TMD factorisation for diffractive jets in photon-nucleus interaction

Siggi Hauksson
IPhT, Saclay

DIS 2024, Grenoble
April 9th 2024

In collaboration with E. Iancu, A. Mueller, D. Triantafyllopoulos, S.Y. Wei

arXiv:2402.14748

Introduction

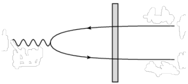
- Color glass condensate (CGC) describes physics of order Q_s in nuclei at high energy.
  [See e.g. Iancu, Venugopalan (2003); Gelis, Iancu, Jalilian-Marian, Venugopalan (2010)]
- Experimental processes also involve hard scales:
  - E.g. Q^2 in DIS and transverse momentum $P_\perp$ (back-to-back jets).
- Do CGC predictions capture physics of hard scales as well?
  - Often captured by collinear and TMD factorization.
- TMD factorization seen in inclusive eA, pA processes, even at NLO.
  [Dominguez, Marquet, Xiao, Yuan (2011); Taels, Altinoluk, Beuf, Marquet (2022); Caucal, Salazar, Schenke, Stebel, Venugopalan (2023)]
- We want to consider diffractive DIS.
  - More sensitive to saturation.
  - Simpler setup to study CGC and factorization.

Introduction

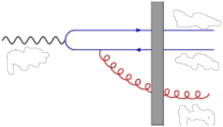
- Typical approach: assume collinear factorization, fit diffractive PDF with data.

$$\frac{d\sigma_D^{\gamma A \to X A}}{d \ln \frac{1}{\beta}} = \left(\frac{4\pi^2 \alpha_{em}}{Q^2} \right) \otimes \beta q^D(\beta, x_P, Q^2)$$

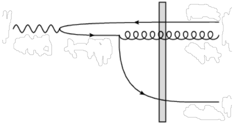
- Our work: only use diagrams in CGC framework and dipole picture.



Exclusive dijet
See e.g. [Golec-Biernat, Wusthoff (1999)]



(2+1) jets, soft gluon
[Iancu, Mueller, Triantafyllopoulos, Wei (2022)]



(2+1) jets, soft quark
NEW

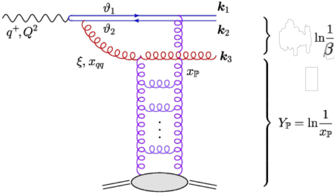
- Two observables: diffractive SIDIS and diffractive dijet production.
  - NEW Establish TMD factorization, e.g. for SIDIS,

$$\frac{d\sigma_D^{\gamma A \to X A}}{d \ln \frac{1}{\beta}\, d^2\mathbf{K}} = \left(\frac{4\pi^2 \alpha_{em}}{Q^2} \right) \otimes \frac{d\beta q^D(\beta, x_P, Q^2)}{d^2\mathbf{K}}$$

  - NEW Universality of diffractive TMDs across different processes.
    See also [Hatta, Xiao, Yuan (2022)]
  - NEW Show emergence of DGLAP at TMD level for target.

- Diffractive DIS using CGC formalism.

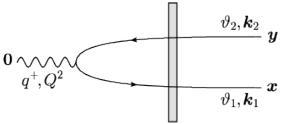
  [For recent NLO work see e.g. Boussarie, Grabovsky, Szymanowski, Wallon (2016); Beuf, Hanninen, Lappi, Mulian, Mantysaari (2022); Fucilla, Grabovsky, Li, Szymanowski, Wallon (2023); Beuf, Lappi, Mantysaari, Paatelainen, Penttala (2024)]



- Look at coherent events: target emerges unbroken.
  - Small transverse momentum exchange.
  - Colorless exchange (pomeron).

