

Odderon at the EIC from exclusive χ_c production

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SB, Dumitru, Kaushik, Motyka, Stebel, 2402.19134

DIS 2024, Grenoble, France, April 9, 2024



HRZZ
Croatian Science
Foundation

Odderon in QCD

. odderon is a state with vacuum quantum numbers which is **C-odd**

Lukaszuk, Nicolescu (1973)

Ewerz (2003)

McNulty, DIS24, Mon 14:00

-> In QCD at least **three gluons** contracted with d_{abc} color factor

. **is pp and $p\bar{p}$ scattering same at high energy?**

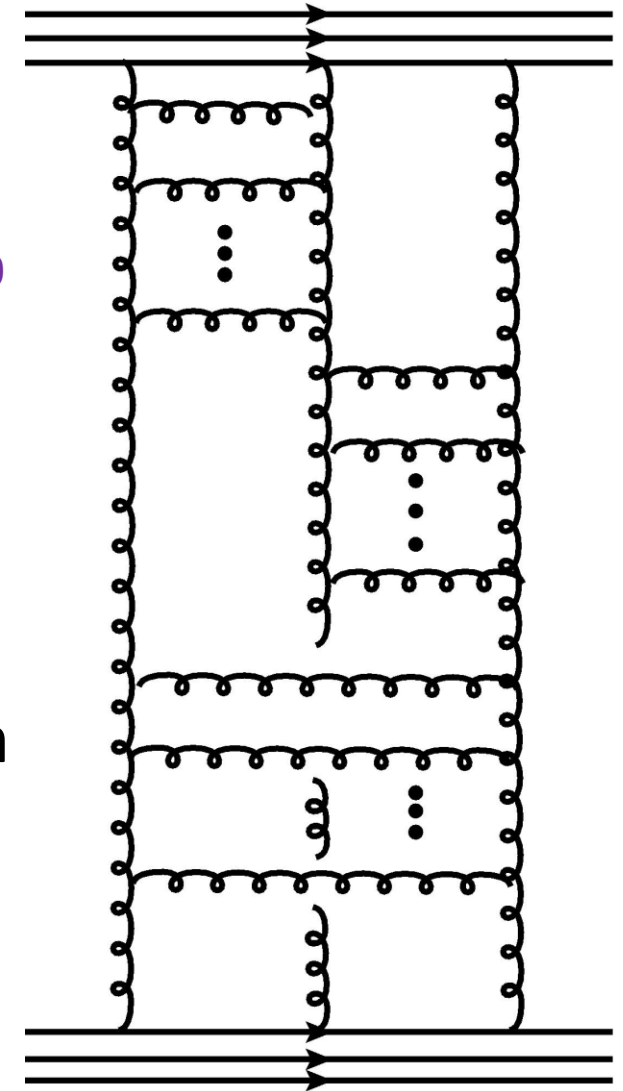
. Bartels-Jaroszewicz-Kwiecinski-Praszalowicz (BJKP) equation was constructed

-> pairwise BFKL ladders between three (reggeized) gluons

Bartels (1979)

Jaroszewicz (1980)

Kwiecinski, Praszalowicz (1980)



Odderon: a modern perspective

Kovchegov, Szymanowski, Wallon (2004)

Hatta, Iancu, Itakura, McLerran (2005)

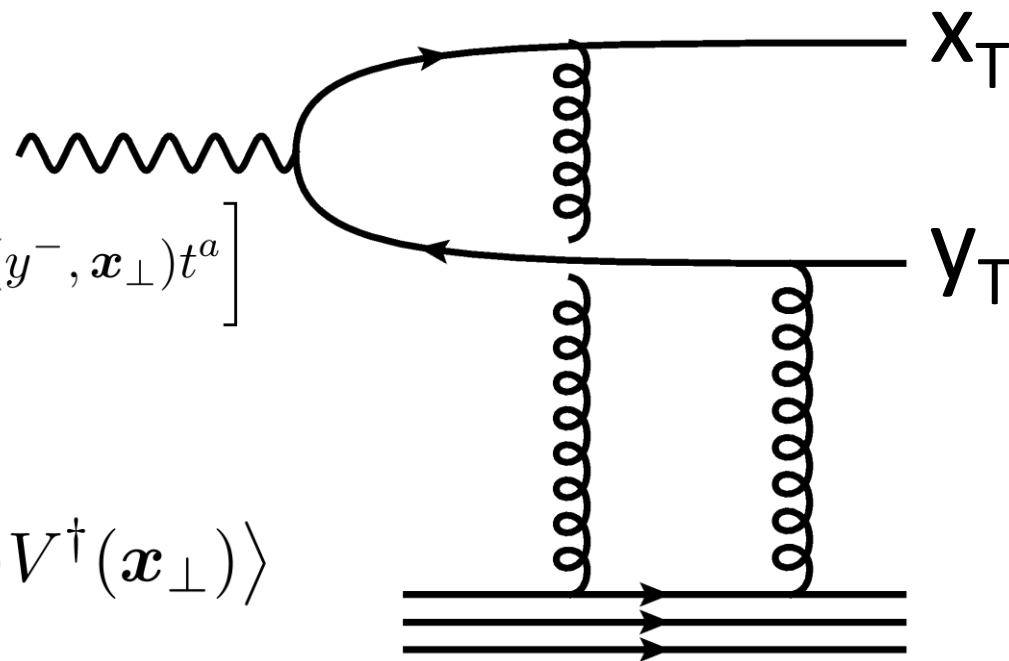
Jeon, Venugopalan (2005)

Lappi, Ramnath, Rummukainen, Weigert (2016)

. dipole S-matrix

$$\mathcal{D}(\mathbf{x}_\perp, \mathbf{y}_\perp) = \frac{1}{N_c} \text{tr} \langle V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) \rangle$$

$$V(\mathbf{x}_\perp) = \mathcal{P} \exp \left[-ig \int dy^- A^{+,a}(y^-, \mathbf{x}_\perp) t^a \right]$$



. odderon as the imaginary part

$$\mathcal{O}(\mathbf{x}_\perp, \mathbf{y}_\perp) = -\frac{1}{2iN_c} \text{tr} \langle V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) - V(\mathbf{y}_\perp) V^\dagger(\mathbf{x}_\perp) \rangle$$

. expand the
Wilson line

$$\mathcal{O}(\mathbf{x}_\perp, \mathbf{y}_\perp) = -\frac{g^3}{24N_c} d^{abc} (\alpha^a(\mathbf{x}_\perp) - \alpha^a(\mathbf{y}_\perp)) (\alpha^b(\mathbf{x}_\perp) - \alpha^b(\mathbf{y}_\perp)) (\alpha^c(\mathbf{x}_\perp) - \alpha^c(\mathbf{y}_\perp))$$

. charge conjugation $\mathbf{x}_\perp \leftrightarrow \mathbf{y}_\perp \rightarrow$ odderon flips sign (C-odd)

Odderon: a modern perspective

$\mathbf{r}_\perp = \mathbf{x}_\perp - \mathbf{y}_\perp$ (dipole size)

$\mathbf{b}_\perp = \frac{1}{2}(\mathbf{x}_\perp + \mathbf{y}_\perp)$ (impact parameter)

-> odderon linear in \mathbf{r}_\perp -> need another vector: $\mathbf{r}_\perp \cdot \mathbf{b}_\perp$

