Impact factor for forward η_c meson production at NLO

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Perturbative instability of quarkonium total cross sections

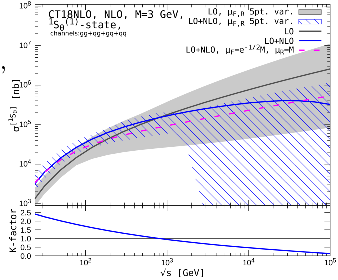
Inclusive η_c-hadroproduction (CSM)

[Mangano *et.al.*, '97, ..., Lansberg, Ozcelik, '20]

$$p+p \to c\bar{c}\left[{}^1S_0^{[1]}\right]+X,\ \text{LO: } g(p_1)+g(p_2) \to c\bar{c}\left[{}^1S_0^{[1]}\right],$$

$$\sigma(\sqrt{s_{pp}}) = f_i(x_1,\mu_F)\otimes f_j(x_2,\mu_F)\otimes\hat{\sigma}(z),$$

where $z = \frac{M^2}{\hat{s}}$ with $\hat{s} = (p_1+p_2)^2$.



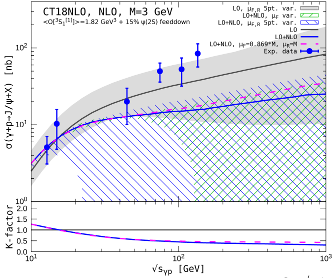
Inclusive J/ψ-photoproduction (CSM)

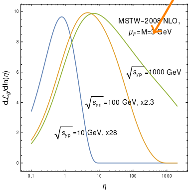
[Krämer, '96, ...,Colpani Serri *et.al*, '21]

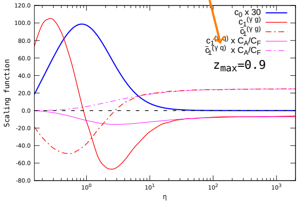
$$\gamma+p \to c\bar{c}\left[{}^3S_1^{[1]}\right]+X,\ \text{LO: } \gamma(q)+g(p_1) \to c\bar{c}\left[{}^3S_1^{[1]}\right]+g,$$

$$\sigma(\sqrt{s_{\gamma p}}) = f_i(x_1,\mu_F)\otimes\hat{\sigma}(\eta),$$

where $\eta = \frac{\hat{s}-M^2}{M^2}$ with $\hat{s} = (q+p_1)^2$, $z = \frac{pP}{qP}$.



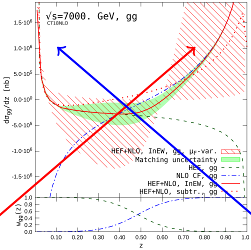


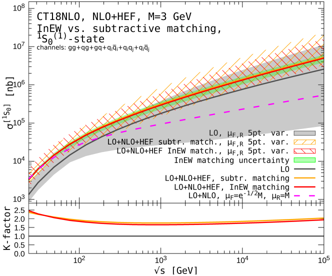


Matching with NLO

The HEF is valid in the **leading-power** in $M^2/\hat{s}$, so for $\hat{s} \sim M^2$ we match it with NLO CF by the *Inverse-Error Weighting Method* [Echevarria *et.al.*, 18'].

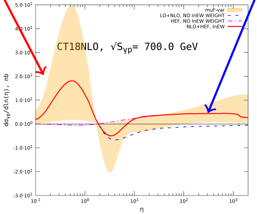
η_c**-hadroproduction**, $z = M^2/\hat{s}$:

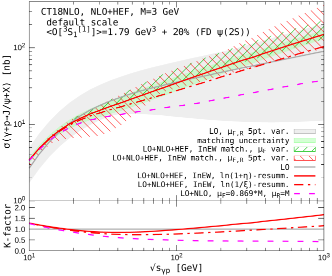




NLO **HEF**

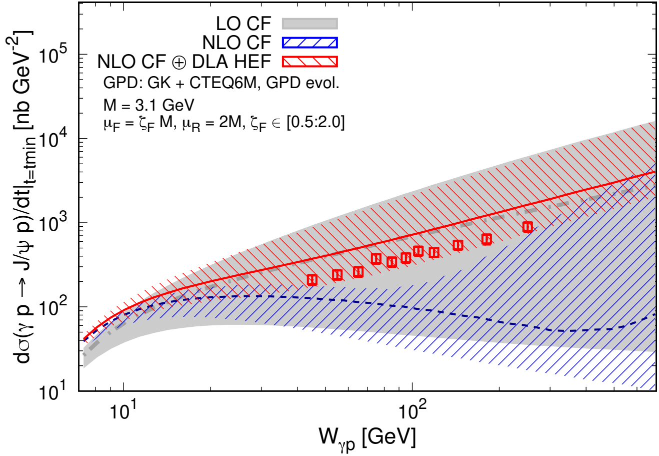
J/ψ**-photoproduction**, $\eta = (\hat{s} - M^2)/M^2$:





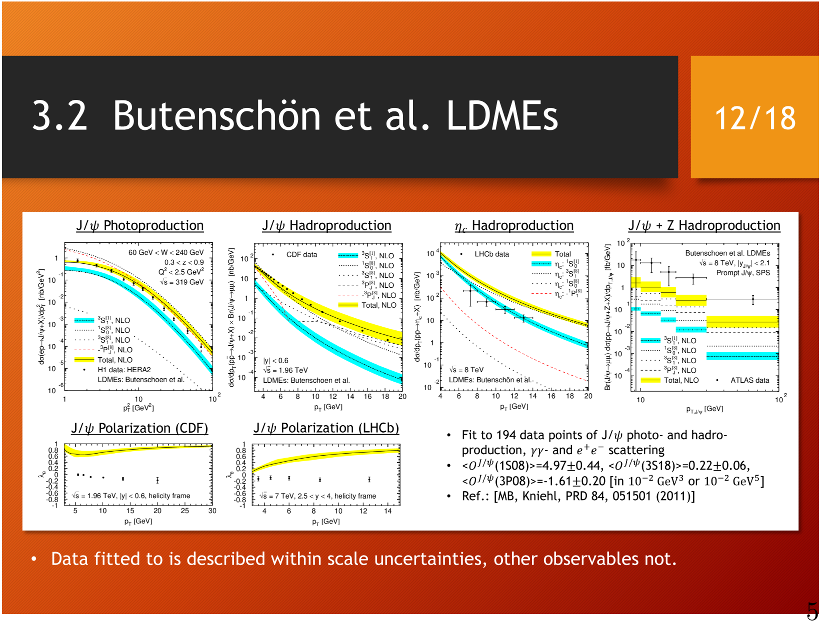
Exclusive J/ψ photoproduction in CF+HEF

See the talk of Saad Nabeebaccus in the WG4.