- Elastic scattering: Cross section goes like $\mathcal{T}^2$ (sensitive to strong scattering and saturation).
  - E.g. in MV model $\mathcal{T}(R) = 1 - \exp\left(-\frac{R^2 Q_A^2}{4} \ln \frac{4}{R^2\Lambda^2}\right)$ [McLerran, Venugopalan (1994)]

- Characterize rapidity gap with $\beta = Q^2/(Q^2 + M_X^2) \sim \frac{1}{2}$.
- $\beta = x_{\mathrm{Bj}}/x_{\mathbb{P}}$ where $x_{\mathrm{Bj}} = q^-/P_N^-$ and $x_{\mathbb{P}} = p_{\mathbb{P}}^-/P_N^-$
  - Fraction of longitudinal momentum carried from pomeron to photon.
  - $\ln\frac{1}{\beta} = \ln\frac{1}{x_{\mathrm{Bj}}} - \ln\frac{1}{x_{\mathbb{P}}}$ can be interpreted as the complementary rapidity gap.



- Wavefunction in dipole picture is $(\theta_1 = k_1^+/q^+, \theta_2 = k_2^+/q^+)$

$$\psi^i_{\lambda_1,\lambda_2} = \delta_{\lambda_1\lambda_2}\sqrt{\frac{q^+}{2}}\frac{ee_f}{(2\pi)^3}\frac{\varphi^{ij}(\theta_1,\lambda_1)\,k_1^j}{k_{1\perp}^2+\overline{Q}^2}$$

- Typical momentum is $k_{1\perp}^2 \sim \overline{Q}^2 = \theta_1\theta_2 Q^2$.
- Add interaction with target in CGC picture: $\mathcal{T}(\mathbf{R}) = \left\langle 1 - \frac{1}{N_c}\mathrm{tr}\left[V(\mathbf{x})V^\dagger(\mathbf{y})\right]\right\rangle$,
- For strong scattering need $R = |\mathbf{x}-\mathbf{y}| \sim 1/k_{1\perp} \sim 1/Q_s$:
  - $k_\perp^2 \sim Q_s^2$ implies $\theta_1\theta_2 \ll 1$: very asymmetric jets (aligned jets).
- $\frac{d\sigma}{d\theta_1 d^2\mathbf{K}} \to \frac{d\sigma}{d\ln\frac{1}{\beta}d^2\mathbf{K}}$: go from projectile perspective (θ_1) to target perspective (β).

Diffractive SIDIS: exclusive dijets

- At leading twist (Q^2 large) get

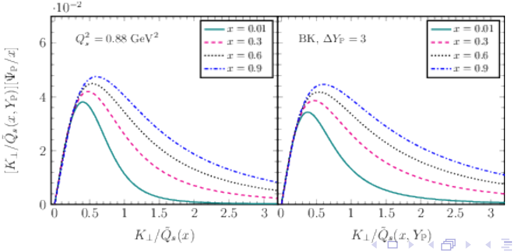
$$\frac{d\sigma}{d\ln\frac{1}{\beta}\,d^2\mathbf{K}} = \frac{4\pi^2\alpha_{em}\sum e_f^2}{Q^2}\,2\frac{dxq_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{d^2\mathbf{K}}\Bigg|_{x=\beta}$$

- We see TMD factorization.
- Here

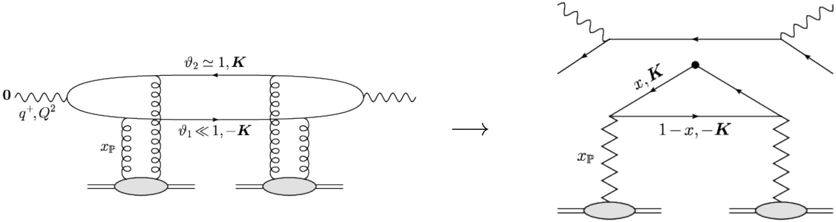
$$\frac{dxq_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{d^2\mathbf{K}} \sim S_\perp\frac{2N_c}{(2\pi)^4}x\begin{cases}1 & \text{for } K_\perp \ll Q_s\\ \frac{(1-x)^2Q_s^4}{K_\perp^4} & \text{for } K_\perp \gg Q_s\end{cases}$$

  - TMD heavily suppressed at $K_\perp \gg Q_s$ because of small dipole (color transparency), $\sigma \sim \mathcal{T}^2$.

