Dipole picture diffractive structure function at NLO

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Outline

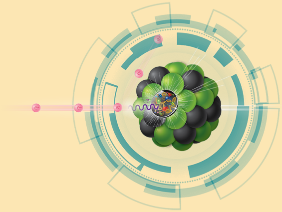
Outline of this talk

- Dipole + eikonal scattering picture of DIS, power counting
- Diffractive DIS
- Contributions to the NLO amplitude
- Technical remarks about the calculation of $F^D_{2,L}$

- Presented in G. Beuf, T. Lappi, H. Mäntysaari, R. Paatelainen, J. Penttala 2401.17251 [hep-ph]
- Partial results G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari, arXiv:2206.13161

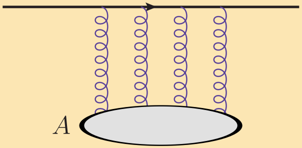
Process of interest

DIS in the high energy saturation regime



High energy collisions as eikonal scattering

Eikonal scattering off target of glue



How to measure small-x glue?

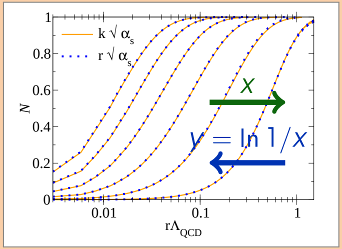
- Dilute probe through target color field
- At high energy interaction is **eikonal**, $\perp$ coordinate conserved in scattering

- Amplitude for quark: **Wilson line**

$$\mathbb{P}\exp\left\{-ig\int^{x^+} \mathrm{d}y^+ A^-(y^+,x^-,\mathbf{x})\right\} \underset{x^+\to\infty}{\approx} V(\mathbf{x}) \in \mathrm{SU}(N_c)$$

- Amplitude for color dipole

$$\mathcal{N}(r=|\mathbf{x}-\mathbf{y}|) = 1 - \left\langle \frac{1}{N_c} \operatorname{tr} V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle$$

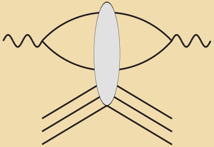


- $r=0$: color transparency, $r \gg 1/Q_s$: saturation , nonperturbative!

Dipole picture of DIS

Limit of small x, i.e. high γ^*-target energy

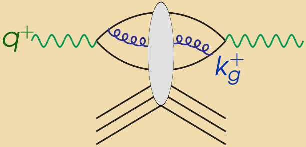
Leading order



- $\gamma^* \to q\bar{q}$ in vacuum
- $q\bar{q}$ interacts eikonally with target

"Dipole model": Nikolaev, Zakharov 1991 ; Mueller

Leading Log: add **soft** gluon



$\Longrightarrow$ Large log $\int_{x_{Bj}} dk_g^+ / k_g^+ \sim \ln \frac{1}{x_{Bj}}$

BK-evolution of target

Balitsky 1995, Kovchegov 1999

NLO

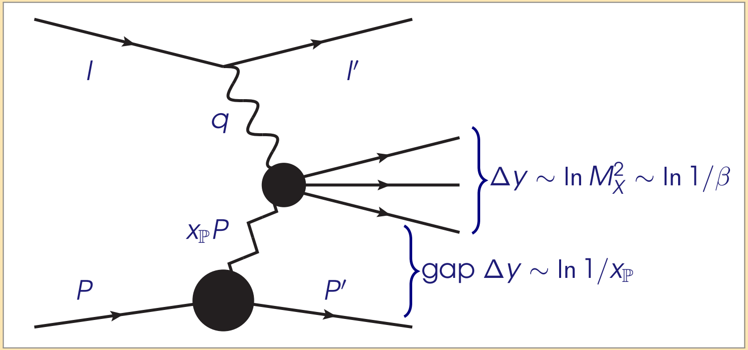
The same gluon with full kinematics

Calculational framework: **Light Cone quantize** γ^*

Diffractive DIS

Inclusive diffraction, kinematics

$\gamma^* + A \to X + A$, differential in M_X



- Momentum transfer $t = (P - P')^2$
- Gap size $x_{\mathbb{P}}$, target evolution rapidity $\sim \ln 1/x_{\mathbb{P}}$
- Diffractive system mass M_X^2, $\beta = Q^2/(Q^2 + M_X^2)$
- Virtuality Q^2
- Lower $x_{\mathbb{P}}$ than dijets (e.g. at EIC)

