Odderon Mechanism for Transverse Single Spin Asymmetry in $p^{\uparrow}p$ and $p^{\uparrow}A$ Collisions

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DIS 2024

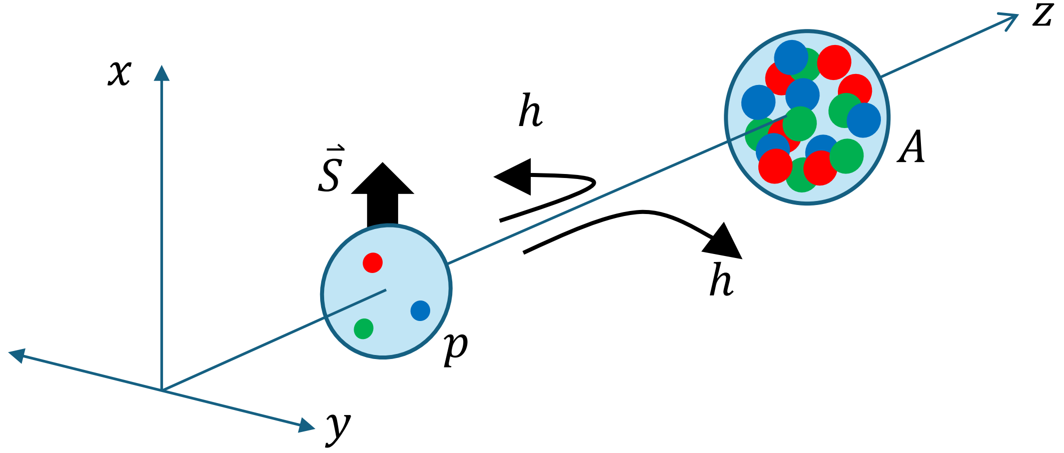
April, 9th 2024.

S. Benić, D. Horvatić, A. Kaushik and EAV, PRD, **106**, 114025 (2022).

S. Benić and EAV, in preparation

Transverse Single Spin Asymmetries (TSSA)

- Left-right asymmetry of produced particles in collisions involving polarized hadrons



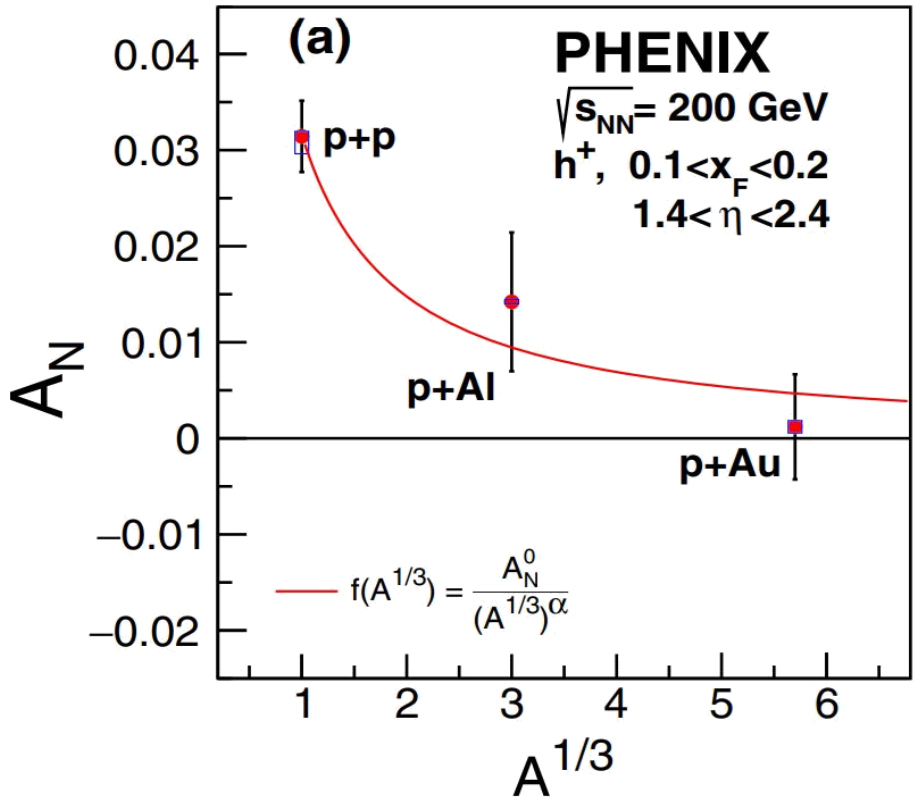
$$A_N \equiv \frac{N_L - N_R}{N_L + N_R} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\Delta\sigma}{2d\sigma_{unp}}$$

$$A_N \sim (\vec{S} \times \vec{P}_h) \cdot \vec{P} \sim \sin(\varphi_h - \varphi_S)$$

Naively T-odd quantity!

- Requires helicity flip → $A_N \sim \frac{m_q}{P_{h\perp}}$ → in pQCD there is no TSSA?

TSSA in $p^{\uparrow}A \to hX$ - experiments



PHENIX Collaboration, C. Aidala et. al., Phys.Rev.Lett. **123**, 122001 (2019).

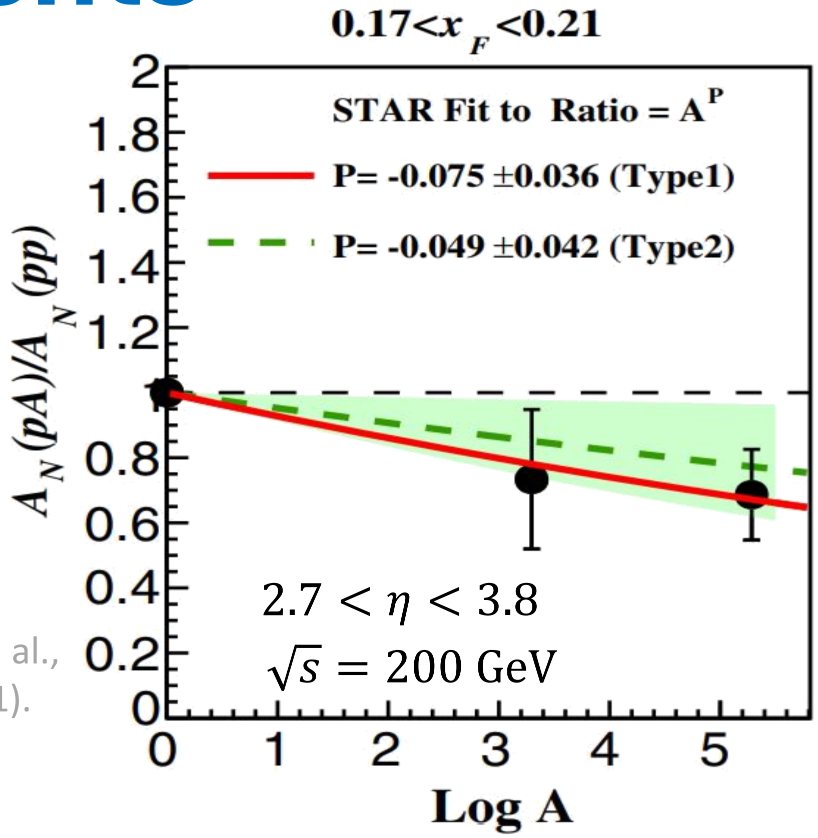
$1.8 < P_{hT} < 7.0$ GeV (integrated)

$0.004 \le x \le 0.1$

$A_N \sim A^{(-1/3)\alpha}$

$\alpha = 1.10^{+0.75}_{-0.41}$

Significant A suppression!