- Numerics: MV model with BK evolution (coll. improved + running coupling). ($\tilde{Q}_s^2 = (1-x)Q_s^2$)



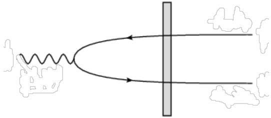
Diffractive SIDIS: exclusive dijets



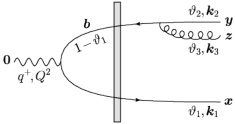
- Do calculation from point of view of projectile (θ_1) but interpret from point of view of target (β).
- Diffractive TMD measures quark taken from pomeron.
- The quark is absorbed by the photon to give the quark we measure (hard process).

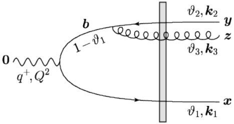
Hard dijets: (2+1) jets

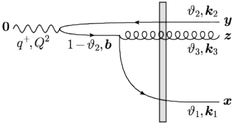
- How to produce diffractive hard dijets ($P_\perp \gg Q_s$)?
- Exclusive dijets (formally LO): suppressed as $1/P_\perp^6$.
- (2+1) jets (formally NLO): goes like $1/P_\perp^4$
  - Suppressed by α_s but dominates at large $P_\perp$



- Need a quark with $K_\perp \sim Q_s$.
- Effectively a $q\bar{q}$ dipole which interacts strongly with the target.

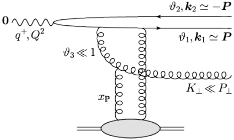


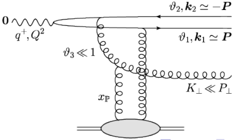




- Have two momentum scales: $K_\perp \sim Q_s$ and $P_\perp \gg Q_s$ of measured quark.
- Equally important as a soft gluon and a gg dipole.

[Iancu, Mueller, Triantafyllopoulos (2021); Iancu, Mueller, Triantafyllopoulos, Wei (2022)]

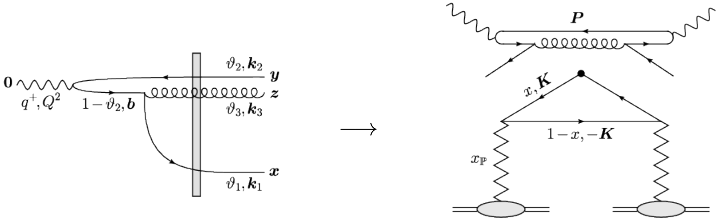


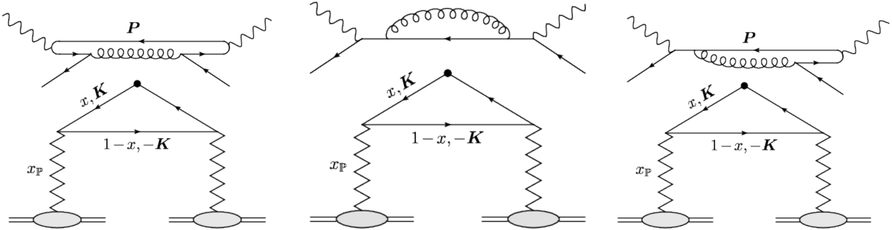


- Calculate $\frac{d\sigma}{d\theta_1 \, d\theta_2 \, d\theta_3 \, d^2K \, d^2P}$ for $P \gg Q_s$, $K \sim Q_s$.
- Energy denominators give $K^2 \sim \mathcal{M}^2 = \theta_1 \left(P^2/\theta_2\theta_3 + Q^2\right)$ so $\theta_1 \sim K^2/Q^2 \ll 1$.
- Suggests that quark can be seen as part of target.
- Need to change variables from θ_1 to x:

$$x x_{\mathbb{P}} = x_{\bar{q}g}$$

  - x is momentum fraction carried by quark from pomeron.





- With the new variable x, we see TMD factorization for (2+1) jets.

