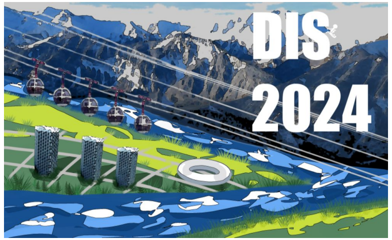
Small-x Quark and Gluon Helicity Contributions to the Proton Spin Puzzle

Yossathorn (Josh) Tawabutr

University of Jyväskylä,
Department of Physics, Centre of Excellence in Quark Matter

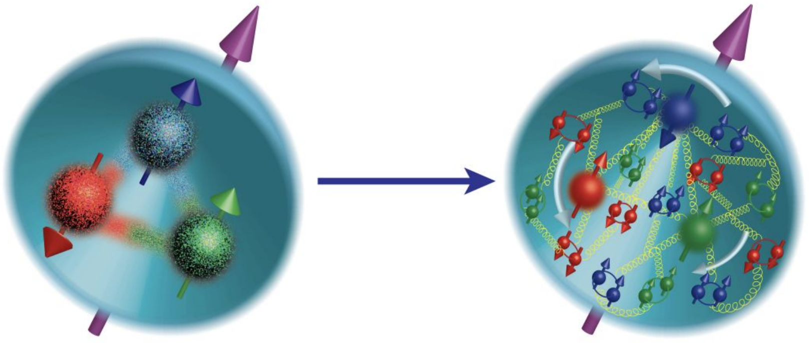
In collaboration with:

Y. Kovchegov, F. Cougoulic, A. Tarasov
D. Pitonyak, M. Sievert, N. Baldonado
D. Adamiak, W. Melnitchouk, N. Sato



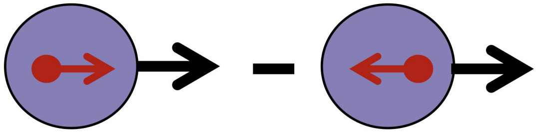
Based on: 2204.11898, 2306.01651, 2308.07461, and earlier publications

Proton Spin



- In the past, proton spin was thought to be the sum of constituent quarks spins.
- Now, we believe it to be the sum of spins of valence quarks, sea quarks and gluons, together with their orbital angular momenta (OAM).

Helicity PDF



- Helicity-dependent generalization of PDFs
- For each parton f,

$$\Delta f(x,Q^2) = f(x,Q^2,+) - f(x,Q^2,-)$$

- For quarks, we often consider the "flavor singlet" quark hPDF:

$$\Delta\Sigma(x,Q^2) = \sum_{q=u,d,s} \left[\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2)\right]$$

and the "flavor non-singlet" quark hPDF: $\Delta q^-(x,Q^2) = \Delta q(x,Q^2) - \Delta\bar{q}(x,Q^2)$

- Gluon hPDF: $\Delta G(x,Q^2)$

Proton Helicity Sum Rule

- Jaffe-Manohar sum rule: $$\frac{1}{2} = S_q + S_G + L_q + L_G$$

  where the helicity of quarks (S_q) and gluons (S_G) are

  $$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\,\Delta\Sigma(x,Q^2) \quad \text{and} \quad S_G(Q^2) = \int_0^1 dx\,\Delta G(x,Q^2)$$

- In the late 1980's, EMC measurement implied that $S_q \approx 0.05$, much lower than what would have been (1/2) had all the proton spin been carried by the constituent quarks.

$$\frac{1}{2} = S_q + S_G + L_q + L_G$$

Current Knowledge of Proton Helicity

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \Delta\Sigma(x, Q^2)$$

$$S_G(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

- More recently, the proton spin carried by quarks and gluon are estimated to be

$$S_q(Q^2 = 10\ \mathrm{GeV}^2) \approx \frac{1}{2}\int_{0.001}^{1} dx\, \Delta\Sigma(x, 10\ \mathrm{GeV}^2) \in [0.15, 0.20]$$

$$S_G(Q^2 = 10\ \mathrm{GeV}^2) \approx \int_{0.05}^{1} dx\, \Delta G(x, 10\ \mathrm{GeV}^2) \in [0.13, 0.26]$$

- They do not add to 1/2. The missing spin can come from:
  - Orbital angular momenta, L_q and L_G.
  - Small-x region of $\Delta\Sigma$ and ΔG. Scattering experiments can only access finitely small x. The limit will improve with EIC.

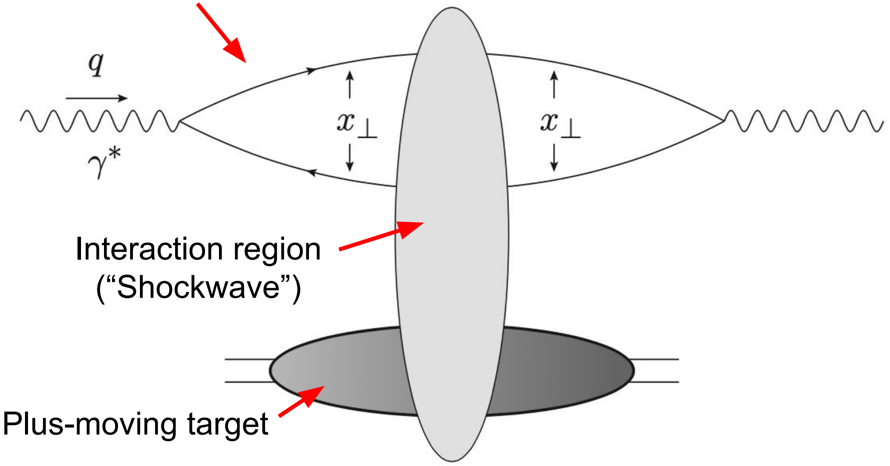
DIS at Small x: The Dipole Picture

- Unpolarized PDF and structure functions, $F_1(x, Q^2)$ and $F_2(x, Q^2)$, relate to the **s-matrix** of dipole-target scattering:

$$S(\underline{x}_1, \underline{x}_0, s) \equiv S_{10}(s) = \frac{1}{N_c}\left\langle \text{tr}\left[V_{\underline{1}} V_{\underline{0}}^{\dagger}\right]\right\rangle(s)$$