$2.7 < \eta < 3.8$

$\sqrt{s} = 200$ GeV

Star Collaboration, J. Adam et. al., Phys.Rev.D. **103**, 072005 (2021).

$1.5 < P_{hT} < 2.0$ GeV

$2.0 < P_{hT} < 3.0$ GeV…

$P_{hT} < 6.0$ GeV

$x < 0.005$

$A_N \sim A^{-0.027 \pm 0.005}$

TSSA as a quest for an i

M.D.Sievert, arxiv:1407.4047

- By using CPT symmetry, the Dirac structure of the cross section has the form:

$$\sigma_n \sim a + \chi b$$

a, spin independent part, is real!

b, spin dependent part, is completely imaginary!

Spin always comes with γ_5

- Cross section is real → we need another i
- Twist-3 contribution to polarized cross section:

Loop corections are higher order in α_S

$$\Delta\sigma \sim D_2 \otimes G_{F3} \otimes G_2 \otimes H_{pole} + iD_3 \otimes h_2 \otimes G_2 \otimes H + D_2 \otimes h_2 \otimes G_3 \otimes H_{pole}$$

- Real twist-3 ETQS functions for polarized projectile
- Phase from propagator cut

C. Kouvaris et. all, PRD **74**, 114013 (2006).

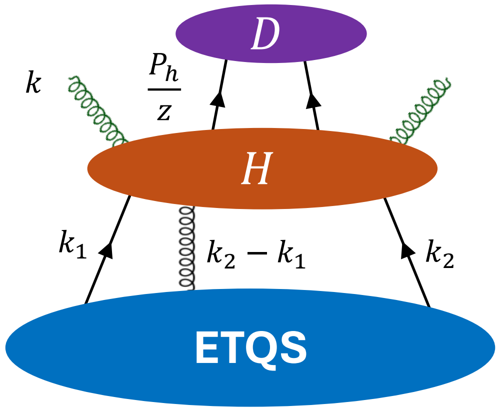
- Transversity PDF for polarized projectile
- Phase from imaginary part of twist-3 fragmentation function

A. Metz and D. Pitonyak, PLB **723**, 365 (2013).

- Transversity PDF for polarized projectile and twist-3 in target
- Phase from propagator cut

Y. Kanazawa and Y. Koike, PLB **478**, 121-126 (2000).

Pole-ETQS in hybrid approach



- Polarized proton described by real ETQS functions:

Y. Hatta et. all,
PRD **94**, 054013 (2016).

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1 + i\mu(x_2 - x_1)} \langle PS_\perp | \bar{\psi}_j(0)[0, \mu n] g F^{\alpha\,\beta}(\mu n) n_\beta [\mu n, \lambda n] \psi_i(\lambda n) | PS_\perp \rangle$$

$$= \frac{M_N}{4} (\not{P})_{ij} \epsilon^{\alpha P n S_\perp} G_F(x_1, x_2) + i \frac{M_N}{4} (\gamma_5 \not{P})_{ij} S_\perp^\alpha \tilde{G}_F(x_1, x_2)$$

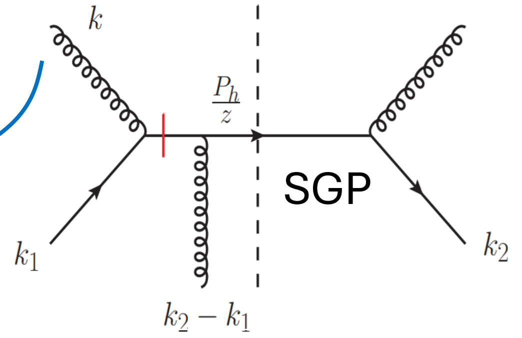
J. W. Qiu and G.F. Sterman, PRL **67**, 2264 (1991).
A.V. Efremov and O.V. Teryaev, PLB **150**, 383 (1985).

- Non-polarized target described by **Color Glass Condensate (CGC)**

$$(Q_S^A)^2 = A^{\frac{1}{3}} (Q_S^p)^2$$

- Phase obtained by propagator cut:

$$\frac{1}{p^2 + i\epsilon} = P\left(\frac{1}{p^2}\right) - i\pi\delta(p^2)$$



$$\frac{\mathrm{d}\Delta\sigma^{SGP}}{\mathrm{d}y_h \mathrm{d}^2 P_{h\perp}} = \frac{\pi M x_F}{2(N_C^2 - 1)} \epsilon^{\alpha\beta} S_{\perp\beta} \int_{x_F}^{1} \frac{\mathrm{d}z}{z^3} D(z) \left\{ -\frac{1}{(P_{h\perp}/z)^2} \right.$$
$$\times \frac{\partial}{\partial(P_h^\alpha/z)} \left(\frac{P_{h\perp}^2}{z^2} F(x_g, P_{h\perp}/z) \right) G_F(x, x)$$
$$\left. + \frac{2P_{h\alpha}/z}{(P_{h\perp}/z)^2} F(x_g, P_{h\perp}/z) x \frac{\mathrm{d}}{\mathrm{d}x} G_F(x, x) \right\}$$

Eikonalized interaction with target:

- F represents real part of dipole distribution function

$$\mathcal{D}(\boldsymbol{x}_\perp, \boldsymbol{x'}_\perp) \equiv \frac{1}{N_C} \mathrm{tr}(V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{x'}_\perp))$$

$$\frac{A_N^{pA}}{A_N^{pp}} \approx 1$$

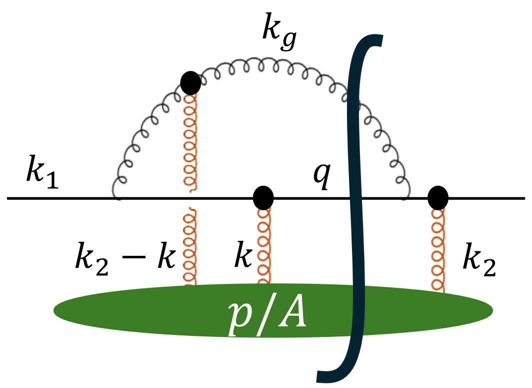
CGC-odderon: new mechanism for TSSA

- Odderon in CGC = imaginary part of dipole distribution:

$$\mathcal{D}(\boldsymbol{x}_\perp, \boldsymbol{x'}_\perp) \equiv \frac{1}{N_C}\mathrm{tr}(V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{x'}_\perp)) \qquad \mathcal{D}(\boldsymbol{x}_\perp, \boldsymbol{x'}_\perp) \equiv \mathcal{P}(\boldsymbol{x}_\perp, \boldsymbol{x'}_\perp) + i\mathcal{O}(\boldsymbol{x}_\perp, \boldsymbol{x'}_\perp)$$

- Phase from odderon and PV of propagators?

