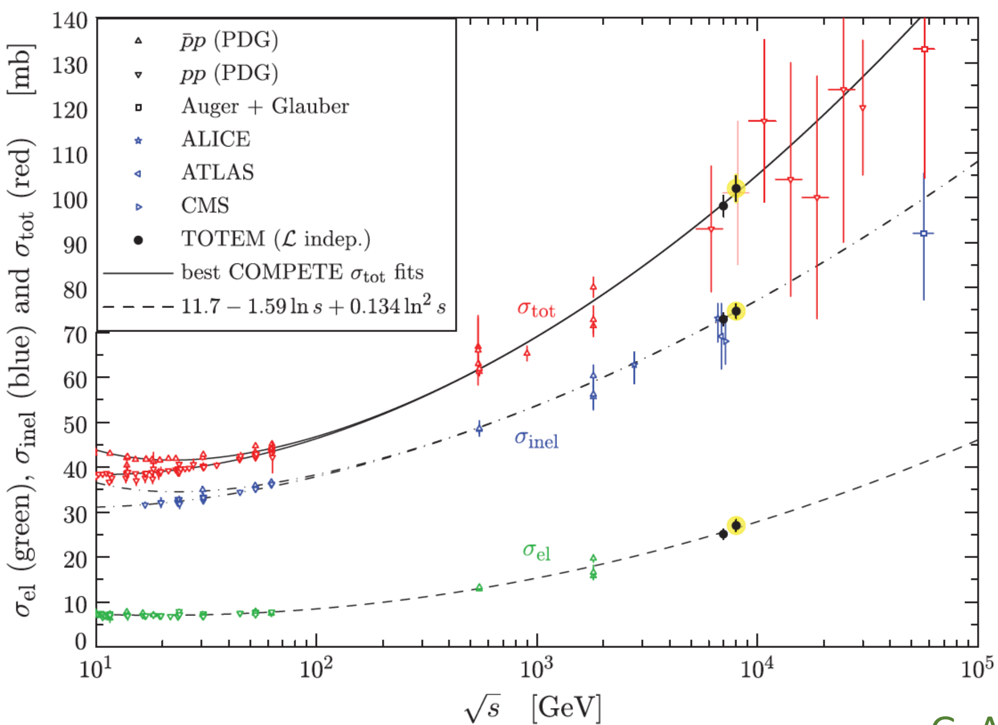


The fact that geometrical scaling is violated in the dip region has been attributed to the vanishing of the imaginary part of the scattering amplitude at the dip.

As a consequence, the dip must disappear at higher energies ~ 300 GeV

Scaling at the LHC?

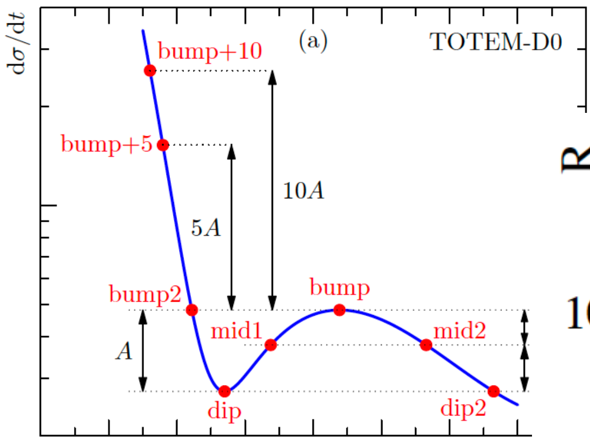
	elastic	inelastic	total	$\frac{\text{elastic}}{\text{inelastic}}$	ρ
ISR	$W^{0.1142\pm0.0034}$	$W^{0.1099\pm0.0012}$	$W^{0.1098\pm0.0012}$	$W^{0.0043\pm0.0036}$	$0.02 - 0.095$
LHC	$W^{0.2279\pm0.0228}$	$W^{0.1465\pm0.0133}$	$W^{0.1729\pm0.0163}$	$W^{0.0814\pm0.0264}$	$0.15 - 0.10$