Brackets: Averaging over target's state, including spin

Minus-moving dipole



Interaction region ("Shockwave")

Plus-moving target

where

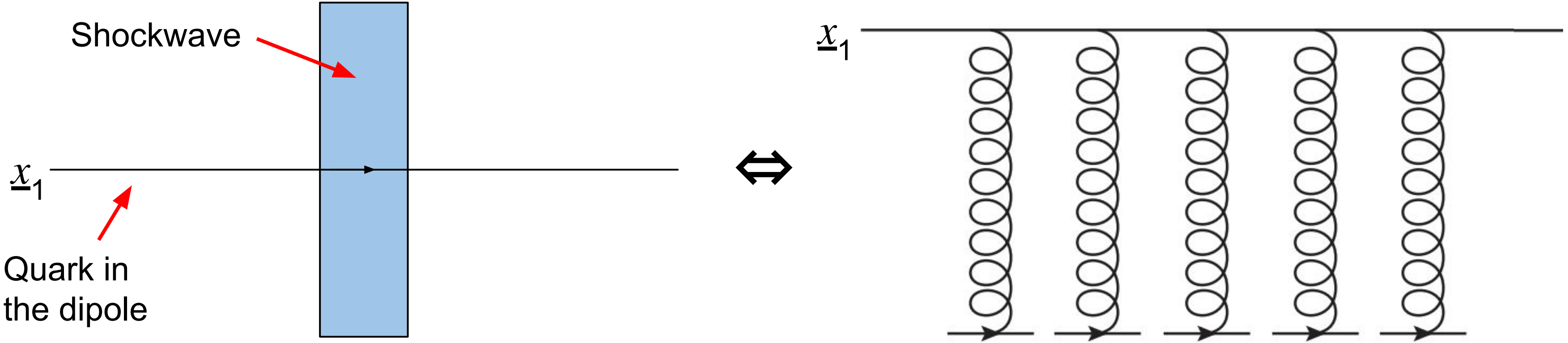
$$V_{\underline{1}}[x_f^-, x_i^-] \equiv V_{\underline{x}_1}[x_f^-, x_i^-] = \mathcal{P}\exp\left[ig\int_{x_i^-}^{x_f^-} dx^- \, A^+(0^+, x^-, \underline{x}_1)\right]$$

