Global fits of proton PDFs with non-linear corrections

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in Quark Matter

Reminder - DGLAP evolution

$$Q^2 \frac{d}{dQ^2} \begin{pmatrix} f_i(x,Q^2) \\ f_g(x,Q^2) \end{pmatrix} = \sum_j \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \begin{pmatrix} P_{q_i q_j}\left(\frac{x}{\xi}\right) & P_{q_i g}\left(\frac{x}{\xi}\right) \\ P_{g q_j}\left(\frac{x}{\xi}\right) & P_{gg}\left(\frac{x}{\xi}\right) \end{pmatrix} \begin{pmatrix} f_j(\xi,Q^2) \\ f_g(\xi,Q^2) \end{pmatrix}$$

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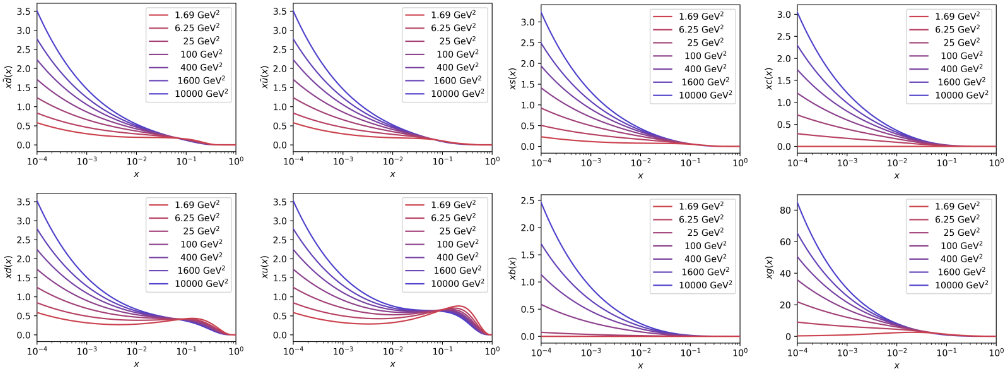

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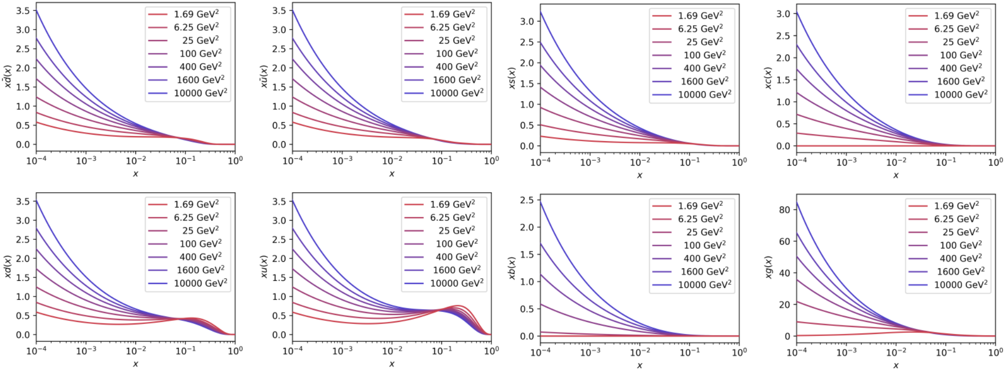

PROBLEM

Rapidly rising gluon at small x and large Q^2 violates unitarity.

GLUON RECOMBINATION — GLR-MQ EQUATION

[Phys. Rep. 100 (1983) 1, Nucl. Phys. B268 (1986) 427]

Based on double-leading-logarithmic approximation in Q^2 and $\frac{1}{x}$

Gluon recombination — GLR-MQ equation

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Based on double-leading-logarithmic approximation in Q^2 and $\frac{1}{x}$

$$\frac{dx_B G(x_B, Q^2)}{d\ln Q^2} = \text{linear terms} - 5.05 \left(\frac{\alpha_s}{RQ}\right)^2 \int_{x_B}^{x_0} \frac{dx_1}{x_1} \left[x_1 G^2(x_1, Q^2)\right]^2$$

$$\frac{dx_B S(x_B, Q^2)}{d\ln Q^2} = \text{linear terms} - 0.0010625 \left(\frac{\alpha_s}{RQ}\right)^2 \left[x_1 G^2(x_1, Q^2)\right]^2 - 0.32 \frac{\alpha_s}{Q^2} \int_{x_B}^{x_0} \frac{dx_1}{x_1} \frac{x_B}{x_1} P_{MQ}^{GG\to q\bar{q}} x_1 H(x_1, Q^2)$$

with

$$\frac{dx_1 H(x_1, Q^2)}{d\ln Q^2} = - 5.05 \left(\frac{\alpha_s}{RQ}\right)^2 \int_{x_B}^{x_0} \frac{dz}{z} \left[z G^2(z, Q^2)\right]^2$$