-> odderon as an off-forward amplitude (GTMD)

-> vanishes at $b_T=0$

Kovchegov, Szymanowski, Wallon (2004)
 Hatta, Iancu, Itakura, McLerran (2005)
 Motyka (2006)

$$\mathcal{D}(\mathbf{r}_\perp, \mathbf{b}_\perp) = 1 - \mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp) + i\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)$$

. non-linear (Balitsky-Kovchegov) evolution equation (written here in local approx)

$$\frac{\partial \mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2} \int_{\mathbf{r}_{1\perp}} \frac{\mathbf{r}_\perp^2}{\mathbf{r}_{1\perp}^2 \mathbf{r}_{2\perp}^2} \left[\mathcal{O}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp) + \mathcal{O}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) - \mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp) - \mathcal{N}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp)\mathcal{O}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) - \mathcal{O}(\mathbf{r}_{1\perp}, \mathbf{b}_\perp)\mathcal{N}(\mathbf{r}_{2\perp}, \mathbf{b}_\perp) \right]$$

linear piece:
equivalent to BJKP

saturation corrections:
acts to suppress the
odderon at high energy

$$Y = \log(1/x)$$

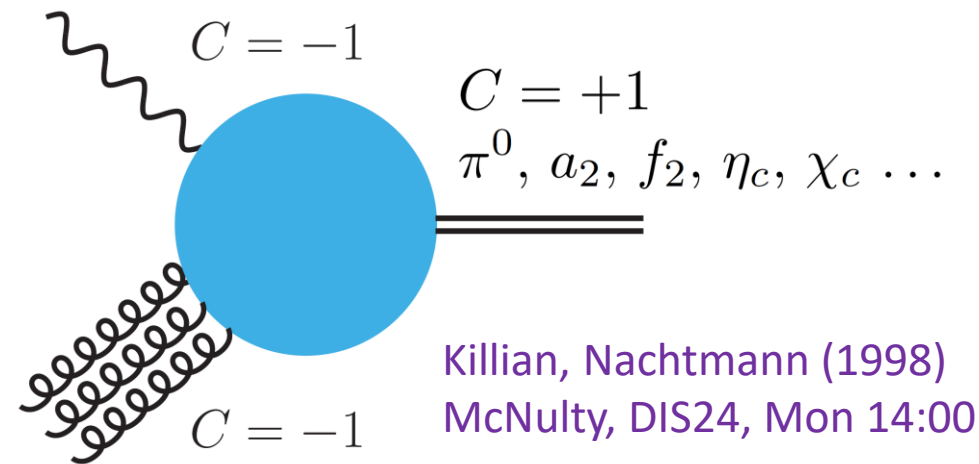
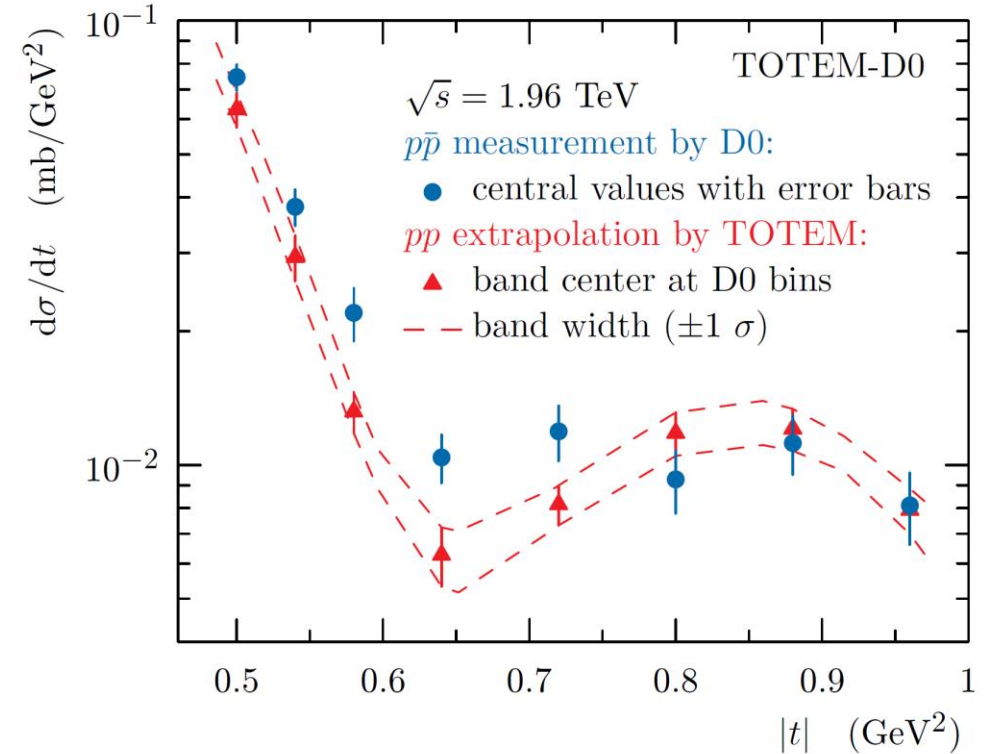
Odderon searches

- recently exciting news by the 5-sigma odderon discovery in $p\bar{p}$ vs pp
- interpretation in terms of a hard (p)QCD odderon?

➔ how about discovering the odderon in DIS?

- exclusive reactions that tag onto the negative C-parity in the target
- in DIS C=+1 meson/quarkonia in the final state
- HERA: null result from H1 collaboration
 $\sigma(\gamma^*p \rightarrow \pi^0 N^*) < 49 \text{ nb}$ (similar for a_2 and f_2)

TOTEM, D0 (2021)



Killian, Nachtmann (1998)
 McNulty, DIS24, Mon 14:00

Motivations for this work

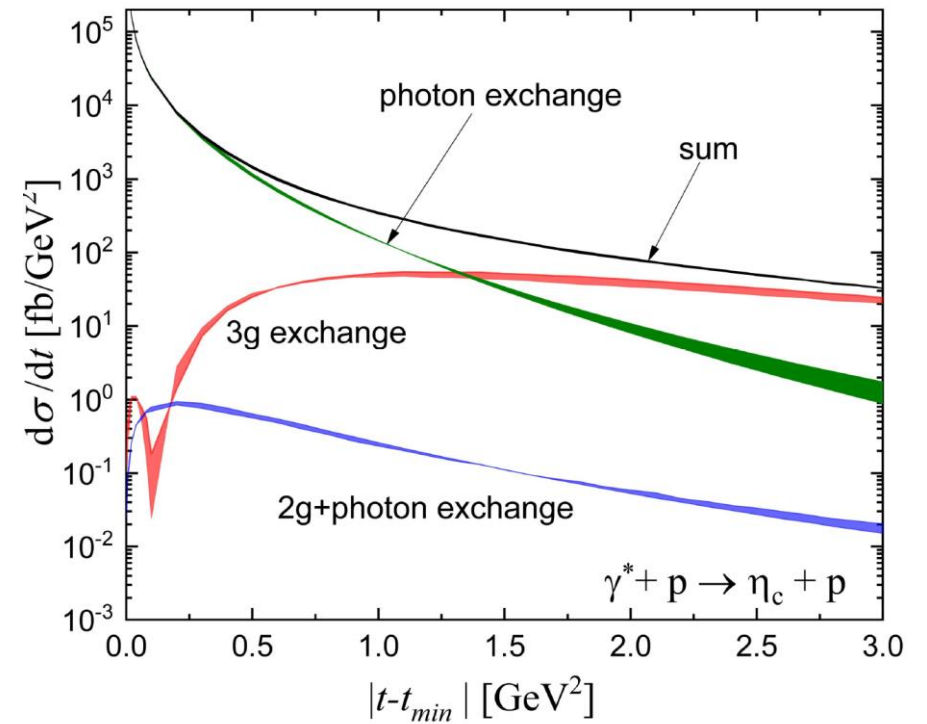
- from late 90's theorists explore exclusive η_c
- **no experimental detection so far**