Y. V. Kovchegov and M. D. Sievert, PRD **86**, 034028 (2012).



- Asymmetry calculated at parton level (up to NLO), i.e. $q^\uparrow A$ collisions

- Polarized cross section:

$$E_q\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3q} \propto i\alpha_S \int_{\boldsymbol{k}_\perp \boldsymbol{k}_{2\perp}} \int_{\boldsymbol{r}_\perp \boldsymbol{b}_\perp \boldsymbol{r'}_\perp} \mathcal{H}(\boldsymbol{r}_\perp, \boldsymbol{r'}_\perp, \boldsymbol{S}_\perp)$$
$$\times \langle \mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r'}_\perp, \boldsymbol{b'}_\perp) - \mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)\mathcal{P}(\boldsymbol{r'}_\perp, \boldsymbol{b'}_\perp)\rangle$$

$$\boldsymbol{r}_\perp = \boldsymbol{x}_\perp - \boldsymbol{y}_\perp$$
$$\boldsymbol{b}_\perp = \frac{1}{2}(\boldsymbol{x}_\perp + \boldsymbol{y}_\perp)$$
$$\boldsymbol{r}'_\perp = \boldsymbol{y}_\perp - \boldsymbol{x}'_\perp$$
$$\boldsymbol{b}'_\perp = \frac{1}{2}(\boldsymbol{x}'_\perp + \boldsymbol{y}_\perp)$$

- Hard factor has initial stat collinear divergence:

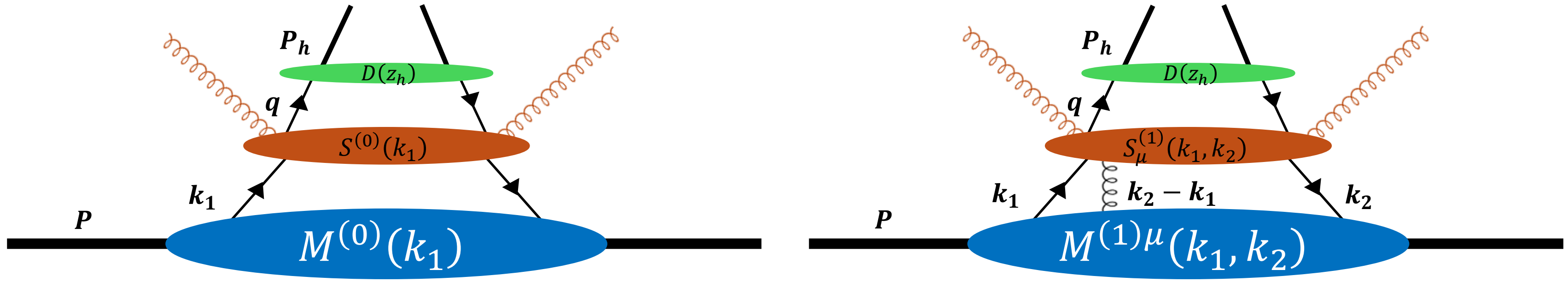
$$\boldsymbol{v}_{1\perp} = \boldsymbol{q}_\perp - z\boldsymbol{k}_{1\perp} - z\boldsymbol{k}_{2\perp}$$
$$\boldsymbol{v}_{2\perp} = \boldsymbol{q}_\perp - z\boldsymbol{k}_{1\perp} - \boldsymbol{k}_\perp$$

$$\mathcal{H} \sim i\frac{\boldsymbol{v}_{1\perp} \times \boldsymbol{S}_\perp}{\boldsymbol{v}_{1\perp}^2 \boldsymbol{v}_{2\perp}^2} \longrightarrow$$ Regulated by quark mass m_q

$$A_N \propto A^{-7/6}$$

From quarks to parton distributions…

- Twist-3 expansion of hadronic tensor:



$$W=\int_z \frac{D(z)}{z^2}w(z)$$

$$w(z)=\int_{k_1} M^{(0)}(k_1)S^{(0)}(k_1)+\int_{k_1,k_2} M_\mu^{(1)}(k_1,k_2)S^{(1)\mu}(k_1,k_2)$$

$$M^{(0)}(k_1)\sim\langle P,S|\bar\psi\psi|P,S\rangle$$

$$M_\lambda^{(1)}(k_1,k_2)\sim\langle P,S|\bar\psi A_\lambda\psi|P,S\rangle$$

- Full twist-3 polarized cross section:

H. Eguchi, Y. Koike and K. Tanaka, NPB **763**, 198 (2007).

$$E_h\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3P_h}=\frac{1}{2(2\pi)^3}\int\frac{\mathrm{d}z_h}{z_h^2}D(z_h)\left\{\frac{M_N}{2}\int\mathrm{d}x_1 g_T(x_1)\mathrm{Tr}\left[\gamma_5\not{S}_\perp S^{(0)}(x_1)\right]+\right.$$
$$\frac{M_N}{2}\int\mathrm{d}x_1\mathrm{Tr}\left[\left(\gamma_5\not{p}_p S_\perp^\lambda g_{1T}^{(1)}(x_1)+\epsilon^{\lambda\bar n n S_\perp}\not{p}_p f_{1T}^{(1)}(x_1)\right)\left(\frac{\partial S^{(0)}(k_1)}{\partial k_{1\perp}^\lambda}\right)_{k_1=x_1P}\right]+$$
$$\left.\frac{iM_N}{4}\int\mathrm{d}x_1\mathrm{d}x_2\mathrm{Tr}\left[\left(\not{p}_p\epsilon^{\bar n n\lambda S_\perp}\frac{G_F(x_1,x_2)}{x_1-x_2}+i\gamma_5\not{p}_p S_\perp^\lambda\frac{\tilde G_F(x_1,x_2)}{x_1-x_2}\right)S_\lambda^{(1)}(x_1,x_2)\right]\right\}$$

Intrinsical $g_T\sim\langle P,S|\bar\psi\psi|P,S\rangle$

Kinematical $g_{1T}^{(1)},f_{1T}^{(1)}\sim\langle P,S|\bar\psi\partial_\perp\psi|P,S\rangle$

Dynamical $G_F\sim\langle P,S|\bar\psi F\psi|P,S\rangle$

Wandzura – Wilczek approximation

S. Wandzura and F. Wilczek, PLB **72**, 195 (1977).

- Total cross section has twist-2 part coming from g_T:

$$g_T(x) = \int_x^1 \frac{dx'}{x'} \Delta_q(x') + \cdots$$ Helicity PDF!

QCD E.O.M.: $x g_T(x) \approx g_{1T}^{(1)}(x) + \text{Twist-3}$

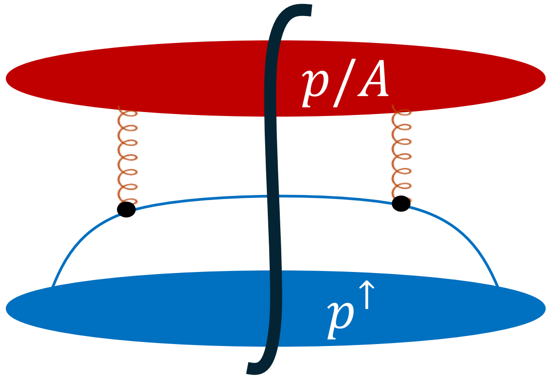
- WW: discard all genuine twist-3 contributions

S. Benić, D. Horvatić, A. Kaushik and EAV, PRD, **106**, 114025 (2022).

$$E_h \frac{d\Delta\sigma}{d^3P_h} \simeq \frac{1}{2(2\pi)^3} \frac{M_N}{2} \int \frac{dz_h}{z_h^2} D(z_h) \int dx_1 g_T(x_1) \times \left(S_\perp^\lambda \frac{\partial}{\partial k_{1\perp}^\lambda} \text{tr}[\gamma_5 \not{k}_1 S^{(0)}(k_1)] \right)_{k_1 = x_1 P}$$