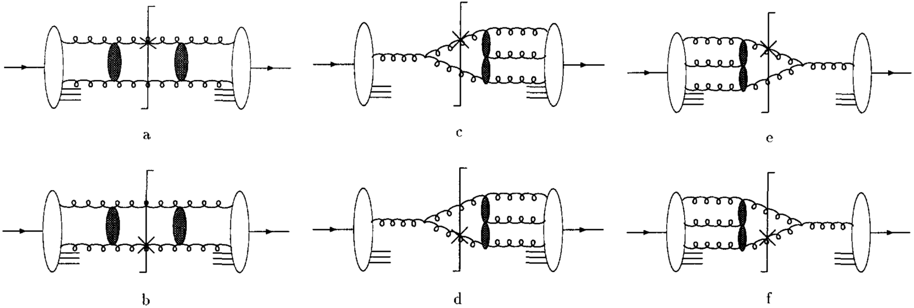
- $\frac{dx_B S(x_B, Q^2)}{d\ln Q^2}$ does not appear naturally; requires special treatments, i.e. mixing in NLL contributions
- **Violates momentum sum rules**

Zhu + Ruan approach

[Nucl. Phys. B 559 (1999), 378-392]

Based on leading logarithmic approximation in Q^2

- Valid over the entire x range
- Includes transitions to quarks (and can be extended to $q\bar{q} \to G$, etc.)



- $2 \to 2$ diagrams lead to antiscreening (a,b)
- $2 \to 3$ diagrams lead to screening (c,d,e,f)
- Same recombination functions, but different kinematic regimes

Zhu + Ruan approach

[Nucl. Phys. B 559 (1999), 378-392]

$$\frac{dx_B G(x_B,Q^2)}{d\ln Q^2} = \text{linear terms} + \frac{9}{32\pi^2}\left(\frac{1}{RQ}\right)^2 \int_{x_B/2}^{1/2} dx_1 x_B x_1 G^2(x_1,Q^2) \sum_i P_i^{GG\to G(x_1,x_B)}$$
$$- \frac{9}{16\pi^2}\left(\frac{1}{RQ}\right)^2 \int_{x_B}^{1/2} dx_1 x_B x_1 G^2(x_1,Q^2) \sum_i P_i^{GG\to G(x_1,x_B)}$$
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- The parameter R can be interpreted as the size of the transverse area where gluon overlap leads to non-linear corrections.

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- The parameter R can be interpreted as the size of the transverse area where gluon overlap leads to non-linear corrections.

$$\left.\begin{aligned} \frac{d\int_0^1 dx_B x_B G(x_B,Q^2)}{d\ln Q^2} &= 0 \\ \frac{d\int_0^1 dx_B x_B q(x_B,Q^2)}{d\ln Q^2} &= 0 \end{aligned}\right\} \Rightarrow \text{Momentum is conserved.}$$

Nonlinear evolution in practice - HOPPET

[Comput.Phys.Commun. 180 (2009) 120-156]

Problem: PDF fitting requires DGLAP evolution to be performed thousands of times as fast as possible

- Convolution codes like HOPPET are highly optimized to solve the linear DGLAP evolutions quickly

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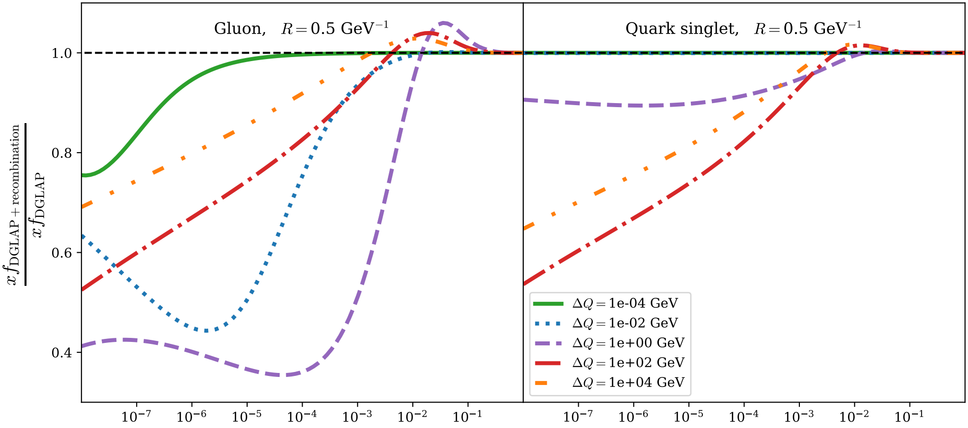
Solution: Treat $G^2(x, Q^2)$ as a separate flavour $\rightarrow$ calculation just 20% slower than regular DGLAP