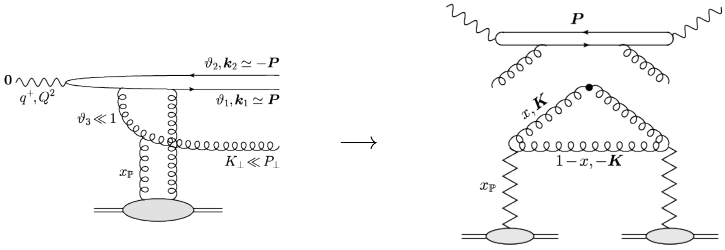
$$\frac{d\sigma}{d\theta_2 d\theta_3 d^2\mathbf{P} d^2\mathbf{K} d\ln(1/x)} = \frac{4\pi^2\alpha_{em}\sum e_f^2}{Q^2}\mathcal{H}_T(\theta_2,\theta_3,P_\perp^2,\tilde{Q}^2)\frac{dxq_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{d^2\mathbf{K}}$$

where

$$\mathcal{H}_T(\theta_2,\theta_3,P_\perp^2,\tilde{Q}^2) = \delta(\theta_2+\theta_3-1)\frac{\alpha_s C_F}{\pi^2}\frac{\tilde{Q}^2[(P_\perp^2+\tilde{Q}^2)^2+\theta_2^2\tilde{Q}^4+\theta_3^2 P_\perp^4]}{P_\perp^2(P_\perp^2+\tilde{Q}^2)^3}; \quad \tilde{Q}^2 = \theta_2\theta_3 Q^2$$

- Same quark TMD as we saw in SIDIS: Universality
- Similar TMD factorization seen for soft gluon process: gluon TMD

Hard dijets: (2+1) jets



- With the new variable x, we see TMD factorization for (2+1) jets.

$$\frac{d\sigma}{d\theta_2 d\theta_3 d^2\mathbf{P} d^2\mathbf{K} d\ln(1/x)} = \frac{4\pi^2\alpha_{em}\sum e_f^2}{Q^2}\mathcal{H}_T(\theta_2,\theta_3,P_\perp^2,\tilde{Q}^2)\frac{dxq_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{d^2\mathbf{K}}$$

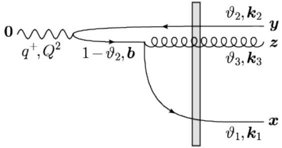
where

$$\mathcal{H}_T(\theta_2,\theta_3,P_\perp^2,\tilde{Q}^2) = \delta(\theta_2+\theta_3-1)\frac{\alpha_s C_F}{\pi^2}\frac{\tilde{Q}^2[(P_\perp^2+\tilde{Q}^2)^2+\theta_2^2\tilde{Q}^4+\theta_3^2 P_\perp^4]}{P_\perp^2(P_\perp^2+\tilde{Q}^2)^3}; \quad \tilde{Q}^2 = \theta_2\theta_3 Q^2$$

- Same quark TMD as we saw in SIDIS: Universality
- Similar TMD factorization seen for soft gluon process: gluon TMD

Diffractive SIDIS: (2+1) jets

- How do (2+1) jets contribute to diffractive SIDIS?



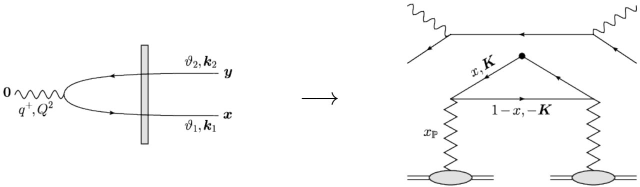
$\longrightarrow$?