Czyzewski, Kwiecinski, Motyka, Sadzikowski (1997)

Bartels, Braun, Colferai, Vacca (2001)

Dumitru, Stebel (2019)

SB, Horvatić, Kaushik, Vivoda (2023)



1. issues with η_c detection (small branching ratios to hadronic channels)

-> consider other C=+1 quarkonia: χ_{cJ} (J = 0, 1, 2) family

• χ_c is a P-wave so it is above J/ψ -> main decay mode $\chi_{cJ} \rightarrow J/\psi \gamma$ (BR ~ 30% for χ_{c1} !)

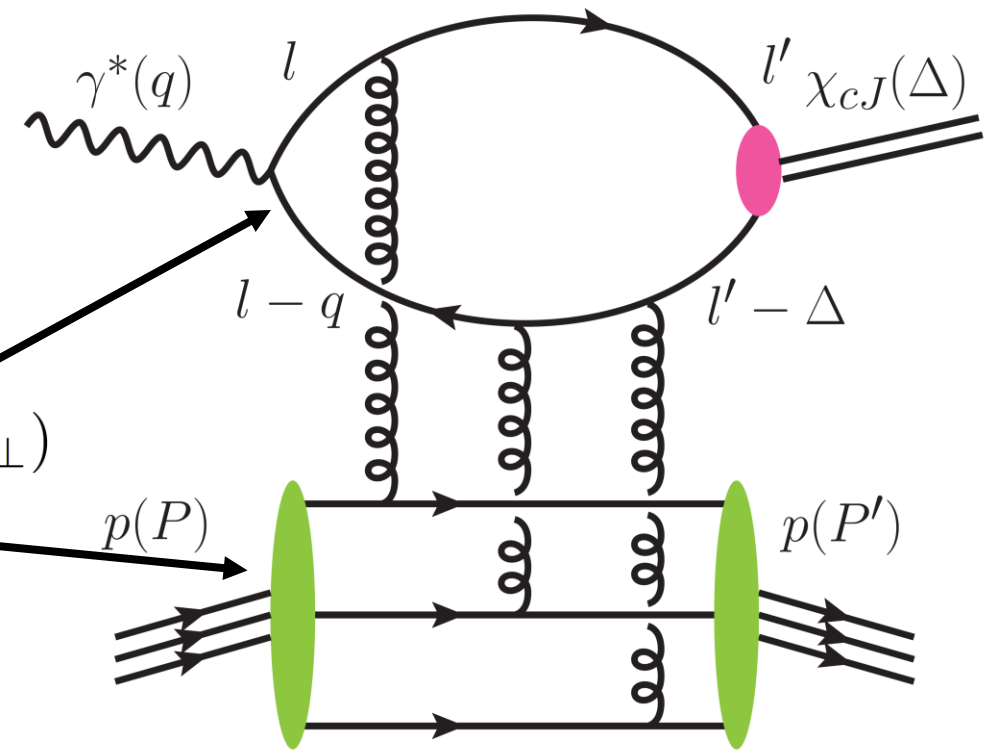
• about 56 χ_{c1} 's and ~12 χ_{c2} 's **detected** (!) near threshold with GlueX [Pentchev, DIS2023, GHP 2023](#)

2. odderon cross sections **in general small** but **EIC luminosity is high**

-> **discovery at the EIC?**

Amplitude

$$\gamma^*(q)p(P) \rightarrow \mathcal{H}(\Delta)p(P')$$



$$\langle \mathcal{M}_{\lambda\bar{\lambda}} \rangle = 2q^- N_c \int_{\mathbf{r}_\perp \mathbf{b}_\perp} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \boxed{i\mathcal{O}(\mathbf{r}_\perp, \mathbf{b}_\perp)} \mathcal{A}_{\lambda\bar{\lambda}}(\mathbf{r}_\perp, \Delta_\perp)$$

. reduced amplitude

$$\mathcal{A}_{\lambda\bar{\lambda}}(\mathbf{r}_\perp, \Delta_\perp) = \int_z \int_{\mathbf{l}_\perp \mathbf{l}'_\perp} \sum_{h\bar{h}} \Psi_{\lambda, h\bar{h}}^\gamma(\mathbf{l}_\perp, z) \Psi_{\bar{\lambda}, h\bar{h}}^{\mathcal{H}*}(\mathbf{l}'_\perp - z\Delta_\perp, z) e^{i(\mathbf{l}_\perp - \mathbf{l}'_\perp + \frac{1}{2}\Delta_\perp) \cdot \mathbf{r}_\perp}$$

. perturbative photon wave function $\Psi_{\lambda, h\bar{h}}^\gamma(\mathbf{k}_\perp, z) \equiv \sqrt{z\bar{z}} \frac{\bar{u}_h(k) e q_c \not{\epsilon}(\lambda, q) v_{\bar{h}}(q-k)}{\mathbf{k}_\perp^2 + \varepsilon^2}$

$$\varepsilon \equiv \sqrt{m_c^2 + z\bar{z}Q^2} \quad z = \frac{k^-}{q^-} \quad \bar{z} \equiv 1 - z$$

C-even charmonia wave functions

scalar function: **boosted Gaussian ansatz**

$$\Psi_{\bar{\lambda}, h \bar{h}}^{\mathcal{H}}(\mathbf{k}_{\perp}, z) \equiv \frac{1}{\sqrt{z \bar{z}}} \underbrace{\bar{u}_h(k) \Gamma_{\bar{\lambda}}^{\mathcal{H}}(k, k') v_{\bar{h}}(k')}_{\text{spin structure}} \phi_{\mathcal{H}}(\mathbf{k}_{\perp}, z)$$

Forshaw, Sandapen, Shaw (2004)
Kowalski, Motyka, Watt (2006)

. covariant ansatz

$$\Gamma_{\bar{\lambda}}^{\mathcal{H}}(k, k') = \left\{ \begin{array}{l} 1, \\ i\gamma_5 \not{E}(\bar{\lambda}, \Delta_0), \\ \frac{1}{4} (\gamma_{\mu}(k_{\nu} - k'_{\nu}) + \gamma_{\nu}(k_{\mu} - k'_{\mu})) E^{\mu\nu}(\bar{\lambda}, \Delta_0), \end{array} \right. \left. \begin{array}{l} \mathcal{H} = S \\ \mathcal{H} = A \\ \mathcal{H} = T \end{array} \right\} \chi_{\mathcal{CJ}}$$

polarization vector fo spin 1

spin 2 -> coupling to the energy momentum tensor

spin 2 polarization tensor: in terms of $E^{\mu}(\bar{\lambda}, \Delta_0)$ via Clebsch-Gordans