- Can be integrated into xFitter to allow PDF fitting [Eur.Phys.J.C 75 (2015) 7, 304]

Results - Q-dependence of non-linear corrections

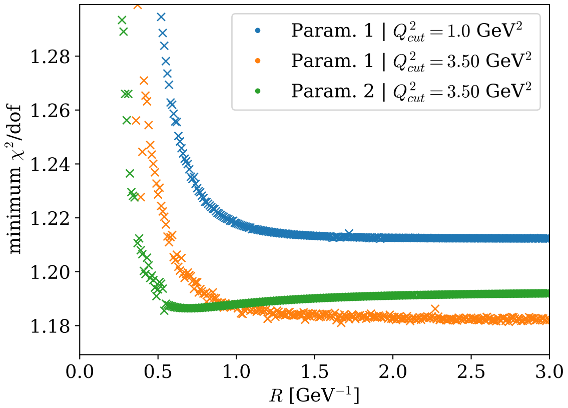
- Non-linear corrections applied to the evolution of a fixed set of PDFs (CJ15)



- Largest correction at $Q = \mathcal{O}(1)$ GeV for gluons and $Q = \mathcal{O}(100)$ GeV for quarks

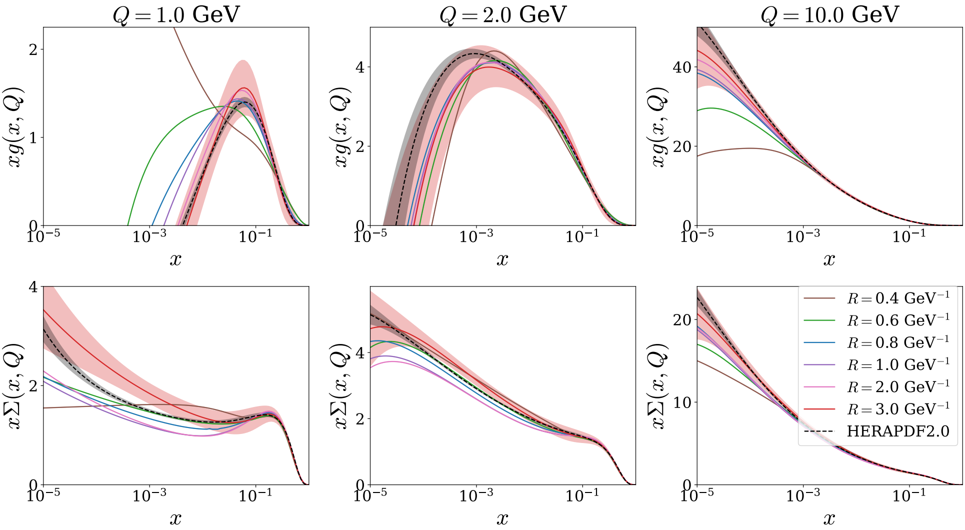
R-dependent PDF fits

- HERAPDF2.0 parameterization and methodology, NNLO
- 1568 / 1636 data points (BCDMS, HERA and NMC DIS)