Why η_c ?

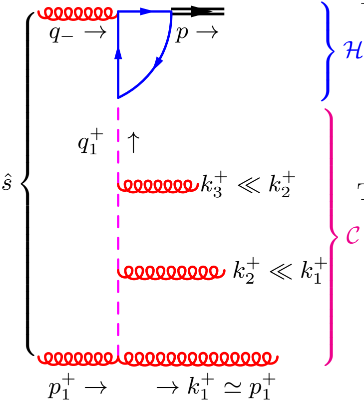
Slides from M. Butenschön at **QaT-2021**:



High-Energy Factorization, forward η_c hadroproduction

The **LLA** ($\sum_n \alpha_s^n \ln^{n-1}$) formalism [Collins, Ellis, '91; Catani, Ciafaloni, Hautmann, '91,'94]

Physical picture in the **LLA**:



The LLA in $\ln(p_1^+/p^+)$:

$$\hat{\sigma}_{\rm HEF}(p_+/p_1^+, z) \propto \int\limits_0^\infty dq_1^+ \int\limits_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{q_1^+}{p_1^+}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \mathcal{H}(q_1^+, \mathbf{q}_{T1}^2, z),$$

The LLA in $\ln(\hat{s}/M^2)$:

$$\hat{\sigma}_{\rm HEF}(\hat{s}/M^2, z) \propto \int\limits_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{\hat{s}}{M^2}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \int\limits_0^\infty dq_1^+ \mathcal{H}(q_1^+, \mathbf{q}_{T1}^2, z).$$

Two kinds of LLA are equivalent up to NLL terms because

$$\frac{\hat{s}}{M^2} = \frac{q_- p_1^+}{M^2} = \frac{M^2 + \mathbf{p}_T^2}{M^2} \frac{p_1^+}{q_1^+} = \frac{M^2 + \mathbf{p}_T^2}{M^2} \frac{p_1^+}{p^+}.$$

The Leading Order

The LLA resummation formula for $\mathbf{p}_T^2$ and $z = p_-/q_-$-differential partonic cross section:

$$\frac{d\hat{\sigma}_{ig}^{(\mathrm{LLA})}}{dz d\mathbf{p}_T^2} = \frac{1}{2M^2}\int \frac{d^2\mathbf{q}_T}{\pi}\mathcal{C}_{ig}\left(\frac{\hat{s}}{M^2}, \mathbf{q}_T^2, \mu_F, \mu_R\right)\mathcal{H}_{gg}^{(\mathrm{LO})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2),$$
$$= \frac{1}{2M^2}\mathcal{C}_{ig}\left(\frac{\hat{s}}{M^2}, \mathbf{p}_T^2, \mu_F, \mu_R\right)\mathcal{H}_{gg}^{(\mathrm{LO})}(\mathbf{p}_T^2)\delta(1-z),$$

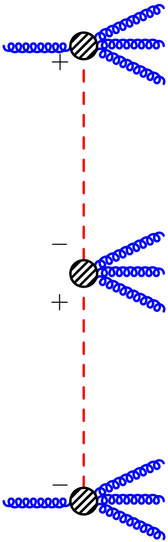
with [Kniehl, Vasin, Saleev, '06]

$$\mathcal{H}_{gg}^{(\mathrm{LO})} = \frac{32\pi^3\alpha_s^2(\mu_R)M^4}{N_c^2(N_c^2-1)(M^2+\mathbf{p}_T^2)^2}\frac{\left\langle \mathcal{O}\left[{}^1S_0^{[1]}\right]\right\rangle}{M^3}\delta(1-z)\delta(\mathbf{q}_T^2-\mathbf{p}_T^2),$$

where $\left\langle \mathcal{O}\left[{}^1S_0^{[1]}\right]\right\rangle = 2N_c|R(0)|^2/(4\pi)$.

In this talk we will compute $\mathcal{H}_{gg}^{(\mathrm{NLO})}$, which includes **virtual** and **real-emission** corrections.

The Gauge-Invariant EFT for Multi-Regge processes in QCD



- Reggeized gluon fields $R_\pm$ carry $(k_\pm, \mathbf{k}_T, k_\mp = 0)$: $\partial_\mp R_\pm = 0$.
- **Induced interactions** of particles and Reggeons [Lipatov '95, '97; Bondarenko, Zubkov '18]:

$$L = \frac{i}{g_s} \text{tr} \left[R_+ \partial_\perp^2 \partial_- \left(W\left[A_-\right] - W^\dagger \left[A_-\right] \right) + (+ \leftrightarrow -) \right],$$

with $W_{x_\mp}[x_\pm, \mathbf{x}_T, A_\pm] = P \exp \left[\frac{-ig_s}{2} \int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T) \right] = \left(1 + ig_s \partial_\pm^{-1} A_\pm\right)^{-1}$.