. transversality condition

$$\Delta_0 \cdot E(\bar{\lambda}, \Delta_0) = 0 \quad \Delta_0 = k + k'$$

Berger, Donnachie, Dosch, Nachtmann (2000)
SB, Dumitru, Kaushik, Motyka, Stebel (2024)

Final amplitudes: scalar harmonia

SB, Dumitru, Kaushik, Motyka, Stebel (2024)

. reduced amplitudes → off-forward phase $\delta_{\perp} = \frac{1}{2}(z - \bar{z})\Delta_{\perp}$

$$\mathcal{A}_0(\mathbf{r}_{\perp}, \Delta_{\perp}) = eq_c \int_z e^{-i\delta_{\perp} \cdot \mathbf{r}_{\perp}} \mathcal{A}_L(r_{\perp})$$

$$\mathcal{A}_{\lambda=\pm 1}(\mathbf{r}_{\perp}, \Delta_{\perp}) = eq_c \lambda e^{i\lambda\phi_r} \int_z e^{-i\delta_{\perp} \cdot \mathbf{r}_{\perp}} \mathcal{A}_T(r_{\perp})$$

$$\mathcal{A}_L(r_{\perp}) \equiv -\frac{2}{\pi} m_c Q(z - \bar{z}) K_0(\epsilon r_{\perp}) \phi_S(r_{\perp}, z)$$

$$\mathcal{A}_T(r_{\perp}) \equiv \frac{i\sqrt{2}}{2\pi} \frac{m_c}{z\bar{z}} \left[(z - \bar{z})^2 \epsilon K_1(\epsilon r_{\perp}) \phi_S(r_{\perp}, z) - K_0(\epsilon r_{\perp}) \frac{\partial \phi_S}{\partial r_{\perp}} \right]$$

. numerics-ready full amplitudes (after removing overall phase)

odderon harmonics
(we keep only k=0 mode)

$$\tilde{\mathcal{M}}_L = 8\pi N_c eq_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^{\infty} r_{\perp} dr_{\perp} \mathcal{O}_{2k+1}(r_{\perp}, \Delta_{\perp}) \mathcal{A}_L(r_{\perp}) \text{sgn}(z - \bar{z}) J_{2k+1}(r_{\perp} \delta_{\perp})$$

$$\tilde{\mathcal{M}}_T = 4\pi i N_c eq_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^{\infty} r_{\perp} dr_{\perp} \mathcal{O}_{2k+1}(r_{\perp}, \Delta_{\perp}) \mathcal{A}_T(r_{\perp}) [J_{2k}(r_{\perp} \delta_{\perp}) - J_{2k+2}(r_{\perp} \delta_{\perp})]$$

Final amplitudes: axial vector harmonia

$$\mathcal{A}_{LL}(r_{\perp}) = 0 \quad \longrightarrow \quad \text{no contribution when photon and axial harmonia are longitudinally polarized}$$

$$\mathcal{A}_{LT}(r_{\perp}) \equiv \frac{\sqrt{2}}{\pi} Q K_0(\varepsilon r_{\perp}) \frac{\partial \phi_{\mathcal{A},T}}{\partial r_{\perp}}$$

$$\mathcal{A}_{TL}(r_{\perp}) \equiv \frac{\sqrt{2}}{\pi} \frac{1}{z\bar{z}} \frac{1}{M_{\mathcal{A}}} \left[-m_c^2 K_0(\varepsilon r_{\perp}) \frac{\partial \phi_{\mathcal{A},L}}{\partial r_{\perp}} + \varepsilon K_1(\varepsilon r_{\perp}) \nabla_{\perp}^2 \phi_{\mathcal{A},L} \right]$$

$$\mathcal{A}_{TT}(r_{\perp}) \equiv -\frac{i}{\pi} \frac{z - \bar{z}}{z\bar{z}} \left[\frac{\partial \phi_{\mathcal{A},T}}{\partial r_{\perp}} \varepsilon K_1(\varepsilon r_{\perp}) - m_c^2 K_0(\varepsilon r_{\perp}) \phi_{\mathcal{A},T} \right] \quad \text{only polarization preserving transition}$$

$$\tilde{\mathcal{M}}_B = 4\pi i N_c e q_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^{\infty} r_{\perp} dr_{\perp} \mathcal{O}_{2k+1}(r_{\perp}, \Delta_{\perp}) \mathcal{A}_B(r_{\perp}) [J_{2k}(r_{\perp} \delta_{\perp}) - J_{2k+2}(r_{\perp} \delta_{\perp})], \quad B = TL, LT$$

$$\tilde{\mathcal{M}}_{TT} = 8\pi N_c e q_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^{\infty} r_{\perp} dr_{\perp} \mathcal{O}_{2k+1}(r_{\perp}, \Delta_{\perp}) \mathcal{A}_{TT}(r_{\perp}) \operatorname{sgn}(z - \bar{z}) J_{2k+1}(r_{\perp} \delta_{\perp}),$$

SB, Dumitru, Kaushik, Motyka, Stebel (2024)

Final amplitudes: tensor harmonia

$$\widetilde{\mathcal{M}}_B = -4\pi N_c e q_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^{\infty} r_{\perp} dr_{\perp} \mathcal{O}_{2k+1}(r_{\perp}, \Delta_{\perp}) \mathcal{A}_B(r_{\perp}) \\ \times \text{sgn}(z - \bar{z}) [J_{2k+3}(r_{\perp} \delta_{\perp}) + J_{2k-1}(r_{\perp} \delta_{\perp})], \quad B = LT2, TTf,$$

$$\widetilde{\mathcal{M}}_B = 4\pi i N_c e q_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^{\infty} r_{\perp} dr_{\perp} \mathcal{O}_{2k+1}(r_{\perp}, \Delta_{\perp}) \mathcal{A}_B(r_{\perp}) \\ \times [J_{2k}(r_{\perp} \delta_{\perp}) - J_{2k+2}(r_{\perp} \delta_{\perp})], \quad B = TT2p, LT, TL,$$

$$\widetilde{\mathcal{M}}_B = 8\pi N_c e q_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^{\infty} r_{\perp} dr_{\perp} \mathcal{O}_{2k+1}(r_{\perp}, \Delta_{\perp}) \mathcal{A}_B(r_{\perp}) \text{sgn}(z - \bar{z}) J_{2k+1}(r_{\perp} \delta_{\perp}), \quad B = TTp, LL,$$

$$\widetilde{\mathcal{M}}_{TT2f} = 4\pi i N_c e q_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^{\infty} r_{\perp} dr_{\perp} \mathcal{O}_{2k+1}(r_{\perp}, \Delta_{\perp}) \mathcal{A}_{TT2f}(r_{\perp}) [J_{2k+4}(r_{\perp} \delta_{\perp}) - J_{2k-2}(r_{\perp} \delta_{\perp})],$$

$$\mathcal{A}_{LT2}(r_{\perp}) \equiv \frac{1}{\pi} (z - \bar{z}) Q K_0(\varepsilon r_{\perp}) \left(\frac{\partial^2 \phi_{\mathcal{T}, T2}}{\partial r_{\perp}^2} - \frac{1}{r_{\perp}} \frac{\partial \phi_{\mathcal{T}, T2}}{\partial r_{\perp}} \right),$$