$$x_{Bj} = x_{\mathbb{P}} \beta$$

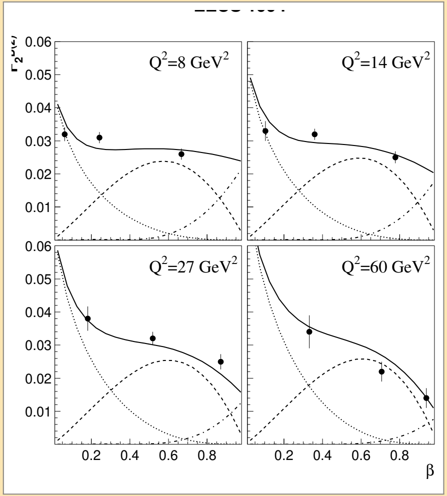
This talk: $x_{\mathbb{P}}$ small, β not.

Dependence on β, i.e. M_X

${M_X}^2$ = photon remnants.

Essential regimes:

- Large $\beta \to 1$ — small M_X:
  longitudinal $\gamma^* \to q\bar{q}$
- Medium $\beta \sim 0.5$ — ${M_X}^2 \sim Q^2$:
  transverse $\gamma^* \to q\bar{q}$
- Small $\beta \ll 1$ — large ${M_X}^2$:
  higher Fock states ($q\bar{q}g$ etc.)



LO $q\bar{q}$ and leading ln Q^2 $q\bar{q}g$

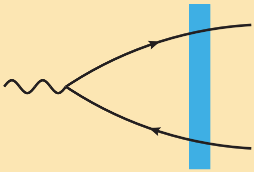
Golec-Biernat& Wüsthoff hep-ph/9903358

NLO amplitude for diffractive DIS

Diffractive DIS at leading order

- Earlier: factorized impact parameter dependence
- Full kinematics G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari, arXiv:2206.13161

$$\frac{\mathrm{d}\sigma^{D}_{\lambda,q\bar{q}}}{\mathrm{d}{M_X}^2\,\mathrm{d}|t|} = \frac{N_c}{4\pi}\int_0^1 \mathrm{d}z \int_{\mathbf{x}_0\mathbf{x}_1\bar{\mathbf{x}}_0\bar{\mathbf{x}}_1} \mathcal{I}^{(2)}_{\boldsymbol{\Delta}}\mathcal{I}^{(2)}_{M_X} \times \sum_{f,h_0,h_1}\left(\widetilde{\psi}_{\gamma^*_\lambda\to q_{\bar{0}}\bar{q}_{\bar{1}}}\right)^{\dagger}\left(\widetilde{\psi}_{\gamma^*_\lambda\to q_0\bar{q}_1}\right)\boxed{\left[S^{\dagger}_{\bar{0}\bar{1}}-1\right]\left[S_{01}-1\right]}$$



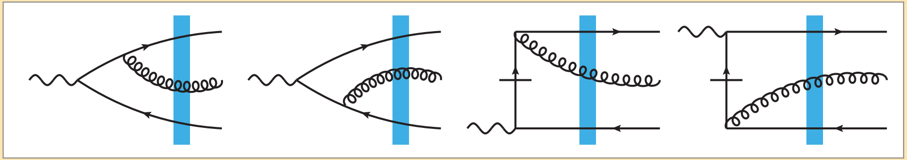
(Blue: shockwave target)

- $q\bar{q}$ crossing shockwave: dipole S_{01}
- "Transfer functions:" relate coordinates at shockwave to:
  - Momentum transfer $t = -\boldsymbol{\Delta}^2$ $\mathcal{I}^{(2)}_{\boldsymbol{\Delta}} = \frac{1}{4\pi}J_0\left(\sqrt{|t|}\,\|z\mathbf{x}_{\bar{0}0} - (1-z)\mathbf{x}_{\bar{1}1}\|\right)$
  - Invariant mass $\mathcal{I}^{(2)}_{M_X} = \frac{1}{4\pi}J_0\left(\sqrt{z(1-z)}M_X\|\bar{\mathbf{r}}-\mathbf{r}\|\right)$