- Will see that (2+1) jets give DGLAP evolution of target TMD!
- Change from θ_2, θ_3 to the diffractive variable β.

$$\frac{d\sigma}{d\ln(1/\beta)d^2\mathbf{P}} = \int d\theta_2 d\theta_3 \int \frac{dx}{x} \beta\delta\left(\beta - x\frac{\tilde{Q}^2}{\tilde{Q}^2 + P_\perp^2}\right) \int d^2\mathbf{K}\, \frac{d\sigma}{d\theta_2 d\theta_3 d^2\mathbf{P} d^2\mathbf{K} d\ln(1/x)}$$

- Dominant contribution comes from $\theta_2 \approx 1 \gg \theta_3 \sim \frac{P_\perp^2}{Q^2} \gg \theta_1 \sim \frac{Q_s^2}{Q^2}$.
  - Can move both gluon and quark to the target.
- Also $\mathbf{k}_2 \approx -\mathbf{k}_3 \approx \mathbf{P}$ and $\mathbf{k}_1 \approx \mathbf{K}$ with $\mathbf{K} \ll \mathbf{P}$.
  - Kinematics of DGLAP evolution.

- Recall exclusive dijet contribution to SIDIS:

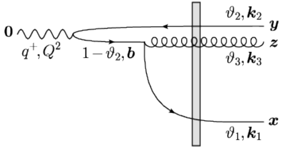


- Will see that (2+1) jets give DGLAP evolution of target TMD!
- Change from θ_2, θ_3 to the diffractive variable β.

$$\frac{d\sigma}{d\ln(1/\beta)d^2\mathbf{P}} = \int d\theta_2 d\theta_3 \int \frac{dx}{x} \beta\delta\left(\beta - x\frac{\tilde{Q}^2}{\tilde{Q}^2 + P_\perp^2}\right) \int d^2\mathbf{K} \frac{d\sigma}{d\theta_2 d\theta_3 d^2\mathbf{P} d^2\mathbf{K} d\ln(1/x)}$$

- Dominant contribution comes from $\theta_2 \approx 1 \gg \theta_3 \sim \frac{P_\perp^2}{Q^2} \gg \theta_1 \sim \frac{Q_s^2}{Q^2}$.
  - Can move both gluon and quark to the target.
- Also $\mathbf{k}_2 \approx -\mathbf{k}_3 \approx \mathbf{P}$ and $\mathbf{k}_1 \approx \mathbf{K}$ with $\mathbf{K} \ll \mathbf{P}$.
  - Kinematics of DGLAP evolution.

- How do (2+1) jets contribute to diffractive SIDIS?



$\longrightarrow$?

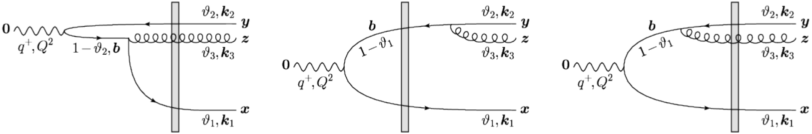
- Will see that (2+1) jets give DGLAP evolution of target TMD!
- Change from θ_2, θ_3 to the diffractive variable β.

$$\frac{d\sigma}{d\ln(1/\beta)d^2\mathbf{P}} = \int d\theta_2 d\theta_3 \int \frac{dx}{x}\beta\delta\left(\beta - x\frac{\tilde{Q}^2}{\tilde{Q}^2 + P_\perp^2}\right)\int d^2\mathbf{K}\ \frac{d\sigma}{d\theta_2 d\theta_3 d^2\mathbf{P} d^2\mathbf{K} d\ln(1/x)}$$

- Dominant contribution comes from $\theta_2 \approx 1 \gg \theta_3 \sim \frac{P_\perp^2}{Q^2} \gg \theta_1 \sim \frac{Q_s^2}{Q^2}$.
  - Can move both gluon and quark to the target.
- Also $\mathbf{k}_2 \approx -\mathbf{k}_3 \approx \mathbf{P}$ and $\mathbf{k}_1 \approx \mathbf{K}$ with $\mathbf{K} \ll \mathbf{P}$.
  - Kinematics of DGLAP evolution.