- Expansion of the Wilson line generates **induced vertices**:

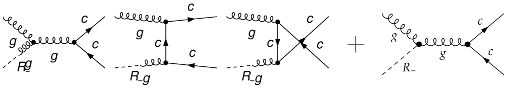
$$\text{tr} \left[R_+ \partial_\perp^2 A_- + (-ig_s)(\partial_\perp^2 R_+)(A_- \partial_-^{-1} A_-) \right.$$
$$\left. + (-ig_s)^2 (\partial_\perp^2 R_+)(A_- \partial_-^{-1} A_- \partial_-^{-1} A_-) + O(g_s^3) + (+ \leftrightarrow -) \right].$$

- The *Eikonal propagators* $\partial_\pm^{-1} \to -i/(k^\pm)$ lead to **rapidity divergences**, which are regularized by tilting the Wilson lines from the light-cone [Hentschinski, Sabio Vera, Chachamis *et. al.*, '12-'13; M.N. '19]:

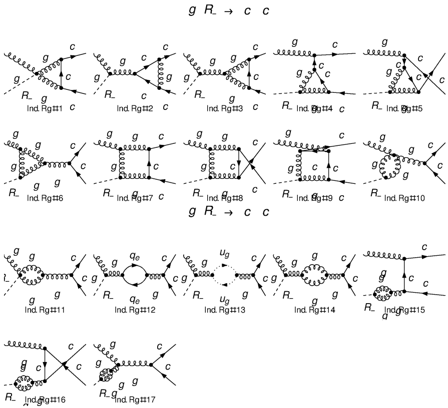
$$n_\pm^\mu \to \tilde{n}_\pm^\mu = n_\pm^\mu + r n_\mp^\mu, \; r \ll 1: \; \tilde{k}^\pm = \tilde{n}^\pm k.$$

The terms for conversion of the result into any other regularisation scheme for RDs can be easily computed.

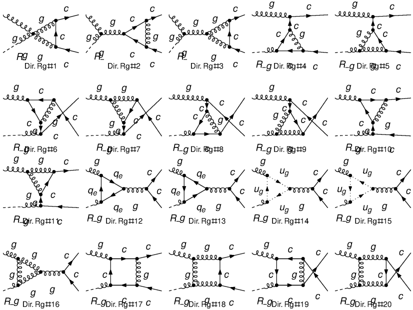
$Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$ and $c\bar{c}\left[{}^3S_1^{[8]}\right]$ @ 1 loop

Interference with LO: 

Induced Rgg coupling diagrams:



Some Rg-coupling diagrams:



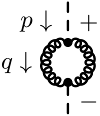
and so on...

- Diagrams had been generated using custom `FeynArts` model-file, projector on the $c\bar{c}\left[{}^1S_0^{[1]}\right]$-state is inserted
- heavy-quark momenta $= p_{\mathcal{Q}}/2 \Rightarrow$ need to resolve linear dependence of quadratic denominators in some diagrams before IBP
- IBP reduction to master integrals has been performed using `FIRE`
- Master integrals with linear and massless quadratic denominators are expanded in $r \ll 1$ using Mellin-Barnes representation. The differential equations technique is used when the integral depends on more than one scale of virtuality.
- In presence of the linear denominator the massive propagator can be converted to the massless one:

$$\frac{1}{((\tilde{n}_+ l) + k_+)(l^2 - m^2)} = \frac{1}{((\tilde{n}_+ l) + k_+)(l + \kappa\tilde{n}_+)^2} + \frac{2\kappa\left[(\tilde{n}_+ l) + \frac{m^2 + \tilde{n}_+^2 \kappa^2}{2\kappa}\right]}{((\tilde{n}_+ l) + k_+)(l + \kappa\tilde{n}_+)^2(l^2 - m^2)}$$

$\Rightarrow$ all the masses can be moved to integrals with **only quadratic propagators**.

Rapidity divergences and regularization.

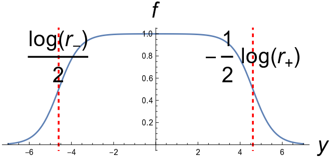


$$= g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{\left(\mathbf{p}_T^2 (n_+ n_-)\right)^2}{q^2 (p-q)^2 q^+ q^-}, \quad \int \frac{dq^+ dq^- (\ldots)}{q^+ q^-} = \int_{y_1}^{y_2} dy \int \frac{dq^2 (\ldots)}{q^2 + \mathbf{q}_T^2}$$

the regularization by explicit cutoff in rapidity was originally proposed [Lipatov, '95] ($q^\pm = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$, $p^+ = p^- = 0$):

$$\delta_{ab} \mathbf{p}_T^2 \times \underbrace{C_A g_s^2 \int \frac{\mathbf{p}_T^2 d^{D-2} \mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

The square of regularized Lipatov vertex:



$$\Gamma_{+\mu-} \Gamma_{+\nu-} P^{\mu\nu} = \frac{16 \mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2}{\mathbf{k}_T^2} f(y),$$

$$\longleftarrow f(y) = \frac{1}{(re^{-y} + e^{y})(re^{y} + e^{-y})},$$

$$\int_{-\infty}^{+\infty} dy \ f(y) = -\log r + O(r)$$

Result: $Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$ @ 1 loop

Result [MN, '23] for $2\Re\left[\frac{H_{\mathbf{1L\ x\ LO}}(\mathbf{q}_T) - (\textbf{On-shell mass CT})}{(\alpha_s/(2\pi))H_{\text{LO}}(\mathbf{q}_T)}\right]$:

$$ {}^1S_0^{[1]} : \left(\frac{\mu^2}{\mathbf{q}_T^2}\right)^\epsilon \left\{ -\frac{N_c}{\epsilon^2} + \frac{1}{\epsilon}\left[N_c \left(\ln\frac{q_-^2}{\mathbf{q}_T^2 r} + \frac{25}{6} \right) - \frac{2n_F}{3} - \frac{3}{2N_c} \right] \right\} + F_{{}^1S_0^{[1]}}(\mathbf{q}_T^2/M^2) $$

$$ \begin{aligned} F_{{}^1S_0^{[1]}}(\tau) &= -\frac{10}{9}n_F + \Re[C_F F^{(C_F)}_{{}^1S_0^{[1]}}(\tau) + C_A F^{(C_A)}_{{}^1S_0^{[1]}}(\tau)], \\ F^{(C_F)}_{{}^1S_0^{[1]}}(\tau) &= F^{(C_F)}_{{}^1S_0^{[8]}}(\tau), \end{aligned} $$

while $F^{(C_A)}_{{}^1S_0^{[1]}}(\tau) \neq F^{(C_A)}_{{}^1S_0^{[8]}}(\tau)$.