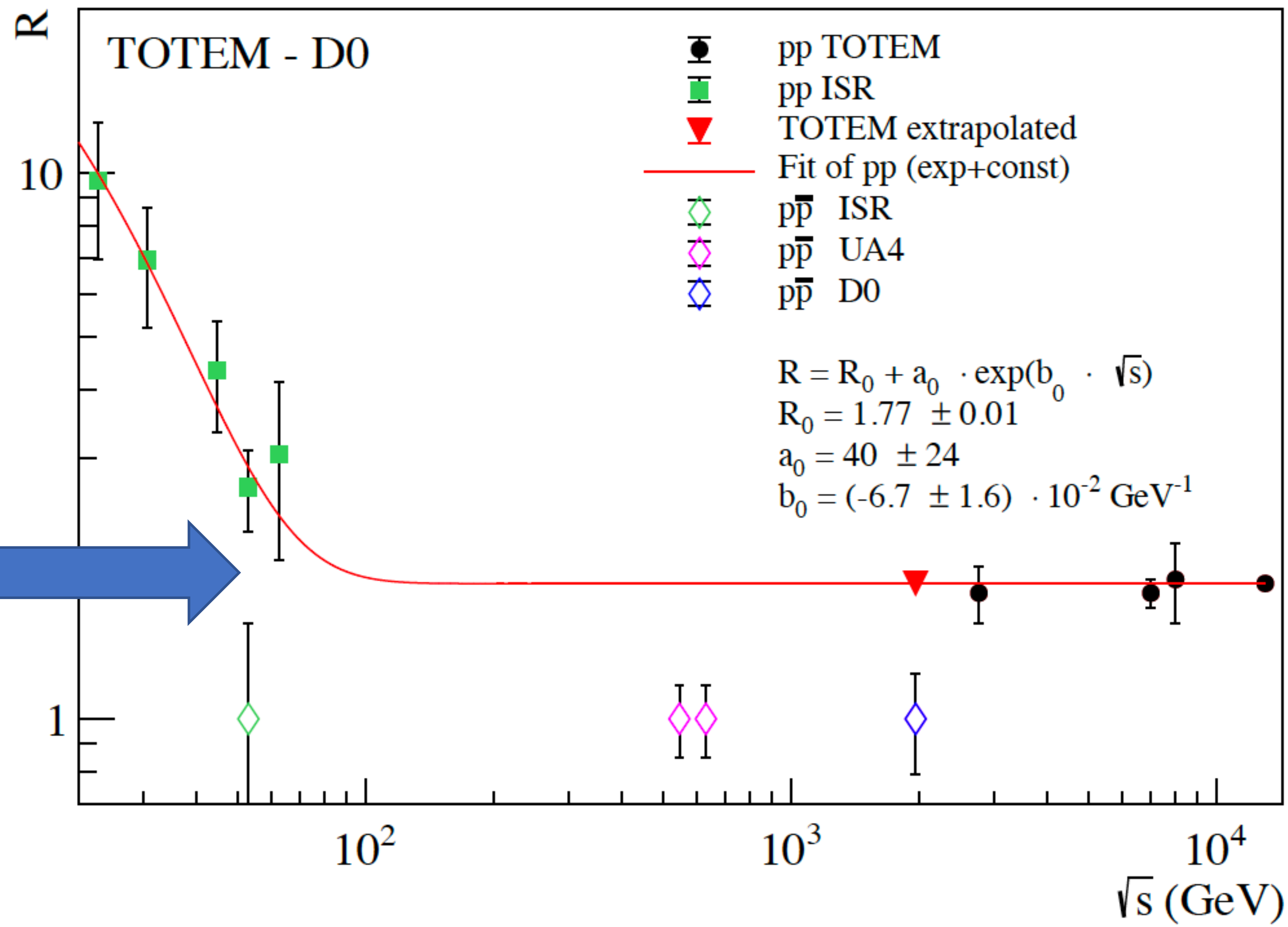


?

G. Antchev [TOTEM] PRL 111 (2013) 012001

Bump/Dip behaviour

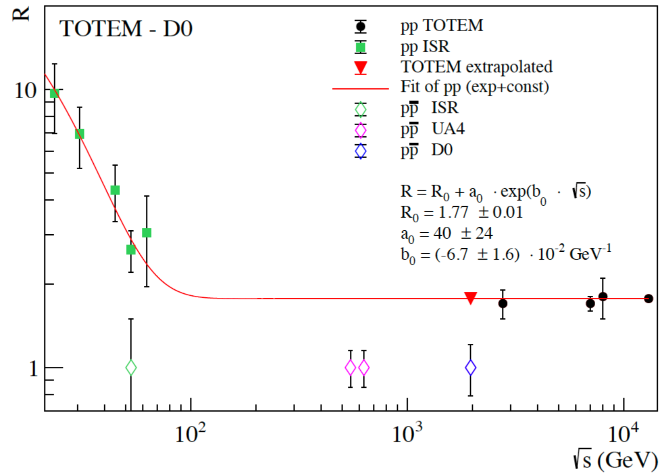




Hope for scaling
at the LHC

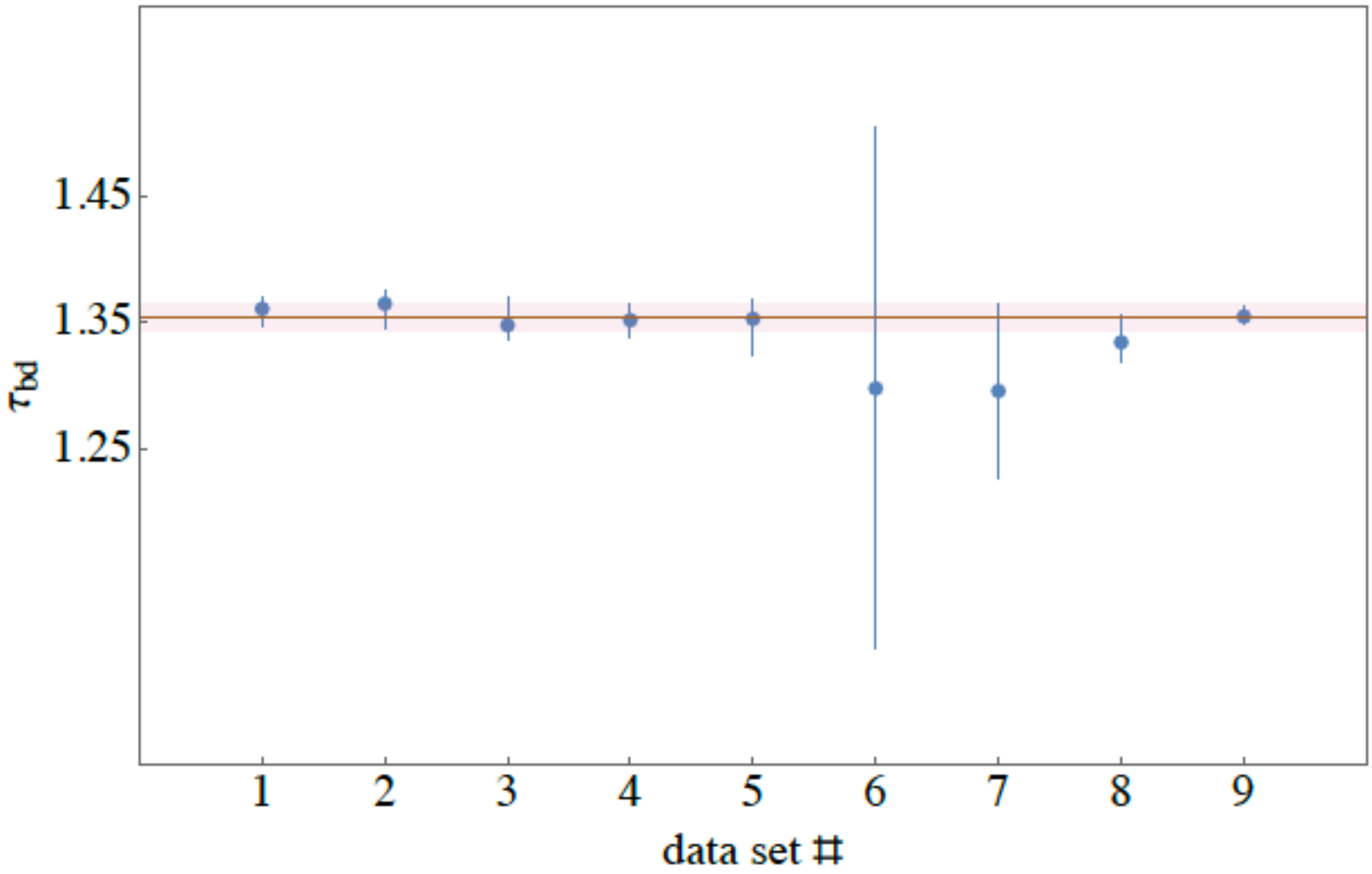
V.M. Abazov [TOTEM and D0] PRL 102 (2020) 062003 (Royon odderon paper)