$$\mathcal{A}_{TT2,p}(r_{\perp}) \equiv -\frac{i}{\sqrt{2\pi}} \frac{1}{z\bar{z}} \left((z^2 + \bar{z}^2) \varepsilon K_1(\varepsilon r_{\perp}) \left(\frac{\partial^2 \phi_{\mathcal{T}, T2}}{\partial r_{\perp}^2} - \frac{1}{r_{\perp}} \frac{\partial \phi_{\mathcal{T}, T2}}{\partial r_{\perp}} \right) + m_c^2 K_0(\varepsilon r_{\perp}) \frac{\partial \phi_{\mathcal{T}, T2}}{\partial r_{\perp}} \right),$$

$$\mathcal{A}_{TT2,f}(r_{\perp}) \equiv \frac{i\sqrt{2}}{\pi} \varepsilon K_1(\varepsilon r_{\perp}) \left(\frac{\partial^2 \phi_{\mathcal{T}, T2}}{\partial r_{\perp}^2} - \frac{1}{r_{\perp}} \frac{\partial \phi_{\mathcal{T}, T2}}{\partial r_{\perp}} \right),$$

$$\mathcal{A}_{LT}(r_{\perp}) \equiv -\frac{i}{2\pi} Q M_{\mathcal{T}} (3 - 4(z^2 + \bar{z}^2)) K_0(\varepsilon r_{\perp}) \frac{\partial \phi_{\mathcal{T}, T}}{\partial r_{\perp}},$$

$$\mathcal{A}_{TT,p}(r_{\perp}) = -\frac{\sqrt{2} M_{\mathcal{T}}}{4\pi} \frac{z - \bar{z}}{z\bar{z}} \left[m_c^2 K_0(\varepsilon r_{\perp}) \phi_{\mathcal{T}, T}(r_{\perp}, z) - (z - \bar{z})^2 \varepsilon K_1(\varepsilon r_{\perp}) \frac{\partial \phi_{\mathcal{T}, T}}{\partial r_{\perp}} \right],$$

$$\mathcal{A}_{TT,f}(r_{\perp}) = -\frac{\sqrt{2} M_{\mathcal{T}}}{\pi} (z - \bar{z}) \varepsilon K_1(\varepsilon r_{\perp}) \frac{\partial \phi_{\mathcal{T}, T}}{\partial r_{\perp}},$$

$$\mathcal{A}_{LL}(r_{\perp}) = -\frac{\sqrt{2} Q}{\sqrt{3\pi}} (z - \bar{z}) K_0(\varepsilon r_{\perp}) (3\nabla_{\perp}^2 - 2m_c^2) \phi_{\mathcal{T}, L}(r_{\perp}, z),$$

$$\mathcal{A}_{TL}(r_{\perp}) = \frac{i}{2\pi\sqrt{3}} \frac{1}{z\bar{z}} \left[\varepsilon K_1(\varepsilon r_{\perp}) (z - \bar{z})^2 (3\nabla_{\perp}^2 - 2m_c^2) \phi_{\mathcal{T}, L} - m_c^2 K_0(\varepsilon r_{\perp}) \frac{\partial \phi_{\mathcal{T}, L}}{\partial r_{\perp}} \right].$$

all overlaps calculated, all amplitudes found

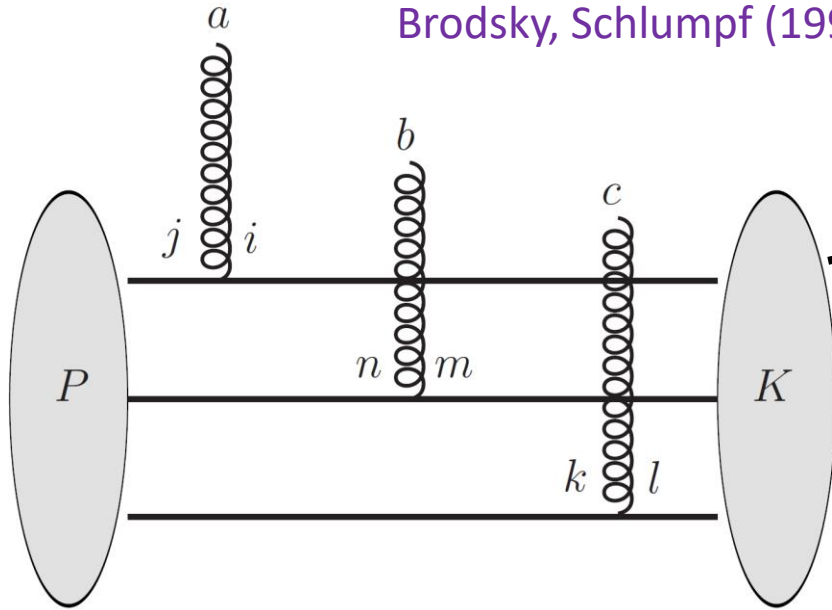
SB, Dumitru, Kaushik, Motyka, Stebel (2024)

Odderon initial condition

- . a microscopic quark model of the proton

Dumitru, Miller, Venugopalan (2018)

Brodsky, Schlumpf (1994)



an example of a 3-body contribution that becomes relevant at **high-t**

-> **Landshoff mechanism**: gluons can give a high-t kick to the proton without breaking it

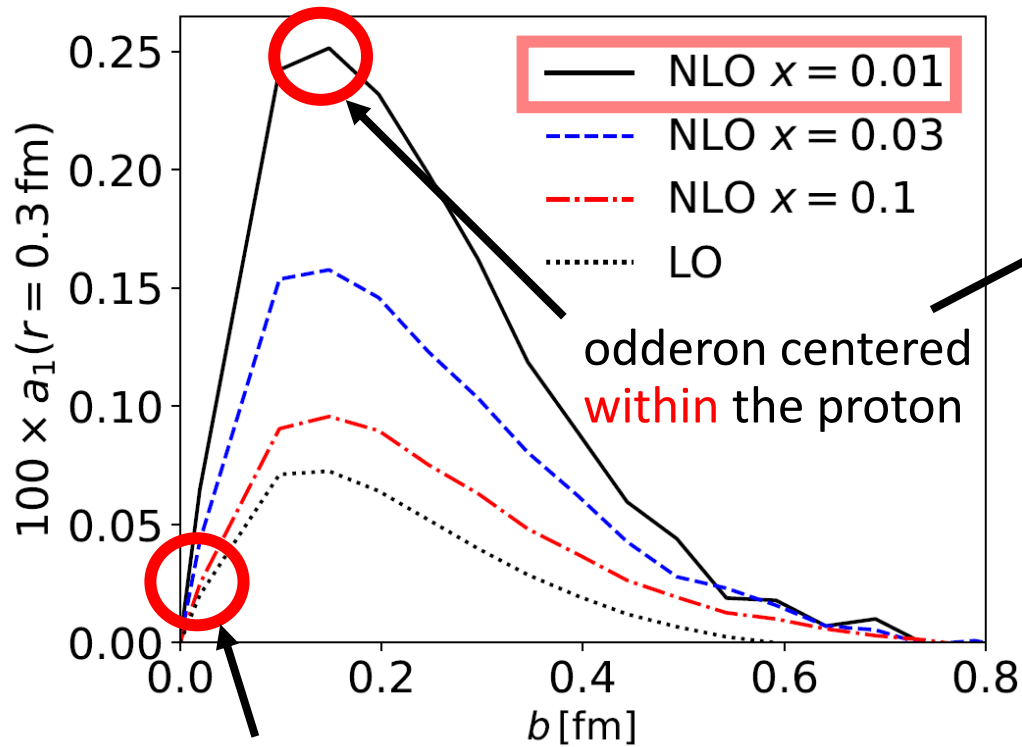
Landshoff (1974)