After UV renormalization:

$$ \mathcal{H}^{(\text{NLO, V})}_{gj}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \mathcal{H}^{(\text{LO})}_{gg}(\mathbf{q}_T^2, \mathbf{p}_T^2, z)\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\mu_R^2}{\mathbf{p}_T^2}\right)^\epsilon $$

$$ \times\left\{ -\frac{C_A}{\epsilon^2} - \frac{C_A}{\epsilon}\left(\ln\frac{r\mathbf{p}_T^2}{q_-^2} + 1\right) + \beta_0 \ln\frac{\mu_R^2}{\mathbf{p}_T^2} + F_{{}^1S_0^{[1]}}(\mathbf{p}_T^2/M^2) \right. $$

$$ \left. -2C_F\left(2 + \frac{3}{2}\ln\frac{\mathbf{p}_T^2}{m_c^2}\right) + O(r,\epsilon) \right\}. $$

Real-emission correction

The real-emission contribution to the coefficient function is given by:

$$\mathcal{H}_{gg}^{(\text{NLO, R})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int \frac{d\Omega_{2-2\epsilon}}{(2\pi)^{1-2\epsilon}} \frac{\tilde{H}_{Rg}(\mathbf{q}_T, \mathbf{p}_T, z)}{z(1-z)\mathbf{q}_T^2},$$

where the **function** $\tilde{H}_{Rg}(\mathbf{q}_T, \mathbf{p}_T, z)$ **is very complicated.** The following subtraction term:

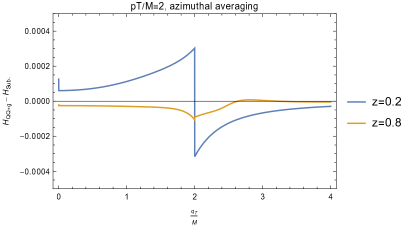
$$\mathcal{J}_{Rj}^{(\text{sub.})} = \frac{2C_A}{\mathbf{k}_T^2}\left[\frac{1-z}{(1-z)^2 + r\frac{\mathbf{k}_T^2}{q_-^2}} + z(1-z) + 2\frac{\mathbf{k}_T^2\mathbf{q}_T^2 - (\mathbf{k}_T\mathbf{q}_T)^2}{z\mathbf{k}_T^2\mathbf{q}_T^2} - \frac{3\mathbf{k}_T^2\mathbf{q}_T^2 - 2(\mathbf{k}_T\mathbf{q}_T)^2}{\mathbf{k}_T^2\mathbf{q}_T^2}\right],$$

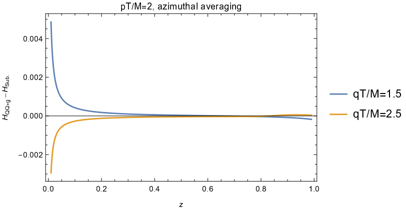
captures it's singular behaviour in:

- **Regge limit:** $z \to 1$, $\mathbf{k}_T = \mathbf{q}_T - \mathbf{p}_T$ – fixed,
- **Collinear limit:** $\mathbf{k}_T \to 0$, z-fixed
- **Soft limit:** $\mathbf{k}_T \to 0$, $z \to 1$

Real-emission correction, finite part

$$\mathcal{H}^{(\text{fin.})}_{gj}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}^{(\text{LO})}_{gg}(\mathbf{p}_T^2)\int\limits_0^{2\pi}\frac{d\phi}{2\pi}\left[\frac{\tilde{H}_{Rj}(\mathbf{q}_T, \mathbf{p}_T, z)}{z(1-z)\mathbf{q}_T^2} - \mathcal{J}^{(\text{sub.})}_{Rj}(\mathbf{q}_T, \mathbf{p}_T, z, r=0)\right]$$





This contribution is finite for $\mathbf{k}_T \to 0$ and $z \to 1$ and can be safely convoluted with the resummation factor or unintegrqted-PDF in $\mathbf{q}_T$ and gluon PDF in z.

Integrated subtraction term

$$\mathcal{H}_{gj}^{(\text{int. sub.})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\alpha_s(\mu_R)}{2\pi}\frac{\Omega_{2-2\epsilon}\mu^{2\epsilon}}{(2\pi)^{1-2\epsilon}}\int d^{2-2\epsilon}\mathbf{k}_T \mathcal{J}_{Rj}^{(\text{sub.})}(\mathbf{q}_T, \mathbf{q}_T - \mathbf{k}_T, z, r) \times \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{q}_T - \mathbf{k}_T, \mathbf{p}_T) = \mathcal{H}_{gj}^{(\text{int. sub. I})} + \mathcal{H}_{gj}^{(\text{int. sub. II})}$$

$$\mathcal{H}_{gj}^{(\text{int. sub. I})} = \frac{\alpha_s(\mu_R)C_A}{\pi}\int \frac{d^2\mathbf{k}_T}{\mathbf{k}_T^2}\left[\frac{1}{(1-z)_+} + (\ldots) - \delta(1-z)\frac{1}{2}\ln\frac{r\mathbf{k}_T^2}{q_-^2}\right]\left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2}\delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T)\right]\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)$$

$$\mathcal{H}_{gj}^{(\text{int. sub. II})} = \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)\delta(\mathbf{p}_T^2 - \mathbf{q}_T^2)\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^\epsilon \times\left\{-\frac{1}{\epsilon}P_{gg}(z) + 2C_A\frac{1-z}{z} + \delta(1-z)\left[\frac{C_A}{\epsilon^2} + \frac{\beta_0}{2}\frac{1}{\epsilon} + \frac{C_A}{\epsilon}\ln\frac{r\mathbf{p}_T^2}{q_-^2} - \frac{\pi^2}{6}C_A\right] + O(\epsilon^2)\right\}.$$