An observation



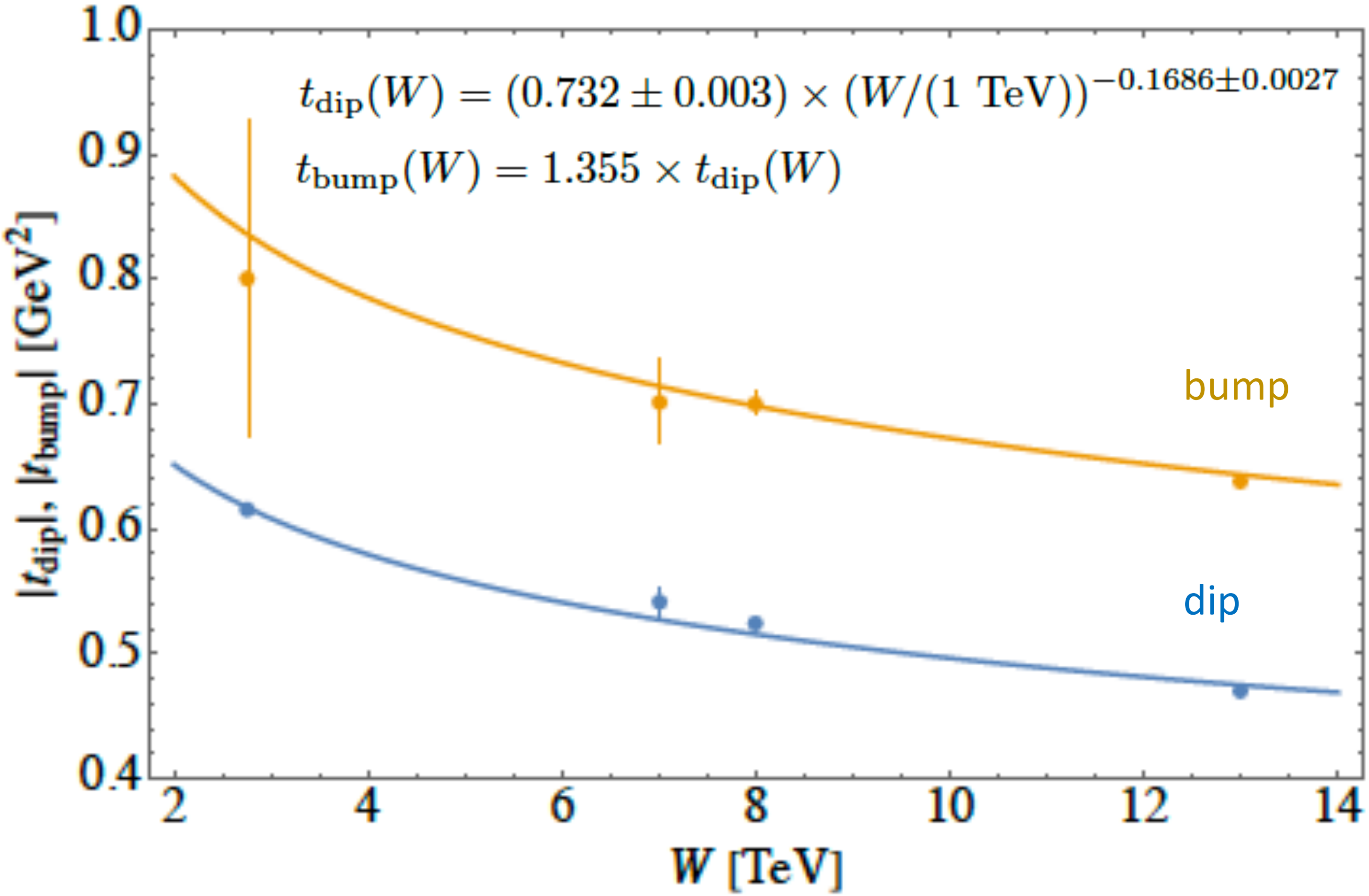
	#	W	dip		bump		ratios	
			$\|t\|_{\mathrm{d}}$	error	$\|t\|_{\mathrm{b}}$	error	$t_{\mathrm{b}}/t_{\mathrm{d}}$	error
LHC [TeV]	9	13.00	0.471	$^{+0.002}_{-0.003}$	0.6377	$^{+0.0006}_{-0.0006}$	1.355	$^{+0.008}_{-0.005}$
	8	8.00	0.525	$^{+0.002}_{-0.004}$	0.700	$^{+0.010}_{-0.008}$	1.335	$^{+0.021}_{-0.016}$
	7	7.00	0.542	$^{+0.012}_{-0.013}$	0.702	$^{+0.034}_{-0.034}$	1.296	$^{+0.069}_{-0.069}$
	6	2.76	0.616	$^{+0.001}_{-0.002}$	0.800	$^{+0.127}_{-0.127}$	1.298	$^{+0.206}_{-0.206}$
ISR [GeV]	5	62.50	1.350	$^{+0.011}_{-0.011}$	1.826	$^{+0.016}_{-0.039}$	1.353	$^{+0.016}_{-0.029}$
	4	52.81	1.369	$^{+0.006}_{-0.006}$	1.851	$^{+0.014}_{-0.018}$	1.352	$^{+0.012}_{-0.014}$
	3	44.64	1.388	$^{+0.003}_{-0.007}$	1.871	$^{+0.031}_{-0.015}$	1.348	$^{+0.023}_{-0.011}$
	2	30.54	1.434	$^{+0.001}_{-0.004}$	1.957	$^{+0.013}_{-0.028}$	1.365	$^{+0.010}_{-0.020}$
	1	23.46	1.450	$^{+0.005}_{-0.004}$	1.973	$^{+0.011}_{-0.018}$	1.361	$^{+0.009}_{-0.013}$