- $S^{(0)}$ is calculated in perturbation theory; no L.O. contribution

- Partonic channels in NLO:
  1. $q \to q$ real
  2. $q \to q$ virtual
  3. $g \to q\bar{q}$
  4. $q \to g$ real
  5. $g \to g$ virtual
  6. $g \to gg$



➢ **Odderon does not contribute to TSSA in WW approximation up to NLO**

Non-pole ETQS – odderon contribution

S. Benić and EAV, in preparation

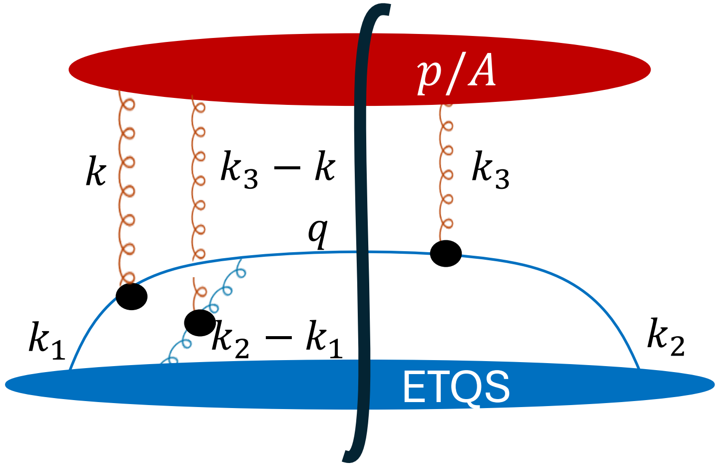
- Non-pole L.O. Full twist-3 polarized cross section:

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{iM_N}{8(2\pi)^3} \int \frac{dz_h}{z_h^2} D(z_h) \int dx_1 dx_2 \, \mathrm{Tr}\left[\left(\not{P}_p \epsilon^{\bar{n}n\lambda S_\perp} \frac{G_F(x_1,x_2)}{x_1-x_2} + i\gamma_5 \not{P}_p S_\perp^\lambda \frac{\tilde{G}_F(x_1,x_2)}{x_1-x_2}\right) S_\lambda^{(1)}(x_1,x_2)\right]$$

- ETQS functions obey the following symmetry properties:

$$G_F(x_1,x_2) = G_F(x_2,x_1) \qquad \tilde{G}_F(x_1,x_2) = -\tilde{G}_F(x_2,x_1)$$

- $S_\lambda^{(1)}(x_1,x_2)$ is calculated from diagrams involved:



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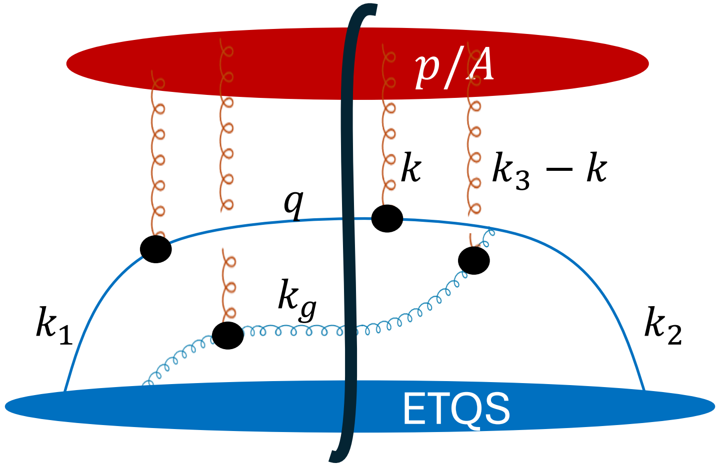

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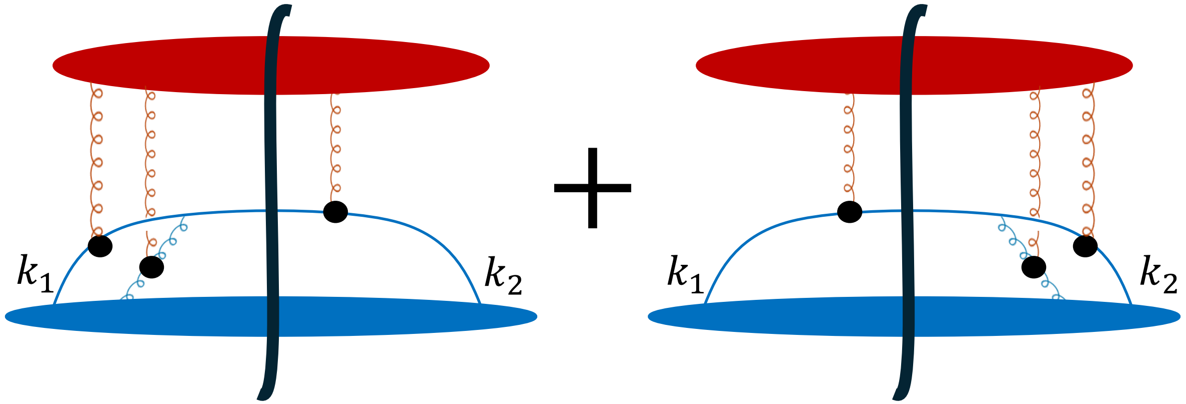
Calculation of $S_\lambda^{(1)}(x_1, x_2)$

- It is given by a sum of diagram and complex conjugated diagram
  - ➢Keeping in mind ETQS attachments!

$\bar{S}_\lambda^{(1)L}(x_2, x_1)$

$S_\lambda^{(1)}(x_1, x_2) =$



$= S_\lambda^{(1)L}(x_1, x_2) + S_\lambda^{(1)R}(x_1, x_2)$

- We have integrals over x_1 and x_2:

$$\int_{x_1,x_2}\left(\frac{\not{P}_p \epsilon^{\bar{n}n\lambda S_\perp} G_F(x_1,x_2)}{x_1 - x_2} + \frac{i\gamma_5 \not{P}_p S_\perp^\lambda \tilde{G}_F(x_1,x_2)}{x_1 - x_2}\right)\left(S_\lambda^{(1)}(x_1,x_2)\right) = \int_{x_1,x_2}\left\{\frac{\not{P}_p \epsilon^{\bar{n}n\lambda S_\perp} G_F(x_1,x_2)}{x_1 - x_2}\left(S_\lambda^{(1)L}(x_1,x_2) - \bar{S}_\lambda^{(1)L}(x_1,x_2)\right) + \frac{i\gamma_5 \not{P}_p S_\perp^\lambda \tilde{G}_F(x_1,x_2)}{x_1 - x_2}\left(S_\lambda^{(1)L}(x_1,x_2) + \bar{S}_\lambda^{(1)L}(x_1,x_2)\right)\right\}$$

Connected diagram:

- Amplitudes:

$$T^{\mu}_{qg}(x_1, x_2, \boldsymbol{k}_\perp) = x_2(x_2 - x_1)\gamma^{\nu}(\not{k}_1 + \not{k})\gamma^{+}\frac{d^{\mu}_{\nu}(q - k_1 - k)}{(x_1\boldsymbol{q}_\perp - x_2\boldsymbol{k}_\perp)^2}$$

$$\mathcal{M}^{\mu a}_{qg\to q}(x_1, x_2) = \int_{\boldsymbol{k}_\perp, \boldsymbol{x}_\perp, \boldsymbol{y}_\perp} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp} e^{i(\boldsymbol{k}_{3\perp}-\boldsymbol{k}_\perp)\cdot\boldsymbol{y}_\perp} T^{\mu}_{qg}(x_1, x_2, \boldsymbol{k}_\perp) t_b V(\boldsymbol{x}_\perp) U^{ba}(\boldsymbol{y}_\perp)$$

$$\mathcal{M}_{q\to q} = -i\int_{\boldsymbol{k}_\perp, \boldsymbol{x}_\perp, \boldsymbol{y}_\perp} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp} e^{i(\boldsymbol{k}_{3\perp}-\boldsymbol{k}_\perp)\cdot\boldsymbol{y}_\perp}\gamma^{+} V(\boldsymbol{x}_\perp)$$

Adjoint Wilson line:

$$U^{ba}(\boldsymbol{y}_\perp) = 2\mathrm{tr}\left(t^{b} V(\boldsymbol{y}_\perp) t^{a} V^{\dagger}(\boldsymbol{y}_\perp)\right)$$

- Formula for $S_L^{(1)\mu a}$:

$$S_L^{(1)\mu a}(x_1, x_2) = \frac{1}{2P^{+}}(2\pi)\delta(q^{+} - x_2 P^{+})\left\langle \bar{\mathcal{M}}_{q\to q}\not{q}\,\mathcal{M}^{\mu a}(x_1, x_2)\right\rangle$$

- Color factor is: $$\frac{2}{N_C^2-1}\left\langle \mathrm{tr}\left(V^\dagger(\boldsymbol{x}_\perp)t^b V(\boldsymbol{x}_\perp)U^{ba}(\boldsymbol{y}_\perp)t^a\right)\right\rangle$$
- After some SU(N) Fiertz algebra: $$\frac{1}{N_C^2-1}\left\langle N_C^2\mathcal{D}(\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp)\mathcal{D}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)-\mathcal{D}(\boldsymbol{x}_\perp,\boldsymbol{x}'_\perp)\right\rangle$$
- Here: $\mathcal{D}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)=\frac{1}{N_C}\mathrm{tr}\left(V(\boldsymbol{x}_\perp)V^\dagger(y_\perp)\right)$ → $\langle\mathcal{D}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\rangle=\frac{1}{2}\sum_s\frac{\langle PS|\mathcal{D}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)|PS\rangle}{\langle PS|PS\rangle}$

$$S^{(1)\mu}=-\frac{1}{2P^+}\frac{1}{N_C^2-1}(2\pi)\delta(q^+-x_2P^+)\int_{\boldsymbol{k}_\perp,\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp}e^{i\boldsymbol{k}_\perp\cdot(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp)}e^{i\boldsymbol{q}_\perp\cdot(\boldsymbol{y}_\perp-\boldsymbol{x}'_\perp)}$$

$$\left[\left(\gamma^+\not{q}T^\mu_{qg}(\boldsymbol{k}_\perp)\right)\left\langle N_C^2\mathcal{D}(\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp)\mathcal{D}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)-\mathcal{D}(\boldsymbol{x}_\perp,\boldsymbol{x}'_\perp)\right\rangle\right.$$

$$\left.\pm\left(\bar{T}^\mu_{qg}(\boldsymbol{k}_\perp)\not{q}\gamma^+\right)\left\langle N_C^2\mathcal{D}(\boldsymbol{y}_\perp,\boldsymbol{x}_\perp)\mathcal{D}(\boldsymbol{x}'_\perp,\boldsymbol{y}_\perp)-\mathcal{D}(\boldsymbol{x}'_\perp,\boldsymbol{x}_\perp)\right\rangle\right]$$

$\tilde{G}_F$ ← (first term)

G_F ← (second term)

Dipole piece

- In SGP we reproduced results from

Y. Hatta et. all,
PRD **94**, 054013 (2016).

- Non pole contribution:

$$S^{(1)\mu}_{dipole} = \frac{1}{2P^+}\frac{1}{N_C^2-1}(2\pi)\delta(q^+ - x_2P^+)\int_{\boldsymbol{x}_\perp,\boldsymbol{x}'_\perp} e^{i\boldsymbol{q}_\perp\cdot(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)}$$
$$\left(\gamma^+ \not{q} T^\mu_{qg}(\boldsymbol{q}_\perp) \pm \bar{T}^\mu_{qg}(\boldsymbol{q}_\perp)\not{q}\gamma^+\right)\langle\mathcal{D}(\boldsymbol{x}_\perp,\boldsymbol{x}'_\perp)\rangle$$

- Utilizing C-parity of Dirac traces

G_F part: $\mathrm{Tr}\left(\not{p}\bar{T}^\mu_{qg}(\boldsymbol{k}_\perp)\not{q}\gamma^+\right) = \mathrm{Tr}\left(\not{p}\gamma^+\not{q}T^\mu_{qg}(\boldsymbol{k}_\perp)\right)$

$\tilde{G}_F$ part: $\mathrm{Tr}\left(\gamma^5\not{p}\bar{T}^\mu_{qg}(\boldsymbol{k}_\perp)\not{q}\gamma^+\right) = -\mathrm{Tr}\left(\gamma^5\not{p}\gamma^+\not{q}T^\mu_{qg}(\boldsymbol{k}_\perp)\right)$

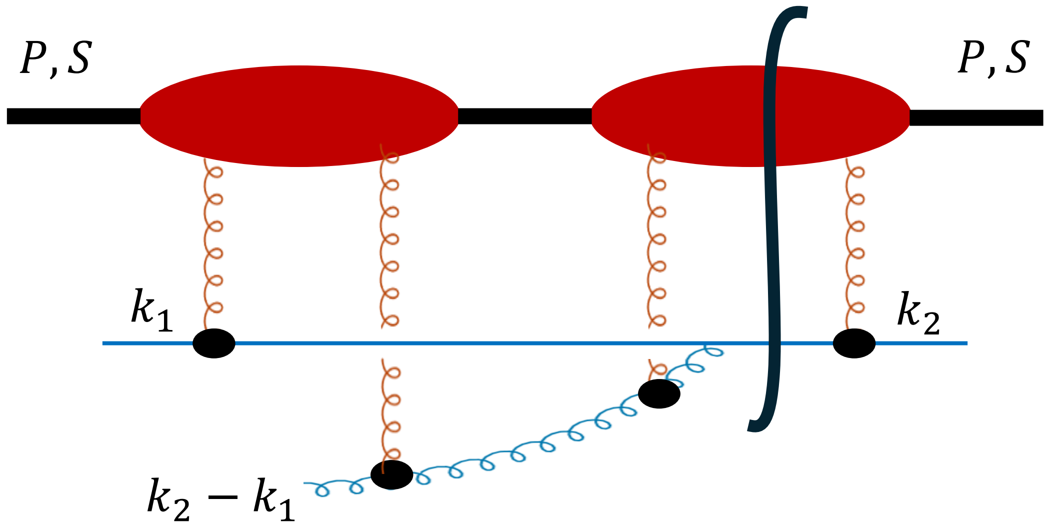
- Dipole piece vanishes under the Dirac trace!