Diffractive SIDIS: (2+1) jets

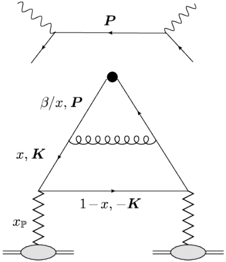
- Very different diagrams



- In the end get a strikingly simple answer.

$$\frac{d\sigma}{d\ln(1/\beta)d^2\mathbf{P}} = \frac{4\pi^2\alpha_{em}\sum e_f^2}{Q^2}\frac{\alpha_s}{2\pi^2}$$
$$\times\frac{1}{P_\perp^2}\int_{x_{\min}}^1 \frac{dx}{x}\frac{\beta}{x}P_{qq}\left(\frac{\beta}{x}\right) xq_{\mathbb{P}}(x, x_{\mathbb{P}}, (1-x)P_\perp^2)$$

  - DGLAP evolution for TMD of target.
  - From target point of view: Gluon emission before photon absorption.

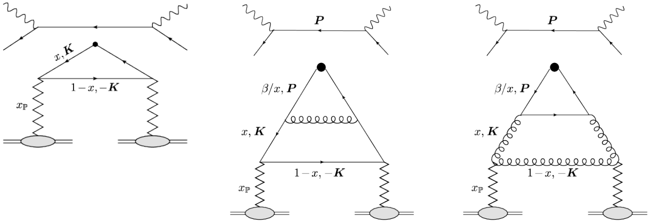


- $P_{qq}(z) = C_F(1+z^2)/(1-z)$ is the usual splitting function.
- Also less important contribution from final-state gluon emission.

Diffractive SIDIS: (2+1) jets

- SIDIS cross section is of the form

$$\frac{d\sigma}{d\ln(1/\beta)d^2\mathbf{P}} = \frac{4\pi^2\alpha_{em}\sum e_f^2}{Q^2} 2\frac{dxq_{\mathbb{P}}^{\text{total}}(x, x_{\mathbb{P}}, P_\perp^2)}{d^2\mathbf{P}}\Bigg|_{x=\beta}$$



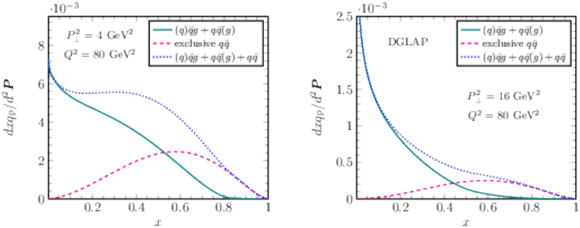
Exclusive dijet | (2+1) jet, soft quark | (2+1) jet, soft gluon

- TMD from exclusive dijet goes like $1/P_\perp^4$
- TMD from (2+1) jet goes like $1/P_\perp^2$.
  - Vanishes faster as $\beta \to 1$.
  - Soft gluon contribution also described by DGLAP evolution of target.

Diffractive SIDIS: (2+1) jets

- SIDIS cross section is of the form

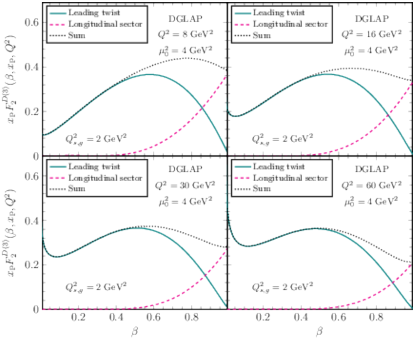
$$\frac{d\sigma}{d\ln(1/\beta)d^2\mathbf{P}} = \frac{4\pi^2\alpha_{em}\sum e_f^2}{Q^2} 2 \frac{dxq_{\mathbb{P}}^{\text{total}}(x, x_{\mathbb{P}}, P_\perp^2)}{d^2\mathbf{P}}\bigg|_{x=\beta}$$

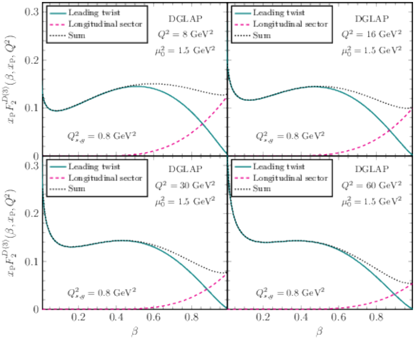


- TMD from exclusive dijet goes like $1/P_\perp^4$
- TMD from (2+1) jet goes like $1/P_\perp^2$.
  - Vanishes faster as $\beta \to 1$.
  - Soft gluon contribution also described by DGLAP evolution of target.