An observation

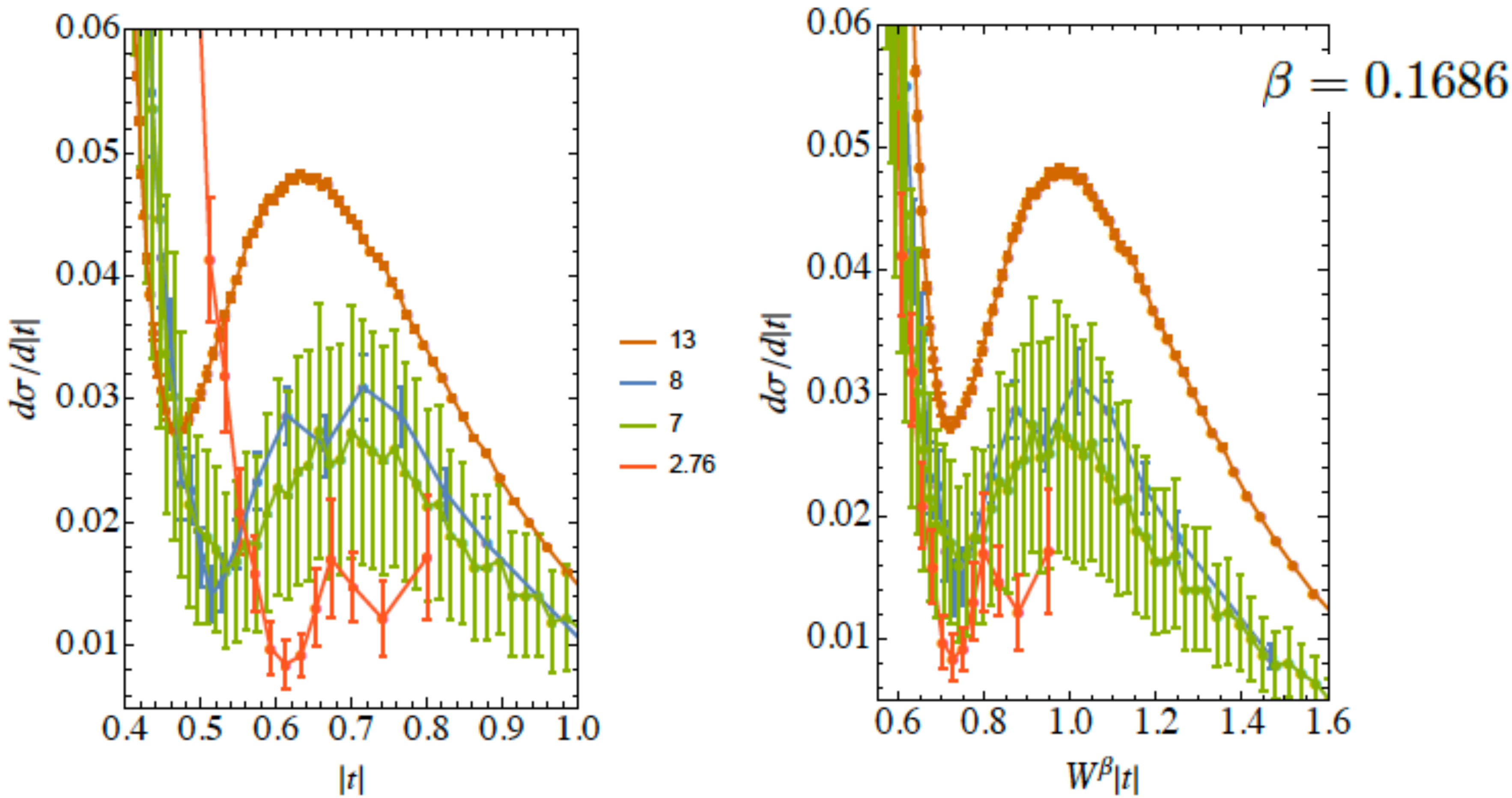


An observation

The fact that $t_{\text{bump}}/t_{\text{dip}}$ = const. implies: $\tau = f(s)|t|$



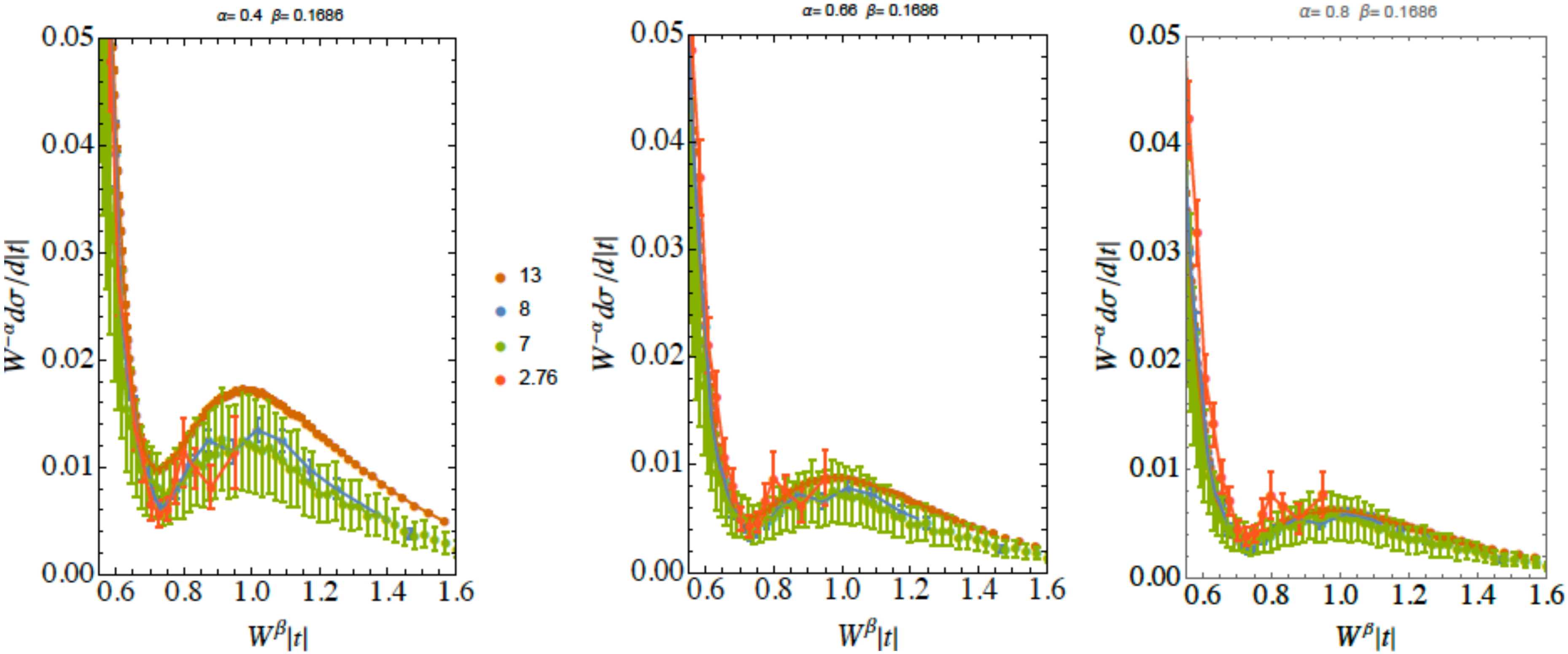
Scaling at the LHC – first step



Bump and dip positions are superimposed. Now we have to superimpose bump and dip values.