Double - dipole piece

- Utilizing again C-symmetry on Dirac trace, we have the formula for cross section:

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{-iM_N}{8(2\pi)^2(2P^+)} \frac{N_C^2}{N_C^2-1} \int \frac{\mathrm{d}z_h}{z_h^2} D(z_h) \int \mathrm{d}x_1 \mathrm{d}x_2 \frac{\delta(q^+ - x_2P^+)}{x_1 - x_2} \int_{\boldsymbol{k}_\perp, \boldsymbol{x}_\perp, \boldsymbol{x}'_\perp, \boldsymbol{y}_\perp} e^{i\boldsymbol{k}_\perp\cdot(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)} e^{i\boldsymbol{q}_\perp\cdot(\boldsymbol{y}_\perp - \boldsymbol{x}'_\perp)}$$

$$\times \left[\langle \mathcal{D}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)\mathcal{D}(\boldsymbol{y}_\perp, \boldsymbol{x}'_\perp)\rangle - \langle \mathcal{D}(\boldsymbol{x}'_\perp, \boldsymbol{y}_\perp)\mathcal{D}(\boldsymbol{y}_\perp, \boldsymbol{x}_\perp)\rangle\right]\left[G_F(x_1, x_2)\mathcal{H}(x_1, x_2, \boldsymbol{k}_\perp) + \tilde{G}_F(x_1, x_2)\tilde{\mathcal{H}}(x_1, x_2, \boldsymbol{k}_\perp)\right]$$



$$\mathcal{H} = 8(P^+)^2 x_2(x_2 + x_1) \frac{\epsilon^{(x_1\boldsymbol{q}_\perp - x_2\boldsymbol{k}_\perp)\boldsymbol{S}_\perp}}{(x_1\boldsymbol{q}_\perp - x_2\boldsymbol{k}_\perp)^2}$$

$$\tilde{\mathcal{H}} = 8(P^+)^2 x_2(x_2 - x_1) \frac{\epsilon^{(x_1\boldsymbol{q}_\perp - x_2\boldsymbol{k}_\perp)\boldsymbol{S}_\perp}}{(x_1\boldsymbol{q}_\perp - x_2\boldsymbol{k}_\perp)^2}$$

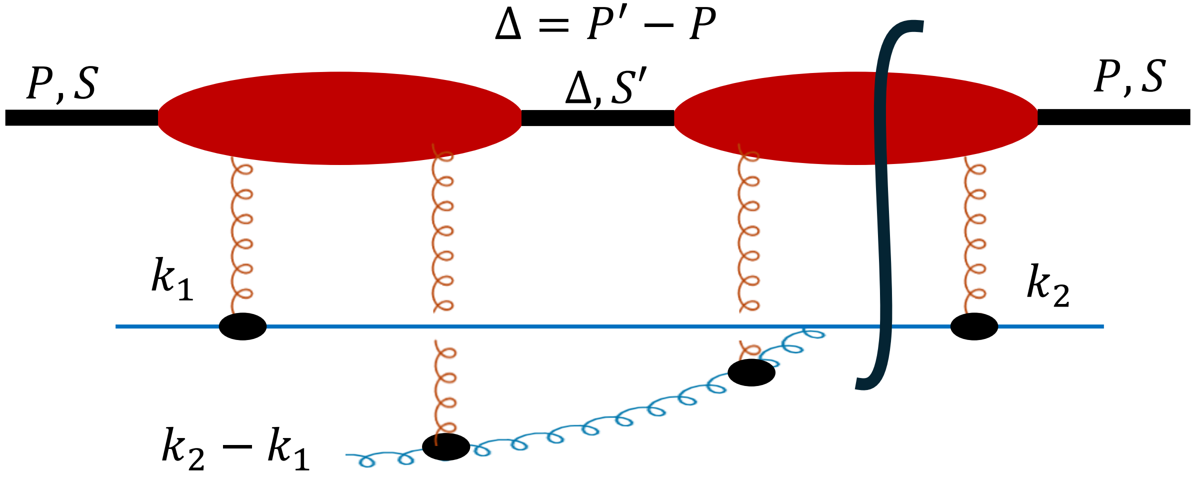
Hard factors are **not collinearly divergent**!

GTMDs and spin dependent odderon

- Adding a complete set of states in the CGC average:

$$\langle \mathcal{D}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)\mathcal{D}(\boldsymbol{y}_\perp, \boldsymbol{x}'_\perp)\rangle = \frac{1}{2\langle PS|PS\rangle}\sum_{SS'}\int \frac{d^2P'_\perp dP'^-}{(2\pi)^3 2P'^-}$$

$$\times \langle PS|\mathcal{D}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)|P'S'\rangle\langle P'S'|\mathcal{D}(\boldsymbol{y}_\perp, \boldsymbol{x}'_\perp)|PS\rangle$$



- By switching to $\boldsymbol{r}_\perp$ and $\boldsymbol{b}_\perp$ coordinates we can use the decomposition:

$$\int_{\boldsymbol{r}_\perp} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp}\langle P'S'|\mathcal{D}(\boldsymbol{r}_\perp)|PS\rangle = (2\pi)^4\delta(P^- - P'^-)\frac{P^-}{2M}\frac{g^2}{N_C\left(\boldsymbol{k}_\perp^2 - \frac{1}{4}\boldsymbol{\Delta}_\perp^2\right)}\bar{u}(P',S')\left[F^g_{1,1} - i\frac{\sigma^{i-}k^i_\perp}{P^-}F^g_{1,2} + i\frac{\sigma^{i-}\Delta^i_\perp}{P^-}F^g_{1,3}\right]u(P,S)$$

GTMDs

S. Meissner, et. all, JHEP **08**, 056 (2009)
S. Brattacharya, et. all, 1805.05219 (2018.)
R. Boussarie, Y. Hatta, L. Szymanowski and S. Wallon, PRL **124**, 172501 (2020)

- After some algebra ($\mathcal{P}^2$ and $\mathcal{O}^2$ cancel due to minus relative sign):

$$\int_{\boldsymbol{k}_\perp,\boldsymbol{r}_\perp,\boldsymbol{r}'_\perp,\boldsymbol{b}_\perp} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{r}'_\perp}\langle\mathcal{O}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{P}(\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp)+\mathcal{P}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{O}(\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp)\rangle\frac{(x_1\boldsymbol{q}_\perp - x_2\boldsymbol{k}_\perp)\times\boldsymbol{S}_\perp}{(x_1\boldsymbol{q}_\perp - x_2\boldsymbol{k}_\perp)^2}$$

$$\approx\frac{(2\pi)^6 g^4}{4N_C^2}\int_{\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp}\frac{1}{\boldsymbol{\kappa}_\perp^2\boldsymbol{q}_\perp^2}\left\{\frac{1}{M^2}(\boldsymbol{\kappa}_\perp\cdot\boldsymbol{\Delta}_\perp)\left[f_{1,1}(\boldsymbol{q}_\perp)g_{1,1}(\boldsymbol{\kappa}_\perp)+\frac{1}{2}E(\boldsymbol{q}_\perp)g_{1,2}(\boldsymbol{\kappa}_\perp)\right]\right.$$

$$\left.-\frac{1}{M^2}(\boldsymbol{q}_\perp\cdot\boldsymbol{\Delta}_\perp)\left[f_{1,1}(\boldsymbol{\kappa}_\perp)g_{1,1}(\boldsymbol{q}_\perp)+\frac{1}{2}E(\boldsymbol{\kappa}_\perp)g_{1,2}(\boldsymbol{q}_\perp)\right]\right\}\frac{(x_1\boldsymbol{q}_\perp - x_2\boldsymbol{k}_\perp)\times\boldsymbol{S}_\perp}{(x_1\boldsymbol{q}_\perp - x_2\boldsymbol{k}_\perp)^2}$$

Spin-independent pomeron

Spin-independent odderon

Spin-dependent odderon

- ➢ Completely new contribution!
- ➢ Same structure as in BK equation!