$$V_{\underline{1}} \equiv V_{\underline{1}}[\infty, -\infty]$$

Lightcone (unpolarized) Wilson line

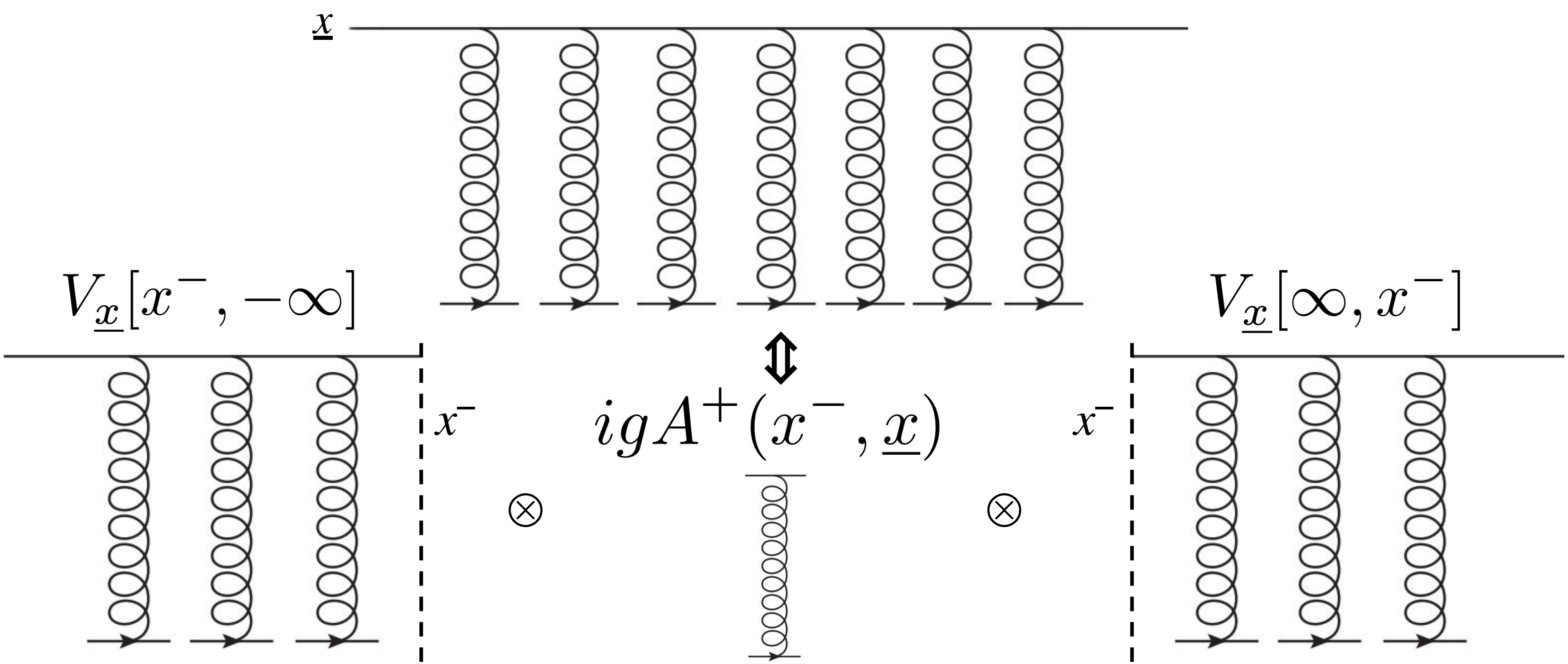
Unpolarized Dipole Amplitude

- Parton **unpolarized PDF,** $\Sigma(x, Q^2)$ and $G(x, Q^2)$, relate to **unpolarized dipole amplitude,** $S_{10}(s) = \frac{1}{N_c}\left\langle \mathrm{tr}\left[V_{\underline{1}} V_{\underline{0}}^{\dagger}\right]\right\rangle(s)$, which obeys BFKL/BK/JIMWLK evolution.
- Quark going through the shockwave at $\underline{x}_1$: unpolarized Wilson line, .
- Multiple parton exchanges at **eikonal** level (leading order in x).



Unpolarized Wilson Line

$$V_{\underline{x}_1}[x_f^-, x_i^-] = \mathcal{P} \exp \left[ig \int_{x_i^-}^{x_f^-} dx^- \, A^+(0^+, x^-, \underline{x}_1) \right]$$

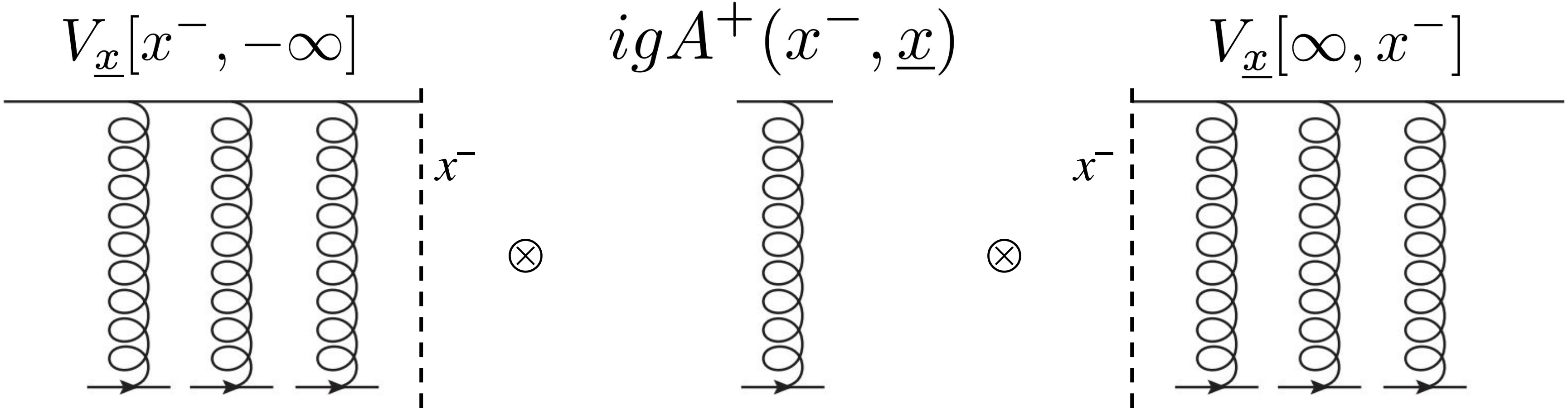


Unpolarized Wilson Line

$$V_{\underline{x}_1}[x_f^-, x_i^-] = \mathcal{P} \exp \left[ig \int_{x_i^-}^{x_f^-} dx^- \, A^+(0^+, x^-, \underline{x}_1) \right]$$

- Eikonal vertex insertion:

$$V_{\underline{x}} = ig \int_{-\infty}^{\infty} dx^- V_{\underline{x}}[\infty, x^-] \, A^+(x^-, \underline{x}) \, V_{\underline{x}}[x^-, -\infty]$$

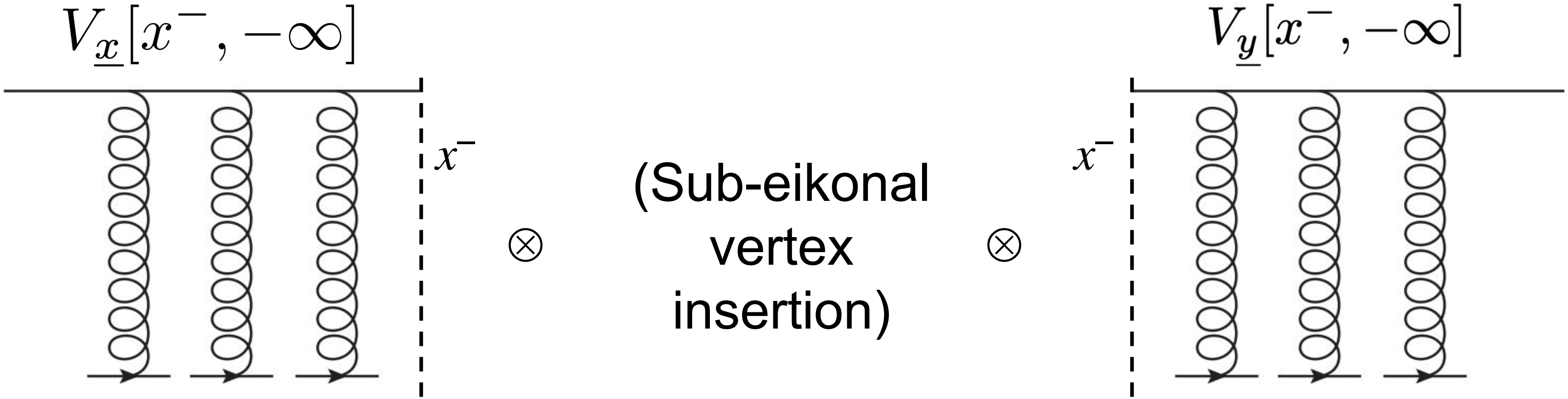


Polarized Wilson Line

$$V_{\underline{x}_1}[x_f^-, x_i^-] = \mathcal{P} \exp \left[ig \int_{x_i^-}^{x_f^-} dx^- \, A^+(0^+, x^-, \underline{x}_1) \right]$$