- . model computation **fixes the overall sign of the odderon**
- . evolution linear in O -> **odderon sign not changed by evolution**

Odderon evolved

. In the numerical approach the proton is described by $|qqq\rangle + |qqqg\rangle$ Fock state computation by Dumitru, Mantysaari and Paatelainen

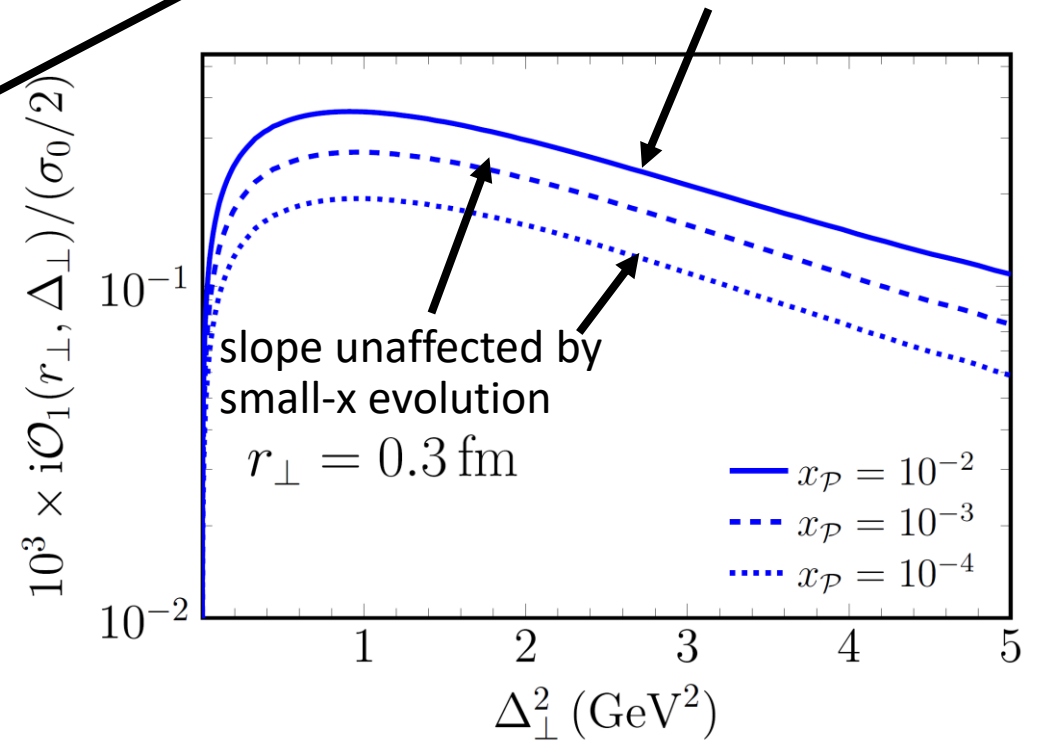
we take $\alpha_s(2m_c) \sim 0.25$



➔

Fourier transform

Landshoff mechanism



vanishes at the origin

Dumitru, Mantysaari, Paatelainen (2022)
 Dumitru, Mantysaari, Paatelainen (2023)

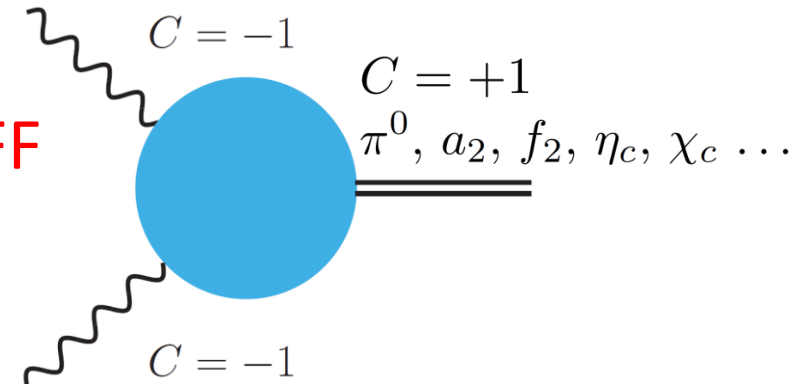
SB, Horvatic, Kaushik, Vivoda (2023)
 SB, Dumitru, Kaushik, Motyka, Stebel (2024)

The Primakoff process

- . usually we do not care about QED contributions to QCD process
- . but in case of odderon QCD cross section is small ($\sim \alpha_s^6$)
- > **Primakoff process is a serious background** to the odderon searches
- . replace odderon with photon exchange

$$\gamma^*(q)\gamma^*(\ell) \rightarrow \mathcal{H}(\Delta)$$

QED charge FF



$$\mathcal{O}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) \rightarrow 8\pi i q_c \alpha \sin\left(\frac{\mathbf{\Delta}_\perp \cdot \mathbf{r}_\perp}{2}\right) \frac{F_1(\mathbf{\Delta}_\perp)}{\Delta_\perp^2}$$

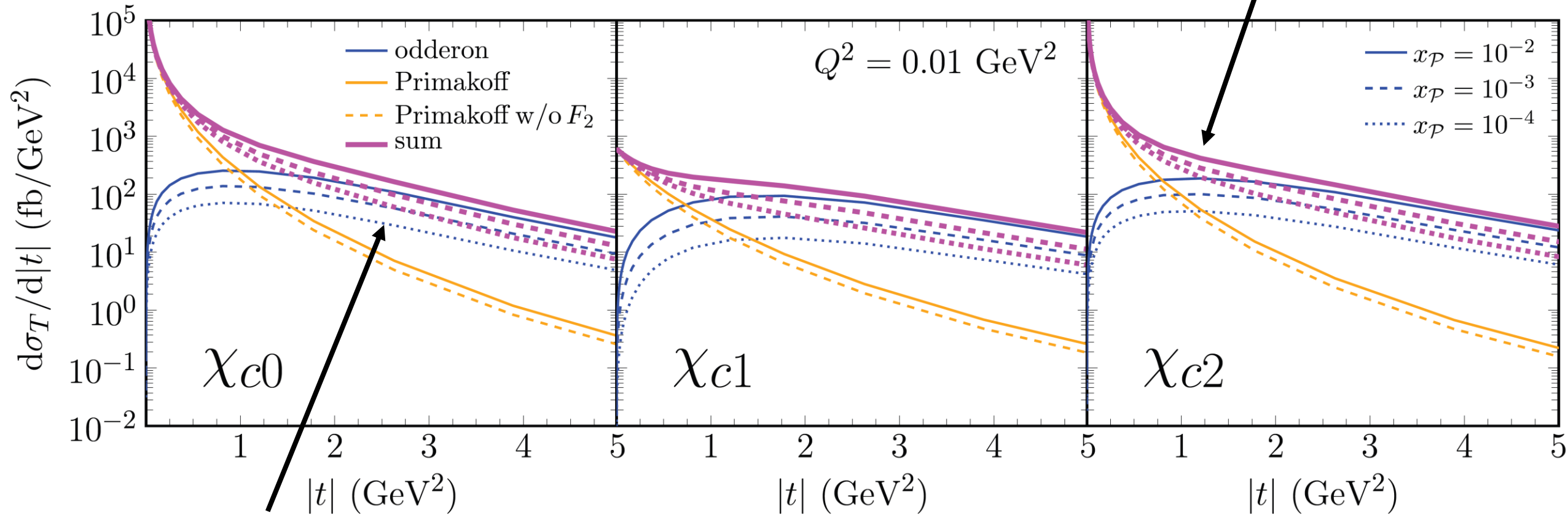
- . In numerical computations we **also take into account Pauli (spin-flip) FF** (up to 50% correction at finite t)