Diffractive structure function

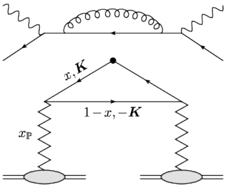
- $\frac{d\sigma}{d\ln(1/\beta)d^2\mathbf{P}} \to \frac{d\sigma}{d\ln(1/\beta)}$: Integrate out $\mathbf{P}$ to get diffractive structure function F_2^D.
- Evaluate for two cases:
  - Large nucleus: $Q_s^2 = 2\,\mathrm{GeV}^2$
  - "proton": $Q_s^2 = 0.8\,\mathrm{GeV}^2$
- DGLAP evolution starts at $\mu_0^2 = 2Q_s^2$, see back-up.
- See clear differences (different starting point for DGLAP).

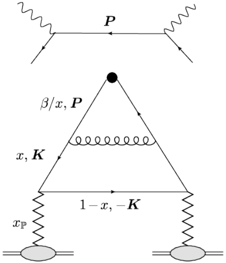




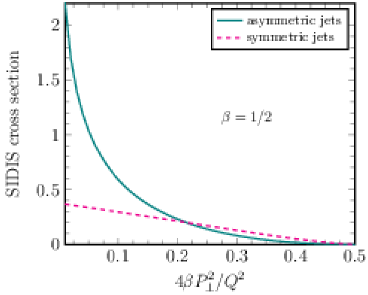
Conclusions

- Single framework for calculations (CGC and dipole picture).
- Show TMD factorization for diffractive hard dijet production.
  - (2+1) jet process.
- Show TMD factorization for diffractive SIDIS.
  - Exclusive dijets: quark TMD.
  - (2+1) jets: DGLAP evolution of target TMD.
- Get explicit expression for diffractive PDFs, TMDs valid for large nuclei at high energy.
  - Universality of TMDs, PDFs.
- Further developments:
  - Diffractive SIDIS: TMD factorization at NLO?
  - Inclusive SIDIS: DGLAP evolution?
  - Sudakov effect, CSS evolution of TMDs.
    [See also Marquet, Xiao, Yuan (2009)]
  - Phenomenology.





- Our (2+1) jet diagrams give two contributions:
  - Asymmetric jets: $\theta_2 \to 1$, $(x-\beta)/\beta \sim 1$, DGLAP evolution of target.
  - Symmetric jets: $\theta_2, \theta_3 \sim 1/2$, $x \to \beta$.
- Interpret as final-state emission of a gluon from measured antiquark.
- Only comes from diagrams where antiquark emits gluon.
- Symmetric jet contribution is smaller.



One step in DGLAP evolution for SIDIS

- We derive exactly that

$$\frac{d\sigma}{d^2\mathbf{P}\, d\ln(1/\beta)} = \frac{4\pi^2\alpha_{em}\sum e_f^2}{Q^2}\ 2\frac{dxq_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2)}{d^2\mathbf{P}}\Big|_{x=\beta}$$

where

$$\frac{dxq_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2)}{d^2\mathbf{P}} = \frac{dxq_{\mathbb{P}}^{LO}(x, x_{\mathbb{P}}, P_\perp^2)}{d^2\mathbf{P}}$$
$$+ \frac{\alpha_s}{2\pi^2}\frac{1}{P_\perp^2}\int_{x_{\min}}^1 dz\ \left[P_{qq}(z)\frac{x}{z}q_{\mathbb{P}}^{LO}\left(\frac{x}{z}, x_{\mathbb{P}}, P_\perp^2\right) + P_{qg}(z)\frac{x}{z}G_{\mathbb{P}}^{LO}\left(\frac{x}{z}, x_{\mathbb{P}}, P_\perp^2\right)\right]$$

- Contribution from virtual diagrams suppressed (goes like $1/P_\perp^4$).
- Lower limit $x_{\min} = \beta + 4\beta\frac{P_\perp^2}{Q^2}$ from kinematics.