- Insertion of leading helicity-dependent vertex, which is
  - Sub-eikonal, i.e. (1/s)-suppressed
  - Not necessarily diagonal in transverse position
  - Denoted $V_{\underline{x},\underline{y}}^{\text{pol}}$

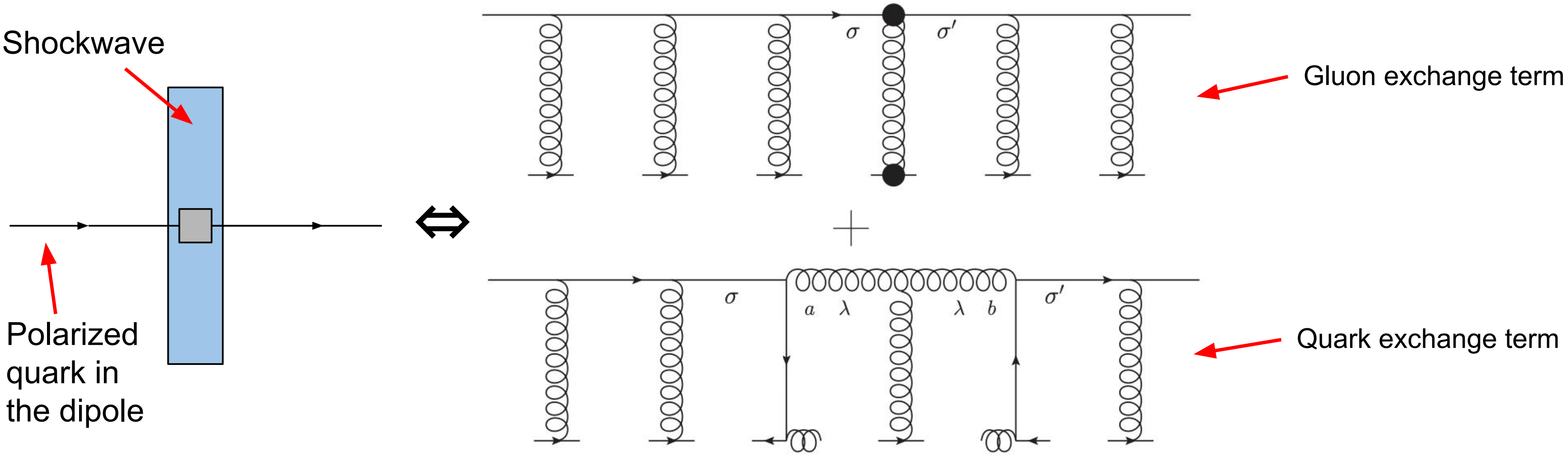
[Cougoulic, Kovchegov (YK), Tarasov, Tawabutr (JT), 2204.11898 & predecessors]



Polarized Wilson Line

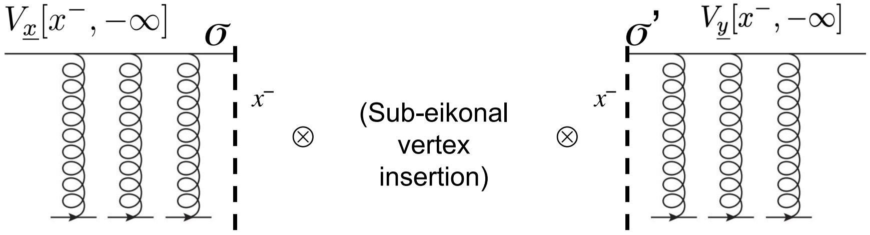
[Cougoulic, Kovchegov (YK), Tarasov, Tawabutr (JT), 2204.11898 & predecessors]

- Helicity-dependent quark line going through the shockwave corresponds to multiple eikonal parton exchanges, except for **one** helicity-dependent exchange, which is **sub-eikonal** (suppressed by an extra factor of x).



[Cougoulic, YK, Tarasov, JT, 2204.11898 & predecessors]

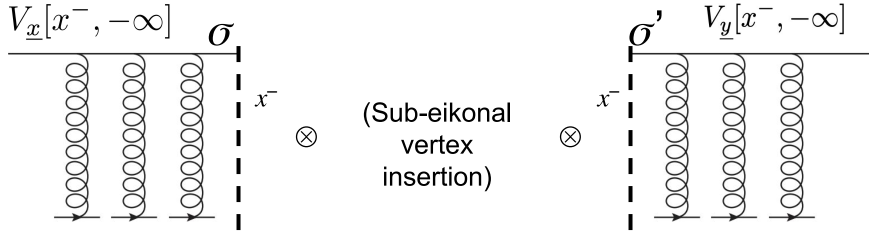
Polarized Wilson Line



Polarized Wilson line	Type 1	Type 2
Helicity structure	$\sigma\delta_{\sigma,\sigma'}$	$\delta_{\sigma,\sigma'}$
Gluon exchange	$\sim F^{12}\,\delta^2(\underline{x}-\underline{y})$	$\sim \overleftarrow{\underline{D}} \cdot \overrightarrow{\underline{D}}$
Quark exchange	$\sim \psi\,(\gamma^+\gamma_5)\,\bar{\psi}\,\delta^2(\underline{x}-\underline{y})$	N/A
Adjoint dipole	$\widetilde{G}(x_{10}, zs)$	$G_2(x_{10}, zs)$
Fundamental dipole	$Q(x_{10}, zs)$	$G_2(x_{10}, zs)$