$$\langle \mathcal{M}_{\lambda\bar{\lambda}}(\gamma^* p \rightarrow \mathcal{H} p) \rangle = -n_\mu \mathcal{M}_{\lambda\bar{\lambda}}^\mu(\gamma^* \gamma^* \rightarrow \mathcal{H}) \left[\frac{eF_1(\ell_\perp)}{\ell_\perp^2} \delta_{hh'} + \frac{eF_2(\ell_\perp)}{\ell_\perp^2} \frac{\ell_\perp}{2m_N} h e^{ih\phi_\ell} \delta_{h,-h'} \right]$$

t-distributions

. odderon important after $|t| \sim 1 \text{ GeV}^2$, low t-region dominated by Primakoff

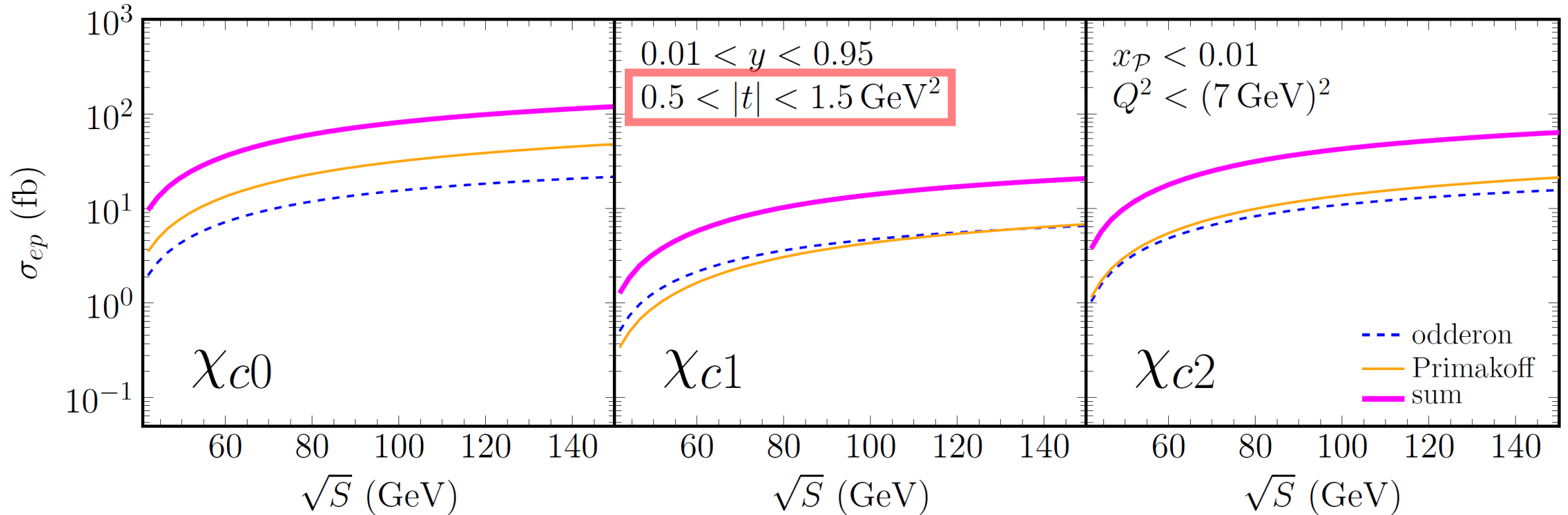
coherent sum of photon+odderon



Landshoff mechanism

photon and odderon interfere **constructively**

Total electroproduction cross section



SB, Dumitru, Kaushik, Motyka, Stebel (2024)

. note: at the EIC proton detection is up to $p_T \sim 1.3 \text{ GeV}$

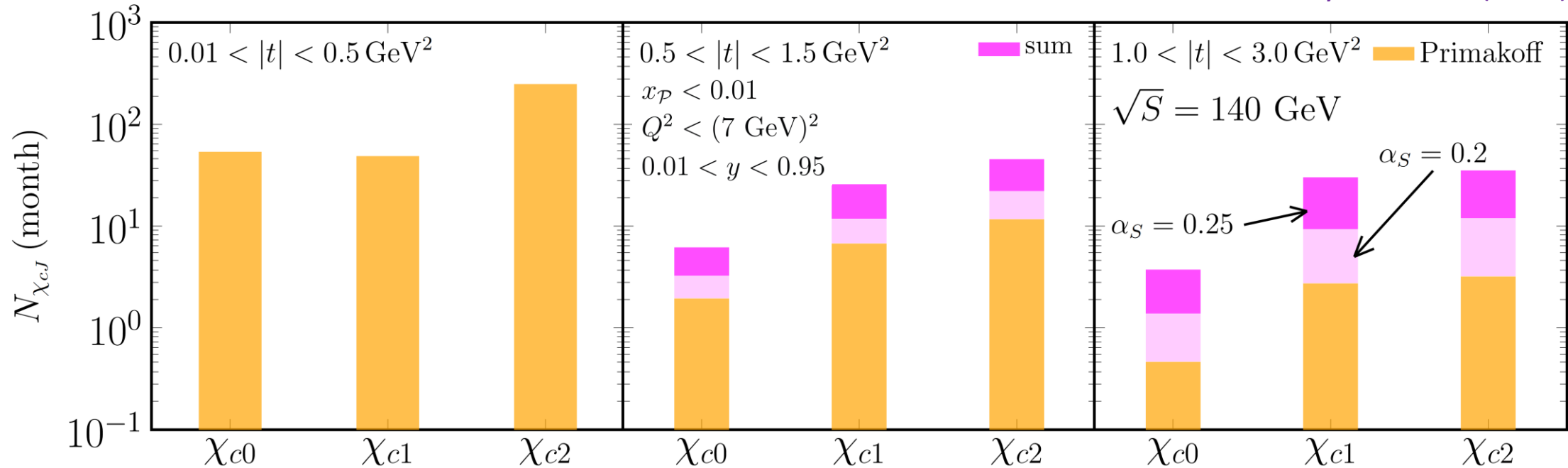
EIC Yellow report

Van Hulse: DIS24, Mon, 14:30

Expected number of events at the EIC

- detection channel: $\chi_{cJ} \rightarrow J/\psi\gamma, J/\psi \rightarrow l^+l^-$