Scaling at the LHC – second step

$\alpha = 0.4$ 0.66 0.8

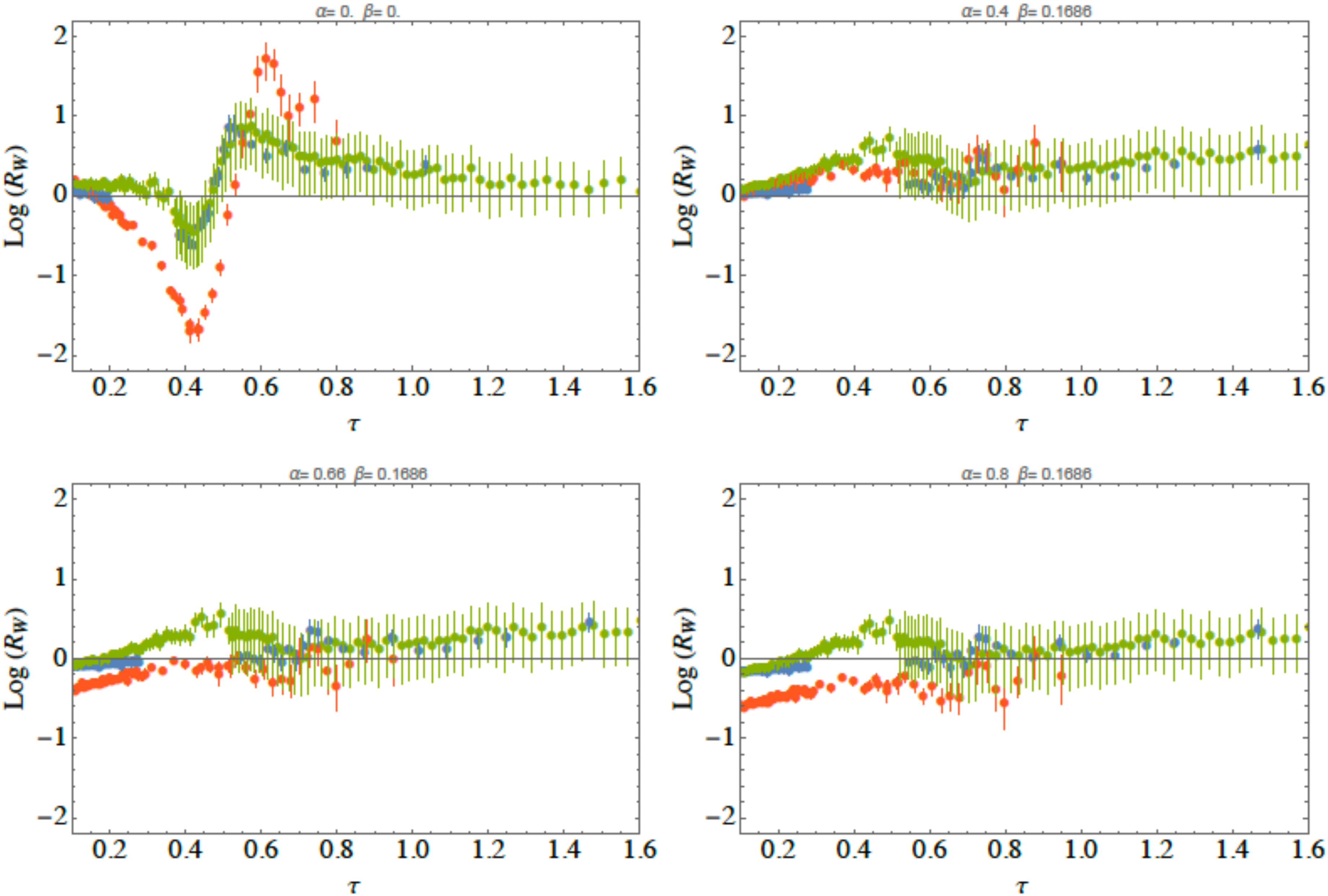


$$\frac{d\sigma_{\rm el}}{d|t|}(\tau) \rightarrow \left(\frac{W}{1\rm TeV}\right)^{-\alpha} \frac{d\sigma_{\rm el}}{d|t|}(\tau)$$

Scaling at the LHC – second step ratio method

$$W_{\mathrm{ref}} = 13\ \mathrm{TeV}$$

$$R_W(\tau_i) = \frac{d\tilde{\sigma}_{\mathrm{el}}/d|t|(W_{\mathrm{ref}}, \tau_i)}{d\tilde{\sigma}_{\mathrm{el}}/d|t|(W, \tau_i)}$$



α= 0. β= 0.
α= 0.4 β= 0.1686
α= 0.66 β= 0.1686
α= 0.8 β= 0.1686
Log (R_W)
τ

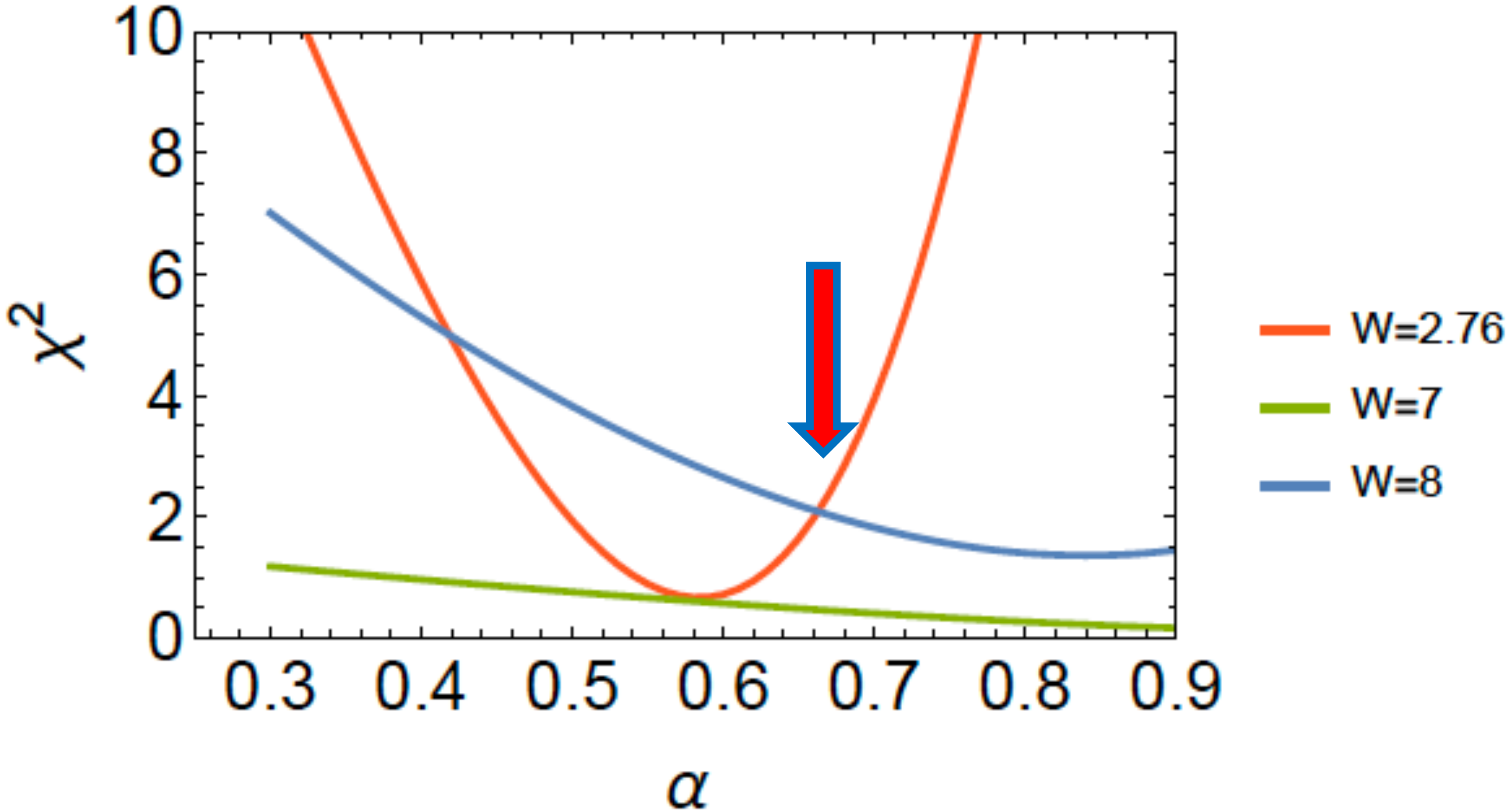
A few observations

- poor quality of lower energy data
- hard to find the best value of α “by an eye”
- try χ^2

$$\chi^2(W) = \frac{1}{N_W}\sum_{i=1}^{N_W}\left(\frac{R_W(\tau_i)-1}{\delta R_W(\tau_i)}\right)^2$$

$$0.35\ \mathrm{GeV}^2 < |t| < 1.5\ \mathrm{GeV}^2$$

Fixing α



A few observations

- poor quality of lower energy data
- hard to find the best value of α "by eye"
- try χ^2
- best value of α is determined by the lowest energy data
- 7 TeV data have large errors, χ^2 is flat
- small $|t|$ and large $|t|$ points do not scale as well as at the ISR
- no universal power for τ and normalization
- no problem with scaling in the dip region

Other scaling laws

Physics Letters B 830 (2022) 137141

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Scaling properties of elastic proton-proton scattering at LHC energies

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[c] Department of Physics, Penn State University, University Park, PA 16802, USA

$$\frac{1}{s^{\alpha/2}} \frac{d\sigma_{\mathrm{el}}}{d|t|}(s,t) = \Phi(\tau) \qquad \tau = s^a t^b$$

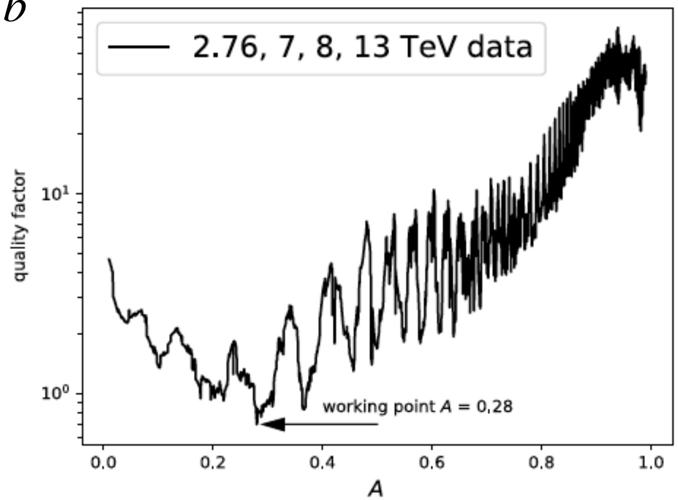
Baldenegro, Royon, Staśto

$$\frac{1}{s^{\alpha/2}} \frac{d\sigma_{\rm el}}{d|t|}(s,t) = \Phi(\tau) \qquad \tau = s^a t^b$$

Using quality factor method they find

$$\alpha \simeq 0.61 \qquad a \simeq 0.065 \qquad b \simeq 0.72$$

$A = 1 - b$



W TeV	χ^2 this paper	[11]
2.76	1.99	1.40
7	0.47	0.64
8	2.11	2.96

Other scaling laws

$$\frac{1}{s^{\alpha/2}}\frac{d\sigma_{\rm el}}{d|t|}(s,t)=\Phi(\tau) \qquad \tau=s^a t^b$$

In terms of variable τ positions of dips (and bumps) should be the same at *all* energies. We know from $t_{\rm bump}/\, t_{\rm dip}$= const. that $t_d=s^{\beta/2}B_{\rm dip}$

Hence,

$$\tau_{\rm d}=s^a t_{\rm d}^b=s^{a+b\,\beta/2}B_{\rm dip}^b$$

is energy independent. Therefore

$$a+b\,\beta/2=0$$

Other scaling laws

$$a + b\,\beta/2 = 0 \qquad (*)$$

Experimental fact at the LHC energies

$$\beta = -0.1686$$

Baldenegro, Royon, Staśto fit:

$$a \simeq 0.065 \qquad b \simeq 0.72$$

Substituting their b to the constraint (*)

$$a = 0.061 \pm 0.001$$

6% off

Amplitude parametrizations

One commonly uses two exponent parametrizations of elastic amplitude

$$\mathcal{A}(s,t) = i\left(\mathcal{A}_1(s,t) + \mathcal{A}_2(s,t)e^{i\phi}\right)$$

with

$$\mathcal{A}_i(s,t) = N_i(s)\, e^{-B_i(s)|t|}$$

Solving $t_{\mathrm{bump}}/\ t_{\mathrm{dip}} = \mathrm{const.}$ condition gives

$$N_i(s) = n_i N(s) \quad \text{and} \quad B_i(s) = b_i B(s)$$

Summary and Conclusions

- Ratio $(t_{\rm b}/t_{\rm d}) = 1.355$ is constant from Ithe SR to the LHC
- This implies scaling variable $\tau = f(s)|t|$: all dips and bumps have the same position
- At the ISR x-sections: total, elastic and inelastic have the same energy dependence
- This leads to the concept of geometrical scaling $\Omega(s,b) = \Omega\left(b/R(s)\right)$
- Not true at the LHC, but ....
- Cross-section ratios bump/dip: approximately constant at the LHC (not at the ISR)
- Cross-section values scaled by $g(s)$ supperimpose data for all energies
- Qualitative measure of alignement: ratios of scaled x-sections or quality factor
- Family of different scalings: $\tau = s^a t^b$ where $a + b\,\beta/2 = 0$ with $\beta = -0.1686$
- Constant ratio $(t_{\rm b}/t_{\rm d})$ constrains phenomenological parametrizations of $\mathcal{A}(s,t)$

Summary and Conclusions

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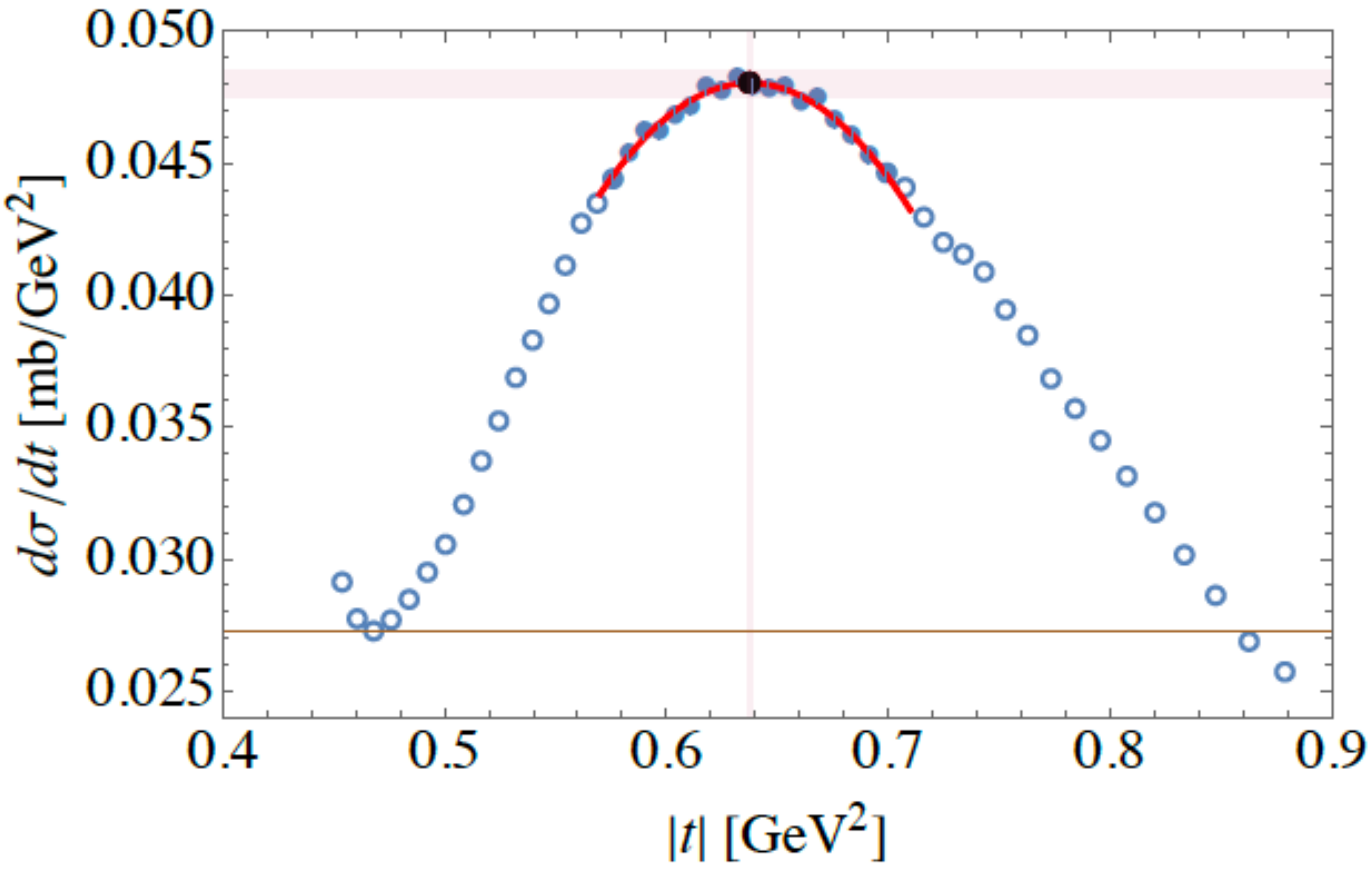
Thank you