[Cougoulic, YK, Tarasov, JT, 2204.11898 & predecessors]

Polarized Wilson Line



$$Q(x_{10}, zs) \sim \left\langle \mathrm{tr}\left[V_{\underline{0}} V_{\underline{1}}^{\mathrm{pol}[1]\dagger}\right] + \mathrm{tr}\left[V_{\underline{1}}^{\mathrm{pol}[1]} V_{\underline{0}}^{\dagger}\right]\right\rangle$$

$$\widetilde{G}(x_{10}, zs) \sim \left\langle \mathrm{Tr}\left[U_{\underline{0}} U_{\underline{1}}^{\mathrm{pol}[1]\dagger}\right] + \mathrm{Tr}\left[U_{\underline{1}}^{\mathrm{pol}[1]} U_{\underline{0}}^{\dagger}\right]\right\rangle$$

$$G_2(x_{10}, zs) \sim \left\langle \mathrm{tr}\left[V_{\underline{0}} V_{\underline{1}}^{\mathrm{pol}[2]\dagger}\right] + \mathrm{tr}\left[V_{\underline{1}}^{\mathrm{pol}[2]} V_{\underline{0}}^{\dagger}\right]\right\rangle$$

Brackets now include $\frac{1}{2}\sum_S S$ of proton helicity

Polarized Wilson line	Type 1	Type 2
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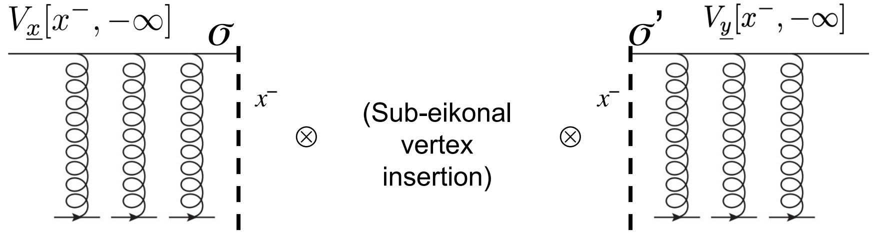

$$\Delta\Sigma(x, Q^2) = -\frac{N_c N_f}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2}, \frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \left[Q(x_{10}^2, zs) + 2\, G_2(x_{10}^2, zs)\right]$$

$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s \pi^2} \left[\left(1 + x_{10}^2 \frac{\partial}{\partial x_{10}^2}\right) G_2\left(x_{10}^2, zs = \frac{Q^2}{x}\right)\right]_{x_{10}^2 = \frac{1}{Q^2}}$$

$$g_1(x, Q^2) = -\sum_f \frac{N_c Z_f^2}{4\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2}, \frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \left[Q(x_{10}^2, zs) + 2\, G_2(x_{10}^2, zs)\right]$$

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Small-x Asymptotics with Quark Exchanges (Large N_c&N_f)

- At small x, gluons dominate → $N_c \gg 1$
- Still important to include quark exchanges (~ N_f / N_c) for helicity evolution
- Flavor non-singlet hPDF:

$$\Delta q^-(x, Q^2) = \Delta q(x, Q^2) - \Delta \bar{q}(x, Q^2) \sim \left(\frac{1}{x}\right)^{\sqrt{\alpha_s N_c/\pi}}$$

[YK, Pitonyak, Sievert, 1610.06197]

- Flavor singlet hPDF:

$$\Delta\Sigma(x, Q^2) = \sum_{q=u,d,s} \left[\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)\right]$$

$$\sim \Delta G(x, Q^2) \sim g_1(x, Q^2) \sim \left(\frac{1}{x}\right)^{3.43\sqrt{\alpha_s N_c/2\pi}}$$

[Adamiak, YK, JT, 2306.01651]

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Smaller than 1

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Smaller than 1

[YK, Pitonyak, Sievert, 1610.06197]

- Flavor singlet hPDF (with N_f = 3):

$$\Delta\Sigma(x, Q^2) = \sum_{q=u,d,s} \left[\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)\right]$$

$$\sim \Delta G(x, Q^2) \sim g_1(x, Q^2) \sim \left(\frac{1}{x}\right)^{3.43\sqrt{\alpha_s N_c/2\pi}}$$

Exceed 1 for $\alpha_s \gtrsim 0.18$

Infinite spin from small x???

[Adamiak, YK, JT, 2306.01651]

Corrections to the DLA Evolution

- So far, helicity evolution resums $\alpha_s \ln^2(1/x)$.
- Potentially significant **single-log corrections,** resumming $\alpha_s \ln(1/x)$.
  - Convoluting with unpolarized dipoles, which obey BK evolution
  - Likely to include saturation mechanism
  - See [YK, Tarasov, JT, 2104.11765] and upcoming work

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- Recently, a **running coupling correction** (daughter dipole prescription) is employed to the DLA evolution in a global fit with polarized DIS & SIDIS data.
- KPS-CTT evolution (with rc) starts at $x_0 = 0.1$.
- At larger x, employ generalized Born-level initial condition:

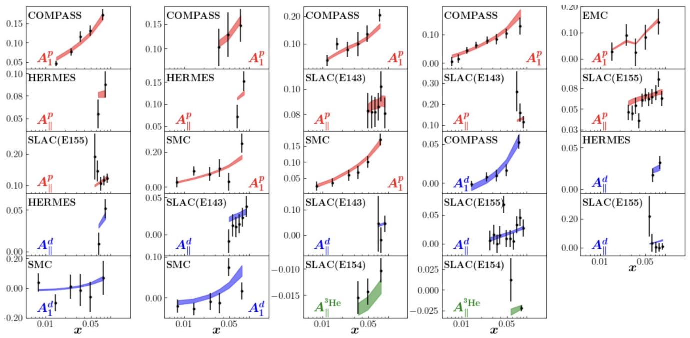
[Adamiak et al, 2308.07461]

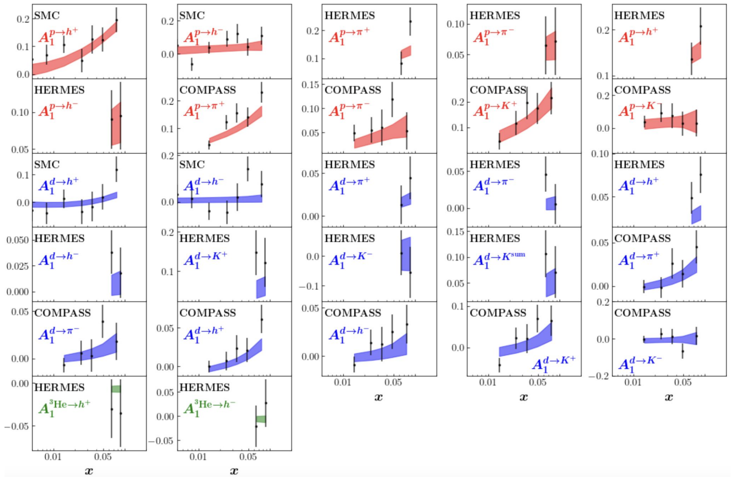
Dipole ~ a ln(rapidity) + b ln(dipole size) + c

Global Fit

[Adamiak et al, 2308.07461]

- Polarized DIS and SIDIS data (A_1, $A_\parallel$, A_1^h) from SLAC, EMC, SMC, COMPASS and HERMES at $0.005 \le x \le 0.1$ and $1.69\ \text{GeV}^2 \le Q^2 \le 10.4\ \text{GeV}^2$.
  - Include proton, deuteron and helium-3 targets
  - For SIDIS, include $\pi^\pm$, $K^\pm$ and unidentified charged hadron productions
- In total, N_{pts} = 226 data points
- Overall, χ^2 / N_{pts} = 1.03





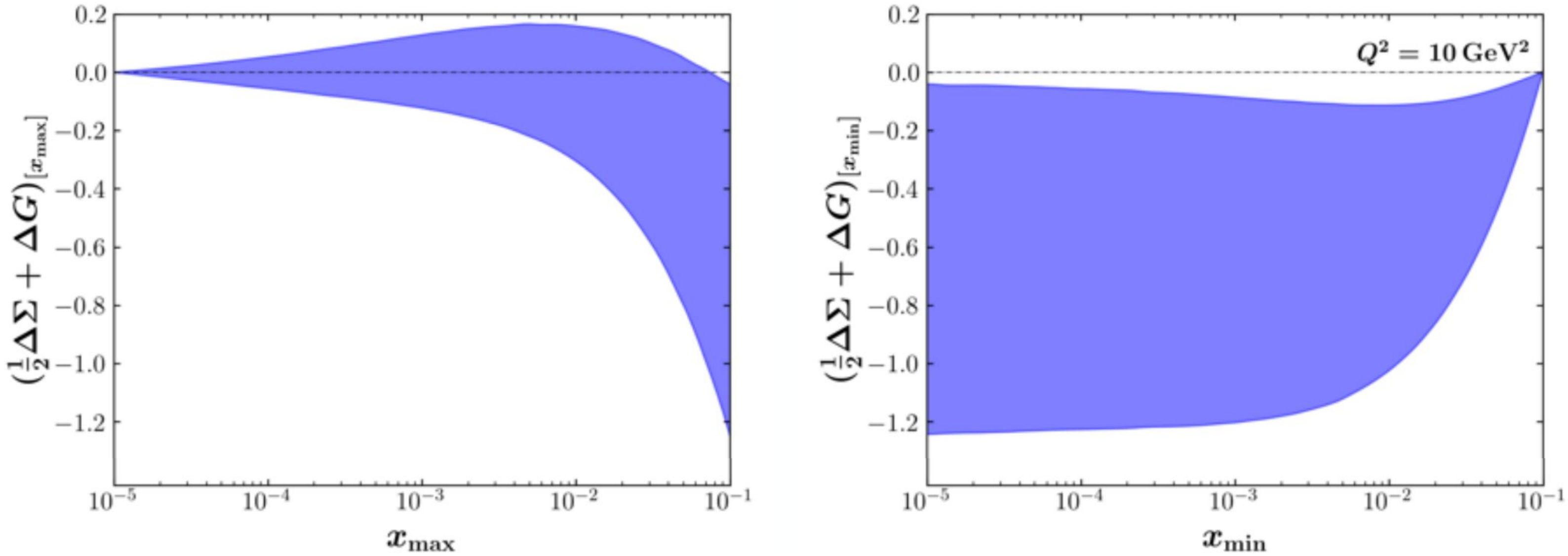
Global Fit

[Adamiak et al, 2308.07461]

$$S_q + S_G \approx \int_{10^{-5}}^{0.1} \mathrm{d}x \left(\frac{1}{2}\Delta\Sigma + \Delta G\right)(x) = -0.64 \pm 0.60$$