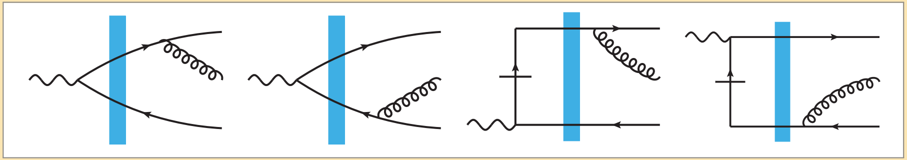
Now to NLO

NLO radiative corrections

- Emission before target

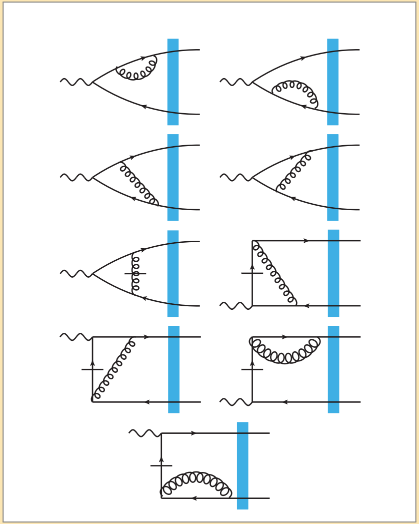


  - Squares already in G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari `arXiv:2206.13161`
  - Contain leading $\ln Q^2$ contribution
- Emission after target



- Interferences $\Longrightarrow$ simplify with some of the virtual corrections

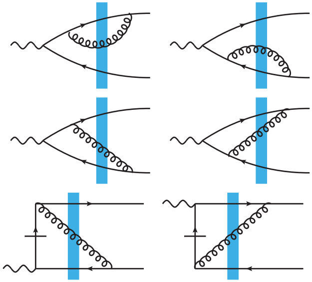
NLO virtual



- Vertex corrections:
  known 1-loop $\gamma \to q\bar{q}$ wavefunction

See also Boussarie et al 2014: diffractive jets,
also Caucal et al 2021 inclusive

NLO virtual



- Vertex corrections:
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- Gluon crosses shockwave, but not the cut:
  - Loop corrections to amplitude,
    tree level wavefunctions
  - 3-point operator of Wilson lines
  - BK/JIMWLK evolution of LO amplitude

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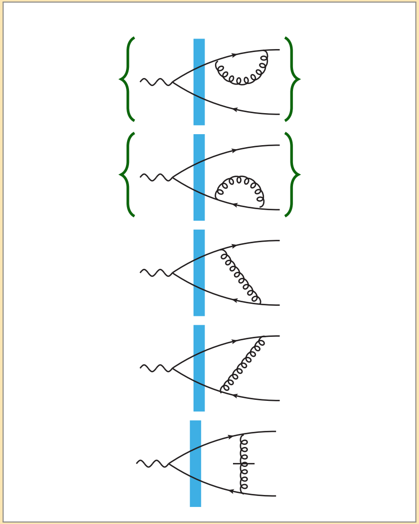

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  - BK/JIMWLK evolution of LO amplitude
- Final state interactions
  (Propagator corrections $\{\}$ $\to$
  State renormalization, in fact = 0 in dim. reg.)

See also Boussarie et al 2014: diffractive jets,

also Caucal et al 2021 inclusive

NLO cross section

Calculation in 2401.17251 [hep-ph]

We have calculated all these contributions

- Diffractive structure function:
  clean, [perturbative = experimental] final state definition M_X!
  (No fragmentation function, jet definition)
  $\Longrightarrow$ Divergences must cancel
- Explicit expressions will not fit in the slides, but there in 2401.17251 [hep-ph]

Rest of talk: two features of the calculation:

- Divergence structure
- Treatment of energy denominators

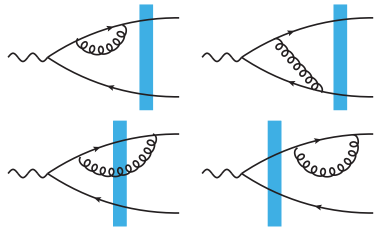
Regularization and divergences

Regularization procedure

- Transverse momentum in $2 - 2\varepsilon$ dimensions $\Longrightarrow$ $\frac{1}{\varepsilon}$ divergences, collinear or UV
- Longitudinal k^+: cutoff $k^+ > \alpha$, $\alpha \to 0$ $\Longrightarrow$ $1/\alpha$, $\ln^2 \alpha$, $\ln \alpha$ divergences