Here:

$$F_{1,1}^g = f_{1,1} - i\frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{\Delta}_\perp)}{M^2}g_{1,1}$$

$$F_{1,2}^g = -\frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{\Delta}_\perp)}{M^2}f_{1,2} + ig_{1,2}$$

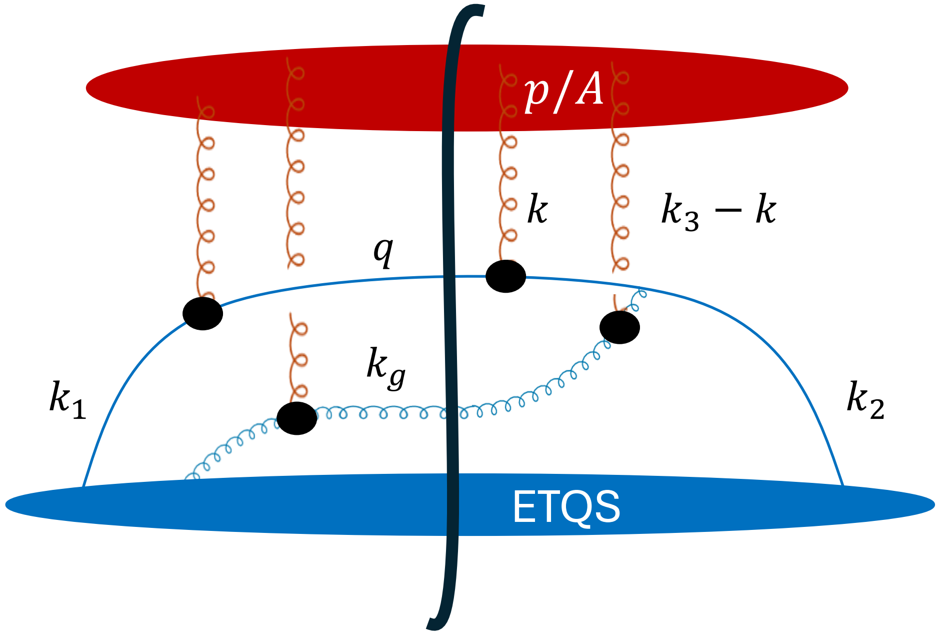
$$F_{1,3}^g = f_{1,3} - i\frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{\Delta}_\perp)}{M^2}g_{1,3}$$

GTMD E: $E(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp) = 2f_{1,3}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp) - f_{1,1}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp) + \frac{\boldsymbol{\kappa}_\perp^2}{M^2}f_{1,2}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp)$

- Combination of two non spin flip and two spin flip terms!

Disconnected diagram

- Similar strategy: only now we have outgoing gluon (quark) that is not observed
- We also have two adjoint Wilson lines: possible sextupole?



- Amplitudes: $T^{\mu}_{qg}(\boldsymbol{k}_\perp, x_2, x_1) = \frac{x_1 - x_2}{x_2}\gamma^+(\not{q} - \not{k})\gamma^\nu \frac{d^\mu_\nu(k_2 - q + k)}{(\boldsymbol{k}_\perp - \boldsymbol{q}_\perp)^2}$

$$\mathcal{M}_{qg\to qg}(x_1, x_2) = i(2\pi)\delta\left(k_g^+ - x_2P^+ + x_1P^+\right)\left(2k_g^+\right)\gamma^+ \int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{x}_\perp} e^{i\boldsymbol{k}_{g\perp}\cdot\boldsymbol{y}_\perp} V(\boldsymbol{x}_\perp)U^{ba}(\boldsymbol{y}_\perp)$$

$$\mathcal{M}^{\mu}_{q\to qg}(x_2, x_1) = i\int_{\boldsymbol{k}_\perp,\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp} e^{i(\boldsymbol{q}_\perp + \boldsymbol{k}_{g\perp} - \boldsymbol{k}_\perp)\cdot\boldsymbol{y}_\perp}\, T^{\mu}_{qg}(\boldsymbol{k}_\perp, x_2, x_1)V(\boldsymbol{x}_\perp)t^bU^{ab}(\boldsymbol{y}_\perp)$$

- $S^{1\mu}$ with integration over final state gluon:

$$S^{(1)\mu}(x_1,x_2) = \frac{(2\pi)}{2P^+}\int \frac{\mathrm{d}^3 k_g}{(2\pi)^3 2k_g^+} d_\nu^\mu(k_g)\delta(q^+ + k_g^+ - k_2^+)$$

$$\times \left[\langle \bar{\mathcal{M}}^\nu{}_{q\to qg}(x_2,x_1)\not{q}\mathcal{M}_{qg\to qg}(x_1,x_2)\rangle \pm \langle \bar{\mathcal{M}}_{qg\to qg}(x_1,x_2)\not{q}\mathcal{M}^\nu_{q\to qg}(x_2,x_1)\rangle\right]$$

- μ and ν have to be transverse → only $\boldsymbol{k}_{g\perp}$ dependence is in phase
- Adjoint Wilson lines colapse in δ^{ab}

$$S^{(1)\mu}_{dipole} = -\frac{(2\pi)}{2P^+}\delta(q^+ - x_1 P^+)\int_{\boldsymbol{x}_\perp,\boldsymbol{x}'_\perp} e^{i\boldsymbol{q}_\perp\cdot(\boldsymbol{x}_\perp - \boldsymbol{x}'_\perp)}$$

$$\left(\bar{T}^\mu_{qg}(\boldsymbol{q}_\perp,x_2,x_1)\not{q}\gamma^+ \pm \gamma^+\not{q}T^\mu_{qg}(\boldsymbol{q}_\perp,x_2,x_1)\right)\langle\mathcal{D}(\boldsymbol{x}'_\perp,\boldsymbol{x}_\perp)\rangle$$

This is zero under trace!