Rapidity factorisation schemes

The $\ln r$-regularisation is equivalent to the cut in rapidity, **for HEF** we need to cut in "projectile light-cone component" k_+:

$$\mathcal{H}_{gj}^{(\text{HEF-sch.})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\bar{\alpha}_s}{2\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{q}_T^2, \mathbf{p}_T^2, z)\left[-\frac{1}{\epsilon}\left(\frac{\beta_0}{2} - C_A\right) + \frac{8}{3}C_A - \frac{5}{3}\beta_0 - \frac{\pi^2}{3}C_A\right]$$
$$-\frac{\alpha_s C_A}{\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)\int \frac{d^2\mathbf{k}_T}{\mathbf{k}_T^2}\left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2}\delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T)\right]$$
$$\times\left(-\frac{1}{2}\ln r + \ln\frac{|\mathbf{k}_T|}{\Lambda_+}\right),$$

where $\Lambda_+ \simeq q_1^+ = (M^2 + \mathbf{p}_T^2)/q_-$. The **blue** terms come from R self-energy.
In BFKL we cut in $\ln(s_{\eta_c g}/s_0)$:

$$\mathcal{H}_{gj}^{(\text{BFKL-sch.})}(\mathbf{q}_T^2, z, \mathbf{p}_T^2) = \frac{\bar{\alpha}_s}{2\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{q}_T^2, \mathbf{p}_T^2, z)\left[-\frac{1}{\epsilon}\left(\frac{\beta_0}{2} - C_A\right) + \frac{8}{3}C_A - \frac{5}{3}\beta_0\right]$$
$$-\frac{\alpha_s C_A}{\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)\int \frac{d^2\mathbf{k}_T}{\mathbf{k}_T^2}\left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2}\delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T)\right]$$
$$\times\left(-\frac{1}{2}\ln r + \ln\frac{q_-}{\sqrt{s_0}}\right).$$

Impact factor, HEF scheme

$$\mathcal{H}_{gg}^{(\text{NLO, analyt.})}(\mathbf{q}_T, z, \mathbf{p}_T) = \mathcal{H}_{gj}^{(\text{int. sub. I})} + \mathcal{H}_{gj}^{(\text{int. sub. II})} + \mathcal{H}_{gj}^{(\text{NLO, V})} + \mathcal{H}_{gj}^{(\text{HEF-sch.})}$$

$$= \frac{\alpha_s C_A}{\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int \frac{d^2\mathbf{k}_T}{\mathbf{k}_T^2} \left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2} \delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T) \right] \left[\frac{1}{(1-z)_+} \right.$$

$$\left. + z(1-z) + 2\frac{\mathbf{k}_T^2\mathbf{q}_T^2 - (\mathbf{k}_T\mathbf{q}_T)^2}{z\mathbf{k}_T^2\mathbf{q}_T^2} - \frac{3\mathbf{k}_T^2\mathbf{q}_T^2 - 2(\mathbf{k}_T\mathbf{q}_T)^2}{\mathbf{k}_T^2\mathbf{q}_T^2} + \delta(1-z) \ln\left(\frac{M^2 + \mathbf{p}_T^2}{\mathbf{k}_T^2} \right) \right]$$

$$+ \frac{\alpha_s C_A}{2\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \delta(\mathbf{q}_T^2 - \mathbf{p}_T^2) \left\{ - \ln \frac{\mu_F^2}{\mathbf{p}_T^2} P_{gg}(z) + 2C_A \frac{1-z}{z} \right.$$

$$\left. + \delta(1-z) \left[-\frac{\pi^2}{2} C_A + \frac{8}{3} C_A - \frac{5}{3}\beta_0 - 2C_F \left(2 + \frac{2}{3} \ln \frac{\mathbf{p}_T^2}{m_c^2} \right) + \beta_0 \ln \frac{\mu_R^2}{\mathbf{p}_T^2} + F_{{}^1S_0^{[1]}}(\mathbf{p}_T^2/M^2) \right] \right\}$$

This result should be added to the $\mathcal{H}_{gg}^{(\text{fin.})}$.

Impact factor, BFKL scheme

$$\mathcal{H}_{gg}^{(\text{NLO, analyt., BFKL})}(\mathbf{q}_T, z, \mathbf{p}_T) = \mathcal{H}_{gj}^{(\text{int. sub. I})} + \mathcal{H}_{gj}^{(\text{int. sub. II})} + \mathcal{H}_{gj}^{(\text{NLO, V})} + \mathcal{H}_{gj}^{(\text{BFKL-sch.})}$$

$$= \frac{\alpha_s C_A}{\pi} \mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2) \int \frac{d^2\mathbf{k}_T}{\mathbf{k}_T^2} \left[\delta^{(2)}(\mathbf{q}_T - \mathbf{k}_T - \mathbf{p}_T) - \frac{\mathbf{p}_T^2}{\mathbf{p}_T^2 + \mathbf{k}_T^2}\delta^{(2)}(\mathbf{q}_T - \mathbf{p}_T)\right] \left[\frac{1}{(1-z)_+}\right.$$

$$\left. + z(1-z) + 2\frac{\mathbf{k}_T^2\mathbf{q}_T^2 - (\mathbf{k}_T\mathbf{q}_T)^2}{z\mathbf{k}_T^2\mathbf{q}_T^2} - \frac{3\mathbf{k}_T^2\mathbf{q}_T^2 - 2(\mathbf{k}_T\mathbf{q}_T)^2}{\mathbf{k}_T^2\mathbf{q}_T^2} + \delta(1-z)\ln\left(\frac{\sqrt{s_0}}{|\mathbf{k}_T|}\right)\right]$$

$$+ \frac{\alpha_s C_A}{2\pi}\mathcal{H}_{gg}^{(\text{LO})}(\mathbf{p}_T^2)\delta(\mathbf{q}_T^2 - \mathbf{p}_T^2)\left\{-\ln\frac{\mu_F^2}{\mathbf{p}_T^2}P_{gg}(z) + 2C_A\frac{1-z}{z}\right.$$

$$\left. + \delta(1-z)\left[-\frac{\pi^2}{6}C_A + \frac{8}{3}C_A - \frac{5}{3}\beta_0 - 2C_F\left(2 + \frac{2}{3}\ln\frac{\mathbf{p}_T^2}{m_c^2}\right) + \beta_0\ln\frac{\mu_R^2}{\mathbf{p}_T^2} + F_{{}^1S_0^{[1]}}(\mathbf{p}_T^2/M^2)\right]\right\}$$

This result should be added to **the same** $\mathcal{H}_{gg}^{(\text{fin.})}$.