1. UV $\frac{1}{\varepsilon}$ and $\frac{1}{\varepsilon} \ln \alpha$ divergences: $\gamma^* \to q\bar{q}$ vertex, gluon crossing shock, wavefunction renormalization

1. UV $\frac{1}{\varepsilon}$ e.g. in



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2. Collinear $\frac{1}{\varepsilon}$:
   wavef. renormalization, final state emission

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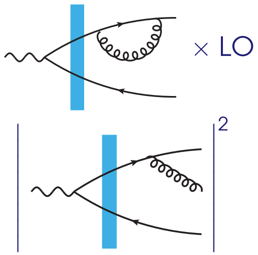

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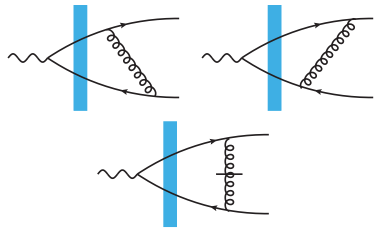
3. $1/\alpha$ e.g. in



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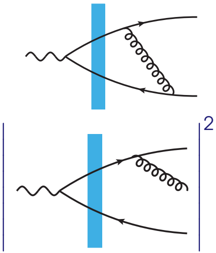
2. Collinear $\frac{1}{\varepsilon}$: wavef. renormalization, final state emission
3. $1/\alpha$ cancels between normal and instantaneous exchange
4. $\ln^2 \alpha$ from final state exchange and emission (${M_X}$ restriction matters here!)

4. $\ln^2 \alpha$ e.g. in



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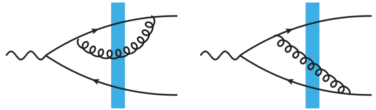
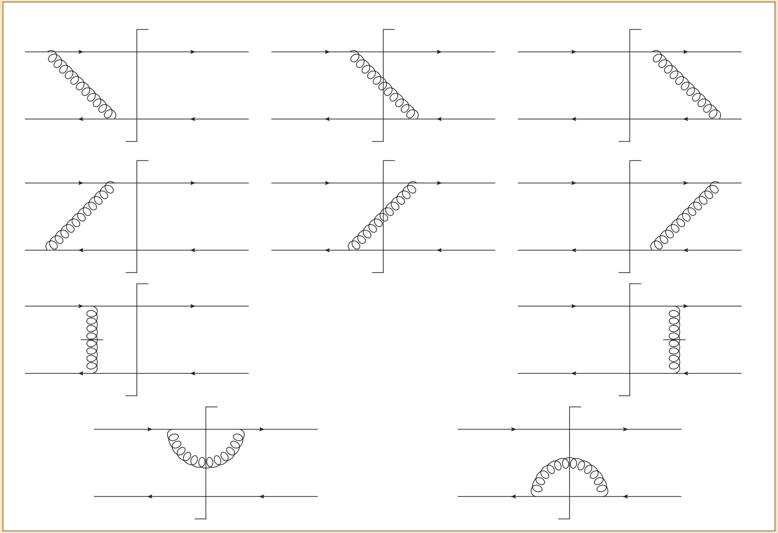
1. UV $\frac{1}{\varepsilon}$ and $\frac{1}{\varepsilon} \ln \alpha$ divergences: $\gamma^* \to q\bar{q}$ vertex, gluon crossing shock, wavefunction renormalization
2. Collinear $\frac{1}{\varepsilon}$: wavef. renormalization, final state emission
3. $1/\alpha$ cancels between normal and instantaneous exchange
4. $\ln^2 \alpha$ from final state exchange and emission (*M_X restriction matters here!*)
5. Remaining $\ln \alpha$ absorbed into BK/JIMWLK

5. $\ln \alpha$ e.g. in



Final state corrections

How to dig out different types of divergences?