Significant spin from small x

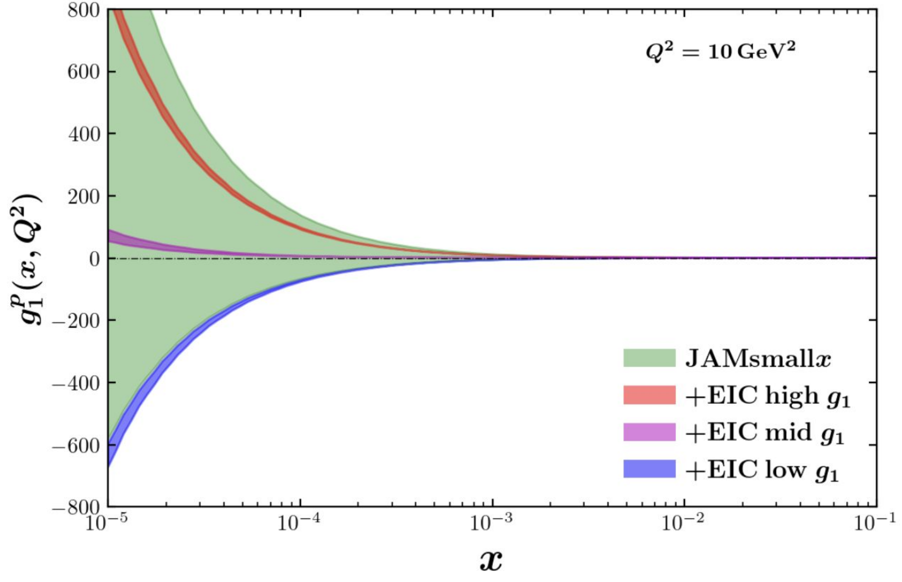
Integrated within [10^{-5}, x_{max}] | Integrated within [x_{min}, 0.1]

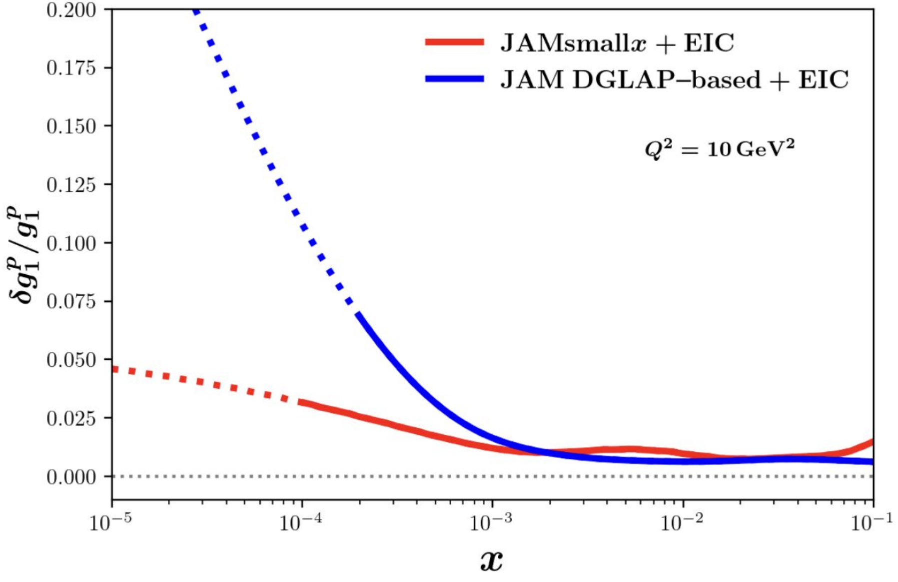


Future EIC Impact

[Adamiak et al, 2308.07461]

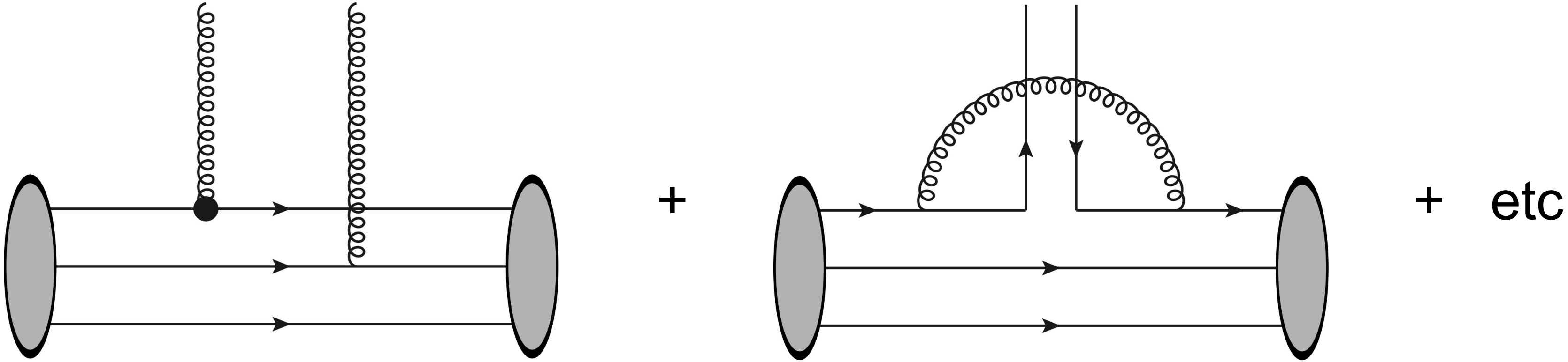
- Significant reduction of uncertainty at small x with future EIC data.