Full DGLAP evolution

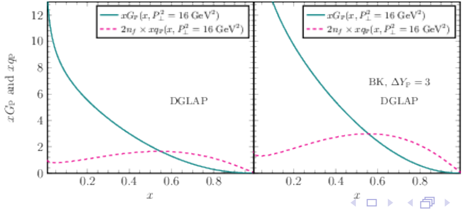
- Our derivation suggests that

$$\frac{d\sigma}{d^2\mathbf{P}d\ln(1/\beta)} = \frac{4\pi^2\alpha_{em}\sum e_f^2}{Q^2}\, 2\frac{dxq_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2)}{d^2\mathbf{P}}\Big|_{x=\beta}$$

where

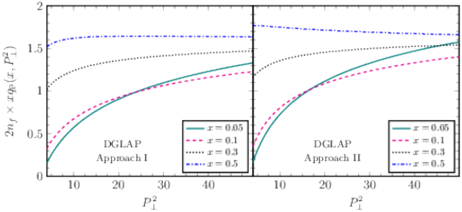
$$\frac{dxq_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2)}{d^2\mathbf{P}} = \frac{dxq^{LO}_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2)}{d^2\mathbf{P}}$$
$$+ \frac{\alpha_s}{2\pi^2}\frac{1}{P_\perp^2}\int_x^1 dz\, \left[P_{qq}(z)\frac{x}{z}q_{\mathbb{P}}\left(\frac{x}{z}, x_{\mathbb{P}}, P_\perp^2\right) + P_{qg}(z)\frac{x}{z}G_{\mathbb{P}}\left(\frac{x}{z}, x_{\mathbb{P}}, P_\perp^2\right)\right]$$

- Unlike usual DGLAP have a source term.
- To derive more rigourously would need virtual diagrams as well.

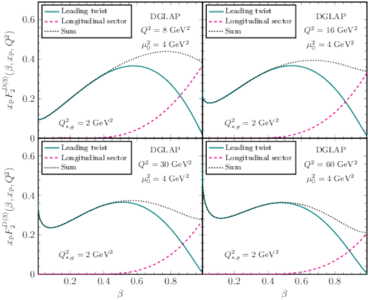


Full DGLAP evolution

- Two schemes for DGLAP evolution:
  - Similar results.



- Contribution to SIDIS:
  - Goes like $d\sigma/d\ln(1/\beta)/d^2K \sim 1/Q^4$, not leading twist.
  - Because photon vertex is $\sim \theta(1-\theta)$ so asymmetric jets suppressed.
  - However, dominates over transverse sector when $\beta \to 1$ because transverse sector vanishes.

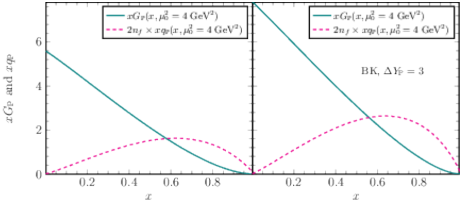


- Contribution to hard dijets:
  - Comparable in size to transverse sector.
  - Factorizes like transverse sector.

$$\frac{d\sigma}{d\theta_2 d\theta_3 d^2\mathbf{P} d^2\mathbf{K} d\ln(1/x)} = \frac{4\pi^2\alpha_{em}\sum e_f^2}{Q^2}\mathcal{H}_L(\theta_2,\theta_3,P_\perp^2,\tilde{Q}^2)\frac{dxq_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{d^2\mathbf{K}}$$

Results for diffractive PDFs

Without DGLAP:



With DGLAP: