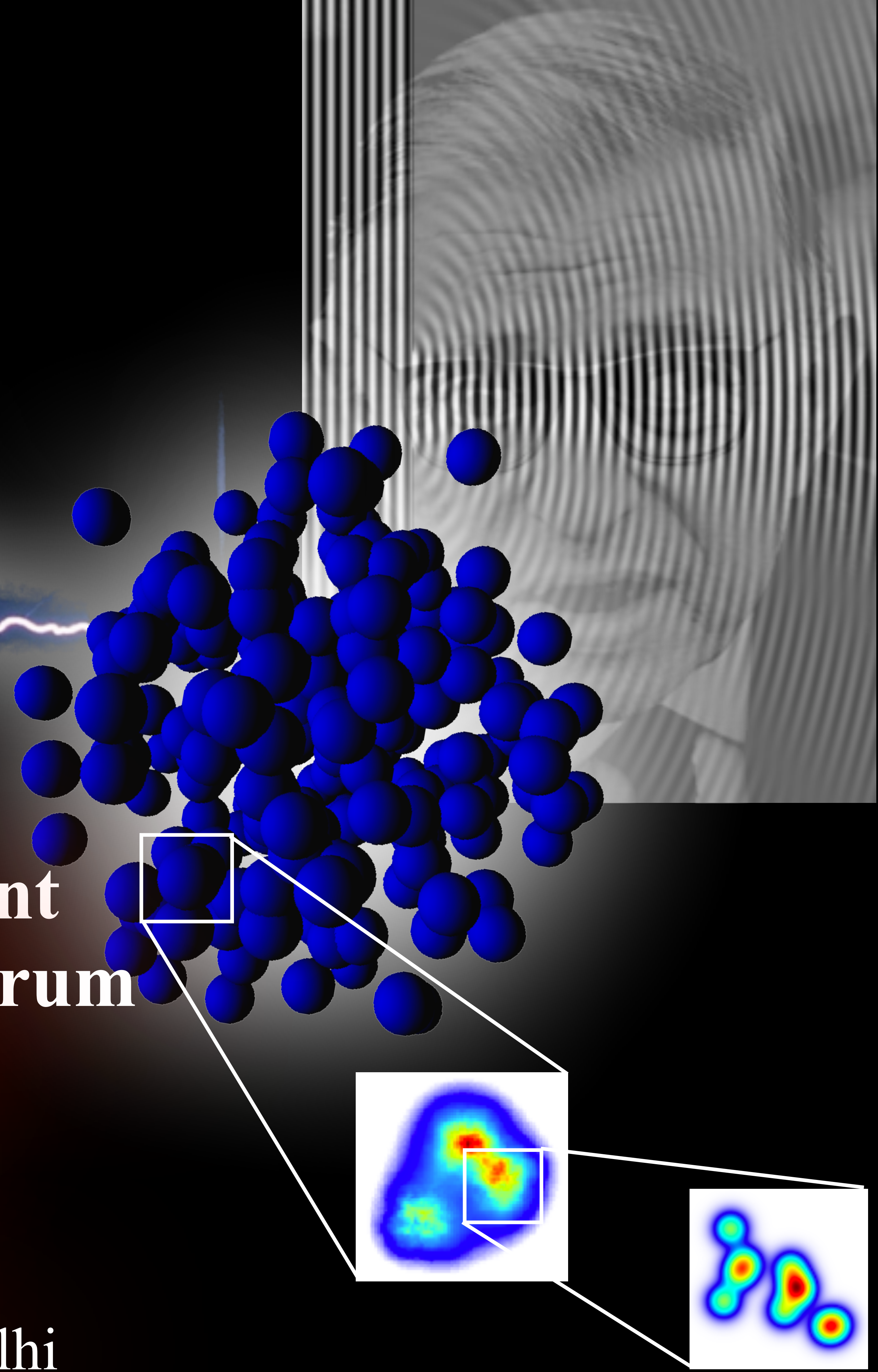
“Describing the incoherent exclusive diffraction *t*-spectrum with hotspot evolution”

DIS2024, Grenoble

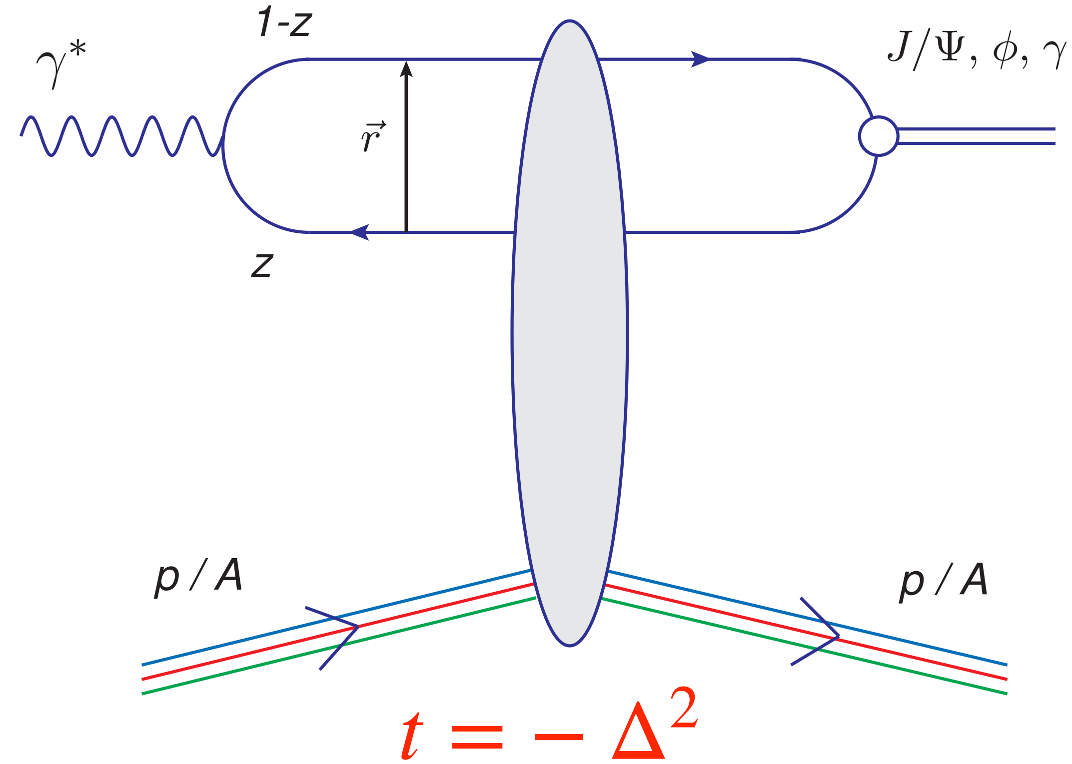
April 10, 2024

Tobias Toll with Arjun Kumar
Indian Institute of Technology Delhi



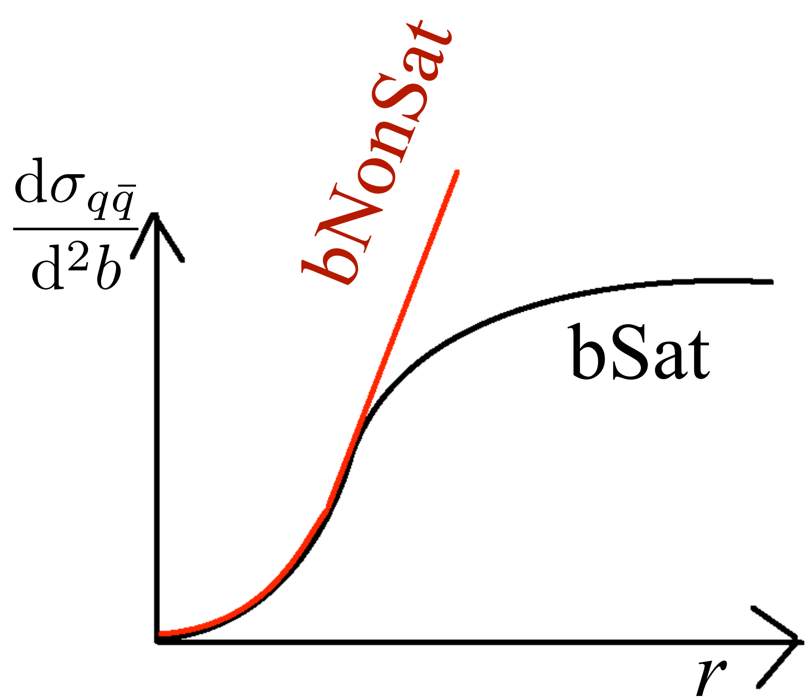
H. Kowalski, L. Motyka, G. Watt, Phys.Rev.D 74 (2006) 074016, arXiv: hep-ph/0606272

Exclusive diffraction in the Dipole Model



$$\frac{d\sigma^{\gamma^* p \to Vp}}{dt} = \frac{1}{16\pi} \left| \mathscr{A}_{T,L}^{\gamma^* p \to Vp} \right|^2$$

$$\mathscr{A}_{T,L}^{\gamma^* p \to Vp}(x_{I\!P}, Q^2, \Delta) = i \int 2\pi r \, dr \int \frac{dz}{4\pi} \int d^2\vec{b} \left(\Psi_V^* \Psi\right)(r, z) J_0([1-z] r\Delta) e^{-\vec{b}\cdot\vec{\Delta}} \frac{d\sigma_{q\bar{q}}}{d^2\vec{b}}(x_{I\!P}, r, \vec{b})$$



$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)\right)\right]$$

$$\frac{d\sigma_{q\bar{q}}^{\text{nosat}}}{d\mathbf{b}_2} = \frac{\pi^2}{N_C} r^2 \alpha_S(\mu^2) x g(x, \mu^2) T(b)$$

H. Kowalski, L. Motyka, G. Watt, Phys.Rev.D 74 (2006) 074016, arXiv: hep-ph/0606272

Exclusive diffraction in the Dipole Model

$$\mathcal{A}_{T,L}^{\gamma^* p \to Vp}(x_{I\!P}, Q^2, \Delta) = i \int 2\pi r \mathrm{d}r \int \frac{\mathrm{d}z}{4\pi} \int \mathrm{d}^2\vec{b} \left(\Psi_V^* \Psi\right)(r,z) J_0([0.5 - z] r\Delta) e^{-\vec{b}\cdot\vec{\Delta}} \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, \vec{b})$$

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)\right)\right] = 2\left[1 - \exp\left(-\frac{\Omega}{2}\right)\right]$$

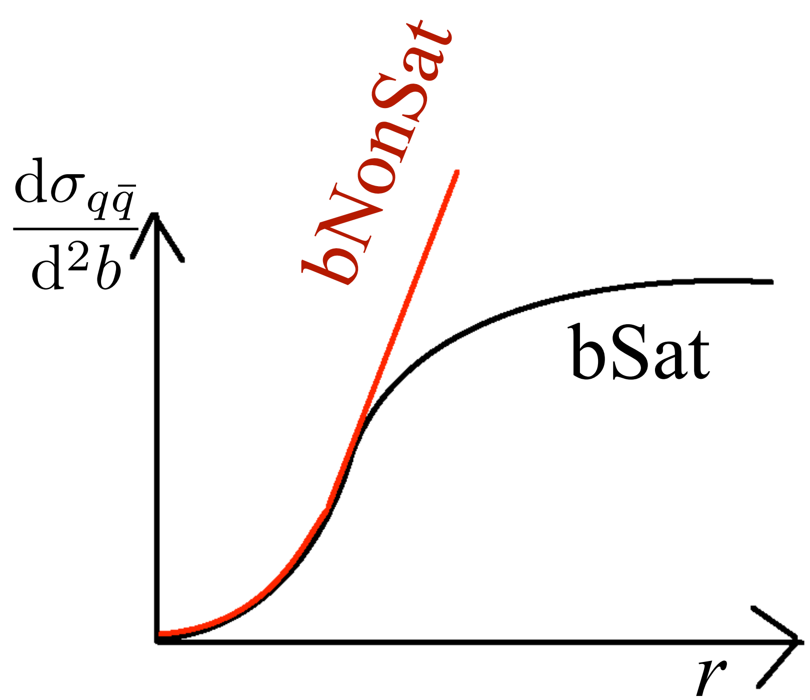
Saturation Scale:

$$Q_S^2 = \frac{2}{r_S^2}$$

$$1 = \frac{\pi^2}{N_C} {r_S}^2 \alpha_S(\mu^2(r_S)) x g(x, \mu^2(r_S)) T(b)$$

$$\mu^2(r) = \frac{C}{r^2} + \mu_0^2$$

$$Q_S^2 \simeq T(b)$$

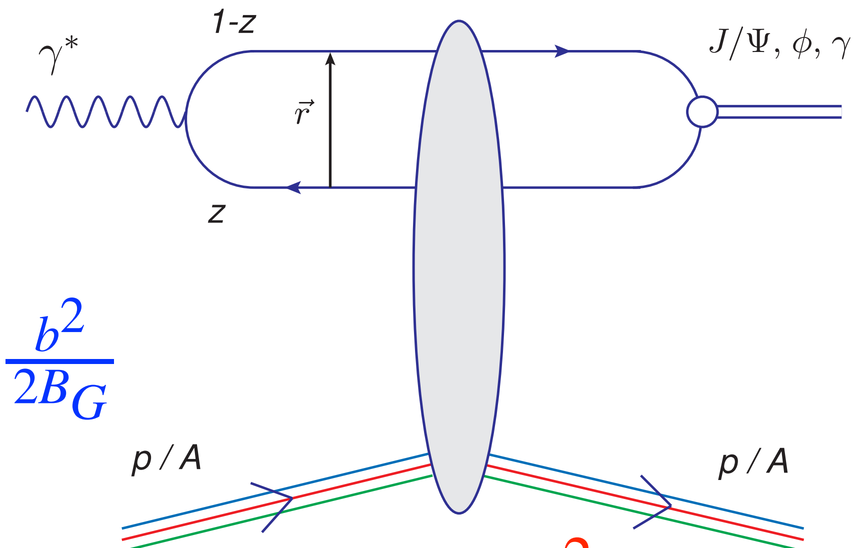


$$\frac{\mathrm{d}\sigma_{\mathrm{q\bar{q}}}^{\mathrm{nosat}}}{\mathrm{d}\mathbf{b}} = \frac{\pi^2}{N_C} r^2 \alpha_{\mathrm{S}}(\mu^2) x g(x, \mu^2) T(b)$$

H. Kowalski, L. Motyka, G. Watt, Phys.Rev.D 74 (2006) 074016, arXiv: hep-ph/0606272

Exclusive diffraction in the Dipole Model

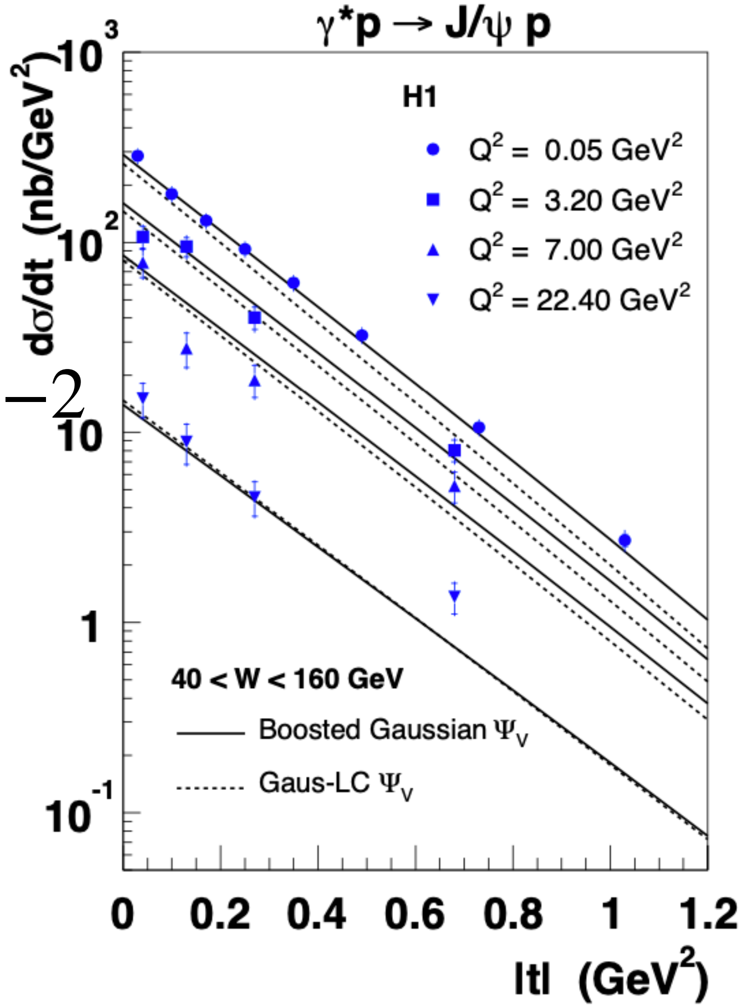
$$\frac{d\sigma_{\text{nosat}}}{dt} \propto \left| \mathscr{F}\text{ourier}(T(b)) \right|^2$$

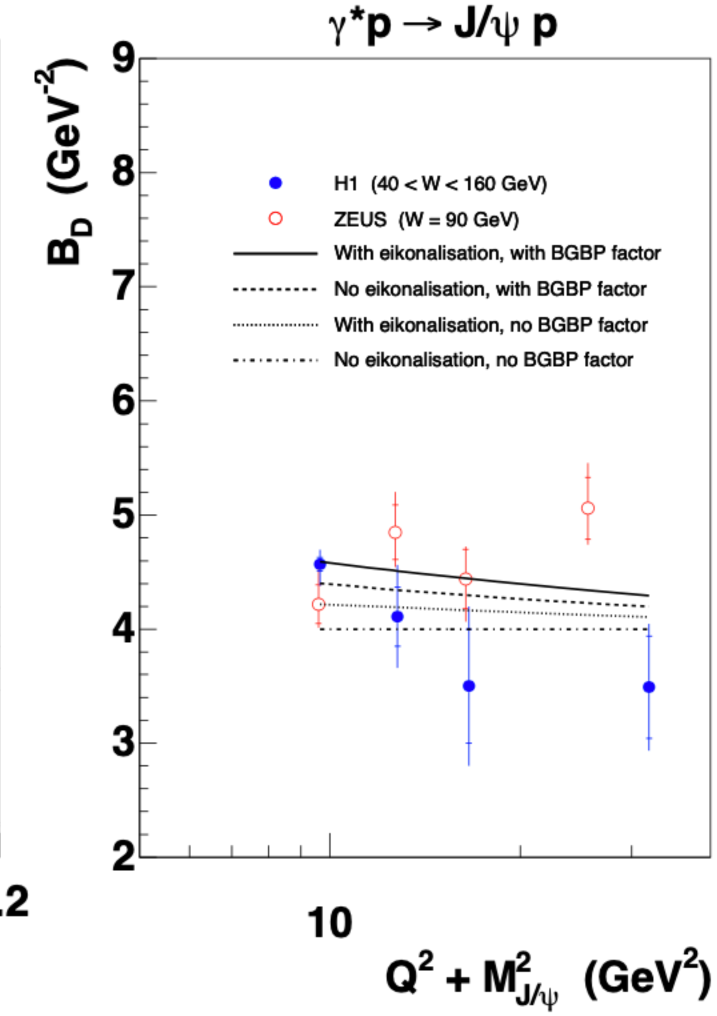


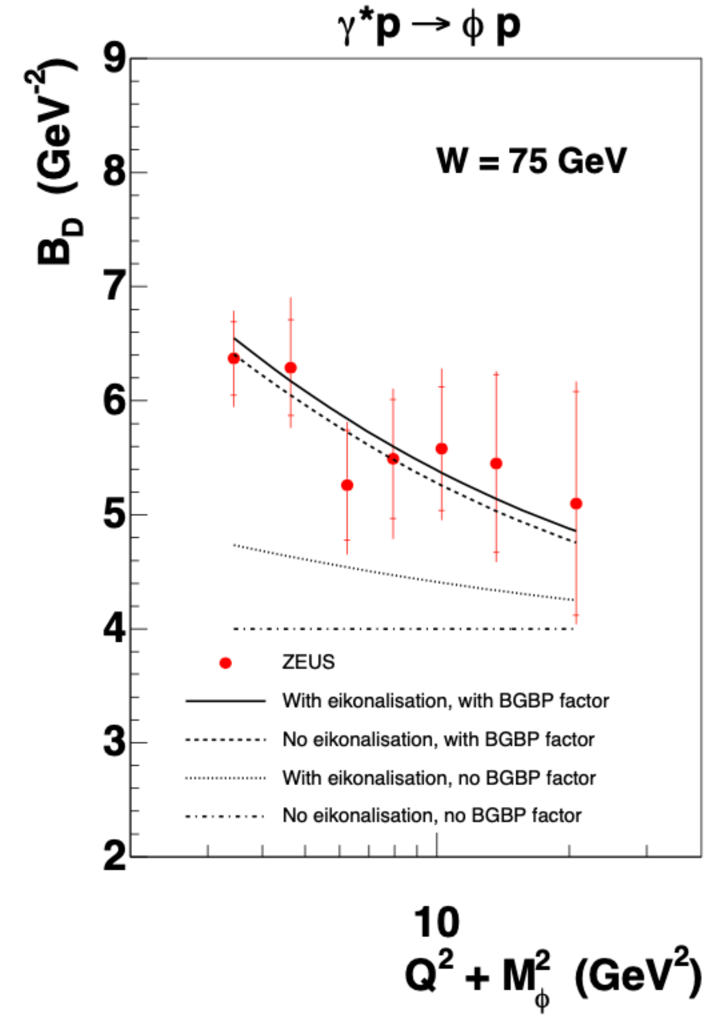
$$\frac{d\sigma}{dt} \propto e^{Bt} = e^{-B\Delta^2} \qquad T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

$$t = -\Delta^2$$

$$B_G = 4\ \text{GeV}^{-2}$$







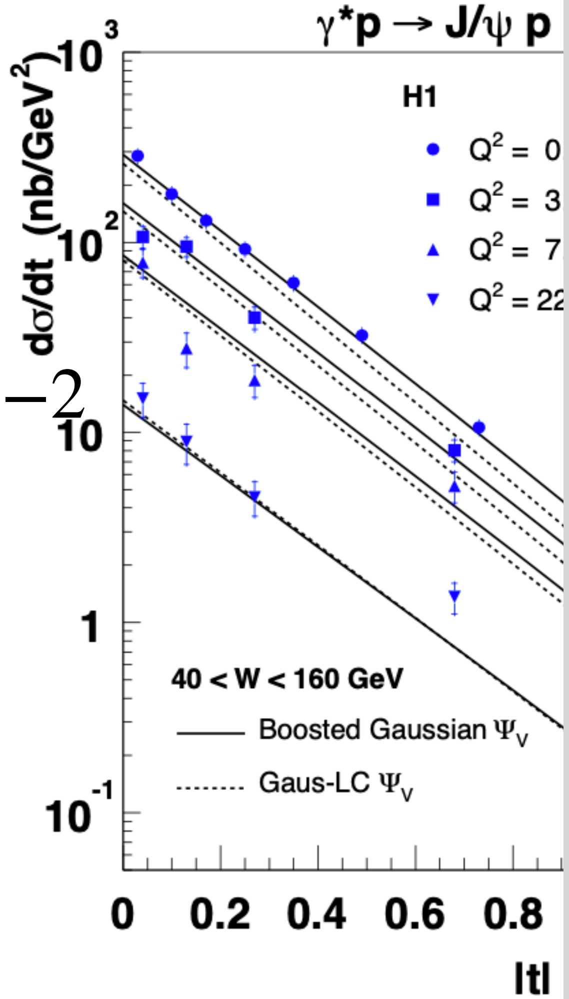
H. Kowalski, L. Motyka, G. Watt, Phys.Rev.D 74 (2006) 074016, arXiv: hep-ph/0606272

Exclusive diffrac

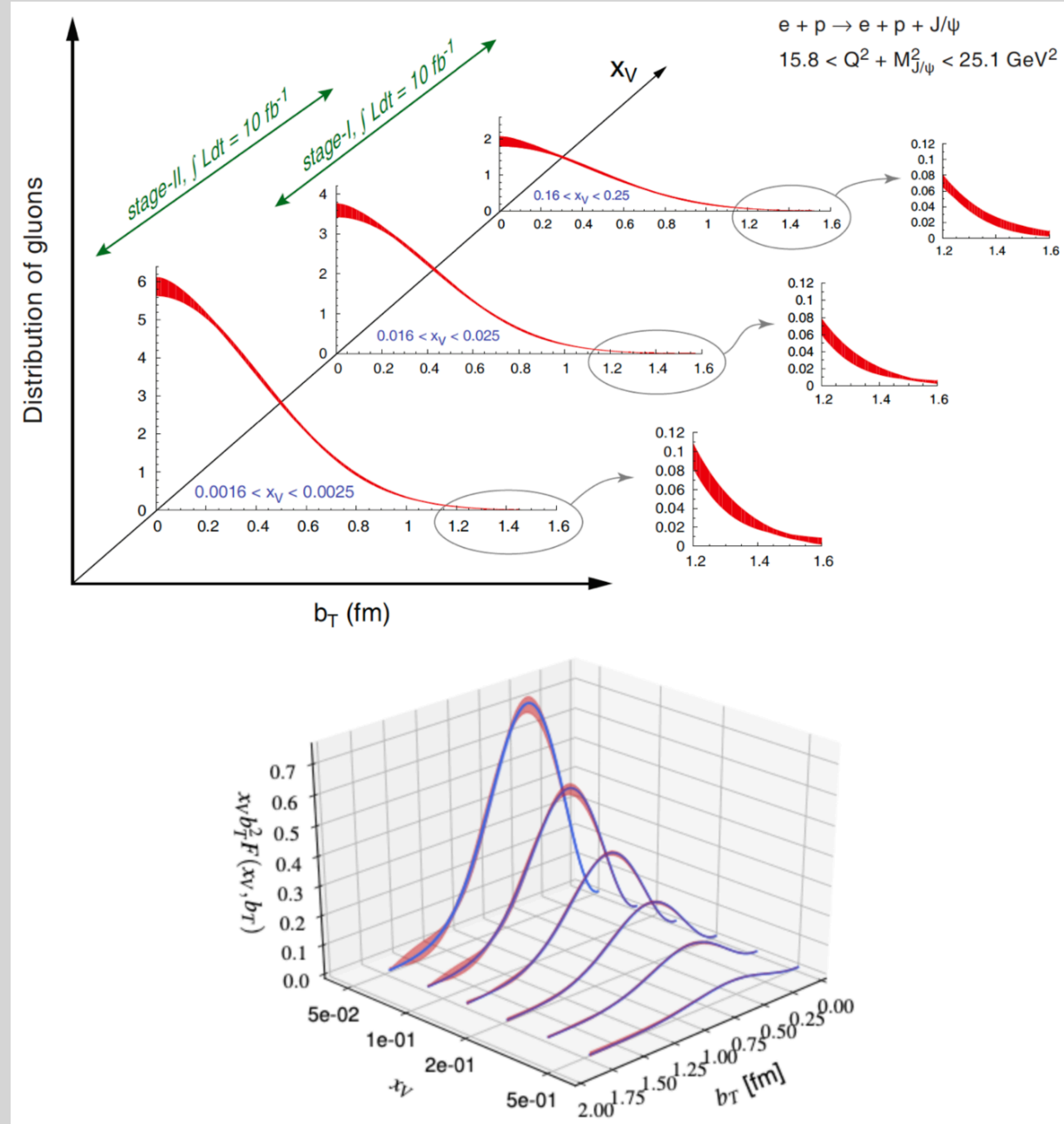
$$\frac{d\sigma_{\rm nosat}}{dt} \propto \Big| \mathscr{F}\text{ourier}(T$$

$$\frac{d\sigma}{dt} \propto e^{Bt} = e^{-B\Delta^2} \qquad T_p$$

$$B_G = 4\ \mathrm{GeV}^{-2}$$



GPDs with EIC:

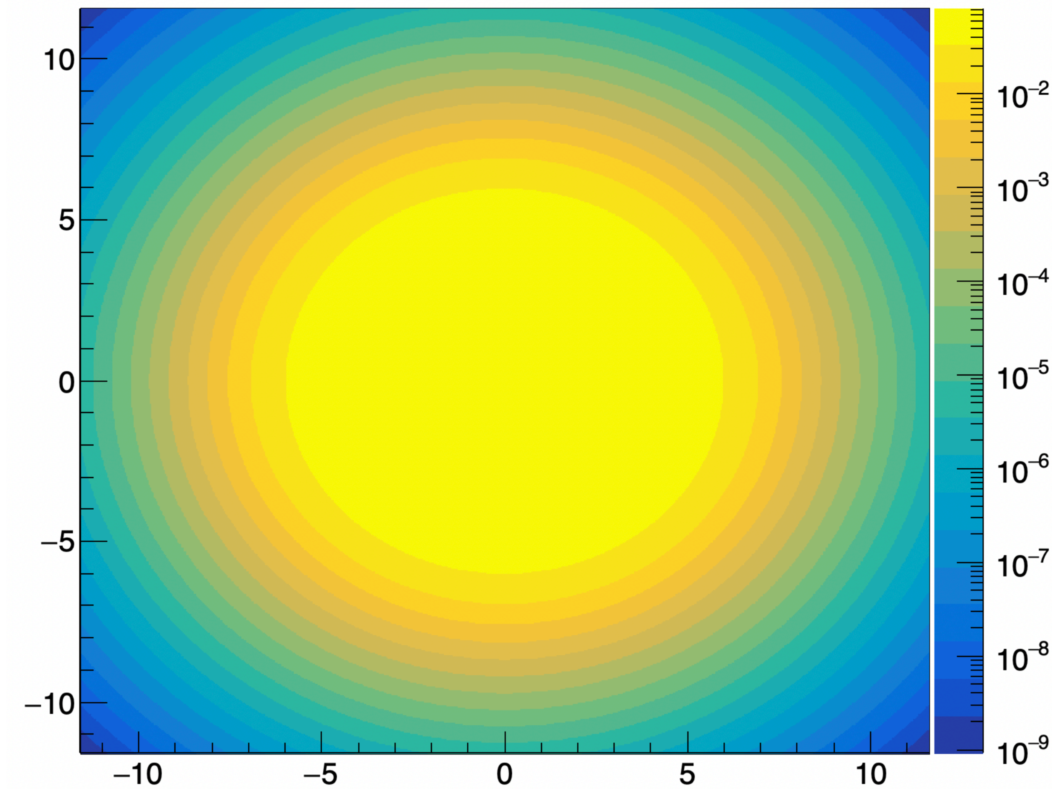


Nucl. Phys. A 1026 (2022) ; arXiv:2103.05419

The nucleus thickness

Naively, use a Woods-Saxon distribution:

$$T_A(\vec{b}) = \int \mathrm{d}z \frac{\rho_0}{1 + \exp\left(\frac{\sqrt{\vec{b}^2 + z^2} - R_0}{d}\right)}$$



$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)\right)\right]$$

Nuclear PDF

The nucleus thickness

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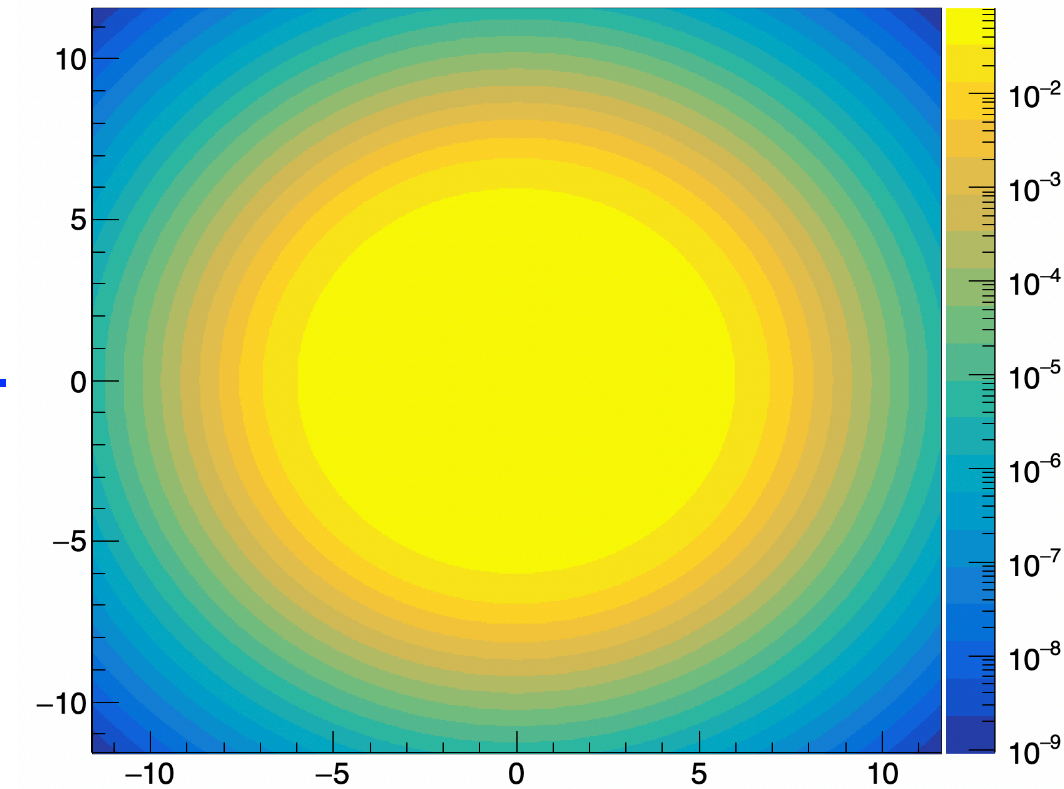


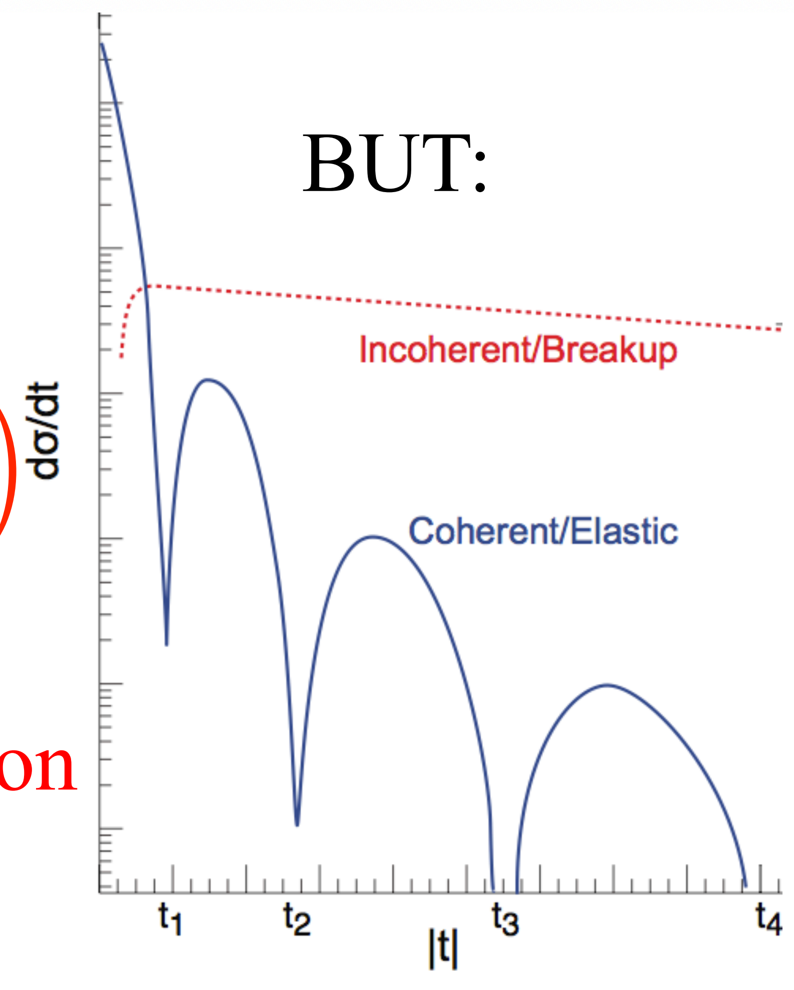
$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)\right)\right]$$

BUT:

$$\frac{d\sigma}{dt} = \frac{1}{16\pi}\left\langle |\mathscr{A}|^2 \right\rangle = \frac{1}{16\pi}\left|\langle \mathscr{A} \rangle\right|^2 + \frac{1}{16}\left(\left\langle |\mathscr{A}|^2 \right\rangle - \left|\langle \mathscr{A} \rangle\right|^2\right)$$

Coherent Incoherent

Averaged over initial state gluon distribution

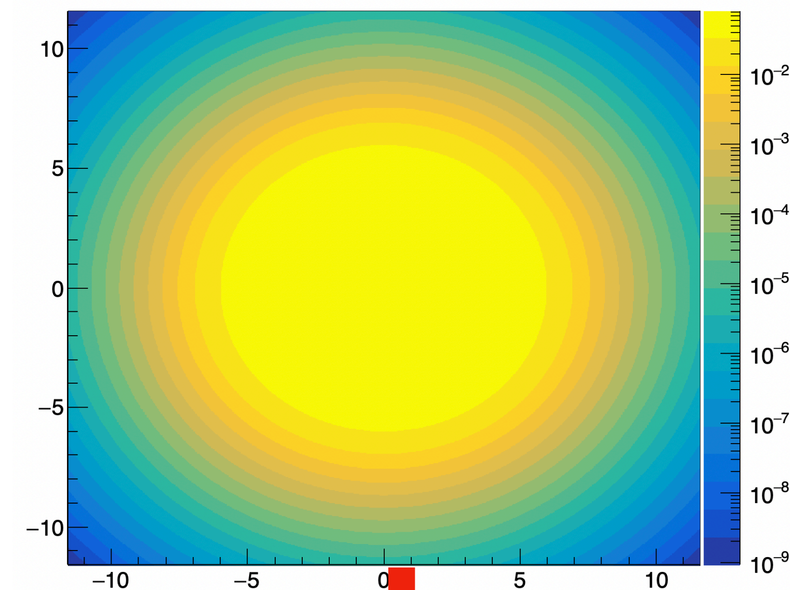


The nucleus as a collection of nucleons

TT, Thomas Ullrich
Phys.Rev.C 87 (2013) 2, 024913, arXiv: 1211.3048
Comput.Phys.Commun. 185 (2014) 1835-1853 arXiv:1307.8059

Independent scattering approximations:

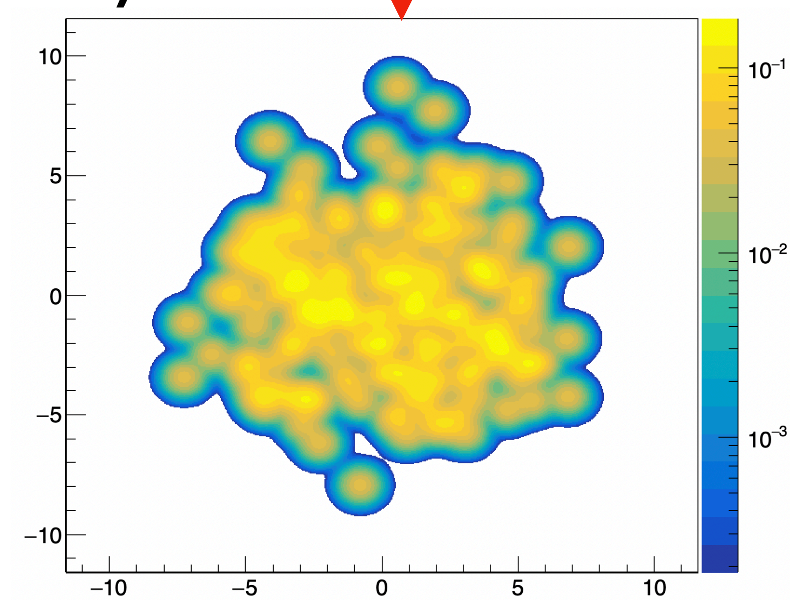
$$1 - \frac{1}{2}\frac{\mathrm{d}\sigma^{(A)}_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, \vec{b}) = \prod_{i=1}^{A}\left(1 - \frac{1}{2}\frac{\mathrm{d}\sigma^{(p)}_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, |\vec{b} - \vec{b}_i|)\right)$$



Proton PDF

$$\frac{1}{2}\frac{\mathrm{d}\sigma^{(A)}_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, \vec{b}) = 1 - \exp\left(\frac{\pi^2}{2N_C} r^2 \alpha_S(\mu^2) xg(x, \mu^2) \sum_{i=1}^{A} T_p(|\vec{b} - \vec{b}_i|)\right)$$

$$T_A(\vec{b}) = \int \mathrm{d}z \frac{\rho_0}{1 + \exp\left(\frac{\sqrt{\vec{b}^2 + z^2} - R_0}{d}\right)}$$



$$T_A(\vec{b}) = \sum_{i=1}^{A} T_p(|\vec{b} - \vec{b}_i|) \qquad T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

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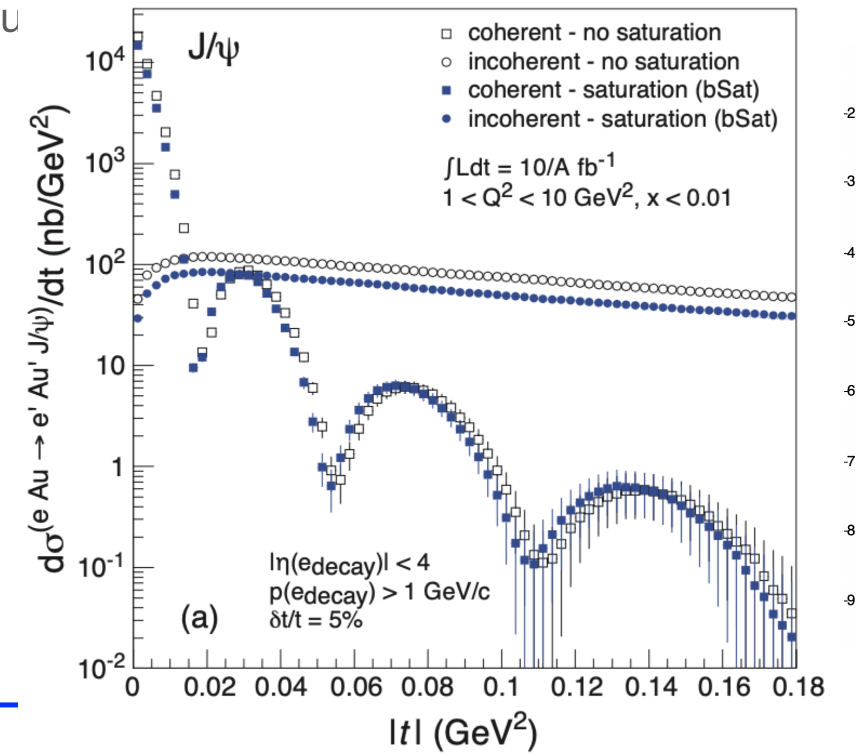
Comput.Phys.Commu

Independent scattering approximations:

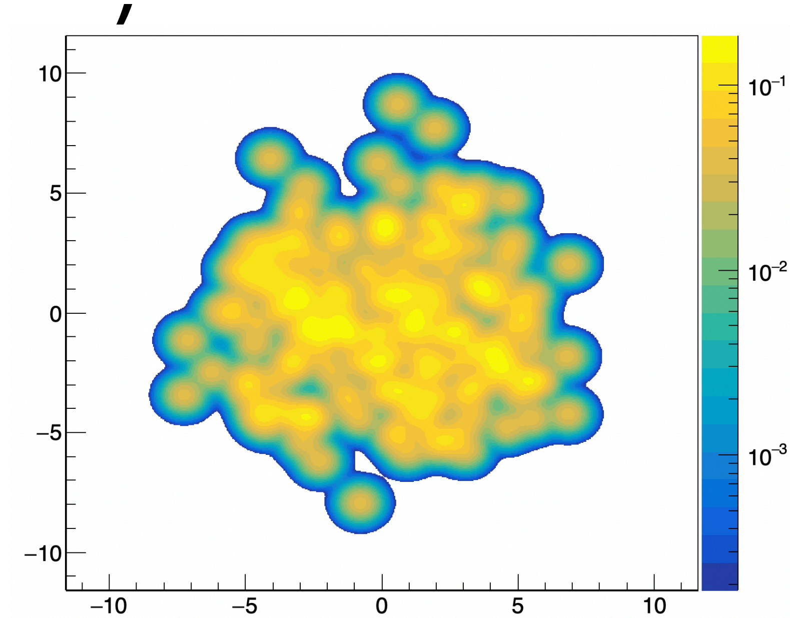
$$1-\frac{1}{2}\frac{d\sigma^{(A)}_{q\bar{q}}}{d^2\vec{b}}(x_{I\!P},r,\vec{b})=\prod_{i=1}^{A}\left(1-\frac{1}{2}\frac{d\sigma^{(p)}_{q\bar{q}}}{d^2\vec{b}}(x_{I\!P},r,|\vec{b}-\vec{b}_i|)\right)$$

Proton PDF

$$\frac{1}{2}\frac{d\sigma^{(A)}_{q\bar{q}}}{d^2\vec{b}}(x_{I\!P},r,\vec{b})=1-\exp\left(\frac{\pi^2}{2N_C}r^2\alpha_S(\mu^2)xg(x,\mu^2)\sum_{i=1}^{A}T_p(|\vec{b}-\right.$$



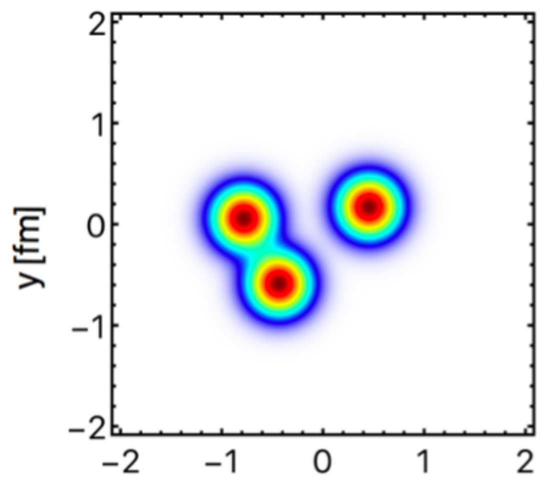
$$T_A(\vec{b})=\int dz\frac{\rho_0}{1+\exp\left(\frac{\sqrt{\vec{b}^2+z^2}-R_0}{d}\right)}$$



$$T_A(\vec{b})=\sum_{i=1}^{A}T_p(|\vec{b}-\vec{b}_i|)\qquad T_p(b)=\frac{1}{2\pi B_G}e^{-\frac{b^2}{2B_G}}$$

Hotspot model for incoherent *ep*-scattering

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x,\mu^2) T(b)\right)\right]$$

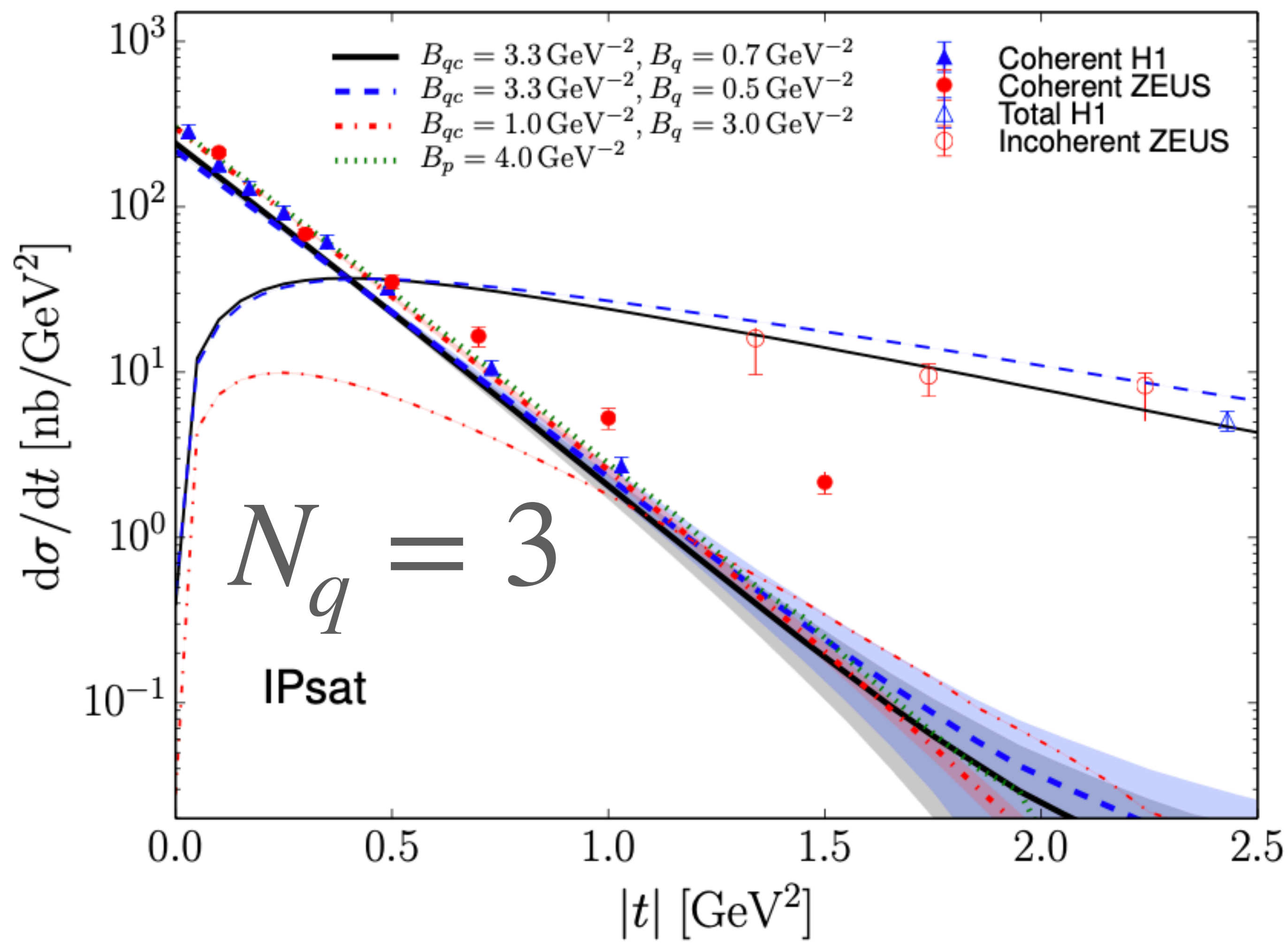


$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

↓

$$T_p(b) = \frac{1}{2\pi N_q B_q} \sum_{i=1}^{N_q} e^{-\frac{(\vec{b} - \vec{b}_i)^2}{2B_q}}$$

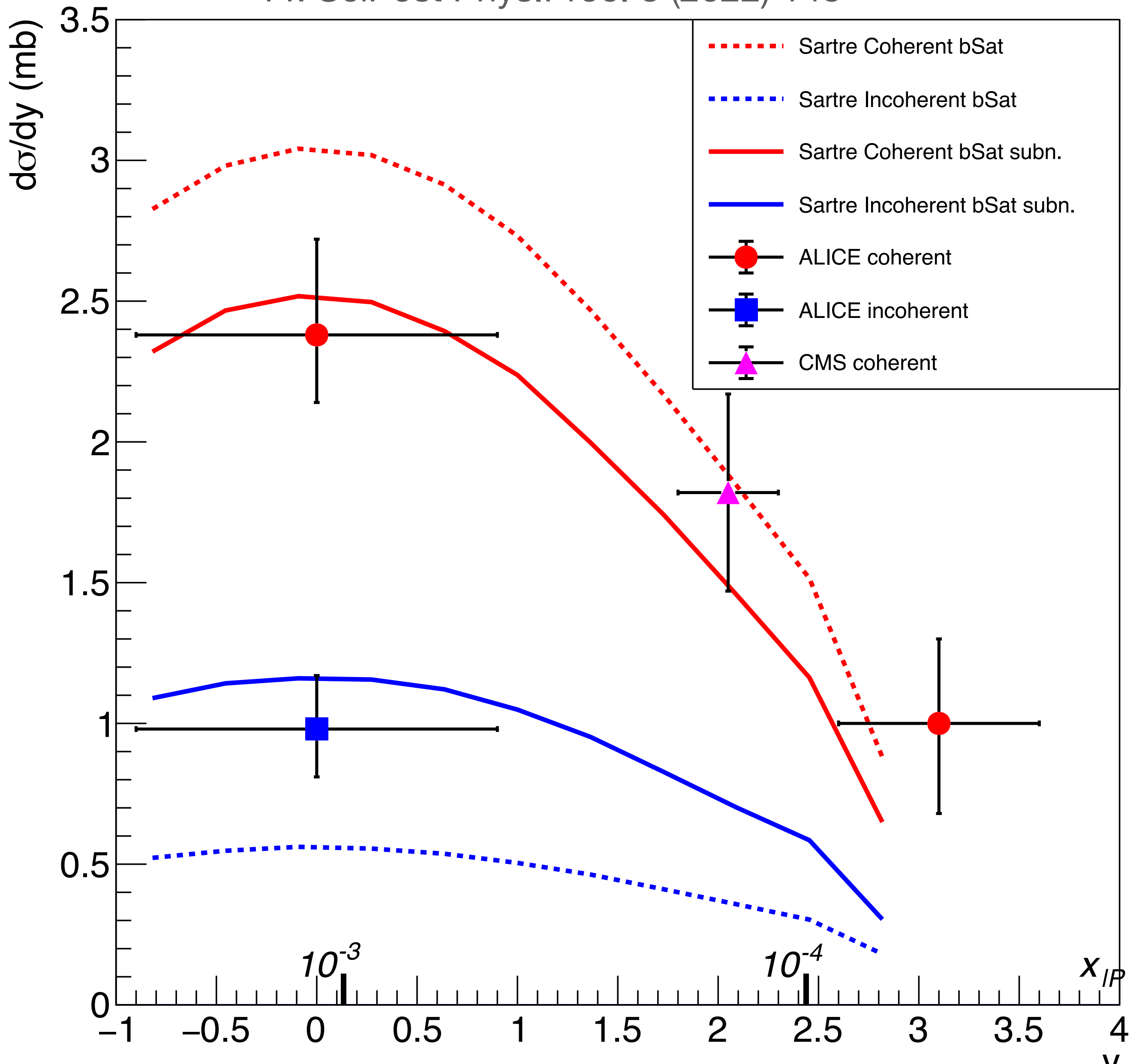
$\vec{b}_i$ with a Gaussian distribution of width B_{qc}



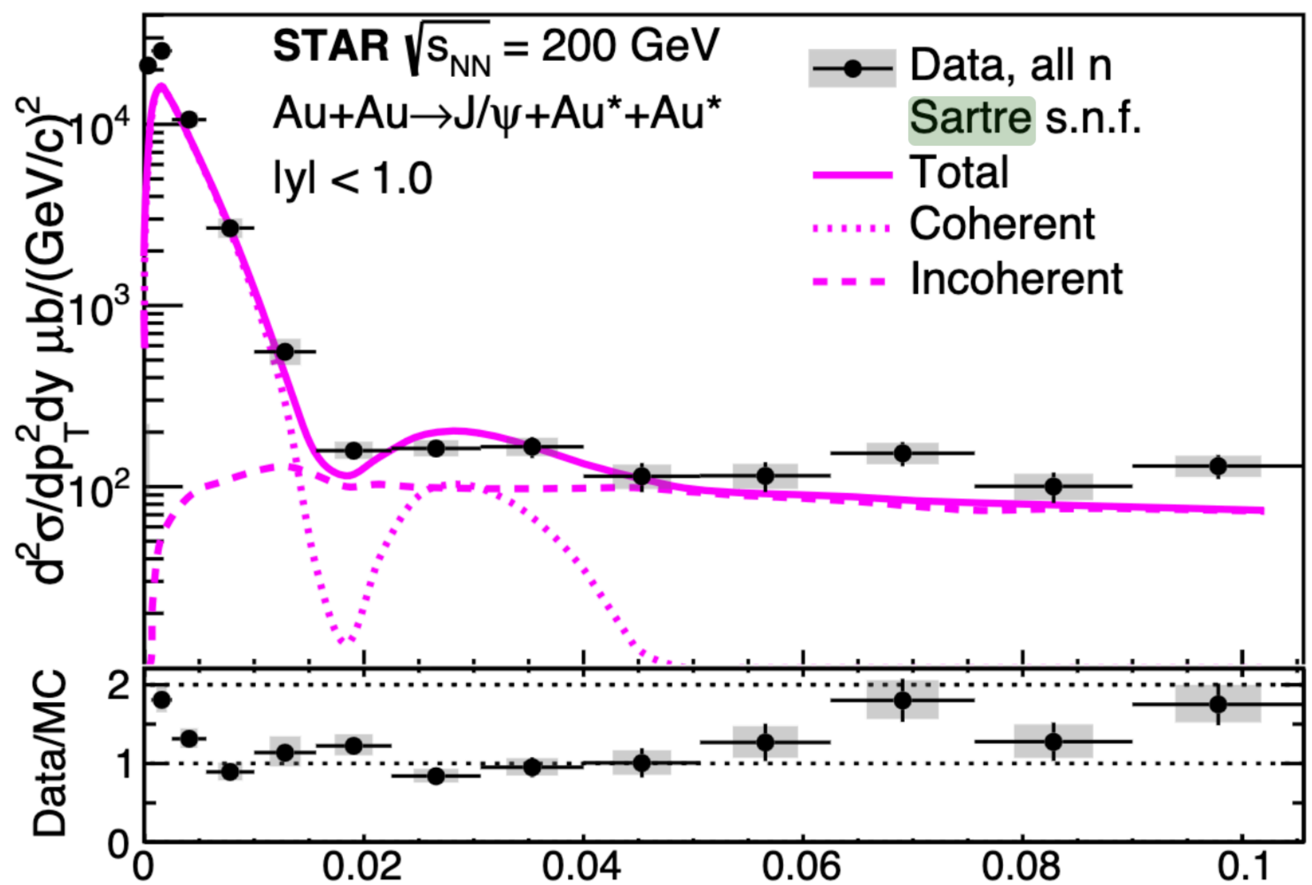
 H. Mäntysaari and B. Schenke Phys. Rev. Lett., 117(5):052301, 2016.

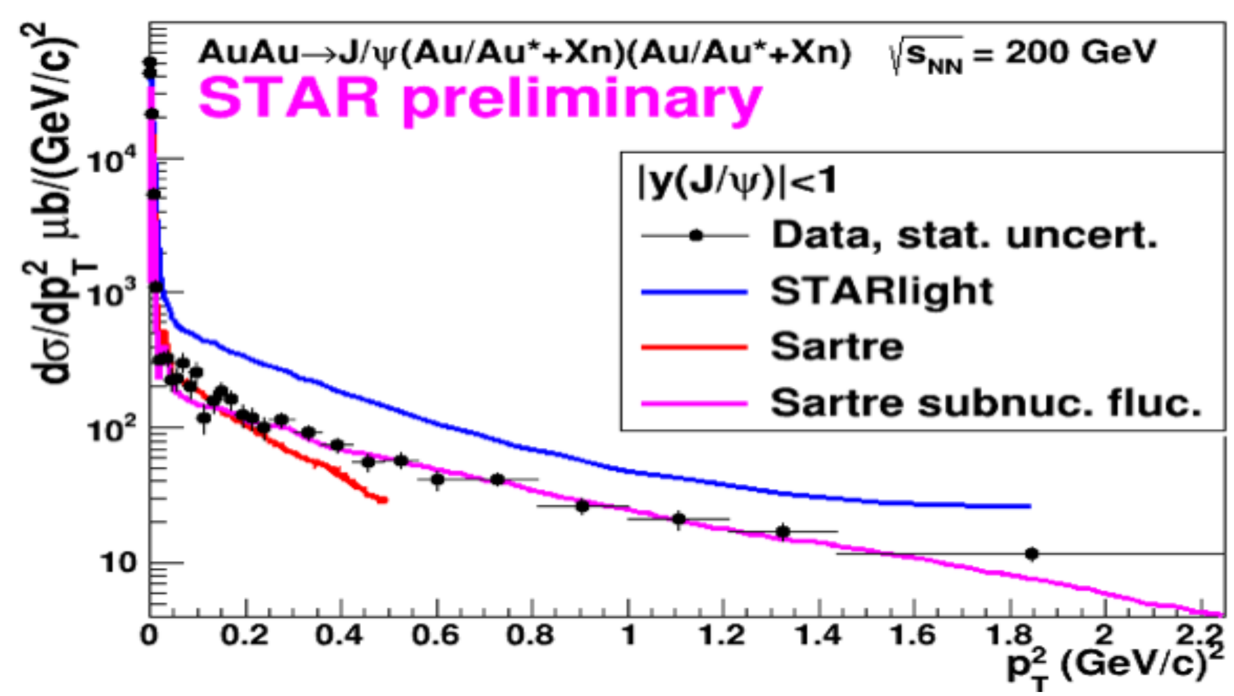
A-A UPC at the LHC & RHIC

TT: SciPost Phys.Proc. 8 (2022) 148



STAR Collaboration, e-Print: 2311.13632 [nucl-ex]

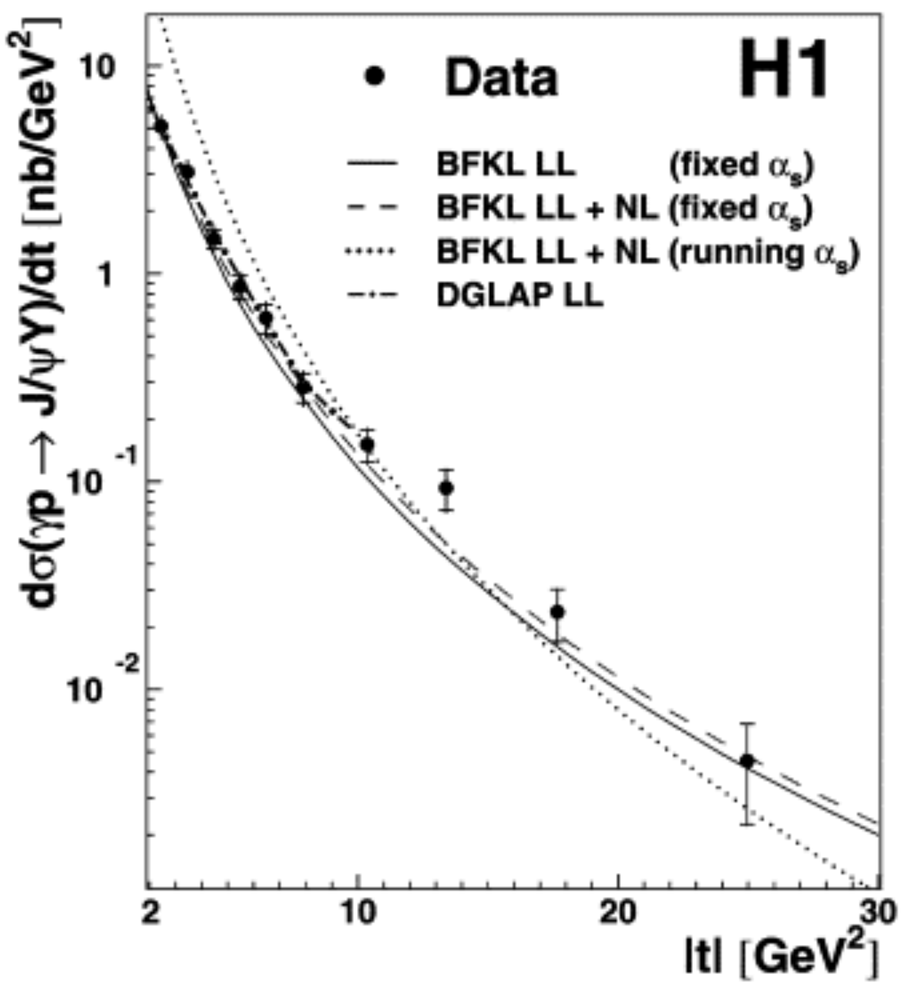




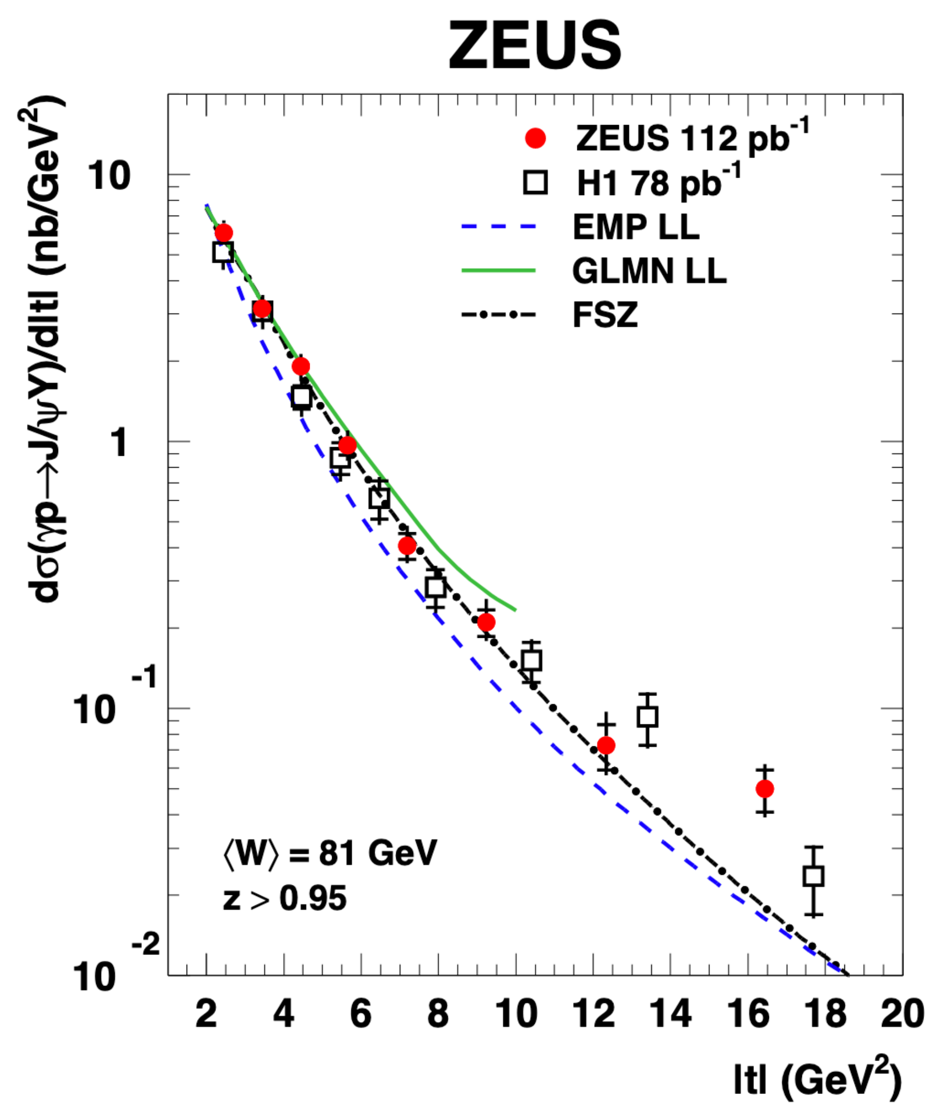
Even though coherent events dominate, the large $|t|$ tails have a significant effect on the cross sections!

Subnucleon structure becomes important for $|t| > 0.2$ GeV^2

Large $|t|$?



Phys. Lett. B 568 (2003) 205–218



JHEP 05 (2010) 085

Hotspot model: Non-perturbative phenomenology. Only valid for $|t| \lesssim 1$ GeV^2.

What about larger $|t|$?

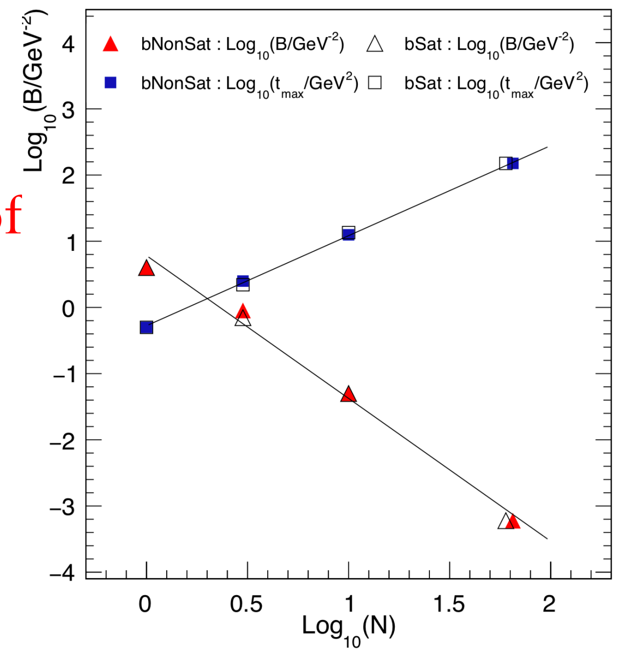
Arjun Kumar, TT, Eur.Phys.J.C 82 (2022) 9, 837, arXiv: 2106.12855

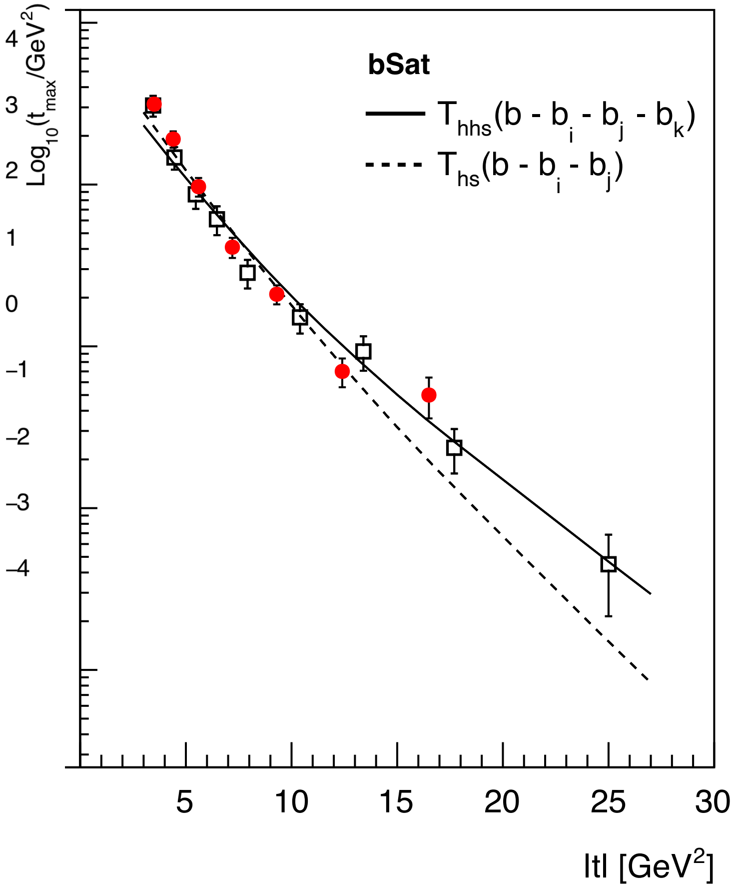
Insights

1. *t*-spectrum can be described by a self-similar structure of hotspots within hotspots

2. Small-*x* partons are maximally entangled (described by the same wave function)

S. Demirci, T. Lappi, S. Schlichting
Phys. Rev. D 106 (7) (2022) 074025:
"While the t-dependence of the cross section is well reproduced in our model, the relative normalization between the coherent and the incoherent cross sections points to the need for additional fluctuations in the proton."

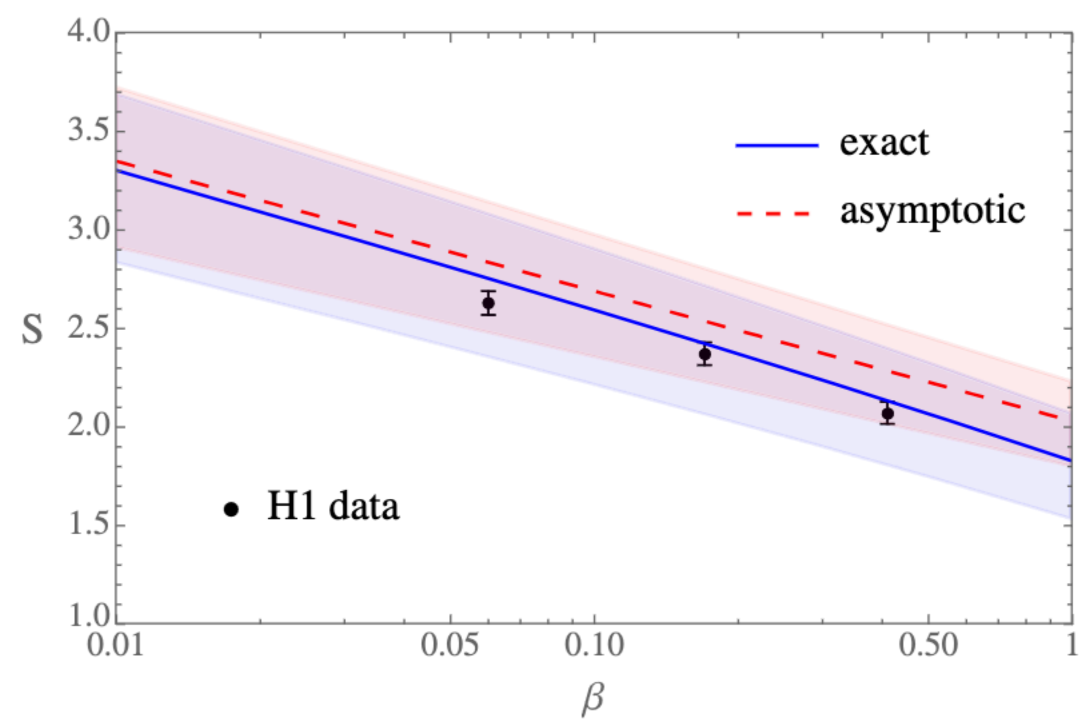




This suggests that we can describe the hotspot
t-spectrum with a linear,
scale-independent (in $\log|t|$) evolution

Picture: Transverse part of gluon wavefunction probed with areal resolution $\delta b^2 \sim \frac{1}{|t|}$

Wavefunction collapses into this area.
Increased resolution appears as hotspots splittings.



Probing the Onset of Maximal Entanglement inside the Proton in Diffractive Deep Inelastic Scattering,
Hentschinski, Kharzeev, Kutak, Tu: Phys.Rev.Lett. 131 (2023) 24, 241901

A. Kumar, TT, arXiv: 2403.13631

Hotspot Evolution

Initial State at $t = t_0$:

$$T_p(\vec{b}) = \frac{1}{N_q} \sum_{i=1}^{N_q} T_q(|\vec{b} - \vec{b}_i|)$$

$$T_q(\vec{b}) = \frac{1}{2\pi B_q} e^{-\frac{b^2}{2B_q}}$$

Inital State Parameters:

$B_{qc} = 3.1\ \mathrm{GeV}^{-2}$

$B_q = 1.25\ \mathrm{GeV}^{-2}$

$N_q = 3$

Probability of a hotspot created at t_0 splitting at $|t| > |t_0|$

$$\frac{\mathrm{d}P_{\mathrm{split}}}{\mathrm{d}t} = \frac{\alpha}{|t|} \frac{t - t_0}{t}$$

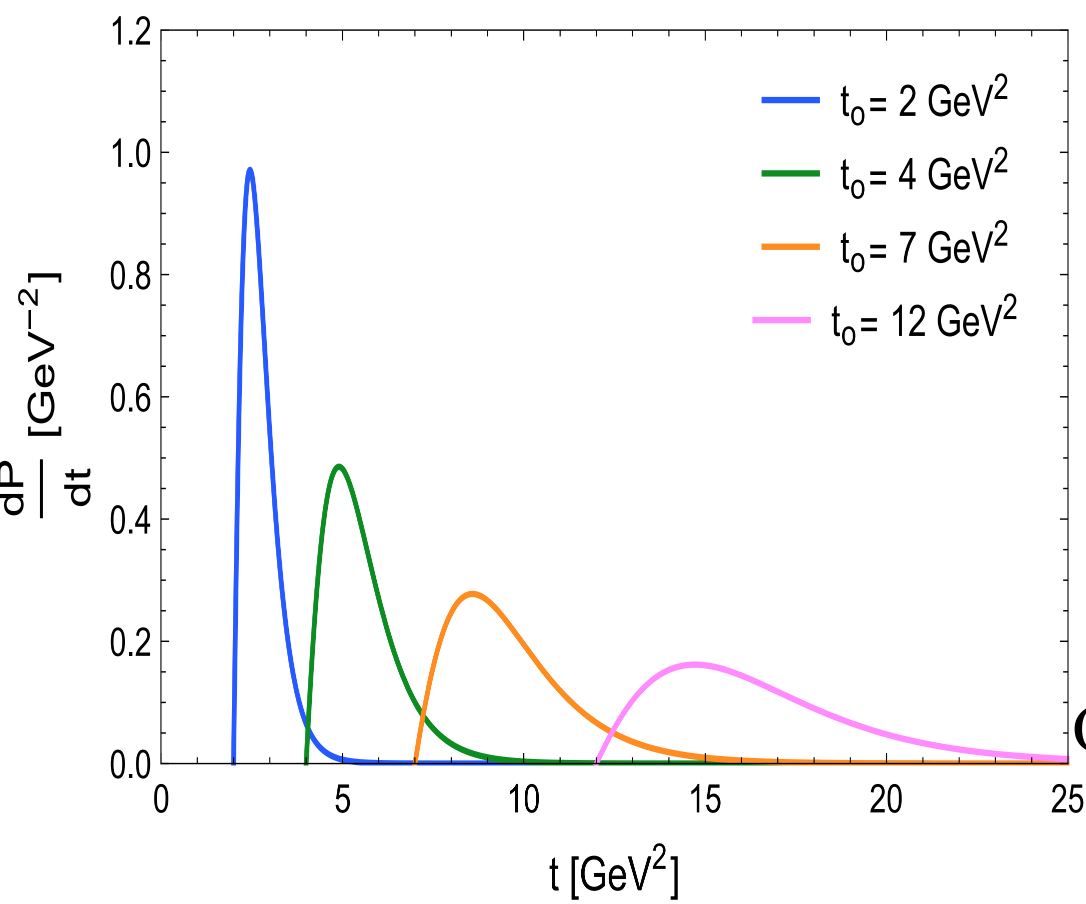
$$\frac{\mathrm{d}P_{\mathrm{nosplit}}}{\mathrm{d}t} = \exp\left(-\int_{t_0}^{t} \mathrm{d}t' \frac{\mathrm{d}P_{\mathrm{split}}}{\mathrm{d}t'}\right)$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\alpha}{|t|} \frac{t - t_0}{t} \exp\left[-\alpha\left(\frac{t_0}{t} - \ln\frac{t_0}{t} - 1\right)\right]$$

A. Kumar, TT, arXiv: 2403.13631

Hotspot Evolution

We consider a parton shower-like evolution based on resolution, where a hotspot may split into two as the resolution increases.



Probability of a hotspot created at t_0 splitting at $|t| > |t_0|$

$$\frac{dP}{dt} = \frac{\alpha}{|t|} \frac{t - t_0}{t} \exp\left[-\alpha\left(\frac{t_0}{t} - \ln\frac{t_0}{t} - 1\right)\right]$$

Generate offspring $\vec{b}_{i,j}$ from parent $T_{\text{parent}}(\vec{b}_{i,j})$.

Conserve Normalisation in each splitting.

Offspring hotspots i, j created at distance $d_{ij} = |\vec{b}_i - \vec{b}_j|$,

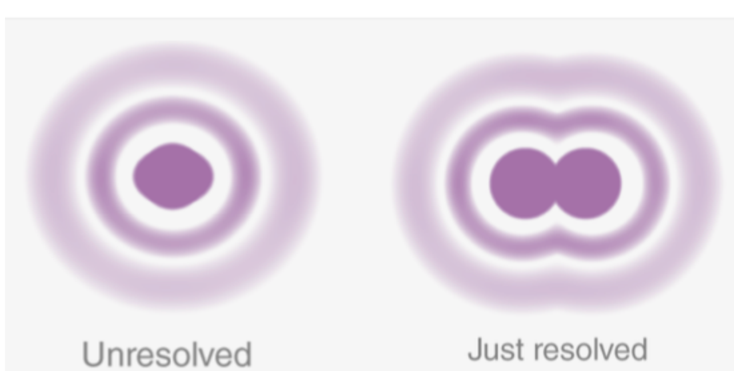
with widths $B_{i,j} = \frac{1}{|t|}$

Conditions for resolution:

Probe resolution: $d_{ij} > \frac{2}{|\vec{\Delta}|}$ Geometry: $d_{ij} > 2\sqrt{B_{i,j}}$

Reject if not resolved.

This becomes an effective hotspot repulsion.

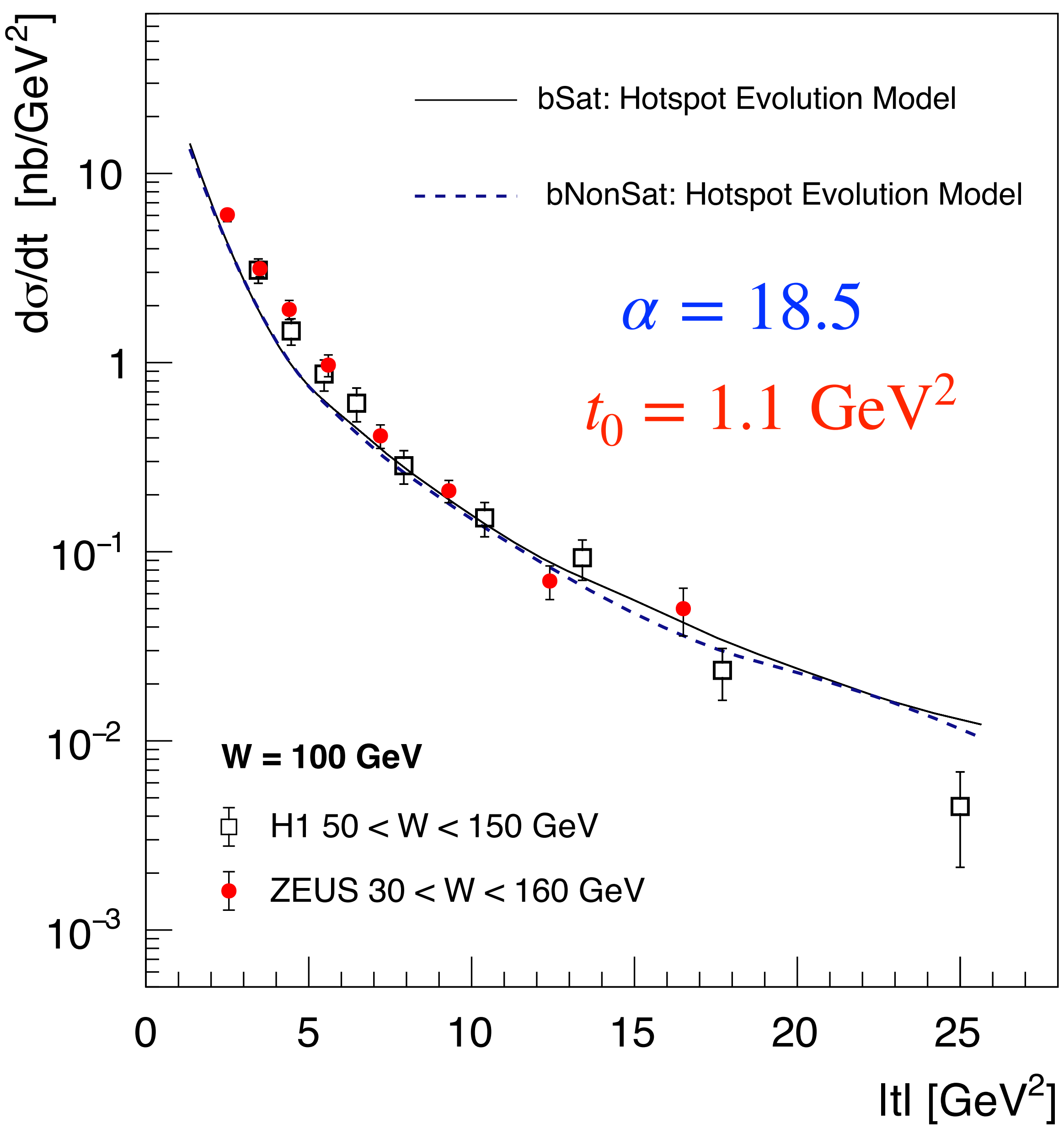


Hotspot repulsion:
J. L. Albacete, H. Petersen, A. Soto-Ontoso, Phys. Lett. B 778 (2018)

A. Kumar, TT, arXiv: 2403.13631

Hotspot Evolution

$$\frac{dP}{dt} = \frac{\alpha}{|t|}\frac{t-t_0}{t}\exp\left[-\alpha\left(\frac{t_0}{t} - \ln\frac{t_0}{t} - 1\right)\right]$$



Models can describe all data points for $|t| > |t_0| = 1.1\ \mathrm{GeV}^2$ with only one extra parameter α.

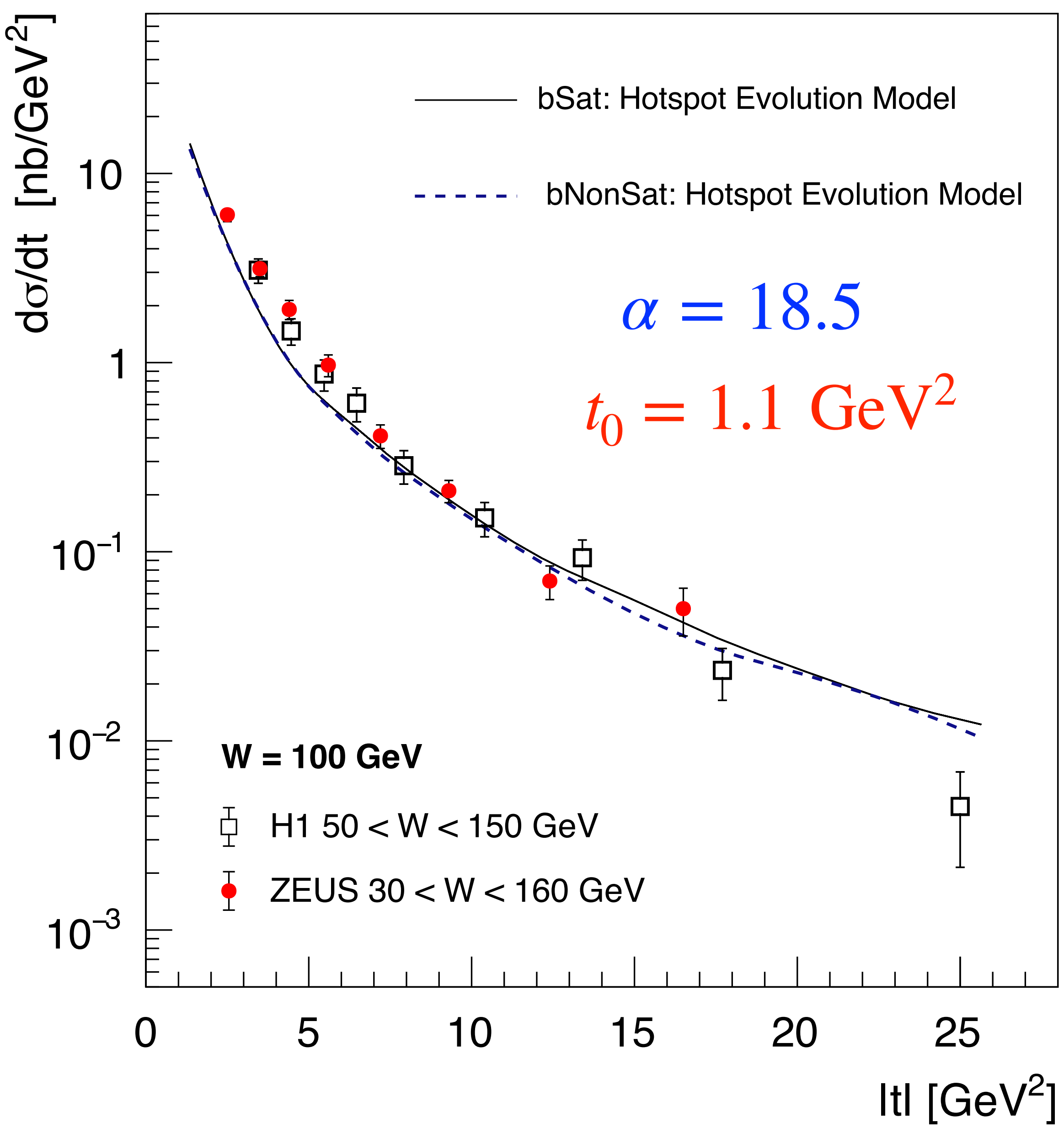
Checked that description unchanged for $|t_0| \in [0.8, 1.2]\ \mathrm{GeV}^2$

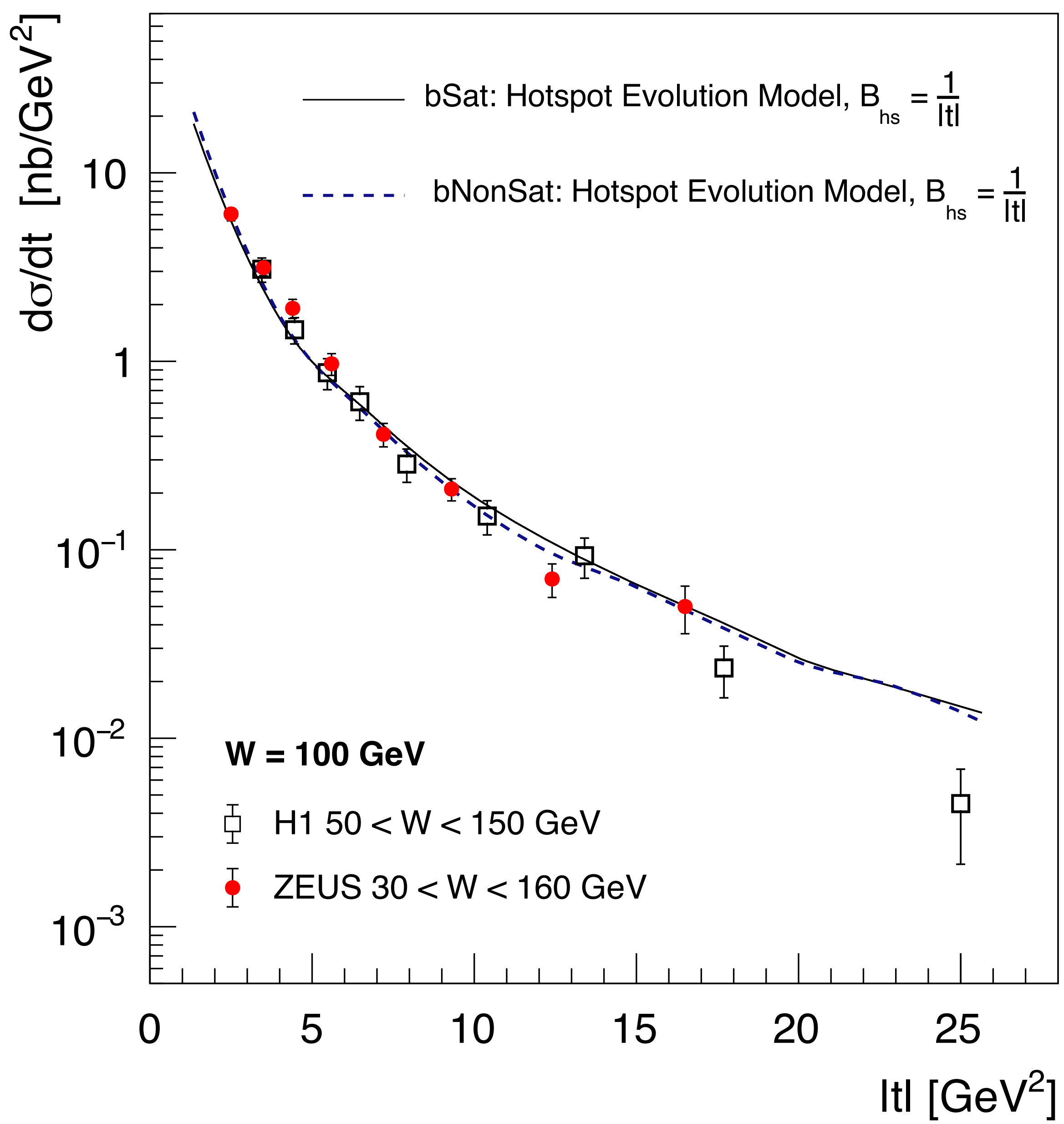
Sources of event-by-event fluctuations:
Number of hotspots
Hotspot Width
Normalisation

Saturation?

Hotspot Evolution

$$\frac{dP}{dt} = \frac{\alpha}{|t|}\frac{t-t_0}{t}\exp\left[-\alpha\left(\frac{t_0}{t} - \ln\frac{t_0}{t} - 1\right)\right]$$

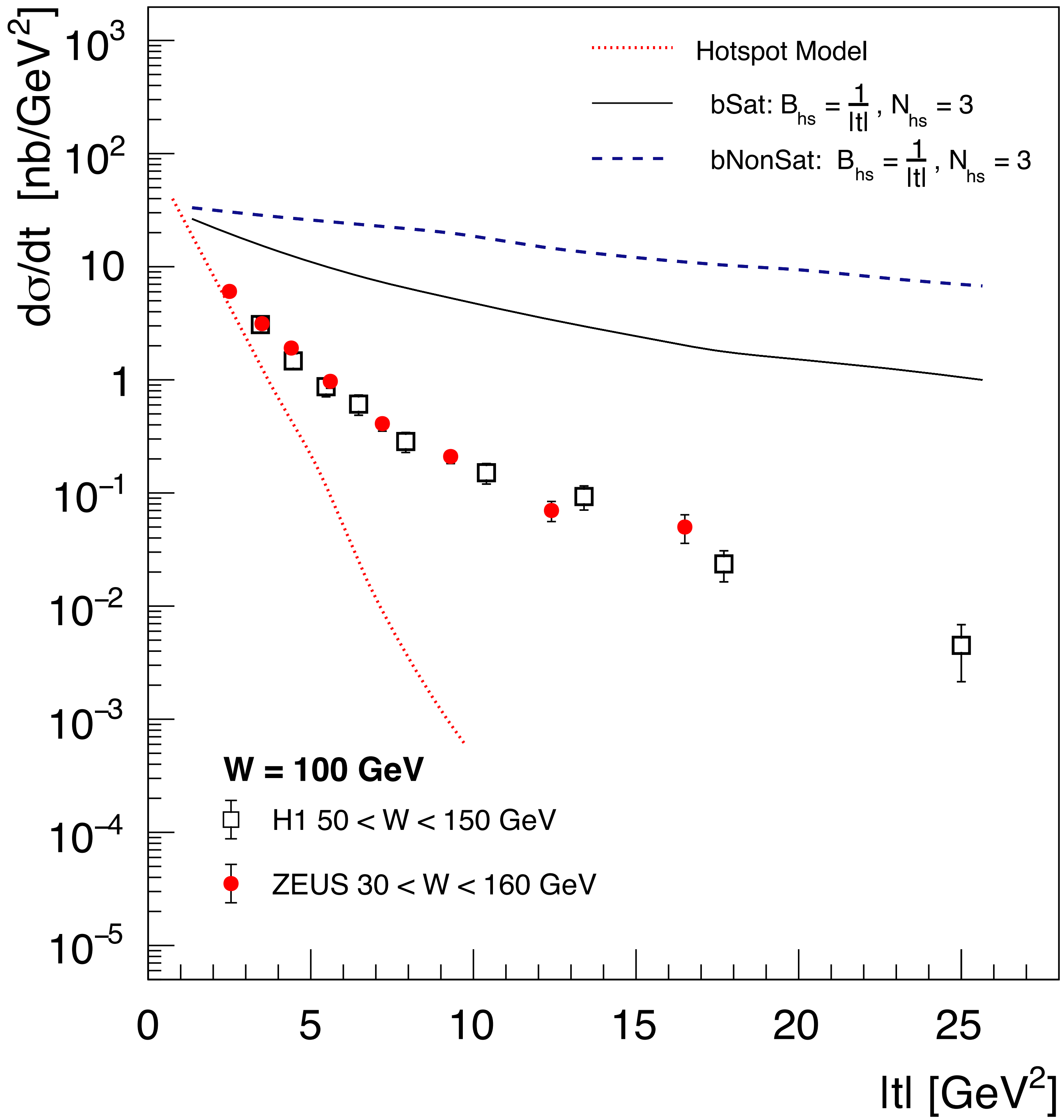




A. Kumar, TT, arXiv: 2403.13631

Hotspot Evolution

Saturation?



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Saturation Scale

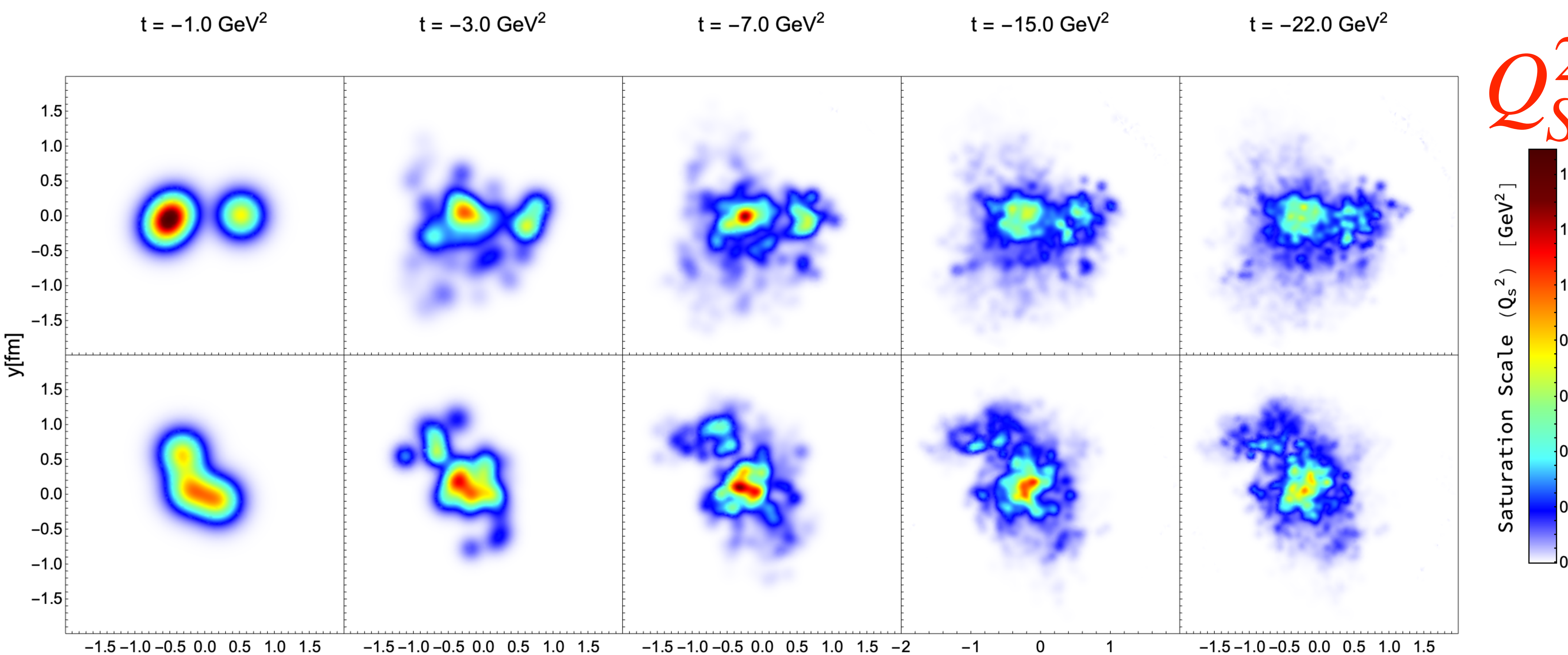
$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\alpha}{|t|}\frac{t-t_0}{t}\exp\left[-\alpha\left(\frac{t_0}{t} - \ln\frac{t_0}{t} - 1\right)\right]$$

$$Q_S^2 \simeq T(b)$$

$$T(b) \rightarrow T(b, t)$$

$\alpha = 18.5$

$t_0 = 1.1\ \mathrm{GeV}^2$

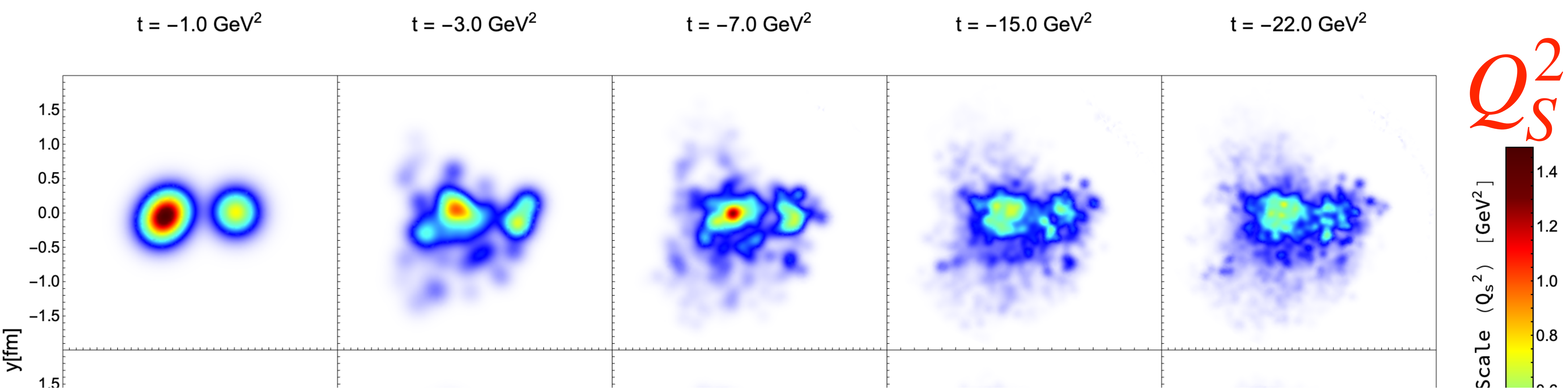


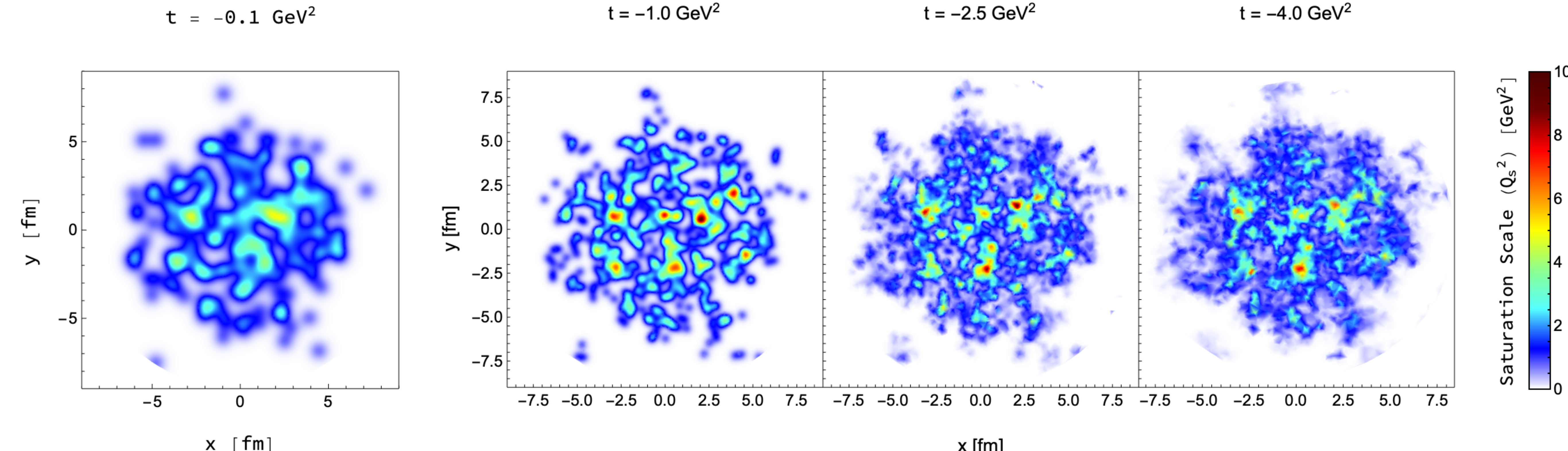
Q_S^2

A. Kumar, TT, arXiv: 2403.13631

Saturation Scale

$T(b) \to T(b,t)$ $\quad Q_S^2 \simeq T(b)$ $\quad Q^2_{S,\mathrm{Pb}} \sim 7 Q^2_{S,p}$





Summary & Outlook

We have developed a "classical evolution" of the hotspot model to large $|t|$ using t as a parameter of resolution. $T(b) \rightarrow T(\vec{b}) \rightarrow T(\vec{b}, t)$

We can describe the entire t-spectrum with 1 extra parameter

For large $|t|$ the physics is perturbative, in principle possible to calculate the hotspot shape and splitting function from first principle (0 extra parameters)

Then, the parameters of the initial state (hotspot model) can be constrained from the large $|t|$ evolution (N_q, B_q, B_{qc} …)

Further measurements possible at the EIC:

Different final states $\rho, \phi, J/\psi$

Different initial states, *p, Ca, Zr, Pb*

Multidimensional in t, W, Q^2