SB, Dumitru, Kaushik, Motyka, Stebel (2024)



- we predict **excess** in odderon events over Primakoff background

- for χ_{c1} (30% BR to $J/\psi + \gamma$): with EIC luminosity $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

expect **~20 events/month** (only Primakoff ~5 events/month)

Concluding remarks

- . can the 'hard' odderon (ggg exchange) be discovered at the EIC?
- . our suggestion: exclusive χ_c production
- . odderon signal enhancement thanks to a constructive photon-odderon interference
- > event excess above the Primakoff background
- . we predict about a few dozen events/month at the EIC (max energy, max luminosity)
- . was not possible at HERA.. (luminosity $\sim 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$)
- . need moderate to high $|t|$ and low Q^2 (opposite from GPDs)

The Primakoff process in the $t \rightarrow 0$ limit

. **Strong constraint** on the Primakoff cross section at $t \rightarrow 0$ by the $\gamma\gamma$ width

For $J \neq 1$
(Landau-Yang theorem)

$$\lim_{|t| \rightarrow 0} |t| \frac{d\sigma(\gamma p \rightarrow \mathcal{H} p)}{d|t|} = \frac{8\pi\alpha(2J+1)\Gamma(\mathcal{H} \rightarrow \gamma\gamma)}{M_{\mathcal{H}}^3}$$

. Extracted $\gamma\gamma$ widths in accordance with the literature

Li, Li, Vary (2022)

. Charmonia model wave function \rightarrow boosted Gaussian

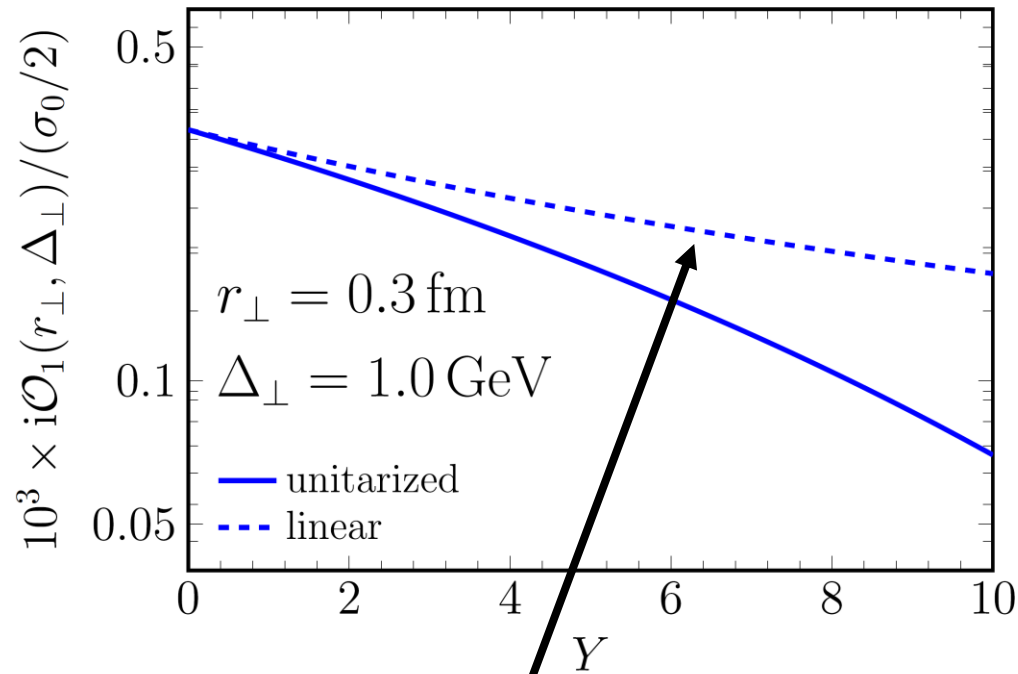
$$\phi_{\mathcal{H},B}(r_{\perp}, z) = \mathcal{N}_{\mathcal{H},B} z \bar{z} \exp\left(-\frac{m_c^2 \mathcal{R}_{\mathcal{H}}^2}{8z\bar{z}} - \frac{2z\bar{z}r_{\perp}^2}{\mathcal{R}_{\mathcal{H}}^2} + \frac{1}{2}m_c^2 \mathcal{R}_{\mathcal{H}}^2\right)$$

Forshaw, Sandapen, Shaw (2004)
Kowalski, Motyka, Watt (2006)

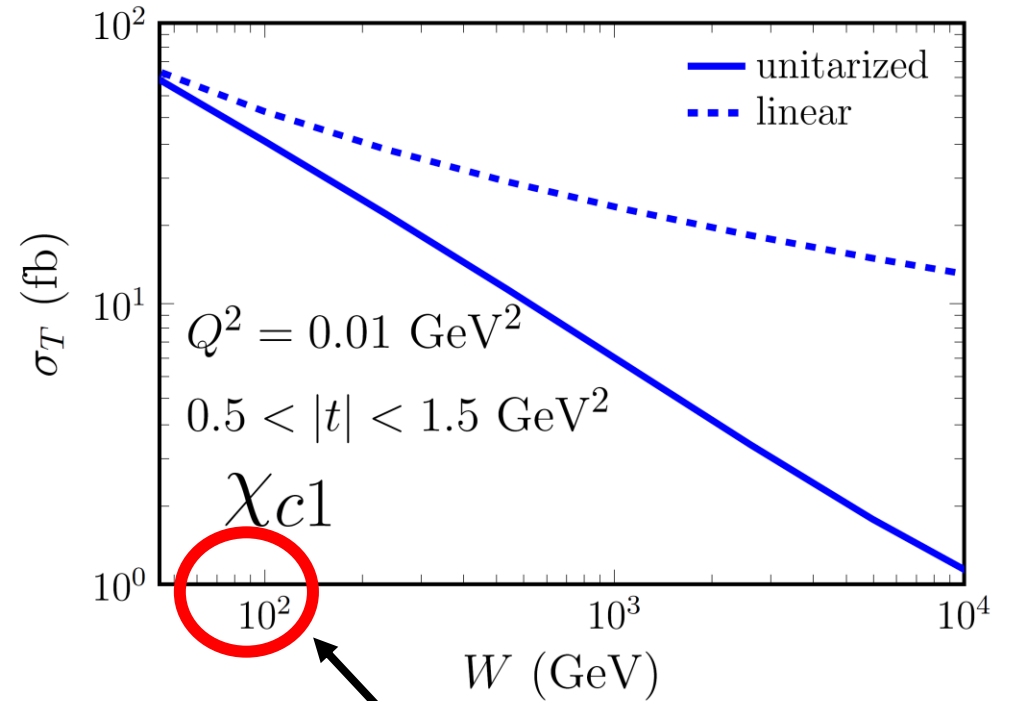
. We fit the radius and the normalization parameters to the $\gamma\gamma$ width and the WF normalization

SB, Dumitru, Kaushik, Motyka, Stebel (2024)

W distributions linear vs nonlinear evolution



Dropping the unitarity/saturation corrections
 Odderon evolution with Y much slower \rightarrow BLV
 asymptotics? (difficult to see numerically..)



May not matter much
 at the EIC..

SB, Dumitru, Kaushik, Motyka, Stebel (2024)