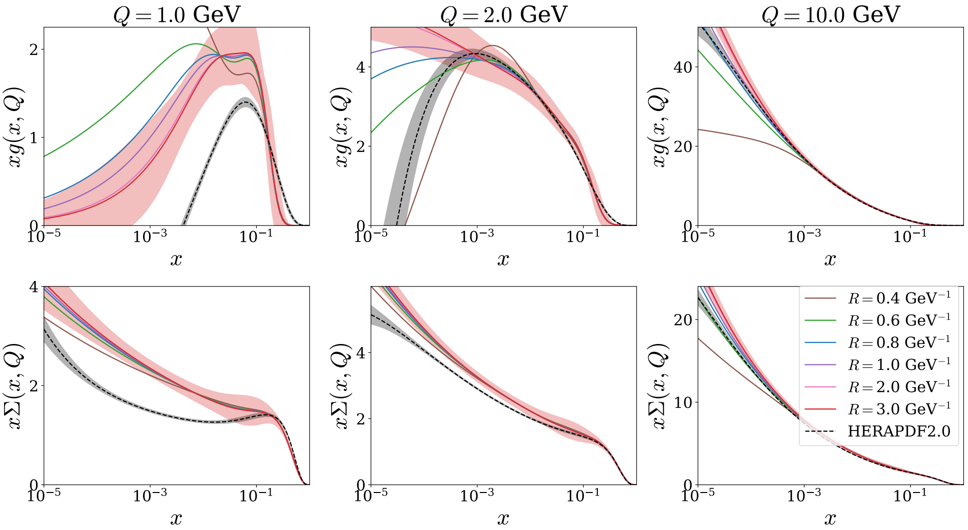


The entire procedure has also been repeated with a different parameterization for the gluon PDF, but no significant differences were observed (see paper for details)

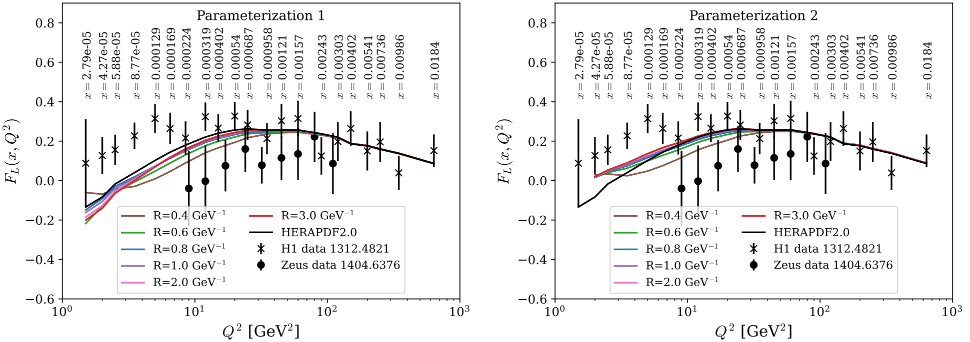
R-dependent PDF fits - Parameterization 1



R-dependent PDF fits - Parameterization 2

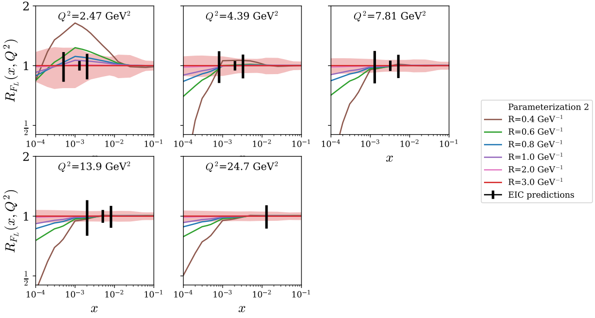


Comparing to HERA F_L data

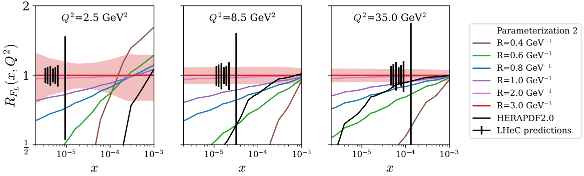


- Recombination does not alleviate tensions with low-Q^2 F_L data (unlike small-x resummation corrections)
- Parameterization 1 leads to negative F_L at low Q, which is unphysical

Comparing to F_L predictions for future experiments



[E. Aschenauer et al.,`https://www.phenix.bnl.gov/WWW/publish/elke/EIC/EIC-R&D-Tracking/Meetings/fl.pdf`]



[P. Agostini et al., J.Phys.G 48 (2021) 11, 110501]

- Future data will be more sensitive to recombination effects

Conclusions

- Gluon recombination offers a possible explanation for saturation
- GLR(-MQ) equations violate momentum conservation $\rightarrow$ calculation by Zhu+Ruan avoids this problem
- Implemented in HOPPET + xFitter to produce new global proton PDF fits
- Current DIS data shows no signs of gluon recombination $\rightarrow$ Lower bound $R > 0.5\,\text{GeV}^{-1}$

Conclusions and Outlook

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Outlook

- Tools (modified HOPPET) and results (LHAPDFs) will be made available
- EIC, and especially LHeC data may put tighter constraints on strength of non-linear effects