Backup slides

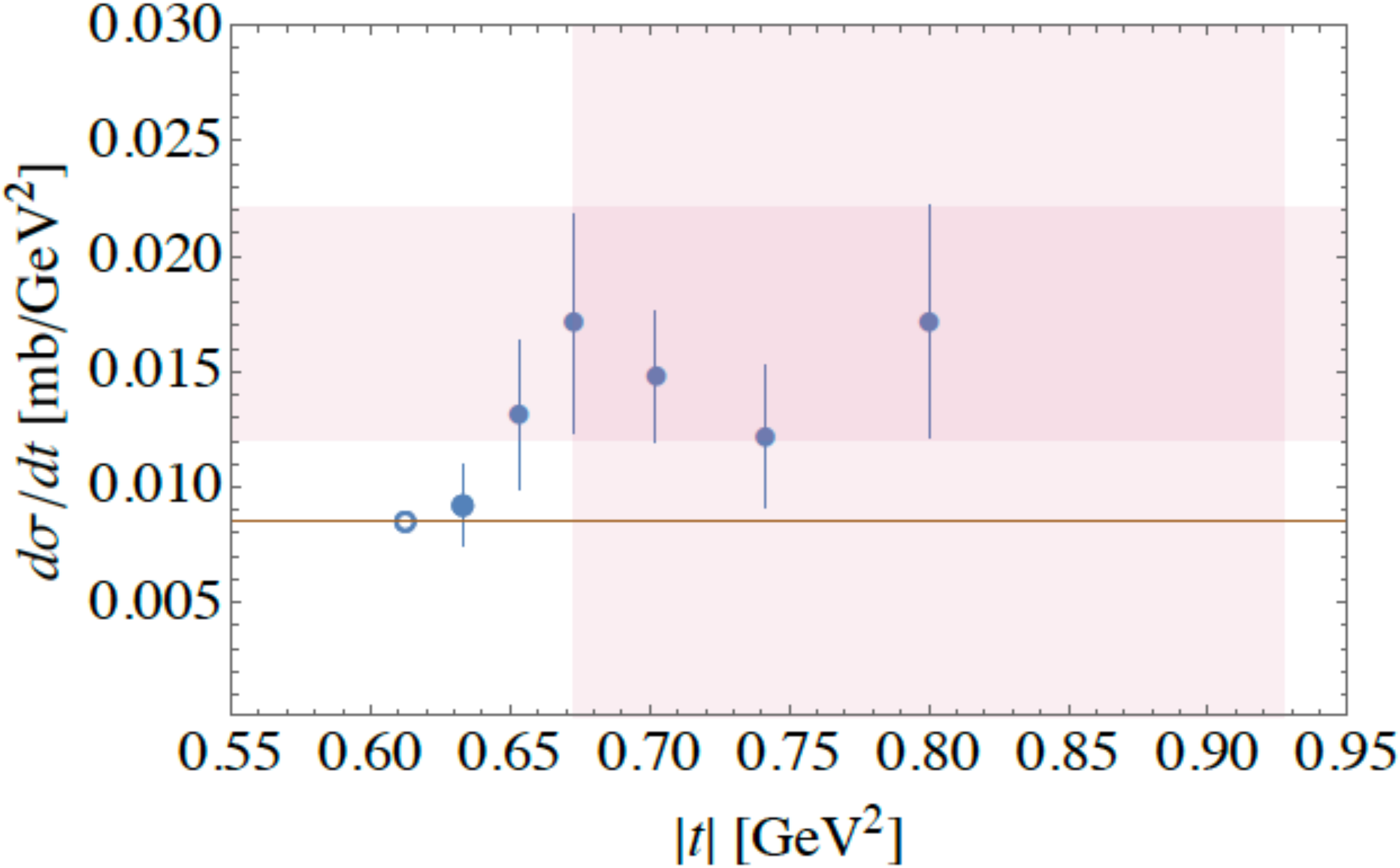
Fitting dips and bumps

13 TeV

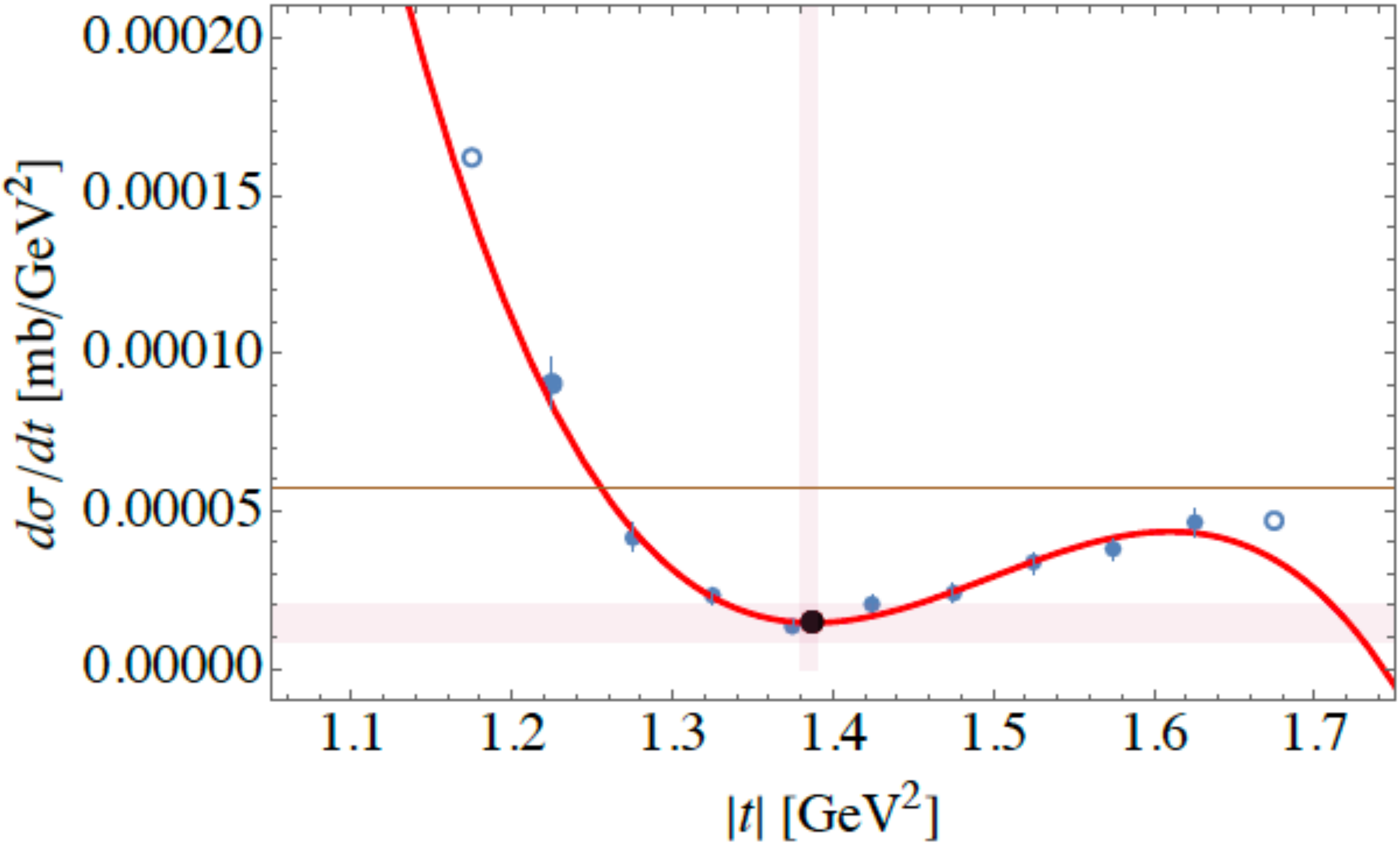


Fitting dips and bumps

2.76 TeV



Fitting dips and bumps



40.64 GeV

Quality factor

$$\mathrm{QF} = \left[\Sigma_i \frac{(v_{i+1} - v_i)^2 \times \Delta v_{i+1} \times \Delta v_i}{(u_{i+1} - u_i)^2 + \epsilon^2}\right]$$

$$\Delta v_i \leftarrow \Delta y_i$$

$$y_i \rightarrow v_i = \log y_i$$

One shifts and rescales variables u_i and v_i so that they are betwween 0 and 1

$$x_i \rightarrow u_i = \log x_i$$