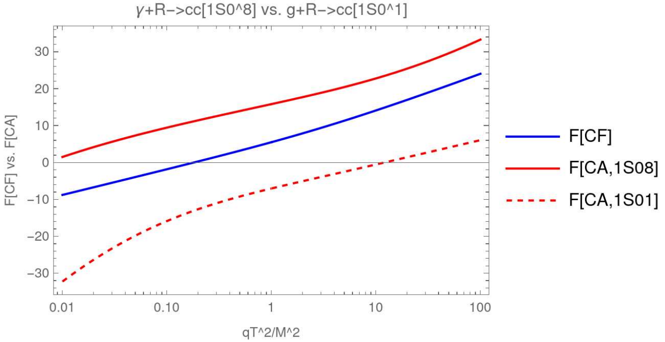
Conclusions and outlook

- The complete NLO HEF coefficient function (impact factor) for the $g + R \to c\bar{c}[{}^1S_0^{[1]}]$ process is computed, including one-loop and real-emission corrections
- The computation for other NRQCD-factorisation intermediate states: $c\bar{c}[{}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_J^{[1,8]}]$ are in progress. The $c\bar{c}[{}^3S_1^{[1]}]$ is more challenging.
- The result in HEF scheme is useful for the resummation of $\ln \hat{s}/M^2$ corrections in CF coefficient function
- The result in BFKL scheme is useful for the study of double-η_c froduction at large rapidity separation
- The result in the "shockwave" scheme, corresponding to the cut in "projectile" light-cone component (k^-) is easy to obtain. However this is $1R$-exchange only.
- The same computaion technology can be applied to the central production vertices $RR \to c\bar{c}[n]$.

Thank you for your attention!

The C_F coefficient

$$F^{(C_F)}_{{}^1S_0^{[8]}}(\tau) = \frac{\mathcal{L}_2 + \mathcal{L}_7(1-2\tau)}{\tau+1}$$
$$+\frac{1}{6(\tau+1)(2\tau+1)^2}\{144L_1\tau^2 + 144L_1\tau + 36L_1 - 16\pi^2\tau^3 - 72\tau^3 + 72\tau^3\log(2)$$
$$-156\tau^2 + 12\tau^2\log^2(2\tau+1) + 168\tau^2\log(2) - 24\left(3\tau^2+5\tau+2\right)\tau\log(\tau+1)$$
$$+12\pi^2\tau - 108\tau + 12\tau\log^2(2\tau+1) + 3\log^2(2\tau+1) + 132\tau\log(2)$$
$$+18(\tau+1)(2\tau+1)^2\log(\tau) + 4\pi^2 - 24 + 36\log(2)\}$$



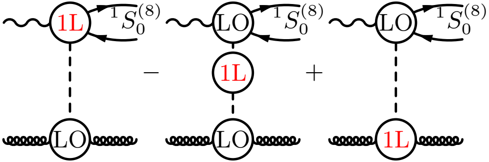
The C_A coefficient for $Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$

$$
\begin{aligned}
F^{(C_A)}_{{}^1S_0^{[1]}}(\tau) \quad = \quad & \frac{1}{(\tau-1)(\tau+1)^3}\{2\mathcal{L}_1\left(\tau^2+\tau-2\right)(\tau+1)^3+\tau\Big[2\mathcal{L}_5\left(\tau(\tau+1)\left(\tau^2-2\right)+1\right) \\
& -\mathcal{L}_7\left(\tau^2+\tau-1\right)-\left(\mathcal{L}_2(\tau+2)(\tau+1)^2\right)+\mathcal{L}_6(\tau(\tau(6-(\tau-4)\tau)+4)-1)\Big] \\
& +2\mathcal{L}_3(\tau-1)(\tau+1)^3+2\mathcal{L}_5+\mathcal{L}_7\} \\
- \quad & \frac{1}{18(\tau-1)(\tau+1)^3}\{6\pi^2\tau^5-36\tau^5\log(2)\log(\tau+1)+36\tau^5\log(\tau+1)\log(\tau+2)+63\pi^2\tau^4 \\
& -98\tau^4-63\tau^4\log^2(\tau+1)+9\tau^4\log^2(2\tau+1)-63\tau^4\log^2(2)+54\tau^4\log(2)\log(\tau+1) \\
& -36\tau^4\log(\tau+1)+36\tau^4\log(\tau+1)\log(\tau+2)+36\tau^4\log(2)+138\pi^2\tau^3-196\tau^3 \\
& -72\tau^3\log^2(\tau+1)+36\tau^3\log^2(2\tau+1)-72\tau^3\log^2(2)+144\tau^3\log(2)\log(\tau+1) \\
& -36\tau^3\log(\tau+1)-72\tau^3\log(\tau+1)\log(\tau+2)-36\tau^3\log(2)+18\pi^2\tau^2 \\
& -18\tau^2\log^2(\tau+1)+45\tau^2\log^2(2\tau+1)-18\tau^2\log^2(2)+108\tau^2\log(2)\log(\tau+1) \\
& +36\tau^2\log(\tau+1)-72\tau^2\log(\tau+1)\log(\tau+2)-36\tau^2\log(2) \\
& -18\left(4\tau^4+5\tau^3+\tau^2-3\tau-1\right)\log^2(\tau)+18\log(\tau)\Big[\tau^5\log(2)-\tau^4(\log(4)-2) \\
& -\tau^3\log(4)-2\tau^2(1+\log(4))-\left(\tau^4-4\tau^3-6\tau^2-4\tau+1\right)\tau\log(\tau+1)-\tau\log(8)-\log(4)\Big] \\
& -120\pi^2\tau+196\tau+36\tau\log^2(\tau+1)+18\tau\log^2(2\tau+1)+36\tau\log^2(2)+9\log^2(\tau+1) \\
& -36\tau\log(2)\log(\tau+1)+36\tau\log(\tau+1)+36\tau\log(\tau+1)\log(\tau+2)+36\tau\log(2) \\
& -36(\tau-1)(\tau+1)^3\log(\tau-1)(\log(2)-\log(\tau+1))-18\log(2)\log(\tau+1) \\
& +36\log(\tau+1)\log(\tau+2)-69\pi^2+98+9\log^2(2)\}
\end{aligned}
$$