- Integration over final state quark will give dipole in adjoint representation → **no asymmetry from disconnected diagram**

Estimate of Nuclear dependence (toy model)

- Pomeron and odderon models ($T(\boldsymbol{b}_\perp)$ is a profile function normalized to target surface area):

$$\mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = T(\boldsymbol{b}_\perp) e^{-\frac{1}{4}Q_S^2 r_\perp^2}$$

T. Lappi and H. Mäntysaari, PRD **88**, 114020 (2013).

$$\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = -\frac{3}{128}\frac{N_C^2-4}{(N_C^2-1)^2}\frac{Q_S^3 R^3}{\alpha_S^3 A^2} R \frac{\mathrm{d}T(\boldsymbol{b}_\perp)}{\mathrm{d}b_\perp} Q_S^3 r_\perp^3 \cos(\phi_{rb}) e^{-\frac{1}{4}Q_S^2 r_\perp^2}$$

S. Jeon and R. Venugopalan, PRD **71**, 125003 (2005).

- Polarized cross section:

$$\mathrm{d}\Delta\sigma \sim M_N \frac{\boldsymbol{q}_\perp \times \boldsymbol{S}_\perp}{q_\perp^2}\frac{R^4}{A^2}\left(\frac{q_\perp^2}{Q_S^2}-2\right) e^{-\frac{q_\perp^2}{Q_S^2}} \times \int_{\boldsymbol{\Delta}_\perp} \Delta_\perp^2 T^2(\boldsymbol{\Delta}_\perp) \int_{\boldsymbol{\kappa}_\perp} \frac{1}{Q_S^2} e^{-\frac{\kappa_\perp^2}{Q_S^2}} \sim A^{-\frac{5}{6}}$$

Node!

$$f_{1,1}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) \approx \frac{2N_C}{(2\pi)^3 g^2} k_\perp^2 \mathcal{P}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)$$

$$\frac{1}{M^2} g_{1,1}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) \approx \frac{2N_C}{(2\pi)^3 g^2} k_\perp^2 \mathcal{O}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)$$

- Unpolarized cross section (propotional only to pomeron):

$$\mathrm{d}\sigma \sim R^2 \frac{1}{Q_S^2} e^{-\frac{q_\perp^2}{Q_S^2}} \sim A^{\frac{1}{3}}$$

➢ Asymmetry: $A_N \sim \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}\sigma} \sim A^{-\frac{7}{6}}$

Nuclear dependence is the same as in:
Y. V. Kovchegov and M. D. Sievert,
PRD **86**, 034028 (2012).

Conclusion and further tasks:

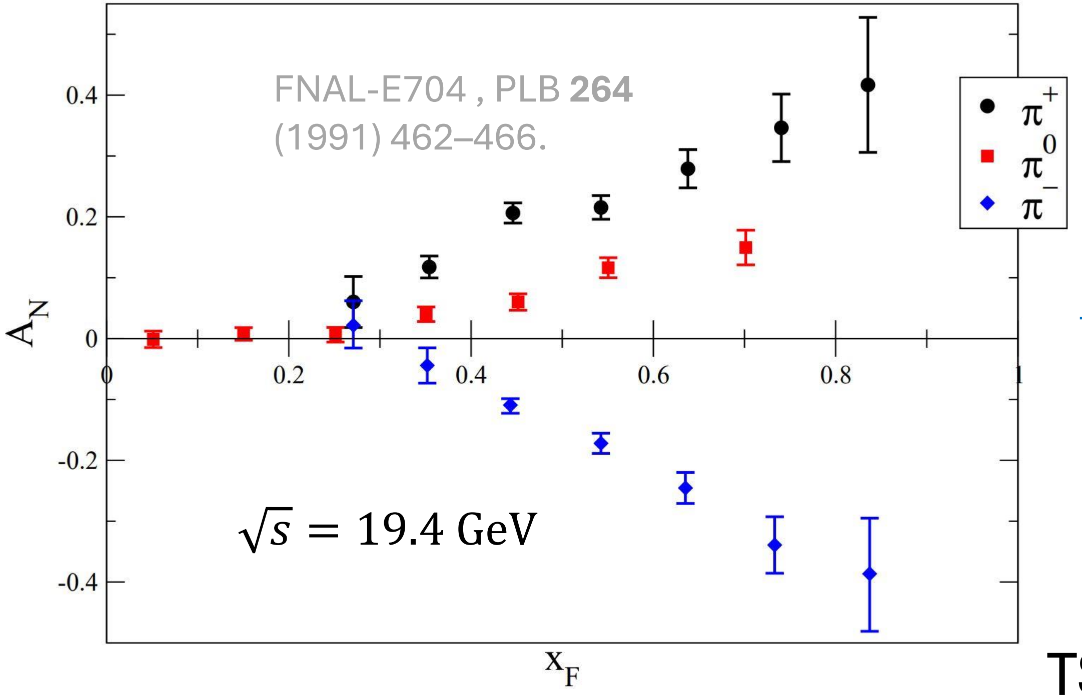
✓We calculated a non-zero odderon contribution to TSSA in $p^{\uparrow}A$

✓We obtained a contribution that comes from the spin-dependent odderon

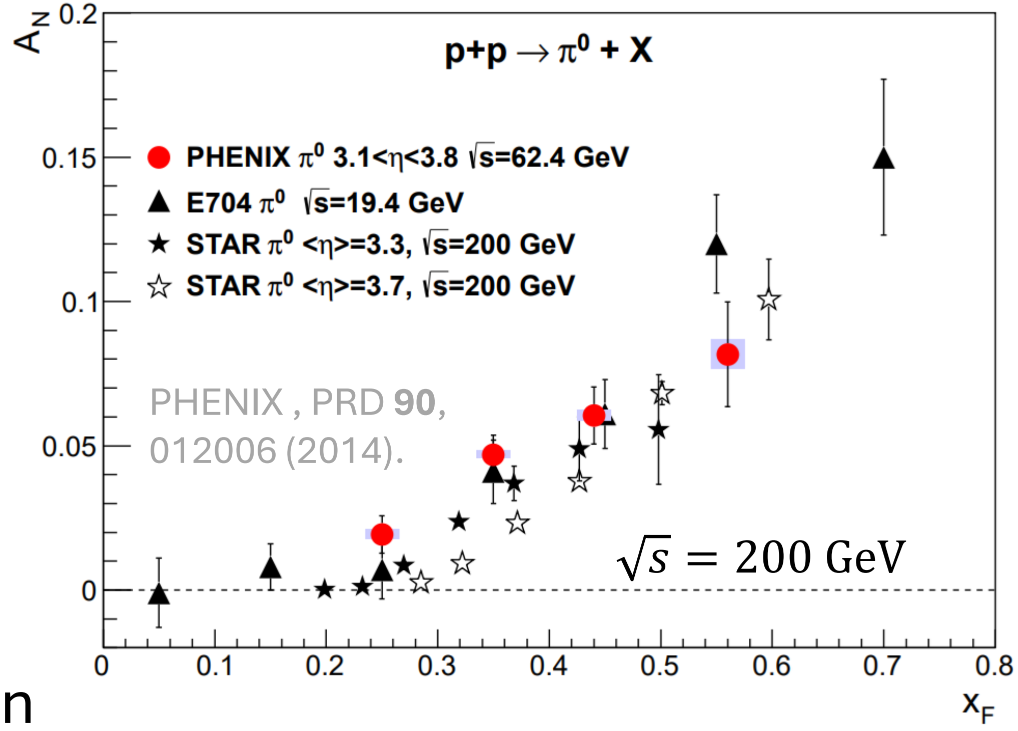
➢Evolution of target distributions and numerical calculation of integrals

➢For the momentum fraction integral we need a model for ETQS function (never obtained in the literature!)

TSSA in $p^{\uparrow}p \to hX$ - experiments



Larger energy



TSSA is largest in forward region!

$$x_F = \frac{P_h^z}{\sqrt{s}}$$

- Measured at different $\sqrt{s}$

Fixed target!
- PRL, **36**, 929 (1976). $\sqrt{s} = 4.9$ GeV
- PRD, **65**, 092008 (2002). $\sqrt{s} = 6.6$ GeV
- PLB, **264**, 462 (1991). $\sqrt{s} = 19.4$ GeV

Collider!
- PRL, **101**, 042001 (2008). $\sqrt{s} = 62.4$ GeV
- PRD, **90**, 012006 (2014). $\sqrt{s} = 200$ GeV