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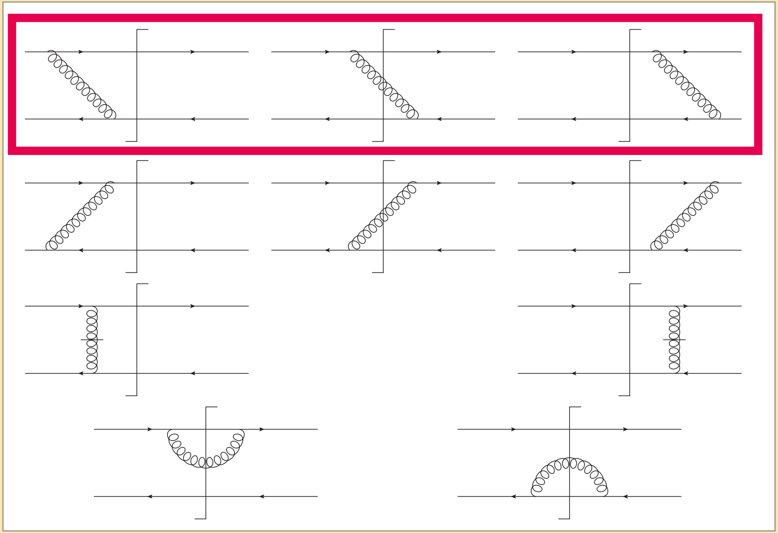
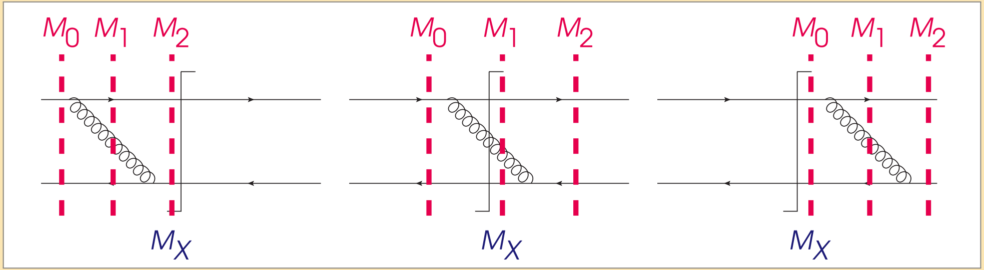


As an example: consider 1st row

Combine energy denominators

"Beuf trick": write M_X delta function as imaginary part of "propagator"



$$\frac{\delta(M_X{}^2 - M_2^2)}{(M_2^2 - M_1^2 + i\delta)(M_2^2 - M_0^2 + i\delta)} + \frac{\delta(M_X{}^2 - M_1^2)}{(M_1^2 - M_0^2 + i\delta)(M_1^2 - M_2^2 - i\delta)} + \frac{\delta(M_X{}^2 - M_0^2)}{(M_0^2 - M_1^2 - i\delta)(M_0^2 - M_2^2 - i\delta)}$$

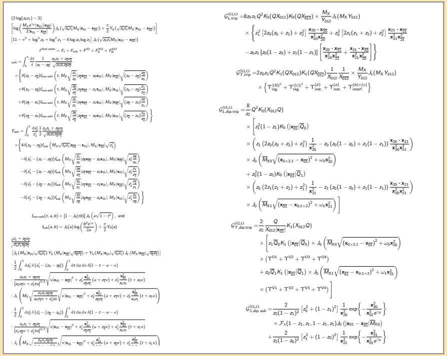
(Note: sign of $i\delta$ essential)

$$= \frac{1}{2\pi i}\left[\frac{1}{(M_X{}^2 - M_0^2 - i\delta)(M_X{}^2 - M_1^2 - i\delta)(M_X{}^2 - M_2^2 - i\delta)} - \text{c.c.}\right]$$

- Then express numerator (⊥ momentum dot products) in terms of M_0^2, M_1^2, M_2^2
- Combine before integration—then separate different divergence types

Conclusions

- Diffractive DIS total cross section = diffractive structure function F_2^D **calculated in dipole picture fully at NLO**
- General impact parameter dependence
- Reproduces earlier large M_X, ln Q^2 limits, used so far in phenomenology
- Future:
  - Numerical implementation
  - Inclusion in global fits



(The result: logs, Bessel functions, polynomials in z_i's)