Global Fit: Next Step

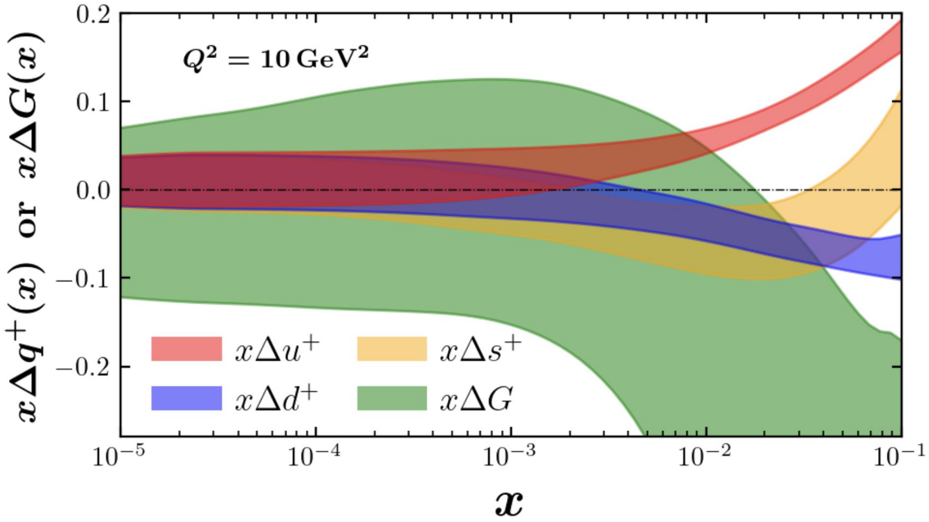
- To allow the data to fix the total helicity, we need a more deterministic IC.
- Model proton target at moderate x with 3 valence quarks, c.f. [Dumitru et al 2010.11245, 2303.16339].
- Stay tuned

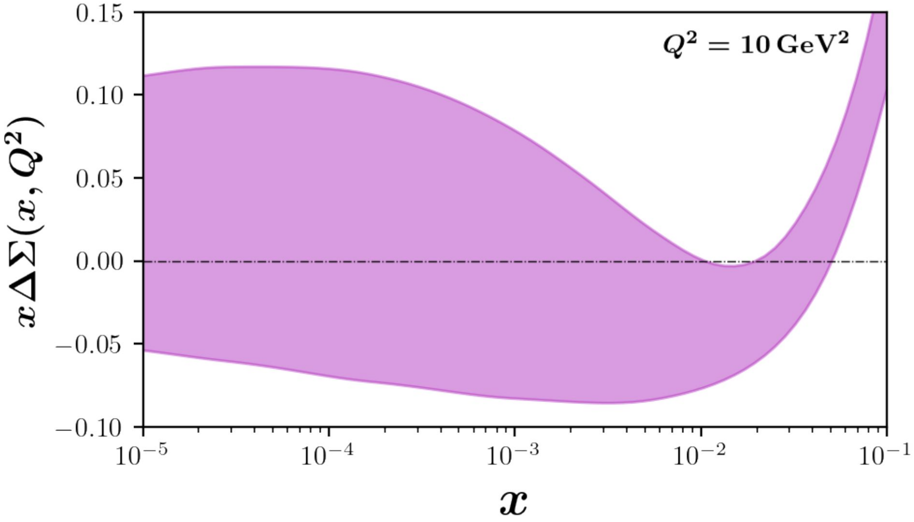


Conclusion

- Already at DLA, KPS-CTT evolution provides a promising small-x description of parton helicity, with potential improvement from future EIC results.
- Future work:
  - More deterministic initial condition using a valence-quark wave function
  - Improved global fit that includes pp particle production data
  - Complete single-logarithmic corrections, which will incorporate saturation
- The framework can be modified to calculate OAM's [YK, Manley, 1901.07453, 2310.18404] and other TMD's [YK, Santiago, 2108.03667, 2209.03538, 2310.02231].
- The framework has been generalized to helicity-JIMWLK evolution and helicity-dependent extension to MV model [Cougoulic, YK, 1910.04268, 2005.14688].

Global Fit: hPDF Results





Global Fit: Data Points

Data set (A_1)	Target	N_{pts}	χ^2/N_{pts}
SLAC (E142) [141]	^{3}He	1	0.60
EMC [146]	p	5	0.20
SMC [147, 149]	p	6	1.29
	p	6	0.53
	d	6	0.67
	d	6	2.26
COMPASS [150]	p	5	1.02
COMPASS [151]	p	17	0.74
COMPASS [152]	d	5	0.88
HERMES [153]	n	2	0.73
Total		59	0.91

Data set ($A_\parallel$)	Target	N_{pts}	χ^2/N_{pts}
SLAC(E155) [144]	p	16	1.28
	d	16	1.62
SLAC (E143) [143]	p	9	0.56
	d	9	0.92
SLAC (E154) [142]	^{3}He	5	1.09
HERMES [154]	p	4	1.54
	d	4	0.98
Total		63	1.19

Dataset (A_1^h)	Target	Tagged Hadron	N_{pts}	χ^2/N_{pts}
SMC [148]	p	h^+	7	1.03
	p	h^-	7	1.45
	d	h^+	7	0.82
	d	h^-	7	1.49
HERMES [158]	p	π^+	2	2.39
	p	π^-	2	0.01
	p	h^+	2	0.79
	p	h^-	2	0.05
	d	π^+	2	0.47
	d	π^-	2	1.40
	d	h^+	2	2.84
	d	h^-	2	1.22
	d	K^+	2	1.81
	d	K^-	2	0.27
	d	$K^+ + K^-$	2	0.97
HERMES [159]	^{3}He	h^+	2	0.49
	^{3}He	h^-	2	0.29
COMPASS [156]	p	π^+	5	1.88
	p	π^-	5	1.10
	p	K^+	5	0.42
	p	K^-	5	0.31
COMPASS [157]	d	π^+	5	0.50
	d	π^-	5	0.78
	d	h^+	5	0.90
	d	h^-	5	0.86
	d	K^+	5	1.50
	d	K^-	5	0.78
Total			104	1.01