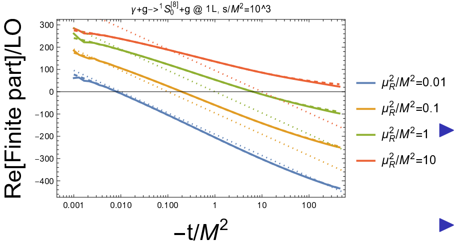
$$
\begin{aligned}
L_1 &= \sqrt{\tau(1+\tau)}\ln\left[1+2\tau+2\sqrt{\tau(1+\tau)}\right], \\
\mathcal{L}_1 &= \mathrm{Li}_2\left(\frac{1}{\tau}+1\right) \\
\mathcal{L}_2 &= \mathrm{Li}_2\left(\frac{1}{-2\tau-1}\right) \\
\mathcal{L}_3 &= \mathrm{Li}_2\left(\frac{1}{\tau}\right)+\mathrm{Li}_2\left(\frac{\tau-1}{\tau+1}\right)-\mathrm{Li}_2\left(\frac{\tau+1}{2\tau}\right)+\frac{\mathrm{Li}_2\left(\frac{1}{4}\right)}{2}+\mathrm{Li}_2(-2) \\
\mathcal{L}_4 &= \mathrm{Li}_2\left(1+\frac{1}{\tau}\right)+\mathrm{Li}_2\left(\frac{1}{\tau}\right)+\mathrm{Li}_2\left(\frac{\tau-1}{\tau+1}\right)-\mathrm{Li}_2\left(\frac{\tau+1}{2\tau}\right)+\frac{\mathrm{Li}_2\left(\frac{1}{4}\right)}{2}+\mathrm{Li}_2(-2) \\
\mathcal{L}_5 &= \mathrm{Li}_2\left(-\frac{1}{\tau+1}\right)-\mathrm{Li}_2(\tau+2)+\frac{1}{2}\mathrm{Li}_2\left(\frac{2\tau+1}{2\tau+2}\right) \\
\mathcal{L}_6 &= -\mathrm{Li}_2\left(-\frac{2\tau+1}{\tau^2}\right)+\mathrm{Li}_2\left(-\frac{-2\tau^2+\tau+1}{2\tau^2}\right)+\mathrm{Li}_2\left(\frac{1}{2}-\frac{\tau}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{\tau}\right) \\
&\quad -\mathrm{Li}_2\left(\frac{\tau-1}{2\tau}\right)-\mathrm{Li}_2(-\tau)+\mathrm{Li}_2\left(\frac{1-\tau}{\tau+1}\right) \\
\mathcal{L}_7 &= \mathrm{Li}_2(-2\tau-1)-\mathrm{Li}_2\left(\frac{2\sqrt{\tau}}{\sqrt{\tau}-\sqrt{\tau+1}}\right)-\mathrm{Li}_2\left(\frac{2\sqrt{\tau}}{\sqrt{\tau}+\sqrt{\tau+1}}\right)
\end{aligned}
$$

$R\gamma \to c\bar{c}\left[{}^1S_0^{[8]}\right]$ @ 1 loop, cross-check

In the combination of 1-loop results in the EFT:



the $\ln r$ cancels and it should reproduce the the Regge limit($s \gg -t$) of the *real part* of the $2 \to 2$ 1-loop QCD amplitude:



Solid lines – QCD, dashed lines – EFT, dotted lines – $-2C_A \ln(-t/\mu_R^2)\ln(s/M^2)$

$$\gamma + g \to c\bar{c}\left[{}^1S_0^{(8)}\right] + g.$$

- The $2 \to 2$ QCD 1-loop amplitude can be computed numerically using `FormCalc` (with some tricks, due to Coulomb divergence)
- The Regge limit of $1/\epsilon$ divergent part agrees with the EFT result
- For the finite part agreement within few % is reached, need to push to higher s

Quarkonium in the potential model

Cornell potential:

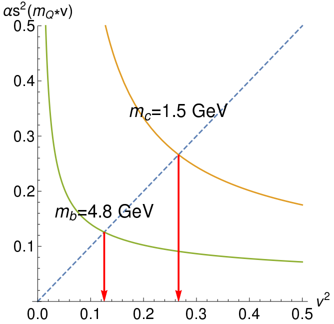
$$V(r) = -C_F \frac{\alpha_s(1/r)}{r} + \sigma r,$$

neglect linear part, because quarkonium is "small" (~ 0.3 fm) $\rightarrow$ Coulomb wavefunction (for effective mass $\frac{m_1 m_2}{m_1+m_2} = \frac{m_Q}{2}$):

$$R(r) = \frac{\sqrt{m_Q^3 \alpha_s^3 C_F^3}}{2} e^{-\frac{\alpha_s C_F}{2} m_Q r}$$

$$\langle v^2 \rangle = \frac{C_F^2 \alpha_s^2}{2}, \langle r \rangle = \frac{3}{2C_F} \frac{1}{m_Q v}$$

$$\Rightarrow \boxed{\alpha_s^2(m_Q v) \simeq v^2}$$



Non-relativistic QCD

The velocity-expansion for quarkonium eigenstate is carbon-copy of corresponding arguments from atomic physics (hierarchy of E-dipole/M-dipole with ΔS/M-dipole transitions):

$$\begin{aligned} |J/\psi\rangle &= O(1)\left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|c\bar{c}\left[{}^3P_J^{(8)}\right]+g\right\rangle \\ &+ O(v^{3/2})\left|c\bar{c}\left[{}^1S_0^{(8)}\right]+g\right\rangle + O(v^2)\left|c\bar{c}\left[{}^3S_1^{(8)}\right]+gg\right\rangle + \ldots, \end{aligned}$$

for validity of this arguments, we should work in *non-relativistic EFT*, dynamics of which conserves number of heavy quarks. In such EFT, $Q\bar{Q}$-pair is produced in a point, by local operator:

$$\mathcal{A}_{\rm NRQCD} = \langle J/\psi + X|\, \chi^\dagger(0)\kappa_n\psi(0)\,|0\rangle\,,$$

Different operators "couple" to different Fock states:

$$\chi^\dagger(0)\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1S_0^{(1)}\right]\right\rangle,\ \chi^\dagger(0)\sigma_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle,$$
$$\chi^\dagger(0)\sigma_i T^a\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(8)}\right]\right\rangle,\ \chi^\dagger(0)D_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1P_1^{(8)}\right]\right\rangle, \ldots$$

squared NRQCD amplitude (=LDME):

$$\sum_X |\mathcal{A}|^2 = \langle 0|\, \underbrace{\psi^\dagger\kappa_n^\dagger\chi a^\dagger_{J/\psi} a_{J/\psi}\chi^\dagger\kappa_n\psi}_{\mathcal{O}_n^{J/\psi}}\,|0\rangle = \left\langle \mathcal{O}_n^{J/\psi}\right\rangle,$$

Exclusive J/ψ photoproduction

$$p(P) + \gamma(q) \to J/\psi(p) + p(P'),\ q^2 \simeq 0,$$

Kinematics (skewness):

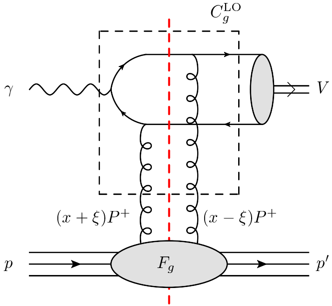
$$\xi = \frac{p^+}{2P^+} = \frac{M}{4E_p} e^{y(J/\psi)},$$

Factorisation formula ($P \simeq P'$):

$$A = \int_{-1}^{1} \frac{dx}{x} F_g(x, \xi, \mu_F) C_g(x),$$

$$\sigma \propto |A|^2.$$

Figure from
hep-ph/1507.06942



Exclusive J/ψ photoproduction at NLO

Partonic energy ($t = (P - P')^2 \simeq 0$):

$$\hat{s} = M^2 \frac{x+\xi}{2\xi} \gg M^2 \text{ if } \xi \ll x \ll 1,$$

NLO γg amplitude at $\xi \ll 1$ [Ivanov, Schaefer, Szymanowsky; Gracey, Jones, Teubner] :

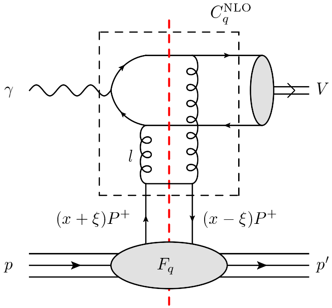
$$\text{Im}A_{\text{NLO}} \sim \hat{\alpha}_s \ln \frac{M^2}{4\mu_F^2} \int_\xi^1 \frac{dx}{x} F_g(x, \xi, \mu_F) + \ldots$$

The GPD F_g is relatively flat as function of x so $\int_\xi^1 \frac{dx}{x} \sim \ln 1/\xi$.

Also in Mellin space

$$\hat{\alpha}_s \int_\xi^1 \frac{dx}{x} \to \frac{\hat{\alpha}_s}{N}.$$

Figure from hep-ph/1507.06942



Treatment of the instability

Work in torgress, together with Jean-Philippe Lansberg, Chris Flett, Saad Nabebacus and Jakub Wagner.

- Simplest solution: choose $\hat{\mu}_F = \frac{M}{2}$ (+ some other less conventional tricks...) [Jones, Martin, Ryskin, Teubner, 2016; ...]
- HEF resummation of $\hat{\alpha}_s^n/N^n$ corrections [Ivanov 2007]
- **My proposal:** one have to do matching of the HEF-resummed $C(x)$ at $\xi \ll x \ll 1$ and NLO CPM at $x \sim 1$.
- The closed formula for for the coefficient function at $x \ll 1$ can be derived in DLA:

$$\frac{2C_{\perp g}^{\rm HEF}(x)}{-i\pi\hat{\alpha}_s F_{\rm LO}} = \frac{1}{|x|}\sqrt{\frac{L_\mu}{L_x}}\left\{ I_1\left(2\sqrt{L_x L_\mu}\right) - 2\sum_{k=1}^{\infty} {\rm Li}_{2k}(-1)\left(\frac{L_x}{L_\mu}\right)^k I_{2k-1}\left(2\sqrt{L_x L_\mu}\right)\right\},$$

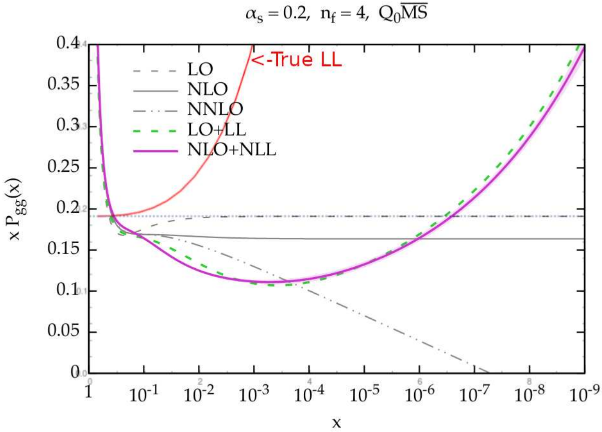
where $L_\mu = \ln[M^2/(4\mu_F^2)]$, $L_x = \hat{\alpha}_s \ln 1/|x|$ and Bessel functions $I_n(2\sqrt{L_\mu L_x})$ turn into $J_n(2\sqrt{-L_\mu L_x})$ if $L_\mu < 0$.

Backup: DGLAP P_{gg} at small z

LO: $P_{gg}(z) = \frac{2C_A}{z} + \ldots \Leftrightarrow \gamma_N = \frac{\hat{\alpha}_s}{N}$

Plot from hep-ph/1607.02153 with my curve (in red) for the **strict LLA:**

$\frac{\hat{\alpha}_s}{N}\chi_{LO}(\gamma_{gg}(N)) = 1 \Rightarrow \gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \ldots$



The "LO+LL" and "NLO+NLL" curves represent a form of matching between DGLAP and BFKL expansions, in a scheme by Altarelli, Ball and Forte which is more complicated than the **strict LL or NLL approximation**.

Effect of anomalous dimension beyond LO

Effect of taking **full LLA** for $\gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \ldots$ together with NLO PDF.